Conditional Investment-Cash Flow Sensitivities and Financing Constraints

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Abstract

Kaplan and Zingales (QJE, 1997) study the unconditional sensitivity of investment to cash flow in a static demand for capital framework. In contrast we study the sensitivity of investment to cash flow conditional on measures of q in an adjustment costs framework. This is more closely related to the empirical literature on investment and financing constraints.

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1. Introduction

Kaplan and Zingales (1997) study the sensitivity of investment to the availability of internal finance for firms that face different cost premia for external finance, in a one-period model with no costs of adjusting the capital stock. In this framework investment is chosen so that the marginal revenue product of capital is equated with the user cost of capital, i.e. the relevant first order condition is the traditional 'neoclassical' marginal productivity condition that describes the demand for capital in a static framework (cf. Jorgenson, 1963). Kaplan and Zingales (1997) show that investment may be more sensitive to the availability of internal funds for firms that face a lower cost premium for external funds, if the marginal revenue product of capital is sufficiently convex.

Whilst interesting, this analysis is far removed from the framework of most empirical studies of investment and financing constraints, in the tradition of Fazzari, Hubbard and Petersen (1988). These studies typically regress a measure of investment on a measure of q as well as a measure of cash flow, i.e. they estimate the sensitivity of investment to cash flow *conditional* on q or, in some cases, a wider set of control variables. These empirical specifications recognise that, even in the absence of financing constraints, investment is likely to be subject to adjustment costs that prevent the capital stock adjusting continuously to maintain equality between the marginal revenue product and user cost of capital. The relevant first order condition in a model with strictly convex adjustment costs is that which equates the marginal costs of an additional unit of investment with the shadow value of an additional unit of installed capital (see, for example, Abel, 1980). Notice that the curvature of the marginal revenue product of capital plays no direct role in this condition. Interestingly, the special case of this model that delivers the linear relationship between investment and q, which dominates the empirical literature, requires marginal adjustment costs that are linear in investment.

We study the sensitivity of investment to the availability of internal finance in a simple model with quadratic adjustment costs. We distinguish between two types of cost premia for external funds: a cost premium that is increasing in the level of external finance used; and a cost premium that is fixed, independent of the level of external finance used. In the former case there are two financial regimes: an unconstrained regime in which investment is financed internally and the shadow value of capital, or marginal q, remains a sufficient statistic for current investment; and a constrained regime in which external funds are the marginal source of finance, and investment displays excess sensitivity to windfall fluctuations in the availability of internal funds. In this constrained regime, there is a straightforward monotonic relationship between the conditional investment-cash flow sensitivity and the severity of the capital market 'imperfection', as measured by the slope of the cost schedule for external funds. That is, if we consider two otherwise identical firms with the same adjustment cost function, supply of internal funds, *and* marginal q, the sensitivity of investment to a windfall increase in cash flow will be greater for the firm that faces a more steeply sloping cost of external funds schedule.

In the model with a fixed cost premium for external finance, there are three financial regimes: an unconstrained regime in which investment is financed internally; a constrained regime in which available internal funds are exhausted but the firm chooses to use no external funds; and an external finance regime in which external funds are the marginal source of finance. In this case, if a firm is in the constrained regime, investment increases dollar-for-dollar with small windfall increases in cash flow, regardless of the size of the fixed cost premium for internal funds. If a firm is in the external finance regime, investment is insensitive to small windfall increases in cash flow, but may be increased by large cash flow shocks that shift it into a different regime. In this model we get a weaker result that, if we consider two otherwise identical firms with the same adjustment cost function, supply of internal funds and marginal q, the sensitivity of investment to a windfall increase in cash flow will be no lower for the firm that faces a higher cost premium for external finance, and will be strictly greater in response to some cash flow shocks.

These results indicate that at a given level of the shadow value of capital or *marginal* q, otherwise identical firms will display (weakly) greater sensitivity of investment to cash flow if they face a greater cost premium for the use of external finance. We also study the relationship between marginal q and *average* q in these models, to assess the extent to which empirical studies may succeed in controlling for variation in marginal q by including a standard measure of average q, in the presence of financing constraints. In the model with a fixed cost premium for external finance, the equality between marginal q and average q continues to hold under the same conditions derived by Hayashi (1982) in the absence of financing constraints. In this model, otherwise identical firms will therefore display weakly greater sensitivity of investment to cash flow, at a given level of average q, if they face a higher cost premium for external funds. In the model with an increasing cost premium for external finance, however, the equality between marginal q and average q breaks down.

The remainder of the paper is organised as follows. Section 2 reviews the sensitivity of investment to windfall fluctuations in cash flow in a static demand for capital framework, and illustrates the result highlighted by Kaplan and Zingales (1997). Section 3 outlines our basic model with convex adjustment costs and discusses the sensitivity of investment to cash flow conditional on marginal q in two special cases. Section 4 considers the relationship between marginal q and average q in these two models. Section 5 concludes.

2. A static model

In a setting with no adjustment costs for capital, the first order condition describing the evolution of the optimal capital stock equates the marginal revenue product of capital (MPK) to the user cost of capital (u). The user cost of capital represents the minimum rate of return required for the investment to be value increasing, and reflects the cost of finance. If the firm faces a higher cost for using external funds than for using internal funds, this will be reflected in a higher required rate of return on investment financed from external sources.[1] This sit-

[1] See Hubbard (1998), for example, for a discussion of why this 'pecking order' assumption may be relevant.

uation is depicted for a case with an increasing marginal cost of external finance in Figure 1.

Here the cost of capital for investment financed internally is denoted u_{INT}. If the firm wants to finance investment spending beyond the level denoted by C, the firm must use increasingly expensive external sources. This increasing cost premium for external funds is reflected in the upward sloping segment of the cost of capital schedule u beyond the investment level C. There are two financing regimes in this framework. If the firm has a marginal revenue product schedule MPK_1 its desired investment spending is low relative to the availability of low cost internal funds.[2] It finances its preferred level of investment spending I_1 internally, and this level of investment is insensitive to windfall fluctuations in cash flow. More precisely, an increase in the availability of internal funds that leaves the marginal revenue product of capital schedule unchanged, but increases the level of investment that can be financed internally to $C' > C$, has no effect on the optimal level of investment spending for firms in this unconstrained regime.

In contrast, if the firm has the marginal revenue product schedule MPK_2, its desired investment spending exceeds its supply of low cost internal funds. In this case it finances additional investment beyond C by using more expensive external sources, but the increasing cost of external finance influences its optimal level of investment spending. This firm chooses the level of investment I_2 where the first order condition equating the marginal revenue product and user cost of

[2] The figure is drawn for a given inherited level of the capital stock, so there is a one-to-one association between current investment and the current level of the capital stock.

capital is satisfied. An otherwise identical firm with the same marginal revenue product MPK_2 and the same cost premium schedule for external finance, but with a much greater supply of low cost internal funds, would instead choose the higher level of investment I_3. This indicates that the level of investment spending is sensitive to windfall fluctuations in the availability of internal funds, for firms in this financially constrained regime. This sensitivity is illustrated in Figure 2. With more internal funds available, the firm is required to use less external finance, faces a lower required rate of return for all investment levels above C, and optimally chooses a higher level of investment $I_2^{'}$. Note that in this model with an increasing marginal cost of external finance, the increase in investment spending $(I_2^{'} - I_2)$ is typically smaller than the windfall increase in cash flow $(C' - C)$.

It would appear that the investment spending of firms in the constrained regime will display greater sensitivity to fluctuations in cash flow for firms that face a greater cost premium for external funds, or what might be termed a more severe financing constraint. This is indeed possible, as illustrated in Figure 3. A firm with the marginal product schedule MPK and cost of external funds schedule u_L increases its investment spending from I_L to $I_L^{'}$ in response to a windfall increase in cash flow from C to $C^{'}$. An otherwise identical firm with the more steeply sloping cost of external funds schedule u_H increases its investment spending by the larger amount $(I_H^{'} - I_H)$ as a result of the same cash flow shock. Thus we might expect to find evidence of greater investment-cash flow sensitivity among samples of firms that face a higher cost premium for the use of external sources of finance.

