How Valuable Is Exchange Rate Flexibility? Optimal Monetary Policy under Sectoral Shocks

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Abstract

The paper explores the optimal monetary policy reaction to productivity shocks in an open economy. Whereas earlier studies assume that countries specialize in producing particular goods, I enrich the analysis by allowing for incomplete specialization. I confirm the finding of Obstfeld and Rogoff (2000) –who build on Friedman (1953) –that a flexible exchange rate is highly valuable in delivering the optimal response to country-specific shocks. Its value is, however, much smaller when shocks are sector-specific, because exchange rate fluctuations then lead to misallocations between different firms within a sector. The limitation on the value of flexibility is sizable even when specialization is high.

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1 Introduction

The central role of a flexible exchange rate in allowing economies to optimally adjust to shocks has received a renewed attention in the theoretical literature. Using a micro-founded general equilibrium model well suited for a welfare analysis, Obstfeld and Rogoff (2000a) analyze the optimal monetary policy response to productivity shocks in an open economy. Although one may conjecture a need for the monetary authorities to take the effect of their actions on other countries into account, Obstfeld and Rogoff show that this is not the case. They find that an inward looking monetary policy, in which the authorities are preoccupied solely with reacting to their domestic shocks, delivers the best possible outcome. A flexible exchange rate is a key piece of the puzzle as its impact on relative prices brings the economy to an efficient allocation, a confirmation of Friedman's (1953) insight that a flexible exchange rate can bring the economy around the obstacle of rigid prices.

A central assumption underlying the value of the exchange rate as an adjustment mechanism is that it affects the prices faced by consumers, who then adjust their consumption patterns. The impact of exchange rate fluctuations on consumer prices can however be limited, as pointed by Devereux and Engel (2000) and Engel (2001). The limited pass-through of exchange rate fluctuations into prices in turn significantly affects the optimal monetary policy. Corsetti and Pesenti (2001) undertake a detailed analysis of this aspect and find that an international dimension of monetary policy is restored when the pass-through is incomplete. In the extreme case where the exchange rate has no impact on consumer prices, they show that the optimal monetary regime is a pegged exchange rate. The best policy is to suppress the fluctuations in the exchange rate as they lead to no beneficial reallocation of consumption and only translate in volatile profits which lead firms to charge high prices.

This paper explores a second caveat to the benefits of exchange rate volatility. We maintain the assumption that exchange rate fluctuations are entirely passed-through to consumer prices and lead to a reallocation of consumption. Our emphasis is on the sectoral structure of the economy. All existing contributions are built on a restrictive assumption, namely that there is complete *sectoral specialization* among countries. Under the usual Dixit-Stiglitz specification of preferences, consumers allocate their purchases first across types of goods that are poor substitutes (textiles and cars for instance), and then across highly substitutable brands for each type. Under the prevalent assumption of complete sectoral specialization there is no distinction between the sectoral and international dimensions of the economy, as a particular type is produced only in one country. This is a serious limitation in light of the work by Stockman (1988) who points that a substantial share of economic fluctuations falls along sectoral, as opposed to national, lines.

Our model allows for incomplete sectoral specialization, where most, but not all, firms in a country produces brands of a given type, with the remaining minority producing brands of another term. The situation is mirrored in the foreign country.[1] Our setup allows us to contrast the optimal monetary policy reaction to *country specific* productivity shocks with the reaction to *sector specific* shocks.

We find that under country specific shocks the results of Obstfeld and Rogoff (2000a) remains valid, with monetary policy being inward looking and the flexible exchange rate delivering the efficient cross-country allocation. Things are however sharply different under sectoral shocks, with the value of a flexible exchange rate being quite small. Intuitively, the efficient response to a sectoral productivity shock is for the relative price between sectors to change, but for the relative price between firms in a given sector to remain constant. A flexible exchange rate cannot deliver both. As long as one country is predominant in one sector, the exchange rate can deliver the efficient relative price change between sectors. The same exchange rate fluctuation however alters the relative price between firms of different nationality producing the same type of good, which is inefficient. As the substitutability between different brands of a given type is large, the inefficient exchange rate fluctuations translate into sizable misallocation of resources. This cost leads the monetary authorities to keep the exchange rate fluctuations within strict bounds, substantially reducing the value of flexibility.

We show that the limits placed on the exchange rate fluctuations are significant even when sectoral specialization is very high, but incomplete. It would be misleading to thing that a highly specialized economy is sufficiently similar to a completely specialized one to justify ignoring the sectoral dimension. Such a conjecture would be based solely on the size of the sectors and ignore the substitutability between brands within a sector. As long as brands

[1]We maintain the usual assumption of complete specialization at the brand level, as each brand is produced only by one firm. Therefore, a country exports some brands of each type while importing other brands of the same type, but no brand is simultaneously produced in different countries.

are close substitutes, the inefficient exchange rate fluctuations lead to significant misallocation and cannot be ignored. A numerical illustration of our results shows that the value of a flexible exchange rate when specialization is very high but incomplete (with 95% of firms in a sector being located in a given country) is only half as high as when specialization is complete (with 100% of firms in a sector being located in a given country).

The paper also explores the interaction between monetary policy and firms, and looks at which combinations of the degree of sectoral specialization and exchange rate volatility are equilibria, taking an approach similar to Corsetti and Pesenti (2002). We find two possible equilibria. In the first there are no exchange rate fluctuations and no sectoral specialization. In the absence of exchange rate fluctuations firms are indifferent between producing a brand of one type or another, so a fixed exchange rate supports any sectoral structure. In particular it supports a structure where there is no specialization and all countries are identical. This structure in turn justify the maintaining of a fixed exchange rate. Another equilibrium is a case where monetary policy is inward looking, with large the exchange rate fluctuations, and sectoral specialization is complete. When the exchange rate fluctuates, the competitiveness of a firm vis-a-vis its competitors is more volatile if it competes against foreign firms than domestic firms. This induces firms to switch to producing the type of good that is already dominated by domestic producers. This leads to complete sectoral specialization, which in turn justify the inward looking monetary policy. We can show that the equilibrium with complete specialization and a flexible exchange rate is more beneficial than the equilibrium with no specialization and a fixed exchange rate.

The paper also makes a methodological contribution by using a general method. Following Sutherland (2001) we derive tractable second order approximations of the model around a deterministic steady state. This allows us to avoid the specific functional forms required to derive closed forms solutions, while still capturing the effect of second order moments on first order moments through the risk premia included in prices, an aspect that is lost under linear approximations.

The paper is organized as follows: section 2 presents the structure of the model. The solution is derived in section 3. Section 4 analyzes the optimal monetary response to productivity shocks, with section 5 providing a numerical illustration. Section 6 discusses the possibility of multiple equilibria, with section 7 concluding. Throughout the analysis we focus on the main results and leave the detailed derivations into an Appendix.

2 Structure of the model

2.1 Geographic and production structure

The geographic structure is taken from Tille (2002). The world is made of two countries, home and foreign. For simplicity we normalize the world size to unity and assume that both countries are of the same size, $1/2$. Each country is inhabited by a representative consumer, and is host to a continuum of firms with mass $1/2$. The model is characterized by monopolistic competition: there is a unit mass continuum of differentiated brands of consumption goods, and each firm is the sole producer of one particular brand.

Two types of goods are available for consumption, denoted by A and B (textiles and cars for example), with each type consisting of a continuum of brands of mass $1/2$. The central feature of this paper is to allow for a general production structure by not restricting each country to produce brands of only one type. Instead different brands from both types can be produced in both countries. More specifically we define the *degree of sectoral specialization* $\gamma \in [0.5, 1]$, assuming without loss of generality that the home country specializes to some extent in producing type A. We distribute firms and brands along a unit interval, with firms on the $[0, 1/2)$ interval being in the home country and firms on the $[1/2, 1]$ interval being in the foreign country. The home firms located on the $[0, 0.5\gamma)$ interval produce brands of type A, whereas the home firms located on the $[0.5\gamma, 0.5)$ interval produce brands of type B. Turning to the foreign country, firms located on the $[0.5, 0.5\,(2-\gamma))$ interval produce brands of type A, while firms located on the $[0.5\,(2-\gamma)\,, 1]$ interval produce brands of type B. The structure is summarized in Table 1:

Table 1: Production structure

Range	Country	Type produced	Mass
0 - 0.5γ	Home	A	0.5γ
0.5γ - 0.5	Home	B	$0.5\,(1-\gamma)$
0.5 - $0.5\,(2-\gamma)$	Foreign	A	$0.5\,(1-\gamma)$
$0.5\,(2-\gamma)$ - 1	Foreign	B	0.5γ

Our setup implies that a fraction $\gamma \in [0.5, 1]$ of firms in the home country produce brands of type A, and the same fraction of foreign firms produce brands of type B. Figure 1 illustrates various cases, with the left [right] squares representing the mass of firms in the home [foreign] country. The top panel illustrates the situation under complete sectoral specialization ($\gamma = 1$) where each country produces only one type. In the middle panel, sectoral specialization is partial ($0.5 < \gamma < 1$): most, but not all, home firms produce brands of type A, the situation being mirrored in the foreign country. Finally the bottom panel shows the case where there is no sectoral specialization ($\gamma = 0.5$) and the production structure is the same in both countries.

2.2 Dynamics

For simplicity, we abstract from any dynamics by considering a one period model with uncertainty. The timing of events within the period goes as follows: at the beginning of the period firms commit to a price in their own currency at which they will sell their goods, with the commitment taking place before shocks are realized. Once prices are set, productivity shocks are realized and the monetary authorities can adjust monetary policy in response. Consumers then purchase goods from the firms at the posted prices, with the consumer prices for imported goods being adjusted for any exchange rate fluctuations. Firms meet the demand at the price they posted.

The assumption of a static model is certainly restrictive. This is however a common assumption, either explicitly (Obstfeld and Rogoff (2000a,b)), or implicitly by shutting down the dynamics through the functional forms and asset structure (Corsetti and Pesenti (2001), Devereux and Engel (2000)).[2]

2.3 The consumer problem

The representative consumer in the home country maximizes the following utility:

$$U = E \ln C - \kappa E L \tag{1}$$

where C is a consumption index detailed below and L is costly labor effort. E is the expectations operator defined across all possible states of nature. The specification (1) is quite specific but allows us to derive the key results of the model while keeping the algebraic complexity to a minimum.

Our assumption of a representative consumer precludes any analysis of the distributive effects of monetary policy. This can seem puzzling as Tille (2002) shows the importance of such effects in a setup with incomplete asset markets.

[2] Our analysis assumes complete asset markets are complete, so that there would be no dynamics in a multi period model.

We abstract from redistributive issues in order to focus on the (in)ability of monetary policy to perform an optimal stabilization exercise by bringing the economy around the inefficiency of sticky prices. Any limitation in the power of monetary policy therefore stems from the production structure of the economy and not from any distributive concern, making our results more easily comparable with earlier contributions.

The consumption index C is defined across the unit mass of available brands, in three steps illustrated in Figure 2. The first step separates the aggregate index into two consumption indexes of types:

$$C = 2\left(C_A\right)^{0.5}\left(C_B\right)^{0.5}$$

where C_A and C_B are subindexes of the consumption of goods of type A and type B respectively. We assume that the elasticity of substitution between types is equal to 1, so that our model nests existing contributions such as Corsetti and Pesenti (2001).

In the second step, the type indexes are subdivided into origin subindexes:

$$\begin{aligned} C_A &= \left[(\gamma)^{\frac{1}{\lambda}}\left(C_{A,H}\right)^{\frac{\lambda-1}{\lambda}} + (1-\gamma)^{\frac{1}{\lambda}}\left(C_{A,F}\right)^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}} \\ C_B &= \left[(1-\gamma)^{\frac{1}{\lambda}}\left(C_{B,H}\right)^{\frac{\lambda-1}{\lambda}} + (\gamma)^{\frac{1}{\lambda}}\left(C_{B,F}\right)^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}} \end{aligned}$$

where $C_{i,H}$ and $C_{i,F}$ are indexes of home and foreign goods of type i respectively. For instance $C\left(A,H\right)$ is a consumption indexes of brands of type A produced in the home country. The weights on the subindexes reflect the degree of sectoral specialization γ. Under complete sectoral specialization ($\gamma = 1$), all brands of type A are produced in the home country and all brands of type B are produced in the foreign country: $C_A = C_{A,H}$ and $C_B = C_{B,F}$. The elasticity of substitution between origins for a given type is given by λ.

The third and final step divides the origin indexes across particular brands:

$$\begin{aligned} C_{A,H} &= \left[\left(\frac{2}{\gamma}\right)^{\frac{1}{\theta}} \int_0^{0.5\gamma} \left(C_{A,H}\left(z\right)\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \\ C_{A,F} &= \left[\left(\frac{2}{1-\gamma}\right)^{\frac{1}{\theta}} \int_{0.5}^{0.5(2-\gamma)} \left(C_{A,F}\left(z\right)\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \end{aligned}$$

$$C_{B,H} = \left[\left(\frac{2}{1-\gamma} \right)^{\frac{1}{\theta}} \int_{0.5\gamma}^{0.5} \left(C_{B,H}(z) \right)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

$$C_{B,F} = \left[\left(\frac{2}{\gamma} \right)^{\frac{1}{\theta}} \int_{0.5(2-\gamma)}^{1} \left(C_{B,F}(z) \right)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

where $C_{i,H}(z)$ $[C_{i,F}(z)]$ indicate the consumption of home [foreign] brand z of type i. The weights in the indexes reflects the allocation of brands across countries and types as described in Table 1. The elasticity of substitution between brands is given by θ.

Throughout the analysis, we assume that the elasticity of substitution between brands is at least as large as the elasticity between origins, which in turn is at least as large as the elasticity between types:[3]

$$\theta \geq \lambda \geq 1$$

In each state of nature the home household optimally allocates consumption across types, origins and brands, with the allocation reflecting the relative prices. The resulting demands are given in Table 2:

Table 2: Consumption allocation

$C_{A,H}(z)$	$=$	$\left[\frac{P_{A,H}(z)}{P_{A,H}}\right]^{-\theta} \left[\frac{P_{A,H}}{P_A}\right]^{-\lambda} \frac{P}{P_A} C$
$C_{A,F}(z)$	$=$	$\left[\frac{P_{A,F}(z)}{P_{A,F}}\right]^{-\theta} \left[\frac{P_{A,F}}{P_A}\right]^{-\lambda} \frac{P}{P_A} C$
$C_{B,H}(z)$	$=$	$\left[\frac{P_{B,H}(z)}{P_{B,H}}\right]^{-\theta} \left[\frac{P_{B,H}}{P_B}\right]^{-\lambda} \frac{P}{P_B} C$
$C_{B,F}(z)$	$=$	$\left[\frac{P_{B,F}(z)}{P_{B,F}}\right]^{-\theta} \left[\frac{P_{B,F}}{P_B}\right]^{-\lambda} \frac{P}{P_B} C$

The prices in Table 2, all expressed in home currency, are defined as follows: $P_{i,H}(z)$ $[P_{i,F}(z)]$ is the price of a home [foreign] brand z of type i, $P_{i,F}$ $[P_{i,F}]$ is the price index across all a home [foreign] brands of type i, P_i is the price index across all brands of type i from all origins, and P is the consumer price index. All price indexes represent the minimal expenditure required to

[3] We also assume that $\theta > 1$ to ensure a well-defined solution.

purchase one unit of the index in question, and are given in Table 3:

Table 3: Price indexes		
$P_{A,H}$	$=$	$\left[\frac{2}{\gamma}\int_0^{0.5\gamma}\left[P_{A,H}\left(z\right)\right]^{1-\theta}dz^H\right]^{\frac{1}{1-\theta}}$
$P_{A,F}$	$=$	$\left[\frac{2}{1-\gamma}\int_{0.5}^{0.5(2-\gamma)}\left[P_{A,F}\left(z\right)\right]^{1-\theta}dz^F\right]^{\frac{1}{1-\theta}}$
P_A	$=$	$\left[\gamma\left[P_{A,H}\right]^{1-\lambda}+\left(1-\gamma\right)\left[P_{A,F}\right]^{1-\lambda}\right]^{\frac{1}{1-\lambda}}$
$P_{B,H}$	$=$	$\left[\frac{2}{1-\gamma}\int_{0.5\gamma}^{0.5}\left[P_{B,H}\left(z\right)\right]^{1-\theta}dz^H\right]^{\frac{1}{1-\theta}}$
$P_{B,F}$	$=$	$\left[\frac{2}{\gamma}\int_{0.5(2-\gamma)}^{1}\left[P_{B,F}\left(z\right)\right]^{1-\theta}dz^F\right]^{\frac{1}{1-\theta}}$
P_B	$=$	$\left[\left(1-\gamma\right)\left[P_{B,H}\right]^{1-\lambda}+\gamma\left[P_{B,F}\right]^{1-\lambda}\right]^{\frac{1}{1-\lambda}}$
P	$=$	$\left(P_A\right)^{0.5}\left(P_B\right)^{0.5}$

The consumption allocation for the foreign representative consumer is similar. We denote foreign variables, including foreign currency prices, with asterisks. We assume that the law of one price holds, so the home currency price of a particular brand is equal to the foreign currency price adjusted by the exchange rate, with S representing the amount of home currency required to purchase one unit of foreign currency. For instance:

$$P_{A,F}\left(z\right)=SP_{A,F}^{*}\left(z\right)$$

As the composition of the consumption baskets is the same in both countries, the law of one price translates at the level of price indexes, so that any home currency price index is equal to the corresponding foreign currency price index adjusted for the exchange rate. In particular purchasing power parity holds:

$$P=SP^{*}$$

Having derived the composition of consumption basket, we now turn to the determination of its level. Each household holds domestic money and equity in domestic firms. In addition, the home and foreign households can trade a complete set of contingent commodities paying off in home currency. The budget constraint of the home consumer for a particular state of the world u is written as:

$$P_uC_u+\sum_t q_tB_t=W_uL_u+\Pi_u+B_u+\left(M_0-M_u\right)+T_u \tag{2}$$

The left hand side of (2) indicates the use of funds, namely consumption spending and the purchase of securities at the beginning of the period, with q_t and B_t representing the price and holding, respectively, of a security paying off 1 unit in home currency in state t. The right hand side indicates the sources of funds: the wage income, with W being the nominal wage, the profits from the household's ownership of all home firms (Π), the income from contingent securities (B), the initial cash balances net of final balances ($M_0 - M$), and lump-sum transfers from the government (T). (2) is simplified by assuming that the government repays any seignorage revenue through a lump-sum transfer, so that $(M_0 - M) + T = 0$.

