Efficient Risk Sharing within a Catastrophe Insurance Pool*

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Paper presented at the NBER Insurance Project Workshop, Cambridge, MA

January 31-February 1, 2003

Abstract

This paper examines optimal catastrophic risk sharing arrangements within a pool when the financial resources may be insufficient to pay all valid claims in full. Under this threat of default on payment, the mutuality principle and the standard allocation of aggregate risk based on the individual risk tolerances are shown to hold within a sub-pool of agents who decide to stay in the pool following the occurrence of a cataclysmic event. In the context of the insurance market, this constraint precludes from the obtainment of a first-best optimal participating policy displaying full insurance with a variable premium. The second-best optimal insurance contract is shown to provide full (marginal) coverage above an *ex post* variable deductible. This deductible is such that the total claims paid to the policyholders and the financial resources of the pool are equalized. This innovative contract contrasts with current catastrophe insurance schemes based on pro rated indemnification should the insured losses exceed the capital available. Catastrophe insurance pools as market enhancing instruments are investigated.

Key words : cataclysm, limited liability, partial insurance, participating policy, variable deductible

* This paper is the result of fruitful discussions with Brian D. Wright.

1. Introduction

Past decades have shown a dramatic increase in catastrophic losses due to natural disasters. Seven of the ten most costly insured property losses have occurred since 1990. Hurricane Andrew, the largest U.S. natural catastrophe on record, cost insurers $18.6 billion in 1992. The second largest catastrophe is the Northridge earthquake in 1994, with insured losses of $13.8 billion, and the third most costly event is Typhoon Mireille in Japan in 1991 ($6.7 billion). Insured losses over the period 1989-1995 totaled almost $75 billion (after adjusting for house inflation), more than five times the average real insured losses during the prior four decades (Lewis and Murdock 1999). In addition, scenarios constructed by catastrophe modeling firms suggest that a hurricane in Florida could cost up to $76 billion and an earthquake in New Madrid could cause losses higher than $100 billion. Beside natural catastrophes, insured losses caused by man-made disasters are taking on a new dimension. The terrorist attack against WTC and Pentagon on 11 September 2001 caused property and business interruption losses estimated at $19 billion (plus $17-39 billion in liability and life insurance). Insured losses due to this terrorist event are much higher than those caused by the bomb explosions in London's city in 1993 ($907 million), in Manchester in 1996 ($744 million) or in garage of WTC in 1993 ($725 million). Regarding technical risks, the explosion in a fertilizer factory in Toulouse (France) on September 2001 costs about $1.4 billion (Swiss Re 2002). Concerning environmental risks, the cost to insurers of toxic waste cleanup may reach $30-50 billion (Cutler and Zeckhauser 1999). Therefore, it is clear that the financing of both natural and man-made catastrophes is a growing problem for both insurance and capital markets.

At first sight, the international insurance and reinsurance insurance markets may be able to pay for such mega-catastrophes. The property-liability insurance industry's equity capital is approximately $350 billion in the US and $550 worldwide. However, it is widely admitted that insurance and reinsurance are not the most efficient way to handle extremely large and infrequent loss event due to market imperfections. Froot (2001) provides several explanations for the paucity of catastrophe risk sharing. Cummins, Doherty and Lo (2002) conduct a detailed analysis of the insurers' financial capacity to pay catastrophic losses. They estimate that the insurance and reinsurance industry could pay up to 93% of insured losses for a $100 billion catastrophe, 84% for a $200 billion catastrophe and 78% for a $300 billion catastrophe. This means that there still would be

$7 billion, $32 billion and $66 billion in unpaid valid claims, respectively. In addition, such catastrophes would cause a large number of insurer insolvencies rising disproportionately with the size of the catastrophic loss.

The potential losses from catastrophes have lead researchers to investigate how innovative financial instruments could fill the gap capacity in the provision of insurance against catastrophic events, and more particularly natural disasters. Approximately $13 billion of these capital market insurance solutions (e.g., cat bonds/swaps, PCS options, contingent capital) have been issued for the financing of natural disasters since 1996 (Swiss Re 2001). Another financial risk class that is of interest to insurers is weather derivatives. Almost $12 billion in capacity has been created since their inception in 1997 (Swiss Re 2002). However in the present state of capital markets, these products, when they exist, are still confidential or no longer traded due to low trading volume (e.g., catastrophe (PCS) options or area yield options offered by the CBOT). On the supply side, the catastrophic-loss index contracts are of financial investors' interest because of their marginal correlation with market risks. They thus open opportunities for more efficient portfolio selection. Hence, the additional, almost completely uncorrelated diversification possibility shifts the so-called "efficient frontier" upward.[1] However, September 11 event has revealed a stronger correlation between underwriting and investment risks than previously presumed. This unprecedented simultaneous shock on both the asset and liability sides of insurance companies has highlighted that a significant positive correlation between underwriting and investment risks may limit the ability of innovative financial products to provide new risk transfer alternatives. This effect is presumably not limited to terrorist attacks. Extraordinary natural catastrophes may also lead to indirect disruptions of economic activities on a significant global scale, in addition to their direct damage impact (Atchleitner, Biebel and Wichels 2002). In addition, the uncertainty about the catastrophic loss distribution, making these risks difficult to assess, and the lack of a widely accepted pricing model make investors reluctant to trade these products. On the demand side, the high risk premiums and the perception among insurers that catastrophe options are subject to unacceptable levels of basis risk is the main obstacle to the development of this market (Cummins, Lalonde and Phillips 2002). As a consequence, alternative risk transfer (ART) solutions aiming at

[1] Kielholz and Durrer (1997) show several optimal portfolio structures in a global context.

bringing catastrophe exposures directly to the capital market may be unable to supplement the limited capacity of reinsurance markets in order to cover losses caused by a mega-catastrophe. Regarding man-made disasters, alternative financial solutions are very confidential, or even unavailable, in the present state of capital markets. However, the insurance and capital markets are expected to develop in the near future alternative risk transfer products addressing these events, and especially terrorism, using an approach similar to the financing of natural catastrophes (Swiss Re 2002). Following the example of cat bonds to deal with natural disasters, investor may issue *terror bonds* under which the triggering conditions would depend on a predefined terrorist event.