Kaplan and Zingales (1997) have noted that this conclusion depends heavily on the presumed linearity of the marginal revenue product schedule in Figure 3. The opposite result is possible if the firm's marginal revenue product of capital is sufficiently convex. This case is illustrated in Figure 4, where the firm facing the higher cost of external funds schedule u_H increases investment by less in response to the cash flow shock than the firm which faces the lower cost premium reflected in u_L.

Kaplan and Zingales correctly conclude that there is not necessarily a monotonic relationship between the sensitivity of investment to windfall fluctuations in the availability of internal finance and the slope of the cost of external finance schedule in a static demand for capital model of this type. They provide a formal analysis of a one-period investment problem with no adjustment costs. In the next section we consider whether a similar result holds in a dynamic investment problem with strictly convex costs of adjustment, which is the basis for the investment-q relation adopted by much of the empirical research in this area, including that presented by Kaplan and Zingales (1997) themselves.

3. A dynamic model with adjustment costs

We study a standard investment problem where the firm chooses investment to maximise the value of its equity V_t given by

$$V_t = E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left(D_{t+s} - N_{t+s} \right) \right\} \tag{3.1}$$

where D_t denotes dividends paid in period t, N_t denotes the value of new equity issued in period t, $\beta < 1$ is the one-period discount factor assumed constant for simplicity, and $E_t[.]$ denotes an expected value given information available at time t.

Dividends and new equity are linked to the firm's net revenue Π_t each period by the sources and uses of funds identity

$$D_t - N_t = \Pi_t - \phi(N_t) \tag{3.2}$$

where $\phi(N_t)$ represents additional costs imposed by issuing new equity. We follow Kaplan and Zingales (1997) in not considering debt finance explicity, so that issuing new equity is the only source of external finance considered. Formally we treat $\phi(N_t)$ as a transaction fee that must be paid to third parties when new shares are issued. Less formally we can also think of these costs reflecting differential tax treatments, agency costs, or losses imposed on existing shareholders when the firm issues new shares in markets characterised by asymmetric information.[3] We assume $\phi(0) = 0$ and $\phi'_t = \frac{d\phi(N_t)}{dN_t} \geqslant 0$.

Following the q literature, we assume $\Pi_t = \Pi(K_t, I_t)$ where $K_t = (1-\delta)K_{t-1} + I_t$ is the stock of capital in period t, I_t is gross investment (which may be positive or negative), and δ is the rate of depreciation. The dependence of net revenue on investment reflects the presence of adjustment costs, which are assumed to be strictly convex in I_t.

The firm maximises V_t subject to this capital accumulation constraint and to

[3] See, for example, Myers and Majluf (1984).

non-negativity constraints on dividends and new equity issues, with shadow values λ_t^D and λ_t^N. The problem can be expressed as

$$V_t(K_{t-1}) = \max_{I_t, N_t} \left\{ \begin{array}{c} \Pi\left((1-\delta)K_{t-1} + I_t, I_t\right) - \phi(N_t) \\ +\lambda_t^D \left[\Pi\left((1-\delta)K_{t-1} + I_t, I_t\right) + N_t - \phi(N_t)\right] + \lambda_t^N N_t \\ +\beta E_t \left[V_{t+1}\left((1-\delta)K_{t-1} + I_t\right)\right] \end{array} \right\} \quad (3.3)$$

Letting $\lambda_t^K = \frac{1}{1-\delta}\left(\frac{\partial V_t}{\partial K_{t-1}}\right)$ denote the shadow value of inheriting $\frac{K_{t-1}}{1-\delta}$ units of capital from the previous period, or equivalently of having one additional unit of installed capital at time t, the first order condition for optimal investment can be written as

$$-\Pi_{It} = \frac{\lambda_t^K}{1+\lambda_t^D} \quad (3.4)$$

where $-\Pi_{It} = \frac{-\partial \Pi_t}{\partial I_t}$ is strictly increasing in the level of investment I_t. If the non-negativity constraint on dividends is not binding ($\lambda_t^D = 0$), this simply equates the marginal costs of investing in an additional unit of capital with the shadow value of that additional unit. Along the optimal path, the evolution of this shadow value of capital is described by the intertemporal condition

$$\lambda_t^K = (1+\lambda_t^D)\Pi_{Kt} + (1-\delta)\beta E_t\left[\lambda_{t+1}^K\right] \quad (3.5)$$

where $\Pi_{Kt} = \frac{\partial \Pi_t}{\partial K_t}$. The first order condition for optimal new share issues implies

$$\lambda_t^D = \frac{\phi_t' - \lambda_t^N}{1-\phi_t'} \quad (3.6)$$

In the case where new shares are issued ($N_t > 0$) and $\lambda_t^N = 0$, this simplifies to give

$$\lambda_t^D = \frac{\phi_t'}{1-\phi_t'} \quad (3.7)$$

To study the implications we focus on two special cases. The first assumes a strictly increasing cost premium for external finance, similar to the case considered in the previous section. The second considers a different specification of the capital market imperfection, in which there is a fixed cost premium per unit of new equity issued.

3.1. An increasing cost premium

To simplify, we assume that $\phi(N_t) = \left(\frac{\phi}{2}\right).N_t^2$ where ϕ is now a parameter that specifies the slope of the cost premium for external finance. In this case $\phi_t' = \phi N_t$. In the case where new shares are issued, this gives $\frac{1}{1+\lambda_t^D} = 1 - \phi N_t$.

The first order condition for investment is then depicted in Figure 5, adapted from Hayashi (1985), which is drawn for a given level of the shadow value of capital λ_t^K. The adjustment cost function used to obtain the linear relationship between investment rates and q makes marginal adjustment costs linear in the investment rate $(\frac{I_t}{K_t})$, giving the linear marginal cost schedules depicted here. As before, levels of investment spending up to C can be financed using low cost internal funds. More precisely, for $I < C$ the firm issues no new equity $(N = 0)$ and pays strictly positive dividends ($D > 0$ and $\lambda_t^D = 0$). For $I > C$, the firm issues new equity ($\lambda_t^N = 0$), pays zero dividends ($D = 0$), and λ_t^D is obtained from the first order condition for new equity issues (3.7). Here this gives

$$\frac{\lambda_t^K}{1 + \lambda_t^D} = \lambda_t^K (1 - \phi N_t) \tag{3.8}$$

as noted above.

In this model there are again two financing regimes. For a given level of the shadow value of capital or marginal q,[4] a firm with the adjustment cost function $-\Pi_{I1}$ is in the unconstrained regime and chooses the investment rate $\frac{I_1}{K}$ at which the first order condition (3.4) is satisfied.[5] A firm with the adjustment cost function $-\Pi_{I2}$ is in the constrained regime and chooses the investment rate $\frac{I_2}{K}$. This firm would choose a higher level of investment if it was less dependent on expensive external finance; if its supply of internal funds was high enough, it would choose the investment rate $\frac{I_3}{K}$. This sensitivity of investment to windfall changes in cash flow for firms in the constrained regime is illustrated in Figure 6. Here a windfall increase in cash flow is one which leaves expected future profitability and hence the shadow value of an additional unit of capital (λ_t^K) constant.

Figure 7 considers this investment-cash flow sensitivity for two otherwise identical firms, with the same adjustment cost function, availability of internal funds and shadow value of capital, but subject to different cost schedules for external funds. One firm faces a low cost premium represented by ϕ_L, whilst the other firm faces a much higher cost premium represented by ϕ_H. In the constrained regime, a given windfall increase in the availability of internal finance will clearly have a larger impact on the investment spending of the firm that faces the more steeply increasing cost of external finance schedule, and whose investment *conditional*

[4]Marginal q is usually expressed as the ratio of the shadow value of an additional unit of capital (λ_t^K) to the purchase price of a unit of capital. Here we normalise the price of capital goods to unity for simplicity.

[5]The distinction between the two financial regimes may alternatively be illustrated for a firm with a given adjustment cost schedule by considering different levels of the shadow value of capital.

on marginal q is therefore affected much more by reliance on external sources of funds.

This illustrates the main result of this section. In the model with quadratic adjustment costs and a strictly increasing cost of new equity, there is a simple monotonic relationship between the sensitivity of investment to windfall fluctuations in cash flow and the severity of the financing constaint, as reflected in the slope of the cost schedule for external funds, for otherwise identical firms in the financially constrained regime. The result is obtained by holding constant the shadow value of capital or marginal q. If we could measure marginal q and control for the likely endogeneity of cash flow, then at least to a linear approximation this is the kind of conditional cash flow sensitivity estimated in regression specifications that relate investment rates to cash flow and q. The extent to which marginal q may be measured by the standard measure of average q in models with financing constraints is considered in section 4 below.