The labor supply is obtained by maximizing (1) with respect to L, subject to (2):

$$W = \kappa PC \tag{3}$$

The optimal choice of contingent securities portfolio by the home and foreign households leads to the usual result that the ratio of marginal utilities of consumption is equal to the real exchange rate:

$$\frac{C}{C^*} = \frac{SP^*}{P}$$

As the purchasing power parity holds, consumption in perfectly insured between the home and the foreign household:

$$C = C^* \tag{4}$$

The last component of the consumption side of the model is given by the equilibrium in the money markets, which stipulate that consumption spending has to be covered by nominal balances:

$$M = PC \quad , \quad M^* = P^*C^* \tag{5}$$

(5) shows that by changing the amount of money in circulation the monetary authorities affect the nominal consumption spending. Such equilibria can be derived either as cash-in-advance constraints or through a money in the utility function approach.

Combining the money market equilibrium for the home and foreign countries (5) with the complete consumption insurance (4) and the fact that purchasing power parity holds, we show that the exchange rate is simply the ratio of the money supplies:

$$S = \frac{M}{M^*} \tag{6}$$

2.4 Optimal price setting

After solving the consumer problem, we turn to the optimal price setting by firms. Firms use a linear technology with labor as the only input, and a productivity term that is subjected to random shocks:

$$\begin{aligned} Y_{A,H}(z) &= K_{A,H}L_{A,H}(z) \quad , \quad Y_{A,F}(z) = K_{A,F}L_{A,F}(z) \\ Y_{B,H}(z) &= K_{B,H}L_{B,H}(z) \quad , \quad Y_{B,F}(z) = K_{B,F}L_{B,F}(z) \end{aligned}$$

where $Y_{i,H}(z)$ $[Y_{i,F}(z)]$ is the output by a home [foreign] firm producing brand z of type i, $L_{i,H}(z)$ $[L_{i,F}(z)]$ is the labor input used in the process and $K_{i,H}$ $[K_{i,F}]$ is the technology parameter common to all home [foreign] firms producing brands of type i.

Our setup allows the productivity shocks to be specific to a particular sector in a particular country. This nests the cases where productivity levels are country specific ($K_{A,H} = K_{B,H}$, $K_{A,F} = K_{B,F}$) or sector specific ($K_{A,H} = K_{A,F}$, $K_{B,H} = K_{B,F}$).

The demands faced by each firm are derived by aggregating the consumption allocation rules of Table 2 across the home and the foreign households, recalling that each is of mass $1/2$ and that consumption is equalized through (4):

$$\begin{aligned} Y_{A,H}(z) &= \left[\frac{P_{A,H}(z)}{P_{A,H}}\right]^{-\theta}\left[\frac{P_{A,H}}{P_A}\right]^{-\lambda}\frac{PC}{P_A} \\ Y_{A,F}(z) &= \left[\frac{P_{A,F}(z)}{P_{A,F}}\right]^{-\theta}\left[\frac{P_{A,F}}{P_A}\right]^{-\lambda}\frac{PC}{P_A} \\ Y_{B,H}(z) &= \left[\frac{P_{B,H}(z)}{P_{B,H}}\right]^{-\theta}\left[\frac{P_{B,H}}{P_B}\right]^{-\lambda}\frac{PC}{P_B} \\ Y_{B,F}(z) &= \left[\frac{P_{B,F}(z)}{P_{B,F}}\right]^{-\theta}\left[\frac{P_{B,F}}{P_B}\right]^{-\lambda}\frac{PC}{P_B} \end{aligned} \tag{7}$$

The optimization problem faced by firms is to choose the price they charge prior to the uncertainty on the productivity shocks and any subsequent monetary reaction being resolved. Firms choose their prices to maximize the expected discounted value of profits, with the discount factor reflecting the marginal utility of income by the owner of the firm.

A home firm producing a brand of type A chooses a price in home currency $P_{A,H}(z)$, charged for both domestic and export sales,[4] to maximize:

$$E\frac{1}{PC}\left[P_{A,H}(z)-\frac{W}{K_{A,H}}\right]\left[\frac{P_{A,H}(z)}{P_{A,H}}\right]^{-\theta}\left[\frac{P_{A,H}}{P_A}\right]^{-\lambda}\frac{PC}{P_A}$$

Note that in equilibrium all home firms producing type A are identical, so $P_{A,H}(z)=P_{A,H}$. Using the labor supply (3) and the money market equilibrium (5), the optimal price is then given by:

$$P_{A,H}=\frac{\theta\kappa}{\theta-1}\frac{EM\left(K_{A,H}\right)^{-1}\left(P_A\right)^{\lambda-1}}{E\left(P_A\right)^{\lambda-1}} \tag{8}$$

(8) shows the price is set at its certainty equivalent level, $\frac{\theta\kappa}{\theta-1}EM\left(K_{A,H}\right)^{-1}$, only when $\lambda=1$ or P_A is uncorrelated with $M\left(K_{A,H}\right)^{-1}$. Intuitively the price is determined by two terms: the marginal cost (nominal wage adjusted by productivity: $M\left(K_{A,H}\right)^{-1}$), and the price competitiveness vis-a-vis other producers of type A ($\left(P_A\right)^{\lambda-1}$): if P_A is high, the relative price of the home firm is low and it sells a relatively large quantity. Note that competitiveness issues are more important the higher the substitutability, λ, between home and foreign goods of type A.

(8) indicates that the price exceeds the certainty equivalent when the marginal cost and the competitiveness are positively correlated. Intuitively, firms have to balance ex-ante between cases where they would like to increase their price ex-post because they face a high marginal cost, and cases where they would like to reduce their prices because of a low marginal cost. Firms do not care about all possible states of nature equally, but put a higher weight on states where they have a lot of customers, as a suboptimal price then leads to more foregone revenue. States where home firms are competitive, that is P^A is large, are states where there are many customers. If the marginal cost tends to be higher than average in these states, firms will be more likely to wish for a higher price ex-post than a lower price and charge an ex-ante price above the certainty equivalent.

Turning to a foreign producer of type A, her goal is to set a price in foreign currency $P^*_{A,F}(z)$ to maximize the expected discounted profits:

$$E\frac{1}{P^*C}\left[P^*_{A,F}(z)-\frac{W^*}{K_{A,F}}\right]\left[\frac{P^*_{A,F}(z)}{P^*_{A,F}}\right]^{-\theta}\left[\frac{P_{A,F}}{P_A}\right]^{-\lambda}\frac{PC}{P_A}$$

[4] If the firm could choose different home currency prices for domestic and foreign sales, it would nevertheless set them equal as the demand elasticity is the same in both markets.

The optimal price is derived as:

$$P^*_{A,F} = \frac{\theta\kappa}{\theta-1}\frac{EM^*\left(K_{A,F}\right)^{-1}\left(P_A\right)^{\lambda-1}\left(S\right)^{1-\lambda}}{E\left(P_A\right)^{\lambda-1}\left(S\right)^{1-\lambda}} \tag{9}$$

The intuition is similar as for $P_{A,H}$, with the competitiveness now captured by $\left(P_A\right)^{\lambda-1}\left(S\right)^{1-\lambda}$.

The optimal prices for home and foreign firms producing type B are derived following similar steps as:

$$P_{B,H} = \frac{\theta\kappa}{\theta-1}\frac{EM\left(K_{B,H}\right)^{-1}\left(P_B\right)^{\lambda-1}}{E\left(P_B\right)^{\lambda-1}} \tag{10}$$

$$P^*_{B,F} = \frac{\theta\kappa}{\theta-1}\frac{EM^*\left(K_{B,F}\right)^{-1}\left(P_B\right)^{\lambda-1}\left(S\right)^{1-\lambda}}{E\left(P_B\right)^{\lambda-1}\left(S\right)^{1-\lambda}} \tag{11}$$

3 Solution of the model

3.1 Methodology

The usual approach used in solving the stochastic 'New open economy macroeconomics' models is to derive the full-blown closed form solution (Obstfeld and Rogoff (2000a,b), Corsetti and Pesenti (2001), Devereux and Engel (2000)). This method captures the key dimension, namely the impact of the second order moments on the preset prices. As forward looking firms have to commit to prices before the shocks are observed, the ex post volatility in the economy leads them to incorporate a risk premium in their prices. The second order moments of the model then significantly affect the first order moments, with higher prices resulting in lower expected consumption.

Solving the model in closed form however requires restrictive assumption on the functional forms used, such as a unit elasticity of substitution between home and foreign goods. The contributions that have analyzed more general setups are limited to a certainty equivalent analysis, as the need to take first order approximations around a deterministic steady state precludes them from capturing the impact of second order moments on preset prices.

This paper uses a new approach that gets around the limitations of both the closed form and linear approximation approaches. It allows us to avoid restrictive parametrizations while still capturing the impact of second order moments on prices. Our model being too general to be solved in a closed form, we take approximations around a deterministic steady state. We however do not limit ourselves to first order expansions, but derive second order expansions following Sutherland (2001).[5] We take second order approximations of both the welfare criterion and the positive equations in the model. The method is then more general than the Rotemberg and Woodford method (Woodford (2000)) which limits the expansions of the positive equations to a first order. For a discussion on the applicability of the Rotemberg and Woodford method to an open economy see Benigno and Benigno (2001).

The approximations in the model are taken around a deterministic steady state in which productivity is the same for all firms ($K_{A,H} = K_{B,H} = K_{A,F} = K_{B,F} = K_0$). All firms then produce the same quantity $(\theta - 1)(\theta\kappa)^{-1} K_0$, which is also equal to the consumption level. All home currency prices are equal to $(\theta - 1)^{-1} \theta\kappa M_0 (K_0)^{-1}$. The approximations are expressed in terms of log deviations from the steady state, denoted by Sans Serif letters:

$$\mathsf{x} = \ln X - \ln X_0$$

The stochastic dimension of the model stems from the presence of productivity shocks. The only assumption we need to make about them is that the expected value of the logarithms of the shocks around the steady state is zero:

$$E\mathsf{k}_{A,H} = E\mathsf{k}_{A,F} = E\mathsf{k}_{B,H} = E\mathsf{k}_{B,F} = 0 \tag{12}$$

The randomness of the productivity feeds into a randomness of the monetary stance, as we allow each country to react to the productivity shocks. For simplicity we assume that the monetary authority in both country perfectly observe the realization of the shocks. Throughout the paper we abstract from discretion and assume that monetary policy is conducted according to a rule through which the monetary stance in each country react to all shocks:

$$\begin{aligned} \mathsf{m} &= \Omega_{A,H}\mathsf{k}_{A,H} + \Omega_{A,F}\mathsf{k}_{A,F} + \Omega_{B,H}\mathsf{k}_{B,H} + \Omega_{B,F}\mathsf{k}_{B,F} \\ \mathsf{m}^* &= \Omega^*_{A,H}\mathsf{k}_{A,H} + \Omega^*_{A,F}\mathsf{k}_{A,F} + \Omega^*_{B,H}\mathsf{k}_{B,H} + \Omega^*_{B,F}\mathsf{k}_{B,F} \end{aligned} \tag{13}$$

where the Ω's are constant parameters. The rules (13) and the process (12) imply that there is no first order bias in monetary policy:

$$E\mathsf{m} = E\mathsf{m}^* = 0 \tag{14}$$

[5] Applications of the method to more complex closed economy models are found in Scmitt-Grohe and Uribe (2001) and Sims (2000).

Throughout the analysis, we make extensive use of the following relations between the level of a variable and the log deviations around the steady state, derived in the Appendix:

$$X = X_0 \left[1 + \mathsf{x} + \frac{1}{2}\mathsf{x}^2\right] \tag{15}$$

3.2 Flexible prices

Before undertaking the analysis with sticky prices, we derive the solution of the model under flexible prices as a benchmark. When firms can adjust their prices after observing the shocks and any monetary reaction, the optimal price rules (8)-(11) hold not only in expectation but in ex-post terms:

$$\begin{aligned} P_{A,H} &= \frac{\theta\kappa}{\theta-1} M \left(K_{A,H}\right)^{-1} \quad , \quad P^*_{A,F} = \frac{\theta\kappa}{\theta-1} M^* \left(K_{A,F}\right)^{-1} \\ P_{B,H} &= \frac{\theta\kappa}{\theta-1} M \left(K_{B,H}\right)^{-1} \quad , \quad P^*_{B,F} = \frac{\theta\kappa}{\theta-1} M^* \left(K_{B,F}\right)^{-1} \end{aligned} \tag{16}$$

The relative flexible prices then simply reflects differentials in productivity. For instance:

$$\frac{P_{A,H}}{SP^*_{A,F}} = \frac{K_{A,F}}{K_{A,H}}$$

Combining this with the consumption demands listed in Table 2, we see that the relative consumption levels also reflect the productivity differences:

$$\frac{C_{A,H}(z)}{C_{A,F}(z)} = \left[\frac{K_{A,H}}{K_{A,F}}\right]^{\lambda} \quad , \quad \frac{C_{B,H}(z)}{C_{B,F}(z)} = \left[\frac{K_{B,H}}{K_{B,F}}\right]^{\lambda}$$

Furthermore the flexible price allocation can be shown optimal in the sense that it corresponds to the allocation chosen by a benevolent worldwide planner, the detailed derivation being presented in the Appendix. In other words, the best outcome that monetary policy can hope for is to replicate the flexible price allocation.

Two particular cases are especially noteworthy. When productivity shocks are sector specific ($K_{A,H} = K_{A,F} = K_A$, $K_{B,H} = K_{B,F} = K_B$), the relative price between a home and a foreign brand of a given type is unity, while the relative price between types reflects the productivity differential:

$$\frac{P_{A,H}}{SP^*_{A,F}} = \frac{P_{B,H}}{SP^*_{B,F}} = 1 \quad , \quad \frac{P_A}{P_B} = \frac{K_B}{K_A} \tag{17}$$

When productivity shocks are country specific ($K_{A,H} = K_{B,H} = K_H$, $K_{A,F} = K_{B,F} = K_F$), the relative price between a home and a foreign brand of a given type reflects the productivity differentials:

$$\frac{P_{A,H}}{SP^*_{A,F}} = \frac{P_{B,H}}{SP^*_{B,F}} = \frac{K_F}{K_H} \quad , \quad \frac{P_A}{P_B} = \left[\frac{\gamma (K_H)^{\lambda-1} + (1-\gamma)(K_F)^{\lambda-1}}{(1-\gamma)(K_H)^{\lambda-1} + \gamma (K_F)^{\lambda-1}}\right]^{\frac{1}{1-\lambda}} \tag{18}$$

3.3 Approximations of prices

We now derive the second order approximations of the preset prices (8)-(11). We focus on the results, with the details being provided in the Appendix. Starting with a home firm producing a brand of type A, we can show that:

$$\mathsf{p}_{A,H} = \frac{1}{2}E\left(\mathsf{m} - \mathsf{k}_{A,H}\right)^2 + (\lambda - 1)(1-\gamma) E\mathsf{s}\left(\mathsf{m} - \mathsf{k}_{A,H}\right) \tag{19}$$

The price can deviate from the steady state value through two channels. First, with a set price, fluctuations in the marginal cost translate into fluctuations in the markup. This leads the risk averse firm owner to incorporate a risk premium, $E\left(\mathsf{m} - \mathsf{k}_{A,H}\right)^2$, in the price. The second term reflect competitiveness issues. A home firm faces a stronger demand when it is more competitive thanks to a depreciation of the domestic currency ($\mathsf{s} > 0$). The firm owner then cares more about states where the home currency is depreciated. If such states happen to be states when the marginal cost is high ($E\mathsf{s}\left(\mathsf{m} - \mathsf{k}_{A,H}\right) > 0$), the firms owner sets a higher price ex-ante.

