On the cyclical behavior of employment, unemployment and labor force participation*

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Abstract: In this paper I evaluate to what extent a real business cycle (RBC) model that incorporates search and leisure decisions can simultaneously account for the observed behavior of employment, unemployment and out-of-the-labor-force. This contrasts with the previous RBC literature, which analyzed employment or hours fluctuations by either lumping together unemployment and out-of-the-labor-force into a single non-employment state or assuming a fixed labor force participation. Once the three employment states are explicitly introduced I find that the RBC model generates highly counterfactual labor market dynamics.

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1. Introduction

In the real business cycle (RBC) literature it is standard to analyze labor market dynamics by focusing on the behavior of employment and hours worked. Thus, RBC models typically lump together unemployment and out-of-the-labor-force into a single nonemployment state and analyze variations in employment and hours worked by either studying a work-leisure decision (e.g. Hansen [9] and Prescott [15]) or a work-home production decision (e.g. Greenwood and Hercowitz [8] and Benhabib, Rogerson, and Wright [3]). While this approach led to important advances in business cycle theory and considerable success in accounting for fluctuations in employment and hours worked, it abstracts from one of the main characteristics of labor markets that labor economists typically emphasize: search frictions. Despite the preponderant role that search plays in the labor literature it is surprising how little attention it has received in the RBC literature. The only exceptions are Andolfatto [2], Merz [11], [12] and Den Haan, Ramey and Watson [5], who studied versions of the Mortensen-Pissarides [13] matching framework, and Gomes, Greenwood and Rebelo [7], who analyzed a version of the Lucas-Prescott [10] islands framework.

The basic finding in all of these papers is that an RBC model that incorporates search frictions can account for salient features of U.S. business cycles and outperform the standard model in several ways. While this is an important result, none of the above papers attempted to explain the joint behavior of employment, unemployment and labor force participation (Merz [11], [12] and Gomes, Greenwood and Rebelo [7] assumed a fixed labor force, while Andolfatto [2] and Den Haan, Ramey and Watson [5] lumped together unemployment and out-of-the-labor-force into a single nonemployment state).[1] Explaining the joint behavior of employment, unemployment and labor force participation is important not only for obtaining a better understanding of labor market dynamics, but to test the empirical plausibility of the search and leisure decisions embodied in a model. Consider, for example, the models by Merz [11], [12] or Gomes, Greenwood and Rebelo [7] that allow agents to search and enjoy leisure while they are unemployed but that restrict them to stay in the labor force. If the main reason

[1] Strictly speaking, Den Haan, Ramey and Watson [5] lumped together unemployment and out-of-the-labor-force workers that claim to want a job.

why agents become unemployed in those models is to enjoy leisure (i.e. if intertemporal substitution in leisure is the main factor driving employment fluctuations), a significant number of agents may want to leave the labor force in order to enjoy even more leisure. Thus, most of the flows from employment to unemployment during a recession could end up being flows from employment to out-of-the-labor-force once a labor force participation margin is allowed for, generating highly counterfactual behavior. Lumping together unemployment and out-of-the-labor-force into a single nonemployment state (as in Andolfatto [2] and Den Haan, Ramey and Watson [5]) may hide similar problems.

The purpose of this paper is to evaluate to what extent a RBC model that incorporates search and leisure decisions can account for the joint behavior of employment, unemployment and labor force participation. The benchmark model is a version of one used by Alvarez and Veracierto [1], which in turn is based on the Lucas and Prescott [10] equilibrium search model.[2] Output, which can be consumed or invested, is produced by a large number of islands that use capital and labor as inputs into a decreasing returns to scale production technology. Contrary to the deterministic steady state analysis of Alvarez and Veracierto [1], the islands are subject both to idiosyncratic and aggregate productivity shocks. At the beginning of each period agents must decide whether to work in the island where they are currently located or to search for a new employment opportunity. An important difference with Lucas and Prescott [10] is that agents have the choice of making their search directed or undirected. Another difference is that the model incorporates an out of the labor force margin and physical capital.

Parameter values are chosen so that the deterministic steady state of the model economy reproduces important observations from the National Income and Product Accounts (NIPA) and key labor market statistics. Aggregate productivity shocks in turn are selected to match the behavior of measured Solow residuals. Under such parameterization, I find that the model fails to account for the behavior of employment, unemployment and labor force participation. The search and leisure decisions embodied in this version of the neoclassical growth model generate drastically counterfactual behavior: 1) employment fluctuates as much as the labor

[2] Even though most of the analysis is done in the Lucas-Prescott islands framework, I also provide results for a version of the Mortensen-Pissarides matching framework.

force while in the data it is three times more variable, 2) unemployment fluctuates as much as output while in the data it is six times more variable, and 3) unemployment is acyclical while in the data it is strongly countercyclical. These results are obtained under a wide variety of specifications for the search technology.

Even though the paper fails to account for U.S. observations, it fails in an informative way. The paper shows that the empirical performance of an RBC model can be quite poor once unemployment and endogenous labor force participation are explicitly introduced. The paper thus questions the ability of previous RBC models to account for labor market fluctuations. In addition, the paper suggests that a successful business cycle model, whatever that may end up being, will have to give a much more important role to fluctuations in search decisions than to fluctuations in leisure.

The paper is organized as follows: Section 2 describes the model economy. Section 3 describes a deterministic steady state equilibrium. Section 4 parameterizes the model. Section 5 presents the results, and Section 6 concludes the paper. A detailed Appendix discusses the computational methodology.

2. The benchmark economy

The economy is populated by a representative household constituted by a large number of members with names in the unit interval. The household's preferences are given by:

$$E\sum_{t=0}^{\infty}\beta^t\left\{\ln c_t + A\left(\frac{h_t^{1-\phi}-1}{1-\phi}\right)-\psi D_t\right\} \tag{2.1}$$

where c_t is consumption of a market good, h_t is consumption of a home good, D_t is the number of agents that do directed search, $0<\beta<1$ is the subjective time discount factor, and $\phi>0$. Notice that there is a disutility cost $\psi\geq 0$ incurred by each agent that performs directed search.

The market good, which can be consumed or invested, is produced in a continuum of islands. Each island has a production function given by

$$y_t = e^{(1-\varphi)a_t}z_t n_t^{\gamma}k_t^{\varphi}$$

where y_t is production, n_t is the labor input, k_t is the capital input, z_t is an idiosyncratic productivity shock, a_t is an aggregate productivity level common to all islands, $\gamma > 0$, $\dot{\varphi} > 0$, and $\gamma + \varphi < 1$. The idiosyncratic productivity shock z_t follows a finite Markov process with transition matrix Q, where $Q(z, z')$ is the probability that $z_{t+1} = z'$ conditional on $z_t = z$. Realizations of z_t are assumed to be independent across islands.

The aggregate productivity level evolves according to the following AR(1) process:

$$a_{t+1} = \rho_a a_t + \varepsilon_{t+1}, \tag{2.2}$$

where $0 < \rho_a < 1$ and ε_{t+1} is i.i.d., normally distributed, with variance σ_a^2 and zero mean.

Capital is assumed to be freely mobile across islands, but not labor. At the beginning of every period there is a given distribution of agents across islands. An island cannot employ more than the total number of agents x present in the island at the beginning of the period. If an agent stays in the island in which he is currently located, he produces market goods and starts the following period in the same location. Otherwise, the agent leaves the island and becomes nonemployed.

A nonemployed agent has three alternatives. The first alternative is to search for a new employment opportunity using an undirected search technology. If the agent uses this technology he gets zero production during the current period, but becomes randomly assigned to an island at the beginning of the following period. Since agents that perform undirected search have no control over which islands they will arrive to, they are assumed to arrive uniformly across all islands in the economy.

The second alternative for a nonemployed agent is to perform directed search. The directed search technology entails a disutility cost. However, agents in this technology learn the current idiosyncratic shocks of all islands in the economy and are allowed to pick which island to arrive to in the following period.

The third alternative is to leave the market sector in order to specialize in home production. The home production technology is described by the following linear function:

$$h_t = 1 - \pi_S[U_t + D_t] - \pi_N N_t \tag{2.3}$$

where U_t is the number of agents that do undirected search, N_t is the number of agents that are employed in the market sector, π_S, and π_N denote the amount of time required by the search and production technologies, respectively, and $0 < \pi_S \leq \pi_N \leq 1$.[3] Each household member is endowed with one unit of time. Market production, directed search, undirected search and out-of-the-labor-force are assumed to be mutually exclusive activities, that is, an agent cannot be in more than one of these activities in any given period of time. Hereon, I will refer to $1 - U_t - D_t - N_t$ as the number of agents being "out-of-the-labor-force".

In order to describe the aggregate feasibility conditions for this economy, it will be important to index each island according to its individual state: the idiosyncratic productivity level of the island z and the number of agents available at the beginning of the period x. Feasibility requires that the island's employment level $n_t(x, z)$ do not exceed the number of agents initially available:

$$n_t(x, z) \leq x. \tag{2.4}$$

The number of agents in the island at the beginning of the following period, is given by

$$x' = n_t(x, z) + U_t + d_t\,(x, z)$$

where U_t is total undirected search in the economy and $d_t(x, z)$ is the number of agent that direct their search to this particular island. Observe that this equation uses the fact that undirected searchers become uniformly distributed across all islands in the economy.

The law of motion for the distribution μ_t of islands across idiosyncratic productivity levels and available agents is then given by

$$\mu_{t+1}\,(X', Z') = \int_{\{(x,z):n_t(x,z)+U_t+d_t(x,z)\in X'\}} Q(z, Z')\mu_t(dx, dz) \tag{2.5}$$

for all X' and Z'. This equation states that the total number of islands with a number of agents in the set X' and a productivity shock in the set Z' is given by the sum of all islands that transit from their current shocks to a shock in Z' and choose an employment level such

[3] Given that home production only requires labor, I will refer interchangeably to "home production" and "leisure".

that $x' = n_t(x,z) + U_t + d_t(x,z)$ is in X'.

Aggregate employment is then

$$N_t = \int n_t(x,z)\mu_t(dx,dz), \tag{2.6}$$

the number of agents that do directed search is

$$D_t = \int d_t(x,z)\mu_t(dx,dz), \tag{2.7}$$

and aggregate capital is

$$K_t = \int k_t(x,z)\mu_t(dx,dz). \tag{2.8}$$

In turn, aggregate feasibility for the market good is given by

$$c_t + K_{t+1} - (1-\delta)\,K_t \leq \int e^{(1-\varphi)a_t} z_t n_t(x,z)^{\gamma} k_t(x,z)^{\varphi} \mu_t(dx,dz). \tag{2.9}$$

Assuming complete markets, a competitive equilibrium can be obtained by solving the social planner's problem, which is given by maximizing (2.1) subject to equations (2.2), (2.3), (2.4), (2.5), (2.6), (2.7), (2.8), and (2.9).

3. Deterministic steady state

This section describes the steady state conditions for a version of the economy where the aggregate productivity shock a_t is set to its unconditional mean of zero. The purpose is to describe the different decision margins underlying a competitive equilibrium and to show that the employment adjustments at the islands level are of the S-s type. This property will make the computation of a stochastic equilibrium feasible.[4]

To start with observe that, since consumption is constant at steady state, the interest

[4] Solving for a stochastic competitive equilibrium is a complicated task because the state space is highly dimensional. The appendix describes in detail the computational strategy used in the paper, as well as a formal derivation of the steady state conditions described in this section.

rate is given by $1/\beta$. Hence the rental rate of capital is given by

$$r = \frac{1}{\beta} - 1 + \delta. \tag{3.1}$$

In what follows it will be assumed that there are competitive labor markets within islands. Hence firms equate the marginal productivity of labor and capital to the wage rate w and the rental rate r, respectively:

$$w = z\gamma n^{\gamma-1} k^{\varphi} \tag{3.2}$$

$$r = z n^{\gamma} \varphi k^{\varphi-1} \tag{3.3}$$

Substituting (3.3) in (3.2), we get the following wage function:

$$w(n,z) = z\gamma n^{\gamma-1} \left(\frac{z n^{\gamma} \varphi}{r} \right)^{\frac{\varphi}{1-\varphi}}$$

where n is the employment level of the island.

Let consider the decision problem of an agent that begins a period in an island of type (x,z) and must decide whether to stay or leave the island, taking the employment level of the island $n(x,z)$, the number of agents directing their search to the island $d(x,z)$ and the aggregate number of agents doing undirected search U as given. If the agent decides to stay, he earns the competitive wage rate $w(n(x,z),z)$ and begins the following period in the same island. If the agent decides to leave, he becomes nonemployed and obtains a value of θ (to be determined below). His problem is then described by the following Bellman equation:

$$v(x,z) = \max \left\{ \theta, w\left(n(x,z),z\right) + \beta \sum_{z'} v\left(n(x,z) + U + d(x,z), z'\right) Q\left(z,z'\right) \right\}$$

where $v(x,z)$ is the expected value of beginning a period in an island of type (x,z).

At equilibrium, the employment rule $n(x,z)$ must be consistent with individual decisions. In particular, if the state of the island is such that $v(x,z) > \theta$ (agents are strictly better-off staying than leaving), then all agents stay, that is:

$$n(x,z) = x$$

On the other hand, if $v(x,z) = \theta$ (agents are indifferent between staying or leaving) then some agents leave until

$$n(x,z) = \bar{n}\left(z\right),$$

where $\bar{n}\left(z\right)$ satisfies:

$$\theta = w(\bar{n}\left(z\right), z) + \beta \sum_{z'} v\left(\bar{n}\left(z\right) + U, z'\right) Q\left(z, z'\right). \tag{3.4}$$

Observe that the employment rule is then given by

$$n(x,z) = \min\left\{x, \overline{n}\left(z\right)\right\}. \tag{3.5}$$

That is, if the number of agents in the island is less than $\overline{n}\left(z\right)$ then everybody stays. On the other hand, if the number of agents in the island exceeds $\overline{n}\left(z\right)$ then some agents leave until employment equals $\overline{n}\left(z\right)$.

If nobody directs its search to the island, the next period number of agents available will be given by $n(x,z) + U$. However if the expected value of being in the island is sufficiently large, some agents will direct their search to it. In particular, there will be an upper bound σ on the expected value of an island. If the expected value of the island is less than σ nobody will direct his search to it. But the arrival of directed searchers will preclude the expected value of the island to exceed σ. Let $\underline{n}\left(z\right)$ be the number of agents such that $\underline{n}\left(z\right) + U$ makes directed searchers indifferent between arriving to the island or not:

$$\sigma = \sum_{z'} v(\underline{n}\left(z\right) + U, z')Q(z, z'). \tag{3.6}$$

Then, the number of agents directing their search to the island is given by

$$d(x,z) = \max\left\{\underline{n}\left(z\right), n(x,z)\right\} - n(x,z). \tag{3.7}$$

That is, if employment $n(x,z)$ is less than $\underline{n}(z)$ the expected value of the island would exceed σ. This attracts directed searchers until the number of agents doing directed search to the island equals $\underline{n}(z) - n(x,z)$.

Observe from the above discussion that we can write the equilibrium value v as the solution to the following Bellman equation:

$$v(x,z) = \max\left\{\theta,\ z\gamma x^{\gamma-1}\left[\frac{zx^{\gamma}\varphi}{r}\right]^{\frac{\varphi}{1-\varphi}} + \beta\min\left[\sigma, \sum_{z'} v(x+U,z')Q(z,z')\right]\right\} \tag{3.8}$$

Given $n(x,z)$ and $d(x,z)$, the invariant distribution of islands μ must satisfy the following recursion:

$$\mu\left(X',Z'\right) = \int_{\{(x,z):n(x,z)+U+d(x,z)\in X'\}} Q(z,Z')\mu(dx,dz). \tag{3.9}$$

Obtaining the capital rule $k(x,z)$ from $n(x,z)$ and equation (3.3), the steady state aggregate capital, employment, directed search and consumption levels are then given by

$$K = \int k(x,z)\mu(dx,dz) \tag{3.10}$$

$$N = \int n(x,z)\mu(dx,dz) \tag{3.11}$$

$$D = \int d(x,z)\mu(dx,dz) \tag{3.12}$$

$$c = \int zn_t(x,z)^{\gamma}k(x,z)^{\varphi}\mu(dx,dz) - \delta K \tag{3.13}$$

respectively.

The last equilibrium conditions determine the labor force participation decision, the value of nonemployment θ and the upper bound on the expected value of an island σ. With respect to labor force participation decisions we have that

$$\theta = \pi_N A\left(1 - \pi_S U - \pi_S D - \pi_N N\right)^{-\phi} c + \beta\theta \tag{3.14}$$

must hold. This condition states that an agent located in an island where some agents leave must be indifferent between staying in the island, receiving a value equal to θ, and working at home for one period and becoming nonemployed the following period. The value of working at home for one period is the marginal utility of the home good multiplied by the hours π_N freed from employment, divided by the marginal utility of consumption.

Assuming that there is undirected search at equilibrium, the following condition must hold:

$$\pi_S A \left(1 - \pi_S U - \pi_S D - \pi_N N\right)^{-\phi} c + \beta\theta = \beta \int v(x,z)\mu(dx,dz). \tag{3.15}$$

This condition states that a nonemployed agent must be indifferent between leaving the labor force and doing undirected search. The right hand side of this equation gives the expected value of doing undirected search: it is the expected value under the invariant distribution (since undirected searchers arrive uniformly across all islands) discounted by the factor β (since the arrival takes place the following period). The left hand side is the value of spending one period doing home production and becoming nonemployed the following period.

Assuming that directed search also takes place at equilibrium, unemployed agents must be indifferent between both search alternatives. This requires that

$$\beta\sigma - \psi c = \beta \int v(x,z)\mu(dx,dz) \tag{3.16}$$

The right hand side is the expected value of doing undirected search. The left hand side is the expected value of doing directed search σ discounted by the factor β minus the disutility of doing directed search expressed in consumption units.

4. Parameterization

This section describes the steady state observations used to select the parameters of the model. The parameters to be chosen are β, A, ϕ, ψ, γ, φ, π_S, π_N, δ, the values for the idiosyncratic productivity shock z, the transition matrix Q, and the parameters determining the driving process for the aggregate productivity shock: ρ and σ_ε^2. The time period selected for the model is one month. A short time period is called for in order to match the relatively short average duration of unemployment observed in U.S. data.

The curvature of home production in the utility function ϕ and the disutility from directed search ψ will be taken as free parameters in the experiments below. As a consequence I will postpone discussing their values until the next section. The calibration of other parameters are conditional on the choices for ϕ and ψ. As a consequence in this section I discuss the observations used to calibrate them, but I will provide actual values only after ϕ and ψ are determined.

The stock of capital in the market sector K is identified with business capital, that is, with plant, equipment and inventories. As a result, investment in business capital I is associated in the National Income and Product Accounts with fixed private non-residential investment plus change in business inventories. Considering that the depreciation rate is related to steady state I and K according to

$$\delta = \frac{I}{K},$$

the average I/K ratio over the period 1967:Q1 to 1999:Q4 gives a monthly depreciation rate $\delta = 0.00659$.

In turn, consumption c is identified with consumption of non-durable goods and services (excluding housing services). Output is then defined as the sum of these consumption and investment measures. The average monthly capital-output ratio K/Y corresponding to the period 1967:Q1 to 1999:Q4 is 25.8.

The interest rate in the model economy is given by

$$1 + i = \frac{1}{\beta}$$

As a consequence $\beta = 0.9967$ is chosen to reproduce an annual interest rate of 4 percent, roughly the average between the return on equity and the return on treasury bills in the U.S. economy.

The Cobb-Douglas production function and the competitive behavior assumption implies that φ equals the share of capital in market output. That is,

$$\left(\frac{1}{\beta} - 1 + \delta\right)\frac{K}{Y} = \varphi.$$

Given the previous values for β, δ, and $\frac{K}{Y}$, it follows that $\varphi = 0.2554$. On the other hand, $\gamma = 0.64$ is selected to reproduce the labor share in National Income.

The idiosyncratic productivity levels z and the transition matrix Q are chosen to approximate (by quadrature methods) the following AR(1) process:

$$\log z_{t+1} = \rho_z \ln z_t + \varepsilon^z_{t+1}$$

where ε^z_{t+1} is i.i.d., normally distributed, with zero mean and variance σ^2_z.[5] Since the stochastic process for the idiosyncratic productivity shocks is a crucial determinant of the unemployment rate and the average duration of unemployment, ρ_z and σ^2_z will be selected later on to reproduce an unemployment rate of 6.2 percent and an average duration of unemployment equal to one quarter, which correspond to U.S. observations. The weight for the utility of home production A in turn will be selected to reproduce a labor force participation equal to 74 percent (the average ratio between the labor force and the size of the population between 16 and 65 years old). The actual values for ρ_z, σ^2_z and A will depend on the values for ϕ and ψ, which are taken as free parameters.

The rest of the parameters to calibrate are the time requirements for search π_S and for employment π_N. There is no reliable data on the amount of time that unemployed agents spend searching, but a reasonable guess is that they spend half the time that employed agents spend working, i.e. $\pi_S = 0.5\pi_N$.[6] I also impose that total hours spent in market activities must be equal to 0.33, which is consistent with the evidence provided by Ghez and Becker [6] and is the magnitude commonly used in the RBC literature. Given an unemployment

[5] Only three values for z will be allowed in the computations. While this may not seem a large number, it leads to a considerable amount of heterogeneity: the support of the invariant distribution will be over one thousand (x, z) pairs in most of the experiments reported below.

[6] This is the assumption that Andolfatto ([2]) used, but none of the results in the paper change if $\pi_S = \pi_N$ or $\pi_S = 0.1\pi_N$ are used instead.

rate of 6.2 percent and a labor force participation of 74 percent, this requires that $\pi_N = 0.46$ and $\pi_S = 0.23$. The mix of directed-undirected search will be determined by the disutility of directed search ψ, which is taken as a free parameter.

Finally, using the measure of output described above and a labor share of 0.64, measured Solow residuals are found to be as highly persistent but somewhat more variable than Prescott [15]: the standard deviation of quarterly technology changes is 0.009 instead of 0.0076.[7] As a consequence, $\rho_a = 0.98$ and $\sigma_a^2 = 0.009^2/3$ are chosen here.

5. Results

In order to evaluate the behavior of the model economy, the first column of Table 1 reports U.S. business cycle statistics. Before any statistics were computed, all time series were logged and detrended using the Hodrick-Prescott filter. The empirical measures for output Y, consumption c, investment I and capital K reported in the table correspond to the measures described in the previous section, and cover the period between 1967:Q1 and 1999:Q4. The table shows some well known facts about U.S. business cycle dynamics: that consumption and capital are less variable than output while investment is much more volatile, and that consumption and investment are strongly procyclical while capital is acyclical. The variability of labor relative to output (0.57) is lower than usual among other things because it refers to employment instead of total hours worked. What is important in Table 1 is the variability of unemployment, which is 6.25 times the variability of output, and the variability of the labor force, which is only 0.20 times the variability of output. While employment is strongly procyclical, labor force participation is only weakly procyclical. On the contrary, unemployment is strongly countercyclical: its correlation with output is -0.83. Note that even though unemployment is a small fraction of the labor force, its behavior is key in generating a much larger variability in employment than in labor force participation.

Before reporting the results for the benchmark economy I will proceed to analyze the

[7] Proportionate changes in measured Solow residual are defined as the proportionate change in aggregate output Y minus the sum of the proportionate change in aggregate employment N times the labor share γ, minus the sum of the proportionate change in aggregate capital K times $(1-\gamma)$.

business cycles of different environments. The analysis will help relate the paper to the previous literature and better understand the results for the benchmark economy.

5.1. Fixed labor force

5.1.1. Fixed labor force and utility from search

As was pointed out in the introduction, several of the papers that analyzed RBC models with unemployment assumed that the labor force is fixed and that agents enjoy leisure while they search. To facilitate comparisons with this literature, I study a version of the economy where the labor force is fixed at 0.74 (same magnitude as the benchmark economy) and preferences are given by:

$$E\sum_{t=0}^{\infty}\beta^t \left\{\ln c_t + B \times U_t\right\}$$

where $B > 0$. These preferences correspond to the employment lotteries model analyzed by Hansen [9]. For the time being I abstract from directed search, the only type of search allowed is undirected (that is, the disutility of directed search ψ is assumed to be large).

In this case there are three parameters (the utility of search B, the persistence of the idiosyncratic shocks ρ_z and the variance of the idiosyncratic shocks σ_z^2) to determine two observations: the unemployment rate and the average duration of unemployment. Later on I will show that these parameters are key determinants of the cyclical behavior of employment. In this section I report business cycle fluctuations under the most favorable specification for these parameters (in terms of matching the relative variability of employment most closely). The second column in Table 1 reports results for $B = 1.09$, $\rho_z = 0.63$, and $\sigma_z^2 = 0.0004$. The statistics correspond to averages across 100 simulations of 408 periods each (corresponding to the 136 quarters of data). Before computing these statistics, the monthly data generated by the model was aggregated to a quarterly time period and then logged and detrended using the Hodrick-Prescott filter. Comparing the business cycles generated by this version of the model with those of the U.S. economy, we see that consumption fluctuates less than output in both economies but that it is considerably smoother in the model than in the U.S: its relative volatility is 0.32 instead of 0.57. Investment is about 4.5 times as variable as output in both economies, and it is strongly procyclical in both. Employment fluctuates the same amount in the model as in the data (parameters were selected to generate this result) and is strongly procyclical both in the model and the data. Given the fixed labor force, the behavior of employment leads to a highly variable and countercyclical unemployment, actually a bit too variable and countercyclical compared to the data (8.75 versus 6.25 and -0.96 versus -0.83, respectively). Overall, these results indicate that this version of the model in principle can be reconciled with salient features of business cycle data.

In this economy there are two channels that generate fluctuations in employment. One is the standard channel: when there is a good aggregate shock it is a bad time to enjoy leisure, so agents substitute leisure intertemporally and supply more employment. The second channel arise from the search decisions: when there is a good aggregate shock it is a bad time to search for a good idiosyncratic productivity shock, so agents accept employment more easily and leave the islands less frequently. An important question is which channel is the most important for generating the relatively large employment fluctuations reported here. The following section addresses this particular issue.

5.1.2. Inelastic labor force, no utility from search

In order to evaluate the importance of intertemporal leisure substitution in the previous section, I analyze an identical environment except that search is assumed to provide no leisure ($B = 0$). In order for the model to reproduce the unemployment rate and average duration observed in the data when unemployment does not provide utility, the persistence and variance of the idiosyncratic shocks must be made much larger than before ($\rho_z = 0.795$, and $\sigma_z^2 = 0.0221$), that is, agents must face stronger "idiosyncratic" reasons to search.

The third column of Table 1 reports results for this case. We see that the relative variability and persistence of consumption, capital and investment are not very different from the second column. The big difference is with respect to labor dynamics. Instead of fluctuating 57% of output, employment fluctuates only 3% when utility does not provide leisure. In the same vein, instead of fluctuating 8.75 times more than output, unemployment fluctuates only 41% when utility has no direct value. In both cases the correlations with output are significantly reduced. These results show that the assumption that agents enjoyed leisure while unemployed was key to generating the large fluctuations in employment reported in the previous section. It is true that when the economy receives a good aggregate shock it is a bad time to search and employment increases, but this is a weak channel. When agents do not derive utility from unemployment the idiosyncratic productivity shocks have to be so persistent and their innovations so large, that when an agent decides to leave or stay in an island it is due to strong idiosyncratic reasons (it is from comparing large and persistent differences in idiosyncratic productivities across islands). In this context the aggregate shocks can do very little to affect the reallocation of labor.

The lack of response of the search decisions to aggregate shocks could depend on the rigidities of the undirected search technology. The following section explores how the results change when the search technology is allowed to be more flexible.

5.1.3. Inelastic labor force and directed search margin

I extend the version in the previous section by introducing a directed search margin.[8] Preferences are now given by

$$E\sum_{t=0}^{\infty}\beta^{t}\left\{\ln c_t-\psi D_t\right\}$$

where $\psi>0$ (the model in the previous section assumed ψ to be a large number).

There is a good reason to believe that this version of the model can give rise to larger fluctuations in employment than in the previous version. Suppose that a good aggregate productivity shock hits the economy. It is still true that agents tend to accept employment more easily and leave less frequently because it is a good time to work instead of search. However, we saw in the previous section that this margin is not very responsive. The additional margin introduced here however is that, after a good aggregate shock hits the economy, some of the agents that were doing undirected search are now willing to pay the disutility cost and do directed search in order to become employed much faster. Since the disutility cost enters linearly, there will be a high willingness to substitute effort intertemporally and the shifts from undirected search to directed could be large. Since variations in directed search have an immediate impact on employment, this could lead to higher employment variability.

[8] Allowing for directed search introduces a search intensity decision.

The disutility parameter ψ is an important determinant of the steady state ratio of directed searchers to undirected searchers (D/U). Since it is not clear what an empirically relevant magnitude for D/U should be, I here treat ψ as a free parameter. In what follows I report results under the most favorable case that I found in the experiments. This is the case where ψ generates a D/U ratio equal to 0.20.[9] The last column of Table 1 shows the results. We see that when the directed search margin is introduced both employment and unemployment become twice as variable (the variability of employment increases from 0.03 to 0.06 while the variability of unemployment increases from 0.41 to 0.98), verifying the intuition provided above. However, the employment and unemployment fluctuations generated by the search decisions are still extremely small.

Lowering ψ further does not continue to improve the ability of the model to generate higher fluctuations in employment. When ψ is lowered, there are more unemployed agents doing directed search, which lowers the average duration of unemployment and the unemployment rate. To match the unemployment rate and the average duration of unemployment observed in the data, the persistence and variance of the idiosyncratic shocks must be increased substantially. This reduces the responsiveness of the search decisions to aggregate productivity shocks.

Given this background, I turn to evaluate the cyclical behavior of economies with endogenous labor force participation.

5.2. Flexible labor force

5.2.1. The benchmark economy with directed search

This section constitutes the core of the paper. It shows results for the benchmark economy with elastic labor force participation that was parametrized in Section 4. Similarly to the previous section, directed search is allowed for. Preferences are given by

$$E\sum_{t=0}^{\infty}\beta^t\left\{\ln c_t + A\left[\frac{(1-\pi_S[U_t+D_t]-\pi_N N_t)^{1-\phi}-1}{1-\phi}\right]-\psi D_t\right\}.$$

[9] The values for ψ, ρ_z and σ_z^2 used in this section are 5.95, 0.833 and 0.020, respectively.

Observe that, under the benchmark parametrization, being unemployed provides more leisure than being employed ($\pi_S < \pi_N$) but the maximum amount of leisure is obtained by staying out of the labor force ($\pi_S > 0$).

The disutility of directed search is chosen to reproduce the same directed to undirected search ratio D/U as in the previous section (0.20), maximizing its contribution to employment fluctuations. While the parameter A is picked to reproduce a labor force participation equal to 74 percent, the curvature parameter ϕ is not pinned down from steady state observations. Since it is an important determinant of the variability of the labor force I choose it to reproduce the labor force variability observed in the U.S. economy.[10] The key question will be how well the model reproduces the cyclical behavior of the rest of the variables and, in particular, the behavior of employment and unemployment.

The second column of Table 2 ("Directed search, disutility cost") reports the results. Similarly to the previous cases considered, the benchmark economy does a relatively good job in reproducing the business cycle behavior of consumption, capital and investment observed in U.S. data. However, the model fails badly in terms of the labor market dynamics that it generates. There are three main problems: 1) employment fluctuates as much as the labor force while employment is three times more variable than the labor force in the U.S. economy, 2) unemployment is only slightly more variable than output while it is six times more volatile than output in the data, and 3) unemployment is acyclical while it is strongly countercyclical in the U.S.

The previous cases analyzed provide some intuition for these results. Similarly to the economy of Section 5.1.3, the idiosyncratic productivity shocks in the benchmark economy are variable and persistent enough that the decisions to search for better idiosyncratic productivities are virtually independent of the aggregate productivity level.[11] Thus, most of the employment fluctuations are due to intertemporal substitution effects on leisure. But given that aggregate productivity shocks are highly persistent, that agents enjoy more leisure

[10] Everything considered, I end with the following parameter values: $A = 0.30$, $\phi = 5$, $\psi = 3.58$, $\rho_z = 0.83$ and $\sigma_z^2 = 0.012$.

[11] In the benchmark economy unemployment provides some leisure, so the idiosyncratic shocks do not need to be as persistent and variable as in section 5.1.3, but they are still quite persistent and variable.

being out-of-the-labor-force than being unemployed and that it is relatively easy to find employment (the average duration of unemployment is only one quarter), when agents decide to become nonemployed in order to enjoy leisure, they choose to leave the labor force instead of becoming unemployed. As a consequence, most of the variations in employment in the benchmark economy are reflected in fluctuations in labor force participation instead of unemployment.

To understand the lack of cyclicality of unemployment it is important to observe that when agents enter the labor force they must first go through unemployment. Thus increases in labor force are initially accompanied by increases in unemployment. This tends to make unemployment move procyclically. However, when a positive aggregate shock hits the economy, unemployed agents accept jobs more easily and are more willing to do directed search. This tends to make unemployment move countercyclically. When both effects are considered unemployment behaves acyclically. This is more easily seen in Figure 1, which shows several impulse response functions to a positive innovation in aggregate productivity equal to one (monthly) standard deviation. The figure shows that in the first quarter after the innovation, employment increases substantially due to the higher job acceptance rate. However, there is a spike in unemployment due to the large increase in labor force participation. In the second quarter after the shock, unemployment plunges while employment continues to increase due to the higher job acceptance rate.[12] The sharp reversal in the response of unemployment leads to its acyclical behavior.

Finally, the relatively small variation in unemployment can be understood by the fact that at times when agents are more willing to accept low productivity jobs (that is, when a positive aggregate shock hits the economy), the flows from out of the labor force into unemployment increase, dampening the decrease in unemployment.

In order to test the robustness of the above results to different specifications for the directed search technology, I consider the case where directed search does not entail a disutility cost but requires resources. In particular, each agent that performs directed search must now pay ν units of the consumption good. With this change in the directed search technology

[12] Observe that the number of agents doing directed search increases substantially in response to the aggregate productivity shock, increasing the hazard rate from unemployment.

the aggregate feasibility condition becomes

$$c_t + K_{t+1} - (1-\delta) K_t + \nu D_t \leq \int e^{(1-\varphi)a_t} z_t n_t(x,z)^{\gamma} k_t(x,z)^{\varphi} \mu_t(dx, dz).$$

The resource cost parameter ν is chosen to reproduce the same directed to undirected search ratio D/U as before (0.20).[13] The results, shown in the third column of Table 2, are very similar to those in the second column. The only sizable difference is with respect to unemployment, which is more variable when the directed search cost is in terms of resources. The reason for this is that when a positive aggregate productivity shock hits the economy there are more resources available to facilitate directed search, making the shift from undirected to directed search a bit stronger. But the difference is not big. We see that the business cycles generated by the benchmark economy are surprisingly robust to the nature of the directed search technology.

5.3. A Mortensen-Pissarides matching function framework

Before concluding, this section evaluates to what extent the above results extend to the Mortensen and Pissarides [13] framework.[14] To this end, the benchmark economy is modified as follows. Similarly to the economy with no directed search, the agents that arrive to the islands sector are assumed to be uniformly distributed across all the islands. However, the number of agents that arrive to the islands sector is not given by the total number of agents that are unemployed but determined by a matching function that depends on the number of agents unemployed and the number of vacancies posted. More specifically, the matching function is given as follows:

$$M_t = \Omega U_t^{\eta} V_t^{1-\eta}$$

[13] Except for ψ, which now equals zero, all other parameters remain the same.

[14] Since many of the previous studies that analyzed unemployment in RBC models used the Mortensen-Pissarides framework (e.g. Andolfatto [2], Den Haan, Ramey and Watson [5] and Merz [11], [12]), this section is extremely relevant.

where M_t is the number of matches, U_t is the number of unemployed agents, and V_t is the number of vacancies posted.[15] Posting a vacancy entails a resource cost equal to λ.

The social planner's problem is then given by the following problem:[16]

$$\max E \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + A \left[\frac{(1 - \pi_S U_t - \pi_N N_t)^{1-\phi} - 1}{1-\phi} \right] \right\}$$

subject to

$$\begin{aligned}
c_t + K_{t+1} - (1-\delta) K_t + \lambda V_t &\leq \int e^{(1-\varphi)a_t} z_t n_t(x,z)^{\gamma} k_t(x,z)^{\varphi} \mu_t(dx,dz) && (5.1)\\
n_t(x,z) &\leq x && (5.2)\\
N_t &\geq \int n_t(x,z)\mu_t(dx,dz) && (5.3)\\
K_t &\geq \int k_t(x,z)\mu_t(dx,dz) && (5.4)\\
M_t &\leq \Omega U_t^{\eta} V_t^{1-\eta} && (5.5)\\
\mu_{t+1}(X', Z') &= \int_{\{(x,z): n_t(x,z)+U_t+M_t \in X'\}} Q(z,Z')\mu_t(dx,dz) && \\
a_{t+1} &= \rho_a a_t + \varepsilon_{t+1} && (5.6)\\
& a_0,\ \mu_0,\ K_0 \text{ given.} && (5.7)
\end{aligned}$$

Observe that, relative to the benchmark economy with no directed search, three new parameters are introduced in this section: Ω, λ, and η. While the rest of the parameters are calibrated to the same observations as in the previous cases, the following criteria is used to select the parameters associated to the matching technology. Since Blanchard and Diamond [4] present evidence of constant returns to scale in the matching function and estimate the elasticity of the matching function with respect to unemployment and vacancies to be 0.40

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[15] Economic interpretation requires that $M_t < \min\{U_t, V_t\}$. This can be achieved by specifying the matching function to be $M_t = \min\left\{\Omega U_t^{\eta} V_t^{1-\eta}, U_t, V_t\right\}$. However, the additional restrictions were never binding in the experiments.

[16] Similarly to Den Haan, Ramey and Watson [5] and contrary to Andolfatto [2] and Merz [11], [12], the Mortensen-Pissarides framework considered here allows for time varying idiosyncractic productivity shocks and endogenous employment termination. Similarly to Den Haan, Ramey and Watson [5] and Merz [12], but contrary to Andolfatto [2] and Merz [11], I abstract from a search intensity margin.

and 0.60, respectively, a value of $\eta = 0.40$ is used here. In turn, the resource cost of posting a vacancy λ is chosen to reproduce an average vacancy duration equal to 45 days, which according to Ours and Ridder [14] corresponds to the Dutch economy. Finally, since the parameter Ω is a key determinant of the fraction of unemployed agents that arrive to the islands sector and there is no direct evidence on such magnitude, I treat it as a free parameter. In what follows, I use a value for Ω that generates a steady-state ratio M/U equal to one-half, i.e. only half of the agents that are unemployed arrive to the islands sector.[17] However, similar results are obtained with values of Ω that generate a M/U ratio equal to 0.75 or 0.95.[18]

The results for this version of the Mortensen-Pissarides framework are reported in the last column of Table 2. We see that the behavior of consumption, investment and capital are the same as in the benchmark economy of the previous section. With respect to the behavior of employment, unemployment and labor force participation we see that the matching-function economy is also very similar to the benchmark economy. However the correlation of unemployment with output becomes positive (0.24) and the variability of unemployment relative to output is even smaller than in the benchmark economy with a resource cost of directed search (1.62 versus 2.03), which was already too small compared to U.S. data. In addition, employment and the labor force continue to fluctuate the same amount, which is also highly counterfactual. These results suggest that the difficulty of RBC models to account for the observed cyclical behavior of employment, unemployment and labor force participation extends to the Mortensen-Pissarides class of models.

6. Conclusions

In this paper I analyzed a RBC model that makes an explicit distinction between unemployment and out-of-the-labor-force. I found that the model has serious difficulties in reproducing

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[17] In this case, parameter values are as follows: $A = 0.313$, $\phi = 5$, $\lambda = 3.61$, $\Omega = 0.85$, $\eta = 0.40$, $\rho_z = 0.77$ and $\sigma_z^2 = 0.062$.

[18] Observe that the benchmark economy with no directed search corresponds to the case where $\eta = 1$ and $\Omega = 1$. In this case the M/U ratio is equal to one.

the type of labor market dynamics observed in U.S. data. The model delivers three highly counterfactual results: 1) employment fluctuates as much as the labor force while in the data it is three times more variable, 2) unemployment fluctuates as much as output while in the data it is six times more variable, and 3) unemployment is acyclical while in the data it is strongly countercyclical. To match the unemployment rate and the average duration of unemployment observed in U.S. data, the persistence and variability of the idiosyncratic shocks must be made so large that the search decisions end up responding too little to aggregate productivity shocks. Thus, most of the employment fluctuations become the result of intertemporal substitution effects on leisure. But given that aggregate productivity shocks are highly persistent, that agents enjoy more leisure being out-of-the-labor-force than being unemployed and that it is relatively easy to find employment, when agents decide to enjoy leisure they choose to leave the labor force instead of become unemployed. As a consequence, most of the variations in employment are reflected in fluctuations in labor force participation instead of unemployment. This explains the poor empirical performance of the model.

Despite the failure of the model the paper provides a very important lesson. It shows that the empirical performance of a RBC model that relies on persistent aggregate productivity shocks and high intertemporal substitution in leisure can become quite poor once unemployment and endogenous labor force participation are explicitly introduced. Thus the paper questions the ability of previous RBC models to account for labor market fluctuations. The fact that the model fails under a wide variety of search technologies makes its point stronger.

The key question that the paper left unanswered is "what type of model would generate empirically reasonable labor market dynamics?". An obvious answer seems to be a RBC model with adjustment costs to labor force participation. In fact, it is straightforward to verify that introducing adjustment costs of this sort to the benchmark model can generate empirically relevant labor market dynamics. The reason is quite simple. When there is a bad aggregate productivity shock and agents are willing to substitute towards leisure, the adjustment costs in labor force participation induce agents to become unemployed instead of leaving the labor force. Under large adjustment costs and a high intertemporal substitution in leisure, the model ends up behaving quite similarly to the economy of Section 5.1.1, generating empirically reasonable labor market dynamics.

However this answer cannot satisfy us. There are no good economic reasons to justify this type of adjustment costs in a RBC model. While there may be many out-of-the-labor-force activities subject to large adjustment costs (such as child rearing), these are not the type of activities that are relevant for understanding fluctuations at business cycle frequencies. What matters is the type of activities that unemployed agents undertake when they are not searching. If the RBC model allows agents to intertemporally substitute these activities very easily, it is not clear why there should be large costs to stop searching (leaving the labor force) and doing more of these same activities? Large adjustment costs in labor force participation and a high intertemporal substitution in leisure appear to be mutually inconsistent assumptions.

These reasons suggest that a successful model, whatever that may end up being, will have to shift the source of employment fluctuations from intertemporal substitution in leisure towards search decisions. If fluctuations in labor force participation are small (because it is difficult to substitute leisure intertemporally) but search decisions respond to aggregate shocks in a significant way, the labor market dynamics thus generated may become much more satisfactory. Finding such a model promises to be a challenging area of research.

At this point, two possible alternatives seem worth pursuing. One possibility is to introduce some key labor market policies that this paper has abstracted from. An unemployment insurance system seems particularly relevant. Since unemployment insurance provides agents additional incentives to remain unemployed, the model would not require a high persistence and variability of idiosyncratic shocks to reproduce the unemployment rate and the average duration of unemployment observed in the U.S. economy, making search decisions much more responsive to aggregate shocks. In addition, even under the low intertemporal substitution in leisure that would be needed to generate small fluctuations in labor force participation, the shifts from employment to unemployment could become large because agents would be substituting competing sources of income over time (relatively constant unemployment benefits against time varying wages). The challenge will be to generate reasonable labor market dynamics with unemployment benefit levels that are justified both by the nature of the environment considered and by actual U.S. policies.

Another possibility would be to introduce a reallocation shock that changes the variance of the idiosyncratic productivity shocks over time. Since the distribution of idiosyncratic shocks is an important determinant of the search decisions, this type of shocks can lead to important variations in unemployment that are not the direct consequence of changes in labor force participation. If the variance of the idiosyncratic shocks is negatively correlated to the aggregate productivity shock, depressions would be accompanied by high incentives to search and unemployment fluctuations could become much larger and countercyclical. In fact, it is not hard to come up with examples where a version of the benchmark economy subject to both aggregate and reallocation shocks generates empirically reasonable labor market dynamics. The challenge will be to verify that this holds under an empirically plausible process for the aggregate and reallocation shocks. Exploring these possibilities will be the focus of future research.

A. Appendix

A.1. Deterministic planning problem

In this section I characterize the solution to the planning problem corresponding to the deterministic version of the benchmark economy, i.e. where the aggregate productivity shock a_t is set to its unconditional mean of zero. The aggregate shock will be reintroduced later on.

Instead of indexing islands according to their current number of agents available and their current productivity levels, it will be convenient to index them according to their history of shocks $z^t = (z_0, z_1, ..., z_t)$ and the number of agents available in the island at date 0. Let $q_t(z^t, x_0)$ be the time t distribution of islands across productivity histories z^t and number of agents available at date 0. For simplicity, I will assume that q_0 has a finite support over pairs (z_0, x_0). Later on I will show that there is no loss of generality in this assumption. The social planner's problem is then given by

$$MAX\sum_{t=0}^{\infty}\beta^t \left\{ \ln c_t + A\left[\frac{(1-\pi_S[U_t+D_t]-\pi_N N_t)^{1-\phi}-1}{1-\phi}\right] - \psi D_t \right\}$$

subject to

$$
\begin{aligned}
c_t + K_{t+1} - (1-\delta) K_t &\leq \sum z_t n_t(z^t, x_0)^\gamma k_t(z^t, x_0)^\varphi q_t\left(z^t, x_0\right) && \text{(A.1)} \\
n_t(z^t, x_0) &\leq n_{t-1}(z^{t-1}, x_0) + U_{t-1} + d_{t-1}\left(z^{t-1}, x_0\right) \\
N_t &\geq \sum n_t(z^t, x_0) q_t\left(z^t, x_0\right) && \text{(A.2)} \\
D_t &\geq \sum d_t(z^t, x_0) q_t\left(z^t, x_0\right) && \text{(A.3)} \\
K_t &\geq \sum k_t(z^t, x_0) q_t\left(z^t, x_0\right) && \text{(A.4)} \\
n_0(z_0, x_0) &\leq x_0 \\
q_{t+1}(z^{t+1}, x_0) &= q_t(z^t, x_0) Q(z_t, z_{t+1}) && \text{(A.5)} \\
& \quad q_0,\ K_0 \text{ given} && \text{(A.6)}
\end{aligned}
$$

where the sums in equations (A.1), (A.2), (A.3) and (A.4) are over histories z^t and initial number of agents x_0.

Let the Lagrange multipliers for the above restrictions be $\beta^t\alpha_t$, $\beta^t\alpha_t\xi_t(z^t, x_0)q_t\left(z^t, x_0\right)$, $\beta^t\alpha_t\omega_t$, $\beta^t\alpha_t s_t$, $\beta^t\alpha_t r_t$ and $\alpha_0\xi_0(z_0, x_0)q_0\left(z_0, x_0\right)$, respectively. Then, the first order conditions for c_t, K_{t+1}, $n_t(z^t, x_0)$, U_t, $d_t(z^t, x_0)$, N_t, D_t and $k_t(z^t, x_0)$ are the following:

$$c_t^{-1} = \alpha_t$$

$$\alpha_t = \beta\alpha_{t+1}\left[r_{t+1} + 1 - \delta\right]$$

$$\xi_t(z^t, x_0) + \omega_t = z_t\gamma n_t(z^t, x_0)^{\gamma-1} k_t(z^t, x_0)^\varphi + \beta\frac{\alpha_{t+1}}{\alpha_t}\sum_{z_{t+1}} \xi_{t+1}((z^t, z_{t+1}), x_0) Q(z_t, z_{t+1})$$

$$\pi_S A\left(1 - \pi_S[U_t + D_t] - \pi_N N_t\right)^{-\phi} = \beta\alpha_{t+1}\sum_{z^{t+1}, x_0} \xi_{t+1}(z^{t+1}, x_0) q_{t+1}\left(z^{t+1}, x_0\right)$$

$$s_t \geq \beta\frac{\alpha_{t+1}}{\alpha_t}\sum_{z_{t+1}} \xi_{t+1}((z^t, z_{t+1}), x_0) Q(z_t, z_{t+1}) \quad (= \text{ if } d_t(z^t; x_0) > 0)$$

$$\pi_N A\left(1 - \pi_S[U_t + D_t] - \pi_N N_t\right)^{-\phi} = \alpha_t\omega_t$$

$$\pi_S A\left(1 - \pi_S[U_t + D_t] - \pi_N N_t\right)^{-\phi} = \alpha_t s_t - \psi$$

$$r_t = z_t n_t(z^t, x_0)^\gamma \varphi k_t(z^t, x_0)^{\varphi-1}.$$

Define

$$\begin{aligned} \theta_t &= \frac{\omega_t}{1-\beta} \\ \widetilde{v}_t(z^t, x_0) &= \xi_t(z^t, x_0) + \theta_t \end{aligned}$$

From the first order condition for $n_t(z^t, x_0)$ we obtain

$$\begin{aligned} \widetilde{v}_t(z^t, x_0) &= z_t \gamma n_t(z^t, x_0)^{\gamma-1} k_t(z^t, x_0)^{\varphi} \\ &\quad + \beta \frac{\alpha_{t+1}}{\alpha_t} \sum_{z_{t+1}} \widetilde{v}_{t+1}((z^t, z_{t+1}), x_0) Q(z_t, z_{t+1}) + \beta \left(\theta_t - \frac{\alpha_{t+1}}{\alpha_t} \theta_{t+1} \right) \end{aligned}$$

Observe that

$$n_t(z^t, x_0) < n_{t-1}(z^{t-1}, x_0) + U_{t-1} + d_{t-1}\left(z^{t-1}, x_0\right)$$

implies that

$$\widetilde{v}_t(z^t, x_0) = \theta_t$$

Also from first order condition for $d_t(z^t, x_0)$

$$s_t + \beta \frac{\alpha_{t+1}}{\alpha_t} \theta_{t+1} \geq \beta \frac{\alpha_{t+1}}{\alpha_t} \sum_{z_{t+1}} \widetilde{v}_{t+1}((z^t, z_{t+1}), x_0) Q(z_t, z_{t+1})$$

with equality if $d_t(z^t, x_0) > 0$.

Defining

$$\begin{aligned} x_t\left(z^t, x_0\right) &= n_{t-1}(z^{t-1}, x_0) + U_{t-1} + d_{t-1}\left(z^{t-1}, x_0\right) \\ v_t\left(x_t\left(z^t, x_0\right), z_t\right) &= \widetilde{v}_t(z^t, x_0) \end{aligned}$$

we have that v_t and x_t satisfy

$$v_t(x_t\left(z^t, x_0\right), z_t) = \max \left\{ \begin{array}{c} \theta_t, \\ z_t \gamma x_t(z^t, x_0)^{\gamma-1} \left[\frac{z_t x_t(z^t,x_0)^\gamma \varphi}{r_t}\right]^{\frac{\varphi}{1-\varphi}} \\ +\min \left[\begin{array}{c} s_t + \beta \frac{\alpha_{t+1}}{\alpha_t}\theta_{t+1}, \\ \beta \frac{\alpha_{t+1}}{\alpha_t} \sum_{z_{t+1}} v_{t+1}(x_t\left(z^t, x_0\right) + U_t, z_{t+1}) Q(z_t, z_{t+1}) \end{array} \right] \\ +\beta\left(\theta_t - \frac{\alpha_{t+1}}{\alpha_t}\theta_{t+1}\right) \end{array} \right\}$$

A.2. Deterministic steady state

A steady state is an initial state (q_0, K) such that the solution to the social planner problem corresponding to that initial state has the following characteristics:

$$\begin{aligned} c_t &= c \\ K_t &= K \\ U_t &= U \\ N_t &= N \\ D_t &= D \end{aligned}$$

for every t. Also there is a measure μ and three functions v, n, and d, such that

$$\begin{aligned} q_t\left(z^t, x_0\right) &= \mu\left(x_t\left(z^t, x_0\right), z_t\right) \\ \widetilde{v}_t(z^t, x_0) &= v\left(x_t\left(z^t, x_0\right), z_t\right) \\ n_t(z^t, x_0) &= n\left(x_t\left(z^t, x_0\right), z_t\right) \\ d_t(z^t, x_0) &= d\left(x_t\left(z^t, x_0\right), z_t\right) \end{aligned}$$

for all t, z^t, and x_0. In steady state there is a constant distribution of islands across available workers x and productivity levels z, employment in an island depends only on (x, z), and the number of agents doing directed search in an island also depends only on (x, z).

In steady state we then have that

$$r = \frac{1}{\beta} - 1 + \delta$$

$$v(x,z) = \max\left\{\theta,\ z\gamma x^{\gamma-1}\left[\frac{zx^{\gamma}\varphi}{r}\right]^{\frac{\varphi}{1-\varphi}} + \min\left[s+\beta\theta, \beta\sum_{z'} v(x+U,z')Q(z,z')\right]\right\}$$

$$\pi_S A\left(1-\pi_S[U+D]-\pi_N N\right)^{-\phi} c = \beta\int v(x,z)\mu(dx,dz) - \beta\theta$$

$$\pi_N A\left(1-\pi_S[U+D]-\pi_N N\right)^{-\phi} c = \omega$$

$$\pi_S A\left(1-\pi_S[U+D]-\pi_N N\right)^{-\phi} c = s - \psi c$$

$$r = zn(x,z)^{\gamma}\varphi k(x,z)^{\varphi-1}$$

Observe that

$$\begin{aligned}
\omega &= \pi_N A\left(1-\pi_S[U+D]-\pi_N N\right)^{-\phi} c \\
s - \psi c &= \omega\frac{\pi_S}{\pi_N} \\
\omega\frac{\pi_S}{\pi_N} &= \beta\int v(x,z)\mu(dx,dz) - \beta\theta \\
\omega &= \theta\left(1-\beta\right)
\end{aligned}$$

Define σ such that

$$\beta\sigma = s + \beta\theta$$

Then,

$$\begin{aligned}
\beta\sigma &= \theta\left(1-\beta\right)\frac{\pi_S}{\pi_N} + \psi c + \beta\theta \\
\theta\left(1-\beta\right)\frac{\pi_S}{\pi_N} + \beta\theta &= \beta\int v(x,z)\mu(dx,dz) \\
\theta &= \frac{\pi_N A\left(1-\pi_S[U+D]-\pi_N N\right)^{-\phi} c}{\left(1-\beta\right)}
\end{aligned}$$

The steady state conditions are then the following

$$r = \frac{1}{\beta} - 1 + \delta \tag{A.7}$$

$$v(x,z) = \max\left\{\theta,\ z\gamma x^{\gamma-1}\left[\frac{zx^{\gamma}\varphi}{r}\right]^{\frac{\varphi}{1-\varphi}} + \beta\min\left[\sigma, \sum_{z'} v(x+U,z')Q(z,z')\right]\right\} \tag{A.8}$$

$$\theta = \ z\gamma\overline{n}\left(z\right)^{\gamma-1}\left[\frac{z\overline{n}\left(z\right)^{\gamma}\varphi}{r}\right]^{\frac{\varphi}{1-\varphi}} + \beta\sum_{z'} v(\overline{n}\left(z\right)+U,z')Q(z,z') \tag{A.9}$$

$$\sigma = \sum_{z'} v(\underline{n}\left(z\right)+U,z')Q(z,z') \tag{A.10}$$

$$n(x,z) = \min\left\{x, \overline{n}\left(z\right)\right\} \tag{A.11}$$

$$d(x,z) = \max\left\{\underline{n}\left(z\right), n(x,z)\right\} - n(x,z) \tag{A.12}$$

$$\mu\left(X',Z'\right) = \int_{\{(x,z):\max\{\underline{n}(z),n(x,z)\}+U\in X'\}} Q(z,Z')\mu(dx,dz) \tag{A.13}$$

$$\beta\sigma = \theta\left(1-\beta\right)\frac{\pi_S}{\pi_N} + \psi c + \beta\theta \tag{A.14}$$

$$\theta\left(1-\beta\right)\frac{\pi_S}{\pi_N} + \beta\theta = \beta\int v(x,z)\mu(dx,dz) \tag{A.15}$$

$$\theta = \frac{\pi_N A\left(1-\pi_S[U+D]-\pi_N N\right)^{-\phi} c}{(1-\beta)} \tag{A.16}$$

$$r = zn(x,z)^{\gamma}\varphi k(x,z)^{\varphi-1} \tag{A.17}$$

$$c + \delta K = \int zn_t(x,z)^{\gamma}k(x,z)^{\varphi}\mu(dx,dz) \tag{A.18}$$

$$K = \int k(x,z)\mu(dx,dz) \tag{A.19}$$

$$N = \int n(x,z)\mu(dx,dz) \tag{A.20}$$

$$D = \int d(x,z)\mu(dx,dz) \tag{A.21}$$

Equations (A.7) through (A.21) correspond to equations (3.1) through (3.16) in the text.

B. Stochastic equilibrium

This section describes how to compute the equilibrium business cycle dynamics of the benchmark economy subject to stochastic shocks. The method has close similarities with the one described in Veracierto [16].

From equation (2.9) we have that

$$c_t = \int e^{(1-\varphi)a_t} z_t n_t(x,z)^{\gamma} k_t(x,z)^{\varphi} \mu_t(dx,dz) - K_{t+1} + (1-\delta) K_t \tag{B.1}$$

Substituting equations (2.6) and (2.7) in (2.3) gives

$$h_t = 1 - \pi_S [U_t + \int d_t(x,z)\mu_t(dx,dz)] - \pi_N \int n_t(x,z)\mu_t(dx,dz) \tag{B.2}$$

Observe that conditional on a_t, n_t, K_t, and μ_t, the optimal allocation of capital across islands k_t is obtained as a solution to the following static problem:

$$\max_{k_t} \left\{ \int e^{(1-\varphi)a_t} z_t n_t(x,z)^{\gamma} k_t(x,z)^{\varphi} \mu_t(dx,dz) \right\} \tag{B.3}$$

subject to

$$K_t = \int k_t(x,z)\mu_t(dx,dz)$$

Substituting the solution k_t to this problem into equation (B.1) and then substituting the resulting expression together with equations (B.2) and (2.7) into the one-period return function

$$R = \ln c_t + A\left(\frac{h_t^{1-\phi} - 1}{1-\phi}\right) - \psi D_t$$

allows to write the return function R as a function of $(a_t, \mu_t, K_t, n_t, d_t, U_t, K_{t+1})$.

The social planner's problem can then be written as

$$V(a_t, \mu_t, K_t) = \max_{\{n_t, d_t, U_t, K_{t+1}\}} \left\{ R\left(a_t, \mu_t, K_t, n_t, d_t, U_t, K_{t+1}\right) + \beta E V(a_{t+1}, \mu_{t+1}, K_{t+1}) \right\} \tag{B.4}$$

subject to

$$\mu_{t+1}(X', Z') = \int_{\{(x,z): n_t(x,z)+d_t(x,z)+U_t \in X'\}} Q(z, Z')\mu_t(dx, dz) \tag{B.5}$$

$$n_t(x, z) \leq x$$

$$a_{t+1} = \rho_a a_t + \varepsilon_{t+1} \tag{B.6}$$

The high dimensionality of the state space seems to preclude any possibility of computing a solution to this problem. However, the (S,s) nature of the employment adjustment rule at the islands level will allow to substitute the above problem by one that has linear constraints and is a good approximation to the original problem in a neighborhood of the deterministic steady state, lending itself to standard linear-quadratic methods.

Before proceeding to the transformed problem it will be useful to establish the following Lemma. Hereon, any variable superscripted with a star ($*$) will refer to its deterministic steady state value.

Lemma B.1. *The deterministic steady state distribution μ^* has a finite support given by the vector*

$$\mathbf{x}^* = (\bar{n}^*(z) + m \times U^*)_{\substack{m=1,\ldots,\overline{M}(z) \\ z=z_{\min},\ldots,z_{\max}}} \cup (\underline{n}^*(z) + m \times U^*)_{\substack{m=1,\ldots,\underline{M}(z) \\ z=z_{\min},\ldots,z_{\max}}}$$

where $\overline{M}(z)$ is the lowest natural number that satisfies

$$\bar{n}^*(z) + \overline{M}(z) \times U^* > \bar{n}^*(z_{\max})$$

and $\underline{M}(z)$ is the lowest natural number that satisfies

$$\underline{n}^*(z) + \underline{M}(z) \times U^* > \bar{n}^*(z_{\max})$$

Proof: *It follows from the fact that every time that an island of type (x, z) has a next period number of agents different from $x' = x + U^*$, it must be either $x' = \bar{n}^*(z) + U^*$ or $x' = \underline{n}^*(z) + U^*$.*

Hereon, I will refer to $\mathbf{x}^*(j)$ as the jth element of $\mathbf{x}^*$ and the total number of elements in

$\mathbf{x}^*$ will be denoted by J. In what follows, it will be useful to classify the elements of $\mathbf{x}^*$ into three sets: 1) those that correspond to islands that in the previous period let some agents go (set $\mathcal{L}^*$), 2) those that correspond to islands that in the previous period were targeted by directed searchers (set $\mathcal{G}^*$), and 3) those that correspond to islands that in the previous period did not let anybody go nor attracted any directed searchers (set $\mathcal{I}^*$). That is, for $j = 1, ..., J$:

$$\begin{aligned} j &\in \mathcal{L}^*, \text{ if } \mathbf{x}^*(j) = \bar{n}^*(z) + U^* \text{ for some } z \\ j &\in \mathcal{G}^*, \text{ if } \mathbf{x}^*(j) = \underline{n}^*(z) + U^* \text{ for some } z \\ j &\in \mathcal{I}^*, \text{ if } \mathbf{x}^*(j) = \mathbf{x}^*(j-1) + U^* \end{aligned} \tag{B.7}$$

Observe that equation (B.7) implicitly assumes a particular ordering of the elements of $\mathbf{x}^*$.

With this background, let turn back to the social planner's problem (B.4). Given the (S,s) nature of the adjustment rules at the island level, there is no loss of generality in constraining the social planner to choose an employment function n_t of the following form:

$$n_t(x, z) = \min \{\bar{n}_t(z), x\} \tag{B.8}$$

and a directed search function d_t of the following form

$$d_t(x, z) = \max \{\underline{n}_t(z), n_t(x, z)\} - n_t(x, z). \tag{B.9}$$

Suppose that the state variable μ_t has a finite support $\mathbf{x}_t$ of dimension J (same dimension as $\mathbf{x}^*$), that $\mathbf{x}_t$ is close to $\mathbf{x}^*$ and that

$$\begin{aligned} \mu_t(\mathbf{x}_t(j), z) &= \mu^* \left(\mathbf{x}^*(j), z\right), \text{ for every } z \text{ and every } j = 1, ..., J \qquad \text{(B.10)} \\ \mu_t &= 0, \text{ everywhere else.} \end{aligned}$$

In addition, assume that $\bar{n}_t$, $\underline{n}_t$ and U_t are close to their steady state values $\bar{n}^*$, $\underline{n}^*$ and U^*.

Then, the next period finite support $\mathbf{x}_{t+1}$ will be given by

$$\mathbf{x}_{t+1}(j)=\left\{\begin{array}{c}\bar{n}_t(z)+U_t,\text{ if } j\in\mathcal{L}^*,\\ \underline{n}_t(z)+U_t,\text{ if } j\in\mathcal{G}^*\\ \mathbf{x}_t(j-1)+U_t,\text{ if } j\in\mathcal{I}^*\end{array}\right\},\text{ for } j=1,...,J, \tag{B.11}$$

where z in the first line of the equation satisfies that $\mathbf{x}^*(j)=\bar{n}^*(z)+U^*$, and z in the second line satisfies that $\mathbf{x}^*(j)=\underline{n}^*(z)+U^*$. By continuity, $\mathbf{x}_{t+1}$ will be close to $\mathbf{x}^*$ and μ_{t+1} will satisfy that

$$\begin{aligned}\mu_{t+1}(\mathbf{x}_{t+1}(j),z) &= \mu^*\left(\mathbf{x}^*(j),z\right),\text{ for every } z \text{ and every } j=1,...,J\\ \mu_{t+1} &= 0,\text{ everywhere else.}\end{aligned}$$

Assuming that the optimal variables a_t, $\mathbf{x}_t$, K_t, $\bar{n}_t$, $\underline{n}_t$, U_t fluctuate in a small neighborhood of their deterministic steady state values, the original social planner's problem (B.4) can then be replaced by the following transformed problem:

$$V(a_t,\mathbf{x}_t,K_t)=\max_{\{\bar{n}_t,\underline{n}_t,U_t,K_{t+1}\}}\left\{\widetilde{R}\left(a_t,\mathbf{x}_t,K_t,\bar{n}_t,\underline{n}_t,U_t,K_{t+1}\right)+\beta EV(a_{t+1},\mathbf{x}_{t+1},K_{t+1})\right\} \tag{B.12}$$

subject to (B.6) and (B.11).

The return function $\widetilde{R}$ in (B.12) is given by the value of the return function R in (B.4) that corresponds to the discrete distribution μ_t defined by the finite support $\mathbf{x}_t$ and (B.10), the employment rule n_t defined in (B.8), and the directed search decisions d_t defined in (B.9). The advantage of working with the transformed problem (B.12) instead of the original problem (B.4) is that its constraints describe linear laws of motion. Since all the endogenous arguments of $\widetilde{R}$ take positive values in the deterministic steady state, a second order Taylor expansion around the deterministic steady state can be performed to obtain a quadratic return function. This leaves a standard linear-quadratic structure that can be solved using standard techniques. The assumption that a_t, $\mathbf{x}_t$, K_t, $\bar{n}_t$, $\underline{n}_t$, U_t fluctuate in a small neighborhood of their deterministic steady state values is satisfied in all the experiments reported in this paper.

Table 1

U.S. and Fixed labor force economies

Relative standard deviation (σ_x/σ_Y)				
Variable	U.S	Utility of search	No utility of search	Directed search
output	1.00	1.00	1.00	1.00
consumption	0.57	0.32	0.35	0.35
investment	4.28	4.60	4.37	4.36
capital	0.43	0.32	0.31	0.31
employment	0.57	0.57	0.03	0.06
unemployment	6.25	8.75	0.41	0.98
labor force	0.20	0.0	0.0	0.0
labor productivity	0.63	0.47	0.98	0.94
Correlation with output $(\rho_{x,Y})$				
Variable	U.S.	Utility of search	No utility of search	Directed search
output	1.00	1.00	1.00	1.00
consumption	0.80	0.89	0.92	0.93
investment	0.91	0.99	0.99	0.99
capital	0.05	0.17	0.18	0.17
employment	0.81	0.98	0.76	0.88
unemployment	-0.83	-0.96	-0.76	-0.88
labor force	0.39	0.0	0.0	0.0
labor productivity	0.84	0.97	1.00	1.00

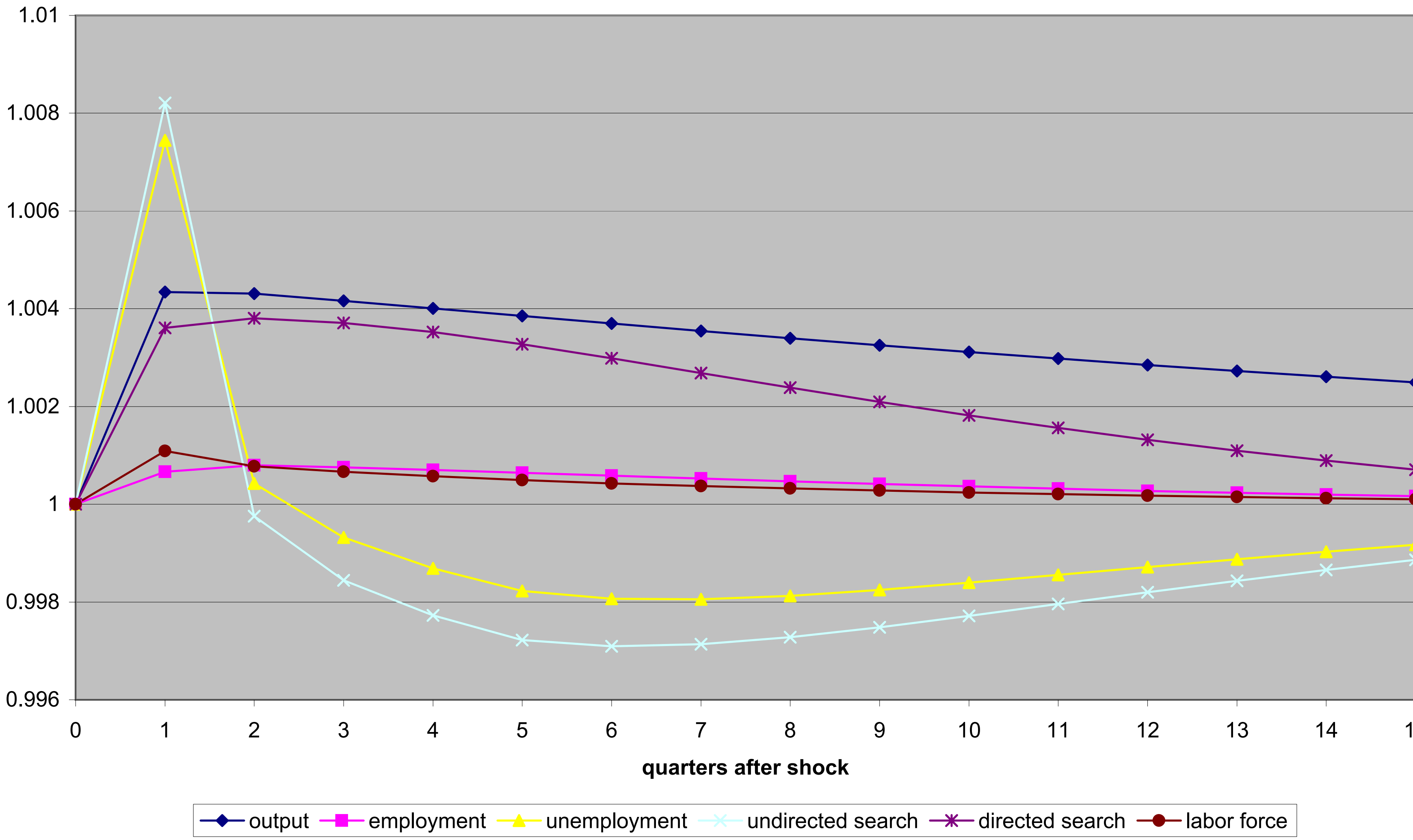
Table 2

U.S. and Flexible labor force economies

Relative standard deviation (σ_x/σ_Y)				
Variable	U.S	Directed search (disutility cost)	Directed search (resource cost)	Matching Function
output	1.00	1.00	1.00	1.00
consumption	0.57	0.34	0.33	0.34
investment	4.28	4.42	4.30	4.30
capital	0.43	0.31	0.31	0.31
employment	0.57	0.20	0.20	0.19
unemployment	6.25	1.38	2.03	1.62
labor force	0.20	0.20	0.22	0.21
labor productivity	0.63	0.81	0.81	0.82
Correlation with output $(\rho_{x,Y})$				
Variable	U.S.	Directed search (disutility cost)	Directed search (resource cost)	Matching Function
output	1.00	1.00	1.00	1.00
consumption	0.80	0.92	0.92	0.92
investment	0.91	0.99	0.99	0.99
capital	0.05	0.17	0.18	0.15
employment	0.81	0.98	0.98	0.97
unemployment	-0.83	0.07	0.08	0.24
labor force	0.39	0.92	0.88	0.93
labor productivity	0.84	1.00	1.00	1.00

Figure 1

Impulse response functions for benchmark economy



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