This simple monotonic relationship could of course be overturned by introducing sufficient curvature into the marginal adjustment cost schedule $-\Pi_{It}$. This is perfectly consistent with the investment model considered here, but would be inconsistent with the linear specification found in most of the empirical literature on financing constraints and investment. If this possibility were to be taken seriously, the shape of the adjustment cost function would need to be reflected in the functional form specified in the empirical analysis.

3.2. A fixed cost premium

In this section we consider a different specification of the external finance premium, in which external finance is more costly than internal finance, but is available at a fixed cost premium that does not increase with the amount of external funds used. Formally this can be thought of as a fixed brokerage fee per unit of new equity issued. The motivation for considering this alternative stems from the relationship between marginal q and average q, discussed in the next section.

Here we assume that $\phi(N_t) = \phi.N_t$ where ϕ is again a parameter that reflects the size of the cost premium for external finance. In this case $\phi'_t = \phi$, and in the case where new shares are issued, this gives $\frac{1}{1+\lambda_t^D} = 1 - \phi$.

In the static framework reviewed in section 2, this specification gives a step function for the cost of capital, and a similar result is found for the model with convex adjustment costs. Although apparently simpler, this formulation gives three distinct financial regimes, which are illustrated in Figure 8. For a given level of the shadow value of capital (λ_t^K), a firm with the adjustment cost function $-\Pi_{I1}$ is again in an unconstrained regime where investment is insensitive to windfall fluctuations in cash flow. This firm chooses the investment rate $\frac{I_1}{K}$; investment spending is financed from internal funds, with no new equity issues $(N_t = 0)$ and strictly positive dividend paymsnts $(D_t > 0, \lambda_t^D = 0)$. A firm with the adjustment cost function $-\Pi_{I2}$ is in a constrained regime where both dividend paymens and new share issues are zero. Here investment spending is constrained to the level of available internal funds (C), and locally investment spending will fluctuate dollar-for-dollar with windfall changes in cash flow for firms in this regime. A firm with the adjustment cost function $-\Pi_{I3}$ is in a third regime where additional investment is financed by issuing new equity. The higher cost of external finance influences the optimal level of investment chosen, as indicated by the first order condition (3.4). Here λ_t^D is given from the optimality condition for new share issues (3.7), so that

$$\frac{\lambda_t^K}{1+\lambda_t^D} = \lambda_t^K(1-\phi) \tag{3.9}$$

and this firm chooses the investment rate $\frac{I_3}{K}$. If the same firm had access to a sufficiently higher level of internal funds, it would choose the higher investment rate $\frac{I_5}{K}$. Locally, however, investment spending is insensitive to small windfall fluctuations in cash flow for firms in this regime, as illustrated in Figure 9. The shock to the availability of internal funds must be large enough to move such firms from the third 'external finance' regime to the second 'constrained' regime in order for their level of investment spending to be affected, as illustrated in Figure 10.

Depending on which regime a firm is in prior to a windfall increase in cash, and on the size of the shock, there are six different paths along which the firm's investment spending may be affected. We find that investment displays excess sensitivity to cash flow shocks if the firm is initially in the external finance regime and is moved to either of the other regimes, or if the firm is initially in the constrained regime.

When we consider the impact of windfall cash flow shocks on the investment spending of otherwise identical firms that are subject to different cost premia for external finance, there are still more possible combinations to consider. We find several cases in which the effect on investment is strictly greater for the firm with the higher cost premium; two of these possibilities are illustrated in Figures 11 and 12. For the case in Figure 11, the cash flow shock increases the investment rate for the low cost premium (ϕ_L) firm from $\frac{I_3}{K}$ to $\frac{C^{'}}{K}$, whilst the same cash flow shock increases investment for the high cost premium (ϕ_H) firm from the lower rate $\frac{C}{K}$ also to $\frac{C^{'}}{K}$. For the case in Figure 12, the cash flow shock increases investment for the ϕ_L firm from $\frac{I_3}{K}$ to$\frac{I_3^{'}}{K}$, whilst the same shock increases investment for the ϕ_H firm from $\frac{C}{K}$ to $\frac{I_3^{'}}{K}$. There is also a case here in which each firm's investment increases dollar-for-dollar with the windfall increase in cash flow, as illustrated in Figure 13. However if we compare otherwise identical firms with the same adjustment cost schedule, supply of internal funds, and shadow value of capital, we find no case in which the effect on investment is strictly greater for the firm with the lower cost premium.

Thus we find that in the model with quadratic adjustment costs and a fixed cost premium for new equity finance, there is a weakly monotonic relationship between the sensitivity of investment to windfall fluctuations in cash flow and the severity of the financing constaint, as reflected in the size of the cost premium for external funds, for otherwise identical firms. The result is again obtained by holding the shadow value of capital or marginal q constant. The following section considers the relationship between marginal q and average q in these models, and hence the extent to which econometric studies may in fact be able to condition on *marginal* q in the presence of financing constraints.

4. Marginal q and average q

Hayashi (1982) showed that for a firm with a linear homogeneous revenue function $\Pi(K_t, I_t) = \Pi_{Kt}K_t + \Pi_{It}I_t$, the first order condition (3.4) and the intertemporal condition (3.5) can be combined in the absence of financing constraints ($\lambda_t^D \equiv 0$) to obtain

$$\lambda_t^K = \frac{V_t}{(1-\delta)K_{t-1}} \tag{4.1}$$

where V_t is the maximised value of the firm. Given that we have normalised the price of capital goods to unity for simplicity, this states that marginal q is equal to average q, or the ratio of the maximised value of the firm to the replacement cost of its inherited capital stock. At least in the absence of share price bubbles,[6] the numerator of this average q ratio can be measured using the firm's stock market valuation. In the absence of financing constraints, econometric specifications can in principle condition on marginal q in the benchmark case of a linear homogeneous revenue function and strictly convex costs of adjustment.

Combining these optimality conditions in the same way in our model with costly external finance yields the equality

$$\lambda_t^K(1-\delta)K_{t-1} = E_t\left\{\sum_{s=0}^{\infty}\beta^s\left(1+\lambda_{t+s}^D\right)\Pi_{t+s}\right\} \tag{4.2}$$

With no cost premium for external finance ($\phi(N_t) \equiv 0$), the shadow value of internal funds (λ_t^D) is identically zero, and the sources and uses of funds identity (3.2) shows that net revenue Π_{t+s} equals the net cash distribution to stockholders

[6] See Bond and Cummins (2001) for further discussion.

$(D_{t+s} - N_{t+s})$, so that the right hand side of (4.2) simplifies to the value of the firm V_t as in (3.1). More generally, we need to consider the relationship between $\left(1 + \lambda^D_{t+s}\right) \Pi_{t+s}$ and this net distribution to stockholders.

First consider the model with a fixed cost premium, as in section 3.2. For firms in the unconstrained regime in period $t + s$, we have $N_{t+s} = 0, D_{t+s} > 0$ and $\lambda^D_{t+s} = 0$. Using the sources and uses of funds identity (3.2) shows that in this regime

$$\left(1 + \lambda^D_{t+s}\right) \Pi_{t+s} = \Pi_{t+s} = D_{t+s} - N_{t+s} \tag{4.3}$$

For firms in the constrained regime in period $t+s$, we have $N_{t+s} = 0$ and $D_{t+s} = 0$, so that here

$$\left(1 + \lambda^D_{t+s}\right) \Pi_{t+s} = \Pi_{t+s} = D_{t+s} - N_{t+s} = 0 \tag{4.4}$$

For firms in the external finance regime we have $N_{t+s} > 0, D_{t+s} = 0$ and $\lambda^D_{t+s} = \frac{\phi}{1-\phi}$. This gives $(1 + \lambda^D_{t+s}) = \frac{1}{1-\phi}$, so that

$$\left(1 + \lambda^D_{t+s}\right) \Pi_{t+s} = \frac{\Pi_{t+s}}{1 - \phi} \tag{4.5}$$

For firms in this regime, the sources and uses of funds identity simplifies to

$$-N_{t+s} = \Pi_{t+s} - \phi N_{t+s} \tag{4.6}$$

giving

$$\Pi_{t+s} = -N_{t+s}(1 - \phi) \tag{4.7}$$

or

$$\frac{\Pi_{t+s}}{1 - \phi} = -N_{t+s} \tag{4.8}$$

Combining (4.5) and (4.8) shows that in this regime we also have

$$\left(1+\lambda_{t+s}^{D}\right)\Pi_{t+s} = -N_{t+s} = D_{t+s} - N_{t+s} \tag{4.9}$$

Since this equality holds regardless of which of the three regimes the firm happens to be in, we have

$$\lambda_t^K(1-\delta)K_{t-1} = E_t\left\{\sum_{s=0}^{\infty}\beta^s\left(1+\lambda_{t+s}^{D}\right)\Pi_{t+s}\right\} = E_t\left\{\sum_{s=0}^{\infty}\beta^s\left(D_{t+s}-N_{t+s}\right)\right\} = V_t \tag{4.10}$$

Consequently in this model we still have equality between marginal q and average q, under the same assumption of a linear homogeneous revenue function required in the absence of financing constraints.