We derive approximation for the other three individual prices (9)-(11) in a similar way and get:

$$\mathsf{p}^*_{A,F} = \frac{1}{2}E\left(\mathsf{m}^* - \mathsf{k}_{A,F}\right)^2 + \gamma(\lambda - 1) E(-\mathsf{s})\left(\mathsf{m}^* - \mathsf{k}_{A,F}\right) \tag{20}$$

$$\mathsf{p}_{B,H} = \frac{1}{2}E\left(\mathsf{m} - \mathsf{k}_{B,H}\right)^2 + \gamma(\lambda - 1) E\mathsf{s}\left(\mathsf{m} - \mathsf{k}_{B,H}\right) \tag{21}$$

$$\mathsf{p}^*_{B,F} = \frac{1}{2}E\left(\mathsf{m}^* - \mathsf{k}_{B,F}\right)^2 + (1-\gamma)(\lambda - 1) E(-\mathsf{s})\left(\mathsf{m}^* - \mathsf{k}_{B,F}\right) \tag{22}$$

Our next step is to derive the values of the prices indexes presented in Table 3, expressed as log approximations. The consumer price index is log linear in its components, so the following relation holds exactly with u indexing the state of nature as the consumer price index is not preset:

$$\mathsf{p}_u = \frac{1}{2}\mathsf{p}_{Au} + \frac{1}{2}\mathsf{p}_{Bu} \tag{23}$$

The type price indexes, P_A and P_B are however not log linear. Following steps detailed in the Appendix, we can express their values as second order approximations:

$$\mathsf{p}_{Au} = \gamma \mathsf{p}_{A,H} + (1-\gamma)\left(\mathsf{s}_u + \mathsf{p}^*_{A,F}\right) - \frac{1}{2}\gamma(1-\gamma)(\lambda-1)(\mathsf{s}_u)^2 \tag{24}$$

$$\mathsf{p}_{Bu} = (1-\gamma)\mathsf{p}_{B,H} + \gamma\left(\mathsf{s}_u + \mathsf{p}^*_{B,F}\right) - \frac{1}{2}\gamma(1-\gamma)(\lambda-1)(\mathsf{s}_u)^2 \tag{25}$$

3.4 Consumption and effort

We now derive log approximations of the consumption and effort. From the money market equilibria (5) and the optimal risk sharing (4), we see that in each state consumption is log linear in its two components, the nominal balances and the consumer price index. The following relation holds then exactly:

$$\begin{aligned} \mathsf{c}_u &= \mathsf{m}_u - \mathsf{p}_u \\ C_u &= C_0\left[1 + \mathsf{c}_u\right] \end{aligned} \tag{26}$$

Deriving the expected effort is more complex. The effort exerted by the home household is the sum of the effort exerted in producing the brands of type A and type B (expressing the result in per-capita terms):

$$\begin{aligned} EL &= 2\int_0^{0.5\gamma} EL_{A,H}(z)\,dz + 2\int_{0.5\gamma}^{0.5} EL_{B,H}(z)\,dz \\ &= \gamma EL_{A,H} + (1-\gamma)EL_{B,H} \end{aligned}$$

where we used the fact that all firms producing a given type in a given country are identical. Following steps presented in the Appendix, we write the expected effort in producing brands of type A as a second order approximation:

$$EL_{A,H} = \frac{\theta-1}{\theta\kappa}\left[1 + \mathsf{Q}_{A,H}\right]$$

where:

$$\begin{aligned} \mathsf{Q}_{A,H} &= -(\lambda-1)(1-\gamma)\left(\mathsf{p}_{A,H}-\mathsf{p}^*_{A,F}\right) \\ &\quad -\frac{2\gamma-1}{2}(\lambda-1)^2(1-\gamma)E(\mathsf{s})^2 \end{aligned} \tag{27}$$

(27) shows that expected effort is not constant, unless there is complete sectoral specialization ($\gamma = 1$) or the substitutability between home and foreign goods of type A is unity ($\lambda = 1$).

The expected effort for the production of type B by the home households is derived in the Appendix as:

$$\begin{aligned} \mathsf{Q}_{B,H} &= -(\lambda-1)\gamma\left(\mathsf{p}_{B,H}-\mathsf{p}^*_{B,F}\right)+\frac{2\gamma-1}{2}(\lambda-1)^2\gamma E(\mathsf{s})^2 \quad (28) \\ EL_{B,H} &= \frac{\theta-1}{\theta\kappa}\left[1+\mathsf{Q}_{B,H}\right] \end{aligned}$$

The expected effort of the foreign household is written as:

$$EL^* = (1-\gamma)EL_{A,F}+\gamma EL_{B,F}$$

The Appendix shows that the two components are written as:

$$\begin{aligned} EL_{A,F} &= \frac{\theta-1}{\theta\kappa}\left[1+\mathsf{Q}_{A,F}\right] \quad , \quad EL_{B,F}=\frac{\theta-1}{\theta\kappa}\left[1+\mathsf{Q}_{B,F}\right] \\ \mathsf{Q}_{A,F} &= (\lambda-1)\gamma\left(\mathsf{p}_{A,H}-\mathsf{p}^*_{A,F}\right)+\frac{2\gamma-1}{2}(\lambda-1)^2\gamma E(\mathsf{s})^2 \quad (29) \\ \mathsf{Q}_{B,F} &= (\lambda-1)(1-\gamma)\left(\mathsf{p}_{B,H}-\mathsf{p}^*_{B,F}\right) \quad (30) \\ &\quad -\frac{2\gamma-1}{2}(\lambda-1)^2(1-\gamma)E(\mathsf{s})^2 \end{aligned}$$

From (27)-(30) we compute the expected effort for all producers of a given type as follows:

$$\begin{aligned} \mathsf{Q}_A &= \gamma\mathsf{Q}_{A,H}+(1-\gamma)\mathsf{Q}_{A,F}=0 \\ \mathsf{Q}_B &= (1-\gamma)\mathsf{Q}_{B,H}+\gamma\mathsf{Q}_{B,F}=0 \end{aligned}$$

The expected average effort for all producers of a given type is always equal to its value in the steady state. Notice that the first right-hand side terms in (27) and (29) show that the expected effort for particular producers are influenced by the terms of trade. For instance, if home producers of type A set high prices ($\mathsf{p}_{A,H} - \mathsf{p}^*_{A,F} > 0$), they reduce their own effort at the expense of the foreign producer. As the aggregate effort Q_A is unaffected, such movements in the terms of trade correspond to a pure zero sum 'beggar-thy-neighbor' dimension where a fixed burden of effort is passed onto foreign producers.

4 Optimal monetary policy

4.1 Objective

The goal of the home and foreign monetary authorities is to choose monetary rules that maximize the expected welfare of the home and foreign household respectively. Recalling (4) the home and foreign objectives are:

$$\begin{aligned} U &= E \ln C - \gamma \kappa E L_{A,H} - (1-\gamma) \kappa E L_{B,H} \\ U^* &= E \ln C - (1-\gamma) \kappa E L_{A,F} - \gamma \kappa E L_{B,F} \end{aligned}$$

These expressions can be expressed in terms of log deviations around the steady state:

$$\begin{aligned} U &= U_0 + E\mathsf{c} - \frac{\theta - 1}{\theta} \left[\gamma \mathsf{Q}_{A,H} + (1-\gamma) \mathsf{Q}_{B,H} \right] \\ U^* &= U_0 + E\mathsf{c} - \frac{\theta - 1}{\theta} \left[(1-\gamma) \mathsf{Q}_{A,F} + \gamma \mathsf{Q}_{B,F} \right] \end{aligned}$$

where $U_0 = \ln C_0 - (\theta - 1) \theta^{-1}$. The objectives consist of an expected consumption component, which is the same in both countries, and an expected effort component which is the opposite in the two countries.[6]

The monetary authorities are then faced with two goals. The first is to undertake an optimal stabilization of the economy so as to maximize the expected consumption term. This is the dimension that is analyzed in existing contribution such as Obstfeld and Rogoff (2000a,b) and Corsetti and Pesenti (2001). The second goal, which is a new dimension, is to pass the burden of effort onto the other country. This is a zero-sum game as any gain by a country comes at the expense of the other.

[6] $\left[\gamma \mathsf{Q}_{A,H} + (1-\gamma) \mathsf{Q}_{B,H} \right] = - \left[(1-\gamma) \mathsf{Q}_{A,F} + \gamma \mathsf{Q}_{B,F} \right]$

We choose to limit our analysis to the first goal. Our focus allows for a more straightforward comparison of our results with existing analyses, as any discrepancies stem solely from differences in the ability of monetary policy to stabilize the economy.[7] Furthermore one can argue about the relevance of the second aspect in practice. Monetary authorities may recognize that it constitutes a zero-sum game and choose to abstain from it. The incentive to pass the effort burden onto the other country is certainly a new and intriguing dimension of monetary policy, and we leave a detailed analysis for future research.

As consumption is always equalized across countries (4), the goal of the home and foreign authorities is the same. There are then no gains from international cooperation, as the Nash equilibrium co-incindes with the in a cooperative solution.

We take the flexible price allocation as a benchmark because corresponds to a central planner's choice. Following steps presented in the Appendix, the objective of the monetary authorities is rewritten as:

$$\begin{aligned} U &= U^* = U_0 + E\mathsf{c} \\ &= U_{flex} - \frac{1}{4}\left[\begin{array}{c} \gamma E\left(\mathsf{m}-\mathsf{k}_{A,H}\right)^2 + \left(1-\gamma\right)E\left(\mathsf{m}^*-\mathsf{k}_{A,F}\right)^2 \\ +\left(1-\gamma\right)E\left(\mathsf{m}-\mathsf{k}_{B,H}\right)^2 + \gamma E\left(\mathsf{m}^*-\mathsf{k}_{B,F}\right)^2 \\ +\left(\lambda-1\right)\gamma\left(1-\gamma\right)\left[\begin{array}{c} E\left[\mathsf{s}-\left(\mathsf{k}_{A,H}-\mathsf{k}_{A,F}\right)\right]^2 \\ +E\left[\mathsf{s}-\left(\mathsf{k}_{B,H}-\mathsf{k}_{B,F}\right)\right]^2 \end{array}\right] \end{array}\right] \end{aligned} \tag{31}$$

where U_{flex} is the expected welfare under flexible prices. (31) shows that the welfare under sticky prices cannot exceeds the welfare under flexible prices: $U \leq U_{flex}$, so the best monetary policy can hope for is to replicate the flexible price allocation.

4.2 The limits of monetary policy

Before computing the optimal monetary policy rules, it is instructive to take a close look at (31). Any deviation from the flexible price welfare level U and U_{flex} stems from two sources. First, with prices preset in producer currency, volatile marginal costs translate into volatile markups, which induces the risk averse firm owners to increase their prices. Second, inefficient exchange rate fluctuations are costly. Recall that in the flexible price allocation, which is efficient, the relative price between a home and a foreign

[7] This focus is similar to the one taken by Canzoneri, Cumby and Diba (2001).

brand of a given type is equal to the productivity differential. When prices are sticky the relative price between a home and foreign brand can change only through fluctuations in the exchange rate. Such fluctuations are efficient only if they match the productivity differentials, as they then deliver the flexible price outcome. Notice that when $\lambda = 1$ exchange rate fluctuations do not entail any cost in addition to the cost of volatile markups. As λ increases however, inefficient exchange rate fluctuations become a serious concern. Intuitively inefficient fluctuations in the relative price of home and foreign brands of a given type matter only to the extent that they are translated into volatile quantities. The volatility in quantities remains moderate when home and foreign brands are poor substitute, but becomes significant when the substitutability increases.

(31) can also be interpreted in terms of price stability, along the lines of Benigno and Benigno (2001). In our static model, the concept of price stability is related to the price that firms would choose ex-post. Monetary policy achieves complete price stability when firms would choose not to change their prices (i.e. monetary policy perfectly gets around the price rigidity). The solution under flexible prices presented in the Appendix shows any ex-post adjustment in prices would simply offset a change in the marginal cost in order to keep the markup constant. For instance, a home producer of type A would choose $\mathsf{p}_{A,H} = \mathsf{m} - \mathsf{k}_{A,H}$. A monetary policy that achieve $\mathsf{m} = \mathsf{k}_{A,H}$ on its own then removes any wish from the firm to change its price. Under this interpretation, (31) shows that the best possible monetary policy can do is to completely stabilize prices throughout the economy.

The limitations of monetary policy can be inferred from (31). Consider first a case where shocks are entirely country specific ($K_{A,H} = K_{B,H} = K_H$, $K_{A,F} = K_{B,F} = K_F$). recalling (6) we simplify (31) to :

$$U = U_{flex} - \frac{1}{4}\left[\begin{array}{c} E\left(\mathsf{m} - \mathsf{k}_H\right)^2 + E\left(\mathsf{m}^* - \mathsf{k}_F\right)^2 \\ +2\left(\lambda - 1\right)\gamma\left(1-\gamma\right)E\left[\left(\mathsf{m} - \mathsf{k}_H\right) - \left(\mathsf{m}^* - \mathsf{k}_F\right)\right]^2 \end{array}\right]$$

We can easily see that monetary policy delivers the efficient outcome when authorities target only the domestic shocks, so there is no international dimension of monetary policy: $\mathsf{m} = \mathsf{k}_H$ and $\mathsf{m}^* = \mathsf{k}_F$. Intuitively, the efficient allocation (18) calls for a change in both the relative prices between home and foreign brands of a given type and the relative price between types. The exchange rate fluctuation under the optimal monetary policy delivers the efficient relative price changes. This confirms of the result by Obstfeld and

Rogoff (2000a,b) that an international coordination of monetary policy is not necessary when exchange rate fluctuations are entirely passed-through to prices, as the ensuing price changes are then efficient.

Our setup shows however that this result if very specific and breaks down as soon as we allow shocks to differ across sectors. For simplicity, consider sector specific shocks ($K_{A,H} = K_{A,F} = K_A$, $K_{B,H} = K_{B,F} = K_B$). (31) then becomes:

$$U = U_{flex} - \frac{1}{4}\left[\begin{array}{c} \gamma E\left(\mathsf{m}-\mathsf{k}_A\right)^2 + (1-\gamma)\,E\left(\mathsf{m}^*-\mathsf{k}_A\right)^2 \\ +(1-\gamma)\,E\left(\mathsf{m}-\mathsf{k}_B\right)^2 + \gamma E\left(\mathsf{m}^*-\mathsf{k}_B\right)^2 \\ +2(\lambda-1)\,\gamma\,(1-\gamma)\,E\left(\mathsf{s}\right)^2 \end{array}\right]$$

Monetary policy clearly cannot deliver the efficient flexible price outcome. (17) shows that the efficient outcome would be to change the relative price between types while keeping the relative price between home and foreign brands of a given type unchanged. The exchange rate cannot achieve both: whereas exchange rate fluctuations can deliver the efficient relative price change between types, they do so at the cost of an inefficient change in the relative price between brands.

4.3 Optimal rules

We now derive the optimal monetary policy rules in the home and the foreign country. As the expected welfare under flexible prices is independent of monetary policy, the goal is to minimize the second order terms in (31). Starting with the home monetary policy, the derivative with respect to the monetary stance in a particular state u is given by:[8]

$$\frac{\partial U}{\partial \mathsf{m}_u} = -\frac{1}{2}\pi_u\left[\begin{array}{c} \gamma\left(\mathsf{m}_u-\mathsf{k}_{A,Hu}\right)+(1-\gamma)\left(\mathsf{m}_u-\mathsf{k}_{B,Hu}\right) \\ +(\lambda-1)\,\gamma\,(1-\gamma)\left[\begin{array}{c} \left(\mathsf{m}_u-\mathsf{m}^*_u\right)-\left(\mathsf{k}_{A,Hu}-\mathsf{k}_{A,Fu}\right) \\ +\left(\mathsf{m}_u-\mathsf{m}^*_u\right)-\left(\mathsf{k}_{B,Hu}-\mathsf{k}_{B,Fu}\right) \end{array}\right] \end{array}\right]$$

Setting this derivative to zero and re-arranging terms, we obtain:

$$\mathsf{m}_u = \frac{\gamma+(\lambda-1)\,\gamma\,(1-\gamma)}{1+2(\lambda-1)\,\gamma\,(1-\gamma)}\mathsf{k}_{A,Hu}$$

[8] The optimal policy could also be computed by replacing m and m^* by linear combinations of the productivity shocks, with undetermined coefficients. The second step is then to take the derivative with respect to each coefficient and set them to zero. This leads to the exact same results.

$$+\frac{(1-\gamma)+(\lambda-1)\gamma(1-\gamma)}{1+2(\lambda-1)\gamma(1-\gamma)}\mathsf{k}_{B,Hu}$$
$$-\frac{(\lambda-1)\gamma(1-\gamma)}{1+2(\lambda-1)\gamma(1-\gamma)}\left(\mathsf{k}_{A,Fu}+\mathsf{k}_{B,Fu}\right)$$
$$+\frac{2(\lambda-1)\gamma(1-\gamma)}{1+2(\lambda-1)\gamma(1-\gamma)}\mathsf{m}_u^*$$

The optimal policy is an expansionary stance following productivity improvements for home producers, but a contractionary stance following similar improvements abroad. Furthermore, the home authority partially match the move by the foreign authorities in order to limit costly deviations of the exchange rate from its efficient level. This feature can be interpreted as a 'fear of floating' feature of optimal monetary policy.

The optimal monetary policy for the foreign country can be computed following similar steps, and we obtain:

$$\begin{aligned}\mathsf{m}_u^* &= \frac{(1-\gamma)+(\lambda-1)\gamma(1-\gamma)}{1+2(\lambda-1)\gamma(1-\gamma)}\mathsf{k}_{A,Fu}\\ &+\frac{\gamma+(\lambda-1)\gamma(1-\gamma)}{1+2(\lambda-1)\gamma(1-\gamma)}\mathsf{k}_{B,Fu}\\ &-\frac{(\lambda-1)\gamma(1-\gamma)}{1+2(\lambda-1)\gamma(1-\gamma)}\left(\mathsf{k}_{A,Hu}+\mathsf{k}_{B,Hu}\right)\\ &+\frac{2(\lambda-1)\gamma(1-\gamma)}{1+2(\lambda-1)\gamma(1-\gamma)}\mathsf{m}_u\end{aligned}$$

The final step is to combine the home and foreign optimal rules to derive Nash equilibrium rules as function of the productivity shocks. The home monetary stance is:

$$\begin{aligned}\mathsf{m}_u &= \frac{\gamma\left[1+(\lambda-1)(1-\gamma)\left[2\gamma+1\right]\right]}{1+4(\lambda-1)\gamma(1-\gamma)}\mathsf{k}_{A,Hu}\\ &+\frac{(1-\gamma)\left[1+(\lambda-1)\gamma\left[2(1-\gamma)+1\right]\right]}{1+4(\lambda-1)\gamma(1-\gamma)}\mathsf{k}_{B,Hu}\\ &-\frac{(\lambda-1)\gamma(1-\gamma)(2\gamma-1)}{1+4(\lambda-1)\gamma(1-\gamma)}\mathsf{k}_{A,Fu}\\ &+\frac{(\lambda-1)\gamma(1-\gamma)(2\gamma-1)}{1+4(\lambda-1)\gamma(1-\gamma)}\mathsf{k}_{B,Fu}\end{aligned} \tag{32}$$

And similarly for the foreign monetary stance:

$$\begin{aligned}
\mathsf{m}_u^* \;=\;& \frac{(1-\gamma)\left[1+(\lambda-1)\,\gamma\left[2\,(1-\gamma)+1\right]\right]}{1+4\,(\lambda-1)\,\gamma\,(1-\gamma)}\mathsf{k}_{A,Fu} \\
&+\frac{\gamma\left[1+(\lambda-1)\,(1-\gamma)\left[2\gamma+1\right]\right]}{1+4\,(\lambda-1)\,\gamma\,(1-\gamma)}\mathsf{k}_{B,Fu} \\
&+\frac{(\lambda-1)\,\gamma\,(1-\gamma)\,(2\gamma-1)}{1+4\,(\lambda-1)\,\gamma\,(1-\gamma)}\mathsf{k}_{A,Hu} \\
&-\frac{(\lambda-1)\,\gamma\,(1-\gamma)\,(2\gamma-1)}{1+4\,(\lambda-1)\,\gamma\,(1-\gamma)}\mathsf{k}_{B,Hu}
\end{aligned} \tag{33}$$

4.4 Special cases

The optimal rules (32) and (33) are fairly complex, but we can gain insight by considering some special cases. The first case is to consider a unit elasticity of substitution between home and foreign brands of a given type ($\lambda = 1$). The derivations are then much simpler, and the model can actually be solved in a closed form without resorting to log approximations.[9] The optimal monetary rules are then:

$$\mathsf{m}_u = \gamma \mathsf{k}_{A,Hu} + (1-\gamma)\,\mathsf{k}_{B,Hu} \quad , \quad \mathsf{m}_u^* = (1-\gamma)\,\mathsf{k}_{A,Fu} + \gamma \mathsf{k}_{B,Fu}$$

There is no international dimension of monetary policy, as it purely inward looking. The home [foreign] authority reacts one for one to the 'home' ['foreign'] productivity shock, which we define as a weighted average of the shocks in the home [foreign] country with the weight reflecting the sectoral structure. This is the result by Obstfeld and Rogoff (2000a).

Another special case is the situation where shocks are country specific ($\mathsf{k}_{A,Hu} = \mathsf{k}_{B,Hu} = \mathsf{k}_{Hu}$, $\mathsf{k}_{A,Fu} = \mathsf{k}_{B,Fu} = \mathsf{k}_{Fu}$). In such a case each authority targets solely its own shock and the efficient exchange rate fluctuations bring the economy to the flexible price allocation.:

$$\mathsf{m}_u = \mathsf{k}_{Hu} \quad , \quad \mathsf{m}_u = \mathsf{k}_{Fu} \tag{34}$$

[9] This simply reflects the fact that the elasticity of substitution between the baskets of home and foreign produced goods is unity. Indeed our model is then only marginally different from the usual specification, because the distinction between home and foreign brands of a given type is irrelevant when when the elasticity of substitution between them is equal to the elasticity between types.

When shocks are sector specific ($\mathsf{k}_{A,Hu} = \mathsf{k}_{A,Fu} = \mathsf{k}_{Au}$, $\mathsf{k}_{B,Hu} = \mathsf{k}_{B,Fu} = \mathsf{k}_{Bu}$) the monetary authorities target a combination of both shocks:

$$\begin{aligned}
\mathsf{m}_u &= \frac{\gamma + 2(\lambda - 1)\gamma(1-\gamma)}{1 + 4(\lambda - 1)\gamma(1-\gamma)}\mathsf{k}_{Au} + \frac{(1-\gamma) + 2(\lambda - 1)\gamma(1-\gamma)}{1 + 4(\lambda - 1)\gamma(1-\gamma)}\mathsf{k}_{Bu} \\
\mathsf{m}_u^* &= \frac{(1-\gamma) + 2(\lambda - 1)\gamma(1-\gamma)}{1 + 4(\lambda - 1)\gamma(1-\gamma)}\mathsf{k}_{Au} + \frac{\gamma + 2(\lambda - 1)\gamma(1-\gamma)}{1 + 4(\lambda - 1)\gamma(1-\gamma)}\mathsf{k}_{Bu}
\end{aligned}$$

The ensuing exchange rate fluctuations are given by:

$$\mathsf{s}_u = \frac{2\gamma - 1}{1 + 4(\lambda - 1)\gamma(1-\gamma)}(\mathsf{k}_{Au} - \mathsf{k}_{Bu})$$

If $\lambda = 1$ the exchange rate fluctuations are equal to the difference between the average shock in the home country and the average shock in the foreign country ($\mathsf{s}_u = (2\gamma - 1)(\mathsf{k}_{Au} - \mathsf{k}_{Bu})$). When λ increases however, the cost of inefficient fluctuations of the exchange rate quickly becomes large, so the optimal monetary policy sharply reduces the exchange rate movements.

4.5 Country vs. sectoral shocks: how apparently similar cases have different policy implications.

Our analysis points that the optimal monetary policy stance is sharply different depending on whether shocks are country or sector specific. This is especially noteworthy as two different shocks can look similar to an observer limiting himself to country wide aggregate.

To illustrate this aspect, consider a case where $\lambda = 6$[10] and $\gamma = 0.95$, so sectoral specialization is very high, but incomplete. Such a high degree of sectoral specialization may suggest that the sectoral dimension of the model is only a secondary aspect and little insight would be lost by considering complete sectoral specialization. We show that this inference would be very misleading by looking at two different shocks. The first is a combination of country shocks, with a large shock in the home country ($\mathsf{k}_{A,H} = \mathsf{k}_{B,H} = 0.95$, $\mathsf{k}_{A,F} = \mathsf{k}_{B,F} = 0.05$). The second is a sectoral shock ($\mathsf{k}_{A,H} = \mathsf{k}_{A,F} = 1$, $\mathsf{k}_{B,H} = \mathsf{k}_{B,F} = 0$). Table 4 compares the optimal monetary reaction to both shocks. Note that they look identical to an observer focusing on country averages ($\mathsf{k}_H = \gamma \mathsf{k}_{A,H} + (1-\gamma)\mathsf{k}_{B,H} = 0.95$ and $\mathsf{k}_F = (1-\gamma)\mathsf{k}_{A,F} + \gamma \mathsf{k}_{B,F} = 0.05$).

[10] That is $\lambda = \theta$ with $\theta = 6$ being a standard value, corresponding to a 20% steady state markup.

Table 4: Optimal monetary policy			
		Country shock	Sectoral shock
Country average of shocks	k_H	0.95	0.95
	k_F	0.05	0.05
Monetary stances	m	0.95	0.73
	m^*	0.05	0.27
Exchange rate	s	0.90	0.46

Despite their apparent similarity, the two cases have sharply different implications in terms of monetary policy. When shocks are country specific, monetary policy is purely inward looking ($\mathsf{m} = \mathsf{k}_H$, $\mathsf{m}^* = \mathsf{k}_F$) and the ensuing fluctuation in the exchange rate is large. When shocks are sectoral however, the monetary expansion is significantly reduced in the home country and expanded in the foreign country. This sharply limits the fluctuation in the exchange rate, with the optimal depreciation of the home currency under sectoral shocks being only half as big as under country shocks.

5 A numerical illustration of the value of exchange rate flexibility

Our analysis shows that whereas exchange rate flexibility is efficient and valuable when productivity shocks are sector specific, it looses its attractiveness when productivity shocks have a sectoral component. We illustrate this point by mean of a numerical example, focusing on a unit productivity shock in sector A ($\mathsf{k}_{A,H} = \mathsf{k}_{A,F} = 1$, $\mathsf{k}_{B,H} = \mathsf{k}_{B,F} = 0$). Figure 3 shows the optimal monetary reaction for the home and foreign countries (m and m^*) represented by the solid and dotted lines respectively. The figure illustrates the whole range of sectoral specialization from the case where both countries are identical ($\gamma = 0.5$) to the case of complete sectoral specialization ($\gamma = 1$). We consider two values for the degree of substitutability between home and foreign brands of a given type: low substitutability ($\lambda = 1$) and high substitutability ($\lambda = 6$).

At the extreme left of the figure ($\gamma = 0.5$) both countries are identical in terms of their sectoral structure. The exchange rate is then powerless as an adjustment tool following productivity shocks. Not only does it generate an inefficient change in the relative price between home and foreign brands of a given type, it does not even move the relative price of types ($\mathsf{p}_A - \mathsf{p}_B = 0$) because the weight of home brands is the same in both baskets. The only use of monetary policy is then to boost the world economy by the amount of the worldwide productivity shock, with both countries moving together to prevent useless and costly exchange rate fluctuations ($\mathsf{m} = \mathsf{m}^* = 0.5$).

The extreme right of the figure ($\gamma = 1$) represents the case of complete specialization where the sectoral dimension boils down to a cross-country dimension. This is the case considered in earlier contributions, and the optimal monetary response to a sectoral (country) shock is for the authorities to target their domestic shocks ($\mathsf{m} = 1$, $\mathsf{m}^* = 0$) and let the exchange rate fluctuations deliver the optimal cross-country allocation. Note that at both extremes the degree of substitutability between home and foreign brands of a given type, λ, is irrelevant.

The interesting dimension of the model emerges when we consider intermediate degrees of sectoral specialization ($0.5 < \gamma < 1$). Start with the case where $\lambda = 1$. As we move away from complete sectoral specialization, the optimal monetary policy is for both the home and foreign authorities to take an expansionary stance, the magnitude being larger in the home country. There is however no international dimension in monetary policy, as each country targets its 'domestic' shock, defined as a weighted average of the sectoral shocks with the weights reflecting the sectoral structure. Graphically this results in the monetary stances changing gradually along a straight line for different degrees of sectoral specialization.

An international dimension nevertheless emerges when home and foreign brands of a given type are close substitutes ($\lambda > 1$). The monetary authorities are caught between two conflicting objectives. On the one hand they want to depreciate the home currency to generate an efficient change in the relative price between types, $\mathsf{p}_A - \mathsf{p}_B$, but on the other hand they want to contain the inefficient changes in the relative price between home and foreign brands of a given type. The optimal monetary policy reflects a balancing between these two objectives. Because brands a close substitutes, the inefficient changes in the relative price between brands leads to substantial quantity misallocation. This leads the authorities to keep the exchange rate fluctuations moderate, unless the degree of sectoral specialization is very high.

The 'fear of floating' dimension is illustrated further in Figure 4 which indicates the exchange rate response under the optimal monetary policy stances described in figure 3. Whereas a higher degree of sectoral specialization only gradually reduces the optimal exchange rate fluctuations when substitutability is low ($\lambda = 1$), the effect is much sharper under a high substitutability. For instance, with $\lambda = 6$, the optimal exchange rate response when the degree of substitutability is very high, though incomplete ($\gamma = 0.95$) is less than half the response under complete specialization ($\gamma = 1$).[11]

Figure 4 shows that whereas a flexible exchange rate always dominate a fixed rate,[12] the amount of exchange rate fluctuations under the optimal monetary policy can be quite subdued. To illustrate this point further, we compare the welfare outcome under the optimal monetary policy with the outcome under an optimal fixed exchange rate, under which the monetary authorities in both countries react to the worldwide productivity shocks. Consider for simplicity that shocks are purely sectoral, uncorrelated and of identical volatility: $E(\mathsf{k}_A)^2 = E(\mathsf{k}_B)^2 = E(\mathsf{k})^2$, $E(\mathsf{k}_A \mathsf{k}_B) = 0$. From (31), the expected welfare under the optimal fixed exchange rate ($\mathsf{m} = \mathsf{m}^* = \frac{1}{2}\mathsf{k}_A + \frac{1}{2}\mathsf{k}_B$) is:

$$U_{PEG} - U_{flex} = -\frac{1}{8}E(\mathsf{k}_A - \mathsf{k}_B)^2 = -\frac{1}{4}E(\mathsf{k})^2$$

Under the optimal rule, the monetary stances are set according to (32)-(33) and we write:

$$U_{FLOAT} - U_{flex} = -\left[\begin{array}{c} \gamma\left[\frac{(1-\gamma)+2(\lambda-1)\gamma(1-\gamma)}{1+4(\lambda-1)\gamma(1-\gamma)}\right]^2 \\ +(1-\gamma)\left[\frac{\gamma+2(\lambda-1)\gamma(1-\gamma)}{1+4(\lambda-1)\gamma(1-\gamma)}\right]^2 \\ +(\lambda-1)\gamma(1-\gamma)\left[\frac{2\gamma-1}{1+4(\lambda-1)\gamma(1-\gamma)}\right]^2 \end{array}\right] E(\mathsf{k})^2$$

Figure 5 illustrates the gain from exchange rate flexibility by showing the value of $U_{FLOAT} - U_{PEG}$, setting the standard deviation of shocks at 10 percent ($E(\mathsf{k})^2 = 0.01$). The values on the vertical axis can be interpreted as the percentage change in expected consumption that would be equivalent to the welfare difference between a floating exchange rate and an optimal peg. For example a value of 0.15 percent indicates that the gain of a floating exchange rate, compared to an optimal peg, represent a 0.15 percent increase in expected consumption. To scale the numbers, moving from an optimal peg to flexible prices would represent the equivalent of a 0.25 percent

[11] When $\gamma = 0.95$, we have $\mathsf{s} = 0.46$, whereas $\mathsf{s} = 1$ when $\gamma = 1$.

[12] A fixed exchange rate is only optimal when $\gamma = 0.5$.

increase in consumption. Recall that under complete sectoral specialization ($\gamma = 1$) the optimal flexible exchange rate brings the economy to the flexible price allocation. In other words, the value of flexibility under complete sectoral specialization is a 0.25 percent increase in consumption. Figure 5 shows that the value of exchange rate flexibility is significantly reduced when sectoral specialization is incomplete, especially if home and foreign brands of a given type are highly substitutable. For instance, if $\lambda = 6$ and sectoral specialization is high but incomplete ($\gamma = 0.95$), a flexible exchange rate is equivalent to a 0.10 percent increase in consumption, less than half the value of flexibility under complete sectoral specialization.

6 Self fulfilling equilibria

We have so far focused on the optimal monetary policy design, taking the sectoral structure of the economy as given. The next step is to relax the assumption of an exogenous sectoral structure by allowing firms to switch from producing a brand of a certain type to producing a brand of the other type. This allows for an exploration of the impact of monetary policy on the incentive by firms to operate in a specific sector.

We leave a full blown characterization of the interaction between firms and monetary authorities for further research, and choose to explore the issue in a simple way by assessing which combinations of sectoral structure and monetary policy are equilibria. More specifically, we compare the expected profits of a home firm producing type A with the expected profits of a home firm producing type B. If the former exceed the later, a home firm producing type B has an incentive to switch sectors, so the situation is not an equilibrium. We conduct a similar analysis for foreign firms.

The analysis detailed in the Appendix shows that two combinations are equilibria, focusing on sectoral shocks for brevity. The first equilibrium is a situation of no sectoral specialization ($\gamma = 0.5$) and an optimal exchange rate peg ($\mathsf{m} = \mathsf{m}^* = 0.5\,(\mathsf{k}_A + \mathsf{k}_B)$). When the exchange rate does not fluctuate, there are no ex-post fluctuations of firms' competitiveness and all firms in a given country are in an identical situation. With the same expected profits in both sectors, firms are indifferent between producing brands of type A and type B. A fixed exchange rate then supports any choice of sectoral specialization, but only one choice ($\gamma = 0.5$) in turn supports a fixed exchange rate.

The second equilibrium is a situation of complete sectoral specialization ($\gamma = 1$) and inward looking monetary policy ($\mathsf{m} = \mathsf{k}_A$, $\mathsf{m}^* = \mathsf{k}_B$). In the presence of exchange rate fluctuations, the expected profits are smaller for a home firm producing type B than for a home firm producing type A. Intuitively, a home firm producing type B faces a mostly foreign competition and its competitiveness is significantly volatile because of the exchange rate fluctuations. By contrast the competitiveness of a home firm producing type A is steadier as it competes mostly against other home firm. This gives an incentive for home firms to produce type A. The situation is mirrored in the foreign country, where firms have an incentive to produce type B. A flexible exchange rate then pushes the firms towards complete sectoral specialization, a structure that in turn supports a flexible exchange rate regime.

The two equilibria described above can easily be ranked, with the flexible exchange rate / complete sectoral specialization equilibrium dominating the peg / no specialization equilibrium as it brings the economy to the flexible price allocation. Our finding of two self-fulfilling equilibria is similar to the analysis by Corsetti and Pesenti (2002) who focus on exchange rate pass-through.

7 Conclusion

This paper re-assesses the benefit of exchange rate flexibility in delivering efficient adjustments to shocks, and shows that the argument supporting a flexible exchange rate may be weaker than previously thought. We expand on the existing literature by allowing for a more general sectoral structure, with each country producing goods of different types. Whereas a flexible exchange rate is useful in adjusting to country specific shocks, its value is much smaller when shocks are sector specific, even when specialization is high.

Our analysis points to the need for further research into assessing whether shocks are predominantly country or sector specific. It indicates that uncertainties regarding the nature of shocks may lead to a 'fear of floating' motivated by the risk of inefficient price movements at the sector level. An interesting avenue for future research is to put the analysis in a setup where the nature of the shocks is uncertain.

8 Appendix

8.1 Level and log deviations

We derive the relation (15) by writing the level of variable X as:

$$X = \exp(\ln X) = \exp(x)$$

Take a second order expansion of this relation with respect to x:

$$\exp(x) = \exp(x_0) + \exp(x_0)(x - x_0) + \frac{1}{2}\exp(x_0)(x - x_0)^2$$

Recall that $x - x_0 = \ln X - \ln X_0 = \mathsf{x}$ to write this relation as:

$$X = X_0 + X_0 \mathsf{x} + \frac{1}{2} X_0 \mathsf{x}^2 = X_0 \left[1 + \mathsf{x} + \frac{1}{2}\mathsf{x}^2\right]$$

8.2 Planner's allocation

A world benevolent planner maximizes an equally weighted average of the home and foreign household welfare. With preferences separable in consumption and effort, the planner does not allocate different consumption levels for the two households, so $C = C^*$. The planner allocation is computed ex-post and the objective is separable across the various states of the world, so we focus on one particular state. The planner chooses the brand specific consumption levels to maximize:

$$\ln C - \kappa \int_0^{0.5\gamma} L_{A,H}(z)\, dz - \kappa \int_{0.5\gamma}^{0.5} L_{B,H}(z)\, dz$$
$$-\kappa \int_{0.5}^{0.5(2-\gamma)} L_{A,F}(z)\, dz - \kappa \int_{0.5(2-\gamma)}^{1} L_{B,F}(z)\, dz$$

subject to the conditions that the output of each brand is equal to its consumption:

$$\begin{aligned} K_{A,H} L_{A,H}(z) &= C_{A,H}(z) \quad , \quad K_{B,H} L_{B,H}(z) = C_{B,H}(z) \\ K_{A,F} L_{A,F}(z) &= C_{A,F}(z) \quad , \quad K_{B,F} L_{B,F}(z) = C_{B,F}(z) \end{aligned}$$

The first order conditions with respect to particular $C_{A,H}(z)$, $C_{B,H}(z)$, $C_{A,F}(z)$ and $C_{B,F}(z)$ are:

$$\frac{1}{C}\frac{\partial C}{\partial C_A}\frac{\partial C_A}{\partial C_{A,H}}\frac{\partial C_{A,H}}{\partial C_{A,H}(z)} = \frac{\kappa}{K_{A,H}}$$

$$
\begin{aligned}
\frac{1}{C}\frac{\partial C}{\partial C_A}\frac{\partial C_A}{\partial C_{A,F}}\frac{\partial C_{A,F}}{\partial C_{A,F}(z)} &= \frac{\kappa}{K_{A,F}} \\
\frac{1}{C}\frac{\partial C}{\partial C_B}\frac{\partial C_B}{\partial C_{B,H}}\frac{\partial C_{B,H}}{\partial C_{B,H}(z)} &= \frac{\kappa}{K_{B,H}} \\
\frac{1}{C}\frac{\partial C}{\partial C_B}\frac{\partial C_B}{\partial C_{B,F}}\frac{\partial C_{B,F}}{\partial C_{B,F}(z)} &= \frac{\kappa}{K_{B,F}}
\end{aligned}
$$

These conditions imply that the consumption allocation is identical for all firms producing a specific brand in a specific country, that is: $C_{A,H}(z) = 2\gamma^{-1}C_{A,H}$, $C_{A,F}(z) = 2(1-\gamma)^{-1}C_{A,F}$, $C_{B,H}(z) = 2(1-\gamma)^{-1}C_{B,H}$ and $C_{B,F}(z) = 2\gamma^{-1}C_{B,F}$. Using the forms of the consumption aggregates, the first order conditions are rewritten as:

$$
\begin{aligned}
\left(\frac{1}{2}\right)^{\frac{\lambda-1}{\lambda}}\frac{1}{C_A}\left[\frac{C_{A,H}(z)}{C_A}\right]^{-\frac{1}{\lambda}} &= \frac{\kappa}{K_{A,H}} \\
\left(\frac{1}{2}\right)^{\frac{\lambda-1}{\lambda}}\frac{1}{C_A}\left[\frac{C_{A,F}(z)}{C_A}\right]^{-\frac{1}{\lambda}} &= \frac{\kappa}{K_{A,F}} \\
\left(\frac{1}{2}\right)^{\frac{\lambda-1}{\lambda}}\frac{1}{C_B}\left[\frac{C_{B,H}(z)}{C_B}\right]^{-\frac{1}{\lambda}} &= \frac{\kappa}{K_{B,H}} \\
\left(\frac{1}{2}\right)^{\frac{\lambda-1}{\lambda}}\frac{1}{C_B}\left[\frac{C_{B,F}(z)}{C_B}\right]^{-\frac{1}{\lambda}} &= \frac{\kappa}{K_{B,F}}
\end{aligned}
$$

Combining these conditions shows that the relative consumption levels reflect the productivity differences in exactly the same way as under flexible prices:

$$
\frac{C_{A,H}(z)}{C_{A,F}(z)} = \left[\frac{K_{A,H}}{K_{A,F}}\right]^{\lambda} \quad , \quad \frac{C_{B,H}(z)}{C_{B,F}(z)} = \left[\frac{K_{B,H}}{K_{B,F}}\right]^{\lambda}
$$

Combining the first order conditions with the form of the consumption indexes, we derive the consumption levels chosen by the planner:

$$
\begin{aligned}
C_{A,H}(z) &= \frac{K_{A,H}}{\kappa}\frac{(K_{A,H})^{\lambda-1}}{\gamma(K_{A,H})^{\lambda-1} + (1-\gamma)(K_{A,F})^{\lambda-1}} \\
C_{A,F}(z) &= \frac{K_{A,F}}{\kappa}\frac{(K_{A,F})^{\lambda-1}}{\gamma(K_{A,H})^{\lambda-1} + (1-\gamma)(K_{A,F})^{\lambda-1}} \\
C_{B,H}(z) &= \frac{K_{B,H}}{\kappa}\frac{(K_{B,H})^{\lambda-1}}{\gamma(K_{B,H})^{\lambda-1} + (1-\gamma)(K_{B,F})^{\lambda-1}}
\end{aligned}
$$

$$C_{B,F}(z) = \frac{K_{B,F}}{\kappa} \frac{(K_{B,F})^{\lambda-1}}{\gamma (K_{B,H})^{\lambda-1} + (1-\gamma)(K_{B,F})^{\lambda-1}}$$

The consumption indexes then follow as:

$$\begin{aligned} C_A &= \frac{1}{2\kappa} \left[\gamma (K_{A,H})^{\lambda-1} + (1-\gamma)(K_{A,F})^{\lambda-1} \right]^{\frac{1}{\lambda-1}} \\ C_B &= \frac{1}{2\kappa} \left[(1-\gamma)(K_{B,H})^{\lambda-1} + \gamma (K_{B,F})^{\lambda-1} \right]^{\frac{1}{\lambda-1}} \\ C &= \frac{1}{\kappa} \left[\begin{array}{c} \gamma (K_{A,H})^{\lambda-1} \\ +(1-\gamma)(K_{A,F})^{\lambda-1} \end{array} \right]^{\frac{1}{2}\frac{1}{\lambda-1}} \left[\begin{array}{c} (1-\gamma)(K_{B,H})^{\lambda-1} \\ +\gamma (K_{B,F})^{\lambda-1} \end{array} \right]^{\frac{1}{2}\frac{1}{\lambda-1}} \end{aligned}$$

This allocation is similar to the one reached under flexible prices. Plugging the optimal flexible prices (16) into the consumption demands of Table 2, and using the form of the consumption indexes, we can show that the consumption levels under flexible prices are identical to the levels in the planner solution, up to a constant reflecting the distortion from monopolistic competition. For example, the aggregate consumption under flexible prices is:

$$C = \frac{\theta - 1}{\theta \kappa} \left[\begin{array}{c} \gamma (K_{A,H})^{\lambda-1} \\ +(1-\gamma)(K_{A,F})^{\lambda-1} \end{array} \right]^{\frac{1}{2}\frac{1}{\lambda-1}} \left[\begin{array}{c} (1-\gamma)(K_{B,H})^{\lambda-1} \\ +\gamma (K_{B,F})^{\lambda-1} \end{array} \right]^{\frac{1}{2}\frac{1}{\lambda-1}}$$

The only difference from the planner allocation is the $\frac{\theta-1}{\theta}$ term.

8.3 Approximations of prices

8.3.1 Optimal individual prices

We now derive the second order approximations of the preset prices (8)-(11). Starting with a home producer of type A, recall that $P_{A,H}$ solves:

$$P_{A,H} E (P_A)^{\lambda-1} = \frac{\theta\kappa}{\theta - 1} EM (K_{A,H})^{-1} (P_A)^{\lambda-1}$$

Define two variables G and F that are set before the shocks are realized:

$$F = E (P_A)^{\lambda-1} \quad , \quad G = EM (K_{A,H})^{-1} (P_A)^{\lambda-1}$$

We write the optimal price as a simple function of these two variables:

$$P_{A,H} = \frac{\theta\kappa}{\theta - 1} \frac{G}{F}$$

Note that this expression is linear in logs, so the following relation in logs deviations from the steady state holds exactly:

$$\mathsf{p}_{A,H} = \mathsf{G} - \mathsf{F}$$

The F and G are however not linear in logs. We start by rewriting them as follows:

$$\begin{aligned} F &= \sum \pi_u \left(P_{Au}\right)^{\lambda-1} = \sum \pi_u f_u \\ G &= \sum \pi_u M_u \left(K_{A,Hu}\right)^{-1} \left(P_{Au}\right)^{\lambda-1} = \sum \pi_u g_u \end{aligned}$$

where u is and index of the states of the world and π_u is the probability of state u. Note that the state specific terms f and g are linear in logs, so the following relations hold exactly:

$$\mathsf{f}_u = (\lambda - 1)\, \mathsf{p}_{Au} \quad , \quad \mathsf{g}_u = \mathsf{m}_u - \mathsf{k}_{A,Hu} + (\lambda - 1)\, \mathsf{p}_{Au}$$

We now turn to the second order expansions, starting with the term F and its components. Using (15) we write:

$$F = F_0 \left[1 + \mathsf{F} + \frac{1}{2}\mathsf{F}^2\right] \quad , \quad f_u = F_0 \left[1 + \mathsf{f}_u + \frac{1}{2}\left(\mathsf{f}_u\right)^2\right]$$

Combining and simplifying we obtain:

$$\mathsf{F} + \frac{1}{2}\mathsf{F}^2 = \sum \pi_u \mathsf{f}_u + \frac{1}{2} \sum \pi_u \left(\mathsf{f}_u\right)^2$$

The next step is to notice that $\mathsf{F}^2 = 0$. Recall that F is a preset variable. As the only stochastic variables are the productivity shocks and the monetary stances, the deviations of F around the steady states will be made of terms of first and second order involving the monetary stances and the shocks:

$$\mathsf{F} = F_1 \left(\mathsf{m}, \mathsf{k}\right) + F_2 \left(\mathsf{m}, \mathsf{k}\right)$$

With F being preset, F_1 reflects the first moments of the stochastic variables, and is a linear combinations of the expected log deviations of both monetary stances and all four productivity shocks. (12) and (14) then imply $F_1 = 0$. Squaring this expression we obtain:

$$\mathsf{F}^2 = \left[F_2 \left(\mathsf{m}, \mathsf{k}\right)\right]^2 = O\left(3\right)$$

where $O(3)$ are terms of order 3 and above that we drop as we limit ourselves to a second order approximation. It follows that $\mathsf{F}^2 = 0$. A similar reasoning can be undertaken for the deviations of any preset variables. Such deviations are of order 2, so the squared deviations are of order higher than 2 and can be dropped.

The next step is to use the expression for $\mathsf{f}_u$to write:

$$\mathsf{F} = (\lambda - 1) \sum \pi_u \mathsf{p}_{Au} + \frac{(\lambda - 1)^2}{2} \sum \pi_u (\mathsf{p}_{Au})^2$$

The term p_{Au} contains elements of first order as well as elements of second order, the later capturing the risk premium contained in the preset prices. Notice however that as we limit ourselves to terms up to order 2, we can see that the only terms relevant for $(\mathsf{p}_{Au})^2$ will be the squared values of the first order terms in p_{Au}. In other words, we can evaluate the second order moments based on a first order approximation of the model. With preset prices, the price index p_{Au} fluctuates only because of the exchange rate. Taking a first order approximation of the expression in Table 2, we write:

$$\mathsf{p}_{Au} = (1 - \gamma) \mathsf{s}_u$$

The home currency price index for type A increases following a depreciation of the home currency because imported goods are more expensive. The magnitude of the change reflects the share of foreign firms in sector A, namely $1 - \gamma$. Using this result we write F as:

$$\mathsf{F} = (\lambda - 1) E\mathsf{p}_A + \frac{(\lambda - 1)^2 (1 - \gamma)^2}{2} E (\mathsf{s})^2$$

We undertake similar steps for G. We take the second order expansions for G and g_u, combine them and use $E\mathsf{G}^2 = 0$ to write:

$$\mathsf{G} = \sum \pi_u \mathsf{g}_u + \frac{1}{2} \sum \pi_u (\mathsf{g}_u)^2$$

Using the solution for g_u and using (12) and (14) leads to:

$$\mathsf{G} = (\lambda - 1) E\mathsf{p}_A + \frac{1}{2} E \left[(\mathsf{m} - \mathsf{k}_{A,H}) + (\lambda - 1) \mathsf{p}_A \right]^2$$

We evaluate the second order terms by taking the first order approximation $\mathsf{p}_{Au} = (1 - \gamma) \mathsf{s}_u$, and obtain:

$$\mathsf{G} = (\lambda - 1) E\mathsf{p}_A + \frac{1}{2} \left[\begin{array}{c} E (\mathsf{m} - \mathsf{k}_{A,H})^2 + (\lambda - 1)^2 (1 - \gamma)^2 E (\mathsf{s})^2 \\ +2 (\lambda - 1) (1 - \gamma) E\mathsf{s} (\mathsf{m} - \mathsf{k}_{A,H}) \end{array} \right]$$

We now combine our results for F and G we derive our expression for the price deviation:

$$\mathsf{p}_{A,H} = \mathsf{G} - \mathsf{F} = \frac{1}{2} E\left(\mathsf{m} - \mathsf{k}_{A,H}\right)^2 + (\lambda - 1)(1 - \gamma) E\mathsf{s}\left(\mathsf{m} - \mathsf{k}_{A,H}\right)$$

We next turn to a foreign firm producing type A. Its price (9) is written as:

$$P_{A,F}^{*} E\left(P_A\right)^{\lambda-1}(S)^{1-\lambda} = \frac{\theta\kappa}{\theta - 1} EM^{*}\left(K_{A,F}\right)^{-1}\left(P_A\right)^{\lambda-1}(S)^{1-\lambda}$$

Define two preset variables G and F as:

$$\begin{aligned} F &= E\left(P_A\right)^{\lambda-1}(S)^{1-\lambda} = \sum \pi_u \left(P_{Au}\right)^{\lambda-1}\left(S_u\right)^{1-\lambda} \\ &= \sum \pi_u f_u \\ G &= EM^{*}\left(K_{A,F}\right)^{-1}\left(P_A\right)^{\lambda-1}(S)^{1-\lambda} = \sum \pi_u M_u^{*}\left(K_{A,Fu}\right)^{-1}\left(P_{Au}\right)^{\lambda-1}\left(S_u\right)^{1-\lambda} \\ &= \sum \pi_u g_u \end{aligned}$$

$P_{A,F}^{*}$ is a log linear function of F and G, and f_u and g_u are log linear functions in their components. The following relations then hold exactly:

$$\begin{aligned} \mathsf{p}_{A,F}^{*} &= \mathsf{G} - \mathsf{F} \\ \mathsf{f}_u &= (\lambda - 1)\left(\mathsf{p}_{Au} - \mathsf{s}_u\right) \quad , \quad \mathsf{g}_u = \mathsf{m}_u^{*} - \mathsf{k}_{A,Fu} + (\lambda - 1)\left(\mathsf{p}_{Au} - \mathsf{s}_u\right) \end{aligned}$$

We next use (15) to expand F and f_u, and combine the results to write:

$$\begin{aligned} \mathsf{F} &= \sum \pi_u \mathsf{f}_u + \frac{1}{2}\sum \pi_u \left(\mathsf{f}_u\right)^2 \\ &= (\lambda - 1)\left(E\mathsf{p}_A - E\mathsf{s}\right) + \frac{1}{2} E\left[(\lambda - 1)\left(\mathsf{p}_A - \mathsf{s}\right)\right]^2 \end{aligned}$$

(6) shows that $\mathsf{s} = \mathsf{m} - \mathsf{m}^{*}$, and (14) then implies $E\mathsf{s} = 0$. The second order terms are computed based on a first order approximation, under which $\mathsf{p}_A - \mathsf{s} = -\gamma\mathsf{s}$, and we get:

$$\mathsf{F} = (\lambda - 1) E\mathsf{p}_A + \frac{1}{2}\gamma^2 (\lambda - 1)^2 E(\mathsf{s})^2$$

The term G is evaluated following similar steps. We first write the second order approximation:

$$\mathsf{G} = (\lambda - 1) E\mathsf{p}_A + \frac{1}{2} E\left[\left(\mathsf{m}^{*} - \mathsf{k}_{A,F}\right) + (\lambda - 1)\left(\mathsf{p}_A - \mathsf{s}\right)\right]^2$$

Evaluating the second moment based on the first order approximation $\mathsf{p}_A - \mathsf{s} = -\gamma \mathsf{s}$, we get:

$$\begin{aligned}\mathsf{G} &= (\lambda - 1) E\mathsf{p}_A + \frac{1}{2}E(\mathsf{m}^* - \mathsf{k}_{A,F})^2 + \frac{1}{2}\gamma^2 (\lambda-1)^2 E(\mathsf{s})^2 \\ &+ \gamma(\lambda-1) E(-\mathsf{s})(\mathsf{m}^* - \mathsf{k}_{A,F})\end{aligned}$$

The optimal price is then written as:

$$\mathsf{p}^*_{A,F} = \frac{1}{2}E(\mathsf{m}^* - \mathsf{k}_{A,F})^2 + \gamma(\lambda-1)E(-\mathsf{s})(\mathsf{m}^* - \mathsf{k}_{A,F})$$

Turning to the price charged by a home firm producing type B (10), we write:

$$P_{B,H} E(P_B)^{\lambda-1} = \frac{\theta\kappa}{\theta-1} EM (K_{B,H})^{-1} (P_B)^{\lambda-1}$$

The steps are similar to the one followed for the derivation of (19), with the second order terms computed using the first order approximation $\mathsf{p}_B = \gamma \mathsf{s}$. The analysis leads to:

$$\mathsf{p}_{B,H} = \frac{1}{2}E(\mathsf{m} - \mathsf{k}_{B,H})^2 + \gamma(\lambda-1)E\mathsf{s}(\mathsf{m} - \mathsf{k}_{B,H})$$

Finally, the price charged by a foreign firm producing type B (11) is written as:

$$P^*_{B,F} E(P_B)^{\lambda-1} (S)^{1-\lambda} = \frac{\theta\kappa}{\theta-1} EM^* (K_{B,F})^{-1} (P_B)^{\lambda-1} (S)^{1-\lambda}$$

The analysis follows the steps used in deriving (20) and we obtain:

$$\mathsf{p}^*_{B,F} = \frac{1}{2}E(\mathsf{m}^* - \mathsf{k}_{B,F})^2 + (1-\gamma)(\lambda-1)E(-\mathsf{s})(\mathsf{m}^* - \mathsf{k}_{B,F})$$

8.3.2 Price indexes

The approximation of the price index for type A, P_A, proceeds as follows. Recall that in every state of nature, u, $P_{A,Fu} = S_u P^*_{A,F}$, and that the prices $P_{A,H}$ and $P^*_{A,F}$ are preset. The price index is written as:

$$[P_{Au}]^{1-\lambda} = \gamma [P_{A,H}]^{1-\lambda} + (1-\gamma)\left[S_u P^*_{A,F}\right]^{1-\lambda}$$

Recall that in the symmetric steady state $P_{A0} = P_{A,H0} = S_0 P^*_{A,F0} = P_0$. We take a second order approximation of the left-hand side as follows:

$$\begin{aligned}
[P_{Au}]^{1-\lambda} &= [P_0]^{1-\lambda} + (1-\lambda)[P_0]^{-\lambda}(P_{Au} - P_0) \\
&\quad -\lambda(1-\lambda)[P_0]^{-\lambda-1}\frac{1}{2}(P_{Au} - P_0)^2 \\
&= [P_0]^{1-\lambda}\left[1 + (1-\lambda)\left(\frac{P_{Au} - P_0}{P_0}\right) - \lambda(1-\lambda)\frac{1}{2}\left(\frac{P_{Au} - P_0}{P_0}\right)^2\right]
\end{aligned}$$

Using (15) this becomes:

$$\begin{aligned}
[P_{Au}]^{1-\lambda} &= [P_0]^{1-\lambda}\left[\begin{array}{c} 1 + (1-\lambda)\left(\mathsf{p}_{Au} + \frac{1}{2}(\mathsf{p}_{Au})^2\right) \\ -\lambda(1-\lambda)\frac{1}{2}\left(\mathsf{p}_{Au} + \frac{1}{2}(\mathsf{p}_{Au})^2\right)^2 \end{array}\right] \\
&= [P_0]^{1-\lambda}\left[1 + (1-\lambda)\mathsf{p}_{Au} + \frac{1}{2}(\lambda-1)^2(\mathsf{p}_{Au})^2\right] + O(3)
\end{aligned}$$

where $O(3)$ represents terms of order 3 and above that we drop. We can take similar approximations of the two components on the right hand side of the price index:

$$\begin{aligned}
[P_{A,H}]^{1-\lambda} &= [P_0]^{1-\lambda}\left[1 + (1-\lambda)\mathsf{p}_{A,H} + \frac{1}{2}(\lambda-1)^2(\mathsf{p}_{A,H})^2\right] \\
\left[S_u P^*_{A,F}\right]^{1-\lambda} &= [P_0]^{1-\lambda}\left[1 + (1-\lambda)\left(\mathsf{s}_u + \mathsf{p}^*_{A,F}\right) + \frac{1}{2}(\lambda-1)^2\left(\mathsf{s}_u + \mathsf{p}^*_{A,F}\right)^2\right]
\end{aligned}$$

Combining our results and cancelling terms we write:

$$\begin{aligned}
\mathsf{p}_{Au} + \frac{1}{2}(1-\lambda)(\mathsf{p}_{Au})^2 &= \gamma\left[\mathsf{p}_{A,H} + \frac{1}{2}(1-\lambda)(\mathsf{p}_{A,H})^2\right] \\
&\quad + (1-\gamma)\left[\left(\mathsf{s}_u + \mathsf{p}^*_{A,F}\right) + \frac{1}{2}(1-\lambda)\left(\mathsf{s}_u + \mathsf{p}^*_{A,F}\right)^2\right]
\end{aligned}$$

From (19) and (20) we know that $\mathsf{p}_{A,H}$ and $\mathsf{p}^*_{A,F}$ are of order 2. As we omit terms of order 3 and above, we can write $(\mathsf{p}_{A,H})^2 = 0$ and $\left(\mathsf{s}_u + \mathsf{p}^*_{A,F}\right)^2 = (\mathsf{s}_u)^2$, as s_u is of order 1. The expansion of the price index is then:

$$\begin{aligned}
\mathsf{p}_{Au} + \frac{1}{2}(1-\lambda)(\mathsf{p}_{Au})^2 &= \gamma\mathsf{p}_{A,H} + (1-\gamma)\left(\mathsf{s}_u + \mathsf{p}^*_{A,F}\right) \\
&\quad + \frac{1}{2}(1-\gamma)(1-\lambda)(\mathsf{s}_u)^2
\end{aligned}$$

The next step is to evaluate the second order term $(\mathsf{p}_{Au})^2$ based on the first order approximation that $\mathsf{p}_{Au} = \gamma \mathsf{p}_{A,H} + (1-\gamma)\left(\mathsf{s}_u + \mathsf{p}^*_{A,F}\right)$. Omitting terms of order 3 and above, we use this approximation to write :

$$(\mathsf{p}_{Au})^2 = (1-\gamma)^2 (\mathsf{s}_u)^2$$

Using this results, we write the final approximation of the price index:

$$\mathsf{p}_{Au} = \gamma \mathsf{p}_{A,H} + (1-\gamma)\left(\mathsf{s}_u + \mathsf{p}^*_{A,F}\right) - \frac{1}{2}\gamma(1-\gamma)(\lambda-1)(\mathsf{s}_u)^2$$

Following similar steps for the price index of type B, P_{Bu}, we obtain:

$$\mathsf{p}_{Bu} = (1-\gamma)\mathsf{p}_{B,H} + \gamma\left(\mathsf{s}_u + \mathsf{p}^*_{B,F}\right) - \frac{1}{2}\gamma(1-\gamma)(\lambda-1)(\mathsf{s}_u)^2$$

8.4 Expected effort

Using the output demands (7) the expected effort for the home household in producing brands of type A is written as:

$$\begin{aligned} EL_{A,H} &= E(K_{A,H})^{-1} Y_{A,H} = E(K_{A,H})^{-1}\left[\frac{P_{A,H}}{P_A}\right]^{-\lambda} \frac{P}{P_A} C \\ &= (P_{A,H})^{-\lambda} EM (K_{A,H})^{-1} (P_A)^{\lambda-1} \end{aligned}$$

Define the ex-ante variables $Q_{A,H} = E(K_{A,H})^{-1} Y_{A,H}$ and $F = EM(K_{A,H})^{-1}$ $(P_A)^{\lambda-1}$, and write:

$$Q_{A,H} = (P_{A,H})^{-\lambda} F = (P_{A,H})^{-\lambda} \sum \pi_u f_u$$

$Q_{A,H}$ is log-linear in its components, as are the state specific f_u's. The following relations then hold exactly:

$$\mathsf{Q}_{A,H} = -\lambda \mathsf{p}_{A,H} + \mathsf{F} \quad , \quad \mathsf{f}_u = \mathsf{m}_u - \mathsf{k}_{A,Hu} + (\lambda-1)\mathsf{p}_{Au}$$

F is not log linear in its components, so we use the usual second order approximations to write:

$$\mathsf{F} = E\mathsf{f} + \frac{1}{2}E(\mathsf{f})^2 = (\lambda-1)E\mathsf{p}_A + \frac{1}{2}E\left[(\mathsf{m} - \mathsf{k}_{A,H}) + (\lambda-1)\mathsf{p}_A\right]^2$$

The second order moments are used based on the first order approximation that $\mathsf{p}_{Au} = (1-\gamma)\,\mathsf{s}_u$, leading to:

$$\begin{aligned}\mathsf{F} &= (\lambda-1)\,E\mathsf{p}_A + \frac{1}{2}E\left(\mathsf{m}-\mathsf{k}_{A,H}\right)^2 + \frac{1}{2}(\lambda-1)^2(1-\gamma)^2 E\left(\mathsf{s}\right)^2 \\ &\quad + (\lambda-1)(1-\gamma)\,E\mathsf{s}\left(\mathsf{m}-\mathsf{k}_{A,H}\right)\end{aligned}$$

We put our results together, and use the expectations of (24), as well as (19) to derive:

$$\begin{aligned}\mathsf{Q}_{A,H} &= -(\lambda-1)(1-\gamma)\left(\mathsf{p}_{A,H}-\mathsf{p}^*_{A,F}\right) \\ &\quad -\frac{2\gamma-1}{2}(\lambda-1)^2(1-\gamma)\,E\left(\mathsf{s}\right)^2\end{aligned}$$

The deviations of expected effort from the steady state are of order 2, and the expected effort level can be easily inferred using:

$$EL_{A,H} = Q_{A,H0}\left[1+\mathsf{Q}_{A,H}\right] = \frac{\theta-1}{\theta\kappa}\left[1+\mathsf{Q}_{A,H}\right]$$

Using the output demands (7) the expected effort for the home household in producing brands of type B is written as:

$$\begin{aligned}EL_{B,H} &= E\left(K_{B,H}\right)^{-1}Y_{B,H} \\ &= \left(P_{B,H}\right)^{-\lambda}EM\left(K_{B,H}\right)^{-1}\left(P_B\right)^{\lambda-1}\end{aligned}$$

Define the ex-ante variables $Q_{B,H} = E\left(K_{B,H}\right)^{-1}Y_{B,H}$ and $F = EM\left(K_{B,H}\right)^{-1}$ $\left(P_B\right)^{\lambda-1}$, we write:

$$Q_{B,H} = \left(P_{B,H}\right)^{-\lambda}F = \left(P_{B,H}\right)^{-\lambda}\sum \pi_u f_u$$

The following relations hold exactly:

$$\mathsf{Q}_{B,H} = -\lambda\mathsf{p}_{B,H} + \mathsf{F} \quad , \quad \mathsf{f}_u = \mathsf{m}_u - \mathsf{k}_{B,Hu} + (\lambda-1)\,\mathsf{p}_{Bu}$$

Following similar steps as for the $\mathsf{Q}_{A,H}$ we write:

$$\begin{aligned}\mathsf{F} &= (\lambda-1)\,E\mathsf{p}_B + \frac{1}{2}E\left(\mathsf{m}-\mathsf{k}_{B,H}\right)^2 + \frac{1}{2}(\lambda-1)^2\gamma^2 E\left(\mathsf{s}\right)^2 \\ &\quad + (\lambda-1)\,\gamma E\mathsf{s}\left(\mathsf{m}-\mathsf{k}_{B,H}\right)\end{aligned}$$

We put our results together, and use the expectations of (25), as well as (21) to derive:

$$\mathsf{Q}_{B,H} = -(\lambda - 1)\gamma\left(\mathsf{p}_{B,H} - \mathsf{p}^*_{B,F}\right) + \frac{2\gamma - 1}{2}(\lambda - 1)^2 \gamma E(\mathsf{s})^2$$

We now turn to the expected effort by the foreign household. The effort used in producing brand of type A is:

$$\begin{aligned} EL_{A,F} &= E(K_{A,F})^{-1} Y_{A,F} \\ &= \left(P^*_{A,F}\right)^{-\lambda} EM(K_{A,F})^{-1}(P_A)^{\lambda-1} S^{-\lambda} \end{aligned}$$

Define the ex-ante variables $Q_{A,F} = E(K_{A,F})^{-1} Y_{A,F}$ and $F = EM(K_{A,F})^{-1}$ $(P_A)^{\lambda-1} S^{-\lambda}$, we write:

$$Q_{A,F} = \left(P^*_{A,F}\right)^{-\lambda} F = \left(P^*_{A,F}\right)^{-\lambda} \sum \pi_u f_u$$

The following relations hold exactly:

$$\mathsf{Q}_{A,F} = -\lambda \mathsf{p}^*_{A,F} + \mathsf{F} \quad , \quad \mathsf{f}_u = \mathsf{m}_u - \mathsf{k}_{A,Fu} + (\lambda - 1)\mathsf{p}_{Au} - \lambda \mathsf{s}_u$$

Following the usual steps we write:

$$\begin{aligned} \mathsf{F} = {} & (\lambda - 1) E\mathsf{p}_A + \frac{1}{2} E(\mathsf{m} - \mathsf{k}_{A,F})^2 + \frac{1}{2}\left[(\lambda - 1)(1 - \gamma) - \lambda\right]^2 E(\mathsf{s})^2 \\ & + \left[(\lambda - 1)(1 - \gamma) - \lambda\right] E\mathsf{s}(\mathsf{m} - \mathsf{k}_{A,F}) \end{aligned}$$

We put our results together, and use the expectations of (24), as well as (20) to derive:

$$\mathsf{Q}_{A,F} = (\lambda - 1)\gamma\left(\mathsf{p}_{A,H} - \mathsf{p}^*_{A,F}\right) + \frac{2\gamma - 1}{2}(\lambda - 1)^2 \gamma E(\mathsf{s})^2$$

We finally the effort exerted by the foreign household in producing brand of type B:

$$\begin{aligned} EL_{B,F} &= E(K_{B,F})^{-1} Y_{B,F} \\ &= \left(P^*_{B,F}\right)^{-\lambda} EM(K_{B,F})^{-1}(P_B)^{\lambda-1} S^{-\lambda} \end{aligned}$$

Define the ex-ante variables $Q_{B,F} = E(K_{B,F})^{-1} Y_{B,F}$ and $F = EM(K_{B,F})^{-1}$ $(P_B)^{\lambda-1} S^{-\lambda}$, we write:

$$Q_{B,F} = \left(P^*_{B,F}\right)^{-\lambda} F = \left(P^*_{B,F}\right)^{-\lambda} \sum \pi_u f_u$$

The following relations hold exactly:

$$\mathsf{Q}_{B,F} = -\lambda \mathsf{p}^*_{B,F} + \mathsf{F} \quad , \quad \mathsf{f}_u = \mathsf{m}_u - \mathsf{k}_{B,Fu} + (\lambda - 1)\, \mathsf{p}_{Bu} - \lambda \mathsf{s}_u$$

Following the usual steps we write:

$$\begin{aligned} \mathsf{F} &= (\lambda - 1)\, E\mathsf{p}_B + \frac{1}{2} E\left(\mathsf{m} - \mathsf{k}_{B,F}\right)^2 + \frac{1}{2}\left[(\lambda - 1)\,\gamma - \lambda\right]^2 E\left(\mathsf{s}\right)^2 \\ &\quad + \left[(\lambda - 1)\,\gamma - \lambda\right] E\mathsf{s}\left(\mathsf{m} - \mathsf{k}_{B,F}\right) \end{aligned}$$

We put our results together, and use the expectations of (25), as well as (22) to derive:

$$\mathsf{Q}_{B,F} = (\lambda - 1)\,(1 - \gamma)\left(\mathsf{p}_{B,H} - \mathsf{p}^*_{B,F}\right) - \frac{2\gamma - 1}{2}\,(\lambda - 1)^2\,(1 - \gamma)\, E\left(\mathsf{s}\right)^2$$

8.5 Welfare expressions

8.5.1 Sticky prices

We show how to express the welfare objective as deviations from the flexible price allocation. Using our results, the objective is written as:

$$\begin{aligned} U &= U_0 + E\mathsf{c} = U_0 + E\mathsf{m} - E\mathsf{p} \\ &= U_0 + E\mathsf{m} - E\mathsf{p} \end{aligned}$$

Using (14) and (19)-(25) we write:

$$\begin{aligned} U &= U_0 - \frac{1}{2} E\mathsf{p}_A - \frac{1}{2} E\mathsf{p}_B \\ &= U_0 - \frac{1}{2}\begin{bmatrix} \gamma \mathsf{p}_{A,H} + (1 - \gamma)\,\mathsf{p}^*_{A,F} - \frac{1}{2}\gamma\,(1 - \gamma)\,(\lambda - 1)\, E\left(\mathsf{s}\right)^2 \\ + (1 - \gamma)\,\mathsf{p}_{B,H} + \gamma \mathsf{p}^*_{B,F} - \frac{1}{2}\gamma\,(1 - \gamma)\,(\lambda - 1)\, E\left(\mathsf{s}\right)^2 \end{bmatrix} \\ &= U_0 - \frac{1}{4}\begin{bmatrix} \gamma E\left(\mathsf{m} - \mathsf{k}_{A,H}\right)^2 + (1 - \gamma)\, E\left(\mathsf{m}^* - \mathsf{k}_{A,F}\right)^2 \\ + (1 - \gamma)\, E\left(\mathsf{m} - \mathsf{k}_{B,H}\right)^2 + \gamma E\left(\mathsf{m}^* - \mathsf{k}_{B,F}\right)^2 \\ + (\lambda - 1)\,\gamma\,(1 - \gamma)\begin{bmatrix} E\left[\mathsf{s} - \left(\mathsf{k}_{A,H} - \mathsf{k}_{A,F}\right)\right]^2 \\ + E\left[\mathsf{s} - \left(\mathsf{k}_{B,H} - \mathsf{k}_{B,F}\right)\right]^2 \end{bmatrix} \\ - (\lambda - 1)\,\gamma\,(1 - \gamma)\left[E\left(\mathsf{k}_{A,H} - \mathsf{k}_{A,F}\right)^2 + E\left(\mathsf{k}_{B,H} - \mathsf{k}_{B,F}\right)^2\right] \end{bmatrix} \end{aligned}$$

8.5.2 Flexible prices

Under flexible prices, the welfare is expressed as (focusing on consumption stabilization):

$$U_{flex} = U_0 + E\mathsf{m}_{flex} - E\mathsf{p}_{flex}$$

We set $E\mathsf{m}_{flex} = 0$ for simplicity, although the monetary stance has no real impact under flexible prices. The individual flexible are given by (8)-(11) in ex-post terms, and are log linear so we write:

$$\begin{aligned} \mathsf{p}_{A,Hu} &= \mathsf{m}_u - \mathsf{k}_{A,Hu} \quad , \quad \mathsf{p}_{A,Fu} = \mathsf{m}_u - \mathsf{k}_{A,Fu} \\ \mathsf{p}_{B,Hu} &= \mathsf{m}_u - \mathsf{k}_{B,Hu} \quad , \quad \mathsf{p}_{B,Fu} = \mathsf{m}_u - \mathsf{k}_{B,Fu} \end{aligned}$$

The price indexes for type A and B are not log linear, so we take second order approximations. Starting withe P_A, we write:

$$[P_{Au}]^{1-\lambda} = \gamma \left[P_{A,Hu}\right]^{1-\lambda} + (1-\gamma) \left[P_{A,Fu}\right]^{1-\lambda}$$

We take a second order approximations of both sides and combine them to get:

$$\begin{aligned} \mathsf{p}_{Au} + \frac{1}{2}(1-\lambda)(\mathsf{p}_{Au})^2 &= \gamma \left[\mathsf{p}_{A,Hu} + \frac{1}{2}(1-\lambda)(\mathsf{p}_{A,Hu})^2\right] \\ &\quad + (1-\gamma)\left[\mathsf{p}_{A,Fu} + \frac{1}{2}(1-\lambda)(\mathsf{p}_{A,Fu})^2\right] \end{aligned}$$

We evaluate the second order terms $(\mathsf{p}_{Au})^2$ based on a first order expansion: $\mathsf{p}_{Au} = \gamma \mathsf{p}_{A,Hu} + (1-\gamma)\,\mathsf{p}_{A,Fu}$, and use our results for $\mathsf{p}_{A,Hu}$ and $\mathsf{p}_{A,Fu}$ to write:

$$\begin{aligned} \mathsf{p}_{Au} &= \gamma\left[\mathsf{m}_u - \mathsf{k}_{A,Hu}\right] + (1-\gamma)\left[\mathsf{m}_u - \mathsf{k}_{A,Fu}\right] \\ &\quad - \frac{1}{2}(\lambda-1)\gamma(1-\gamma)(\mathsf{k}_{A,Hu} - \mathsf{k}_{A,Fu})^2 \end{aligned}$$

Following similar steps for the price index of type B, we obtain:

$$\begin{aligned} \mathsf{p}_{Bu} &= (1-\gamma)\left[\mathsf{m}_u - \mathsf{k}_{B,Hu}\right] + \gamma\left[\mathsf{m}_u - \mathsf{k}_{B,Fu}\right] \\ &\quad - \frac{1}{2}(\lambda-1)\gamma(1-\gamma)(\mathsf{k}_{B,Hu} - \mathsf{k}_{B,Fu})^2 \end{aligned}$$

The expected consumer price index under flexible prices is then written as follows, using (12):

$$\begin{aligned} E\mathsf{p}_{flex} &= \frac{1}{2}E\mathsf{p}_{Au} + \frac{1}{2}E\mathsf{p}_{Bu} \\ &= -\frac{1}{4}(\lambda-1)\gamma(1-\gamma)\left[E(\mathsf{k}_{A,H} - \mathsf{k}_{A,F})^2 + E(\mathsf{k}_{B,H} - \mathsf{k}_{B,F})^2\right] \end{aligned}$$

And the expected welfare under flexible prices is:

$$U_{flex} = U_0 + \frac{1}{4}(\lambda - 1)\gamma(1-\gamma)\left[E(\mathsf{k}_{A,H} - \mathsf{k}_{A,F})^2 + E(\mathsf{k}_{B,H} - \mathsf{k}_{B,F})^2\right]$$

Combining this result with the expected welfare under sticky prices leads to (31).

8.6 Endogenous sectoral structure

We compare the expected discounted profits of different firms, using the marginal utility of income as a discount factor. Starting with a home producer of type A, we write:

$$\Pi_{A,H} = E\frac{1}{PC}\left[P_{A,H} - \frac{W}{K_{A,H}}\right]\left[\frac{P_{A,H}}{P_A}\right]^{-\lambda}\frac{PC}{P_A}$$

Using the optimal pricing (8) we can rewrite this as:

$$\Pi_{A,H} = \frac{1}{\theta}(P_{A,H})^{1-\lambda}E(P_A)^{\lambda-1}$$

We can derive a similar expression for a home producer of type B:

$$\Pi_{B,H} = \frac{1}{\theta}(P_{B,H})^{1-\lambda}E(P_B)^{\lambda-1}$$

Similar steps lead to the following expressions for foreign firms:

$$\begin{aligned}
\Pi^*_{A,H} &= \frac{1}{\theta}\left(P^*_{A,F}\right)^{1-\lambda}E(P_A)^{\lambda-1}(S)^{1-\lambda} \\
\Pi^*_{B,H} &= \frac{1}{\theta}\left(P^*_{B,F}\right)^{1-\lambda}E(P_B)^{\lambda-1}(S)^{1-\lambda}
\end{aligned}$$

These expressions can be rewritten as log approximations. Using our results for the approximations of $E(P_i)^{\lambda-1}$ and $E(P_i)^{\lambda-1}(S)^{1-\lambda}$ we get:

$$\begin{aligned}
\mathsf{\Pi}_{A,H} &= (\lambda-1)\left[-\mathsf{p}_{A,H} + E\mathsf{p}_A + \frac{(\lambda-1)(1-\gamma)^2}{2}E(\mathsf{s})^2\right] \\
\mathsf{\Pi}_{B,H} &= (\lambda-1)\left[-\mathsf{p}_{B,H} + E\mathsf{p}_B + \frac{(\lambda-1)\gamma^2}{2}E(\mathsf{s})^2\right] \\
\mathsf{\Pi}^*_{A,F} &= (\lambda-1)\left[-\mathsf{p}^*_{A,F} + E\mathsf{p}_A + \frac{(\lambda-1)\gamma^2}{2}E(\mathsf{s})^2\right] \\
\mathsf{\Pi}^*_{B,F} &= (\lambda-1)\left[-\mathsf{p}^*_{B,F} + E\mathsf{p}_B + \frac{(\lambda-1)(1-\gamma)^2}{2}E(\mathsf{s})^2\right]
\end{aligned}$$

We focus on the profit differences between firms of different types in a given country:

$$\begin{aligned}
\Pi_{A,H}-\Pi_{B,H} &= (\lambda-1)\left[\begin{array}{c} -(\mathsf{p}_{A,H}-\mathsf{p}_{B,H})+(E\mathsf{p}_A-E\mathsf{p}_B) \\ -\frac{(\lambda-1)(2\gamma-1)}{2}E(\mathsf{s})^2 \end{array}\right] \\
\Pi^*_{A,F}-\Pi^*_{B,F} &= (\lambda-1)\left[\begin{array}{c} -\left(\mathsf{p}^*_{A,F}-\mathsf{p}^*_{B,F}\right)+(E\mathsf{p}_A-E\mathsf{p}_B) \\ +\frac{(\lambda-1)(2\gamma-1)}{2}E(\mathsf{s})^2 \end{array}\right]
\end{aligned}$$

We use the log approximations of the optimal prices and price indexes (19)-(25), and focus on sectoral shocks ($\mathsf{k}_{A,H}=\mathsf{k}_{A,F}=\mathsf{k}_A$, $\mathsf{k}_{B,H}=\mathsf{k}_{B,F}=\mathsf{k}_B$) to write:

$$\begin{aligned}
\frac{\Pi_{A,H}-\Pi_{B,H}}{(\lambda-1)} &= -(1-\gamma)\left(\begin{array}{c} \frac{1}{2}E(\mathsf{m}-\mathsf{k}_A)^2+(\lambda-1)(1-\gamma)E\mathsf{s}(\mathsf{m}-\mathsf{k}_A) \\ -\frac{1}{2}E(\mathsf{m}^*-\mathsf{k}_A)^2-\gamma(\lambda-1)E(-\mathsf{s})(\mathsf{m}^*-\mathsf{k}_A) \end{array}\right) \\
&\quad +\gamma\left(\begin{array}{c} \frac{1}{2}E(\mathsf{m}-\mathsf{k}_B)^2+\gamma(\lambda-1)E\mathsf{s}(\mathsf{m}-\mathsf{k}_B) \\ -\frac{1}{2}E(\mathsf{m}^*-\mathsf{k}_B)^2-(1-\gamma)(\lambda-1)E(-\mathsf{s})(\mathsf{m}^*-\mathsf{k}_B) \end{array}\right) \\
&\quad -\frac{(\lambda-1)(2\gamma-1)}{2}E(\mathsf{s})^2 \\
\frac{\Pi^*_{A,F}-\Pi^*_{B,F}}{(\lambda-1)} &= \gamma\left(\begin{array}{c} \frac{1}{2}E(\mathsf{m}-\mathsf{k}_A)^2+(\lambda-1)(1-\gamma)E\mathsf{s}(\mathsf{m}-\mathsf{k}_A) \\ -\frac{1}{2}E(\mathsf{m}^*-\mathsf{k}_A)^2-\gamma(\lambda-1)E(-\mathsf{s})(\mathsf{m}^*-\mathsf{k}_A) \end{array}\right) \\
&\quad -(1-\gamma)\left(\begin{array}{c} \frac{1}{2}E(\mathsf{m}-\mathsf{k}_B)^2+\gamma(\lambda-1)E\mathsf{s}(\mathsf{m}-\mathsf{k}_B) \\ -\frac{1}{2}E(\mathsf{m}^*-\mathsf{k}_B)^2-(1-\gamma)(\lambda-1)E(-\mathsf{s})(\mathsf{m}^*-\mathsf{k}_B) \end{array}\right) \\
&\quad +\frac{(\lambda-1)(2\gamma-1)}{2}E(\mathsf{s})^2
\end{aligned}$$

As $\mathsf{m}^*=\mathsf{m}-\mathsf{s}$ these expressions simplify as:

$$\begin{aligned}
\Pi_{A,H}-\Pi_{B,H} &= \lambda(\lambda-1)\left[\begin{array}{c} -(1-\gamma)E\mathsf{s}(\mathsf{m}-\mathsf{k}_A)+\gamma E\mathsf{s}(\mathsf{m}-\mathsf{k}_B) \\ -\frac{(2\gamma-1)}{2}E(\mathsf{s})^2 \end{array}\right] \\
\Pi^*_{A,F}-\Pi^*_{B,F} &= \lambda(\lambda-1)\left[\begin{array}{c} \gamma E\mathsf{s}(\mathsf{m}-\mathsf{k}_A)-(1-\gamma)E\mathsf{s}(\mathsf{m}-\mathsf{k}_B) \\ -\frac{(2\gamma-1)}{2}E(\mathsf{s})^2 \end{array}\right]
\end{aligned}$$

We can easily see that when the exchange rate does not fluctuate, all the terms are equal to zero: $\Pi_{A,H}-\Pi_{B,H}=\Pi^*_{A,F}-\Pi^*_{B,F}=0$, so firms are indifferent with respect to their sector. A fixed exchange rate then supports any sectoral allocation, but only the allocation $\gamma=0.5$ supports a fixed exchange rate.