There is a growing literature that investigates how insurance and financial contracts can be designed and combined in order to manage efficiently catastrophic risks (e.g., Doherty and Dionne 1993, Doherty and Schlesinger 2002, Mahul 2002). These papers examine how an appropriate decomposition of insured losses into separate idiosyncratic and systemic components allows for an increase in welfare. For example, Mahul (2002) designs a variable participating policy in which the systemic risk is first filtered through a fully participating contract with a variable premium based on the realized systemic loss, and then it is transferred into capital markets through a derivative contract providing coverage on the variable premium. However, these analyses rest on two main assumptions. First, contracts coping with the systemic component of insurance risks are available on financial markets. Second, the participating insurance policies with a variable premium are not exposed to default on payment; the policyholders always accept to pay in full any *ex post* premium adjustments. It should be noticed that the existence and the efficiency of financial instruments to deal with the systemic part of insurance risks contributes to lowering the risk of default on payment.

It may first be useful to classify catastrophe losses with respect to the financial capacity provided by the (re)insurance and financial markets. Cutler and Zeckhauser (1999) propose the following definitions of catastrophes and cataclysms. A *catastrophe* is “a situation of threatened or actual insolvency in the event of an adverse risk” (p.233). Such an event “is bigger than any one insurer can handle but not big enough to upset the entire insurance market” (p.235). They mention that insured damages must be arbitrarily higher than $25 million.[2] They define a *cataclysm* as an "event that strains worldwide insurance and reinsurance industry reserves". More generally, it occurs " when the size of a loss is large relative to the insurance pool for that category of risk" (p.234). It is arbitrarily defined as an event with $5 billion or more of insured losses. We suggest here an alternative definition of a catastrophe/cataclysm based on the comparison between the size of the insured loss and the financial resources available in the insurance pool under consideration. We term an event as a *catastrophe* when the financial capacity of the pool (including all internal and external market-based solutions, e.g., *ex post* premium adjustments, reinsurance and ART solutions) is sufficient to pay all valid claims in full. A *cataclysm* occurs if the financial capacity of the pool (when all internal and external market-based opportunities to provide additional capacity have been exhausted) is not sufficient to pay all valid claims in full. This means that a cataclysm leads to the insolvency of this pool.

As an illustration, consider the financial structure of the California Earthquake Authority (CEA). As of June 1999, the CEA's claims-paying capacity was $7.2 billion in which almost 72% of the owner insurance industry in California joined the CEA (Jaffee and Russell 2000).[3] If, following an earthquake, the total insured losses are less than $7.2 billion, this natural event is considered as a catastrophe from the CEA's viewpoint because the CEA financial resources are sufficient to pay all valid claims in full. On the contrary, an earthquake causing more than $7.2 billion of insured losses is a cataclysmic event because the financial resources available do not allow the CEA pay all valid claims in full.

This paper deals with the efficient market allocations of disaster risks within a catastrophe insurance pool. From our terminology, this problems can be addressed in two issues. First, what is an efficient *catastrophic* risk sharing arrangement? Second, what is an efficient *cataclysmic* risk sharing treaty?

The first question is addressed in the seminal paper written by Borch (1962).[4] He shows that an efficient risk sharing rule among the individuals depends only on the aggregate wealth, i.e., the sum of individual wealth, not on individual results. This is

[2] For insured property catastrophe other than marine and aviation, Swiss Re defines a catastrophe as an event causing at least $35.1 million in insured property losses (Swiss Re 2002).
[3] The corresponding value based on a hypothetical 100% industry participation is estimated at $10.5 billion.
[4] See, e.g., Gollier (1992) for a survey on the economic theory of risk exchanges.

called the *mutuality* principle. Therefore, an efficient risk sharing arrangement is based on the following pooling rule: agents first give their initial wealth to a pool and then specify a rule for distributing this aggregate wealth among all the members of the pool. Idiosyncratic risks are thus diversified through mutualization. The aggregate (undiversifiable) risk is then shared among the agents according to the following rule: any increment in the pool's wealth should be shared in proportion to individual risk tolerances (Wilson 1968). In the insurance context, each agent may prepay a premium equal to, e.g., her share of the expected aggregate loss. Following a catastrophe, she will receive an indemnity providing full coverage on the individual loss and she will pay an *ex post* premium adjustment if the realized aggregate loss is larger than expected.[5]

The purpose of this paper is about the second issue. It investigates the design of an efficient cataclysmic risk sharing treaty, i.e., how losses should be shared among the agents when the total insured losses exceed the financial resources available in the insurance pool. Many studies point out that the exposure of cataclysmic events has substantial effects on the solvency of insurance and reinsurance companies. Surprisingly, there is a few work on efficient risk sharing arrangements when the available capital is known to be limited. Schlesinger (2000, p.143) notes in his recent survey that this risk of nonperformance is "always present for an insurable risk, but almost universally ignored in insurance theory". Only a few papers examine rational insurance purchasing when contracts are exposed to risk of nonperformance. Doherty and Schlesinger (1990) show in a three-state model that if default, when it occurs, is total, partial insurance coverage is always purchased at a "fair price". Mahul and Wright (2002) reconsider this problem in a multiple-state model and they derive the design of an optimal fair insurance contract design when the threat of insolvency is due to a fall in financial resources and is the only systemic risk faced by the policyholder. This fair insurance policy is shown to display full (marginal) coverage above a positive deductible. However, these two papers to do explicitly relate the risk of nonperformance to the risk of capital shortfall. Jaffee and Russell (1997) examine the demand for partial insurance arising whenever insurers lack sufficient capital to pay claims and but they only focus on pro rated indemnity payments. Hence, the existing literature does not shed light on principles that should govern the

[5] Such a risk sharing rule is efficient if problems of informational asymmetries (moral hazard, adverse selection) are assumed away, as it will be the case throughout this paper. See, e.g., Gollier (2002) for a survey about their consequences on insurability.

design of an insurance contract when capital is limited and when the treat of insolvency is caused by unexpected large insured losses.

One of the main contributions of this paper is to show that, when the financial resources of the pool are not sufficient to pay off valid claims in full, an optimal participating insurance contract displays full (marginal) coverage above an *ex post* variable deductible. This deductible is defined such that the total claims paid to the policyholders and the capital available in the pool are equalized. This form of partial insurance contrasts with current risk sharing rules within catastrophe insurance pools, like the CEA, where the contract displays pro rated indemnification, i.e., *ex post* variable coinsurance, in the event of a cataclysm.

This paper is organized as follows. Section 2 examines efficient risk sharing rules under the threat of default on payment and revisits the mutuality principle as well as the allocation of non-diversifiable risk. Section 3 reconsiders this efficient risk sharing arrangement in the context of the insurance market for identical agents. The optimal prepaid premium is discussed and a numerical illustration is presented. Section 4 deals with catastrophe insurance pools. It proposes an innovative insurance policy design is proposed, examines the role of catastrophe insurance pools as market enhancing, and discusses the role of the government as a provider of (partial) insurance of last resort. Section 5 concludes.

2. Efficient risk sharing under risk of default on payment

Consider a two-date model with n agents, $i = 1,2...,n$, and a continuum of possible future states of nature. For every state q, $x_i(q)$ denotes the initial endowment of agent i. Prior to the realization of $\tilde{q}$, agents face uncertainty related to their future wealth. The group also faces systemic risk since the aggregate wealth $X(q) = \sum_{i=1}^{n} x_i(q)$ is a random variable, except when the individual risks are fully diversifiable, i.e., $X(q) = X$ for all q. We assume that agents maximize the expected value of their utility that is a function of wealth. We denote $u_i(.)$ the von Neumann-Morgenstern utility function of the risk-averse agent i, with $u_i' > 0$ and $u_i'' < 0$, $i = 1,2...,n$.

An allocation of risk is a set of n random variables $\{y_i(q), i=1,...,n\}$ describing the agents' wealth in the state of nature q. This allocation is subject to the physical feasibility constraint:

(1) $$\sum_{i=1}^{n} y_i(q) = X(q) \quad \text{for all } q.$$

For every state of nature q, every agent is assumed to be willing to contribute to this pool up to a certain amount $b_i(q)$ in comparison with her initial endowment $x_i(q)$. Otherwise, the agent leaves the pool and receives $x_i(q) - d_i(q)$, with $b_i(q) \geq d_i(q)$ for all q. Formally, the participation (or enforceability) constraint is

(2) $$y_i(q) = \begin{cases} z_i(q) & \text{if } z_i(q) \geq x_i(q) - b_i(q) \\ x_i(q) - d_i(q) & \text{otherwise.} \end{cases}$$

The functions $b_i(.)$ and $d_i(.)$ are taken as given. They will be discussed hereafter.

For every assignment $\{l_i\}_{i=1,...,n}$, a second-best optimal allocation of risks is the solution to the following maximization problem:

(3) $$\max_{y_1(.),...,y_n(.)} \sum_{i=1}^{n} l_i E u_i\left(y_i(\tilde{q})\right)$$

subject to constraint (1), and constraint (2) for all q and all $i=1,...,n$,

where $E(.)$ denotes the expectation operator.

2.1 The mutuality principle revisited

Problem (3) is solved using variational calculus. We denote $g(q)$ the multiplier associated to the feasibility constraint (1) and $m_i(q)$ the multiplier associated to the participation constraint (2). This leads to the first-order conditions:

(4) $$l_i u_i'\left(y_i(q)\right) + g(q) + m_i(q) = 0 \quad \text{for all } q \text{ and all } i=1,...,n.$$

This can be rewritten as

(5) $$l_i u_i'\left(y_i(q)\right) + m_i(q) = l_i u_j'\left(y_j(q)\right) + m_j(q) \quad \text{for all } q \text{ and all } i, j = 1,...,n.$$

Observe that if $b_i(.) \equiv x_i(.)$ for all $i=1,...,n$, then the participation constraint (2) is never binding, i.e., $m_i(.) \equiv 0$, and thus condition (5) is the so-called Borch condition (Borch 1962). Otherwise, we define the sub-pool $S = \{i=1,...,n / z_i(q) \geq x_i(q) - b_i(q)\}$ and condition (5) becomes

(6) $\quad l_i u_i'(z_i(q)) = l_i u_j'(z_j(q))$ for all q and all $(i, j) \in S$.

The Borch condition thus holds *within* the sub-pool S of agents who decide to stay in the pool following the occurrence of a cataclysmic event. Denote $X_S = \sum_{i \in S} x_i(q)$ the aggregate wealth within the sub-pool S. It is straightforward to show that an optimal allocation of risks within the subset S depends only on the aggregate wealth in this sub-pool, i.e., $z_i(q) \equiv z_i(X_S)$ for all $i \in S$. This means that all diversifiable risks within the sub-pool S have been eliminated.

2.2 The allocation of aggregate risk

We examine how the aggregate risk should be allocated between the members of the sub-pool. Differentiating condition (6) with respect to X_S yields

(7) $$\frac{dz_i}{dX_S} = \frac{T_i(z_i(X_S))}{\sum_{j \in S} T_j(z_j(X_S))},$$

where $T_i(.) \equiv -u_i'(.)/u_i''(.)$ is the index of absolute tolerance towards risk. Condition (7) is a necessary and sufficient condition for an allocation of risks to be *ex ante* Pareto efficient.

The existence of the participation (or enforceability) constraint (2) precludes the obtainment of a first-best optimal risk sharing treaty as derived by Borch (1962) and Wilson (1968). However, this allocation of risks holds within the sub-pool of agents who decide to stay in the pool following the occurrence of a cataclysmic event, i.e., for whom the participating constraint (2) is not binding.

2.3 The participation (or enforceability) constraint

The originality of our model is to explicitly recognize the existence of a risk of default on *ex post* payments; the agents may refuse to fully participate in the *ex post* allocation of wealth within the pool, i.e., to fully share the aggregate risk. This is taken into account through the participation (or enforceability) constraint expressed in (2). This constraint is discussed in the context of the insurance market. The final wealth of agent i is rewritten as $z_i(q) = x_i(q) + I_i(q) - P_i$, where $I_i(q)$ is the indemnity schedule paid to the policyholder in the state of nature q and P_i is the prepaid insurance premium, for $i = 1,...,n$. In constraint (2), d_i is the premium paid in advance by the policyholder, i.e.,

$d_i(q) \equiv P_i$, and $b_i(q)$ is the maximum premium net of the indemnity the policyholder i is willing to pay, for $i = 1,...,n$. Some insurance companies, like the assessment mutuals, do not charge premiums in advance but they rely only on a retroactive assessment to pay claims; in our model, this means that $d_i(q) = 0$. However, she may refuse to pay a retroactive premium higher than the prepaid premium should a catastrophe occur. Such a risk of default on payment is consistent with the comparative rarity of assessment mutuals (Doherty and Dionne 1993). It is also explicitly recognized by Jaffee and Russell (2000) in the case of the CEA; they state that "…the *ex post* assessment is voluntary since all an insured has to do to avoid paying it is to cancel the policy" (p.31). This upper limit on *ex post* premium adjustments can be justified as follows.

First, it may be due to regulatory constraints which restrict the amount of the *ex post* premium surcharge. For example, this surcharge is limited to 20% of premiums under the insurance policy offered by the CEA.

Second, it may be caused by the absence of legal authority to enforce *ex post* payments, or to high transaction costs to enforce these payments when exogenous means of enforcement exist. The literature on voluntary exchange with no exogenous means of enforcement examine efficient allocations in intertemporal models using repeated game (see, e.g., Kimball 1988, Kletzer and Wright 2000). The analysis on dynamic risk sharing is beyond the scope of this paper, but it is useful to illustrate the policyholder's willingness to pay for *ex post* adjustments. When no policyholder can force an *ex post* payment from another, either directly or by appeal to a third party, and the only punishment for noncompliance is expulsion from the pool, the sole incentive for making this payment is the value of belonging to the risk pool. This surplus is defined as the difference between the (discounted expected) utility within the risk pool and the (discounted expected) utility under permanent autarky.

Consider first (almost) independent risks, such as fire or auto risks, defined as high frequency, low consequence (HF/LC) risks. The value of belonging to the pool is relatively high because the agent is exposed to this risk all along her lifetime. In addition, *ex post* premium adjustments are relatively small because, from the Law of large numbers, deviations of average loss from expected loss are small. In other words, the annual loss ratio is close to one. Consequently, the enforceability constraint is not binding and the efficient allocation of diversifiable and undiversifiable components of individual risk, as derived in Borch (1962) and Wilson (1968), holds.

Consider now low frequency, high consequence (LF/HC) risks, i.e., catastrophic risks. The probability of facing a second disaster when a first one has just occurred is assumed to be almost zero.[6] As a consequence, the value of belonging to a risk pool, which provides a coverage for this risk only, is close to zero once a catastrophe has occurred. On the contrary, because of their high severity, deviations of average catastrophic loss from expected catastrophic loss may be huge. In other words, the annual loss ratio is much higher than unity following a disaster and zero otherwise.[7] This means that *ex post* premium adjustments paid by the policyholders may be very high. Consequently, the enforceability constraint will be binding and the policyholders may refuse to pay *ex post* adjusted premiums higher than their full indemnity, i.e., their indemnity under full performance. The Borch principle as well as the standard allocation of aggregate risk clearly cannot apply among all the policyholders, contrary to what it is usually assumed in the literature on risk sharing.

These two polar cases point out that participating insurance policies with variable premiums are efficient in the financing of (almost) independent HF/LS risks but they cannot be implemented in the management of catastrophic risks when there are no means or costly means of enforcement. However, this problem may be partly circumvented by offering a unique contract covering both HF/LS and catastrophic risks. In this case, the value of belonging associated to the idiosyncratic part of the individual risk may increase the willingness to pay for an *ex post* premium following the occurrence of a disaster.

3. Optimal insurance when capital is limited

3.1. Optimal insurance contract design for a fixed prepaid premium

The second-best efficient risk sharing treaty with participation (or enforceability) constraint is reconsidered in the context of the insurance market. This allows us to derive

[6] In our model, there is no uncertainty about the probability of a catastrophic event. This implies that the occurrence of a catastrophe does not lead the agent to update their prior belief upward. Moreover, we assume that the probability that a second catastrophe occurs conditional on the occurrence of a first one is not higher than the probability of the first occurrence. Such an assumption seems plausible for natural disasters like earthquake but it is unlike to hold for cases like terrorist risks. Otherwise, the occurrence of a catastrophe would increase the individual value of belonging to the pool.

[7] California earthquake loss ratios, calculated every year, lie between 0% and 2273% over the period 1971-1994 (Jaffee and Russell 1997).

the design of an optimal participating insurance contract when capital is limited. There are n risk-averse agents who face random losses $l_i(\tilde{q})$, $i = 1,...,n$. We assume that the agents have the same attitude towards risk ($u_i \equiv u$). Individual losses are identically distributed, but not necessarily independent. Each agent is endowed with the same non-random initial wealth w (no wealth inequality). The agent's random initial endowment is thus $x_i(q) = w - l_i(q)$ with $0 \le l_{\min} \le l_i(q) \le l_{\max} \le w$ for all $i = 1,...,n$ and all q.

These agents belong to an insurance pool. This pool offers agent *i*, for $i = 1,...,n$, an insurance contract specified by the couple $[I_i(q), P]$, where $I_i(q)$ is the indemnity paid to the agent i when the state of nature is q and P is the prepaid insurance premium. Observe that the agents pay in advance the same insurance premium because individual losses have been assumed to be identically distributed. The final wealth of the insured agent is thus given by $y_i(q) = x_i(q) + I_i(q) - P$, for $i = 1,...,n$.

With no risk of default on payment, it is well known that an optimal insurance contract would display full insurance against individual losses with a variable premium subject to *ex post* adjustments depending on the realized aggregate loss of the pool $L(q) = \sum_{i=1}^{n} l_i(q)$. This means that $I_i^{**}(q) = l_i(q) - [L(q) - nP]/n$, for $i = 1,...,n$. The aggregate risk is thus equally shared among all the members of the pools. Consequently, the final wealth of every agent is identical and equal to $y_i(q) = w - L(q)/n$, for $i = 1,...,n$. This participating policy with an *ex post* variable premium is widespread in life insurance and it has been recently examined in the context of catastrophe insurance (Doherty and Schlesinger 2002, Mahul 2002). However, it relies on the implicit assumption that the insured agents will always accept to pay in full a retroactive dividend should realized aggregate losses be higher than expected aggregate losses. When a catastrophic event occurs, aggregate losses are, by definition, much higher than expected losses and therefore retroactive payments may be much higher than the prepaid premium, i.e., $L > nP$, where nP is the capital available in the pool.[8] Consequently, agents facing small losses compared to the other members of the pool may have a *negative* indemnity net of the *ex post* premium. Would they accept to pay that? As mentioned by Norberg

[8] Prepaid premiums are assume to be the only financial resources of the insurance pool.

(2002), the only way the insurance company can prevent the aggregate risk is to charge the premiums to the safe side. In practice, this is done by calculating the premiums on the conservative so-called *technical basis* or *first order basis*, which represents a provisional worst-case scenario. However, recent natural and man-made catastrophes have shown that the probable maximum loss (PML) may be well beyond these worst-case scenarios, i.e., the assumed maximum loss potentials.

Suppose that, following the occurrence of a cataclysmic event, the insured agents refuse to pay any *ex post* adjustments in the premium that would exceed their full indemnity: $I_i(q) \geq 0$, for $i = 1,...,n$. This means that $b_i(.) \equiv P$. Such a decision would be rational if the agents think that the probability of a cataclysm occurring once again in their lifetime is zero.[9] The participation (enforceability) constraint expressed in (2) becomes

(8) $$y_i(q) = \begin{cases} z_i(q) & \text{if } z_i(q) \geq x_i(q) - P \\ x_i(q) - P & \text{otherwise,} \end{cases}$$

where S is the sub-pool as defined previously and $b_i(.) = d_i(.) = P$. Under our previous assumptions, we have $z_i(X_S) = X_S / s$ where s is the size of the sub-pool. This can be rewritten as $z_i(L_S) = w - P - [L_S - nP]/s$, where $L_S(q) = \sum_{j \in S} l_j(q)$. Therefore, an efficient indemnity schedule, when positive, is $I_i^*(q) = z_i(L_S) - x_i(q) + P = l_i(q) - [L_S(q) - nP]/s$. Under limited financial resources, an efficient indemnity function thus satisfies

(9) $$I_i^*(q) = \max[l_i(q) - D(q), 0] \text{ with } D(q): \sum_{i=1}^{n} I_i^*(q) = nP,$$

where $q = (l_1, l_2, ..., l_n)$.

When premiums can not be subject to *ex post* adjustments higher than the full indemnity, an optimal insurance contract displays full insurance above a *variable* deductible. This deductible depends on every realized individual loss, and not only on the realized aggregate loss of the pool. The aggregate risk is thus only shared among the members facing a loss, and an optimal risk sharing treaty depends on the level of each individual loss.

[9] Such a behavior may also be due to the inability of people to project benefits over a long period of time (Kleindorfer and Kunreuther 1999).

Suppose a cataclysmic event occurs such that the *ex post* variable deductible is less than the lowest insured loss. The participating constraint is unbinding for every policyholder, i.e., the indemnity net of *ex post* premium adjustments is positive. From equation (9), the optimal deductible satisfies $L(q) - nD(q) = nP$ and the optimal indemnity payment is $I_i^*(q) = \max[l_i(q) - D(q), 0] = l_i(q) - [L(q)/n - P]$. Consequently, the variable deductible insurance contract replicates the first-best insurance policy with a variable premium. It is noteworthy that this replication is not feasible if *ex post* variable coinsurance is implemented.

In the theory of insurance demand, the optimality of full (marginal) coverage above a deductible is a central result due to Arrow (1971). The presence of transaction costs and/or moral hazard provides a rationale for a positive deductible (Raviv 1979, Holmström 1979). We show that such an insurance contract design remains optimal, with no transaction costs or moral hazard, when the total valid claims exceed the financial resources of the pool. However, the deductible is *ex post* adjusted depending the loss faced by all the agents of the pool.

3.2. Optimal prepaid premium

The risk of insolvency faced by the catastrophe insurance pool will clearly depend on the financial resources of this pool. The higher its reserves, the lower the risk of default on payment. In other words, a higher prepaid premium will lead to a higher quality of the insurance product. Given our assumptions, the maximum individual loss is $l_{\max}$ for every member of the pool and thus the maximum aggregate loss of the pool is $L_{\max} = nl_{\max}$. With no transaction cost (e.g., no cost of capital), it would be optimal to fix the prepaid premium at the highest individual loss $l_{\max}$ and then, once the catastrophe event has occurred, to pay an indemnity equal to $[l_{\max} - L/n] \geq 0$. Such a strategy excludes risk of default on payment. The participation (enforceability) constraint is never binding and thus the first-best efficient allocation of risks is found.

However, it may be not possible to ask policyholders to pay such a premium, or they may refuse to pay it. In this more realistic case, the optimal prepaid premium is less than $l_{\max}$ and, consequently, there is a risk of insolvency, i.e., $\text{Prob}[\tilde{L} > nP] > 0$. Optimal indemnity payments will thus display a positive deductible in the case of insolvency.

Obviously, the higher the prepaid premium, the lower the deductible, i.e., the higher the partial payment in the case of insolvency. Therefore, the policyholder faces a tradeoff between the cost of insurance and the quality of the insurance coverage.

In fact, it is not necessary to fix the premium at the maximum individual loss to get a first-best optimal risk sharing rule. Suppose that every agent of the pool faces a loss due to the occurrence of a catastrophic event. The lowest individual loss is denoted $l'_{\min} > 0$. Let the prepaid premium be determined such that the *ex post* variable deductible is equal to the minimum individual loss in the worst scenario, $D = l'_{\min}$. The non-negativity constraint on the full indemnity net of *ex post* premium adjustments is never binding and, consequently, the first-best optimum is replicated.

3.3 An illustrative example

Consider a (re)insurance pool with six agents (insurers, firms…). The initial endowment of each member is 100. Under the hypothetical scenario described in Table 1, their loss lies between 20 and 70 and the aggregate loss of the pool is 270. With no insurance, their final wealth is in the range of 30 and 80. Their prepaid premium is arbitrarily set at 10. For the sake of simplicity, no reinsurance or other alternative risk transfer solutions are available. The financial capacity of the pool is thus equal to the aggregate prepaid premium (60). In our terminology, the catastrophe insurance pool faces a cataclysm because the aggregate loss well exceeds the capital available. We examine how these limited financial resources can be shared among the six members of the pool.

Suppose first that the agents accept to pay any *ex post* premium adjustments. A first-best optimal participating insurance policy displays full coverage and the agents pay a retroactive premium based on the aggregate loss of the pool. This risk sharing arrangement is presented in Table 1. The *ex post* premium is 35 and the indemnity net of this additional premium is negative for agents A and B who face the lowest losses. The final wealth is identical for every agent.

[INSERT TABLE 1 HERE]

Suppose now that the agents refuse to pay any retroactive premium higher than their full indemnity, i.e., they refuse to have a negative indemnity net of the *ex post* premium. We first consider the pro rated indemnification, as implemented in the CEA. In our illustration, this implies that the *ex post* coinsurance rate adjustment is equal to 22.22% (60/270*100); every agent receives an indemnity equal to 22.22% of the full indemnity, i.e., 22.22% of their loss. As presented in Table 2, the final wealth lies between 77.44 for the agent facing the smallest loss to 35.56 for the agent bearing the largest loss. Our innovative participating insurance policy, displaying full coverage above an *ex post* variable deductible, is now implemented. One can easily verify that the *ex post* deductible adjustment such that the total claims paid to the members equalize the insurance pool's financial resources is equal to 40. Therefore, the agents facing a loss lower than this deductible receive no indemnity, while the agents bearing a loss larger than the deductible have an indemnity such that their final wealth is identical. In our illustration, the sub-pool as previously defined is $S = \{C, D, E, F\}$. The agents in this sub-pool have an identical final wealth, while the two other agents have a larger final wealth.

[INSERT TABLE 2 HERE]

The performance of the risk sharing treaties is compared using our illustrative example. The agents' utility function is $u(w) = w^{1-d} / (1-d)$ for $d \neq 1$ and $u(w) = \ln(w)$ for $d = 1$, where d is the index of constant relative risk aversion (CRRA). The welfare of the pool is defined as the sum of the six agent's utility levels. It is computed under the first-best optimal insurance policy displaying full coverage with a variable premium and under the insurance contract that displays either a variable coinsurance rate or a variable deductible. The welfare of the catastrophe insurance pool under the two participating contracts when capital is limited is compared to the welfare of the pool under the first-best optimum. As shown in Table 3., the loss in welfare under the participating insurance contract with a variable deductible is lower that under the participating policy with a variable coinsurance rate. This illustrates the optimality of the variable deductible insurance contract when the pool's financial resources are limited. Table 3 also shows the impact of risk aversion on the welfare losses. The higher the CRRA coefficient, the higher the loss in welfare. This loss is much higher under the variable coinsurance policy than under variable deductible insurance contract as the CRRA coefficient increases; it is 6.50% and 1.71%, respectively, when the CRRA coefficient is equal to 2, while it is 386.16% and 41.01%, respectively, when the CRRA coefficient is 8.

[INSERT TABLE 3 HERE]

The impacts of the prepaid premium on the variable deductible, the variable coinsurance rate and the associated welfare losses are detailed in Table 4. Obviously, the higher the prepaid premium, the lower (higher) the variable deductible (coinsurance rate) and the lower the loss in welfare of the insurance pool compared to the first-best optimum. Given the realized losses within the catastrophe insurance pool in our hypothetical scenario, the agents are fully covered if the prepaid premium is higher than 45 because the financial resources of the pool become sufficient to pay all claims in full. When the prepaid premium is higher than 25, the *ex post* variable deductible is less than the lowest loss and, therefore, the indemnity net of the retroactive premium is non-negative for the six members; the first-best optimal risk sharing treaty is replicated. On the contrary, the pro rated indemnification leads to a sub-optimal allocation of risks, as shown by the positive loss in welfare.

[INSERT TABLE 4 HERE]

4. Catastrophe insurance pools

4.1 Designing an innovative insurance policy

Examples of indemnity schedules proposed by current catastrophe insurance pools with limited liability are presented in Table 5. They explicitly recognize that policyholders may not receive their full indemnity in the event of a cataclysm. When the initial financial capacity of the pool is not sufficient to pay valid claims in full, the pool first may ask the policyholders to paid *ex post* premium adjustments. If this additional capital is still insufficient, indemnity payments are pro rated, i.e., partial insurance displays *ex post* variable coinsurance.

[INSERT TABLE 5 HERE]

Our analysis on optimal contract design when capital is limited allows us to suggest an innovative policy within a catastrophe insurance pool. The indemnity function of agent *i*, $i = 1,...,n$, would be

$$(10) \qquad I_i(l_1,...,l_n) = \min\left[(a - \Delta a)\max\left[l_i - (D + \Delta D), 0\right], C\right] - \Delta P \text{, with } \sum_{i=1}^{n} I_i(l_1,...,l_n) \le K \text{,}$$

where K is the financial resources of the pool (including prepaid premiums), C is the coverage limit, $a \in \left]0,1\right]$ is the coinsurance level and $D \ge 0$ is the deductible.[10] When the financial capacity of the pool is sufficient to pay all the insured losses in full, the indemnity function becomes $I_i(l_1,...,l_n) = \min\left[a \max\left[l_i - D, 0\right], C\right]$. In the event of a cataclysm precluding from paying all valid claims in full, the indemnity payments are first subject to *ex post* premium adjustments, $\Delta P \geq 0$, in order to share the unexpected aggregate loss among all the members of the pool. If these additional financial resources are still insufficient, indemnity payments are downward adjusted through the *ex post* coinsurance rate $\Delta a \geq 0$ and/or the *ex post* deductible, $\Delta D \geq 0$. While the first two *ex post* adjustments ΔP and Δa are explicitly mentioned in the catastrophe insurance pools with limited liability, the *ex post* deductible adjustment is, to our knowledge, an innovative way of spreading catastrophic risks among the policyholders when *ex post* premium adjustments turn out to be insufficient to pay indemnities in full. We have shown that a second-best efficient insurance policy display full (marginal) coverage above an *ex post* variable deductible. This means that we should have $\Delta a = 0$ in expression (10).

4.2. Catastrophe insurance pool as market enhancing

Standard reinsurance contracts covering catastrophe risks are in an "excess-of-loss" form. This means that the reinsurer is obliged to pay up to a fixed-limit amount for all losses in excess of a given deductible. Formally, the indemnity payoff is

(11) $$I^{XL}(l) = \min\left[\max\left(l - D, 0\right), R\right] = \max\left[l - D, 0\right] - \max\left[l - (D + R), 0\right],$$

where D is the deductible ("retention") and R is the upper limit on coverage. As an illustration, consider a insurer that purchases a layer of reinsurance covering \$100 million in catastrophic losses in excess of \$300 million. Under such a policy, the layer is triggered if the insurer's losses from a single catastrophic event during the contract year exceed \$300 million retention. The reinsurance company pays the insurer the amount of any losses in excess of \$300 million, with the loss capped at a limit of \$100 million. The insurer thus cedes its exposure to a single-event catastrophe losses in the \$300-400 million range. Cummins and Mahul (2002) examine the demand for excess-of-loss insurance and they show that the upper limit on coverage induces the policyholder to select a positive deductible even if the insurance policy is sold at an actuarially fair price.

Suppose that this reinsurance company offers this contract to n insurers exposed to identically distributed random losses $\tilde{l}_i$, $i = 1,...,n$. For the sake of simplicity, we assume there is no *ex ante* deductible, $D = 0$. The reinsurer's total risk exposure is thus equal to nR, while every insurer has to bear losses that exceed the upper limit on coverage R.

[10] Coinsurance, $a < 1$, or deductible insurance, $D > 0$, may be optimal in order to mitigate moral hazard.

We suggest an alternative reinsurance contract design that may make insurers better off with the same financial capacity nR using a catastrophe reinsurance pool as a financial intermediary. A catastrophe reinsurance pool is jointly creates by the n insurance companies. This new entity purchases a reinsurance treaty covering nR in catastrophic losses (in excess of 0).[11] It offers insurer i, for $i = 1,...,n$, a reinsurance contract under which the indemnity schedule is

$$(12) \qquad I_i(l_1,...,l_n) = \min\left[\max\left[l_i - \Delta D, 0\right], C\right] \text{ with } \sum_{i=1}^{n} I_i(l_1,...,l_n) \le nR,$$

where *ex post* adjustments are positive, $\Delta D \ge 0$. The reinsurance contract offered by the reinsurance pool may still include an upper limit on coverage but it would be *higher* than that under the standard reinsurance policy, $C > R$; insurers would thus have a larger upper limit on coverage than that available under individual reinsurance policies. However, the retention may be subject to *ex post* adjustments, $\Delta D > 0$, should the total insured losses exceed the financial capacity *nR.*

This innovative contract may also be of interest to deal with the limited supply of catastrophe insurance coverage. The maximum number of policies an insurer would be able to sell depends on its financial capacity. This implies that some agents cannot be insured because of the lack of insurance supply due to this limited financial capacity. A way to cope with this supply shortfall would be to introduce downward adjustments in the indemnity payments in the event of a cataclysm; the insurance policy would display full insurance above an *ex post* variable deductible. When a cataclysm occurs, this deductible is upward adjusted so that the total claims paid to the policyholders and the insurer's financial resources are equalized. Such an allocation of catastrophic risk may be socially more efficient because it would give the agents who have been unable to purchase insurance because of the lack of insurance supply the possibility to transfer very high losses.

[11] The cost of this reinsurance treaty may be higher than the sum of the previous ones because, e.g., the probability that the maximum coverage *nR* is paid becomes higher:

$$\text{Prob}\left[\bigcap_{i=1}^{n}\left(\tilde{l}_i \ge R\right)\right] < \text{Prob}\left[\sum_{i=1}^{n}\tilde{l}_i \ge R\right].$$

4.3. The role of the government as a provider of (partial) insurance of last resort

The government is likely to provide post-cataclysm financial assistance. The role of the government as the insurer of last resort is explicitly recognized in several catastrophe pools, like the Pool Re agency in Great-Britain and the GAREAT agency in France, for terrorism insurance (see the appendix for a description of the French entity).[12]

However, the government may only bear a part of the capital shortfall; the financial resources of the insurance pool may be higher but still insufficient to pay all claims in full. This case allows us to highlight the performance of our innovative variable deductible insurance policy in comparison of the pro rated indemnification. Under the variable coinsurance policy, the additional financial capacity provided by the government allows the pool to increase the *ex post* rate of coinsurance. As a consequence, every policyholder facing an insured loss will get an additional indemnity in proportion of her insured loss. Under the participating insurance policy displaying full (marginal) coverage above a variable deductible, the additional financial capacity is first allocated to the policyholders suffering the highest losses through a decrease in the deductible.

The impact of the post disaster financial assistance on the performance of the two participating insurance policies is illustrated using our previous hypothetical scenario. Results are presented in Table 6. Obviously, the higher this additional financial capacity provided by the government, the higher (lower) the *ex post* variable coinsurance rate (deductible) and the lower the welfare loss of the pool compared to the first-best optimal participating insurance policy with variable premium. When the additional capacity is 20 (60), Table 6 shows that the welfare loss is 14.50% (6.81%) under the variable coinsurance policy and it is 2.32% (0.33%) under the variable deductible insurance contract. This illustrates the fact that this additional financial capacity is used in priority to pay additional indemnities to the policyholders suffering the largest losses; this contributes to reduce the difference in the policyholder’s final wealth. Under variable deductible insurance, there is no welfare loss compared to the first-best optimum when the additional capital generates an *ex post* variable deductible which is less than the lowest loss among the policyholders. It is noteworthy that there is no welfare loss under variable coinsurance only when the additional financial capacity allows to pay full indemnities. Therefore, when this minimum loss is positive, the first-best solution can be replicated using the variable deductible insurance with a lower additional capacity than that under variable coinsurance.

[INSERT TABLE 6 HERE]

5. Conclusion

The financing of catastrophe risks, caused by natural or man-made disasters, is a growing problem for both insurance and financial markets. The introduction of innovative insurance-linked securities is a response to the dramatic increase in catastrophe losses over the last two decades and to the increasing recognition that the insurance and reinsurance industry is unable to provide efficient mechanisms for financing losses caused by low frequency, high consequence events. However, these new financial products provide limited coverage and they are currently almost only available for losses caused by natural catastrophes. As a consequence, the financial resources of the insurance and reinsurance industry, including all internal and external market-based solutions to raise new capital, may be insufficient to pay all valid claims in full in the event of a cataclysm. Such a threat of insolvency is widely mentioned in the literature on the financing of catastrophic risks, quantified in recent studies (Cummins, Doherty and Lo 2002) and explicitly recognized in some current catastrophe insurance pools, like the California Earthquake Authority and the Turkish Catastrophe Insurance Pool. However, to our knowledge, there is no studies that precisely examine an optimal allocation of risks with limited capital.

This paper first analyses an efficient risk sharing rule under a threat of default on payment. The mutuality principle defined by Borch (1962) and the efficient allocation of aggregate risk characterized by Wilson (1968) are shown to hold within a *sub-pool* of agents who decide to stay in the pool following the occurrence of a cataclysmic event. The efficient risk sharing rule is reconsidered in the context of the insurance market. An optimal participating insurance policy is shown to display full insurance above an *ex post* variable deductible. The optimal level of deductibility is such that the total claims paid to the policyholders and the financial resources of the insurance pool are equalized. Consequently, the individual indemnity schedule depends on the realized losses of *all*

[12] It is noteworthy that the natural disaster compensation system in France is not based on a reinsurance pool. Insurers are directly reinsured at the public reinsurance company CCR (Magnan 1995).

members of the pool. This risk sharing arrangement contrasts with the current structure of current catastrophe insurance policies based on pro rated indemnification, i.e., *ex post* variable coinsurance. This innovative insurance contract design allows us to highlight the role of catastrophe insurance pools are market enhancing instruments. With the same financial capacity, they may improve the social welfare by providing a better coverage for larger losses and/or a larger number of agents.

We hope that our results on the financing of catastrophic/cataclysmic risks with limited capital will offer useful information to designers of future catastrophe/cataclysm insurance programs.

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Appendix

Terrorist insurance system in France

GAREAT (Gestion de l'assurance et de la réassurance des risques attentats et actes terroristes) is a co-reinsurance pool created jointly by insurers, reinsurers and a public reinsurer CCR in order to cover terrorist acts in France. Coverage of these risks has been mandatory in France since 1986.

The claims paying capacity of this pool is unlimited. It is decomposed into four layers. The first layer of €250 million is the pool retention. The second layer of reinsurance is of €750 million. The third layer is an option of state loan of €500 million through the public reinsurer CCR. Finally, the fourth layer provides an unlimited state cover through the CCR. Insurers can purchase a partial coverage with a minimum of 20% of the sum insured and €20 million.

Losses are covered by this co-reinsurance pool if the following two criteria are satisfied: (i) the sum insured for direct damage and business interruption exceed €6 million, (ii) risks are located in the French territory (metropolitan France, DOM, TOM and Mayotte). The main exclusions are personal lines, bodily injury, liability policies, pure financial loss arising under technical risk lines, risks covered by transport, marine and aviation policies, nuclear weapons and devices.

The 2002 premium income is estimated at €200 million. They are distributed among the first layer (30%), the second layer (50%) and the third layer of reinsurance (20%).

			No insurance	insurance				
Agents	Initial wealth	Loss	Final wealth	Prepaid premium	Indemnity	*Ex post* premium	Net Indemnity[1]	Final wealth
A	100	20	80	10	20	35	-15	55
B	100	30	70	10	30	35	-5	55
C	100	40	60	10	40	35	5	55
D	100	50	50	10	50	35	15	55
E	100	60	40	10	60	35	25	55
F	100	70	30	10	70	35	35	55
Total	600	270	330	60	270	210	60	330

[1]indemnity net of *ex post* premium adjustments.

Table 1. Insurance contract design with *ex post* variable premium.

				Ex post variable coinsurance		*Ex post* variable deductible	
Agents	Initial wealth	Loss	Prepaid premium	Indemnity	Final wealth	Indemnity	Final wealth
A	100	20	10	4.44	74.44	0.00	70.00
B	100	30	10	6.67	66.67	0.00	60.00
C	100	40	10	8.89	58.89	0.00	50.00
D	100	50	10	11.11	51.11	10.00	50.00
E	100	60	10	13.33	43.33	20.00	50.00
F	100	70	10	15.56	35.56	30.00	50.00
Total	600	270	60	60.00	330.00	60.00	330.00

Table 2. Insurance contract design when capital is limited.

CRRA coefficient	*Ex post* variable coinsurance	*Ex post* variable deductible
1	0.77	0.22
2	6.50	1.71
3	21.01	4.96
4	46.19	9.66
5	86.39	15.73
6	148.44	23.15
8	386.16	42.01
10	932.94	66.72

Table 3. Welfare loss of the pool compared to the first-best optimum (in percentage).

Prepaid premium	*Ex post* variable coinsurance		*Ex post* variable deductible	
	Coinsurance rate (%)	Welfare loss[1] (%)	Deductible	Welfare loss (%)
5	11.11	29.21	50.00	12.25
10	22.22	21.01	40.00	4.96
15	33.33	14.66	32.50	1.77
20	44.44	9.76	26.00	0.45
25	55.56	6.05	20.00	0.00
30	66.67	3.32	15.00	0.00
45	100.00	0.00	0.00	0.00

[1]Loss in welfare compared to the first best optimum (in percentage).

Table 4. Welfare loss of the pool, compared to the first-best optimum, and optimal *ex post* variable coinsurance rate and deductible.

	Rate of coinsurance	Deductible	Upper coverage	*Ex post* premium adjustment[1]	*Ex post* rate of coinsurance adjustment
California Earthquake Authority (Jaffee and Russell 2000)	100%	10 to 15%	\$100,000	up to 20%	$a = L/(K + \Delta P)$
Hawaï Hurricane Relief Fund (Jaffee and Russell 1997)	100%	1 to 10%	\$750,000	up to 5%	$a = L/(K + \Delta P)$
Turkish Catastrophe Insurance Plan (Gurenko 2000)	100%	2%	\$30,000	no	$a = L / K$

[1] In percentage of the prepaid premium.
Note: L: aggregate insured losses. K: financial capacity of the pool (including prepaid premiums). ΔP : aggregate *ex post* premium adjustment.

Table 5. Indemnity schedules proposed by catastrophe insurance pools with limited liability.

Post disaster financial assistance	*Ex post* variable coinsurance		*Ex post* variable deductible	
	Coinsurance rate (%)	Welfare loss[1] (%)	Deductible	Welfare loss (%)
20	29.63	14.5	35.00	2.32
60	44.44	6.81	26.00	0.33
90	55.56	3.65	20.00	0.00
140	74.07	0.97	11.66	0.00
180	88.89	0.15	5.00	0.00
210	100.00	0.00	0.00	0.00

[1]Loss in utility compared to the first best optimum (in percentage).

Table 6. Welfare loss of the pool, compared to the first-best optimum, and optimal *ex post* variable coinsurance rate and deductible.