This equality ceases to hold in models with an increasing marginal cost of external finance. If we consider the model with a quadratic cost of external funds schedule, as in section 3.1, we still find that equation (4.3) holds in the unconstrained regime, but equation (4.9) no longer holds in the constrained regime of this model. Here we have $(1+\lambda_{t+s}^{D}) = \frac{1}{1-\phi N_{t+s}}$ and $\Pi_{t+s} = -N_{t+s}(1-(\frac{\phi}{2})N_{t+s})$, so that

$$\left(1+\lambda_{t+s}^{D}\right)\Pi_{t+s} = -N_{t+s}\left[\frac{1-(\frac{\phi}{2})N_{t+s}}{1-\phi N_{t+s}}\right] \tag{4.11}$$

The last term in square brackets is strictly greater than one for $\phi > 0$ and $N_{t+s} > 0$, so that if the firm is ever anticipated to issue new equity in this model, the equality between V_t and the right hand side of (4.2) breaks down.

5. Conclusions

The last result in section 4 limits the usefulness of the monotonicity result obtained in section 3.1, conditional on marginal q, since it is not clear how an empirical analysis could condition on marginal q in the presence of an upward sloping cost of external finance schedule. However the weak monotonicity result obtained in section 3.2 provides a benchmark specification in which a higher cost premium for one group of firms would be reflected in a greater sensitivity of investment to windfall fluctuations in cash flow, conditional on average q, than would be expected for an otherwise identical group of firms with a lower cost premium for external finance.

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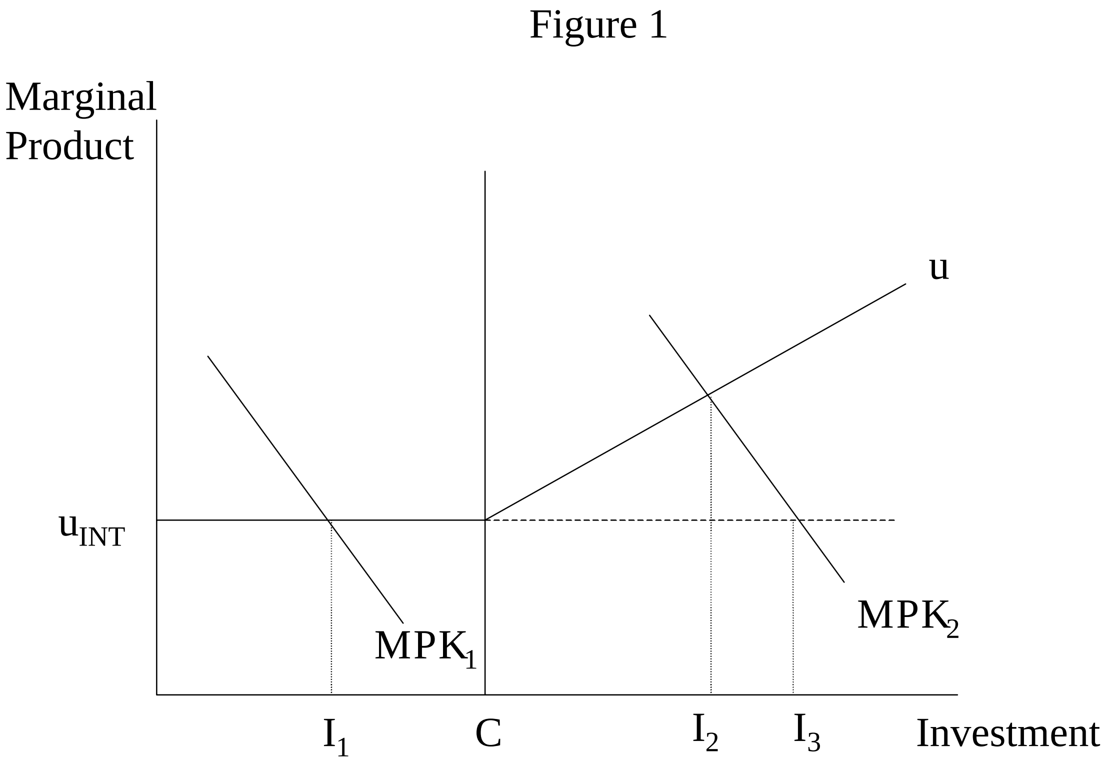
Figure 1
Marginal
Product
u
u_INT
MPK_1
MPK_2
I_1
C
I_2
I_3
Investment

Figure 5.1:

Figure 2

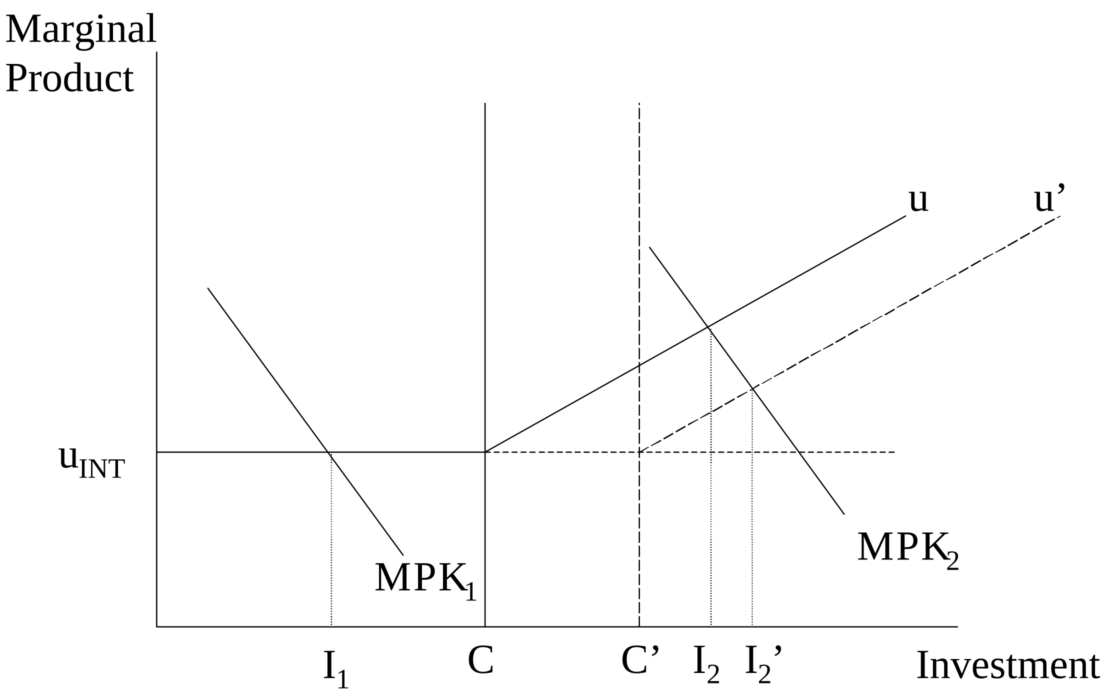

Figure 5.2:

Figure 3

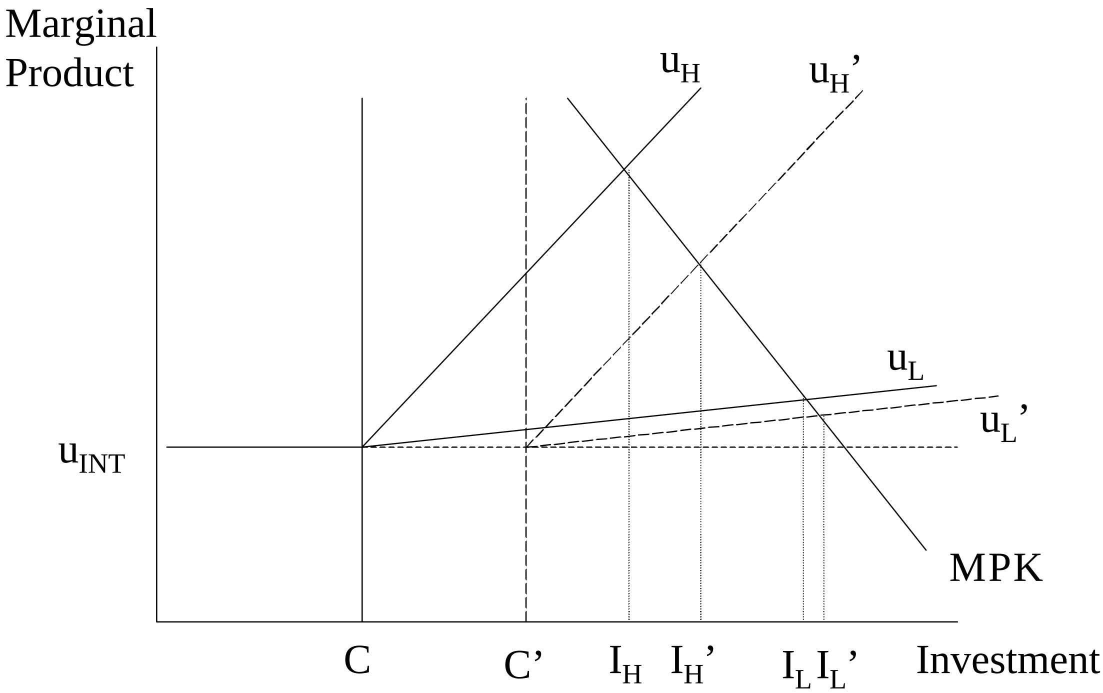

Figure 5.3:

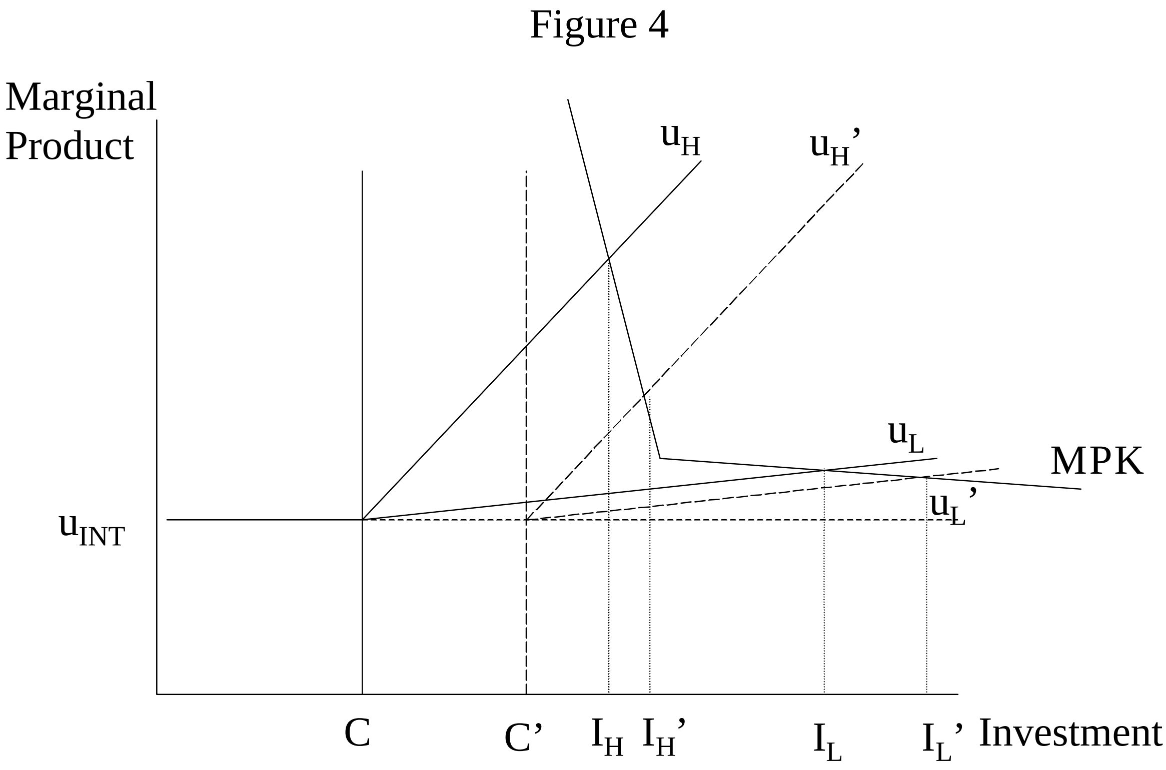
Figure 4
Marginal Product
u_H
u_H'
u_L
MPK
u_L'
u_INT
C
C'
I_H
I_H'
I_L
I_L'
Investment

Figure 5.4:

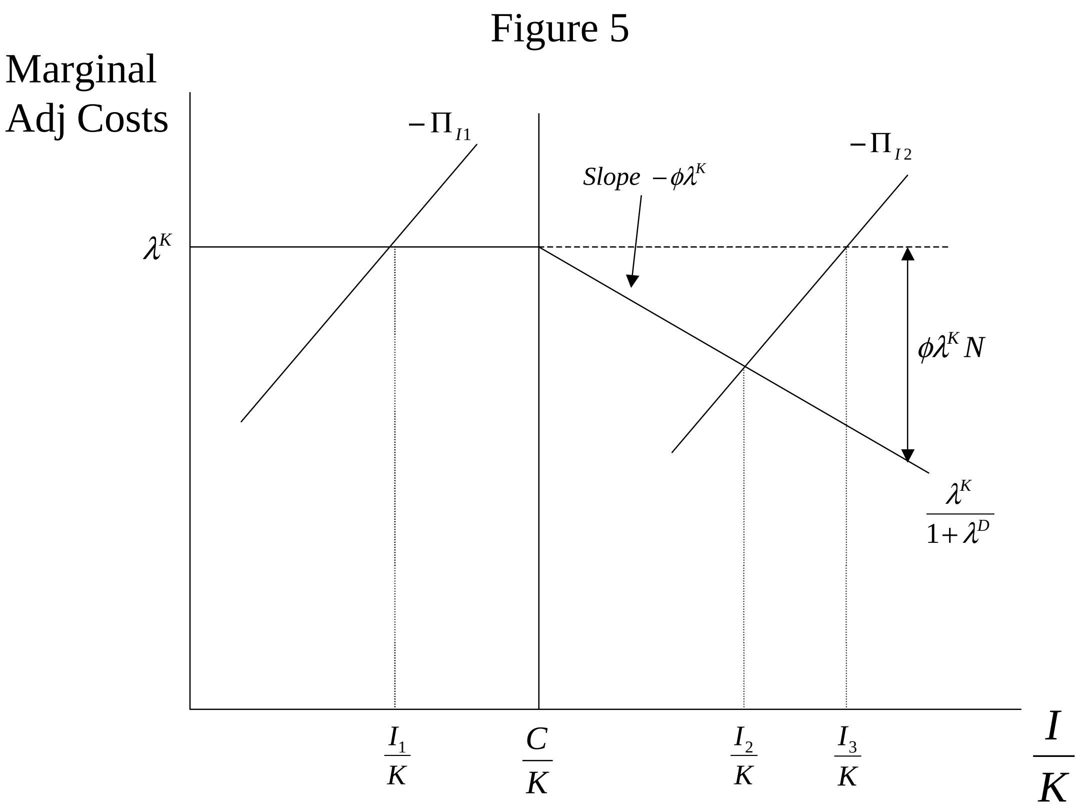
Figure 5
Marginal
Adj Costs
$-\Pi_{I1}$
$-\Pi_{I2}$
Slope $-\phi\lambda^K$
λ^K
$\phi\lambda^K N$
$\frac{\lambda^K}{1+\lambda^D}$
$\frac{I_1}{K}$
$\frac{C}{K}$
$\frac{I_2}{K}$
$\frac{I_3}{K}$
$\frac{I}{K}$

