Identifying the Monetary Transmission Mechanism using Structural Breaks[1]

Andreas Beyer[2]
European Central Bank
Postfach 16 03 19
D-60066
Frankfurt am Main
Andreas.Beyer@ecb.int

Roger E. A. Farmer[3]
UCLA, Dept. of Economics
8283 Bunche Hall
Box 951477
Los Angeles, CA 90095-1477
rfarmer@econ.ucla.edu

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Abstract

We propose a method for estimating a subset of the parameters of a structural rational expectations model by exploiting changes in policy. We define a class of models, midway between a vector autoregression and a structural model, that we call the *recoverable structure*. We provide an application of our method by estimating the parameters of a three equation model of the monetary transmission mechanism using data from 1970:Q1 to 1999:Q4. We estimate a vector autoregression and find that the parameters of this VAR are unstable. However, using our proposed identification method we are able to attribute instability in the parameters of the VAR solely to changes in the parameters of the policy rule. We recover parameter estimates of the recoverable structure and we demonstrate that these parameters are invariant to changes in policy. Since the recoverable structure includes future expectations as explanatory variables our parameter estimates are not subject to the Lucas [24] critique of econometric policy evaluation.

1 Introduction

In this paper we introduce a method for estimating the structural equations of a rational expectations model by exploiting the assumption that economic policy is governed by a rule that may change infrequently and abruptly. Our idea is to use changes in the parameters of the policy rule as instruments to identify the parameters of structural equations that remain constant across different policy regimes. We apply our technique to the problem of identifying the monetary transmission mechanism.

Currently there are two popular approaches to identification of monetary policy. One is to use a vector autoregression (VAR) in which shocks are identified by assumptions about the contemporaneous correlations of the errors. A second approach is to write down a fully specified rational expectations model and to identify its parameters using cross-equation restrictions.[1] We propose an alternative method of identification that is midway between a VAR and a fully identified model. Following Farmer and Uhlig[2] [11] we call this method the estimation of a *recoverable structure.*

In the structural approach one imposes assumptions on the economic environment to achieve identification. Typically, the data are assumed to be generated by the choices of a representative agent with an infinite-horizon time-separable utility function and a period-t utility function, separable in current consumption and real balances. These assumptions imply that the representative agent's consumption Euler equation contains only current consumption, lagged consumption and the real interest rate. When supplemented with a model of aggregate supply, assumptions of this kind generate restrictions on the structure of the unemployment and inflation equations in the class of three-equation models that we will study in this paper.

Suppose, for example, the modeler assumes that real balances do not enter

[1]Sims [34] popularized VAR models in which shocks are identified by ordering the variables and decomposing the VCV matrix with a Cholesky decomposition. Bernanke and Mihov [3], Blanchard and Quah [6], Blanchard [5] and Gali [13] are examples of papers that remain with the VAR framework but identify shocks by placing structural assumptions on the contemporaneous VCV matrix of shocks. A subset of papers that adopt some variant of a full structural approach includes Christiano, Eichenbaum and Evans [7], Gali and Gertler [14], Ireland [16], McCallum and Nelson [26], Rotemburg and Woodford [31] and Smets and Wouters [37].

[2]The term "recoverable-structure" evolved in discussions between Farmer and Uhlig in the summer of 2002. We are grateful to Harald Uhlig for valuable discussions that helped us clarify many of the issues that we confront in this paper.

the Euler equation. This modeling choice implies that the opportunity cost of holding money (represented by the federal funds rate) should not enter the IS curve.[3] Assumptions like this are critical in supplying the restrictions necessary to identify a structural form. But they are not a direct implication of optimizing behavior under rational expectations. In 1980, Sims [34] criticized the "incredible identifying assumptions" of Keynesian models and suggested instead that macroeconomists use vector autoregressions to identify the role of money on economic activity. In our view, most of the restrictions embodied in structural RE models are just as incredible as those of the Keynesian models they replaced and we should not be surprised if models identified by imposing restrictions derived from simple representative agent models are rejected by formal hypothesis tests. Rejection of a particular model does not imply that all rational expectations models are false; rather, it suggests that the econometrician has imposed incorrect identifying restrictions.

Our paper is organized as follows. Section 2 summarizes related work. In Section 3 we introduce a two-equation example and show how to identify the parameters of a demand equation if the parameters of the supply equation change at a known date. Section 4 studies a three equation example and shows that change in one equation is not sufficient to identify all of the parameters of the other two equations in the system. This leads to Section 5 in which we define the concept of the recoverable structure. Sections 6 and 7 extend our ideas to models with forward looking expectations. In Section 8 - 13 we show how to apply our technique to US monetary policy. Section 14 presents a short conclusion.

2 Related Research

Sargent [32] discusses the equivalence of the reduced form representations of rational and non-rational expectations models and he points out that "the empirical evidence from a single estimation period alone...can never settle things between advocates of rules with feedback and rules without feedback" ([32] page 635). Sims [34] takes the view that the rational expectations hypothesis is "more deeply subversive of identification than has yet been recognized". Pesaran [28] contains a fairly comprehensive discussion of identifica-

[3]We use this term in the sense of McCallum and Nelson [26] who refer to the representative agent's Euler equation, supplemented by a market clearing equation, as an "optimizing IS curve".

tion in rational expectations models in which he points out the importance of a-priori information on lag length as a potential way of discriminating between different theories. This paper takes off from Sargent's comments by using information from *different* sample periods to achieve identification.

An early application of structural change to achieve identification is provided by Bean [2]. The closest papers to the present one are by Rigobon [29], [30] and Klaeffling [20]. Rigobon exploits heteroskedasticity to identify the parameters of a structural model. Our concept of a recoverable structure exploits a similar idea although we exploit changes in all of the parameters of the policy rule, not just a change in variance. Klaeffling [20] estimates a class of models that includes future expectations, but he does not allow for the presence of contemporaneous variables in the structure. As a consequence, the parameters of Klaeffling's model confound policy with structure and his approach cannot be applied to study the consequences of a change in the policy rule.

Lubik and Schorfheide [22], [23] have studied structural rational expectations models in which they allow for the possibility of indeterminacy. They place more structure on the model than we are prepared to do, although like Lubik and Schorfheide, our approach allows for both determinate and indeterminate equilibria. Leeper and Zha [21] study a class of "modest policy interventions" that enables them to make predictions of the effects of monetary policy changes within a given regime. We assume instead that the entire regime changes at a discrete point in time. Sims and Zha [36] study regime changes using a markov switching model.

3 Identification Through Exclusion Restrictions

Consider the following two-equation model of demand and supply which is often used to introduce the topic of identification in introductory econometrics texts.[4]

$$\overbrace{\begin{bmatrix} 1 & a_1 \\ 1 & a_2 \end{bmatrix}}^{\mathbf{A}} \overbrace{\begin{bmatrix} q_t \\ p_t \end{bmatrix}}^{Y_t} = \overbrace{\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}}^{\mathbf{B}} Z_t + \overbrace{\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}}^{U_t} \qquad (1)$$

[4]Uppercase letters denote vectors, lowercase letters represent scalars and boldface letters are matrices. We use superscript "T" for the transpose operator.

$$U_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}, \qquad E_{t-1}\left[U_t U_t^T\right] = \mathbf{\Omega}, \qquad E_{t-1}\left[U_t U_{t+k}^T\right] = 0, \qquad k > 0,$$

$$\theta = \left\{ \underset{1\times 1}{a_1}, \underset{1\times m}{B_1}, \underset{1\times 1}{a_2}, \underset{1\times m}{B_2}, \underset{2\times 2}{\mathbf{\Omega}} \right\}.$$

We also assume

Condition 1 (Row Independence) *The rows of*

$$\begin{bmatrix} A & B \end{bmatrix} \tag{2}$$

are linearly independent.

The first row of Equation (1) represents a demand equation and the second row is a supply equation. Condition (2) rules out the trivial case in which the demand and supply equations are the same. Y_t is the 2×1 vector $(q_t, p_t)^T$, q_t is quantity, p_t is price, Z_t is an $m \times 1$ vector of exogenous variables, and θ is a set of parameters of interest. Since the parameters enter symmetrically, in the absence of additional identifying information, the parameters of the demand and supply equations cannot be separately identified.

Suppose however we know that at date T_1, the parameters $\{a_2, B_2\}$ of the supply equation change from $\{a_2^1, B_2^1\}$ to $\{a_2^2, B_2^2\}$ where the superscript 1 or 2 indexes the time period and the subscript 2 indicates supply. Our main idea is to use the change in the parameters of the supply equation, by constructing a series of step-variables, to identify the parameters of the demand equation.

For each variable $\{Y_t, Z_t\}$, our method involves the construction of new *step-variables*, and new *step-errors* defined as follows,

$$\begin{aligned} Y_t^S &= 0, \qquad Z_t^S = 0, \qquad U_t^S = 0, \qquad t = 1, ...T_1, \\ Y_t^S &= Y_t, \qquad Z_t^S = Z_t, \qquad U_t^S = U_t, \qquad t = T_1 + 1, ...T. \end{aligned}$$

Using these transformed variables we can write the *augmented structural form* as

$$\overset{\mathbf{A}^*}{\begin{bmatrix} 1 & a_1 & 0 & 0 \\ 1 & a_2^1 & 0 & a_2^\Delta \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 1 & a_2^2 \end{bmatrix}} \overset{Y_t^*}{\begin{bmatrix} q_t \\ p_t \\ q_t^S \\ p_t^S \end{bmatrix}} = \overset{\mathbf{B}^*}{\begin{bmatrix} B_1 & 0 \\ B_2^1 & B_2^\Delta \\ 0 & B_1 \\ 0 & B_2^2 \end{bmatrix}} \overset{Z_t^*}{\begin{bmatrix} Z_t \\ Z_t^S \end{bmatrix}} + \overset{U_t^*}{\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{1t}^S \\ u_{2t}^S \end{bmatrix}}, \tag{3}$$

where,

$$a_2^{\Delta} \equiv a_2^2 - a_2^1, \quad B_2^{\Delta} \equiv B_2^2 - B_2^1.$$

We use a star to indicate the expanded matrices and the vectors of exogenous and endogenous variables augmented by the step variables. The augmented reduced form of this system is a set of equations

$$\overset{Y_t^*}{\begin{bmatrix} q_t \\ p_t \\ q_t^S \\ p_t^S \end{bmatrix}} = \overset{\mathbf{\Gamma}^*}{\begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \\ 0 & \Gamma_{31} \\ 0 & \Gamma_{32} \end{bmatrix}} \overset{Z_t^*}{\begin{bmatrix} Z_t \\ Z_t^S \end{bmatrix}} + \overset{W_t^*}{\begin{bmatrix} w_t^1 \\ w_t^2 \\ w_t^{1S} \\ w_t^{2S} \end{bmatrix}}, \tag{4}$$

where the parameters of the augmented reduced form are related to those of the augmented structural form by the equation

$$\mathbf{A}^*\mathbf{\Gamma}^* = \mathbf{B}^*,$$

and the reduced form errors are related to the structural errors by the equation,

$$\mathbf{A}^* W_t^* = U_t^*.$$

Identification theory can be organized around the concept of an instrumental variable. Our method transforms a system of two equations in two endogenous variables into an equivalent system of four equations in four endogenous variables. If the parameters are constant in both equations, the step variables and the second two equations are redundant. However, if the parameters of the supply equation change, the newly constructed step variables can be used as instruments to identify parameters of the demand equation. Provided the $1 \times m$ parameter vectors Γ_{31} and Γ_{32} of the reduced form are non-zero, the endogenous variables q_t and p_t will be correlated with Z_t^S.

For the parameters of an equation to be identified, classical identification theory (Fisher [12]) requires that two conditions, the order and the rank condition, should both be satisfied. The order condition requires that there are at least as many instruments as endogenous explanatory variables. The demand equation contains one endogenous variable. Since there are m excluded exogenous variables, (these are the m step-variables Z_t^S) and since each of these variables can be used as an instrument for p_t, the order condition is satisfied.

The rank condition can be expressed in several equivalent ways. One representation of this condition is that the $[3 \times (m+2)]$ matrix

$$M = \begin{bmatrix} B_2^{\Delta} & 0 & a_2^{\Delta} \\ B_1 & 1 & a_1 \\ B_2^2 & 1 & a_2^2 \end{bmatrix}$$

has full row rank.[5] Condition (2) implies that the second two rows are independent of each other. It follows that a_1 is identified if at least one element of $\left\{B_2^{\Delta}, a_2^{\Delta}\right\}$ is non-zero.

We have shown that a change in the parameters of one of the equations in a two-equation model, enables us to identify the parameters of the other equation in the system. A parameter change of this type does not, however, allow us to say anything about the parameters of the equation that breaks. In our example, we cannot identify the parameters of the supply equation since there are no valid instruments to identify them.

4 A Three Equation Model

Our next example is a model with three equations that illustrates how to extend our identification method to larger systems. Consider the model

$$\mathbf{A}Y_t = \mathbf{B}Z_{t-1} + U_t, \tag{5}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & a_{12} & a_{13} \\ a_{21} & 1 & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix},$$

$$Y_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix}, \quad Z_{t-1} = \begin{bmatrix} Y_{t-1} \\ \vdots \\ Y_{t-k} \\ C \end{bmatrix} \quad U_t = \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix}.$$

[5] This is condition (b) from Hamilton [15] page 246 with the following notational difference. We use B for the matrix premultiplying exogenous variables in the structural equations, Hamilton uses Γ. We use A for the matrix premultiplying the endogenous variables, Hamilton uses B. We use Γ for the reduced form coefficients, Hamilton uses Π.

In this example there are three endogenous variables, $Y_t = \{y_{1t}, y_{2t}, y_{3t}\}$ and a vector of predetermined variables Z_{t-1}, which includes a constant C. We assume that the errors U_t have zero mean and covariance matrix $\mathbf{\Omega}$.

To fix ideas, we will give names to the equations and to the variables. Let the three variables represent the unemployment rate, the inflation rate and the interest rate. To name the equations, we normalize Equation 1 by setting the coefficient on unemployment to 1, Equation 2 by setting the coefficient on inflation to 1 and Equation 3 by setting the coefficient on the interest rate to 1. Using this convention we will refer to Equation 1 as the *unemployment equation*, Equation 2 as the *inflation equation* and Equation 3 as the *interest rate equation*. Since we assume that the interest rate is controlled by the Fed we will refer to the interest rate equation as the *policy rule*.

We can represent the reduced form of the model as follows;

$$Y_t = \mathbf{\Gamma} Z_{t-1} + W_t, \tag{6}$$

where

$$\mathbf{A} W_t = U_t, \tag{7}$$

and $\mathbf{\Gamma}$ is related to the structural coefficients by the expression,

$$\mathbf{A\Gamma} = \mathbf{B}.$$

If the structural equations were to remain stable, we would be unable to identify the parameters of the matrix $\mathbf{A}$. But the reduced form coefficients, $\mathbf{\Gamma}$, could be estimated consistently by least squares. Since Z_{t-1} consists of lagged values of Y_t, Equation (6) describes a vector autoregression. The stability of the structure implies that its parameters should remain constant over time.

For many data sets that economists have studied it is difficult or impossible to find extended periods over which the parameters of a vector autoregression remain constant. We believe that in a substantial number of examples of interest, it may be possible to make credible arguments that parameter change can be attributed to a specific cause. For example, the date of the break may coincide with an announcement by the policy authorities or with a change in administration. This is the situation that occurred in 1979 when Paul Volcker took over from G. William Miller as chairman of the Board of Governors of the Fed. A similar event occurred in 1973 with the collapse of the Bretton Woods system of fixed exchange rates. In situations like this, when structural change can credibly be attributed to a change in one structural equation, it may be possible to use this information to identify the remaining equations of the system.

Pursuing this idea, let us assume that the parameters of the policy rule are known to change at date T_1 and let Y_t^S and U_t^S be step variables and step errors defined as in Section 3. As our previous two equation example, we introduce additional parameters $a_{31}^{\Delta}, a_{32}^{\Delta}$ and B_3^{Δ} defined as follows,

$$a_{31}^{\Delta} = a_{31}^2 - a_{31}^1, \quad a_{32}^{\Delta} = a_{32}^2 - a_{32}^1, \quad B_3^{\Delta} = B_3^2 - B_3^1,$$

where the superscript 1 or 2 indexes the first or second subperiod. For the three equation model the *augmented structural form* is defined as follows,

$$\overset{\mathbf{A}^*}{\begin{bmatrix} 1 & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{21} & 1 & a_{23} & 0 & 0 & 0 \\ a_{31}^1 & a_{32}^1 & 1 & a_{31}^{\Delta} & a_{32}^{\Delta} & 0 \\ 0 & 0 & 0 & 1 & a_{12} & a_{13} \\ 0 & 0 & 0 & a_{21} & 1 & a_{23} \\ 0 & 0 & 0 & a_{31}^2 & a_{32}^2 & 1 \end{bmatrix}} \overset{Y_t^*}{\begin{bmatrix} y_{1t} \\ y_t^2 \\ y_t^3 \\ y_t^{1S} \\ y_t^{2S} \\ y_t^{3S} \end{bmatrix}} = \overset{\mathbf{B}^*}{\begin{bmatrix} B_1 & 0 \\ B_2 & 0 \\ B_3^1 & B_3^{\Delta} \\ 0 & B_1 \\ 0 & B_2 \\ 0 & B_3^2 \end{bmatrix}} \overset{Z_{t-1}^*}{\begin{bmatrix} Z_{t-1} \\ Z_{t-1}^S \end{bmatrix}} + \overset{U_t^*}{\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{1t}^S \\ u_{2t}^S \\ u_{3t}^S \end{bmatrix}}. \tag{8}$$

As with out previous example, the parameters a_{12}, a_{13}, a_{21}, a_{23}, B_1, and B_2 do not contain superscripts, reflecting the assumption that they remain constant for the entire sample. Rows 4 and 5 of the augmented structural form represent the same equations as rows 1 and 2, but for the second sub-sample. Rows 3 and 6 describe the third equation which, by assumption, is the policy rule. The parameters a_{31}^1, a_{32}^1, and B_3^1 represent the coefficients of y_{1t}, y_{2t} and Z_{t-1} of the policy rule during the first sub-period and the parameters a_{31}^2, a_{32}^2 and B_3^2 premultiply these same variables during the second sub-period. The parameters a_{31}^{Δ}, a_{32}^{Δ} and B_3^{Δ} are the changes in the parameters at the date of the break.

The assumption of parameter constancy is represented by the absence of the variables y_{1t}^S, y_{2t}^S y_{3t}^S and Z_{t-1}^S from the unemployment and inflation equations. We are aware that most modern macroeconomic models cannot be represented in this way as a consequence of the existence of endogenous expectations. At this point we ask the reader to suspend criticism based on this observation since the model without expectations will allow us to make an important point about identification that will also hold in models with expectational terms. We take up the issue of endogenous expectations in Section 7.

For the same reason that we could not identify the supply equation in our first example, we will not be able to identify the parameters of the policy rule. One might think, however, that we could use the same arguments that were applied in Section 3 to show that the unemployment and the inflation equations *are* identified. After all, the unemployment and the inflation equation each contain two right-hand-side endogenous variables and they each exclude the m exogenous variables, Z_t^S. It follows that the order condition is satisfied for each equation as long as $m \geq 2$. But although the order condition is satisfied, the rank condition fails. This condition is the same for both equations and it requires that the matrix

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ B_3^{\Delta} & a_{31}^{\Delta} & a_{32}^{\Delta} & 0 \\ B_1 & 1 & a_{12} & a_{13} \\ B_2 & a_{21} & 1 & a_{23} \\ B_3^2 & a_{31}^2 & a_{32}^2 & 1 \end{bmatrix}$$

should have full row-rank[6]. Since the first row of $\mathbf{M}$ is zero the rows of $\mathbf{M}$ are not linearly independent and neither the unemployment equation nor the inflation equation is identified. The following section pursues this point further.

5 The Recoverable Structure

Identification fails in our example because changes in the policy rule cannot give us independent information about the way that inflation and unemployment interact with each other. We cannot use information that there has been a break in the policy rule to identify the parameters a_{12} and a_{21}. These are important parameters if our goal is to discriminate between competing theories of macroeconomic behavior. For example, in some popular theories a_{21} would represent the impact of unemployment on current inflation in the Phillips curve. It would be interesting to bring evidence to bear on the magnitude of this parameter, however, it is not something that can be uncovered from the methods discussed in this paper.

There are other questions, however, for which we do not need to know the magnitude of a_{12} or a_{21}. As an example of such a question, consider the

[6] Hamilton [15] page 246 condition (b).

problem faced by a central banker whose goal is to maximize the value of an objective function,

$$J = E_t \sum_{s=t}^{\infty} \beta^{s-t} W\left(y_{1s}, y_{2s}\right). \tag{9}$$

The parameter β is a discount factor and W is a concave function that represents the central banker's aversion to fluctuations in unemployment, y_{1t} and inflation, y_{2t}. We assume that the structure of the economy is given by Equation (5).

To facilitate our exposition, we will partition the structural equations in the following way. First we separate the endogenous variables into two blocks

$$Y_t = \begin{bmatrix} \underset{2\times 1}{Y_{1t}} \\ \underset{1\times 1}{y_{3t}} \end{bmatrix}, \quad Y_{1t} = \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}.$$

We will refer to Y_{1t} as the *structural variables* to distinguish them from y_{3t} that we call the *policy variable*. Second we partition the coefficients in the matrices A and B conformably,

$$\begin{bmatrix} 1 & a_{12} & a_{13} \\ a_{21} & 1 & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} = \begin{bmatrix} \underset{2\times 2}{\mathbf{A}_{11}} & \underset{2\times 1}{A_{13}} \\ \underset{1\times 2}{A_{31}} & \underset{1\times 1}{1} \end{bmatrix}, \quad \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} = \begin{bmatrix} \underset{2\times m}{\mathbf{B}_1} \\ \underset{1\times m}{B_3} \end{bmatrix}.$$

Using this notation we can represent the unemployment and the inflation equations as follows:

$$\overset{\mathbf{A}_{11}}{\begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}} \overset{Y_{1t}}{\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}} + \overset{A_{13}}{\begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}} y_{3t} = \overset{\mathbf{B}_1}{\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}} Z_{t-1} + \overset{U_{1t}}{\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}}, \tag{10}$$

and we can write the policy rule as

$$\overset{A_{31}}{\begin{bmatrix} a_{31} & a_{32} \end{bmatrix}} \overset{Y_{1t}}{\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}} + y_{3t} = B_3 Z_{t-1} + u_{3t}. \tag{11}$$

The problem of the policy maker is to design a rule by picking $2+m$ parameters $\{a_{31}, a_{32}, B_3\}$ to maximize the value of (9) given that the structural variables are related to the policy variable by the structural equations (10). We do not take a stand on the origin of policy "shocks". Since the objective function is concave, the optimal policy will set $\{u_{3t} = 0\}$ for all t. In practice, shocks to the observed policy rule may arise from measurement error or from the reaction of the policy maker to variables that are unobserved by the econometrician.

Within this structure we ask the question: What does the policy maker need to know about the structure of the economy in order to implement an optimal policy? The answer to this question leads us to define a class of models that we call the *recoverable structure*.

Definition 1 (Recoverable Structure) *Let y_{3t} be a policy variable and let Y_{1t} be a 2×1 vector of structural variables. Let the structural variables be governed by the equations*

$$\overset{\mathbf{A}_{11}}{\begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}} \overset{Y_{1t}}{\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}} + \overset{A_{13}}{\begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}} y_{3t} = \overset{\mathbf{B}_1}{\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}} Z_{t-1} + \overset{U_{1t}}{\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}}, \qquad (12)$$

where Z_{t-1} is an $m \times 1$ vector of predetermined variables. The recoverable structure is a pair of equations,

$$\overset{Y_t^1}{\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}} + \overset{\bar{A}_1}{\begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \end{bmatrix}} y_{3t} = \overset{\bar{\mathbf{B}}_1}{\begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}} Z_{t-1} + \overset{\bar{U}_{1t}}{\begin{bmatrix} \bar{u}_{1t} \\ \bar{u}_{2t} \end{bmatrix}}, \qquad (13)$$

that governs the time series evolution of Y_{1t} as a function of Z_{t-1}, the policy variable y_{3t} and the structural shocks $\{u_{1t}, u_{2t}\}$. The $2(1+m)$ coefficients $\{\bar{a}_1, \bar{a}_2, \bar{B}_1, \bar{B}_2\}$ of the recoverable structure are related to the coefficients of the structural model by the following relationships

$$\begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \end{bmatrix} = \begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix},$$

$$\begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix} = \begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$

The shocks to the recoverable structure are related to the structural shocks by the equations,

$$\bar{U}_{1t} = \begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}^{-1} U_{1t}.$$

The recoverable structure is midway between the reduced form and structural form. The following proposition establishes that the recoverable structure summarizes the information about the structure of the economy that the policy maker needs to know in order to form an optimal policy.

Proposition 1 *Consider the problem of a policy maker with objective function*

$$J = E_t \sum_{s=t}^{\infty} \beta^{s-t} W\left(y_{1s}, y_{2s}\right) \tag{14}$$

who controls y_{3t} *by choosing a rule from the class*

$$\overbrace{\left[\begin{array}{cc} a_{31} & a_{32} \end{array}\right]}^{A_{31}} \overbrace{\left[\begin{array}{c} y_{1t} \\ y_{2t} \end{array}\right]}^{Y_{1t}} + y_{3t} = B_3 Z_{t-1} + u_{3t}. \tag{15}$$

To solve this problem it is necessary and sufficient that the policy maker knows the parameters of the recoverable structure.

The proof of this proposition is immediate since to maximize (14) it is necessary and sufficient that the policy maker knows the equation that describes the evolution of Y_{1t} as a function of the control variable y_{3t}. This, by definition, is the recoverable structure.

In Section 4 we established that the structural equations (12) are not identified. Proposition 1 implies that the policy maker does not need to know these equations in order to design an optimal policy. The minimal information that he or she requires is summarized by Equation (13), an Equation that we refer to as the recoverable structure. But are the parameters of the recoverable structure identified when there is a break in the policy rule? To answer this question, consider the following representation of the model in which the structural equations (10) are replaced by the recoverable structure, (12),

$$\overbrace{\left[\begin{array}{c} y_{1t} \\ y_{2t} \end{array}\right]}^{Y_{1t}} + \overbrace{\left[\begin{array}{c} \bar{a}_1 \\ \bar{a}_2 \end{array}\right]}^{\bar{A}} y_{3t} = \overbrace{\left[\begin{array}{c} \bar{B}_1 \\ \bar{B}_2 \end{array}\right]}^{\bar{\mathbf{B}}_1} Z_{t-1} + \overbrace{\left[\begin{array}{c} \bar{u}_{1t} \\ \bar{u}_{2t} \end{array}\right]}^{\bar{U}_{1t}}, \tag{16}$$

$$\overbrace{\left[\begin{array}{cc} a_{31} & a_{32} \end{array}\right]}^{A_{31}} \overbrace{\left[\begin{array}{c} y_{1t} \\ y_{2t} \end{array}\right]}^{Y_{1t}} + y_{3t} = B_3 Z_{t-1} + u_{3t}. \tag{17}$$

Since Equations (16) and (17) have the same reduced form as the original system, they are an equivalent way of representing the model. Suppose that at date T_1 the policy rule changes from $\{a^1_{31}, a^1_{32}, B^1_3\}$ to $\{a^2_{31}, a^2_{32}, B^2_3\}$ and define the step variables Y^S_t and Z^S_t as in Section 3. Define the *augmented recoverable structure* as follows

$$\overbrace{\begin{bmatrix} 1 & 0 & \bar{a}_1 & 0 & 0 & 0 \\ 0 & 1 & \bar{a}_2 & 0 & 0 & 0 \\ a^1_{31} & a^1_{32} & 1 & a^\Delta_{31} & a^\Delta_{32} & 0 \\ 0 & 0 & 0 & 1 & a_{12} & a_{13} \\ 0 & 0 & 0 & a_{21} & 1 & a_{23} \\ 0 & 0 & 0 & a^2_{31} & a^2_{32} & 1 \end{bmatrix}}^{\bar{\mathbf{A}}^*} \overbrace{\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y^S_{1t} \\ y^S_{2t} \\ y^S_{3t} \end{bmatrix}}^{Y^*_t} = \overbrace{\begin{bmatrix} \bar{B}_1 & 0 \\ \bar{B}_2 & 0 \\ B^1_3 & B^\Delta_3 \\ 0 & \bar{B}_1 \\ 0 & \bar{B}_2 \\ 0 & B^2_3 \end{bmatrix}}^{\bar{\mathbf{B}}^*} \overbrace{\begin{bmatrix} Z_{t-1} \\ Z^S_{t-1} \end{bmatrix}}^{Z^*_{t-1}} + \overbrace{\begin{bmatrix} \bar{u}_{1t} \\ \bar{u}_{2t} \\ u_{3t} \\ \bar{u}^S_{1t} \\ \bar{u}^S_{2t} \\ u^S_{3t} \end{bmatrix}}^{\bar{U}^*_t}. \tag{18}$$

We put a bar over the matrices $\bar{\mathbf{A}}^*$, $\bar{\mathbf{B}}^*$ of the augmented recoverable structure to distinguish them from $\mathbf{A}^*$, and $\mathbf{B}^*$ of the augmented structural model. The first and second rows of Equation (18) represent the recoverable structure. The order condition for identification of each of these equations is satisfied since there is a single endogenous variable in each equation and there are m excluded exogenous variables. The rank conditions for identification require that the matrices $\mathbf{M}^{1*}$ and $\mathbf{M}^{2*}$

$$\mathbf{M}^{1*} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ B^\Delta_3 & a^1_{32} & a^\Delta_{31} & a^\Delta_{32} & 0 \\ B_1 & 0 & 1 & a_{12} & a_{13} \\ B_2 & 0 & a_{21} & 1 & a_{23} \\ B^2_3 & 0 & a^2_{31} & a^2_{32} & 1 \end{bmatrix}, \quad \mathbf{M}^{2*} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ B^\Delta_3 & a^1_{31} & a^\Delta_{31} & a^\Delta_{32} & 0 \\ B_1 & 0 & 1 & a_{12} & a_{13} \\ B_2 & 0 & a_{21} & 1 & a_{23} \\ B^2_3 & 0 & a^2_{31} & a^2_{32} & 1 \end{bmatrix}$$

have full row rank. A sufficient condition for $\mathbf{M}^{1*}$, and $\mathbf{M}^{2*}$ to have full rank is that at least one coefficient in the policy rule changes, that is, if $B^\Delta_3, a^\Delta_{31}$ or a^Δ_{32} is non-zero. It follows that the policy maker can, in principle, learn the recoverable structure by changing the policy rule. Since the recoverable structure is the minimal information required to design an optimal policy it follows that optimal policy design is feasible.

6 A Model with Rational Expectations

In this section we introduce a three-equation model in which agents are forward looking and have rational expectations. We will show how to fit this model into the same framework developed in the first part of the paper by defining an additional set of three endogenous variables; these are the expectations of future values $E_t[Y_{t+1}]$ that are determined at date t.

We continue to use Y_t to represent the current values of unemployment, y_{1t}, inflation, y_{2t}, and the interest rate, y_{3t}, and we introduce the notation X_t to represent the vector of six endogenous variables augmented to include expectations,

$$\underset{3\times 1}{Y_t} = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix}, \quad \underset{6\times 1}{X_t} = \begin{bmatrix} Y_t \\ E_t[Y_{t+1}] \end{bmatrix}.$$

We will consider a model in the class

$$\mathbf{A}Y_t + \mathbf{F}E_t[Y_{t+1}] = \sum_{i=1}^{k} \mathbf{B}_i Y_{t-i} + \mathbf{\Phi} C + U_t,$$

which we write more compactly as

$$\begin{bmatrix} \mathbf{A} & \mathbf{F} \end{bmatrix} \begin{bmatrix} Y_t \\ E_t[Y_{t+1}] \end{bmatrix} = \mathbf{B} Z_{t-1} + U_t,$$

where the matrix $\mathbf{B}$ and the predetermined variables Z_{t-1} are defined below. The term C represents a constant and $\mathbf{F}$ is a 3×3 matrix of coefficients that represents the effect of future expectations on each of the three structural equations.

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 & \cdots & \mathbf{B}_k & \mathbf{\Phi} \end{bmatrix}, \quad Z_{t-1} = \begin{bmatrix} Y_{t-1} & \cdots & Y_{t-k} & C \end{bmatrix}^T.$$

We partition the variables Y_t, and Z_{t-1} and the coefficient matrices $\mathbf{A}$ and $\mathbf{B}$ as in Section 5 and we introduce additional terms to describe how expectations enter the partitioned system. These terms are defined below.

$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} = \begin{bmatrix} \underset{2\times 3}{\mathbf{F}_1} \\ \underset{1\times 3}{F_3} \end{bmatrix} = \begin{bmatrix} \underset{1\times 3}{F_{11}} \\ \underset{1\times 3}{F_{12}} \\ \underset{1\times 3}{F_3} \end{bmatrix},$$

$$\mathbf{F}_1 = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \end{bmatrix}, \quad F_3 = \begin{bmatrix} f_{31} & f_{32} & f_{33} \end{bmatrix},$$

$$F_{11} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \end{bmatrix}, \quad F_{12} = \begin{bmatrix} f_{21} & f_{22} & f_{23} \end{bmatrix}.$$

Using this notation we write the structural equations as,

$$\overset{A_{11}}{\begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}} \overset{Y_{1t}}{\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}} + \overset{\mathbf{A}_{13}}{\begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}} y_{3t} \tag{19}$$
$$+ \overset{\mathbf{F}_1}{\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \end{bmatrix}} E_t\left[Y_{t+1}\right] = \overset{\mathbf{B}_1}{\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}} Z_{t-1} + \overset{U_{1t}}{\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}},$$

and we write the policy rule as,

$$\overset{\mathbf{A}_{31}}{\begin{bmatrix} a_{31} & a_{32} \end{bmatrix}} \overset{Y_{1t}}{\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}} + y_{3t} \tag{20}$$
$$+ \overset{F_3}{\begin{bmatrix} f_{31} & f_{32} & f_{33} \end{bmatrix}} E_t\left[Y_{t+1}\right] = B_3 Z_{t-1} + u_{3t}.$$

This model is conceptually no different from the one we have already studied. Its reduced form is found by writing the model in companion form and eliminating the influence of unstable roots. In Appendix A, we show how to write the model in this way.

Sims ([33]) provides Matlab code to find the solution for a model written in companion form. There are three possible cases to consider when deriving this solution. Case (1) is that there exists a unique equilibrium. In this case the Sims code delivers matrices $\mathbf{\Gamma}_1$, $\mathbf{\Gamma}_2$ and $\mathbf{\Theta}$ such that[7]

$$\begin{bmatrix} Y_t \\ E_t\left[Y_{t+1}\right] \end{bmatrix} = \begin{bmatrix} \mathbf{\Gamma}_1 \\ \mathbf{\Gamma}_2 \end{bmatrix} \mathbf{Z}_{t-1} + W_t$$

and

$$W_t = \mathbf{\Theta U}_t.$$

It also possible that (2) there is no stationary equilibrium or (3) there are multiple stationary indeterminate equilibria. If there are multiple equilibria, the matrices $\mathbf{\Gamma}_1$, $\mathbf{\Gamma}_2$ and $\mathbf{\Theta}$ returned by the Sims Matlab algorithm are associated with one of these equilibria.[8] In the case that no equilibrium exists one should infer that the model has been mispecified.

[7] More precisely, the Sims code delivers matrices for the system written in companion form. The matrices $\mathbf{\Gamma}$ and $\mathbf{\Theta}$ referred to in the text can be extracted from the companion form by selecting the appropriate rows of the solution.

[8] Lubik and Schorfheide [22] have written a Gauss version of this code that delivers solutions for the complete set of rational expectations solutions.

7 The Recoverable Structure with Expectations

In this section we show how to apply our identification procedure to a dynamic structural rational expectations model. The introduction of expectations raises the issue discussed by Robert Lucas ([24]) that the parameters of linear models that exclude expectations can be expected to change in response to a change in policy. We propose to handle this issue by introducing expectations as explanatory variables in the recoverable structure. The following definition modifies the recoverable structure to include these terms.

Definition 2 (Recoverable Structure with Expectations) *Let y_{3t} be a policy variable and let Y_{1t} be a 2×1 vector of structural variables. Let the structural variables be governed by the equations*

$$\overbrace{\begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}}^{\mathbf{A}_{11}} \overbrace{\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}}^{Y_{1t}} + \overbrace{\begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}}^{A_{13}} y_{3t} \tag{21}$$
$$+ \overbrace{\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \end{bmatrix}}^{\mathbf{F}_1} E_t\left[Y_{t+1}\right] = \overbrace{\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}}^{\mathbf{B}_1} Z_{t-1} + \overbrace{\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}}^{U_{1t}},$$

where Z_{t-1} is an $m \times 1$ vector of predetermined variables. The recoverable structure is a pair of equations,

$$\overbrace{\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}}^{Y_{1t}} + \overbrace{\begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \end{bmatrix}}^{\bar{A}} y_{3t} + \overbrace{\begin{bmatrix} \bar{f}_{11} & \bar{f}_{12} & \bar{f}_{13} \\ \bar{f}_{21} & \bar{f}_{22} & \bar{f}_{23} \end{bmatrix}}^{\bar{\mathbf{F}}_1} E_t\left[Y_{t+1}\right] \tag{22}$$
$$= \overbrace{\begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}}^{\bar{\mathbf{B}}_1} Z_{t-1} + \overbrace{\begin{bmatrix} \bar{u}_{1t} \\ \bar{u}_{2t} \end{bmatrix}}^{\bar{U}_{1t}}$$

that describes the dependence of Y_{1t} on the predetermined variables Z_{t-1}, the policy variable y_{3t} , the structural shocks $\{u_{1t}, u_{2t}\}$ and expectations $E_t\left[Y_{t+1}\right]$. The $2(4+m)$ coefficients $\left\{\bar{A}, \bar{\mathbf{F}}_1, \bar{\mathbf{B}}_1\right\}$ of the recoverable structure are related to the coefficients of the structural model by the following relationships

$$\begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \end{bmatrix} = \begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix},$$

$$\begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix} = \begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$

$$\begin{bmatrix} \bar{f}_{11} & \bar{f}_{12} & \bar{f}_{13} \\ \bar{f}_{21} & \bar{f}_{22} & \bar{f}_{23} \end{bmatrix} = \begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \end{bmatrix}.$$

The shocks to the recoverable structure are related to the structural shocks by the equation,

$$\bar{U}_{1t} = \begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}^{-1} U_{1t}.$$

We have defined the recoverable structure to be a linear combination of the unemployment and the inflation equations that orthogonalizes the subspace spanned by these two variables. Since the recoverable structure retains the endogenous expectations as explanatory variables one might reasonably ask: Can we find valid instruments to separately identify the parameters $\bar{\mathbf{F}}_1$? Since each equation contains 4 right-hand-side endogenous variables and since there are m excluded predetermined variables the order condition is satisfied for each equation provided that $m \geq 4$. To check the rank condition we write out the augmented recoverable structure in System (23).

$$\overbrace{\begin{bmatrix} 1 & 0 & \bar{a}_{13} & \underset{1\times3}{\bar{F}_{11}} & 0 & 0 & 0 & \underset{1\times3}{\mathbf{0}} \\ 0 & 1 & \bar{a}_{23} & \underset{1\times3}{\bar{F}_{12}} & 0 & 0 & 0 & \underset{1\times3}{\mathbf{0}} \\ a^1_{31} & a^1_{32} & 1 & \underset{1\times3}{F^1_3} & a^{\Delta}_{31} & a^{\Delta}_{32} & 0 & \underset{1\times3}{F^{\Delta}_3} \\ 0 & 0 & 0 & \mathbf{I}_3 & 0 & 0 & 0 & \underset{3\times3}{\mathbf{0}} \\ 0 & 0 & 0 & \underset{1\times3}{0} & 1 & 0 & \bar{a}_{13} & \underset{1\times3}{\bar{F}_{11}} \\ 0 & 0 & 0 & \underset{1\times3}{0} & 0 & 1 & \bar{a}_{23} & \underset{1\times3}{\bar{F}_{12}} \\ 0 & 0 & 0 & \underset{1\times3}{0} & a^2_{31} & a^2_{32} & 1 & \underset{1\times3}{F^2_3} \\ 0 & 0 & 0 & \underset{3\times3}{\mathbf{0}} & 0 & 0 & 0 & \mathbf{I}_3 \end{bmatrix}}^{\bar{\mathbf{A}}^*} \begin{bmatrix} \underset{1\times1}{y_{1t}} \\ \underset{1\times1}{y_{2t}} \\ \underset{1\times1}{y_{3t}} \\ \underset{3\times1}{E_t\left[Y_{t+1}\right]} \\ \underset{1\times1}{y^S_{1t}} \\ \underset{1\times1}{y^S_{2t}} \\ \underset{1\times1}{y^S_{3t}} \\ \underset{3\times1}{E_t\left[Y^S_{t+1}\right]} \end{bmatrix} = \overbrace{\begin{bmatrix} \underset{1\times m}{\bar{B}_1} & \underset{1\times m}{0} \\ \underset{1\times m}{\bar{B}_2} & \underset{1\times m}{0} \\ \underset{1\times m}{B^1_3} & \underset{1\times m}{B^{\Delta}_3} \\ \underset{3\times m}{\mathbf{\Gamma}^1_2} & \underset{3\times m}{\mathbf{\Gamma}^{\Delta}_2} \\ \underset{1\times m}{0} & \underset{1\times m}{\bar{B}_1} \\ \underset{1\times m}{0} & \underset{1\times m}{\bar{B}_2} \\ \underset{1\times m}{0} & \underset{1\times m}{B^2_3} \\ \underset{3\times m}{\mathbf{0}} & \underset{3\times m}{\mathbf{\Gamma}^2_2} \end{bmatrix}}^{\bar{\mathbf{B}}^*} \begin{bmatrix} \underset{m\times12}{Z_{t-1}} \\ \underset{m\times12}{Z^S_{t-1}} \end{bmatrix} \tag{23}$$

$$+ \begin{bmatrix} \underset{6\times1}{W_t} \\ \underset{6\times1}{W^S_t} \end{bmatrix}$$

We use a bar over the matrices $\bar{\mathbf{A}}^*$ and $\bar{\mathbf{B}}^*$ of the augmented recoverable structure to distinguish them from $\mathbf{A}^*$ and $\mathbf{B}^*$ of the augmented structural system. $\bar{F}_{11}$, and $\bar{F}_{12}$ are the first and second rows of $\bar{\mathbf{F}}_1$; these coefficients represent the effects of future expectations on unemployment and inflation.

Under the assumption that expectations are rational, the blocks of equations that determine expectations are found by eliminating the effects of unstable roots from the system written in companion form as described in Appendix A. These blocks of equations are represented as rows 4 and 8 of the augmented recoverable structure. The parameters $\mathbf{\Gamma}_2^1$ and $\mathbf{\Gamma}_2^2$ are coefficient matrices that represent how expectations depend on the predetermined variables in the rational expectations solution under regimes 1 and 2 . $\mathbf{\Gamma}_2^\Delta$ is the change in these coefficients, i.e., $\mathbf{\Gamma}_2^\Delta = \mathbf{\Gamma}_2^2 - \mathbf{\Gamma}_2^1$.

The following argument establishes that the parameters of the unemployment equation in the recoverable structure are identified. The proof that the parameters of the inflation equation are identified is identical since the recoverable structure is symmetric. Using condition (b) from Hamilton [15] page 246, the rank condition for the unemployment equation requires the matrix

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ B_3^\Delta & a_{32}^1 & a_{31}^\Delta & a_{32}^\Delta & 0 & F_3^\Delta \\ \mathbf{\Gamma}^\Delta & 0 & 0 & 0 & 0 & 0 \\ B_1 & 0 & 1 & 0 & \bar{a}_{13} & \bar{F}_{11} \\ B_2 & 0 & 0 & 1 & \bar{a}_{23} & \bar{F}_{12} \\ B_3^2 & 0 & a_{31}^2 & a_{32}^2 & 1 & F_3^2 \\ \mathbf{\Gamma}^2 & 0 & 0 & 0 & 0 & \mathbf{I}_3 \end{bmatrix}$$

to have full row rank. Hence, it is sufficient for identification that at least one of the parameters $B_3^\Delta, a_{31}^\Delta, a_{32}^\Delta$ or F_3^Δ is non-zero.

One might think that the inclusion of three additional endogenous variables would cause difficulties for identification but this is not the case. The effects of endogenous expectations on unemployment are identified because expectations are determined by the reduced form of the entire system and they are, therefore, correlated with the step variables. These variables act as instruments to disentangle the effects of expectations on unemployment from the contemporaneous effects of the interest rate.

Although it is possible to identify how expectations influence unemployment and inflation in the recoverable structure, it is not possible to make a similar argument for the policy equation. Once again, this equation is not identified because, if all of the parameters of the policy rule can change across periods, there are no valid instruments to distinguish changes in expectations from changes in the coefficients of the predetermined variables.

8 A Description of the Data

In this section we turn to an application of our proposed method of identification of a recoverable structure. Our technique requires that one first identifies a structural break in the reduced form of an econometric model. In previous work [4] we studied the behavior of the federal funds rate, i_t, the inflation rate π_t (measured by the annualized rate of change of the quarterly GDP deflator) and the quarterly unemployment rate, u_t. Figure 1 graphs these data for the U.S. economy for the period from 1970:Q1 through 1999:Q3.

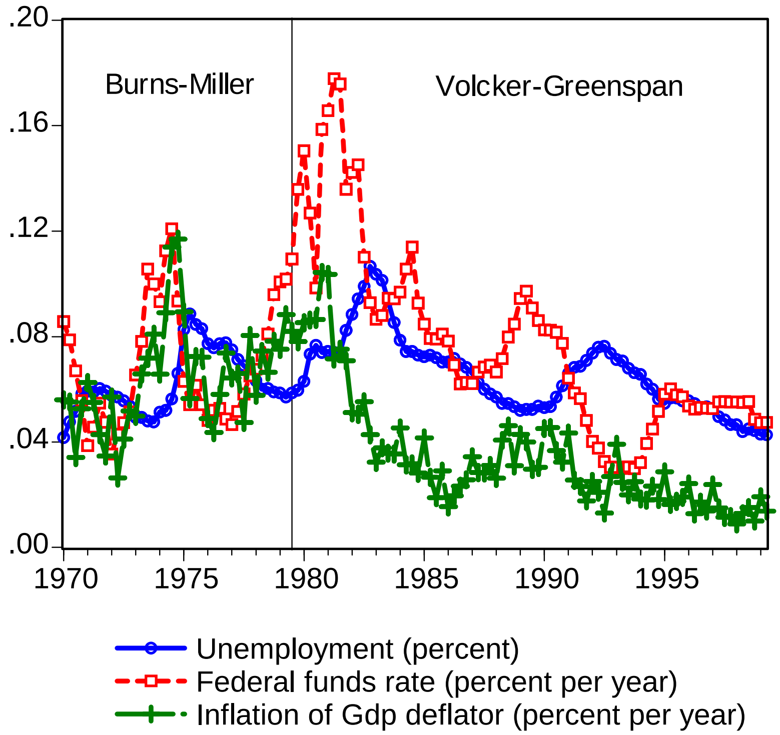

Figure 1: Unemployment, the Federal Funds Rate, and Inflation

In our earlier paper we were unable to fit a stable parameter VAR to data from the entire period. Instead, we found evidence of a structural break in 1979:Q3, a date that coincides with the commencement of Paul Volcker's term as chairman of the Fed. The vertical line in Figure 1 occurs at the date of this break.

Our previous work uncovered evidence that the data are non-stationary but cointegrated. Since non-stationary data may take any value in the real line, we transformed the data by taking a logistic transform of the unemployment rate and the logarithm of the federal funds rate. The transformed variables are defined as follows,

$$y_{1t} = \ln\left(\frac{u_t}{1-u_t}\right), \quad y_{2t} = \pi_t, \quad y_{3t} = \ln(i_t). \tag{24}$$

The domains of the variables u_t, and i_t are

$$u_t \in [0,1], \quad i_t \in [0,+\infty],$$

and the transformations used in (24) map these variables into R. To check the robustness of our results we estimated our model with both transformed and untransformed data and found that our results with both data sets are very close in all important respects.

9 Estimation Method

In this section we discuss the methods used to estimate the model. Since our preliminary tests could not reject the presence of a unit root in all three series, we estimated the model using differenced data and we included a vector of lagged levels on the right-hand-side of each equation. A careful analysis of the low frequency properties of the data for both sub-periods is documented in our working paper [4]. One of the key results in this paper is that the unemployment rate, the inflation rate and the federal funds rate are linked by two cointegrating vectors. One is stable across two policy regimes; the other has shifting coefficients.

It is well known that statistical inference in models with integrated or non-stationary variables is not straightforward. Chi square, F- and t-tests are often only valid when certain conditions apply, e.g. when integrated variables are cointegrated (see e.g. Banerjee et al [1], Johansen [18], Sims et al [35] or Watson [38]). To address this issue we estimated the model in first differences and we modified our identification procedure by using two different instrument sets. In method 1 we chose Z_{t-1}. as follows,

$$Z_{t-1} = \left\{DY_{t-1}, DY_{t-2}, Y_{t-1}, DY^S_{t-1}, DY^S_{t-2}, Y^S_{t-1}, C, C^S\right\},$$

where D is the first difference operator and C^S is a step constant. This method includes the full set of lagged levels as right-hand-side variables and estimating the model in this way is equivalent to estimating the model in levels. Appendix B explains the mapping between these equivalent representations.

In method 2 we excluded non-stationary variables from the set of explanatory variables Z_{t-1} by allowing level information to enter the system only through the effects of a set of estimated cointegrating vectors. In this method we chose

$$Z_{t-1} = \left\{ DY_{t-1}, DY_{t-2}, DY_{t-1}^S, DY_{t-2}^S, CI_{t-1} \right\},$$

where

$$\underset{4\times 1}{CI_{t-1}} = \beta^T \begin{bmatrix} \underset{3\times 1}{Y_{t-1}} \\ \underset{3\times 1}{Y_{t-1}^S} \end{bmatrix},$$

and β^T is a 4×6 vector of cointegrating vectors defined as

$$\beta^T = \begin{bmatrix} y_1 & y_2 & y_3 & y_1^S & y_2^S & y_3^S \\ 1 & -3.91 & -0.41 & 0 & 0 & 0 \\ 0 & -11.57 & 1 & 0 & -4.95 & 0 \\ 0 & 0 & 0 & 1 & -3.91 & -0.41 \\ 0 & 0 & 0 & 0 & -16.52 & 1 \end{bmatrix}.$$

The method we used to normalize the cointegrating space is described in Appendix C. We estimated the cointegrating vectors using Johansen's procedure described in [17] and implemented in Pc-Give [9].

Since we do not observe the expectations $E_t\left[Y_{t+1}\right]$, we replaced these terms with their realizations Y_{t+1} and we added an additional set of error terms $\mathcal{N}_{t+1}$to represent the expectational errors. Equation (25) represents the recoverable structure written in this way.

$$\overset{DY_{1t}}{\begin{bmatrix} Dy_{1t} \\ Dy_{2t} \end{bmatrix}} = -\overset{\bar{A}}{\begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \end{bmatrix}} Dy_t^3 - \overset{\bar{\mathbf{F}}_1}{\begin{bmatrix} \bar{f}_{11} & \bar{f}_{12} & \bar{f}_{13} \\ \bar{f}_{21} & \bar{f}_{22} & \bar{f}_{23} \end{bmatrix}} E_t\left[DY_{t+1}\right] \tag{25}$$
$$+ \overset{\bar{\mathbf{B}}_1}{\begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}} Z_{t-1} + \overset{\bar{U}_{1t}}{\begin{bmatrix} \bar{u}_{1t} \\ \bar{u}_{2t} \end{bmatrix}} + \overset{\mathcal{N}_{1t+1}}{\begin{bmatrix} \eta_{1t+1} \\ \eta_{2t+1} \end{bmatrix}},$$

Y_{1t} is the vector $(y_{1t}, y_{2t})^T$ where y_{1t} is the logistic transform of the unemployment rate, y_{2t} is the inflation rate and y_{3t} is the logarithm of the federal funds rate. The term $\mathcal{N}_{1t+1}$ is defined as

$$\mathcal{N}_{1t+1} = \bar{\mathbf{F}}_1 \left(E_t \left[DY_{1t+1} \right] - DY_{1t+1} \right).$$

Under the rational expectations hypothesis, if there exists a unique rational expectations equilibrium, these errors will be linear functions of the fundamental errors, U_{t+1}, hence Equation (25) is predicted to have a moving average error component. To account for this serial dependence in the residuals we estimated the model with generalized method of moments (GMM) using Eviews 4.0 [10]. If there are multiple rational expectations equilibria, the errors $\mathcal{N}_{1t+1}$ may contain terms that are independent of U_{t+1}. Since GMM can in principle allow for flexible serial correlation in the residuals, our estimation method should be robust to the existence of indeterminacy.

10 Non Identification of the Policy Rule

For the reasons discussed in Section 5, the policy rule is not identified in our framework. Instead, we estimated an equation of the form

$$Dy_{3t} = \Gamma_3 Z_{t-1} + \Gamma_3^{\Delta} Z_{t-1}^S + w_{3t}. \tag{26}$$

We view Equation (26) as a reduced form interest-rate-equation that is a linear combination of the true policy rule and the private sector recoverable structure. There are many possible policy rules that are consistent with our estimates. In some of these rules the government responds contemporaneously to inflation and unemployment, in others it responds to expectations of their future values. All of these rules lead to the same time series behavior for Y_t and an observer using our identification procedure would not be able to distinguish between them.

Suppose that the true policy rule is in the class

$$\begin{aligned} A_{31} DY_{1t} + Dy_{3t} + F_3 E_t \left[DY_{t+1} \right] + A_{31}^{\Delta} DY_{1t}^S \\ + F_3^{\Delta} E_t \left[DY_{t+1}^S \right] = B_3 Z_{t-1} + B_3^{\Delta} Z_{t-1}^S + u_{3t}, \end{aligned} \tag{27}$$

but we estimate Equation (26). Lack of identification implies that there is a set of possible values for $\mathbf{A}_{31}, F_3, \mathbf{A}_{31}^{\Delta}, F_3^{\Delta}, B_3$ and B_3^{Δ} all of which are consistent with the same recoverable structure and with the same reduced form coefficients Γ_3 and Γ_3^Δ. We cannot tell which policy was followed in practise, but we *can* find a set of policies that would have led to the same observable behavior for Y_t as the true policy. The simplest member of this class is the policy

$$\begin{aligned} A_{31} &= 0, \quad A_{31}^\Delta = 0, \\ F_3 &= 0, \quad F_3^\Delta = 0, \\ B_3 &= \Gamma_3, \quad B_3^\Delta = \Gamma_3^\Delta. \end{aligned}$$

11 Two Methods for Identifying the Recoverable Structure

In this section we discuss two different methods for estimating the recoverable structure using alternative instrument sets. To exactly identify these equations we need four instruments in each structural equation since each equation of the recoverable structure contains four right-hand-side endogenous variables. These are the 3×1 vector of expectation terms, $E_t [DY_{t+1}]$ plus the contemporaneous interest rate Dy_{3t}. It follows that exact identification can be achieved by excluding four predetermined variables.

Under method 1 the instruments available to identify the endogenous variable are the 10 excluded predetermined variables

$$Z_{t-1}^S = \left\{ \underset{3\times1}{DY_{t-1}^S}, \underset{3\times1}{DY_{t-2}^S}, \underset{3\times1}{Y_{t-1}^S}, C^S \right\}.$$

Each of these variables can be used to form a moment condition in GMM with both of the structural equations. Since there are four endogenous variables in each equation we have twelve overidentifying restrictions for the system as a whole.

Under method 2 there are also 10 available instruments. These are the 7 step variables

$$Z_{t-1}^S = \left\{ \underset{3\times1}{DY_{t-1}^S}, \underset{3\times1}{DY_{t-2}^S}, C^S \right\}.$$

plus the cointegrated variables

$$\{CI_{2t-1}, CI_{3t-1}, CI_{4t-1}\},$$

where the cointegrating relationships are defined as follows

$$\begin{bmatrix} CI_{1t-1} \\ CI_{2t-1} \\ CI_{3t-1} \\ CI_{4t-1} \end{bmatrix} = \begin{bmatrix} 1 & -3.91 & -0.41 & 0 & 0 & 0 \\ 0 & -11.57 & 1 & 0 & -4.95 & 0 \\ 0 & 0 & 0 & 1 & -3.91 & -0.41 \\ 0 & 0 & 0 & 0 & -16.52 & 1 \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \\ y_{3t-1} \\ y_{1t-1}^S \\ y_{2t-1}^S \\ y_{3t-1}^S \end{bmatrix}.$$

Since the parameters of the recoverable structure are assumed to be constant, Y_{t-1}^S cannot enter these equations. The following proposition establishes that this property, which is an implication of a stable-parameter recoverable structure, implies that the variables CI_{2t-1}, CI_{3t-1} and CI_{4t-1} cannot enter the recoverable structure.

Proposition 2 *Let* $\underset{1\times 6}{\Pi_1} = \begin{bmatrix} \underset{1\times 3}{\Pi_{11}} & \underset{1\times 3}{\Pi_{12}} \end{bmatrix}$ *and* $\underset{1\times 6}{\Pi_2} = \begin{bmatrix} \underset{1\times 3}{\Pi_{21}} & \underset{1\times 3}{\Pi_{22}} \end{bmatrix}$ *represent the coefficients of* $\left\{ \underset{3\times 1}{Y_t}, \underset{3\times 1}{Y_{t-1}^S} \right\}$ *in the two equations of the recoverable structure, and decompose these vectors into two* 1×4 *vectors of loading factors,* α_1 *and* α_2 *and the* 4×6 *matrix of cointegrating vectors* β^T*:*

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \end{bmatrix} \begin{bmatrix} 1 & -3.91 & -0.41 & 0 & 0 & 0 \\ 0 & -11.57 & 1 & 0 & -4.95 & 0 \\ 0 & 0 & 0 & 1 & -3.91 & -0.41 \\ 0 & 0 & 0 & 0 & -16.52 & 1 \end{bmatrix}.$$

If the structural coefficients are stable across regimes then

$$\begin{bmatrix} \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{22} & \alpha_{23} & \alpha_{24} \end{bmatrix} = \mathbf{0}.$$

Proof. Consider the unemployment equation that has loading coefficients $\alpha_1 = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \end{bmatrix}$ (the proof for the inflation equation is the same). The fact that Y_{t-1}^S does not enter the unemployment equation implies that

$$\Pi_{12} = 0.$$

But

$$\Pi_{12} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -4.95 & 0 \\ 1 & -3.91 & -0.41 \\ 0 & -16.52 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1\times 3 \end{bmatrix}, \tag{28}$$

which can be decomposed as

$$[\alpha_{11}] \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha_{12} & \alpha_{13} & \alpha_{14} \end{bmatrix} \begin{bmatrix} 0 & -4.95 & 0 \\ 1 & -3.91 & -0.41 \\ 0 & -16.52 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1\times 3 \end{bmatrix}. \tag{29}$$

Equation (29) implies

$$\begin{bmatrix} \alpha_{12} & \alpha_{13} & \alpha_{14} \end{bmatrix} \begin{bmatrix} 0 & -4.95 & 0 \\ 1 & -3.91 & -0.41 \\ 0 & -16.52 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1\times 3 \end{bmatrix}. \tag{30}$$

Since the submatrix

$$\begin{bmatrix} 0 & -4.95 & 0 \\ 1 & -3.91 & -0.41 \\ 0 & -16.52 & 1 \end{bmatrix}$$

has full rank equation, Equation (30) implies that

$$\begin{bmatrix} \alpha_{12} & \alpha_{13} & \alpha_{14} \end{bmatrix} = \mathbf{0}.$$

Since an equivalent argument holds for the inflation equation, the recoverable structure can contain only the stable cointegrated variable, CI_{1t-1}. ■

Since the cointegrating variables CI_{1t-1}, CI_{2t-2} and CI_{3t-1} are linear combinations of both Y_{t-1} and Y_{t-1}^S, none of these variables can enter the first two equations. In contrast, the variable CI_{1t-1} is a linear combination of Y_{t-1} and Y_{t-1}^S that puts zero weight on the step variables Y_{t-1}^S; this vector *can* therefore enter the structural equations under the null hypothesis of structural stability. We chose to normalize the cointegrating space in this way exactly for the reason that it implies that the parameter restrictions for a stable structure can be represented as simple exclusion restrictions. This issue is discussed further in Appendix C.

12 The Results of Our Estimation Procedure

This section reports the results of our tests of overidentification using GMM to estimate the recoverable structure with two different instruments sets. Under GMM there are a number of options for estimation of the VCV matrix. We experimented with several methods. The results reported in the paper uses a sequential method to simultaneously update estimates of the covariance matrix and estimates of the coefficients. Using one-step methods led to less precise estimates with a higher residual sum of squares, but essentially similar results.[9]

Estimation method	Test statistic	Distribution	P-value
(1) GMM with unrestricted levels	J = (.04) x 118 = 4.72	Chi-squared with 12 degrees of freedom	97%
(2) GMM with cointegrating vectors	J = (.06) x 118 = 6.61	Chi-squared with 12 degrees of freedom	88%

Table 1: J-Statistics for the Recoverable Structure

Point estimates of the parameters are presented in Appendix D. Table 1 reports tests using both identification schemes for the overidentifying restrictions based on Hansen's J-statistic. If the data are stationary, the J-statistic times the number of observations is asymptotically chi-squared with 12 degrees of freedom. For the model with unrestricted levels this statistic is equal to 4.72 which has a p-value of 97% under the null-hypothesis that the data was generated by a stable parameter structural rational expectations model. Since there is reason to doubt that the data are stationary, we also report the results of our estimates in which level information is restricted to enter

[9]We are currently working on two additional estimation strategies that we will report in a future paper. One is to use the Kalman Filter to generate the likelihood function as described in Chapter 13 of Hamilton [15]. The second is to implement a procedure suggested by Kitamura and Phillips [19] that is reported to have better properties than GMM when the data has a near unit root. We find it unlikely that these refinements are likely to modify our main conclusion; that a generic linear rational expectations model provides a good description of the data. However, since the proposed techniques are more efficient than GMM, they are likely to enable us to discriminate more finely between competing refinements of this generic linear model.

the system through the cointegrating vectors. For the model estimated in this way, the corresponding statistic is equal to 6.61 which has a p-value of 88%.

Our failure to reject the overidentifying restrictions implies that the coefficients of the recoverable structure remain constant across the two subsamples. We conclude that the restriction of a stable recoverable structure is not rejected by the data.

13 Determinacy of Equilibrium

Following a paper by Clarida, Gali and Gertler [8] (CGG), there has been considerable discussion in the literature over the uniqueness of equilibrium under alternative policy regimes. CGG estimated a forward looking policy rule using GMM and found evidence that the interest rate had responded much more aggressively to inflation in the Volcker-Greenspan policy regime than under Burns-Miller. In a two-step approach, they took their estimate of the policy rule and plugged it into a calibrated structural model. CGG found that the estimated policy rule under the Burns-Miller policy regime led to an indeterminate equilibrium in their calibrated model. They inferred that, under Burns-Miller, there was a potential for sunspot shocks to add additional instability to the economy. In contrast, their estimate of the Volcker-Greenspan policy rule implied that equilibrium in the calibrated model was unique.

A number of authors have criticized Clarida-Gali-Gertler for their two-step approach. For example, Lubik and Schorfheide [23] point out that identification is a system property and that the validity of the instruments used to identify the policy rule cannot be guaranteed in a partial equilibrium model. Lubik and Schorfheide correct for simultaneity by specifying a structural model and estimating the parameters of the policy rule simultaneously with those of the structure. Using a system estimator, they confirm the CGG result. We find, in contrast, that our point estimates of the parameters for the GMM estimates lead to a unique equilibrium in both regimes.

# Determinate equilibria in 1000 draws	Existence	Uniqueness	Uniqueness conditional on existence
Burns-Miller	598	478	80%
Volcker-Greenspan	672	599	89%

Table 2: Frequency of Existence and Uniqueness in 1000 Simulations

To check the robustness of this finding we drew a sample of 1000 parameter vectors from a normal distribution with a mean and variance estimated by GMM. The means of our random parameter vectors are those reported in Appendix D using estimation method 1. The VCV matrix of the parameters was estimated in Eviews by GMM. For each random draw from the parameter distribution we used an algorithm by Chris Sims,[10] [33] that calculates the solution to a linear rational expectations model. Table 2 reports our findings. In 1000 draws from the parameter vector we found existence of an equilibrium in 598 cases in regime 1 and 672 cases in regime 2. Conditional on existence, 80% of the equilibria in regime 1 and 89% in regime 2 were determinate.

In contrast to our result, Lubik and Schorfheide (LS) report an indeterminate solution for a sub-sample of data running from 1960:Q1 to 1979:Q2. There are a number of differences between the LS study and ours that may explain this discrepancy. First, we used different data. LS used HP filtered GDP, we used unemployment. Second, we studied different data periods; our sample starts in 1970:Q1, theirs in 1960:Q1. Finally, our identification assumptions vary. LS used exclusion restrictions implied by a structural model; we used a policy break to identify the structure.

14 Conclusions

The main idea in this paper is that one can use structural change in one part of the economy to learn something about parts of the economy that do not

[10]These results were obtained with an implementation of Sims algorithm by Lubik and Schorfheide, and an implementation of the QZ decomposition in Gauss due to Paul Soderlind. Schorfheide's code is available at http://www.econ.upenn.edu/~schorf/. Soderlind's code is available at http://www.hhs.se/personal/Psoderlind/. We are indebted to Frank Schorfheide for quickly correcting a bug in an earlier version his program.

change. We provided an example of our approach by studying US monetary policy from 1970 through 1999. We argued that there was a significant break in 1979:Q3 when Chairman Volcker took over the helm from G. William Miller. The regimes of Arthur Burns and that of William Miller can be modeled as a single stable parameter rule. So can the regimes of Paul A. Volcker and that of Alan Greenspan. But the Burns-Miller and Volcker-Greenspan regimes are very different from each other. Given the fact that there was a change in the way that the Fed conducted policy we showed how to use this information to identify some of the parameters of a structural rational expectations model. We refer to the set of parameters that can be identified as the recoverable structure.

Our empirical results provide strong evidence in support of a forward looking rational expectations model with a stable structure. They are fully consistent with the Lucas critique which is a criticism of the parameters of a structural model that does not include future expectations. By including expectations as explanatory variables we were able to recover the "deep parameters" of the structure without imposing incredible identifying assumptions. Our approach can be pursued in a number of directions, some of which we discuss briefly below.

First, our parameter estimates have implications for simple New-Keynesian models of the monetary transmission mechanism. It is not possible to identify the parameters of a New-Keynesian Phillips curve or an "optimization based IS curve" but it *is* possible to ask if existing theories are consistent with our estimates of the recoverable structure. We have begun to explore parsimonious representations of the data and we hope to report further on these explorations in a future working paper. Our initial work suggests that theories of the inflation process that include expectations of future inflation are likely to be successful, in contrast to a number of existing studies that place weight on lagged inflation as the prime explanatory variable in the inflation process. Our initial results also raise some puzzles. In particular, we find that expectations of the future nominal interest rate are highly significant in the inflation equation. We know of no existing models that include this channel, although it is not hard to think of reasons why it might be important.

Second, our work holds out the hope, long since abandoned in many circles, of using optimal control techniques to design an optimal monetary policy. Since the equations of the recoverable structure are truly structural objects, they can be expected to remain invariant to changes in regime. Unlike existing models in this area that often begin with arbitrary identifying assumptions, our approach assumes only that there exists *some* stable structure in which behavior is characterized by agents who use forward looking expectations in forming their optimal decisions. We are pursuing this idea in our current research and we hope to report further on our results in a future working paper.

Appendix A

Consider the model

$$\mathbf{A}Y_t = \sum_{i=1}^{k} \mathbf{B}_i X_{t-i} - \mathbf{F} E_t \left[Y_{t+1} \right] + \mathbf{\Phi} C + U_t. \tag{A1}$$

The companion form referred to in the text is defined as follows,

$$\overbrace{\begin{bmatrix} \mathbf{F} & \mathbf{A} & \cdots & \mathbf{B}_{t-k+2} & \mathbf{B}_{t-k+1} \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \\ 0 & 0 & \cdots & 0 & I \end{bmatrix}}^{\mathbf{A}^*} \overbrace{\begin{bmatrix} E_t \left(Y_{t+1} \right) \\ Y_t \\ \vdots \\ Y_{t-k+2} \\ Y_{t-k+1} \end{bmatrix}}^{X_t^*} =$$

$$+ \overbrace{\begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{B}_{t-k} \\ I & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}}^{\mathbf{A}_1^*} \overbrace{\begin{bmatrix} E_{t-1} \left(Y_t \right) \\ Y_{t-1} \\ \vdots \\ Y_{t-k+1} \\ Y_{t-k} \end{bmatrix}}^{X_{t-1}^*}$$

$$\overbrace{\begin{bmatrix} \mathbf{\Phi} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}^{\mathbf{\Phi}^*} C + \overbrace{\begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}^{\mathbf{\Psi}_u^*} U_t + \overbrace{\begin{bmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix}}^{\mathbf{\Psi}_\eta^*} \zeta_t, \tag{A2}$$

Notice that ζ_t is defined in row 2 of this system to be $Y_t - E_{t-1} \left[Y_t \right]$.

Appendix B

The complete structural model in differences is given by Equation (B1),

$$\mathbf{A}DY_t = \sum_{i=1}^{k-1} \mathbf{B}_i DY_{t-i} - \mathbf{F}E_t\left[DY_{t+1}\right] + \mathbf{\Pi}Y_{t-1} + \mathbf{\Phi}C + U_t. \tag{B1}$$

Consider the following equivalent levels representation,

$$\tilde{\mathbf{A}}Y_t = \sum_{i=1}^{k} \tilde{\mathbf{B}}_i Y_{t-i} - \mathbf{F}E_t\left[Y_{t+1}\right] + \tilde{\mathbf{\Phi}}C + U_t. \tag{B2}$$

The parameters of (B1) are related to those of (B2) by the identities,

$$\begin{aligned} \mathbf{A} &\equiv \left(\tilde{\mathbf{A}} + \mathbf{F}\right), \\ \mathbf{B}_i &\equiv \sum_{i=2}^{k-1} -\tilde{\mathbf{B}}_i, \quad i = 1, ..k-1, \\ \mathbf{\Pi} &\equiv \sum_{i=1}^{k} \tilde{\mathbf{B}}_i + \mathbf{F} - \tilde{\mathbf{A}}. \end{aligned} \tag{B3}$$

This transformation is similar, but not identical, to that which turns a vector autoregression into a vector equilibrium correction model. If the data is stationary, estimating the model in levels is equivalent to estimating (B1). If the data is non-stationary, but cointegrated, the matrix $\mathbf{\Pi}$ will have reduced rank with a representation

$$\underset{3\times 6}{\mathbf{\Pi}} = \underset{3\times 4}{\boldsymbol{\alpha}}\,\underset{4\times 6}{\boldsymbol{\beta}^T},$$

where the $\boldsymbol{\beta}^T$ represent structural cointegrating vectors. By formulating the model in differences, we are able to test hypotheses about the way that the cointegrating vectors enter the structural equations.

Appendix C

In our working paper [4] we showed that unemployment, inflation and the federal funds rate are well described by a VEqCM with two cointegrating vectors for each subperiod. In that study we identified the cointegrating space with the following scheme. Using the notation β_1^1, β_2^1 for the estimated cointegrating vectors for the first subperiod and β_1^2, β_2^2 for the cointegrating vectors over the second subperiod, our identification scheme can be represented as in Equation (C1),

$$\overbrace{\begin{bmatrix} y_1 & y_2 & y_3 \\ 1 & \beta_{12}^1 & 0 \\ 0 & \beta_{22}^1 & 1 \end{bmatrix}}^{t=1...T_1}, \quad \overbrace{\begin{bmatrix} y_1 & y_2 & y_3 \\ 1 & \beta_{22}^2 & 0 \\ 0 & \beta_{22}^2 & 1 \end{bmatrix}}^{t=T_1+1...T}. \tag{C1}$$

Superscripts index regime, and subscripts index elements of the cointegrating vectors β_1 and β_2. This scheme uses exclusion restrictions that set the y_3 coefficient to zero in the first cointegrating vector and the y_1 coefficient to zero on the second cointegrating vector. These "identification" restrictions have no economic content. They are simply a normalization.

Given the definition of the data used in the current study, condition (C1) imposes the following cointegrating vectors on the data set augmented by thestep variables Y_t^S,

$$\begin{bmatrix} y_1 & y_2 & y_3 & y_1^S & y_2^S & y_3^S \\ 1 & \beta_{12}^1 & 0 & 0 & \beta_{12}^2 - \beta_{12}^1 & 0 \\ 0 & \beta_{22}^1 & 1 & 0 & \beta_{22}^2 - \beta_{22}^1 & 0 \\ 0 & 0 & 0 & 1 & \beta_{12}^2 & 0 \\ 0 & 0 & 0 & 0 & \beta_{22}^2 & 1 \end{bmatrix}. \tag{C2}$$

The goal of the current paper is to test structural stability. For this purpose, the normalization in (C2) is inconvenient since it allows the cointegrating space to break across periods. To simplify our tests of parameter constancy, we chose the following equivalent normalization of the vectors that define the cointegrating space,

$$
\begin{bmatrix} y_1 & y_2 & y_3 & y_2^S & y_2^S & y_3^S \\ 1 & \beta_{13}^1 & \beta_{14}^1 & 0 & 0 & 0 \\ 0 & \beta_{23}^1 & 1 & 0 & \beta_{23}^2 - \beta_{23}^1 & 0 \\ 0 & 0 & 0 & 1 & \beta_{13}^1 & \beta_{14}^1 \\ 0 & 0 & 0 & 0 & \beta_{23}^2 & 1 \end{bmatrix} = \begin{bmatrix} y_1 & y_2 & y_3 & y_1^S & y_2^S & y_3^S \\ 1 & -3.91 & -0.41 & 0 & 0 & 0 \\ 0 & -11.57 & 1 & 0 & -4.95 & 0 \\ 0 & 0 & 0 & 1 & -3.91 & -0.41 \\ 0 & 0 & 0 & 0 & -16.52 & 1 \end{bmatrix}. \tag{C3}
$$

The cointegrating vectors identified in this way are graphed in Figure C1. An important feature of our normalization is that the first cointegrating vector remains constant across the two subperiods.

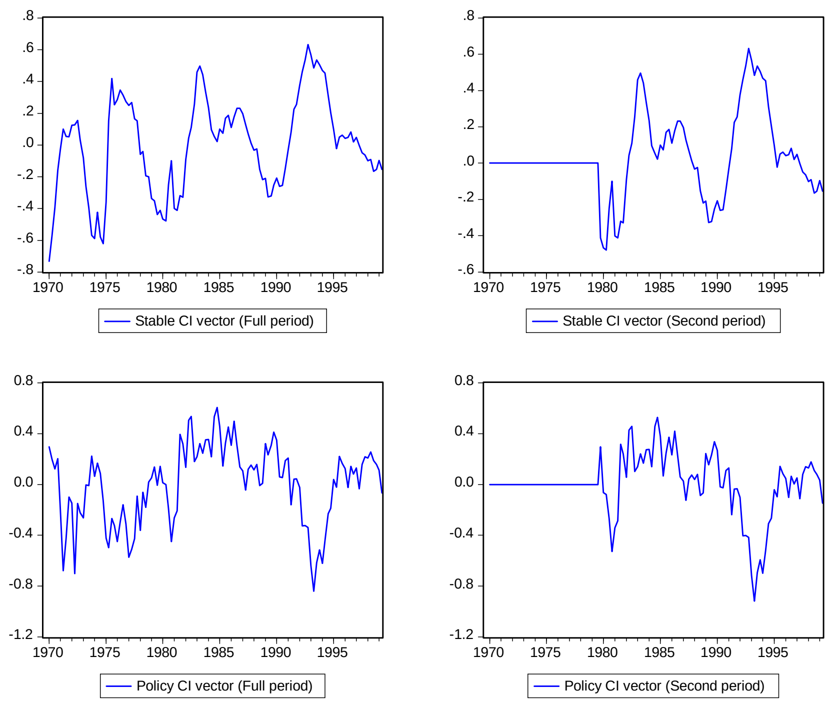

Figure C1: Cointegrated Variables Used in Method 2

Appendix D

This Appendix reports the unrestricted parameter estimates for the two alternative estimation strategies reported in Section 9.

Unemployment Equation										
	(1) GMM LEVELS		**(2) GMM CI**				**(1) GMM LEVELS**		**(2) GMM CI**	
	Coef.	t-stat	Coef.	t-stat			Coef.	t-stat	Coef.	t-stat
Constant	0.054	(0.84)	0.002	(0.55)		Di	0.04	(0.36)	0.084	(0.01)
Du lead 1	***0.47***	(3.60)	***0.98***	(4.98)		Du lag 2	0.12	(1.17)	0.08	(0.72)
Dπ lead 1	0.73	(1.17)	2.54	(1.39)		Dπ lag 2	-0.53	(2.15)	-0.14	(0.33)
Di lead 1	0.12	(1.51)	-0.14	(0.10)		Di lag 2	0.002	(0.06)	0.03	(0.77)
Du lag 1	***0.39***	(2.68)	0.15	(1.01)		u lag 1	-0.005	(0.01)	NA	NA
Dπ lag 1	0.57	(1.76)	1.98	(0.65)		π lag 1	-0.17	(0.81)	NA	NA
Di lag 1	0.02	(0.63)	-0.03	(0.56)		i lag 1	0.02	(2.27)	NA	NA
						CI1	NA	NA	0.02	(0.81)

Table D1: Estimates of the Recoverable Structure - EQ1.

Inflation Equation										
	(1) GMM LEVELS		**(2) GMM CI**				**(1) GMM LEVELS**		**(2) GMM CI**	
	Coef.	t-stat	Coef.	t-stat			Coef.	t-stat	Coef.	t-stat
Constant	-0.02	(0.045)	-0.002	(1.67)		Di	0.04	(0.62)	**0.07**	(2.49)
DU lead 1	0.18	(1.69)	**0.17**	(2.11)		DU lag 2	-0.03	(0.46)	-0.05	(1.26)
Dπ lead 1	0.53	(0.94)	-0.24	(0.58)		Dπ lag 2	-0.07	(0.38)	-0.06	(0.43)
Di lead 1	**-0.16**	(2.36)	**-0.13**	(3.94)		Di lag 2	-0.015	(0.70)	0.03	(1.46)
DU lag 1	-0.12	(0.84)	-0.03	(0.58)		U lag 1	0.009	(0.64)	NA	NA
Dπ lag 1	0.26	(0.87)	0.12	(0.47)		π lag 1	0.033	(0.17)	NA	NA
Di lag 1	-0.28	(0.82)	-0.03	(1.70)		i lag 1	-0.02	(1.76)	NA	NA
						Ci1	NA	NA	**0.02**	2.27

Table D2: Estimates of the Recoverable Structure - EQ2.

Burns-Miller Reduced-Form Interest Rate Equation										
	(1) GMM LEVELS		(2) GMM CI				(1) GMM LEVELS		(2) GMM CI	
	Coef.	t-stat	Coef.	S.E.			Coef.	S.E.	t-stat	S.E.
Constant	***-3.00***	(4.47)	-0.00	(0.01)		Di	NA	NA	NA	NA
Du lead 1	NA	NA	NA	NA		DU lag 2	***-0.69***	(2.14)	***-1.66***	(2.17)
Dπ lead 1	NA	NA	NA	NA		Dπ lag 2	-2.48	(1.04)	***15.18***	(4.11)
Di lead 1	NA	NA	NA	NA		Di lag 2	***-0.31***	(2.32)	***-2.50***	(3.16)
Du lag 1	***-0.78***	(2.41)	***0.19***	(0.33)		u lag 1	***-0.61***	(5.68)	NA	NA
Dπ lag 1	***-5.94***	(3.08)	-0.47	(1.88)		π lag 1	***6.73***	(3.53)	NA	NA
Di lag 1	***0.22***	(2.10)	***1.38***	(2.65)		i lag 1	***-0.35***	(3.27)	NA	NA
						CI1	NA	NA	***-0.24***	(2.15)
						CI2	NA	NA	-0.08	(0.95)

Figure 1: Table D3: Estimates of the Reduced Form - EQ3

Changes to the Interest Rate Equation introduced by Volcker-Greenspan										
	(1) GMM LEVELS		(2) GMM CI				(1) GMM LEVELS		(2) GMM CI	
	Coef.	t-stat	Coef.	t-stat			Coef.	t-stat	Coef.	t-stat
ΔConstant	***2.36***	(3.73)	NA[1]	NA[1]		Δi	NA	NA	NA	NA
ΔDU lead 1	NA	NA	NA	NA		ΔDU lag 2	***0.95***	(2.30)	***1.95***	(2.40)
ΔDπ lead 1	NA	NA	NA	NA		ΔDπ lag 2	4.52	(1.47)	***-12.90***	(3.67)
ΔDi lead 1	NA	NA	NA	NA		ΔDi lag 2	0.24	(0.16)	***2.41***	(3.14)
ΔDU lag 1	-0.56	(1.10)	-1.41	(2.10)		ΔU lag 1	***0.51***	(4.41)	NA	NA
ΔDπ lag 1	3.89	(1.90)	-1.05	(0.49)		Δπ lag 1	-3.50	(1.82)	NA	NA
ΔDi lag 1	-0.17	(0.21)	***-1.26***	(2.18)		Δi lag 1	***2.36***	(3.73)	NA	NA
						CI1	NA	NA	0.14	(1.18)
						CI2	NA	NA	-0.03	(0.30)

[1]For the system with cointegrating vectors GMM did not converge with sequential updating when we included the step dummy in the interest rate equation. We report the estimates with this variable excluded.

Figure 2: Table D4: Estimates of the Reduced Form - EQ3

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