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On the Benefits to Transparency in a Monetary Policy Instrument*

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ABSTRACT

Monetary instruments differ in their *transparency*—how easy it is for the public to monitor the instrument—and their *tightness*—how closely they are linked to inflation. Tightness is always desirable in a monetary policy instrument. When is transparency desirable? We show that transparency is desirable when there is a credibility problem, in that the government cannot commit to its policy. We illustrate our argument by considering a classic question in international economics: Is the exchange rate or the money growth rate the better instrument of monetary policy? Our analysis suggests that the greater transparency of exchange rates means that if both instruments are equally tight, the exchange rate is the preferred one.

Keywords: nominal anchor, exchange rate regime, monetary instrument, time consistency, transparency, inflation targeting, monetary regime

JEL Nos. E5, E52, E61, F33, F41

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"By the simple virtue of being a price rather than a quantity, the exchange rate provides a much clearer signal to the public of the government's intentions and actual actions than a money supply target. Thus, if the public's inflationary expectations are influenced to a large extent by the ability to easily track and continuously monitor the nominal anchor, the exchange rate has a natural advantage" (Calvo and Végh 1999, p. 1589).

"True, the exchange rate has some special properties. In particular, it is easily observable, so the private sector can directly monitor any broken promises by the central bank. But we know of no convincing argument that turns these properties into an explanation for why it would be a more efficient method to achieve credibility to target the exchange rate rather than, say, the money growth rate" (Persson and Tabellini 1994, p. 17).

A classic question in international economics is whether the exchange rate or the money growth rate is the better instrument of monetary policy. A common answer, illustrated by the quotation from Calvo and Végh (1999) above, is that the exchange rate has a natural advantage over the money growth rate as an instrument of monetary policy because the exchange rate is easier for the public to observe; that is, it is more transparent. Skeptics of this view agree that the exchange rate is easier for the public to monitor. However, as Persson and Tabellini (1994) point out in the other quotation above, no clear theoretical argument has been made that explains why the transparency of the exchange rate gives it a natural advantage as a monetary policy instrument. We provide such a theoretical argument here. Our analysis also sheds light on the more general question in monetary economics of why there are benefits to having transparency in a monetary policy instrument. We find that transparency is beneficial because it can help migrate credibility problems.

We build on the analyses of Canzoneri (1985), Zarazaga (1995), and Herrendorf (1997), using a simple model of sustainable monetary policy similar to that of Kydland and Prescott

(1977) and Barro and Gordon (1983). In our model, each period, the government chooses one of two regimes for monetary policy: an exchange rate regime or a money regime. Under the *exchange rate regime*, the government picks the rate of depreciation of the exchange rate with some foreign country as its monetary policy instrument. By choosing this exchange rate, the government sets the mean inflation rate, and realized domestic inflation varies with shocks to the inflation rate in the foreign country.[1] Under the *money regime*, the government picks a money growth rate as its instrument, thus setting the mean inflation rate, and realized inflation varies with domestic inflation shocks. Hence, under both regimes, the government sets the mean inflation rate, and realized inflation varies with exogenous shocks. In both regimes, the government is targeting inflation; it is just using different instruments to implement its target.

The instruments that define these regimes differ in two respects: their tightness and their transparency. One instrument is *tighter* than another if it is more closely linked to inflation.[2] In our setup, the relative tightness of the instruments depends on the relative variance of the foreign and the domestic shocks. One instrument is more *transparent* than another if it more easily observed by the public. In our setup, we assume for simplicity that the exchange rate is perfectly observed while only a noisy signal of the money growth rate is observed. We thus refer to the exchange rate as the *transparent instrument* and the money growth rate as the *opaque instrument*.

Tightness is desirable in an instrument because the government dislikes variability in inflation. We show that transparency is desirable in an instrument only because this characteristic helps mitigate credibility problems that arise when the government cannot commit to its monetary policies.

To emphasize this point, we compare the relative desirability of these instruments in two types of environment. We first consider an environment in which the government can commit to its policies and, hence, has no credibility problems. We show that with commitment, the relative desirability of instruments does not depend on their transparency: the tighter instrument is always preferred. We then consider an environment in which the government has credibility problems because it cannot commit to its policies. In this environment, we show that the relative desirability of instruments depends on both their tightness and their transparency. Tightness is desirable without commitment for the same reasons it is desirable with commitment: a tighter instrument leads to less variable inflation. Transparency is desirable without commitment because it helps mitigate credibility problems. To illustrate this point, we show that the transparent instrument, the exchange rate, may be preferred to the opaque one, the money growth rate, even if money growth is the tighter instrument.

The intuition for our results is as follows. Under either regime, when there is no commitment, the government has a temptation to surprise the public with higher than expected inflation in order to decrease unemployment. In order to achieve a good outcome, the equilibrium strategies must have two features at the same time. They must ensure that the government gets a high payoff when it chooses low inflation and a low payoff when it deviates to high inflation. With a transparent instrument, any deviation is perfectly detectable, there is no conflict between these two features, and the economy need never experience periods with low payoffs for the government. With an opaque instrument, these two features are in conflict. To deter deviations to high money growth, the strategies must ensure that high realizations of inflation are followed by low payoffs for the government. Since high realizations of inflation will occur even if the government does not deviate, with such strategies at least some period of low payoffs for the government must be realized in equilibrium.

The result about the benefits to transparency is easiest to show under the assumptions that inflation is the only signal of money growth and that money growth is never observable. We show that we can allow for other signals or for money growth to be observable with a lag and still obtain our result.

Throughout we use exchange rates and money growth rates as examples of transparent and opaque instruments. Our arguments about the benefits of transparency when there is no commitment apply more generally to other potential instruments. In particular, in countries where the domestic short-term interest rate is a transparent market price, it should have a similar benefit over less transparent instruments. In many less-developed countries with less well-developed capital markets, the exchange rate may be the more relevant alternative to money growth rates.

Canzoneri (1985) discusses what might be observed in the best equilibrium with transparent or opaque instruments when there are credibility problems. Here we extend his analysis. Most interesting to us is what will be observed when the opaque instrument, here money growth, is the preferred one. This will happen when money growth is sufficiently tight. With such an instrument, agents cannot tell whether high realized inflation is the result of the government's choice of a high money growth rate or is simply the result of a large domestic inflation shock. Because of this lack of transparency, the optimal outcome *necessarily* oscillates at random between two extreme phases, with low and high average inflation. This random oscillation along the equilibrium path in the best money regime is analogous to the outcomes obtained by Green and Porter (1984) and Abreu, Pearce, and Stacchetti (1986) in their analyses of equilibrium price wars among oligopolists. Thus, our model predicts that if countries are in the best money regime, there are necessarily periodic bouts of high inflation followed by periodic bouts of low inflation.

Our analysis builds on the seminal contribution of Canzoneri (1985), who assumes that a private information problem arises under a money regime because it is opaque. Canzoneri was the first to use the logic of Green and Porter (1984) to explain periodic bouts of high inflation. (See also the work of Zarazaga (1995), who extends this logic, and Albanesi, Chari, and Christiano (2001), who use multiple Markov equilibria to obtain similar outcomes.)

Our work here is most closely related to Herrendorf's (1997) analysis of fixed versus floating exchange rate regimes and Stokey's (forthcoming) analysis of Markov equilibria with various instruments. Herrendorf (1997) considers an optimal taxation game in which the monetary authority must finance a given amount of spending with a combination of direct taxes and inflation taxes. The monetary authority can choose a transparent fixed exchange rate regime in which it must set some fixed suboptimal exchange rate peg or an opaque money regime in which it is free to choose any level of money it desires. Herrendorf gives an intriguing example in which if the signal of money growth is sufficiently noisy, then the only equilibrium in the money regime is the repeated one-shot equilibrium. Thus, with sufficiently noisy signals, the money regime can be worse than the fixed exchange rate regime with a fixed suboptimal peg. Stokey (forthcoming) builds on our analysis, but focuses on using simple two-state Markov perfect equilibria to study related issues.

There is a related literature which uses signaling models to look at the issue of transparency somewhat differently. Herrendorf (1999) considers an environment with two types of monetary authorities: one type with a commitment technology and one without such a technology. The monetary authority must choose between a transparent fixed exchange rate regime and an opaque floating exchange rate regime. He shows that if the public has sufficiently high beliefs that the monetary authority can commit, then both types choose the fixed exchange rate regime. We think of Herrendorf's (1999) model as applying to countries that are likely to have the power to commit and, hence, do not face significant time inconsistency problems in monetary policy. In contrast, we think of our model as applying to countries that have had chronic problems committing to policies that yield low inflation. Canavan and Tommasi (1997) explore a similar theme to that of Herrendorf (1999) in a model with unobserved types who are required to choose linear strategies. For related work in a domestic context, see the analysis of Backus and Driffill (1985).

Here we have used a simple reduced-form model of money. Chang (1998) and Phelan and Stacchetti (2001) use recursive methods to analyze some general equilibrium macroeconomic models with perfect monitoring.

1. Two Monetary Policy Instruments

Here we present a model of monetary policy in which, each period, the government selects either an exchange rate regime, in which it uses the exchange rate as its policy instrument, or a money regime, in which it uses the money growth rate as its policy instrument.

In the model, time is discrete, and time periods are denoted $t = 0, 1, 2, \ldots$. The economy consists of a government, which dislikes both unemployment and inflation, and a continuum of agents who each choose the rate of change of their individual nominal wages.

The timing of actions within each period is as follows. At the beginning of a period, the government chooses a regime for monetary policy, namely, whether it will use the exchange rate or the money growth rate as its policy instrument in the current period. If it chooses the (crawling peg) exchange rate regime, the government opens a trading desk at which it trades domestic and foreign currency. If it chooses the money regime, the government does not open this desk. The presence or absence of the trading desk is thus an observable indicator of the current regime. After the government's choice of regime, agents choose their nominal wages. Finally, depending on the regime, the government chooses either the rate of depreciation of the exchange rate or the money growth rate. The government is free to switch regimes at the beginning of each period.

For convenience, we will describe the economy for a given period t starting at the end of the period and working backward to the beginning. At the end of the period, depending on the regime, the government chooses the rate of depreciation of the exchange rate or the money growth rate. It takes as given the average rate of wage inflation x set by agents earlier in the period. Unemployment is equal to a constant U plus the gap between average wage inflation x and realized inflation π. The government's per period payoff for a given value of x and a realization of π is

$$(1) \qquad r(x, \pi) = -\frac{1}{2}\left[(U + x - \pi)^2 + \pi^2\right].$$

Under the two regimes, realized inflation is a function of monetary policy as follows. Under the exchange rate regime, the government chooses a rate of change in the exchange rate denoted $e_t = s_t - s_{t-1}$, where s_t is the level of the exchange rate. For simplicity, however, we refer to e_t as the *exchange rate*. Inflation in the home country is given by

(2) $\pi = e + \pi^*$

where π^* is inflation in the foreign country, which has a normal distribution with mean 0 and variance $\sigma^2_{\pi^*}$. Thus, by choosing an exchange rate, the government sets the mean inflation rate to be e, while the variance of domestic inflation is determined by shocks in the foreign country which are outside the domestic government's control. Foreign inflation π^* is observed only after the exchange rate is chosen. We let $g(\pi|e)$ denote the density of realized domestic inflation given the choice of exchange rate e.

Under the money regime, the government chooses a money growth rate μ. Given μ, realized inflation π is given by

(3) $\pi = \mu + \varepsilon$

where ε represents domestic inflation shocks which are normally distributed with mean 0 and variance σ^2_{π}. Thus, by choosing the money growth rate, the government sets the mean inflation rate to be μ, and the variance of domestic inflation is determined by domestic shocks outside of the government's control. We interpret the imperfect connection between money growth and inflation as arising from some combination of the government's imperfect control over actual (as opposed to desired) money growth and a noisy relation between money growth and inflation. We let $f(\pi|\mu)$ denote the density of realized inflation given the choice of money growth rate.

We say that the money growth rate is a *tighter instrument* than the exchange rate if and only if $\sigma^2_{\pi} < \sigma^2_{\pi^*}$. To model the idea that exchange rates are more transparent than money growth rates in that they are more easy to monitor, we assume that under both regimes, agents can see the exchange rate e and the inflation rate π but not the money growth rate μ. Thus, under an exchange rate regime, agents directly see the actions of the government, while under a money regime they do not. In the money regime, inflation serves as a noisy signal of the government's actions. We refer to the exchange rate as the *transparent instrument* and the money growth rate as the *opaque instrument*.

Under both regimes, equations (2) and (3) both hold. In the exchange rate regime, e is the choice variable and the money growth rate μ is endogenously determined, while in the money regime, μ is the choice variable and the exchange rate e is endogenously determined. In these regimes, the government's choice of either e or μ determines the mean inflation rate. In this sense, in both regimes, the government is targeting inflation.

The government's expected per period payoff under an exchange rate e is

$$S(x, e) = \int r(x, \pi) g(\pi|e)\, d\pi$$

and under a money growth rate μ is

$$R(x, \mu) = \int r(x, \pi) f(\pi|\mu)\, d\pi.$$

With our functional forms, these become

$$S(x, e) = -\frac{1}{2}\left[(U + x - e)^2 + e^2\right] - \sigma^2_{\pi^*} \tag{4}$$

$$R(x, \mu) = -\frac{1}{2}\left[(U + x - \mu)^2 + \mu^2\right] - \sigma^2_{\pi}. \tag{5}$$

Notice that the government's payoffs in the two regimes are symmetric with respect to the policy variables e and μ. In particular, the functions S and R differ only with respect to the uncontrollable variances $\sigma^2_{\pi^*}$ and σ^2_{π}, which are constants. Clearly, from (4) and (5) tightness is a desirable characteristic of an instrument. We ensure that the government's payoffs are bounded by assuming that the policies e and μ are bounded above and below by some arbitrarily large constants.

In the middle of each period, each agent chooses the change in the agent's own wage rate $z_t = w_t - w_{t-1}$. For simplicity, we refer to z_t as *individual wages.* We let x_t denote the average change in the wage rate in period t, which, again for simplicity, we refer to as *average wages.* An agent's payoff for a given value of z and a realization of π is

$$(6) \qquad r^A(z, \pi) = -\frac{1}{2}\left[(z-\pi)^2\right].$$

Each agent can choose z differently depending on whether the regime is an exchange rate regime or a money regime. We denote these choices z_e and z_μ. An agent's expected per period payoff under an exchange rate regime with exchange rate e is

$$(7) \qquad S^A(z_e, e) = \int r^A(z_e, \pi) g(\pi|e)\, d\pi = -\frac{1}{2}\left[(z_e - e)^2 + \sigma^2_{\pi^*}\right]$$

while this agent's analogous payoff under a money regime with money growth rate μ is

$$(8) \qquad R^A(z_\mu, \mu) = \int r^A(z_\mu, \pi) f(\pi|\mu)\, d\pi = -\frac{1}{2}\left[(z_\mu - \mu)^2 + \sigma^2_\pi\right].$$

Notice that under either regime, agents aim to choose wages equal to mean inflation, either e or μ, depending on the regime.

Notice also that the objective function of these agents differs from that of the government. In our simple reduced-form model, this difference generates the conflict of interests between the government and the agents that leads to a time inconsistency problem. We think of this setup as a reduced-form way of capturing the tension that occurs in a general equilibrium model in which the government and the agents have the same objectives but there are distortions in the economy that lead to a time inconsistency problem. With some more cumbersome notation we can have the government maximizing the welfare of the private agents and obtain identical results.[3]

The payoff for the government is the discounted value of its expected per period payoffs

$$(9) \qquad (1-\beta)\sum_{t=0}^{\infty}\beta^t[(1-i_t)S(x_{et}, e_t) + i_t R(x_{\mu t}, \mu_t)]$$

where $1 \geq \beta > 0$ is the discount factor and i_t is a variable that indicates the regime chosen in period t, where $i_t = 0$ for the exchange rate regime and $i_t = 1$ for the money regime. Here x_{et} denotes the average wages chosen in period t if an exchange rate regime is chosen and $x_{\mu t}$ denotes the average wages chosen in period t if a money regime is chosen. The discounted payoffs for the agents are written similarly.

2. Two Environments

We examine the potential benefits to tightness and transparency in two environments: when the government can commit to its monetary policy and when it cannot. We conclude tightness has benefits in both environments but transparency is beneficial only when the government cannot commit.

A. With Commitment

We first suppose that the government can commit to a monetary policy once and for all in period 0. We show that when the government can commit to its policies the ranking of instruments does not depend on their transparency: the tighter instrument is always preferred. Here this means that an exchange rate regime is preferred to a money regime if and only if the volatility of foreign inflation shocks is less than that of domestic inflation shocks. Thus, with commitment, transparency has no benefits.

In this environment with commitment, at the beginning of period 0, the government chooses the sequence $\{i_t, e_t, \mu_t\}_{t=0}^{\infty}$ indicating the regime it will follow and the exchange rate or money growth rate it will implement under that regime in each period. After this, in each period t, agents choose wages z_{et} or $z_{\mu t}$, depending on the regime. Given (7) and (8), it is clearly optimal for agents to choose $z_{et} = e_t$ and $z_{\mu t} = \mu_t$; hence, in equilibrium, average wages satisfy

(10) $x_{et} = e_t$ and $x_{\mu t} = \mu_t$.

Here the optimal policies and allocations solve the Ramsey problem of choosing sequences $\{i_t, e_t, \mu_t, x_{et}, x_{\mu t}\}_{t=0}^{\infty}$ to maximize the government's discounted payoff (9) subject to the equilibrium condition on agents' average wages (10). This problem reduces to a sequence of static problems of choosing e and μ to solve $\max_e S(e,e)$ and $\max_\mu R(\mu,\mu)$ and then choosing the regime that leads to the higher payoff. Since the government's payoffs are symmetric with respect to the policy variables, the optimal exchange rate and money growth rate are identical (both 0), and the government simply picks the regime with the lower variance of inflation. We denote this maximum payoff as v^R and refer to it as the *Ramsey payoff*. We summarize this result as

PROPOSITION 1. ONLY TIGHTNESS MATTERS WITH COMMITMENT. *When the government can commit to its monetary policies, the tighter instrument is preferred regardless of its transparency. Thus, with commitment, transparency has no benefits and the exchange rate regime is preferred to a money regime if and only if* $\sigma^2_{\pi^*} \leq \sigma^2_{\pi}$.

Here the optimal policy in both regimes is a constant. This occurs only because, for simplicity, we have abstracted from any source of shocks that would make the policies vary. For example, if the government's per period payoff function is

$$r(x,\pi,\theta) = -\frac{1}{2}\left[(U+x-\pi)^2 + (\pi-\theta)^2\right]$$

where θ follows some stochastic process, then Ramsey policies would be $e(\theta) = \mu(\theta) = \theta$. Here the government has a fluctuating target level of inflation and the Ramsey policies imply stochastically fluctuating exchange rates and money growth rates.

B. Without Commitment

Now we suppose that the government cannot commit to its policies. Instead, in each period, it chooses a regime and then, after agents set their wages, the government chooses the level of its monetary policy instrument. For this environment, we show that transparency has benefits. Specifically, we show that if the exchange rate and money growth are equally tight instruments, then, given any equilibrium in which the government chooses a money regime in some period t, we can construct another equilibrium in which the government chooses instead an exchange rate regime at period t and obtains a strictly higher payoff. Thus, even if money growth is the tighter instrument, an exchange rate regime can be preferred because of its transparency.

In this environment, both the government and agents choose their actions as functions of the observed history of aggregate variables: the choice of regime, the exchange rate, and inflation. In period t, this history is given by $h_t = (i_0, e_0, \pi_0; \ldots; i_{t-1}, e_{t-1}, \pi_{t-1})$. A strategy for the government is a sequence of functions $\sigma^G = \{i_t(h_t), e_t(h_t), \mu_t(h_t)\}$ that map histories into the choice of regime i_t and corresponding exchange rates e_t or money growth rates μ_t. A strategy for agents is a sequence of functions $\sigma^A = \{z_{et}(h_t), z_{\mu t}(h_t)\}_{t=0}^{\infty}$ that map histories into actions z_t, where $z_{et}(h_t)$ is only relevant if $i_t(h_t) = 0$ and $z_{\mu t}(h_t)$ is only relevant if $i_t(h_t) = 1$. We also define a sequence of functions $\sigma^X = \{x_{et}(h_t), x_{\mu t}(h_t)\}_{t=0}^{\infty}$ that record the average wages chosen by agents after each history. Let $\sigma = (\sigma^G, \sigma^A, \sigma^X)$ denote the strategies of the government, the strategies of the agents, and the average wages. Notice that in the histories, we need not record the history of average wages since a deviation by any one agent cannot affect this average. (For details on this point, see, for example, Chari and Kehoe 1990.)

A *perfect equilibrium* in this environment is a collection of strategies σ such that *(i)* after every history h_t, the agents' strategy σ^A is optimal given the government's strategy σ^G and the average of other agents' wages σ^X; *(ii)* after every history h_t, the government's strategy σ^G is optimal given the average of agents' wages σ^X; and *(iii)* after every history h_t, σ^A and σ^X agree.

Clearly, given agents' payoffs (7) and (8), after any history h_t, the agents' best response to the government strategy σ^G is to choose wages $z_{et}(h_t) = e_t(h_t)$ or $z_{\mu t}(h_t) = \mu_t(h_t)$, depending on the regime. Thus, in any perfect equilibrium, average wages must satisfy

$$x_{et}(h_t) = e_t(h_t) \text{ and } x_{\mu t}(h_t) = \mu_t(h_t).$$

That is, in equilibrium, wage inflation must equal expected inflation.

To prove our main result, we formulate the incentive constraint of the government recursively, by drawing on the work of Abreu, Pearce, and Stacchetti (1986, 1990). Their basic idea is as follows. A strategy is a prescription for current actions and all future actions following every possible history. When evaluating the government's incentive constraints, however, we need not specify the whole sequence of future actions for the government and agents that follow every possible current action that the government might take. Rather, we need specify only how the government's payoff from the next period on, namely, its *continuation value*, will vary as the government's current action varies. This simple observation forms the basis for a recursive approach to describing the incentive compatibility constraints for the government.

We first show how strategies can be summarized by a specification of a current action and a continuation value and then use this specification to define a recursive incentive constraint. We begin with the incentives after the money regime has been selected in the current period. Fix a collection of strategies $\sigma = (\sigma^G, \sigma^A, \sigma^X)$, and suppose that in period t following history h_t, the government has chosen a money regime ($i_t(h_t) = 1$) and agents have chosen wages $x_{\mu t}(h_t)$. Since agents observe only inflation $\pi_t = \mu_t + \varepsilon_t$, which is a noisy signal of μ_t, the equilibrium following period t as specified in a collection of strategies σ cannot depend on the government's choice of μ_t directly; it can vary only with inflation π_t. Hence, the government's continuation value from next period on can be summarized by a *continuation value function* $w_t(\pi, h_t)$. This function records the present discounted value of utility for the government that occurs under σ following the history $h_{t+1} = \big(h_t, i_t(h_t) = 1, e_t = \pi_t - \pi_t^*, \pi_t\big)$. (It should be clear that it is feasible but redundant for the equilibrium to be conditioned on the exchange rate $e = \pi - \pi^*$ in addition to realized inflation.)

The incentive constraint for the government in the money regime is as follows. The equilibrium specifies that the government choose $\mu_t(h_t)$ in the current period. Given the current wage chosen by the agents $x_{\mu t}(h_t)$ and the continuation value function $w_t(\pi, h_t)$, the incentive constraint requires that there is no other money growth rate $\mu_t' \neq \mu_t(h_t)$, such that the government could benefit by deviating to μ_t' in the period t and then acting according to its strategy σ^G from period $t+1$ on; that is,

$$
\begin{aligned}
(11) \quad & (1-\beta)R\big(x_{\mu t}(h_t), \mu_t(h_t)\big) + \beta \int w_t(\pi, h_t) f\big(\pi | \mu_t(h_t)\big)\, d\pi \geq \\
& (1-\beta)R\big(x_{\mu t}(h_t), \mu_t'\big) + \beta \int w_t(\pi, h_t) f(\pi | \mu_t')\, d\pi
\end{aligned}
$$

for any possible μ_t'. Notice that here a deviation μ_t' from the specified current action $\mu_t(h_t)$ affects the government's expected discounted payoffs only through how this deviation shifts the distribution of inflation from $f\big(\pi|\mu_t(h_t)\big)$ to $f(\pi|\mu_t')$.

Consider next the incentive constraint given that the government has chosen the exchange rate regime in period t. Given the wages $x_{et}(h_t)$ chosen by agents, this incentive constraint is

$$
(12) \quad (1-\beta)S\big(x_{et}(h_t), e_t(h_t)\big) + \beta w_t\big(e_t(h_t), h_t\big) \geq (1-\beta)S\big(x_{et}(h_t), e_t'\big) + \beta w_t(e_t', h_t)
$$

for any possible e'. Here $w_t(e_t, h_t)$ records the present discounted value of utility for the government that occurs under σ following the history $h_{t+1} = \big(h_t, i_t(h_t) = 0, e_t\big)$.

Notice that in (11) and (12) we are only considering one-shot deviations, namely, changes in the current actions, holding fixed the future strategies. A standard result says that since the payoffs of the government are bounded, these recursive incentive constraints are both necessary and sufficient for full incentive compatibility.

Notice also that if the payoff function specifies a constant in either regime, then there is no deterrence value, and both regimes yield the same one-shot equilibrium outcome (of $\mu_t = U$ and $e_t = U$).

The following proposition shows the precise benefits of the transparent instrument when there is no commitment.

Proposition 2. The Benefits of Transparency. *When the two instruments have equal tightness, the transparent instrument is preferred to the opaque instrument in the following sense. Consider any equilibrium σ in which the money regime is chosen in some period t and the current money growth rate is below the one-shot equilibrium level. Then there is an equilibrium $\tilde{\sigma}$ with higher welfare in which the exchange rate regime chosen in period t and otherwise agrees with the original equilibrium.*

The idea of the proof is the following. To achieve a good outcome, the continuation payoff must have two features. It must deter the government from deviating from the prescribed policy, and it must give the government a high continuation payoff when it does not deviate. With a transparent instrument, any deviation is perfectly detectable, and there is no conflict between these two features: the continuation payoff function can specify the lowest possible continuation when there is any deviation and the highest possible continuation when there is no deviation.

With an opaque instrument, however, the continuation payoff function can depend only on a noisy signal of the policy, so there is a conflict. If this function specifies the highest payoff regardless of the observed noisy signal, then this function fails to have any deterrence value and results in the one-shot equilibrium outcome. If this function builds in any deterrent value by prescribing lower continuation values for some inflation rates, then with positive probability the lower continuation value must be realized even if the government pursues the desired policy. This feature necessarily leads to lower payoffs along the equilibrium path. In this sense, benefits of transparency arise from the ability to tailor the continuation payoff function precisely to deviations: it can give high payoffs only when exactly the right policy is being pursued, and it can give low payoffs when any other policy is used.

Proof of Proposition 2. Let σ be an equilibrium in which the money regime is chosen in period t for some history h_t with agents' wages $x_{\mu t}(h_t)$, money growth rate $\mu_t(h_t)$, and continuation value $w_t(\pi, h_t)$. We construct an equilibrium $\tilde{\sigma}$ with higher welfare as follows.

First set $\tilde{\sigma}$ so that the actions of the agents and the government in every period and history prior to period t are the same as those specified in the original set of equilibrium strategies σ. Next, after history h_t, let $\tilde{\sigma}$ specify that the exchange rate regime is chosen, and let $\tilde{e}_t(h_t) = \mu_t(h_t)$ be the exchange rate. Let agents' wages be $\tilde{x}_{et}(h_t) = \tilde{e}_t(h_t)$ to ensure that the agents' incentive compatibility constraint is satisfied. For all other histories that are possible in period t, set the actions specified under $\tilde{\sigma}$ equal to those specified under σ.

Let $\bar{w}_t(h_t)$ and $\underline{w}_t(h_t)$ denote the highest and the lowest continuation values following h_t under the equilibrium σ, so that $\bar{w}_t(h_t) = \max_\pi w_t(\pi, h_t)$ and $\underline{w}_t(h_t) = \min_\pi w_t(\pi, h_t)$, where $w_t(\pi, h_t)$ is the continuation value from the original equilibrium in which money is used as an instrument in period t. Let the continuation value under $\tilde{\sigma}$ be

$$\tilde{w}_t(e_t, h_t) = \left\{ \begin{array}{l} \bar{w}_t(h_t) \text{ if } e_t = \tilde{e}_t(h_t) \\ \underline{w}_t(h_t) \text{ if } e_t \neq \tilde{e}_t(h_t) \end{array} \right\}$$

and let the future strategies under $\tilde{\sigma}$ correspond to the strategies under σ that support these continuation values. Thus, $\tilde{w}_t(e_t, h_t)$ specifies that if the government chooses the prescribed exchange rate $\tilde{e}_t(h_t)$, then it receives the highest value that it would have received in the original equilibrium in which it chose the money regime, while if it chooses any other value, it receives the lowest value that it would have received in the original equilibrium.

Clearly, to show that our constructed strategies are an equilibrium, we need show only that they satisfy the incentive constraint for the government following h_t when the exchange rate regime is chosen. (For other histories and periods, we simply use the properties of $\tilde{\sigma}$ inherited from σ.) To see that this is true, rewrite the incentive constraint when the exchange rate is used as

$$\text{(13)} \quad (1-\beta)\big[S\big(\tilde{x}_{et}(h_t), e_t'\big) - S\big(\tilde{x}_{et}(h_t), \tilde{e}_t(h_t)\big)\big] \leq \beta\,[\bar{w}_t(h_t) - \underline{w}_t(h_t)]$$

and the incentive constraint when money is used as

$$\text{(14)} \quad (1-\beta)\big[R\big(x_{\mu t}(h_t), \mu_t'\big) - R\big(x_{\mu t}(h_t), \mu_t(h_t)\big)\big] \leq \beta \int w_t(\pi, h_t)\big[f\big(\pi|\mu_t(h_t)\big) - f(\pi|\mu_t)\big]\, d\pi.$$

By construction, the inherited wages in the exchange rate regime equal those of the money regime, $\tilde{x}_{et}(h_t) = x_{\mu t}(h_t)$, and since the two instruments are equally tight, the functions S and R coincide. By construction,

$$\int w_t(\pi, h_t)\big[f\big(\pi|\mu_t(h_t)\big) - f(\pi|\mu_t)\big]\, d\pi \leq \bar{w}_t(h_t) - \underline{w}_t(h_t)$$

so that if (14) holds for any deviation μ_t', then (13) holds for any deviation e_t'.

Finally, along the equilibrium path, the payoffs under our constructed strategies $\tilde{\sigma}$, namely, the left side of (12), are weakly higher than those under σ, namely, the left side of (11) since

$$w_t\big(\tilde{e}_t(h_t), h_t\big) = \bar{w}_t(h_t) \geq \int w_t(\pi, h_t) f\big(\pi|\mu_t(h_t)\big)\, d\pi. \tag{15}$$

Moreover, if money growth rate $\mu_t(h_t)$ is strictly lower than the one-shot equilibrium level, then $w_t(\pi, h_t)$ cannot be constant almost everywhere, and (15) is a strict inequality. *Q.E.D.*

We have shown that given any equilibrium in which money is used as an instrument in some period, there is an equilibrium in which the exchange rate is used as an instrument in that same period which leads to higher welfare. Since our construction works for any equilibrium, it obviously works for the equilibrium with the highest payoff for the government in which money is used in the current period—the *best* equilibrium with money. We next illustrate graphically how the results differ with and without commitment. To do so, it is convenient in the environment without commitment to rank instruments by the best equilibrium when that instrument is used in the current period. With this ranking, we can illustrate the results of Propositions 1 and 2 graphically in Figure 1. There we show how the optimal regime varies with the relative tightness of the instruments. When the government can commit to its policies, the transparent instrument, the exchange rate regime, is preferred if and only if it is the tighter one, so that $\sigma_{\pi^*}^2 < \sigma_{\pi}^2$. This is the region labeled A in the figure. When the government cannot commit to its policies, the transparent instrument is preferred even if the two instruments are equally tight. Thus, the region for which the exchange rate regime is preferred expands to include the region labeled B as well as A.

In proving our result, we have imposed no restrictions on strategies besides the natural ones that arise from the environment. If we restrict strategies in the same way in both regimes, say, to Markov strategies (as in Stokey, forthcoming) or to strategies that allow only reversion to the one-shot equilibrium (as in Canzoneri 1985), then we obtain similar results when we compare the best equilibria within these restricted classes. The logic is identical to that for our main result for an environment with no such restrictions.

3. Relaxing Some Assumptions

In modeling the idea that exchange rates are easier to monitor than money growth rates, we have made the simple but extreme assumptions that inflation is the only signal of the money growth rate and that money growth rates are never observed. Here we show that we can allow for multiple signals or for the money growth rate to be observed with a lag and still find a benefit to transparency.

Suppose first that, in addition to inflation, agents observe another noisy signal of money growth, denoted η. In an environment in which the government has imperfect control over money growth, we might interpret this signal η as the realized money growth rate. Let $f(\pi, \eta|\mu)$ be the density of inflation π and the noisy signal η given the money growth rate μ. Here the government's continuation value can vary only with π and η and can be written as $w(\pi, \eta)$. The government's incentive constraint now becomes

$$(1-\beta)R(x_\mu, \mu) + \beta \int \int w(\pi, \eta) f(\pi, \eta|\mu)\, d\pi d\eta \geq$$

$$(1-\beta)R(x_\mu, \mu') + \beta \int \int w(\pi) f(\pi, \eta|\mu')\, d\pi d\eta$$

for any possible μ'. Proving the analogue of Proposition 2 in this environment is straightforward.

Suppose next that while inflation is the only signal of the money growth rate that agents can observe in the current period, the money growth rate is perfectly observable with a lag, which for simplicity we take to be one period. Specifically, assume that the money growth rate μ_{t-1} is observed after agents set their wages in period t. Here, the history on which agents condition their actions is

$$h_t = (i_0, e_0, \pi_0; i_1, e_1, \pi_1, \mu_0; \ldots; i_{t-1}, e_{t-1}, \pi_{t-1}, \mu_{t-2})$$

and the history for the government is

$$H_t = (i_0, e_0, \pi_0, \mu_0; i_1, e_1, \pi_1, \mu_0; \ldots; i_{t-1}, e_{t-1}, \pi_{t-1}, \mu_{t-1}).$$

The strategies for the agents and the government are defined as functions of these histories in the standard way.

The intuition for why there is a benefit to transparency in this environment is clear. Under the money regime, any deviation in period t is not directly observed in that period. Thus, in period $t+1$, agents can react only to a noisy signal of that action. Of course, by period $t+2$, agents have observed the government's action in period t, and agents at that time can precisely react to any deviation in period t. This lag in the ability to precisely react leads to a tighter incentive constraint under the money regime and thus gives the exchange rate regime its relative advantage. The proof is similar to that for Proposition 2, with the exception that if the government discounts the future sufficiently little, then the incentive constraint in both regimes is slack, and both regimes can attain the Ramsey payoff. In such a case, it is not possible to strictly improve on the best money regime. (For details, see our earlier work, Atkeson and Kehoe 2001.)

4. The Best Equilibria Without Commitment

Here we show what will be observed under the best equilibria. We begin by describing these optimal outcomes under the two regimes. Then we present a formal characterization of the outcomes.

A. Observed Outcomes

When the exchange rate regime is the preferred regime, the equilibrium outcome is simple. In each period, the government chooses an exchange rate regime and sets the exchange rate equal to the best exchange rate policy e^b. If the government deviates from this policy, then the government and agents revert to the actions that implement the worst equilibrium payoff v^w. These actions may correspond to either an exchange rate regime or a money regime, depending on the variances of the shocks. In equilibrium, of course, there are no deviations; hence, the exchange rate is set to e^b in every period, and inflation randomly fluctuates around this mean level e^b.

The equilibrium outcome under the best money regime looks quite different. Under this regime, the government starts by setting the money growth rate equal to some low growth rate μ^b and continues to do that as long as low inflation is realized. Specifically, the government sets the money growth rate to μ^b as long as the domestic inflation shock ε is small enough so that $\mu^b + \varepsilon \leq \pi^b$, where π^b is the relatively low cutoff level of inflation used in the best money regime. In equilibrium, eventually, a large enough domestic inflation shock must occur so that the realized inflation exceeds π^b. After such a shock, the government and agents revert to the actions that implement the worst equilibrium payoff v^w. Thus, under the money regime, the actions that implement the worst equilibrium payoffs are eventually observed. We prove this result later in Proposition 3.

The worst equilibrium payoff v^w can occur under either an exchange rate regime or a money regime, depending on the variances of domestic and foreign inflation shocks. This worst equilibrium payoff is the larger of two payoffs: the worst payoff under an exchange rate regime v_e^w and the worst payoff under a money regime v_μ^w. That is, $v^w = \max\{v_e^w, v_\mu^w\}$. The worst equilibrium payoff is the larger of these two payoffs because, at the beginning of each period, the government can choose which regime it prefers.

It turns out that when the variances are such that a money regime implements the best payoff, that regime also implements the worst payoff. In this worst regime, the government starts by setting the money growth rate equal to some high growth rate μ^w and continues to do that as long as the domestic inflation shock ε is small enough so that $\mu^w + \varepsilon \leq \pi^w$, where π^w is the relatively high cutoff level of inflation used in the worst money regime. When a sufficiently large domestic inflation shock occurs so that realized inflation exceeds π^w, the government and agents revert to the actions that implement the best equilibrium payoff. In this sense, in the worst money regime, extremely high inflation must be realized before average inflation can fall. We prove this result later in Proposition 4.

In Figure 2 we illustrate a typical path of money growth and inflation outcomes observed in the best equilibrium over time when the money regime is used in both the best and worst equilibria. In period 0, agents choose low wages $x_\mu = \mu^b$, the government chooses a low money growth rate μ^b, and realized inflation is this low money growth rate plus the domestic inflation shock $\pi_0 = \mu^b + \varepsilon_0$. In the figure, we assume that realized inflation π_0 is less than the critical value π^b. Hence, in period 1, agents again choose wages $x_\mu = \mu^b$, the government again chooses a low money growth rate μ^b, and realized inflation is $\pi_1 = \mu^b + \varepsilon_1$. The outcomes continue in this fashion, with agents choosing low wages and the government choosing a low money growth rate, until the domestic inflation shock is large enough so that realized inflation exceeds the critical value π^b. In the figure, this occurs in period 4. In period 5, agents choose high wages $x_\mu = \mu^w$, the government chooses high money growth rate μ^w, and realized inflation is $\pi_5 = \mu^w + \varepsilon_{5.}$ This pattern continues until the domestic inflation shock is high enough so that realized inflation exceeds the high critical value π^w. In the figure, this occurs in period 7. In period 8, the outcome reverts back to the pattern of agents choosing low wages and the government choosing a low money growth rate. After that, the outcome cycles stochastically between these two phases, depending on the realizations of the domestic inflation shocks.

We use an argument similar to that in Proposition 2 to characterize the regions of the parameter space in which the exchange rate regime and the money regime are used in the best and worst equilibrium outcomes. When the variances of domestic and foreign inflation shocks are the same, the worst payoff under an exchange rate regime is lower than that under a money regime; that is, $v_e^w < v_\mu^w$. This is because here the current period payoff functions R and S are the same, and the incentive constraint is looser under an exchange rate regime than under a money regime. Hence, when these variances are the same, the worst equilibrium payoff $v^w = \max\{v_e^w, v_\mu^w\}$ is equal to that under a money regime. Clearly, increasing the variance of foreign inflation shocks above that of the domestic shocks reduces v_e^w and leaves v_μ^w unchanged. Hence, $v^w = v_\mu^w$ when the variance of foreign inflation shocks exceeds that of domestic inflation shocks.

In Figure 3, we combine this result with that in Proposition 2 to characterize which regimes are used in the best and worst outcomes in each part of the parameter space. If the variance of foreign shocks is sufficiently high relative to that of domestic shocks, as in region C of the figure, then the government follows a money regime in both the best and the worst equilibria. If the variance of foreign shocks is sufficiently low relative to that of domestic shocks, as in region E, then the government follows an exchange rate regime in both the best and the worst equilibria. When the variances of the two inflation shocks are similar, as in region D, then the government uses an exchange rate regime in the best equilibrium and a money regime in the worst equilibrium. In regions D and E, the observed best outcome is an exchange rate regime with a constant e in every period. The observed best outcome in region C stochastically cycles between the best money regime and the worst money regime as discussed above.

B. Formal Characterization

We focus our formal characterization of the outcomes on those with the best money regime. The characterization of the outcomes with the best exchange rate regime is straightforward.

Let the set $V = [v^w, v^b]$ denote the set of perfect equilibrium payoffs for the government, where v^w and v^b denote the worst and best payoffs. In a perfect equilibrium, the strategies that the government and agents follow from next period on must also be perfect equilibrium strategies of the game starting from that period, and the government's continuation values must lie in the set V of perfect equilibrium payoffs for the government. The *best payoff for the government under a money regime* is thus the solution to the following problem: choose current actions x_μ and μ and a continuation value function $w(\pi) \in V$ to maximize

$$(16) \quad (1-\beta)R(x_\mu, \mu) + \beta \int w(\pi) f(\pi|\mu)\, d\pi$$

subject to the incentive constraints $x_\mu = \mu$ and (11). Substituting $x_\mu = \mu$ and rearranging the incentive constraint, we can write this problem as

$$(17) \quad \max_\mu (1-\beta)R(\mu, \mu) + \beta \int w(\pi) f(\pi|\mu)\, d\pi$$

subject to

$$(18) \quad (1-\beta)\left[R(\mu, \mu') - R(\mu, \mu)\right] \leq \beta \int w(\pi)\left[f(\pi|\mu) - f(\pi|\mu')\right] d\pi.$$

The recursive representation of this regime is the solution to problem (16). To solve this problem, we first replace the incentive constraint (11) with the first-order condition associated with maximizing the left side of this incentive constraint with respect to μ. The resulting constraint is

$$(19) \quad (1-\beta)R_\mu(x_\mu, \mu) + \beta \int w(\pi) f_\mu(\pi|\mu)\ d\pi = 0$$

where $R_\mu(x, \mu) = \partial R(x, \mu)/\partial \mu$ and $f_\mu(\pi|\mu) = \partial f(\pi|\mu)/\partial \mu$. This first-order condition is necessary and sufficient to ensure that (11) holds when the function defined by the left side of (11) is concave in μ. In Proposition 3 below, we simply assume that this approach is valid and characterize the resulting $w(\pi)$. In the lemma below, we show that, given the resulting form of $w(\pi)$, the left side of (11) is concave in μ when the variance of domestic inflation shocks is sufficiently large. In any solution to the problem of maximizing (17) subject to (19), the continuation values necessarily have a *bang-bang* form:

$$(20) \quad w^b(\pi) = \left\{ \begin{array}{l} v^b \ \textit{if} \ \pi \leq \pi^b \\ v^w \ \textit{if} \ \pi > \pi^b \end{array} \right\}.$$

That is, there is a cutoff inflation level π^b such that the optimal continuation value function $w^b(\pi)$ is set to the best payoff v^b if the realized inflation rate is lower than π^b and to the worst payoff v^w if the realized inflation rate is higher than π^b.

Part of the rationale for the optimal continuation value taking the form (20) is intuitive. Since higher money growth rates make higher inflation more likely, in order to discourage the government from choosing a high money growth rate, the continuation value function must specify a low continuation payoff for the government when realized inflation is high. Slightly less intuitive is that the best continuation value function must assign only the best and worst possible equilibrium payoffs. Mechanically, this occurs because both the payoffs and the incentive constraint are linear in the continuation values. Formally, we have the following.

PROPOSITION 3. BANG-BANG CONTINUATIONS FOR BEST. *Under the assumption that the first-order condition approach is valid, the optimal continuation value function has the form of* (20)**.**

Proof. With λ as the multiplier on the government's incentive constraint (19), the term in the Lagrangian that involves $w(\pi)$ is

$$\beta \int w(\pi) \left[1 + \lambda \frac{f_\mu(\pi|\mu)}{f(\pi|\mu)} \right] f(\pi|\mu) \, d\pi.$$

Notice that this term is linear in each value of $w(\pi)$, so that it is optimal to set

$$w^b(\pi) = \left\{ \begin{array}{l} v^b \ if \ \left[1 + \lambda \frac{f_\mu(\pi|\mu)}{f(\pi|\mu)}\right] > 0 \\ v^w \ if \ \left[1 + \lambda \frac{f_\mu(\pi|\mu)}{f(\pi|\mu)}\right] < 0 \end{array} \right\}.$$

These first-order conditions imply that the optimal continuation values are always extreme, that is, either v^b or v^w. The only issue is, for what values of π are the payoffs v^b and v^w assigned? To determine these values, we start by observing that with our assumption of normality, $f_\mu(\pi|\mu) = f(\pi|\mu)(\pi - \mu)/\sigma_\pi$, so that our densities satisfy the monotone likelihood ratio property; that is, the ratio

$$\frac{f_\mu(\pi|\mu)}{f(\pi|\mu)} = (\pi - \mu)/\sigma_\pi$$

is increasing in π. Thus, $w^b(\pi)$ is increasing in π if $\lambda > 0$ and decreasing in π if $\lambda < 0$.

We will show that $\lambda < 0$ and $w^b(\pi)$ is decreasing in π as follows. First, note that at the optimum, $R_\mu(x^b, \mu^b) \geq 0$. This follows since the optimum must weakly improve on the one-shot equilibrium payoff and thus must have a money growth rate less than or equal to the one-shot equilibrium level. That is, $x^b = \mu^b \leq U$. Since $R_\mu(x, \mu) = U + x - 2\mu$, $R_\mu(x^b, \mu^b) \geq 0$. Next, since $R_\mu(x^b, \mu^b) \geq 0$, the incentive constraint (19) implies that

$$\text{(21)} \qquad \int w^b(\pi) f_\mu(\pi|\mu)\, d\pi \leq 0.$$

Since inflation is normally distributed with mean μ, increasing μ increases the distribution of inflation in the sense of first-order stochastic dominance. Thus, increasing μ increases $\int w^b(\pi) f(\pi|\mu)\, d\pi$ when $w^b(\pi)$ is increasing and decreases this integral when $w^b(\pi)$ is decreasing. Thus, to satisfy (21), $w^b(\pi)$ must be decreasing. *Q.E.D.*

In the lemma, proven in the Appendix, we justify our use of the first-order approach. We let ϕ and Φ denote the density and cumulative distribution functions of a standard normal, respectively.

Lemma. First-order approach valid. Given that $w^b(\pi)$ has the bang-bang form (20) and is decreasing, if $\sigma_\pi^2 > \frac{\beta}{1-\beta}(v^b - v^w)\phi(1)/2$, then the incentive constraint (11) is satisfied if and only if the first-order condition (19) holds.

To complete our characterization of the outcome under the best money regime, we must also characterize the outcome under the worst money regime. In the worst money regime, continuation values $w^w(\pi)$ are assigned to give the government the incentive to choose a higher money growth rate than it would choose in the one-shot equilibrium outcome. This entails giving the government high continuation values when high inflation is realized and low continuation values when low inflation is realized. Thus, when the equilibrium reverts to the worst money regime, the government chooses a high money growth rate and keeps choosing this high rate until a sufficiently high level of inflation is realized. This result is proven in the next proposition.

As before, under the assumption that the first-order condition approach is valid, we can write the problem of finding the worst payoff under a money regime as

$$(22) \qquad \min_{\mu,x,w(\pi)} (1-\beta)R(x,\mu) + \beta \int w(\pi) f(\pi,\mu)\, d\pi$$

subject to the constraints $x = \mu$ and (19).

Proposition 4. Bang-Bang Continuations for Worst. *Under the assumption that the first-order approach is valid, the optimal continuation value function for the worst equilibrium in the money regime has the form*

$$(23) \qquad w^w(\pi) = \left\{ \begin{array}{c} v^w \; if \; \pi \leq \pi^w \\ v^b \; if \; \pi > \pi^w \end{array} \right\}$$

for some cutoff inflation rate π^w.

Proof. The proof is similar to that of Proposition 3. Specifically, the first-order condition of the problem (22) with respect to $w(\pi)$ implies that $w^w(\pi)$ has a bang-bang form around some cutoff π^w. To see that $w^w(\pi)$ must be increasing, note that at the optimum, $R_\mu(x^w, \mu^w) \leq 0$, so that the current period payoff for the government is decreased when the government deviates to a higher money growth rate. Accordingly, the incentive constraint (19) implies that

$$\int w^w(\pi) f_\mu(\pi|\mu)\, d\pi \geq 0$$

which gives the result that $w^w(\pi)$ is increasing. *Q.E.D.*

The results confirm the discussion of Canzoneri (1985) that optimal monetary arrangements may involve periodic bouts of high inflation. Interestingly, our analysis shows that such bouts of high inflation are necessarily part of the best equilibrium with an opaque instrument.

5. Conclusion

Here we have considered the benefits to transparency in a model in which the exchange rate is observable and the money growth rate is only observable with noise, at least contemporaneously. We have shown here that a certain price, the exchange rate, has benefits relative over a certain quantity, the money growth rate, as a monetary policy instrument. This basic idea, that prices have an advantage over quantities as policy instruments, might also be applied to a comparison of interest rates and any other quantity instrument that is more difficult to monitor.

Appendix

Proof of Lemma: First-Order Approach Valid

Here we show that the solution to the problem with incentive constraint (11) is satisfied if and only if the first-order condition (19) holds when $\sigma_\pi^2 > \frac{\beta}{1-\beta}(\bar{w} - \underline{w})\frac{\phi(1)}{2}$.

Using (20), we can write the constraint (11) as

$$(24) \quad \mu \in \arg\max_{\mu}(1-\beta)R(x,\mu) + \beta\left\{\bar{w}\Phi\left(\frac{\pi^h - \mu}{\sigma_\pi}\right) + \underline{w}\left[1 - \Phi\left(\frac{\pi^h - \mu}{\sigma_\pi}\right)\right]\right\}.$$

Since $F(\pi^h, \mu) = \Phi\big((\pi^h - \mu)/\sigma_\pi\big)$, we can write the first- and second-order conditions of the maximization problem (24) as

$$(25) \quad (1-\beta)R_\mu(x^h,\mu) - \beta\left(\frac{\bar{w} - \underline{w}}{\sigma_\pi}\right)\phi\left(\frac{\pi^h - \mu}{\sigma_\pi}\right) = 0$$

and for all μ

$$(26) \quad (1-\beta)R_{\mu\mu}(x^h,\mu) - \beta(\bar{w} - \underline{w})\left(\frac{\pi^h - \mu}{\sigma_\pi^2}\right)\phi\left(\frac{\pi^h - \mu}{\sigma_\pi}\right) \le 0$$

which can be written as

$$(27) \quad -2(1-\beta) - \beta\frac{(\bar{w} - \underline{w})}{\sigma_\pi^2}\phi(z)z \le 0$$

for all $z \in [-\infty, \infty]$. The expression $\phi(z)z$ in (27) is minimized at $z = -1$. Since $\phi(-1) = \phi(1)$, the inequality $\sigma_\pi^2 > \frac{\beta}{1-\beta}(\bar{w} - \underline{w})\frac{\phi(1)}{2}$ guarantees that the second-order condition holds globally, and thus, (19) is both necessary and sufficient for (11). *Q.E.D.*

Notes

[1]In assuming that the rate of depreciation is the instrument of policy, we are allowing for any type of crawling peg in an exchange rate regime. Hence, there is no sense in which exchange rates are necessarily fixed in the exchange rate regime. Moreover, our work here is about the choice of two different types of instruments and is silent on any issues concerning the choice of fixed versus flexible exchange rates.

[2]We thank Stokey (forthcoming) for this terminology.

[3]A simple way to do so is to have a continuum of agents indexed by i with objective functions

$$r(z_i, x, \pi) = -\frac{1}{2}[(z_i - \pi)^2 + (U + x - \pi)^2 + \pi^2]$$

and the government objective function be $\int r(z_i, x, \pi)\ di$. Since agents within each country will act identically setting $z_i = x$, we can write the per period expected payoff to the government in the two regimes as

$$S(x, e) = -\frac{1}{2}[(x - e)^2 + (U + x - e)^2 + e^2] - \frac{3}{2}\sigma^2_{\pi^*}$$

$$R(x, \mu) = -\frac{1}{2}[(x - \mu)^2 + (U + x - \mu)^2 + \mu^2] - \frac{3}{2}\sigma^2_{\pi^*}$$

With these objective functions to results will be identical to those in the paper. (See Chari, Kehoe, and Prescott 1989 and Stokey forthcoming for a discussion.)

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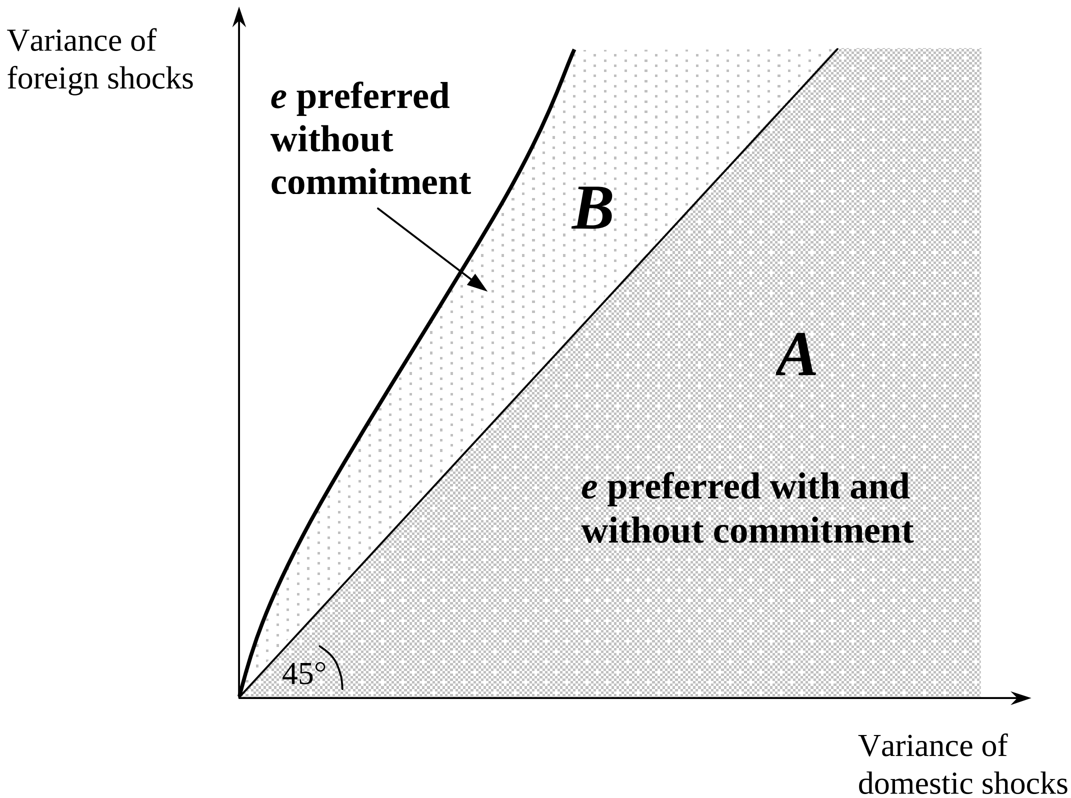
Figure 1

Parameter regions for which an exchange rate regime is preferred to a money regime with and without commitment*



*With commitment, exchange rate regimes are preferred in region *A*, where the variance of domestic inflation shocks is greater than the variance of foreign inflation shocks. With no commitment, exchange rate regimes have an additional advantage; they are preferred in both region *A* and region *B*.

Figure 2

Outcomes with money regime in the best and worst equilibria

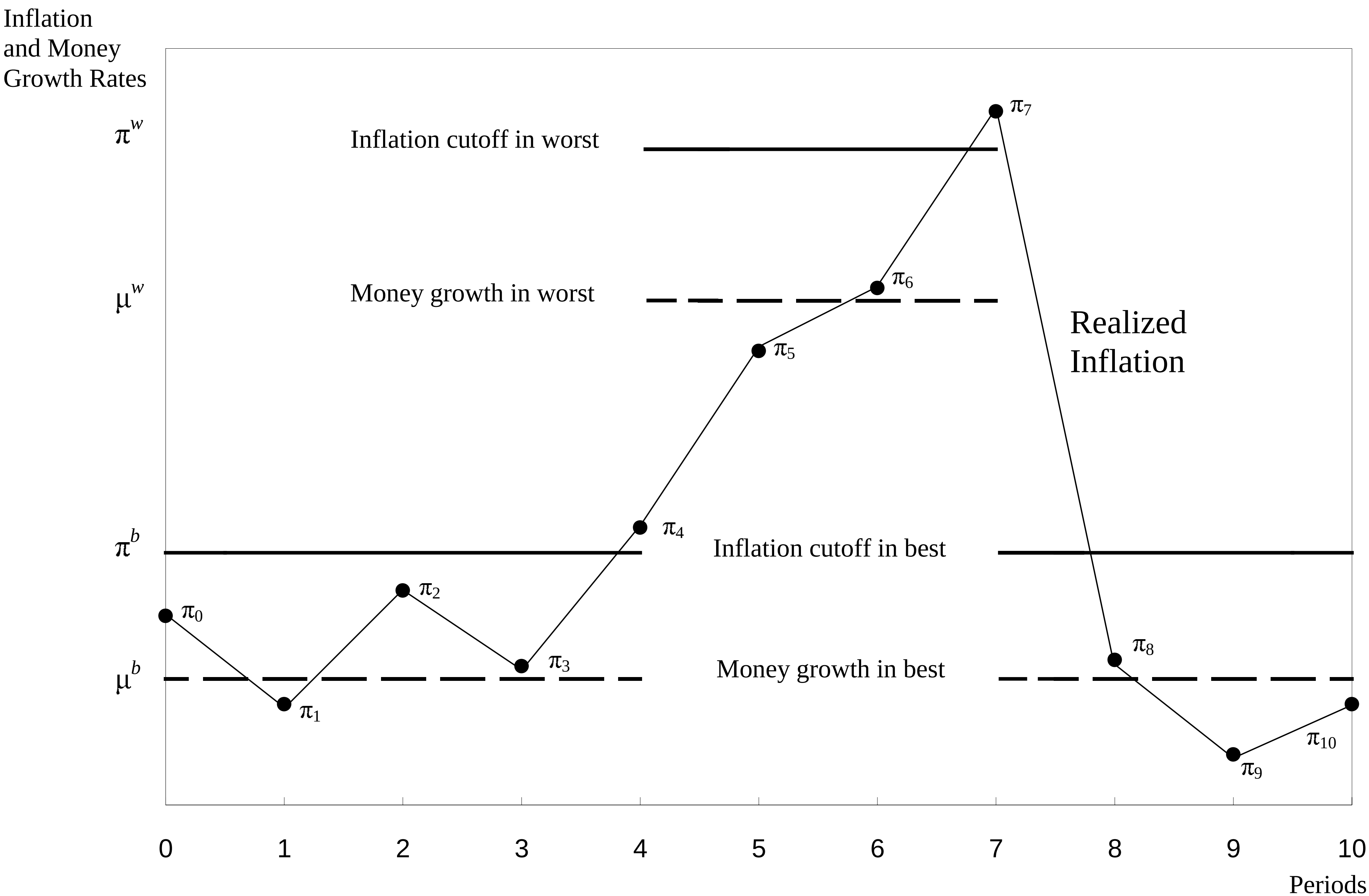
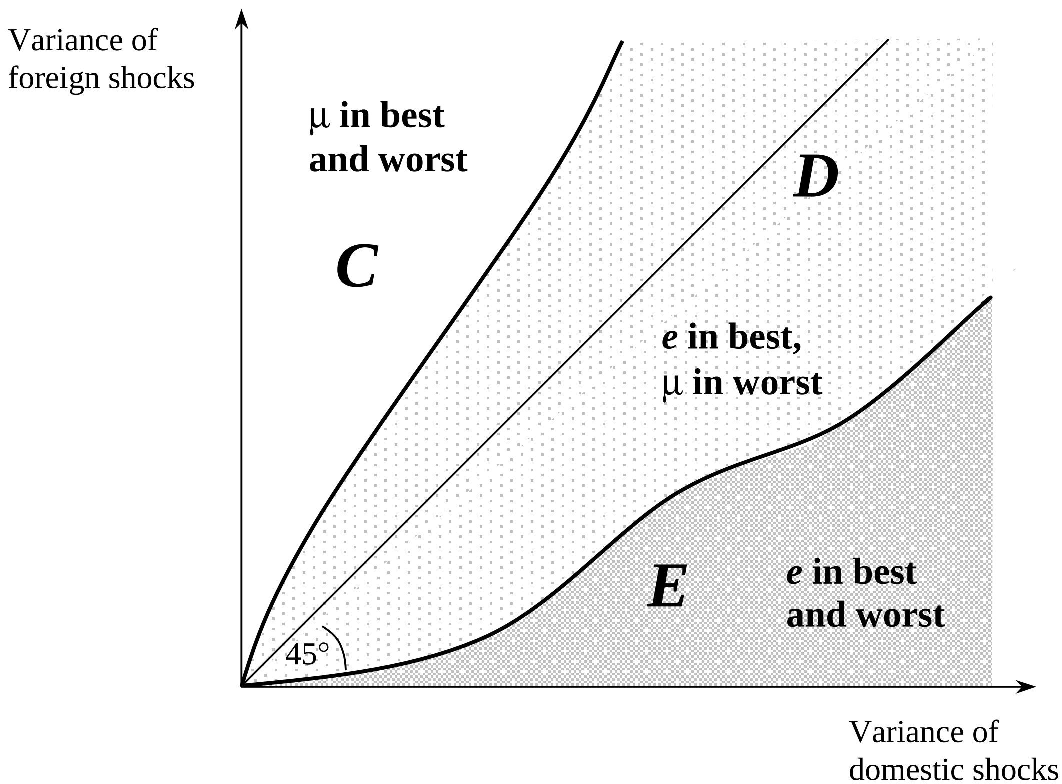


Figure 3

Regimes in the best and worst equilibrium outcomes



*In region *C*, the money regime is followed in the best and the worst equilibria. In region *D*, the exchange rate regime is followed in the best equilibria and the money regime is followed in the worst equilibria. In region *E*, the exchange rate regime is followed both in the best and in the worst equilibria.