Figure 5.5:

Figure 6

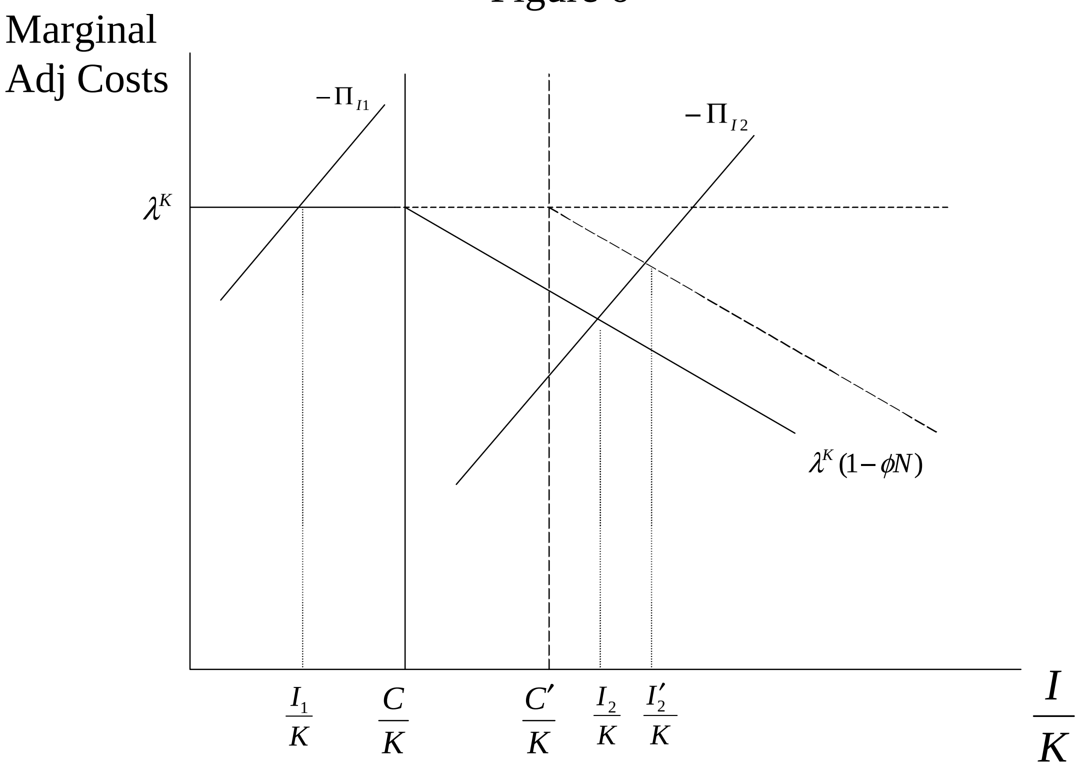

Figure 5.6:

Figure 7

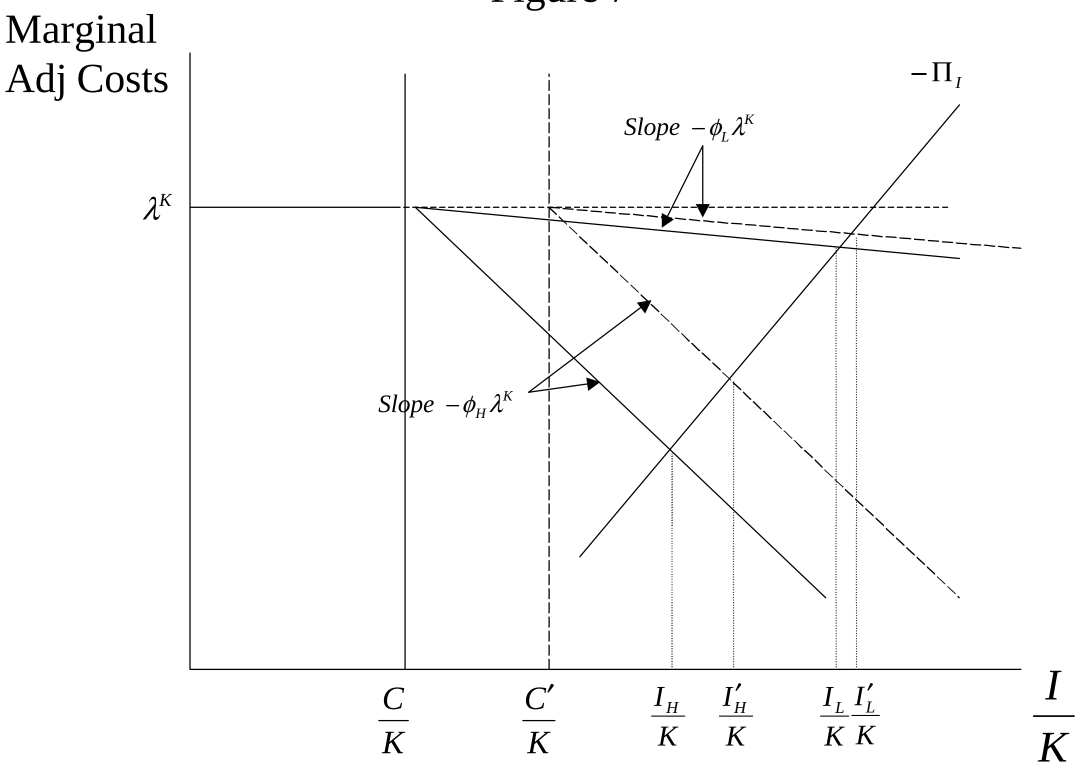

Figure 5.7:

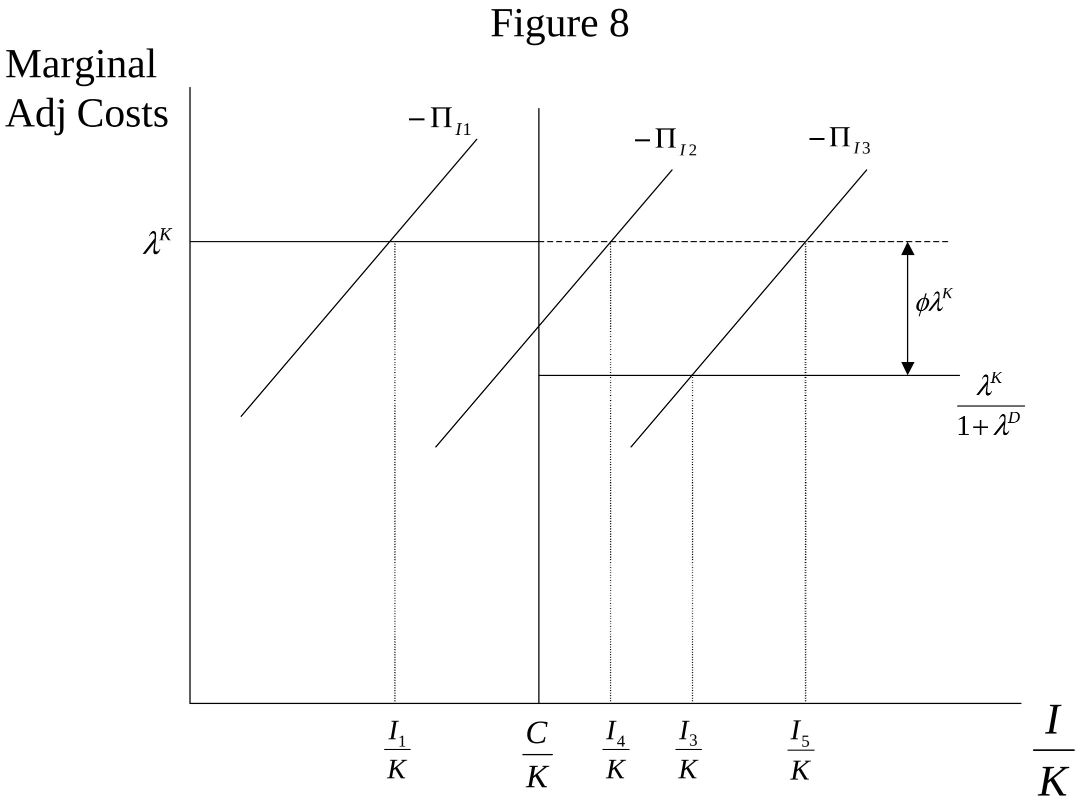
Figure 8
Marginal
Adj Costs
$-\Pi_{I1}$
$-\Pi_{I2}$
$-\Pi_{I3}$
λ^K
$\phi\lambda^K$
$\frac{\lambda^K}{1+\lambda^D}$
$\frac{I_1}{K}$
$\frac{C}{K}$
$\frac{I_4}{K}$
$\frac{I_3}{K}$
$\frac{I_5}{K}$
$\frac{I}{K}$