The second equilibrium is the case where sectoral specialization is complete ($\gamma = 1$) and monetary policy is inward looking ($\mathsf{m} = \mathsf{k}_A$, $\mathsf{m}^* = \mathsf{k}_B$). In this case we have:

$$\begin{aligned} \Pi_{A,H} - \Pi_{B,H} &= \frac{\lambda(\lambda-1)}{2} E(\mathsf{k}_A - \mathsf{k}_B)^2 > 0 \\ \Pi^*_{A,F} - \Pi^*_{B,F} &= -\frac{\lambda(\lambda-1)}{2} E(\mathsf{k}_A - \mathsf{k}_B)^2 < 0 \end{aligned}$$

so home [foreign] firms have an incentive to produce type A [B], thereby sustaining the sectoral specialization that motivates the inward looking monetary policy.

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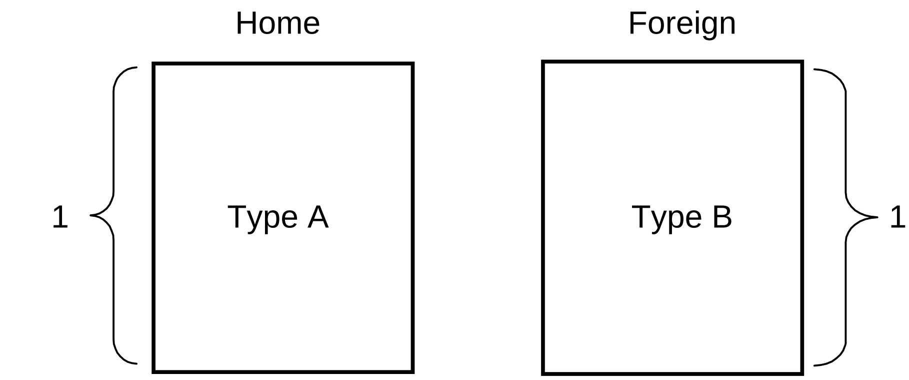
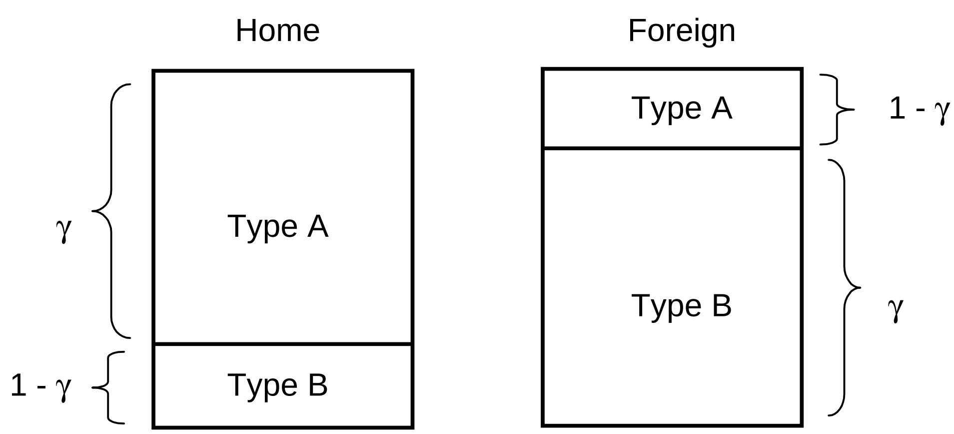
Figure 1: Sectoral specialization

γ **= 1** : Complete sectoral specialization



0 < γ **< 1** : Partial sectoral specialization



γ **= 0.5** : No sectoral specialization

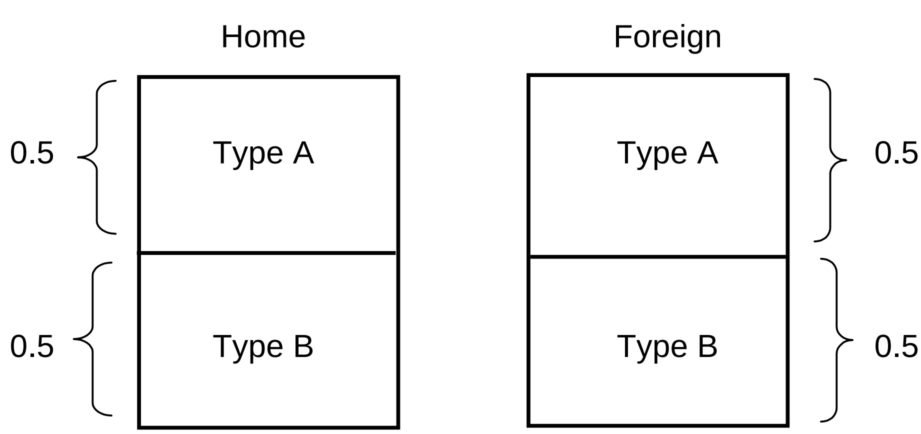
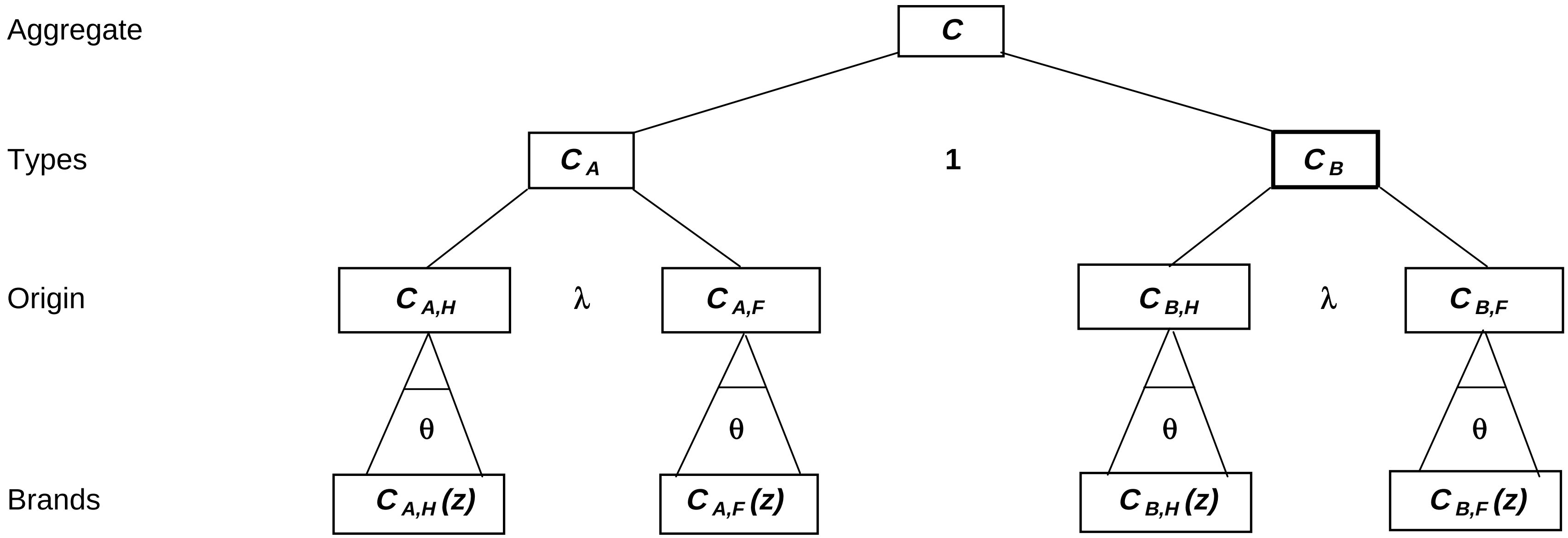


Figure 2: Consumption allocation

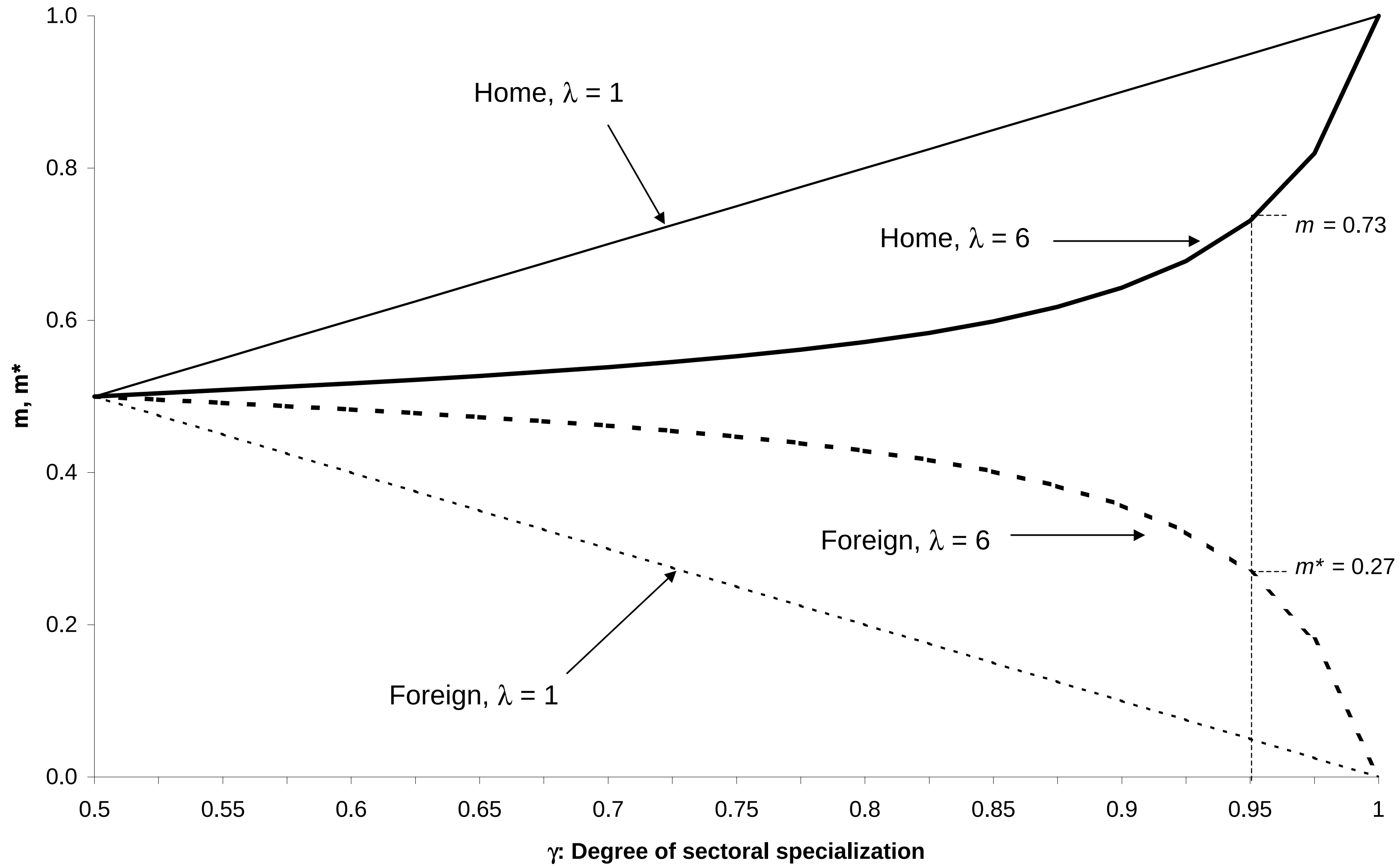


Elasticities of substitution:

- between types (*A* vs. *B*): **1**
- between origins within type (*A*,*H* vs. *A*,*F*): λ
- between brands within origin and type (different *A*,*H(z)*) θ

Ranking of elasticities of substitution: $\theta > \lambda > 1$

Figure 3: Monetary response to a unit shock in sector A



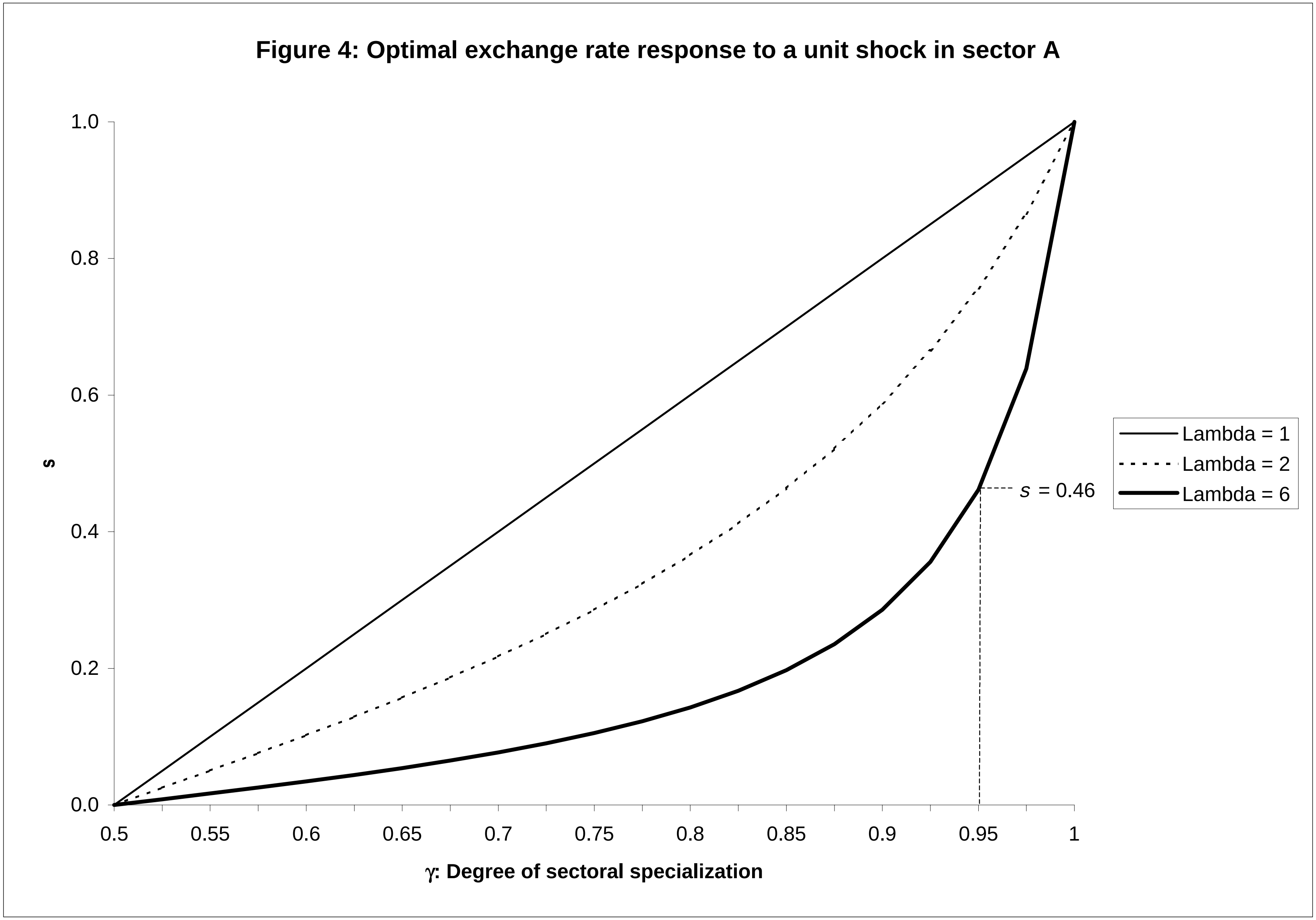


Figure 4: Optimal exchange rate response to a unit shock in sector A
1.0
0.8
0.6
0.4
0.2
0.0
s
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
γ: Degree of sectoral specialization
s = 0.46
Lambda = 1
Lambda = 2
Lambda = 6

Figure 5: Welfare gain from exchange rate flexibility