

AMBIGUITY AND THE INSURANCE OF CATASTROPHIC LOSSES

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January 25, 2003

INTRODUCTION

In a simple insurance contract the policyholder pays a premium in return for a contingent payment from the insurer triggered by a specified event. The supply and demand are conditioned on information on the loss distribution. The policyholder's needs to know whether her reservation price is met (in effect whether the premium is reasonable given the loss distribution risk). And the insurer needs to charge a premium to cover its cost of bearing risk and meet its solvency constraints which also requires knowledge of the loss distribution. Indeed, such knowledge has been identified as a pre-condition for insurance (Berliner). But what happens if the loss distribution is not known? Can insurance markets still function when losses are ambiguous?

Ambiguity is a characteristic of many catastrophic losses. To be sure, the level of ambiguity is not exogenous. Research on natural hazards by modeling firms has vastly improved the estimation of distributions for earthquake and windstorm damages in the 1990's. This investment in modeling came as a response to the needs of the insurers and reinsurers faced with the unprecedented cost of events such as hurricane Andrew and the Northridge earthquake. Terrorism exposure is currently highly ambiguous. The distribution itself can be thought of as the outcome of a non cooperative game against an intelligent and malicious player. And while modeling firms are trying to improve information on terrorist risk, it is clear that this task involves not only data, but development of fundamentally new modeling and estimation techniques. In the meantime, insurers are trying to set premiums, and firms setting their risk management strategies, with only vague ideas of the expected losses.

Our task is to see whether market insurance can occur naturally under conditions of ambiguity and, if so, what contractual and market structure should it assume. We will also address whether the parties to such ambiguous will invest efficiently in loss mitigation. Our focus will be on *ex ante* ambiguity (the inability to specify the loss distribution) rather than *ex post* ambiguity (the difficulty in observing the value of a loss that has already occurred. We will examine a relationship between an insurance company and a reinsurer in which they bargain for coverage against a catastrophic loss such as earthquake, hurricane or major terrorist loss. While previous research has addressed the question as to whether ambiguity would impede the gains to risk sharing between risk averse parties, (Kunreuther et al) we will examine whether ambiguity alone will stimulate risk sharing between risk neutral parties. To do this we look to a risk neutral reinsurer and risk neutral primary insurers who engage in Nash bargaining on sharing an ambiguous risk. We will use Schmeidler's axiomatically derived decision rule to characterize costs of ambiguity to the respective parties. In this rule, the certainty equivalent of

an ambiguous prospect is a linear weighting of its expected value and its worst case scenario.

Our main results are these. Far from impeding insurance contracting, ambiguity alone can provide a sufficient basis for risk sharing of catastrophic losses. The gains from risk sharing between the parties cover a wide range of parameters and derive from two mechanisms. The first is differences in ambiguity aversion between the primary insurer and the reinsurer. Greater ambiguity aversion on the part of the latter motivate reinsurance. This gain is quite intuitive. If the reinsurer dislikes ambiguity less than the insurers, a price can be found such that both gain from the transfer of risk. The second mechanism arises from the catastrophic nature of the loss. Unless the catastrophic event hits all primary insurers to the same degree, the reinsurance mechanism can be used diversify risk within the catastrophic loss state. The reason this provides a basis for trade between risk neutral parties is as follows. The Schmeidler theory focuses attention partly on the worst case. But for a catastrophic event in the worst case differs between the reinsurer and the insurer. Imagine a catastrophic event that will impact some insurers but not others. The worst case for the reinsurer is that the catastrophe occurs and she does not care which primaries are hit. The worst case for any individual primary insurer is that the catastrophe occurs AND it impact him. This difference between the worst state implies that both can gain from a transfer payment in which the primaries not hit by the catastrophe, pay an additional premium to help the reinsurer compensate those insurers that were hit. This additional premium, or assessment, lowers the aversion of the reinsurer to provide reinsurance protection for the ambiguous event.

The opportunity for such diversification within the catastrophic loss state provides the intuition for the third result. The optimal form of the contract is mutual insurance in which insurers pay a premium in the no loss state in return for compensation should they suffer loss. However, insurers are also retroactively assessed according to the aggregate losses faced by the reinsurer. This result follows the long tradition mutualization in the insurance literature dating back to Borch (1962) who showed that such a structure provides a general equilibrium in markets for contingent claims which result later evolved into the capital asset pricing model.

II. INSURANCE MUTUALIZATION UNDER CORRELATION AND AMBIGUITY,

Large losses, such as natural disasters and terrorist attacks, create a number of problems for insurance markets. The first of these, lack of independence, is well known and intensively analyzed. While perfect risk spreading is not possible, a second best solution can be achieved through decomposition of risk. Diversifiable risk can still be distributed widely through the insurance pool with un-diversifiable risk left with the policyholders. This process, mutualization, was first prescribed by Karl Borch in 1962 and forms the basis for organizing other markets such as the capital market.

Catastrophic losses are such as earthquakes, hurricanes and major terrorist events, such as 9/11, produce correlated losses within an insurance portfolio. The same events affects many policyholders. Thus the law of large numbers does not operate and, no matter how many correlated policies the insurer holds, the relative risk will not converge to zero. The insurer is always left with un-diversifiable risk which will be higher the higher the correlations. The

problem of designing insurance markets with correlated risk was addressed in the seminal paper of Borch “Equilibrium in a Reinsurance Market”(1962).¹ Borch showed that, in equilibrium in a market for contingent claims (such as the reinsurance) all players will hold the same portfolio and therefore individual assets will be priced according to their correlation with that portfolio. This has come to be known as the mutualization principle because, in effect, the market organizes as though there were one massive mutual insurance firm. This is easily seen when one considers that the policyholders are the owners of a mutual. With such a mega mutual, all policyholders insure with the mutual, thus hedge away all idiosyncratic risk. However, their ownership of the mutual gives them a residual equity interest in the aggregate social loss. Thus all policyholders (qua shareholders) end up with a *pro rata* share of the same loss portfolio.

Under risk aversion and correlation, the mutual form has the advantage of reducing the insurer’s aversion to insure losses that it cannot completely diversify away because it gets a “second chance” to pass those losses back to policyholders in the form of a dividend adjustment. Since all shareholders also are policyholders, the dividend adjustment is effectively a retroactive premium assessed after aggregate losses are known. To see intuitively why this same concept should be useful for ambiguous losses, consider the two familiar features of the insurance mechanism. First all policyholders contribute *pre-paid* premiums and, second, from this premium pool the insurer is able to make transfers from those who do not, to those who do, suffer loss.

to why conventional insurers might be reluctant to insure losses when they have scant knowledge of the loss distribution. The problem is that, if they do not know the expected loss, how can they calculate an appropriate premium? The problem arises *only* if we consider the insurance premium to be pre-paid. If the premium is paid after the losses are realized, this problem all but disappears.² Ambiguity does not interfere with the more fundamental insurance mechanism of spreading risk. Thus, if a catastrophe hits some policyholders but not others, insurance can still be used to make transfers between those who did, and did not, suffer loss.

Then intuition of the previous paragraph, suggests that ambiguity should not provide a real obstacle to writing insurance contracts, but only should challenge the pre-payment of premiums. We will go further and show that ambiguity can itself be motivation for insurance contracting. Indeed we will show that, even with risk neutral insurers, ambiguity aversion can be a sufficient basis for parties to write risk sharing contracts.

III. CONTRACTING UNDER CONDITIONS OF AMBIGUITY

Contracting theory enjoys a rich literature in economics, law and related disciplines. The classical contracting approach assumes complete and common knowledge about the probability

¹(see Marshall 1974, Gollier 1992, Dionne and Doherty 1993 and Chichilnisky and Heal 1998 for later development of this idea in insurance.

²Retroactive premium might encounter credit risk, but this can be tackled by capital requirement, or by deposit premiums.

distribution governing uncertain states of the world. Based on this distribution, and on the preferences and actions of the parties to the contract, optimal contingent actions and payments are the subject of the resulting contract design problem. This model has been advanced in important ways in the Principal-Agent literature (e.g., Holmstrom (1979), Laffont and Tirole (1988)), and has been a subject of continuing interest in the insurance literature (for a recent review, see Doherty and Mahul (2001)). This was further enriched in the literature on law and economics, e.g., through the work of Williamson (1975) and Klein, Crawford and Alchian (1978). This literature has made great progress in clarifying the incentives for participating in risk sharing and in investments in joint activities among contracting parties.³ However, while this work addresses issues of opportunism (a.k.a. moral hazard) arising from unobservability and information problems, the impact of ambiguity has not received much attention in the literature on the theory of contracts, either in general or in the insurance context of interest here. Rather, the standard assumptions are that parties share a common-knowledge distribution that captures entirely all uncertainties of interest. This may be a reasonable presumption when the transactions of interest involve repeated opportunities for learning, feedback and exchange of information about the valuations held by the parties, but it is unlikely to be reasonable in the area of catastrophic risk, where the processes and actions of the parties giving rise to losses are complex and often unobservable, and where by their very nature feedback in the form of real events is relatively rare. From what we know about the consequences of such ambiguity on real behavior in insurance markets and in the laboratory (e.g., Kunreuther et al. (1993, 1995), Heath and Tversky (1991), Fox and Tversky (1995)), it seems a useful exercise to examine the likely consequences of the existence of significant ambiguity on contracting behavior for risk sharing involving catastrophic losses.

In the next section, we develop a model along the following lines. Two parties, denoted I and R , wish to contract to share the losses that might result from a risk. We think of I as a direct or primary insurer and R as a reinsurer. The losses L are a function of the uncertain state of the world \mathcal{Z} and are denoted by $L(x, y, \mathcal{Z})$, where $x = (x, y)$ is a vector *ex ante* and *ex post* investments by the original bearer of the risk. Parties are assumed to bargain about the nature of the risk transfer and payments and responsibilities before the fact. If they are unable to reach agreement, no risk transfer occurs. If they do, then we assume that the contract they sign is completely enforceable *ex post*. However, some *ex post* moral hazard is assumed in that the insurer I can influence the magnitude of the actual losses *ex post* through exerting effort y (in the insurance context, one might think of this as the effort to accurately assess and control the cost of claims). Both I and R are assumed to be ambiguity averse in a sense to be made more precise below. We examine the impact of ambiguity on the contracting behavior and the efficiency of agreements between I and R about risk mitigation and risk transfers. We only consider here a simple case in which there are only two states of the world, “loss” or “no loss”, in order to keep the level of mathematical detail to a manageable level. The basic results obtained are that differences in ambiguity aversion will affect both the *ex ante* likelihood of coming to agreement between I and R and the level of investments they make, *ex ante* and *ex post*, in controlling and mitigating

³ See also Gul (2001) for a review of recent work on the impact of observability on the outcome of bargaining problems. Gul analyzes, in particular, the consequences of observability and the structure of the bargaining process on the efficiency of outcome-enhancing investments.

losses.

To understand the nature of the model development, it is important to note the approach we use to represent ambiguity and the preference structure of our contracting agents in the face of such ambiguity.⁴ The study of ambiguity began with Frank Knight's distinction between conditions of certainty, risk and uncertainty, with the latter reflecting imperfect knowledge even of the distribution that might reflect unknown states of the world when decisions are made. The term "ambiguity" became prominent with the well-known paper of Ellsberg (1961). Over the intervening years, ambiguity has come to be understood as reflecting a decision context where any of a set of probability distributions might reasonably be argued to govern the outcomes of random states of the world, and where experts or other sources of information relating to the context are unable to provide accurate information about which of these probability distributions is the "right one". Just as in Savage's extension of von Neumann-Morgenstern expected utility theory, the central question in axiomatic foundations of decision making under uncertainty is linking preference representation to beliefs, where in this case beliefs encompass the degree of subjective uncertainty about which probability distribution from the set of feasible such distributions might obtain. The simplest such axiomatic approach will be pursued here, namely that following Schmeidler (1989) in which a non-additive probability measure is associated with the uncertain states. Schmeidler's theory can be shown to imply that the decision maker will undertake a von Neumann-Morgenstern-Savage expected utility evaluation of alternatives, but will do so at the least favorable distribution from among those believed to be reasonable. This calculus can be captured in various ways in general, but we will only be concerned with a two-state example here, so the theory is easy to state and understand. In the two-state world of interest, the effect of the ambiguity theory considered here is that I and R weight the ex post consequences of loss scenarios more heavily than they would absent ambiguity. This is intuitive since the model adopted here is that ambiguity gives rise to an assessment based on the least favorable distribution. However, the consequences of this higher weighting of ex post effects is more subtle than might be suspected at first. It affects not only the ex ante investments in loss reduction, but also interacts with ex post moral hazard as well as the likelihood that I and R will actually be able to come to a profitable agreement about the terms of risk transfer in the first place.

⁴For a review of the normative theory of choice under conditions of ambiguity, see Schmeidler (1989) and Dow and Werlang (1992). For a review and synthesis of the descriptive and experimental literature on ambiguity, see Einhorn and Hogarth (1986) and Fox and Tversky (1995). For applications of ambiguity in the insurance context, see Kunreuther et al. (1993, 1995). Our approach applies the Schmeidler theory to the insurance setting in a bilateral bargaining setting between an assumed insurer (or property owner) and reinsurer (or insurer) who bargain about the terms of sharing an ambiguous risk.

IV. A BARGAINING MODEL FOR RISK TRANSFER UNDER AMBIGUITY

The Basic Model

A group of m identical primary insurers $i \in I$ and a reinsurer R bargain about the possible transfer of a risk from I to R . The states of the world S underlying the definition of the risk are assumed to be uncertain, with ambiguity about the probability of any state occurring captured by a non-additive probability measure P_c defined as in Schmeidler (1989) as follows. For some $c \in [0, 1]$:

$$P_c(S) = 1; \quad P_c(s) = (1-c)p(s), \quad s \in S \quad (1)$$

where $p(s)$ is some proper probability measure defined on S (i.e., $p(s) \geq 0, \sum_s p(s) = p(S) = 1$), where the parameter “ c ”, which is specific to each decision maker, reflects the degree of ambiguity in that decision maker’s beliefs.

For example, in a two-state world in which with $S = \{\text{LOSS}, \text{NO-LOSS}\}$, let $p \in (0,1)$ be the reference probability of the LOSS state. Then the probability measure P_c would be defined as $P_c(\text{LOSS}) = (1-c)p$ and $P_c(\text{NO-LOSS}) = (1-c)(1-p)$. Note that the probability measure P_c is non-additive when $c > 0$, since $P_c(\text{LOSS}) + P_c(\text{NO LOSS}) = (1-c) < 1$.

To keep matters simple, we assume that all agents ($i \in I$ and R) are risk-neutral with possibly different degrees of ambiguity (i.e., different parameters “ c ”) about the resulting loss. Applying the Schmeidler theory to a probability distribution of the form (1), a risk-neutral agent whose beliefs are represented by (1) will maximize a weighted average of the worst possible outcome and the expectation of the additive distribution that arises when there is no ambiguity, with the weight given by $(1-c)$ and c . Thus, in the two-state example of the preceding paragraph, if the final wealth under the LOSS and NO-LOSS states are W_L and W_N , respectively, then the certainty equivalent of this ambiguous lottery with beliefs captured in (1) would be:

$$CE = (1-c)[pW_L + (1-p)W_N] + cW_L \quad (2)$$

We will apply this very simple structure throughout to capture the beliefs of I and R about the results of various contracts. Note that when $c = 0$, the certainty equivalent CE in (2) reduces to the expected value of the lottery at the reference probability distribution determined by “ p ”.

We imagine that a group of primary insurers I and a reinsurer R are engaged in bargaining about the terms and conditions that should apply to the transfer or sharing of an ambiguous risk with loss $L(x, y)$, with x and y investments by each of the identical insurers $i \in I$ made *ex ante* and *ex post* respectively to mitigate the loss, i.e. $L(x, y)$ is decreasing in x and y . We model this process via the Nash Bargaining Solution with the default outcome (i.e., the outcome resulting if no agreement results from this bargaining) as the payoffs arising when I retains the entire risk. Otherwise, we assume a perfectly enforceable contract obtains, which specifies the mitigation investments, x , that are to be made *ex ante* by I . It is assumed here that both parties understand that each agent $i \in I$ will choose an investment, y , that maximizes i ’s profits *ex post* given the (perfectly enforceable) terms of the contract. Thus, denoting the

payoffs, to be specified more fully below, from the contract and the default option as A_i and A_{i0} , respectively, for $i \in \{I, R\}$, agents I and R are assumed to choose a contract that maximizes the Nash Product:

$$\left[\prod_{i \in I} (\Pi_i - \Pi_{i0}) \right] \times [\Pi_R - \Pi_{R0}] \quad (3)$$

Now consider the states of the world underlying the ambiguous risk of interest. There is an uncertain probability p that an “event” will occur such as a terrorist attack. The total social cost of such a loss will be L and some proportion of this will be reinsured by reinsurer R . Conditional on the occurrence, an individual insurer I may suffer a loss of a . The loss to the insurer can be mitigated by ex ante investments in safety by the primary insurer, x , and by ex post investment, y , thus $a = a(x, y)$. The ex ante investment a , is observable by the reinsurer so there is no ex ante moral hazard. However, there is ex post moral hazard because y is not observable and thus the contract cannot be conditioned upon y . We are interested in the mutualization of this risk as a function of the size of the group, i.e., the number, of primary insurers m . The reader may imagine a coastline along served by the m primary insurers, and reinsured by R . If an event occurs, it a social loss $L(x, y)$ will occur, with an insured loss of $a(x, y)$, and this will occur to one and only one of the primary insurers. To take the simplest possible case, we assume that each primary insurer faces the same probability of loss, conditional on the occurrence of L . Thus, defining the conditional probability of a as $B = q(a^*L)p = q^c p$, we assume $q(a^*L) = 1/m$. Thus, each primary insurer faces one of three states: (NO-LOSS), (LOSS-L & NO-LOSS-a), (LOSS-L & LOSS-a).

The reinsurance contract requires that the reinsurer pay to the primary T_a if the primary suffers loss a . T_a is expressed as the net payment in this state (i.e. net of any premium). However, the primary must pay a premium of $T_N \geq 0$ if event L does not occur and T_n if L occurs but a does not. Using Schmeidler’s theory, the certainty equivalent of profit for each risk neutral insurer $i \in I$ is the weighted average of its expected profit and its worst case outcome:

$$\begin{aligned} A_i &= (1-c_i) \left(w_i - x - pq^c(a + y - T_a) - (1-p)T_N - p(1-q^c)T_n \right) + c_i \left(w_i - x - (a + y - T_a) \right) \\ &= (1-c_i) \left(w_i - x - p(a + y - T_a)/m - (1-p)T_N - p(m-1)T_n/m \right) + c_i \left(w_i - x - (a + y - T_a) \right) \end{aligned} \quad (4)$$

Here, assuming identical primary insurers $i \in I$, c_i is the measure of each primary insurer’s aversion to ambiguity with $c_i = 0$ defining indifference to ambiguity and $c_i = 1$ denoting extreme ambiguity aversion. In the latter case, the insurer looks only to the worst case. We will add the restriction $T_n < T_a$. Thus, the assessments for those insurers not suffering loss in state L are lower than the losses by other insurers. Thus, the worst state for any insurer is that L occurs and the insurer suffers a loss of a .

Note that each insurer is exposed to loss $a(x, y)$ resulting from L , with the conditional probability of loss identical for all insurers. However, these insurers’ portfolios are not identical and, for any given event L , exactly one of these insurers will suffer a loss and all others will not.

So, for the reinsurer who reinsures a large number m of primary companies, its total expected loss payouts, conditional on the event, is $m q(a^*L) (T_a) = (m/m)T_a = T_a$. Assuming that the reinsurer writes similar contracts with all primary insurers, we can write the reinsurer's certainty equivalent (*for all reinsurance contracts issued*) after Schmeidler as:

$$A_R = (1-c_R) (w_R - pT_a + (1-p)mT_N + p(m-1)T_n) + c_R (w_R + (m-1)T_n - T_a) \quad (5)$$

Notice that the reinsurer can spread risk in the event of L . Some insurers (in fact precisely one in our model) will have claims and others will not. Since the reinsurer is risk neutral, its expected payoff in the "worst" state is the difference between the expected claims and the expected premiums paid in state L , i.e. $(1-q^c)T_n - q^cT_a = [(m-1)T_n - T_a]/m$. Notice we are assuming that the loss state L cannot result in a gain for the reinsurer, $(1-q^c)T_n - q^cT_a \leq T_N$, or equivalently $[(m-1)T_n - T_a] \leq mT_N$. We will also assume that $T_a \leq a(x, y)$, the loss to the primary insurer (after mitigation, if any). The former constraint assures the worst state for the reinsurer is when the loss L occurs. We will refer to this constraint also as the "mutualization constraint" since (with $T_a \leq a(x, y)$) it implies an upper bound on mutualization payments T_n .

The default options for the contract bargaining problem among I and R are given by:

$$A_{IO} = \text{MAX}_{x \geq 0, y \geq 0} (1-c_I) (w_I - x - p(a+y)/m) + c_I (w_I - x - a - y) \quad (6)$$

$$A_{R0} = w_R \quad (7)$$

where, in (6), x is the level of *ex ante* investment (assumed observable) and y is the level of *ex post* investment by I in the event of a loss when *ex ante* investments by I are x .

Whether or not default occurs, we assume that y is unobservable, and therefore is chosen by each i OI to minimize overall costs *ex post*, i.e.,

$$y^* = y(x) = \text{argmin}_{y \geq 0} [a(x, y) + y - T_a] \quad (8)$$

with F.O.C. for optimality

$$FOC(y): \frac{\partial a}{\partial y} \left(1 - \frac{\partial T_a}{\partial a} \right) + 1 = 0 \quad (9)$$

Similarly, the level of *ex post* investment by I in the default state is characterized by:

$$y_0^* = y_0(x) = \text{argmin}_{y \geq 0} [a(x, y) + y] \quad (10)$$

$$FOC(y_0): \frac{\partial a}{\partial y} + 1 = 0 \quad (11)$$

The above bargaining framework provides a number of insights about the impact of ambiguity on the risk transfer contract between the primary insurers I and R . We optimize the Nash Product (3) in the standard fashion,⁵ subject to the individual rationality constraints $A_i \geq A_{i0}$, $i \in \{I, R\}$. Rather than prove these formally, we simply state these as intuitive results here and interpret them in terms of the framework presented earlier. Because of the complexity of this problem, we will examine a number of special cases. In the first case we will assume that $q=1$, so the state space collapses to two states.

Nash Bargaining with No Diversification in the Loss State, $q^c=1$

In the first special case, suppose there is only one primary insurer so that if the loss state L occurs, then so does the loss state a , i.e. $m = l = q^c$. In particular, the state “ n ” (L but not a) never occurs. Thus, using (4)-(7) and eliminating T_n , the Nash product in (3) can be written in the form:

$$\text{Maximize } [f - b T_N] [g + d T_N] \quad (12)$$

$$\text{where } b = (1 - c_I)(1-p) \quad (13)$$

$$d = (1 - c_R)(1-p)$$

and where f and g are all the remaining terms in (4)-(5), which we note do not depend on T_N . Thus,

$$f = A_I - A_{I0} + (1 - c_I)(1-p) T_N \quad (14)$$

$$g = A_R - A_{R0} - (1 - c_R)(1-p) T_N$$

It is easily verified that the solution for T_N maximizing (12) is unique and is given by:

$$T_N = \frac{df - bg}{2bd} = \frac{(w_I - x - [c_I + (1 - c_I)p][a(x, y) - y - T_a] - \Pi_{I0})}{2(1 - c_I)} + \frac{(w_R - [c_R + (1 - c_R)p]T_a - \Pi_{R0})}{2(1 - c_R)} \quad (15)$$

We can now return to our maximization problem (12) and substitute (15) for the optimal solutions for T_N , obtaining

⁵ The formal derivation assumes only that $a(x, y)$ is jointly convex and strictly decreasing in (x, y) , reasonable assumptions of the impact of mitigating investments on losses.

$$\begin{aligned}
\text{Maximize } & (1-c_R)(1-c_I)(w_I - x - p[a(x, y) + y - T_d]) \\
& + (1-c_R)c_I(w_I - x - [a(x, y) + y - T_d]) - (1-c_I)A_{I0} \\
& (1-c_I)(1-c_R)(w_R - pT_d) + (1-c_I)c_R(w_R - T_d) - c_RA_{R0}
\end{aligned} \tag{16}$$

subject to the *ex post* minimizing conditions characterizing x^* and y^* . Deleting constant terms and rearranging we obtain the following equivalent problem characterizing, with (15) the NBS:

$$\begin{aligned}
\text{Minimize } & (1-c_R)(x + [c_I + (1-c_I)p] [a(x, y) + y - T_d]) \\
& + (1-c_I) [c_R + (1-c_R)p] T_d
\end{aligned} \tag{17}$$

Proposition 1: *When $q^C = m = 1$, the Nash Bargaining contract will involve a positive transfer of wealth from the reinsurer to the insurer in the loss state, $T_a > 0$, if the insurer is more ambiguity averse than the reinsurer, $c_I > c_R$.*

The proof of the proposition arises from the following intuition. First note that the coefficient of T_a in (15) is

$$\text{Coefficient } (T_a) = (1-c_I)[(c_R + (1-c_R)p] - (1-c_R)[(c_I + (1-c_I)p] \tag{18}$$

A bit of algebra shows that this coefficient has the sign of $(c_R - c_I)$. Thus, minimizing (12) implies that $T_a = 0$ if $c_I \leq c_R$; and T_a is a maximum if $c_I > c_R$, where the maximum value for T_a is the value that would completely indemnify the insurer in the event of a loss, i.e. $T_a = a(x, y)$. We see for this simple case that the structure of the optimal transfer of risks between I and R , through contract features and transfer payments, depends fundamentally on the relative perceived ambiguities of these parties to the risk. The party with relatively less ambiguity makes lower transfers to the other party in the loss state, and relatively higher transfers in the “no-loss” state. The magnitude of the transfers is further influenced by the level of mitigation investment made by I , both *ex ante* and *ex post*.

Proposition 2. *When $q^C = m = 1$, the parties to the NBS will choose *ex ante* mitigation to solve:*

$$H(x, y^*(x)) = x + [c_I + (1 - c_I)p][a(x, y^*(x)) + y^*(x)] + \frac{c_R - c_I}{1 - c_R} T_a \tag{19}$$

In particular, in the presence of ambiguity, neither *ex ante* nor *ex post* investments in mitigation are socially optimal.

The proof of the proposition consists of rearranging the characterizing condition (17) by dividing by $(1-c_R)$, yielding (19), which is to be minimized, with $y^*(x)$ given by (8). Concerning the results of this optimization in comparison with the socially optimal levels of *ex ante* and *ex post* investment are given respectively by:

$$\hat{y}(x) \in \operatorname{argmin}[y + a(x, y)]; \quad \hat{x} \in \operatorname{argmin}[x + p(\hat{y}(x) + a(x, \hat{y}(x)))] \tag{20}$$

In general, it is evident that the solutions $y^*(x)$ and x minimizing (19) will not solve (20). As a special case, consider the case where I and R share sufficient information on the risks involved (e.g., via catastrophe modeling) that $c_I = c_R = c$. In this case, the NBS characterizing function H in (19) can be expressed as:

$$h(x,y) = x + [c + (1-c)p][a(x,y) + y] \quad (21)$$

In this case, the NBS solution leads to minimizing *ex ante* investment costs plus an ambiguity adjusted measure of *ex post* losses plus investment costs. Note that the coefficient $[c + (1-c)p] = p$ when ambiguity is 0, i.e. when $c = 0$, and otherwise exceeds p . Note that even in this case $y^*(x) \dots y^{\wedge}(x)$, and therefore $x^{\wedge} \dots x^*$. As expected, the impact of ambiguity is to focus greater attention on *ex post* losses and investment costs. At the same time, *ex post* moral hazard becomes relatively more important under ambiguity as well.

Finally, it is worth noting that the above analysis has proceeded entirely on the basis of the presumption that I and R are able to reach agreement, rather than defaulting. It should be clear from the nature of the problem here that the key elements leading to agreement on feasible transfers are the magnitude of moral hazard by I (as reflected in the efficiency of the choice $y^*(x)$) and the difference in relative ambiguity aversion. As a benchmark case, if $c_I = c_R = c$, then it is intuitive that no agreement can take place.

Nash Bargaining with Diversification in the Loss State, $q^c < 1$

While catastrophic events typically impact a large number of parties, they do not directly affect everyone or, more generally, they affect different parties to different degrees. The non-uniform impact permits the reinsurer to diversify even within the catastrophic loss state. As we will show, this permits gains from risk sharing by risk neutral, ambiguity averse parties, even if their aversion to ambiguity is equal. Some insurers will suffer a loss of $a(x, y)$ when the catastrophic event L occurs but others will escape with no loss. The conditional probability of suffering loss $a(x, y)$ for all insurers is $q^c < 1$. Thus, all insurers now have identical *ex ante* ambiguity adjusted certainty equivalents as shown in (2). We assume $m > 1$ so that $q^c = 1/m < 1$. Notice that our case is simplified and allows precisely one primary to suffer a loss a should the state L occur.

Noting that all primary insurers are identical, we simply denote the generic such by I , leading from (3) to the following Nash product maximization problem of interest:

$$\text{Maximize } \Gamma(T_a, T_n, T_N, x) = [\Pi_I - \Pi_{I0}]^m \times [\Pi_R - \Pi_{R0}] \quad (22)$$

Subject to:

$$(m - 1)T_n - T_a - mT_N \leq 0 \quad (\text{mutualization constraint}) \quad (23)$$

$$T_a - a(x, y^*(x)) \leq 0 \quad (\text{no over insurance}) \quad (24)$$

$$T_a, T_n, T_N, x \geq 0 \quad (25)$$

with profits for I and R given by (4)-(7). The mutualization constraint needs some explanation. It states that the reinsurer should not be better off in the loss, L , state. If binding, this constraint leaves the reinsurer with revenues precisely matched in the “NO- L ” and “ L ” states.

Assume for the moment (we will show this below) that all decision variables are positive at the optimal NBS, solving (22) subject to (23)-(25). It is easily verified that the logarithm of the maximand in (27) is concave in T_a, T_n, T_N , so that on the relevant maximization space (where $f_I^* \geq bT_n$ and $g^* \geq dT_n$), the maximand is quasi-concave. Thus, associating non-negative Lagrange multipliers β and λ with constraints (23)-(24), we have the following FOCs characterizing the premium decision variables T_a, T_n, T_N :

$$(FOC:T_a) \quad \frac{(1-c_I)p + mc_I}{\Pi_I - \Pi_{I0}} - \frac{(1-c_R)p + c_R}{\Pi_R - \Pi_{R0}} + \lambda - \mu = 0 \quad (26)$$

$$(FOC:T_n) \quad -\frac{(1-c_I)p}{\Pi_I - \Pi_{I0}} + \frac{(1-c_R)p + c_R}{\Pi_R - \Pi_{R0}} - \lambda = 0 \quad (27)$$

$$(FOC:T_N) \quad -\frac{(1-c_I)(1-p)}{\Pi_I - \Pi_{I0}} + \frac{(1-c_R)(1-p)}{\Pi_R - \Pi_{R0}} + \lambda = 0 \quad (28)$$

From these, we have the following proposition, solving the case at hand.

Proposition 3: *Assume $c_I > 0$, $c_R > 0$ and $m > 1$. Then, assuming the NBS exists, it is unique and satisfies $T_a = a(x^*, y^*)$, $T_n^* = (mT_N^* + a(x^*, y^*)) / (m-1) > 0$, $T_N^*(T_a, 0) > 0$, with $y^*(0) = 0$ and x^* the solution to:*

$$x^* \in \arg \min_{x \geq 0} \left[x + \frac{(1-c_I)p}{m} a(x, 0) \right] \quad (29)$$

Moreover, if the NBS exists (for a given $m > 1$) when primary insurers I have ambiguity aversion c_p , then the NBS exists for any c_I' for which $c_I' > c_I$. The fact that $T_n^ > 0$ and $T_N^* > 0$ in the NBS means that mutualization is optimal whenever the NBS exists.*

We first note that both (23) and (24) must hold as equalities in the NBS. To this end, we need only show that the associated Lagrange multipliers β and λ are both positive. Adding (26) and (27), we obtain

$$\mu = \frac{(1-c_I)p + mc_I - (1-c_I)p}{\Pi_I - \Pi_{I0}} = \frac{mc_I}{\Pi_I - \Pi_{I0}} > 0 \quad (30)$$

where the final inequality follows from $c_I > 0$ and the assumption that the NBS exists (so that the denominator in (30) is strictly positive). Similarly, adding (27) and (28) we obtain

$$\frac{(1 - c_I)}{\Pi_I - \Pi_{I0}} = \frac{1}{\Pi_R - \Pi_{R0}} \quad (31)$$

so that substitution of (31) for the first term in (27) yields

$$\lambda = \frac{-p}{\Pi_R - \Pi_{R0}} + \frac{(1 - c_R)p + c_R}{\Pi_R - \Pi_{R0}} = \frac{c_R(1 - p)}{\Pi_R - \Pi_{R0}} > 0 \quad (32)$$

where the final inequality follows from $c_R > 0$ and the assumed existence of a NBS. From (30) and (31), we conclude that (23)-(24) must be binding at the NBS. In particular, $T_a^* = a(x^*, y^*)$ so that, from (8), $y^*(x) = 0$ at the NBS, i.e., the primary insurer indulges completely in moral hazard choice ex post.

To obtain T_n^* and T_N^* , we first substitute from (24) to obtain $T_N = [(m-1)T_n - T_d]/m$ which can be used with $y^* = 0$ and $T_a^* = a(x^*, y^*)$ to obtain from (4)-(7) the following expressions for $A_I - A_{I0}$ and $A_R - A_{R0}$ at the optimal values of $T_N = [(m-1)T_n - T_d]/m$, y^* and $T_a^* = a(x^*, y^*)$:

$$\Pi_I - \Pi_{I0} = \left[w_I - x - (1 - c_I)(T_N + pa(x,0) / m) - \Pi_{I0} \right] \quad (33)$$

$$\Pi_R - \Pi_{R0} = mT_N \quad (34)$$

Note, in particular, that since the mutualization constraint (23) holds as an equality the reinsurer's profit (5) reduces to the r.h.s. of (34). This also implies that if the NBS exists the associated solution value $T_N^* > 0$. The optimal solution for T_N is easily obtained now from substituting (33)-(34) into (22) to obtain:

$$\text{Maximize } \left[w_I - x - (1 - c_I)(T_N + pa(x,0) / m) - \Pi_{I0} \right]^m \times [mT_N] \quad (35)$$

Taking logarithms of the maximand in (35) one shows that the result is a strictly concave function in x and T_N , with FOCs for maximization leading to x^* in (29) and

$$T_N^* = \frac{w_I - x^* - (1 - c_I)pa(x^*,0) / m - \Pi_{I0}}{(1 - c_I)(m + 1)} \quad (36)$$

Thus, the solution to (22) is completely characterized and evidently unique, if it exists. Finally, suppose a solution to (22) exists for a given c_I . Then increasing this to c_I' leads clearly from (36) to an increase in T_N^* in (36), which from (34) reflects an increase in reinsurer's surplus. Moreover, substituting T_N^* in (36) into (33) shows that the generic primary insurer's surplus is also increasing in c_I . Thus, increasing ambiguity aversion of the primary insurer leads to a strengthening of the

NBS solution.

Let us now summarize the main results of this section.

- If at NBS exists, it is unique and it involves full insurance, $T_a^* = a(x^*, 0)$ and mutualization. The latter can be seen clearly in that

$$T_n^* = (mT_N^* + a(x^*, y^*)) / (m-1) > T_N^* \text{ if } m > 1$$

Thus, each primary will not suffering in the loss state, is retroactively assessed an additional premium of $T_n^* - T_N^* > 0$

- The more ambiguity averse the primary insurer, the higher the no loss premium T_N^* the primary is willing to pay which increases the reinsurer's surplus and his willingness to complete a bargaining contract.
- In our case of ambiguity aversion and mutualization, there is extreme *ex post* moral hazard, $y^*(x) = 0$. This happens in our case because the one insurer suffering the loss ($q^c = 1/m$), is fully insured and is required to make no *ex post* assessment to contribute to her loss. Any part of the loss not pre-funded through T_N is passed (through the reinsurer) entirely to the remaining $m-1$ insurers not suffering loss. In developing this model we could moderate the *ex post* moral hazard by requiring the unfortunate primary to bear some of the loss.

CONCLUSION

We challenge the notion that ambiguity is a barrier to insurance contracting. On the contrary, we have shown that ambiguity aversion might provide a sufficient basis for reinsurance contracting. The basis for trade arises from differences in ambiguity aversion and from the possibilities for the reinsurer to diversify within the loss state. Differences in ambiguity aversion alone can motivate reinsurance. However, we do not believe that such differences are entirely exogenous. We have mentioned that information on catastrophic losses is generated by modeling firms and by in-house research mostly by reinsurers. As a broad generalization, one can speculate that reinsurers have been the more intensive generators and users of such models and this implies that their aversion to ambiguity is lower than that of the primaries. These are indeed the conditions under which an NBS is likely to exist in our model. Ironically, more equality of access to models could lead to a breakdown in Nash bargaining. Of course, we are only looking at ambiguity aversion here and, with insurer access to modeling, reinsurance might also arise from risk aversion. The second pillar on which a NBS rests is the opportunity for the reinsurer to diversify within the loss state. This prospect leads naturally to a structure for the insurance market which follows the age old mutualization concept. But in the absence of ambiguity, the optimal form for the reinsurance market also followed mutualization. Thus, our results should be of some reassurance to those fearing market failure. The introduction of ambiguity not only reinforces the motivation for risk sharing and does not compromise the optimal contract form.

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