Figure 5.8:

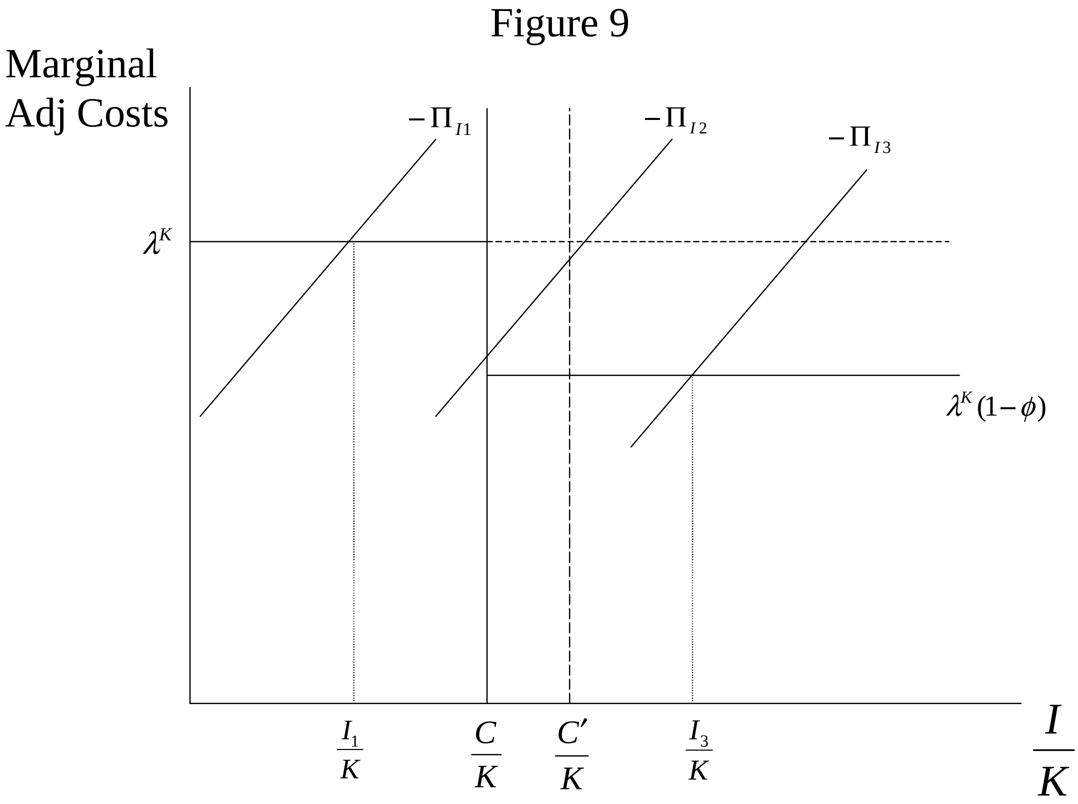
Figure 9
Marginal
Adj Costs
λ^K
$-\Pi_{I1}$
$-\Pi_{I2}$
$-\Pi_{I3}$
$\lambda^K(1-\phi)$
$\frac{I_1}{K}$
$\frac{C}{K}$
$\frac{C'}{K}$
$\frac{I_3}{K}$
$\frac{I}{K}$

Figure 5.9:

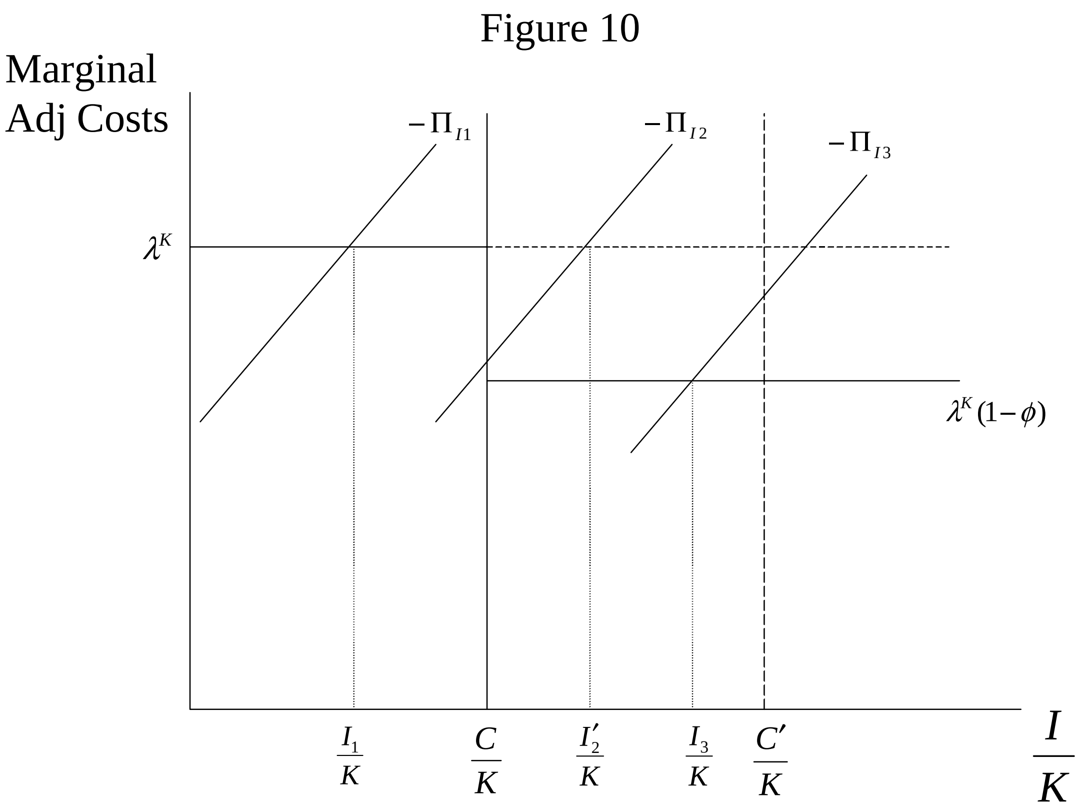
Figure 10
Marginal
Adj Costs
$-\Pi_{I1}$
$-\Pi_{I2}$
$-\Pi_{I3}$
λ^K
$\lambda^K(1-\phi)$
$\frac{I_1}{K}$
$\frac{C}{K}$
$\frac{I_2'}{K}$
$\frac{I_3}{K}$
$\frac{C'}{K}$
$\frac{I}{K}$

Figure 5.10:

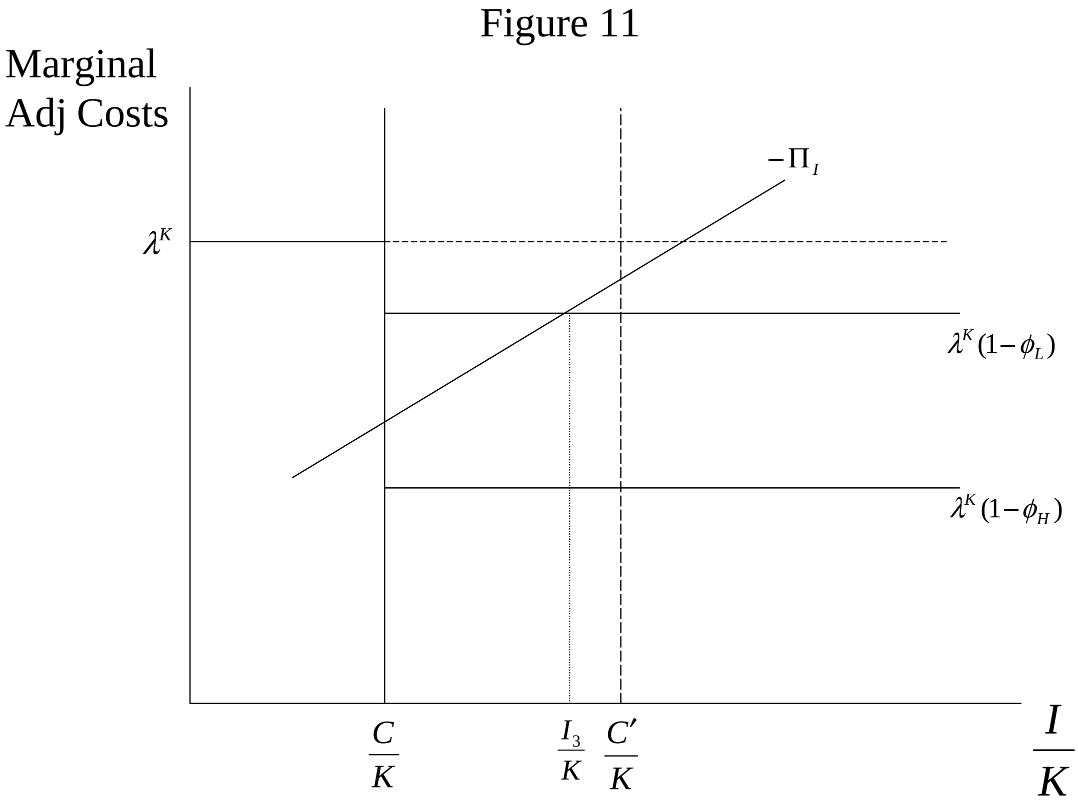

Figure 11
Marginal
Adj Costs
$-\Pi_I$
λ^K
$\lambda^K(1-\phi_L)$
$\lambda^K(1-\phi_H)$
$\frac{C}{K}$
$\frac{I_3}{K}$
$\frac{C'}{K}$
$\frac{I}{K}$

Figure 5.11:

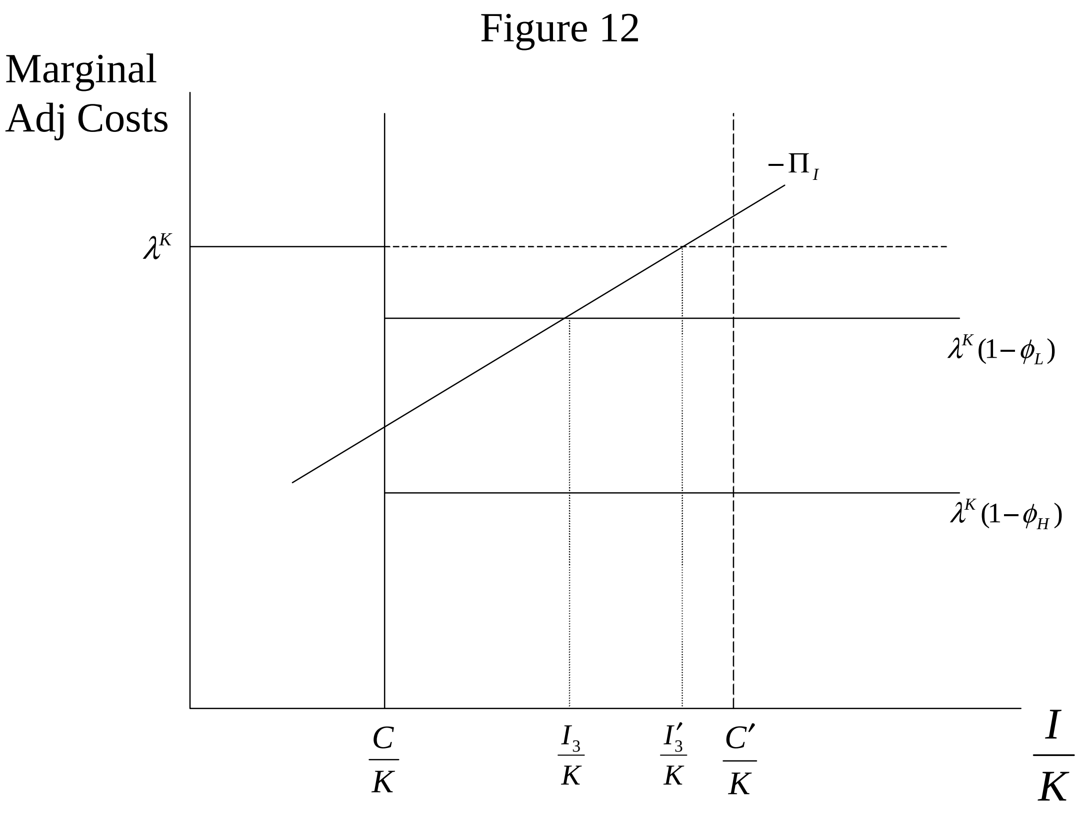
Figure 12
Marginal
Adj Costs
$-\Pi_I$
λ^K
$\lambda^K(1-\phi_L)$
$\lambda^K(1-\phi_H)$
$\frac{C}{K}$
$\frac{I_3}{K}$
$\frac{I_3'}{K}$
$\frac{C'}{K}$
$\frac{I}{K}$

Figure 5.12:

Figure 13

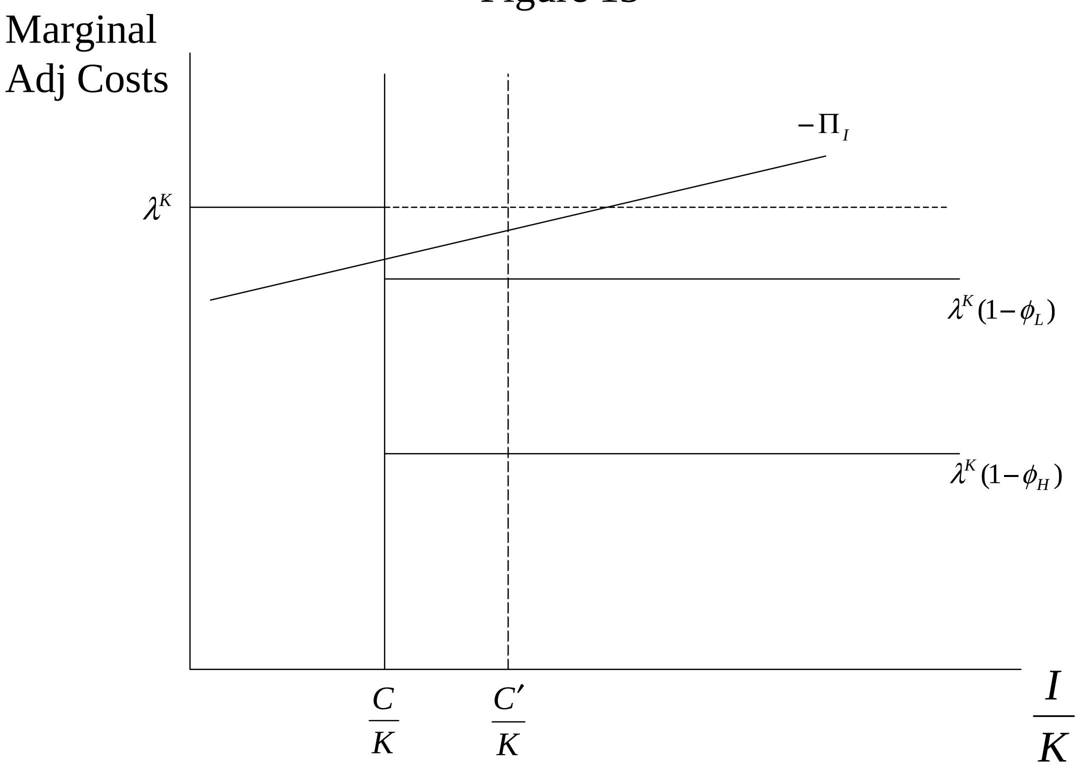

Figure 5.13: