

# IPO Waves and Stock Prices.

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## Abstract

This paper explores why IPO volume changes over time and how it relates to stock prices. We develop a model of optimal IPO timing in which IPO volume fluctuates due to time variation in market conditions. IPO waves in our model are caused by declines in expected market return, increases in expected aggregate profitability, or increases in prior uncertainty about the average future profitability of IPOs. The model has numerous predictions for IPO volume, including that IPO waves are preceded by high market returns and followed by low market returns. These as well as other predictions are supported empirically.

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# 1. Introduction

The number of initial public offerings (IPOs) changes dramatically over time, as shown in Figure 1. For example, 845 firms went public in the U.S. in 1996, but there were only 87 IPOs in 2002, and only 31 IPOs in the first three quarters of 2003. The fluctuation in IPO volume is well known at least since Ibbotson and Jaffe (1975), but its underlying causes are not well understood. The recent trend in the literature is to attribute time variation in IPO volume to market inefficiency, arguing that IPO volume is high when market values of equity are high relative to book values, and that IPO volume is high when the market is “overvalued”.<sup>1</sup> Such an argument assumes that the periodic market mispricing can somehow be detected by the owners of the firms going public but not by the investors providing IPO funds, which seems open to discussion. This paper develops a model in which fluctuation in IPO volume arises in the absence of any mispricing, and in which IPO volume is more closely related to recent changes in stock prices than to the level of stock prices.

In addition to the mispricing argument, the literature provides numerous other insights into IPO volume. Some studies focus on the adverse selection costs of issuing equity (e.g. Myers and Majluf, 1984), and argue that the volume of equity issuance is driven by time variation in the amount of asymmetric information.<sup>2</sup> Other studies develop the corporate control aspect of an IPO (e.g. Zingales, 1995). For example, Benninga, Helmantel, and Sarig (2003) model the tradeoff between private benefits of control and the diversification benefit of going public, and derive implications for optimal IPO timing. This paper abstracts from these important corporate finance issues, and shows that IPO volume can fluctuate in the absence of asymmetric information and private benefits of control.

We develop a model of optimal IPO timing in which IPO volume fluctuates due to time variation in “market conditions,” defined here to have three dimensions: expected market return, expected aggregate profitability, and prior uncertainty about the post-IPO average profitability in excess of market profitability (henceforth, “prior uncertainty”). Market conditions indeed appear to vary in these dimensions. Time variation in expected market return is consistent with empirical evidence on return predictability.<sup>3</sup> Time variation in expected

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<sup>1</sup>See Ritter (1991), Loughran, Ritter, and Rydqvist (1994), Loughran and Ritter (1995), Rajan and Servaes (1997, 2003), Pagano, Panetta, and Zingales (1998), Baker and Wurgler (2000), and Lowry (2003).

<sup>2</sup>See, for example, Lucas and McDonald (1990), Choe, Masulis, and Nanda (1993), Bayless and Chaplinsky (1996), Hoffmann-Burchardi (2001), and Lowry (2003). For other information-based models, see Persons and Warther (1997), Chemmanur and Fulghieri (1999), Subrahmanyam and Titman (1999), Stoughton, Wong, and Zechner (2001), Lowry and Schwert (2002), Benveniste, Busaba, and Wilhelm (2002), and Altı (2003).

<sup>3</sup>See, for example, Keim and Stambaugh (1986), Campbell (1987), and Fama and French (1988).

aggregate profitability is related to business cycles. Time variation in prior uncertainty seems plausible as well. For example, technological revolutions are likely to be accompanied by high prior uncertainty, as they make the prospects of new firms highly uncertain. We show, theoretically and empirically, that IPO volume responds to time variation in all three dimensions of market conditions. Moreover, we note that market conditions are related not only to IPO volume but also to stock prices (i.e. the firms' ratios of market equity to book equity,  $M/B$ ) because they affect the discount rates, expected cash flows, and the cash flow uncertainty of public firms. IPO volume is then naturally related to stock prices as both quantities are driven by the same underlying factors.

Our model considers a special class of agents, "inventors", who invent new ideas that can lead to abnormal profits. Inventors patent each idea upon discovery and start a private firm that owns the patent. Inventors lack the capital necessary to begin production, so they must turn to capital markets. Inventors essentially possess a real option to take their firms public, and they decide when to exercise this option to maximize the value of their patents. When market conditions are constant, it is optimal to go public as soon as the patent is secured. When market conditions vary over time, however, inventors may find it optimal to postpone their IPO in anticipation of more favorable market conditions in the future.

We solve for the optimal time to go public, and show that private firms are attracted to capital markets especially when expected market return is low, when expected aggregate profitability is high, and when prior uncertainty is high. As a result, clusters of IPOs, or "IPO waves", occur after expected market return declines, after expected aggregate profitability increases, or after prior uncertainty increases. When market conditions improve, many inventors exercise their options to go public at about the same time, creating an IPO wave. To analyze the properties of IPO waves implied by the model, we calibrate the model to match some key features of the data on asset prices, profitability, and consumption, and simulate it over a long period of time. In the simulation, one idea is born each period, so that IPO waves do not arise from the clustering of technological inventions in time. Instead, IPO waves arise due to the clustering in the inventors' optimal IPO timing decisions.

Our model is rich in empirical predictions. IPO waves caused by a decline in expected market return should be preceded by high market returns (because prices rise when expected return falls) and followed by low market returns (because expected return has declined).<sup>4</sup> IPO waves caused by an increase in expected aggregate profitability should also be preceded

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<sup>4</sup>Schultz (2003) argues that equity issuers time the market ex post but not ex ante, so that IPO volume is correlated with future returns ex post but not ex ante. In contrast, in our model firms go public after declines in expected market return, so that high IPO volume predicts low market returns also ex ante.

by high market returns (because prices rise as cash flow expectations go up) and followed by high profitability (because expected profitability has risen). Finally, IPO waves caused by an increase in prior uncertainty should be preceded by increased disparity between newly listed firms and seasoned firms in terms of their valuations and return volatilities (because prior uncertainty affects the valuations and volatilities of new firms only).

We test the model's implications using data between 1960 and 2002, and find considerable corroborating evidence. We find support for all three channels (discount rate, cash flow, and uncertainty) through which IPO waves are created in our model. IPO volume is positively related to recent market returns, consistent with a decline in expected market return as well as with an increase in expected aggregate profitability. The discount rate channel is supported by two additional findings: IPO volume is negatively related to future market returns as well as to recent changes in market return volatility, which is itself positively related to expected market return in our model. The cash flow channel is also supported by two additional facts: IPO volume is positively related to changes in aggregate profitability as well as to revisions in analysts' forecasts of long-term earnings growth. Finally, IPO volume is also positively related to recent changes in the excess volatility and valuation of the newly listed firms, consistent with an increase in prior uncertainty.

The above empirical evidence includes a relation between IPO volume and recent changes in stock prices. This relation is due to the endogeneity of IPO timing: firms are induced to go public by improvements in market conditions (e.g. by declines in expected market return or increases in expected aggregate profitability), and these improvements also lift stock prices. IPO volume is related to the level of stock prices as well, but that relation is weaker. IPO volume is not necessarily high when the aggregate M/B is high because the high M/B is a result of cumulative improvements in market conditions, and many private firms that had been waiting for such improvements already went public while M/B was increasing. In the data, IPO volume is indeed significantly related to recent market returns but only insignificantly related to the level of the aggregate M/B, supporting the model.

Our model has implications not only for the aggregate M/B but also for M/Bs of IPOs. IPOs generally command high valuations in our model, partly because IPO timing is endogenous, and partly due to prior uncertainty. This uncertainty increases firm value, as shown by Pástor and Veronesi (2003).<sup>5</sup> A firm's M/B is predicted to decline after the IPO for

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<sup>5</sup>The intuition relies on a simple convexity argument. As an example, consider the Gordon growth model,  $P = D/(r - g)$ , where  $P$  is stock price,  $D$  is tomorrow's dividend,  $r$  is the discount rate, and  $g$  is the growth rate of future dividends. Note that  $P$  is convex in  $g$ . If  $g$  is uncertain,  $P$  is equal to the expectation of the right hand side. This expectation increases with uncertainty about  $g$ , holding other things constant, due to

two reasons. The first reason is learning. Upon observing realized profits, investors update their beliefs and the posterior uncertainty declines, resulting in a gradual decline in M/B over the lifetime of a typical firm. Consistent with this argument, Pástor and Veronesi find empirically that younger firms have higher M/B than older firms, after controlling for other determinants of M/B such as expected return and expected profitability. The second reason is mean reversion in market conditions. Improvements in market conditions induce private firms to go public and at the same time push M/B up, often above its long-term average, but sooner or later market conditions revert to their means, pulling M/B down.

In her empirical investigation, Lowry (2003) concludes that the main determinants of IPO volume are private firms' demands for capital and investor sentiment, with a lesser role for asymmetric information. Our empirical work complements Lowry's. More importantly, this paper develops, analyzes, and tests a novel theoretical model of optimal IPO timing. Jovanovic and Rousseau (2001) present a model in which delaying an IPO is valuable because it allows a private firm to learn about a parameter of its own production function. The idea that investment in general may be delayed due to learning is discussed in the literature on irreversible investment under uncertainty (e.g. Cukierman, 1980, and Bernanke, 1983).<sup>6</sup> In our model, learning about the project does not begin until the IPO, and the option to delay an IPO is valuable due to time variation in market conditions.

The paper is organized as follows. Section 2 describes the setting in which IPO decisions are made. Section 3 discusses the decision to go public and analyzes some properties of optimal IPO timing. Section 4 uses a simulated sample to investigate the properties of IPO waves in our model. Section 5 tests the model's predictions empirically. Section 6 examines some related issues and extensions. Section 7 concludes.

## 2. Model

There are two classes of agents, "inventors" and "investors," who possess identical information and preferences but different endowments. Investors are endowed with a stream of consumption good. Inventors are endowed with the ability to invent ideas that can deliver abnormal profits. Inventors compete for ideas so they always find it optimal to patent a new idea as soon as it is discovered. Upon patenting his idea, an inventor starts a private firm

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the previously mentioned convexity. See Pástor and Veronesi (2003) for a more careful explanation.

<sup>6</sup>The option to delay investment is also studied by Brennan and Schwartz (1985), McDonald and Siegel (1986), Ingersoll and Ross (1992), Dixit (1992), Abel et al (1996), and Berk (1999). See also Shleifer (1986), Gale (1996), Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2001), and Novy-Marx (2003).

that owns the patent. The private firm produces no revenue because the inventor lacks the capital necessary to begin production. This capital is raised in an IPO, in which the private firm is sold to investors. Production begins immediately after the IPO. The main decision faced by the inventor is when to take his firm public and begin production.

Before solving the optimal IPO timing problem in Section 3., it is necessary to describe the production technology and the economic environment in which IPO decisions take place. The key feature of this environment is time variation in market conditions. Market conditions in our model vary in three dimensions, each of which is described in a separate subsection: expected aggregate profitability (Section 2.1.), prior uncertainty (Section 2.2.), and expected market return (Section 2.3.). Given this variation, Section 2.4. solves for the market value of a firm, which is an essential input to the optimization problem analyzed in Section 3.

## 2.1. Time-Varying Profitability

After the IPO, firm  $i$ 's profits are protected by a patent until time  $T_i$ . Let  $\rho_t^i = Y_t^i/B_t^i$  denote the firm's instantaneous profitability at time  $t$ , where  $Y_t^i$  is the earnings rate and  $B_t^i$  is the book value of equity. Motivated by empirical evidence (e.g. Fama and French, 2000), we assume that firm profitability follows a mean-reverting process between the IPO and  $T_i$ :

$$d\rho_t^i = \phi^i (\bar{\rho}_t^i - \rho_t^i) dt + \sigma_{i,0}dW_{0,t} + \sigma_{i,i}dW_{i,t}, \quad (1)$$

where  $W_{0,t}$  and  $W_{i,t}$  are uncorrelated Wiener processes capturing systematic ( $W_{0,t}$ ) and firm-specific ( $W_{i,t}$ ) components of the random shocks that drive the firm's profitability. We also assume that the firm's average profitability  $\bar{\rho}_t^i$  can be decomposed as

$$\bar{\rho}_t^i = \bar{\psi}^i + \bar{\rho}_t.$$

The firm-specific component  $\bar{\psi}^i$ , which we refer to as the firm's average excess profitability, reflects the firm's ability to capitalize on its patent and is assumed constant over time. The common component  $\bar{\rho}_t$ , referred to as expected aggregate profitability, is assumed to exhibit mean-reverting variation that reflects aggregate economic conditions:

$$d\bar{\rho}_t = k_L (\bar{\rho}_L - \bar{\rho}_t) dt + \sigma_{L,0}dW_{0,t} + \sigma_{L,L}dW_{L,t}, \quad (2)$$

where  $W_{0,t}$  and  $W_{L,t}$  are uncorrelated. Periods of high aggregate profitability (which often coincide with economic expansions) are characterized by  $\bar{\rho}_t > \bar{\rho}_L$ , and vice versa. Later in the paper, we show that time variation in  $\bar{\rho}_t$  leads to time variation in IPO volume.

## 2.2. Time-Varying Prior Uncertainty

Average excess profitability  $\bar{\psi}^i$  is unobservable. For any firm  $i$  that goes public at time  $t$ , all agents, inventors and investors, have the same prior belief about  $\bar{\psi}^i$ . Their prior uncertainty,  $\hat{\sigma}_t$ , is the same for all firms going public at time  $t$ , for simplicity. It seems plausible for prior uncertainty  $\hat{\sigma}_t$  to vary over time. For example, uncertainty about the  $\bar{\psi}^i$ 's of new firms is large when the economy has experienced technological advances whose long-term impact is uncertain. Later in the paper, we show that time variation in  $\hat{\sigma}_t$  leads to time variation in IPO volume. To model time variation in  $\hat{\sigma}_t$ , we assume that  $\hat{\sigma}_t$  takes values in the discrete set  $\mathcal{V} = \{v^1, \dots, v^n\}$ , and that it switches from one value to another in each infinitesimal interval  $\Delta$  according to the transition probabilities  $\lambda_{hk}\Delta = \Pr(\hat{\sigma}_{t+\Delta} = v^k | \hat{\sigma}_t = v^h)$ . A discrete state space model allows a convenient solution for optimal IPO timing in Section 3.

All agents, inventors and investors, begin learning about  $\bar{\psi}^i$  as soon as firm  $i$  begins producing at its IPO. Agents learn by observing realized profitability  $\rho_t^i$ , as well as  $c_t$  (defined below),  $\bar{p}_t$ , and  $\rho_t^j$  for all firms  $j$  that are alive at time  $t$ . The prior of  $\bar{\psi}^i$  is assumed to be normal, so the posterior of  $\bar{\psi}^i$  is also normal, with mean  $\hat{\psi}_t^i$  and variance  $\hat{\sigma}_{i,t}^2$ . The dynamics of the posterior moments are given in Lemma 1 in the appendix.  $\bar{p}_t$  is assumed observable.<sup>7</sup>

## 2.3. Time-Varying Expected Market Return

Let  $\mu_t$  denote expected market return at time  $t$ . To generate time-varying  $\mu_t$ , we work with a framework similar to Campbell and Cochrane (1999). In their paper as well as ours,  $\mu_t$  varies over time due to time-varying risk aversion of the representative investor. All agents, inventors and investors, indexed by  $k$ , have habit utility over consumption:

$$U(C_t^k, X_t, t) = e^{-\eta t} \frac{(C_t^k - X_t)^{1-\gamma}}{1-\gamma}, \quad (3)$$

where  $X_t$  is an external habit index,  $\gamma$  regulates the local curvature of the utility function, and  $\eta$  is a time discount parameter. Consider the surplus consumption ratio  $S_t = (C_t - X_t)/C_t$ , where  $C_t = \sum_k C_t^k$  denotes aggregate consumption. Campbell and Cochrane assume that  $s_t = \log(S_t)$  follows a mean-reverting process with time-varying volatility and perfect correlation with unexpected consumption growth. This specification allows Campbell and Cochrane

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<sup>7</sup>Unobservable  $\bar{p}_t$  can be incorporated at the cost of a significant increase in complexity but with little benefit given the objectives of this paper. It can be shown that higher uncertainty about  $\bar{p}_t$  increases expected cash flow but also increases the discount rate, resulting in a relatively small net effect on prices. Veronesi (2000) discusses these effects in a different framework.

to solve for market prices numerically. To obtain analytical solutions for prices, even in the presence of learning, we depart from Campbell and Cochrane and assume that

$$s_t \equiv s(y_t) = a_0 + a_1 y_t + a_2 y_t^2, \quad (4)$$

where  $y_t$  is a state variable driven by the following mean-reverting process:

$$dy_t = k_y (\bar{y} - y_t) dt + \sigma_y dW_{0,t}. \quad (5)$$

Time variation in  $y_t$  generates time variation in both components of  $\mu_t$ , the equity premium and the real risk-free rate. As shown in the appendix, high  $y_t$  implies a low equity premium and a low risk-free rate in the plausible range. We show that time variation in either the equity premium or the risk-free rate leads to time variation in IPO volume in our model.

We assume that markets are dynamically complete, in the sense that shocks to the aggregate state variables  $y_t$  ( $dW_{0,t}$ ),  $\bar{p}_t$  ( $dW_{L,t}$ ), and  $\hat{\sigma}_t$  can be hedged using contingent claims. Firm-specific shocks  $dW_{i,t}$  can instead be hedged using firm equity. Since markets are complete, investors and inventors can perfectly insure each other's consumption. Assuming that their initial endowments are equally valuable, investors and inventors optimally choose identical consumption plans, justifying the existence of a representative agent with preferences given in equation (3). There is a unique stochastic discount factor (SDF)  $\pi_t$ :

$$\pi_t = U_C(C_t, X_t, t) = e^{-\eta t} (C_t S_t)^{-\gamma} = e^{-\eta t - \gamma(c_t + s_t)}, \quad (6)$$

where  $c_t = \log(C_t)$ . This SDF is used to compute the market value of a firm in Section 2.4., and also to value the IPO payoff in Section 3.

In equilibrium, aggregate consumption is given by the sum of all endowments and net payouts in the economy. Computing this sum is complicated because the payouts depend on the inventors' optimal IPO timing. Instead, for tractability, we assume that  $c_t$  follows<sup>8</sup>

$$dc_t = (b_0 + b_1 \bar{p}_t) dt + \sigma_c dW_{0,t}. \quad (7)$$

As other recent studies, we assume that consumption is financed mostly by income (e.g. labor income) that is outside our model, and the resulting process is given in equation (7).

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<sup>8</sup>Consumption growth is allowed to depend on  $\bar{p}_t$  because such a link is plausible ex ante, but none of our results rely on this link. As discussed later, our calibration uses the data-implied value of  $b_1$ . This value turns out to be small ( $b_1 = 0.08$  in Table 1), and  $b_1 = 0$  leads to the same qualitative conclusions throughout.

## 2.4. The Market Value of a Firm

This subsection discusses a closed-form solution for the market value of a firm in the environment described above. This market value is a key input to the IPO timing decision analyzed in Section 3. Our pricing analysis extends the model of Pástor and Veronesi (2003, henceforth PV) to allow for time variation in market conditions.

After its IPO, firm  $i$  earns abnormal profits ( $\bar{\psi}^i$ ) until its patent expires at time  $T_i$ . Any abnormal earnings after  $T_i$  are assumed to be eliminated by competitive market forces, so that the firm's market value at  $T_i$  equals its book value,  $M_{T_i}^i = B_{T_i}^i$ . (See PV, p.1753, for additional discussion.) The firm is assumed to pay no dividends, to be financed only by equity, and to issue no new equity.<sup>9</sup> The firm's market value at any time  $t$  after the IPO but before  $T_i$  is  $M_t^i = E_t[(\pi_{T_i}/\pi_t)B_{T_i}^i]$ . An analytical formula for  $M_t^i$  is provided in Proposition 1 in the appendix, together with expressions for firm  $i$ 's expected return and volatility.

The intuition behind the pricing formula is as follows. A firm's M/B is high if

- (i) the firm's expected profitability is high
- (ii) the firm's discount rate is low
- (iii) uncertainty about the firm's average future profitability is high

Regarding (i), M/B increases with three cash-flow related quantities: expected aggregate profitability,  $\bar{\rho}_t$ , expected excess profitability,  $\hat{\psi}_t^i$ , as well as current profitability,  $\rho_t^i$ . Regarding (ii), we find numerically that M/B increases with the state variable  $y_t$  in the calibrated model. This is sensible – recall from Section 2.3. that high  $y_t$  implies low risk aversion of the representative investor, and thus a low expected market return and high M/B. The facts (i) and (ii) are not surprising, of course. The fact (iii), that M/B increases with  $\hat{\sigma}_{i,t}$ , uncertainty about  $\bar{\psi}^i$ , seems more interesting. The intuition behind this relation, first documented in PV, is provided in footnote 5. For more details on the pricing formula, see the appendix.

Hereafter, we say that market conditions “improve” (“worsen”) when expected market return falls (rises), expected aggregate profitability rises (falls), and prior uncertainty rises (falls). Note that improvements in market conditions raise M/Bs, and vice versa.

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<sup>9</sup>These assumptions are made mostly for analytical convenience; relaxing them would add complexity with no obvious new insights. Given these assumptions, the clean surplus relation implies that book equity grows at the rate equal to the firm's profitability:  $dB_t^i = Y_t^i dt = \rho_t^i B_t^i dt$ .

### 3. Optimal IPO Timing

Having described the environment in which IPO decisions take place, we now turn to the IPO decision itself. Figure 2 summarizes the sequence of events. At time  $t_i$ , an idea is invented and patented by an inventor.<sup>10</sup> Until the patent expires at time  $T_i$ , it enables the owner to earn average excess profitability  $\bar{\psi}^i$ , but production requires capital  $B^{t_i}$  that the inventor does not have. This capital is raised in an IPO. At time  $\tau_i$ ,  $t_i \leq \tau_i \leq T_i$ , the inventor decides to go public and files the IPO. The IPO itself takes place at time  $\tau_i + \ell$ , where the lag  $\ell$  reflects the time required by the underwriter to conduct the “road show”.

In the IPO, the inventor sells the firm to investors for its fair market value  $M_{\tau_i+\ell}^i$ , computed in Section 2.4., and pays a proportional underwriting fee  $f$ . Part of the IPO proceeds,  $B^{t_i}$ , is immediately invested by the inventor and the production begins, generating profits described in equation (1). Only the inventor knows how to invest  $B^{t_i}$  (e.g. what machine to buy or construct), and this knowledge is too complex to be written down and sold, which rules out pre-IPO patent sales. Once the investment  $B^{t_i}$  is made, it is irreversible in that the project cannot be abandoned.<sup>11</sup> The inventor’s payoff from going public is  $M_{\tau_i+\ell}^i(1-f) - B^{t_i}$ , the value of the patent net of fees, which represents the inventor’s human capital.

The inventor chooses the time to go public to maximize the value of his patent, because doing so allows him to maximize his lifetime expected utility from consumption, given in equation (3). Given market completeness, standard results (Cox and Huang, 1989) imply that the maximization problem of inventor  $i$  can be written in its static form as

$$\max_{\{C_t^i, \tau_i\}} E_0 \left[ \int_0^\infty e^{-\eta t} \frac{(C_t^i - X_t)^{1-\gamma}}{1-\gamma} \right] \quad (8)$$

subject to the budget constraint

$$E_0 \left[ \int_0^\infty \frac{\pi_t}{\pi_0} C_t^i \right] \leq E_0 \left[ \frac{\pi_{\tau_i+\ell}}{\pi_0} (M_{\tau_i+\ell}^i(1-f) - B^{t_i}) \right]. \quad (9)$$

The budget constraint says that the present value of the inventor’s lifetime consumption cannot exceed the value of his endowment. It is thus optimal for the inventor to choose  $\tau_i$

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<sup>10</sup>The patent need not be interpreted literally; think of it as a competitive advantage in the marketplace.

<sup>11</sup>The irreversibility assumption can in general be motivated by the results of Abel and Eberly (1996) who find that even if the difference between the purchase and sale prices of capital is relatively small, the results on optimal investment are closer to the case of perfect irreversibility than to perfect reversibility. Schwartz (2001) finds that the option to abandon a patent-protected project often represents a substantial part of the project’s value. This option is not considered here, for tractability (options on options are not easy to value), but if it were, it would further increase IPO valuations, supporting one of our conclusions.

to maximize the endowment value, i.e. to maximize the value of the patent:

$$\max_{\tau_i} E_0 \left[ \frac{\pi_{\tau_i+\ell}}{\pi_0} (M_{\tau_i+\ell}^i (1-f) - B^{t_i}) \right]. \quad (10)$$

This problem is analogous to computing the optimal exercise time of a call option. By securing a patent, the inventor acquires a real option to raise capital in an IPO and invest it in the patented technology. This option is American in the sense that it can be exercised at any time before the patent expires. When contemplating when to exercise the option (i.e. when to go public), the inventor faces a tradeoff. On the one hand, delaying the IPO is costly because it forfeits abnormal profits that can be earned until the patent's expiration. On the other hand, by going public the inventor burns the time value of the option. This value is always positive because market conditions vary over time. Market conditions can in principle worsen so much after the IPO that the firm's cash flow fails to provide a fair rate of return on the initial investment  $B^{t_i}$ . Retaining the option by delaying the IPO offers protection against such an "IPO failure," whose possibility is reflected in the private firm's market value. The IPO generally takes place after market conditions improve because an IPO failure is then less likely due to the persistence in market conditions.

Denoting the optimal stopping time by  $\tau_i^*$ , the patent value at any time  $t$ ,  $t_i \leq t \leq T_i$ , is

$$V(\bar{\rho}_t, y_t, \hat{\sigma}_t, T_i - t) = E_t \left( \frac{\pi_{\tau_i^*+\ell}}{\pi_t} (M_{\tau_i^*+\ell}^i (1-f) - B^{t_i}) \right) \quad (11)$$

We solve for  $V$  and  $\tau_i^*$  numerically. Given market completeness with respect to the aggregate quantities  $\bar{\rho}_t$ ,  $y_t$ , and  $\hat{\sigma}_t$ , the value of the patent  $V$  can be spanned by the existing securities before the IPO. As a result,  $V$  must satisfy the standard Euler equation  $E_t[d(\pi_t V_t)] = 0$ . This condition translates into a system of partial differential equations, one for each possible uncertainty state  $\hat{\sigma}_t \in \mathcal{V} = \{v^1, \dots, v^n\}$ . Using the final condition that the patent is worthless at  $T_i$  if not exercised before, we work backwards to compute  $V_t$  for every combination of the state variables on a fine grid, and then we compute the optimal stopping time  $\tau_i^*$ .

The above discussion assumes that the capital necessary for production is raised in an IPO rather than by borrowing. This is actually a result, not an assumption. The inventor issues equity because he has a strong incentive to diversify.<sup>12</sup> If he instead borrowed and began producing, his entire wealth would become driven by idiosyncratic shocks (see equation 1) that could not be hedged with existing securities, which is clearly suboptimal. The only

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<sup>12</sup>As a practical matter, the inventor may not be able to borrow even if he wanted to because he has no tangible collateral and banks typically do not lend against ideas.

security that can hedge this idiosyncratic risk is a share of the firm’s equity, which is non-tradable before the IPO. Standard risk-sharing arguments then imply that the inventor issues some equity in an IPO. It can be proved formally that it is optimal for the inventor to sell all of his ownership, as assumed above, but it is also easy to show that the model’s implications are identical if the inventor retains any fraction of ownership after the IPO.

The following subsection discusses the numerical solution to the optimal IPO timing problem. The parameters used in the solution are taken from the calibration section 4.1.

### 3.1. When Do Firms Go Public?

Figure 3 plots the pairs of expected market return  $\mu_t$  and expected aggregate profitability  $\bar{\rho}_t$  for which the inventor optimally decides to go public. Each line denotes the locus of points that trigger the IPO decision, or the “entry boundary.” Firms go public when  $\mu_t$  and  $\bar{\rho}_t$  lie inside the “entry region” north-west of the entry boundary. If the idea is born when  $\mu_t$  and  $\bar{\rho}_t$  are inside the entry region, an IPO is filed immediately. Otherwise, the inventor waits until market conditions improve, and an IPO is filed as soon as the entry boundary is reached. If this never happens before the patent expires, the firm never goes public.

The top left panel considers a firm with  $\hat{\psi}_t^i = 0$  and a patent with  $T = 15$  years to expiration.<sup>13</sup> The entry boundary is upward sloping, so if  $\mu_t$  increases,  $\bar{\rho}_t$  must also increase to trigger entry. The entry boundary moves south-east as prior uncertainty  $\hat{\sigma}_t$  increases. Both effects are intuitive. At any point in time, the inventor compares the option value of delaying the IPO with the value of the profits given up by waiting. An IPO is filed when market conditions improve (i.e.  $\mu_t$  decreases,  $\bar{\rho}_t$  increases, or  $\hat{\sigma}_t$  increases) sufficiently so that the option to wait is no longer valuable enough to delay the IPO.

The other three panels of Figure 3 tell the same story, with some additional insights. The top right panel plots the entry boundaries for three different values of expected excess profitability  $\hat{\psi}_t^i$ , with  $\hat{\sigma}_t = 0$ . Higher values of  $\hat{\psi}_t^i$  expand the entry region by shifting the entry boundary south-east, which is intuitive because a more profitable patent has a higher opportunity cost of waiting for an improvement in market conditions. The bottom panels

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<sup>13</sup>The choice of  $T$  should reflect patent duration if we take our model literally. According to the U.S. law, patents issued before June 8, 1995 typically last for 17 years from the date of issuance, while patents granted after June 8, 1995 last for 20 years from the date of filing. The effective life of a patent is often shorter than 20 years, because some products such as drugs require various regulatory approvals before coming to the market, but patent extensions can frequently be obtained to compensate for the time lost in regulatory review (see Schwartz, 2001). Values of  $T$  between 10 and 20 years thus seem reasonable.

focus on the effects of time to the patent’s expiration,  $T$ . As time passes and  $T$  declines from 15 to five years, the entry boundary moves south-east, lowering the hurdle for entry. This is also intuitive, because the option to wait becomes less valuable as the patent’s expiration approaches. Interestingly, this effect is reversed close to the patent’s expiration, as shown in the bottom right panel. The reason is the underwriting fee,  $f$ . As  $T$  declines toward zero,  $M/B$  at the IPO declines to one. When  $M/B$  is sufficiently close to one, the inventor does not exercise his option because the payoff,  $M_{t_i^*+\ell}^i(1-f) - B^{t_i}$ , would be negative. As a result, for  $T$  sufficiently small, the hurdle for entry increases as time passes.

IPOs often command high valuations in our model, for two reasons. First, they occur when  $\mu_t$  is low enough and  $\bar{\rho}_t$  is high enough to be in the entry region (Figure 3). Low  $\mu_t$  and high  $\bar{\rho}_t$  help increase  $M/B$ s for all firms, including IPOs. More often than not,  $\mu_t$  is below and  $\bar{\rho}_t$  above their long-term averages at the time of the IPO. As  $\mu_t$  and  $\bar{\rho}_t$  revert to their central tendencies, the firm’s  $M/B$  declines after the IPO, on average. Second, IPO valuations tend to be higher than the valuations of seasoned firms thanks to prior uncertainty about  $\bar{\psi}^i$ , as discussed earlier. This fact gives a second reason why  $M/B$  declines after the IPO, on average. As soon as the firm begins generating observable profits, the market begins learning about  $\bar{\psi}^i$ , which reduces uncertainty and thus also  $M/B$ . Despite their projected decline, the high IPO valuations are perfectly rational, as IPOs are expected to earn a fair positive rate of return in our model.<sup>14</sup> The  $M/B$  ratios of IPOs do not fall because  $M$  is expected to go down but because  $B$  is expected to go up faster than  $M$ , loosely speaking. Also note that firms that are more profitable and firms that go public sooner after obtaining their patents have higher  $M/B$  at the time of the optimal entry, other things equal.

## 4. IPO Waves

IPO waves arise naturally as a result of optimal IPO timing in time-varying market conditions. This section analyzes the properties of IPO waves in a simulated environment, in which changes in market conditions are conveniently observable. We begin this section by calibrating the model described in Section 2. Then we simulate a long sample from the calibrated model, allowing inventors to time their IPOs according to the optimal rule from Section 3. We analyze the resulting IPO waves, and show that they arise due to time variation in all three dimensions of market conditions. The simulation evidence also provides a

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<sup>14</sup>The literature on the long-run IPO performance debates whether IPOs indeed earn fair expected returns. For important contributions to that literature, see Ritter (1991), Loughran and Ritter (1995), Brav and Gompers (1997), Brav, Geczy, and Gompers (2000), and Schultz (2003), among others.

benchmark for comparison with the empirical evidence in Section 5.

## 4.1. Calibration

This subsection describes the parameters chosen to calibrate the economic environment.<sup>15</sup> All parameters are summarized in Table 1, together with some implied aggregate quantities. We use data on quarterly real aggregate consumption and aggregate profitability between 1966Q1 and 2002Q1 to estimate the parameters for  $c_t$  in equation (7) and for  $\bar{p}_t$  in equation (2). Both series are described in the appendix. The Kalman filter is applied to the discretized versions of the processes. The estimated parameters imply expected consumption growth of 2.37% and volatility of 0.94% per year. For profitability, we obtain  $\bar{p}_L = 12.16\%$  per year,  $k_L = 0.1412$ , and  $\sigma_{LL} = 0.64\%$  per year.<sup>16</sup> We set  $\sigma_{L,0} = 0$ , very close to the unconstrained estimate, which implies zero correlation between  $\bar{p}_t$  and  $y_t$ . As a result, all three state variables that drive IPO volume ( $\bar{p}_t$ ,  $y_t$ , and  $\hat{\sigma}_t$ ) are independent of each other.

The agents' preferences are characterized by the processes for  $s_t$  in equation (4),  $y_t$  in equation (5), and by the utility parameters  $\eta$  and  $\gamma$ . The parameters are chosen to match some basic empirical properties of the market portfolio. Since newly listed firms comprise only a small fraction of the market (e.g. Lamont, 2002), we represent the market by a “long-lived firm” with instantaneous profitability of  $\bar{p}_t$ . The formulas for the long-lived firm's M/B ratio ( $M_t^m/B_t^m$ ), expected return ( $\mu_R^m$ ), and volatility ( $\sigma_R^m$ ) are given in the appendix. The preference parameters are chosen to calibrate  $\mu_R^m$ ,  $\sigma_R^m$ , and  $M_t^m/B_t^m$  to their empirical values for the market, while producing reasonable properties for the real risk-free rate. Our values for  $\bar{y}$  and  $\sigma_y$  imply the average equity premium of 6.8% and market volatility of 15% per year, and the speed of mean reversion  $k_y$  implies a half-life of 9.5 years for  $y_t$ . The long-lived firm's ratio of dividends to book equity is set to 10% per year, which produces an average aggregate M/B of 1.7, equal to the time-series average in the data. The average risk-free rate is 3.3% per year. The volatility of the risk-free rate is 3.9%, which is slightly higher than in the data (as is common in models with habit utility) but still reasonable.

The parameters for individual firm profitability  $\rho_t^i$  in equation (1) are chosen to match the median firm in the data. We use  $\phi^i = 0.3968$ , estimated by PV, who also report an

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<sup>15</sup>Our calibrated model matches some key features of the data on asset prices, profitability, and consumption. Since the models in the IPO literature typically involve two or three periods, few if any of those models can be calibrated to match as many features of the data as our dynamic continuous-time model.

<sup>16</sup>The speed of mean reversion  $k_L$  implies a half-life of about 4.9 years. That is, given any starting value  $\bar{p}_0$ , it takes on average 4.9 years for  $\bar{p}_t$  to cover half the distance between  $\bar{p}_0$  and its central tendency  $\bar{p}_L$ .

8.34% per year median volatility of the AR(1) residuals for individual firm profitability. We decompose this volatility into  $\sigma_{i,0} = 4.79\%$  and  $\sigma_{i,i} = 6.82\%$  per year, which implies a M/B of 1.7 for a firm with 15 years to patent expiration and  $\widehat{\psi}_t^i = 0$  when  $\widehat{\sigma}_t = 0$ ,  $y_t = \bar{y}$ , and  $\rho_t^i = \bar{\rho}_t = \bar{\rho}_L$ . Finally, prior uncertainty  $\widehat{\sigma}_t$  moves along the grid  $\mathcal{V} = \{0, 1, \dots, 12\}$  % per year. The transition probabilities are such that there is 10% probability in any given month of  $\widehat{\sigma}_t$  moving up or down to an adjacent value in the grid. If  $\widehat{\sigma}_t$  hits the boundary of the grid, there is a 20% probability of moving away from the boundary. For comparison, PV compute  $\widehat{\sigma}_t$  of 10% per year as the dispersion of average annual ROE across firms.

The parameters of the IPO timing model are specified as follows. The proportional underwriting fee is set equal to  $f = 0.07$ .<sup>17</sup> The lag between the IPO filing and the IPO itself is set equal to  $\ell = 3$  months.<sup>18</sup> The capital required for production is assumed to be proportional to the book value of the long-lived firm,  $B^{ti} = qB_{t_i}^m$ , with  $q = 0.0235\%$ .<sup>19</sup>

## 4.2. Simulating IPO Waves

Using the parameters from the previous subsection, we simulate a long sample from our economic environment. The pace of technological innovation is assumed to be constant, so that exactly one idea is invented and patented each month. Heterogeneity across ideas is introduced by drawing excess profitability  $\widehat{\psi}_t^i$  randomly from the set  $\{-6, -4, \dots, 4, 6\}$  % per year with equal probabilities. Patent owners decide when to go public by solving the optimal timing problem described in Section 3. To obtain population values for the variables of interest, we simulate 10,000 years (120,000 months) of data.

We define IPO waves as follows. Following Helwege and Liang (2003), we calculate three-month centered moving averages in which the number of IPOs in each month is averaged with the numbers of IPOs in the months immediately preceding and following that month. “Hot markets” are defined as months in which the moving average falls into the top quartile across the whole simulated sample. IPO waves are then defined as all sequences of consecutive

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<sup>17</sup>Chen and Ritter (2000) find that in 91% of the U.S. IPOs raising between \$20 and \$80 million (and in 77% of all IPOs) between 1995 and 1998, the gross spreads received by underwriters were exactly 7%. IPO underpricing can also be incorporated by using a bigger  $f$  without affecting our qualitative results.

<sup>18</sup>Lowry and Schwert (2002) report that the average time between the IPO filing and offer dates between 1985 and 1997 is 72 days. The median is 63 days, the minimum 11 days, and the maximum 624 days.

<sup>19</sup>Every month between January 1960 and December 2002, the book value of new lists (ordinary common shares that first appear on CRSP in that month) is divided by the total book value of equity. The time-series average of the monthly ratios is 0.0235%, excluding the spikes in July 1962 and December 1972 when AMEX and Nasdaq were added to CRSP. The exact value of  $q$  is not important for any of our conclusions. As long as  $q$  is reasonably small, the long-lived firm accounts for the bulk of the market portfolio.

hot-market months.<sup>20</sup> In our simulated 10,000-year-long sample, there are 4,116 IPO waves whose length ranges from one to 17 months, with a median of three months. The maximum number of IPOs in any given month is 51, while the median is one and the average is 0.9.

Since we assume a three-month lag between an IPO filing and the IPO itself, it is also useful to define an IPO “pre-wave” as an IPO wave shifted back in time by three months. Each IPO wave in our model is driven by state variable changes that occur in the respective pre-wave. Let “b” denote the beginning of a wave, or more precisely the last month before the wave begins, and let “e” denote the end of the wave’s last month. An IPO wave then begins at the end of month b and ends at the end of month e, whereas an IPO pre-wave begins at the end of month b-3 and ends at the end of month e-3.

### 4.3. Simulation Evidence Around IPO Waves

Table 2 reports the averages of selected variables around IPO waves. Given the size of the simulated sample, all averages can be treated as population values, so no  $p$ -values are shown. Column 1 of Panel A reports the average change in the given variable during a pre-wave. First, IPO waves are generated by pre-wave changes in expected market return  $\mu_t$ , which declines during a pre-wave by 0.99% per year on average. This is due to declines in expected excess return (0.46%) as well as the risk-free rate (0.53%).  $\mu_t$  declines consistently before the wave and begins increasing shortly before the end of a wave. Second, waves are triggered by changes in expected aggregate profitability  $\bar{\rho}_t$ , which rises by 0.06% per year during a pre-wave, on average. Third, waves are initiated by changes in prior uncertainty  $\hat{\sigma}_t$ , whose average pre-wave increase is 0.33% per year. Table 2 thus illustrates the importance of all three channels (discount rate, cash flow, and uncertainty) in generating IPO waves.

Of the three channels, the discount rate and uncertainty channels have been largely neglected in the literature. The literature has examined the cash flow channel, which seems to be the weakest of the three in Table 2, for two reasons. First,  $\bar{\rho}_t$  exhibits relatively little variation because aggregate profitability data that is used to calibrate the process for  $\bar{\rho}_t$  is relatively stable over time. Second,  $\bar{\rho}_t$  reverts to its mean relatively fast (e.g. faster than the variable  $y_t$  that drives  $\mu_t$ ; see Table 1), so changes in  $\bar{\rho}_t$  are perceived as short-lived. The inventor’s option to wait for an increase in  $\bar{\rho}_t$  is thus less valuable, and  $\bar{\rho}_t$  has a weaker effect on IPO volume than  $\mu_t$  and  $\hat{\sigma}_t$ .

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<sup>20</sup>Rarely, a month with zero IPOs can be designated as the first or last month of a wave if the large IPO volume in the neighboring month inflates the moving average. Such months are excluded from the wave.

Improvements in market conditions lead to higher IPO volume in Table 2 when they draw some outstanding patents into their entry region (Figure 3). To reinforce the intuition, recall the inventor’s tradeoff: delaying the IPO forfeits profits, but it preserves the option to wait for better market conditions. Improvements in market conditions reduce the value of the option to wait because market conditions are mean-reverting, and they also raise the opportunity cost of the profits given up by waiting. Both effects weaken the incentive to delay an IPO. Most of the time, there is a ‘backlog’ of private firms waiting for market conditions to improve. When it happens, many of these firms go public at about the same time. The resulting IPO waves typically last several months, as all private firms rarely go public at exactly the same time because they differ in the time to expiration on their patents as well as in their firm-specific profitability.

Note that IPO waves in our model are caused primarily by changes in market conditions, not levels. Table 2 shows that market conditions are typically only slightly more favorable during the waves than outside the waves. Consider the aggregate M/B, defined as the sum of earnings divided by the sum of book values across all firms, which reflects the level of market conditions. M/B rises during the pre-waves by 0.11 on average, consistent with IPO waves being produced by improvements in market conditions, but the level of M/B during the waves is only slightly higher than outside the waves (1.78 vs 1.76, on average). The reason is an interesting path dependence in IPO volume. Improvements in market conditions induce IPOs, thus depleting the backlog of private firms discussed above. After sufficiently significant improvements in market conditions, there is no backlog left, and IPO volume cannot exceed one per month at the time when M/B is high. Similarly, the backlog of private firms builds up as market conditions get worse, and an improvement in unfavorable market conditions can induce much of the large backlog to go public at the time when M/B is low. The relation between IPO volume and the level of market conditions is likely to get stronger if the model is extended to endogenize the pace of technology (see Section 6.).

#### 4.3.1. Proxies for Changes in Market Conditions

Changes in market conditions are conveniently observable in our simulated environment, but they are unobservable in the data. We therefore need to construct observable proxies for the empirical analysis in Section 5. We describe these proxies here, so that we can evaluate their effectiveness in simulations. One key quantity that is unobservable in the data is expected market return  $\mu_t$ . Its risk-free rate component is observable, but the equity premium is not. One proxy for the equity premium is market return volatility (MVOL). MVOL is highly

correlated with the equity premium in our model because both variables decrease with  $y_t$  in the plausible range. In our simulation, MVOL is indeed highly positively correlated (0.90) with the equity premium, but not with the other two state variables, as the correlation is 0.05 with  $\bar{\rho}_t$ , and zero with  $\hat{\sigma}_t$ . (All correlations are computed for first differences because those are used in the empirical work.) The second proxy for changes in  $\mu_t$  is realized market return, motivated by the fact (e.g. Campbell, 1991) that market returns seem to respond more to news about discount rates than to news about cash flows. High realized market return thus likely reflects a decline in expected market return, and vice versa. In our simulation, realized market returns are indeed highly negatively correlated with changes in  $\mu_t$  (-0.94).

Prior uncertainty  $\hat{\sigma}_t$  is also unobservable in the data. To design a proxy, it is useful to note that both the M/B and the return volatility of IPOs are strongly positively related to  $\hat{\sigma}_t$ , but neither the M/B nor the volatility of the long-lived firm depends on  $\hat{\sigma}_t$ . This distinction suggests two proxies for  $\hat{\sigma}_t$ . One proxy,  $\text{NEWVOL}_t = \sigma_{R,t}^{ipo} - \sigma_{R,t}^m$ , compares the return volatilities of IPOs and the long-lived firm, and the other proxy compares their M/B ratios:  $\text{NEWMB}_t = \log(M_t^{ipo}/B_t^{ipo}) - \log(M_t^m/B_t^m)$ .<sup>21</sup> The intuition that both NEWVOL and NEWMB should increase with  $\hat{\sigma}_t$  is confirmed in our long simulated sample, where NEWVOL and NEWMB exhibit high positive correlations (0.80 and 0.59) with  $\hat{\sigma}_t$ , while their correlations with the other two state variables are much lower: 0.09 with  $\mu_t$  and zero with  $\bar{\rho}_t$  for NEWVOL, -0.29 with  $\mu_t$  and 0.09 with  $\bar{\rho}_t$  for NEWMB. Our proxies for changes in market conditions thus have solid theoretical motivation.

Table 2 examines how these proxies vary around simulated IPO waves. MVOL declines during the pre-waves by 0.47% per year on average, which helps justify MVOL as a proxy for expected market return in the empirical analysis. NEWVOL and NEWMB both increase during the pre-waves, by 2.34% per year and 0.07, which helps justify their role as proxies for prior uncertainty.<sup>22</sup> Realized market return should be unusually high before IPO waves, especially due to declines in expected market return. Indeed, Panel B shows that average return is significantly higher during the pre-waves than outside: 40.30% versus 7.05% per year. Market returns during IPO waves and in the first three post-wave months are relatively

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<sup>21</sup>Jovanovic and Rousseau (2002, 2003) focus on a related quantity, the dispersion in  $Q$  (which is closely related to M/B) across firms. They find that most merger waves in the 20th century were preceded by a rise in this dispersion. They also argue that this dispersion increases due to arrival of new technologies.

<sup>22</sup>Computing NEWVOL and NEWMB requires at least one IPO in the given month. Since only one idea is born each month, our simulated sample includes many months with zero IPOs, especially before IPO waves. To avoid missing observations in the months with the biggest improvements in market conditions, we assume that only one firm with  $T = 15$  and  $\hat{\psi}_t^i = 0$  is born in any month  $t$  into the current market conditions summarized by  $y_t$ ,  $\bar{\rho}_t$ , and  $\hat{\sigma}_t$ . This assumption is made for the purpose of constructing NEWVOL and NEWMB only, and it provides a cleaner assessment of these proxies for  $\hat{\sigma}_t$  than any obvious alternatives.

low, about 9% for total returns, which is less than the 10.27% average outside a wave. There are two reasons behind the lower market returns. First, these returns are expected to be low if the wave is caused by a pre-wave decline in expected return. Second, market conditions typically begin deteriorating during the wave in light of the endogeneity of IPO timing – if market conditions continued to get better, the wave would likely continue as well.

#### 4.4. Regression Analysis

Table 3 further analyzes the determinants of IPO volume. Each column reports the coefficients from a regression of the number of IPOs on the variables listed in the first column, all simulated from our calibrated model. Although the model is simulated at a monthly frequency, all variables are cumulated to the quarterly frequency so that Table 3 matches its empirical counterpart, Table 6. No p-values are shown because all coefficients are highly statistically significant due to the size of the simulated sample (40,000 quarters).

Let us first examine the discount rate channel. As shown in column 1, IPO volume increases after declines in expected market return  $\mu_t$  over the previous two quarters. Column 6 shows that IPO volume also increases after declines in the risk-free rate. As seen from column 5, declines in MVOL (which proxy for declines in  $\mu_t$ ) tend to be followed by more IPOs. The results in column 4 also support the discount rate channel: IPO volume is positively related to past market returns, but negatively related to future and current returns. Realized returns are high while  $\mu_t$  drops, but they are low after the drop stops.

The cash flow and uncertainty channels are also clearly demonstrated in Table 3. As shown in column 2, IPO volume is high after increases in  $\bar{\rho}_t$ , and the results in column 4 are also consistent with the cash flow channel. IPO volume is high after increases in prior uncertainty  $\hat{\sigma}_t$ , as shown in column 3, as well as after increases in NEWMB and NEWVOL (columns 8 and 9), both of which proxy for  $\hat{\sigma}_t$  in our empirical work. Note that IPO volume is positively related to the level of M/B in the previous quarter. This relation is significant statistically but not economically, as shown previously in Table 2.

All regressions in Table 3 include a lag of IPO volume on the right-hand side, to be consistent with the subsequent empirical regressions. This lag is always highly significant, but its removal does not alter any of the above relations. Comparing the  $R^2$ 's in the first three columns, the discount rate channel seems the strongest in our model, while the cash flow channel is the weakest. Note, however, that the  $R^2$ 's are relatively low, between 0.04 and 0.12, because the true relations between IPO volume and the given variables are complex

and nonlinear in our model. We run linear regressions for two reasons – to be consistent with the subsequent empirical regressions, and because they suffice to clearly demonstrate the presence of all three channels that produce IPOs in our model.

## 5. Empirical Analysis

This section empirically investigates the three channels (discount rate, cash flow, and uncertainty) through which time-varying IPO volume is created in our model. The evidence is consistent with the existence of all three channels, as shown below.

### 5.1. Data

The data on the number of IPOs, obtained from Jay Ritter’s website, cover the period January 1960 through December 2002. To avoid potential concerns about nonstationarity (see Lowry, 2003), we deflate the number of IPOs by the number of public firms at the end of the previous month.<sup>23</sup> In the rest of the paper, “the number of IPOs” and “IPO volume” both refer to the deflated series, whose values range from zero to 2.1% per month, with an average of 0.5%. The pattern of time variation in the deflated series looks so similar to the pattern in the raw series plotted in Figure 1 that it is not worth plotting separately.

The data on our proxies for changes in market conditions are also constructed monthly for January 1960 through December 2002, unless specified otherwise. We use all data available to us. Market returns (MKT) are total returns on the value-weighted portfolio of all NYSE, AMEX, and NASDAQ stocks, extracted from CRSP. Market volatility (MVOL) is computed each month after July 1962 as standard deviation of daily market returns within the month. The aggregate M/B ratio (M/B) is the sum of market values of equity across all ordinary common shares divided by the sum of the most recent book values of equity. The real risk-free rate (RF) is the yield on a one-month T-bill in excess of expected inflation, where the latter is the fitted value from an AR(12) process applied to the monthly series of log changes in CPI from the Bureau of Labor Statistics. Aggregate profitability (return on

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<sup>23</sup>All individual stock price data is obtained from the Center for Research in Security Prices (CRSP) at the University of Chicago. We define public firms as ordinary common shares (CRSP sharecodes 10 or 11) with positive market values. The number of CRSP-listed firms jumps in July 1962 and December 1972 due to the addition of Amex and Nasdaq firms. Following Lowry (2003), we use the actual number of public firms after December 1972, but estimate the number of public firms prior to that by assuming that this number grew at the compounded growth rate of 0.45% per year before December 1972.

equity, ROE) is computed quarterly for 1966Q1 through 2002Q1 using Compustat data, as described in the appendix. This measure of profitability follows the definition of  $\rho_t^i$  in Section 2.1. Another measure of cash flow expectations is the I/B/E/S summary data on equity analysts’ forecasts of long-term earnings growth. These forecasts have horizons of five years or more, which makes them suitable given the relatively long-term nature of  $\bar{\rho}_t$ . For each firm and each month, the average forecast of long-term earnings growth is computed across all analysts covering the firm. The forecast of average earnings growth (IBES) is then computed by averaging the average forecasts across all ordinary common shares. The resulting series is available for November 1981 through March 2002. The monthly time series of M/B, MVOL, ROE, and IBES are plotted in Figure 4.

We construct both proxies for prior uncertainty discussed in Section 4.3. New firm excess volatility (NEWVOL) in a given month is computed by subtracting market return volatility from the median return volatility across all new firms, defined as firms whose first appearance in the CRSP daily file occurred over the previous month. A given firm’s return volatility in each month is the standard deviation of daily stock returns within the month. NEWVOL has 464 valid monthly observations in the 486-month period between July 1962 and December 2002. New firm excess M/B ratio (NEWMB) is computed for each month between January 1950 and March 2002 as follows. First, we compute the median M/B across all new firms, defined as those that appeared in the CRSP monthly file over the previous year.<sup>24</sup> NEWMB is computed as the natural logarithm of that median minus the log of the median M/B across all firms. The construction of M/B for individual firms is described in the appendix. NEWMB has eight missing values between January 1960 and March 2002. The monthly time series of NEWVOL and NEWMB are plotted in Figure 5.

In Figure 5, both NEWMB and NEWVOL rise sharply in the late 1990s, and they both decline after 2000. NEWVOL exhibits a remarkable pattern: In 1998, it triples from about 2% per day to about 6%, it remains around 6% through the end of 2000, and then it drops back to about 2% after 2000. Prior uncertainty thus appears to have been unusually high in 1998 through 2000. This is not surprising. The long-term prospects of new firms are uncertain when old paradigms are fading away and a “new era” is being embraced. The high prior uncertainty may have attracted many firms to go public in the late 1990s, and it might also have contributed to the high valuations of many IPOs at that time.

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<sup>24</sup>This definition of new firms ensures availability of their valid M/B ratios. Few firms have valid M/B ratios in the first few months after listing because M/B is computed using lagged book equity, which is often available only on an annual basis and generally available only after market equity becomes available. For both NEWMB and NEWVOL, we require at least three new firms to compute a valid median.

## 5.2. Empirical Evidence Around IPO Waves

Between January 1960 and December 2002, there are 16 IPO waves, whose length ranges from one to 21 months, with a median of five months. Some summary statistics for the 16 waves are shown in Tables 4 and 5. All variables except for the unitless M/B and NEWMB are in percent per year. In Table 4, all but three waves are preceded by above-average positive average market returns during the pre-wave, consistent with the model, and only one (one-month) wave is preceded by a negative return. For all but two waves, MVOL declines during the pre-wave, consistent with a pre-wave decline in expected market return. The wave in 1993-94 appears to be due to the cash flow channel. The waves in 1991, 1992, and especially 1999 were preceded by increases in both NEWVOL and NEWMB, suggesting that these waves may have been caused at least in part by increases in prior uncertainty.

Table 5 reports variable averages across the 16 waves. The  $t$ -statistics, given in parentheses, measure the significance of the difference between the averages within and outside the given period. For example, the  $t$ -statistic for average MVOL during a wave (-3.18) is computed by regressing MVOL on a dummy variable equal to one if the month is part of an IPO wave and zero otherwise. A positive (negative)  $t$ -statistic reveals that the variable's average in the given period is bigger (smaller) than the average in the rest of the sample.

The average pre-wave change in MVOL is significantly negative at -2.81% ( $t = -2.27$ ), consistent with IPO waves being caused by declines in expected market return. M/B, ROE, and IBES all increase before the waves, as the model predicts, but these increases are statistically insignificant. NEWVOL increases significantly during pre-waves ( $t = 2.27$ ), consistent with the uncertainty channel, but NEWMB does not. RF increases insignificantly during pre-waves, whereas our model predicts a pre-wave decrease. Panel B shows that average market returns are high before IPO waves (e.g. 31.17% annualized with  $t = 2.77$  two quarters before a wave), as predicted by the model. Market returns are low during and especially after IPO waves, but they are not significantly lower than in the rest of the sample. The return pattern is similar to the model-predicted pattern observed in Table 2.

Since the averages in Table 5 are computed across only 16 IPO waves, only a few relations are statistically significant. More detailed empirical analysis is therefore performed in the following section, which focuses on IPO volume rather than on IPO waves alone.

### 5.3. Regression Analysis

Each column of Table 6 corresponds to a separate regression, in which the number of IPOs in the current quarter is regressed on proxies for changes in market conditions. Lagged IPO volume is included on the right-hand side to capture persistence in IPO volume that is unexplained due to any potential misspecification in the regressions. Lowry (2003) also includes lagged IPO volume on the right-hand side of her regressions. She also always includes a first-quarter dummy that captures an apparent seasonality in IPO volume, and we follow her treatment. Both variables are significant in each regression. The  $t$ -statistics are in parentheses. Note that Table 6 is an empirical counterpart of Table 3.

First, we test the discount rate channel, in which new IPOs are triggered by declines in expected market return. Column 1 shows that IPO volume is significantly positively related to total market returns over the previous two quarters ( $t = 3.34$  and  $3.25$ ), consistent with both the discount rate and cash flow channels. Moreover, IPO volume is significantly negatively related to market returns in the subsequent quarter ( $t = -2.23$ ), consistent with the discount rate channel. This negative relation is also reported by Lamont (2002), Schultz (2003), and Lowry (2003). The relation with current returns is positive, not negative as in Table 3, but this difference does not contradict the model. IPO waves in the data tend to last longer than our simulated IPO waves, so the actual IPO waves have more overlap than the simulated waves with the declines in expected market return that caused the waves, and therefore also with high realized returns. Column 2 shows that IPO volume is significantly negatively related to current ( $t = -4.41$ ) as well as past ( $t = -3.59$ ) changes in market volatility, again consistent with the discount rate channel. Changes in the risk-free rate in column 3 are positively related to future IPO volume, not negatively as the model predicts. Combined with the results in columns 1 and 2, this positive relation suggests that IPO volume is strongly negatively related to recent changes in the equity premium.

Second, the cash flow channel is also supported by the data. Column 6 shows that IPO volume is positively related to current ( $t = 2.50$ ) as well as future changes in aggregate profitability, suggesting that firms tend to go public when cash flow expectations improve. The same conclusion is reached in column 7: IPO volume is significantly higher ( $t = 5.07$ ) when equity analysts on average upgrade their forecasts of long-term earnings growth.

Third, prior uncertainty also seems to go up before firms go public. In columns 8 and 9, IPO volume is positively related to recent changes in the excess M/B ratio of new firms ( $t = 3.18$  and  $2.35$ ) as well as to recent changes in the excess volatility of new firms ( $t = 2.23$ ),

both of which are strongly associated with prior uncertainty in our model.

Some of the relations described above lose their statistical significance when realized market returns are included in the regression. The reason goes beyond the simple lost-degrees-of-freedom effect. The right-hand-side variables are merely proxies for unobservable changes in market conditions. In reasonably efficient markets, where prices reflect much of the available information, realized returns are the best proxy for changes in market conditions – when market conditions improve, prices go up, and vice versa. It is thus not surprising that including market returns drives some of the weaker proxies below the threshold of significance. The role of these other proxies is simply to provide additional evidence on the likely causes of the observed price changes.

The regressors in Table 6 represent changes in market conditions, while the regressand is the level of IPO volume. Regressing levels on changes is appropriate because the level of IPO volume is affected mainly by changes in market conditions in our model (Section 4.3.). Lowry (2003), who uses the same dependent variable as we do, also suggests using changes in the number of IPOs as a way of avoiding nonstationarity. Using this redefined dependent variable leads to results that are almost identical to those reported in Table 6.

Finally, on a related note, column 4 of Table 6 shows that IPO volume is insignificantly positively related to the level of aggregate M/B at the end of the previous quarter.<sup>25</sup> Recall that our model predicts a positive relation (Table 3), but a very weak one (Table 2), because IPO volume in our model is driven mainly by changes in market conditions, not levels. Column 5 presents a horserace between the levels and changes, in that IPO volume is regressed on M/B, which reflects the level of market conditions ( $\mu_t$  and  $\bar{\rho}_t$ ), as well as on market returns, which reflect changes in market conditions (and which are almost perfectly correlated with changes in M/B). In this regression, market returns remain highly significant but M/B turns insignificantly negative. Put differently, IPO volume is high after a run-up in stock prices, but not necessarily when the level of prices is high. This evidence, which fits the intuition described in Section 4.3., provides additional support for the model.

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<sup>25</sup>Lowry (2003) finds a relation on the border of significance using a different measure of M/B, the equal-weighted average of M/Bs of individual firms.

## 6. Some issues and extensions

### 6.1. The Effect of Market Conditions on Individual Firms

The tenet of this paper is that IPO volume is driven by changes in market conditions (expected market return, expected aggregate profitability, and prior uncertainty). In general, a private firm’s decision to go public depends on the firm’s own expected return, its own expected profitability, and its own prior uncertainty, which we refer to as “firm conditions.” Improvements in market conditions can cause private firms to go public only if they improve their firm conditions. A sensible question is whether firm conditions move together with market conditions for sufficiently many private firms to cause IPO waves.

Our model gives an affirmative answer. The market value of a private firm and the IPO decision both depend only on market conditions and the patent’s time to expiration (equation 11). They do not depend on idiosyncratic risk because there is no production or learning before the IPO. Therefore, firm conditions of all private firms move in lock-step in our model, and changes in market conditions drive IPO volume, as shown in Section 4.

In reality, private firms typically do face idiosyncratic risk, so their firm conditions comove less than they do in our model. To see how this comovement affects IPO waves, we solve a modified version of our model in which private firms face idiosyncratic risk due to pre-IPO learning. We assume that agents observe signals about  $\bar{\psi}^i$  before the IPO, so that  $\hat{\psi}_t^i = E_t(\bar{\psi}^i)$  exhibits firm-specific pre-IPO variation, and the comovement across private firms is weaker than in our model. Even when this comovement is much weaker (e.g. when the average  $R^2$  from the market model regressions for private firms is as low as 0.1 in simulations), we observe IPO waves with properties very similar to those obtained in our model. Specifically, the discount rate and cash flow channels remain highly significant, only the uncertainty channel is weaker because higher uncertainty about pre-IPO signals increases the value of the option to wait. This common result in the literature on irreversible investment (Cukierman, 1980) goes against the effect of prior uncertainty described earlier. Yet, since we find empirically that IPO volume is positively related to increases in proxies for prior uncertainty (Table 6), the uncertainty channel seems robust to pre-IPO learning. Our conclusions therefore hold also under imperfect comovement in firm conditions. We focus mainly on the simpler framework without pre-IPO learning because the modified framework is significantly more complicated and computationally challenging (due to additional state variables) without adding any substantial new insights into the time variation in IPO volume.

In addition to the theoretical analysis summarized above, we look at the comovement in firm conditions empirically. Since firm conditions are unobservable, we examine their comovement indirectly by measuring the comovement in stock prices. This seems reasonable because stock prices move if and only if firm conditions change. Specifically, a firm's M/B depends on the firm's expected profitability and expected return (e.g. Vuolteenaho, 2002), as well as on prior uncertainty (e.g. PV). Since M/B depends on all three components of firm conditions, changes in firm conditions translate into changes in the firm's M/B. For example, if firm conditions improve (i.e. the firm's expected return drops, expected profitability rises, or prior uncertainty rises), the firm's stock return is positive. The higher the degree of comovement in stock prices, the higher the degree of comovement in firm conditions.

The subject of this analysis should be private firms, but reliable data on the market values of private firms is difficult to obtain. As a feasible alternative, we focus on public firms, whose prices are easily available. For all firms with more than three years of data on CRSP between January 1926 and December 2002, we regress firm excess returns on excess market returns. Of the 17,832 firms in our sample, 96.2% have positive market betas estimated from monthly data, of which 74.2% are statistically significant. These results confirm the well known fact that there is significant comovement in stock prices. This implies that there is significant comovement in firm conditions as well, as explained in the previous paragraph.

The average  $R^2$  from the above regressions is 0.13 for monthly returns (0.19 for quarterly, 0.24 for annual). For changes in market conditions to affect IPO volume, it is not necessary that most of the variation in firm conditions be common; it is enough if a significant part of this variation is common. As an illustration, each month, we compute the fraction of firms whose realized excess return has the same sign as the excess market return. The average value of this fraction across all quarters is 0.66, which is significantly larger than 0.5 ( $t = 33.2$ ), the fraction expected by chance. (For quarterly returns, the average is 0.68, and for annual returns, it is 0.70.) That is, when market conditions improve, firm conditions improve for about two thirds of all firms. Given the large number of private firms that could potentially go public, this degree of comovement is likely to significantly affect IPO volume. Indeed, recall from earlier in this section that when we calibrate the modified version of our model to deliver slightly less commonality than in the data (the average  $R^2$  of 0.1 and the same-sign fraction of 0.6), we still observe IPO waves with similar properties.

The price comovement reported above reflects comovement in expected returns and profitabilities, which seems sensible a priori. Expected profitabilities are likely to comove as the economy moves through the business cycle. Expected returns comove in risk-based models

such as the CAPM as long as market betas are reasonably constant over time. Vuolteenaho (2002) decomposes individual stock returns into changes in expected profitability and changes in expected return. He finds that changes in expected returns are highly correlated across firms, more so than changes in expected profitabilities, and concludes that changes in expected returns are “predominantly driven by systematic, marketwide components.”

The comovement in the prices of public firms does not reflect any comovement in prior uncertainty, however. In our model, changes in prior uncertainty affect only the values of private firms because after the IPO, the uncertainty evolves deterministically due to learning (equation 14). Comovement in the prior uncertainties of private firms seems hard to document empirically, but it seems reasonable. Technological advances, for example, are likely to raise prior uncertainty not only in the industries in which these advances are made but also in other industries affected by the new technology. Comovement in prior uncertainty is likely to further strengthen the relation between IPO volume and market conditions.

To summarize, firm conditions exhibit significant comovement across firms, which allows IPO volume to be significantly related to market conditions. This relation is indeed observed in the data. In Table 6, IPO volume is significantly related to various proxies for changes in market conditions, including market returns, which are driven entirely by changes in market conditions. The results in Table 6 can be explained naturally using our model, and there are no obvious alternative explanations. We conclude that the relation between IPO volume and market conditions is strong theoretically as well as empirically.

## 6.2. Extensions

*Pre-IPO Production.* The purpose of an IPO in our model is to raise capital for new investment. Before the IPO, our private firm does not produce. This description applies only to a subset of the observed IPOs, because many real-world IPOs are undertaken by mature firms that produce for years before going public. The fact that production does not begin until the IPO is a result in our model. Producing before the IPO is suboptimal because it exposes the inventor to unhedgeable idiosyncratic risk ( $W_{i,t}$  in equation 1). Put differently, the inventor has a strong incentive to go public as soon as the production begins in order to diversify. Any pre-IPO production must therefore be due to some effects outside our model. Extending our model in this direction is a task for future research.

*Investment.* Our focus is on IPO waves, but our model can in principle be modified to address a broader issue of cyclicity of investment. Suppose public firms can invent ideas or

purchase them from inventors. A public firm solving for the optimal time to make an irreversible investment is considering tradeoffs similar to those of our inventor, and “investment waves” might obtain after market conditions improve.<sup>26</sup> We do not focus on investment by public firms for a number of reasons. First, such firms often invest simply to maintain a competitive stock of physical capital rather than to embark on new projects with uncertain and perishable abnormal profits, making some of the key features of our framework less relevant. For example, prior uncertainty is likely to be lower for the projects of public firms than for the investments made by recent IPOs (although this is not necessarily true for IPOs of mature firms). Second, the investment decisions of public firms may be affected by the firms’ existing projects, a complication that is absent from our model in which inventors have only one project at a time. Third, models of investment typically analyze dynamic properties of investment, whereas we take the firm’s post-IPO investment policy as given and simply assume that profitability follows a mean-reverting process. Fourth, we assume that learning about  $\bar{\psi}^i$  starts when the production begins, which seems better suited for IPOs of start-up companies than for investment by public firms. Learning about a public firm’s new project can take place before the production begins because investors can observe the firm’s other investment projects, whose payoffs are presumably correlated with the new project’s payoffs.

Also note that focusing on IPOs rather than on investment preserves market completeness. New investment projects introduce new idiosyncratic risk ( $W_{i,t}$ ) in the economy. This risk is not spanned by the existing securities, so markets become incomplete unless a new security is issued that can perfectly hedge the new risk. In our model, markets are dynamically complete because each new project is accompanied by an issue of a claim on the project’s cash flows. This issue has a natural interpretation as an IPO of a start-up company. The equity issued in the IPO provides a perfect hedge for the new project because it is a claim on that project only. In contrast, investments by public firms are not accompanied by issues of equity that would provide a perfect hedge. For example, the equity issued in a seasoned equity offering cannot perfectly hedge the new risk because it is a claim to all projects of this public firm, not just the new project. Due to market incompleteness in that case, the SDF may not be unique, which could complicate the analysis.

A separate point related to investment is that while IPOs in our model are accompanied by investment, Pagano et al. (1998) find in Italian data that firms tend to invest especially before their IPOs. Pre-IPO investment can be easily obtained in our model if we allow

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<sup>26</sup>A positive relation between corporate investment and the level of stock prices is reported by Barro (1990) and Baker, Stein, and Wurgler (2003), among others, although some studies find the relation to be insignificant. A positive relation is consistent with the q theory of investment (Brainard and Tobin, 1968).

for “time to build.” Instead of investing at time  $\tau^i + \ell$  (at the IPO), suppose the inventor invests at time  $\tau^i$  (when he decides to go public) using borrowed money. The loan is repaid from the IPO proceeds at time  $\tau^i + \ell$ . Also suppose that it takes  $\ell$  months to build the production technology, so that production does not begin until time  $\tau^i + \ell$ . This modified model produces results identical to ours, except that investment precedes the IPO.

*Industry clustering.* Some IPO waves are driven by many IPOs in just a few industries. This industry concentration of IPOs can be easily explained in a straightforward extension of our model. We assume for simplicity that prior uncertainty  $\hat{\sigma}_t$  is the same for all firms, but this uncertainty is likely to be more similar for firms in the same industry, as technological progress is often industry-specific. For the same reason, the average excess profitabilities  $\bar{\psi}^i$  can also be correlated for firms in the same industry. Increases in industry-specific prior uncertainty or excess profitability can then lead to IPO waves concentrated in the given industry, without triggering IPOs in other industries.

*The pace of technology.* IPO waves are obtained in our model even when new ideas arrive at a constant pace. It could be interesting to endogenize the arrival of new ideas. If capital must be raised to produce an idea, then low cost of capital might accelerate the pace of technological innovation, leading to more ideas and more IPOs. High expected aggregate profitability might also speed up the pace of technology and produce more IPOs. These effects, if present, would link IPO volume to the level of market conditions. In addition, these effects are likely to amplify the variation in IPO volume obtained in our model.

## 7. Conclusions

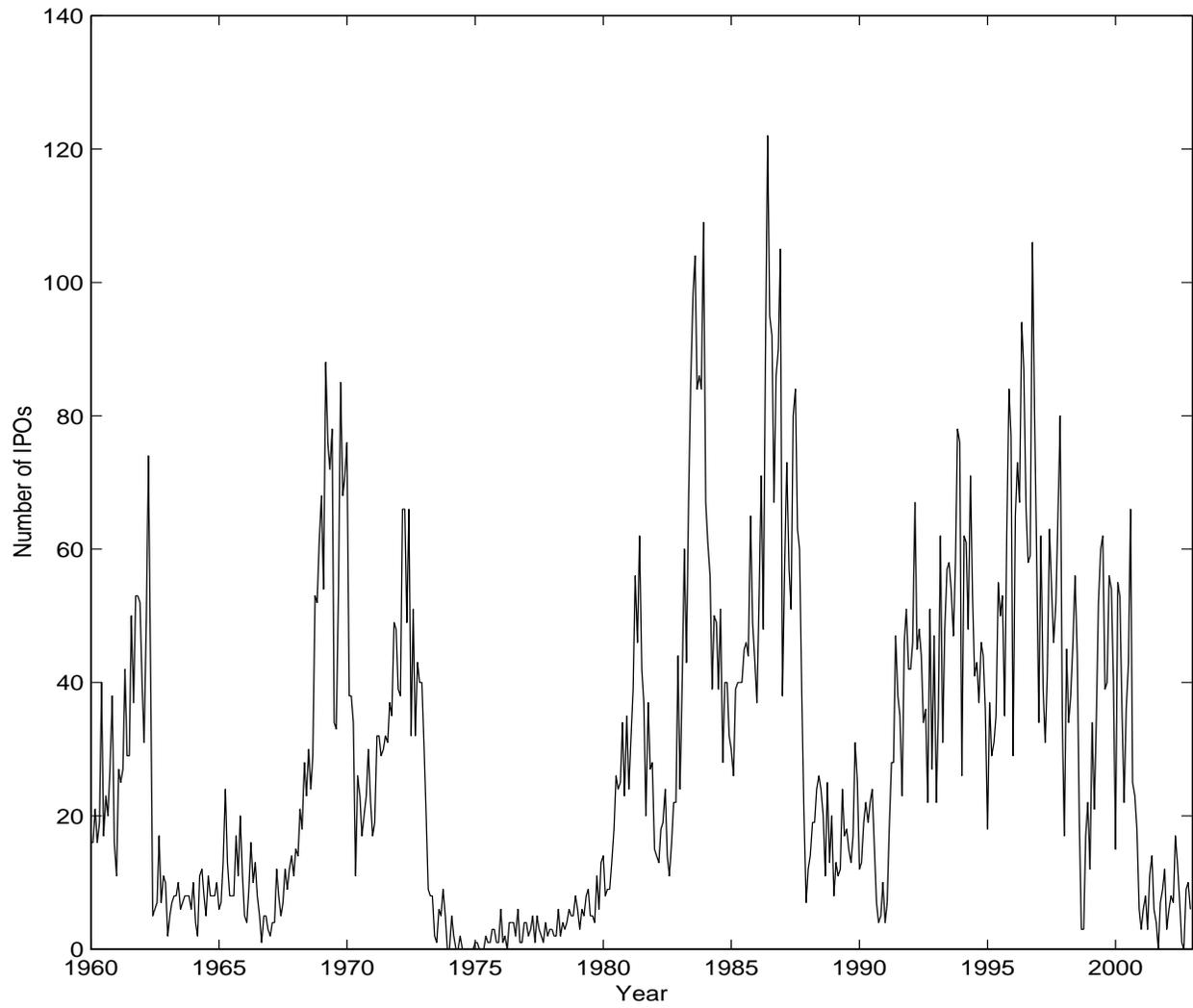
In their recent survey of the IPO literature, Ritter and Welch (2002) conclude that “market conditions are the most important factor in the decision to go public.” We agree, and we point out three dimensions of market conditions that appear especially relevant. Ritter and Welch also state that “perhaps the most important unanswered question is why issuing volume drops so precipitously following stock market drops.” Our model provides a simple answer. When market conditions worsen, stock prices drop and IPO volume declines as private firms choose to wait for more favorable market conditions.

This link between IPO volume and stock prices allows us to offer an asset pricing perspective on what has traditionally been a corporate finance topic. Our model implies that IPO waves should be preceded by high market returns, followed by low market returns, and

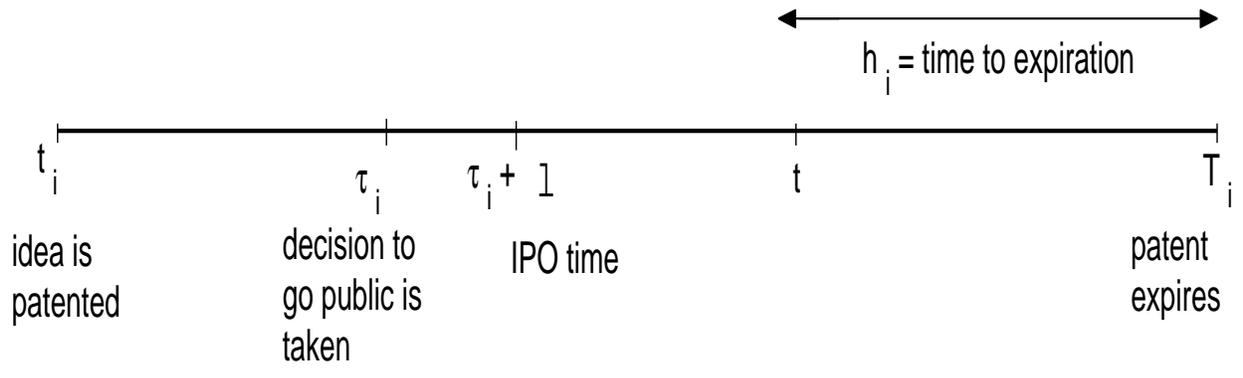
accompanied by increases in aggregate profitability. IPO waves should also be preceded by increased disparity between new firms and old firms in terms of their valuations and return volatilities. IPO volume should be related to changes in stock prices but less so to their levels. All of these implications are confirmed in the data.

The above implications are obtained simply as a result of time-varying market conditions, the most novel of which is prior uncertainty. We show that prior uncertainty helps explain high IPO valuations, and that its increases can lead to IPO waves. According to its proxies, prior uncertainty was unusually high in the “new era” period in the late 1990s. This high prior uncertainty may have attracted many firms to go public in the late 1990s, and it might also have contributed to the high valuations of many IPOs at that time.

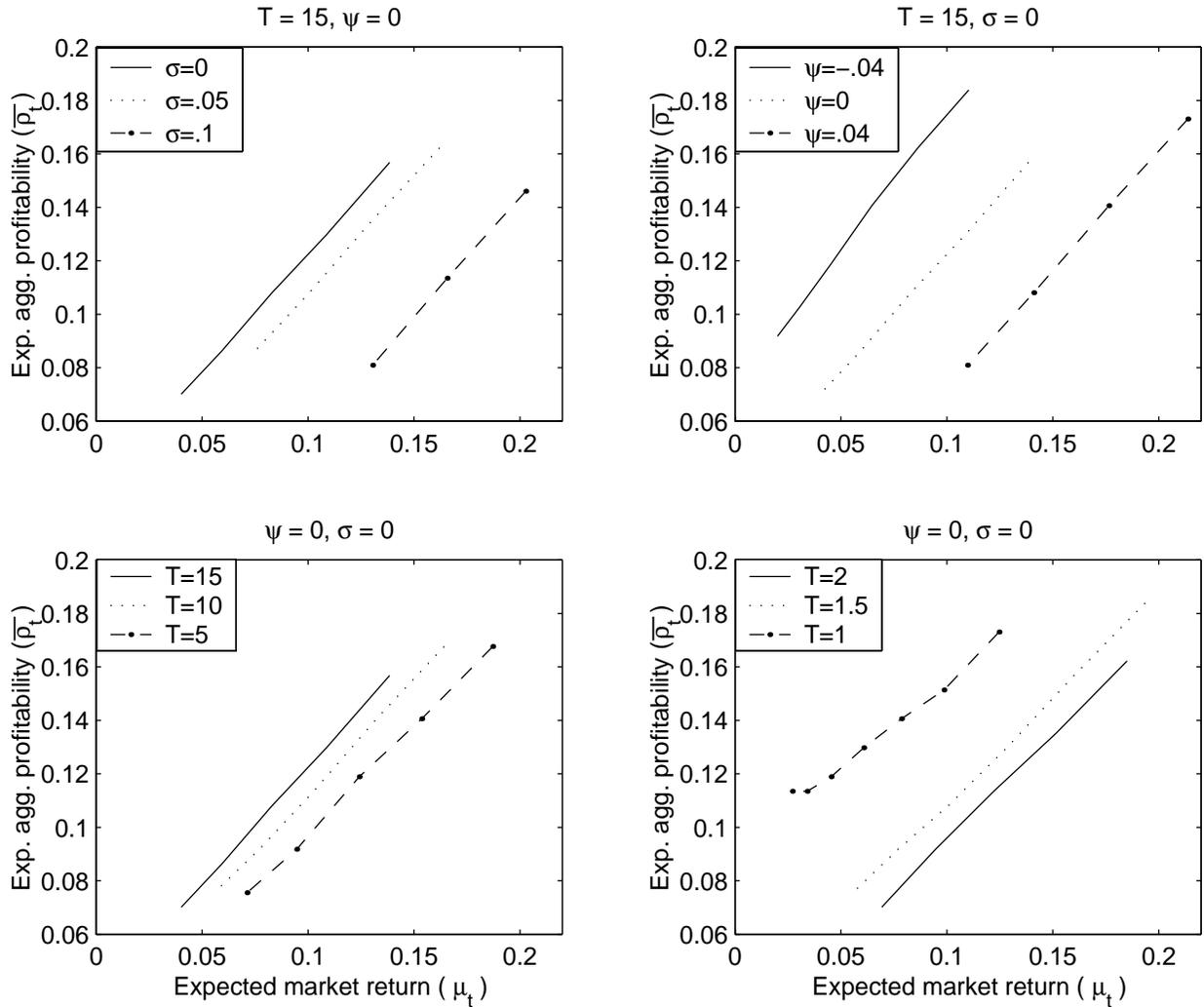
Another dimension of market conditions, expected market return, varies in our model due to time-varying risk aversion of our agents, but it might also covary with investor sentiment. Whether expected returns vary for rational or behavioral reasons, their implications for IPO volume are the same in our model, so we cannot rule out behavioral explanations for IPO volume. However, three of our empirical findings seem more in line with our rational model than with stories based on mispricing. High IPO volume tends to be accompanied by increased aggregate profitability as well as an increased difference between the volatilities of new firms and old firms. Also, IPO volume is significantly related to changes in the aggregate M/B but not to its level. All three facts are consistent with our model but not with the mispricing story in which firms go public in response to market overvaluation. Differentiating mispricing from rational variation in expected returns clearly merits more work. Numerous additional extensions of our model are discussed in Section 6.



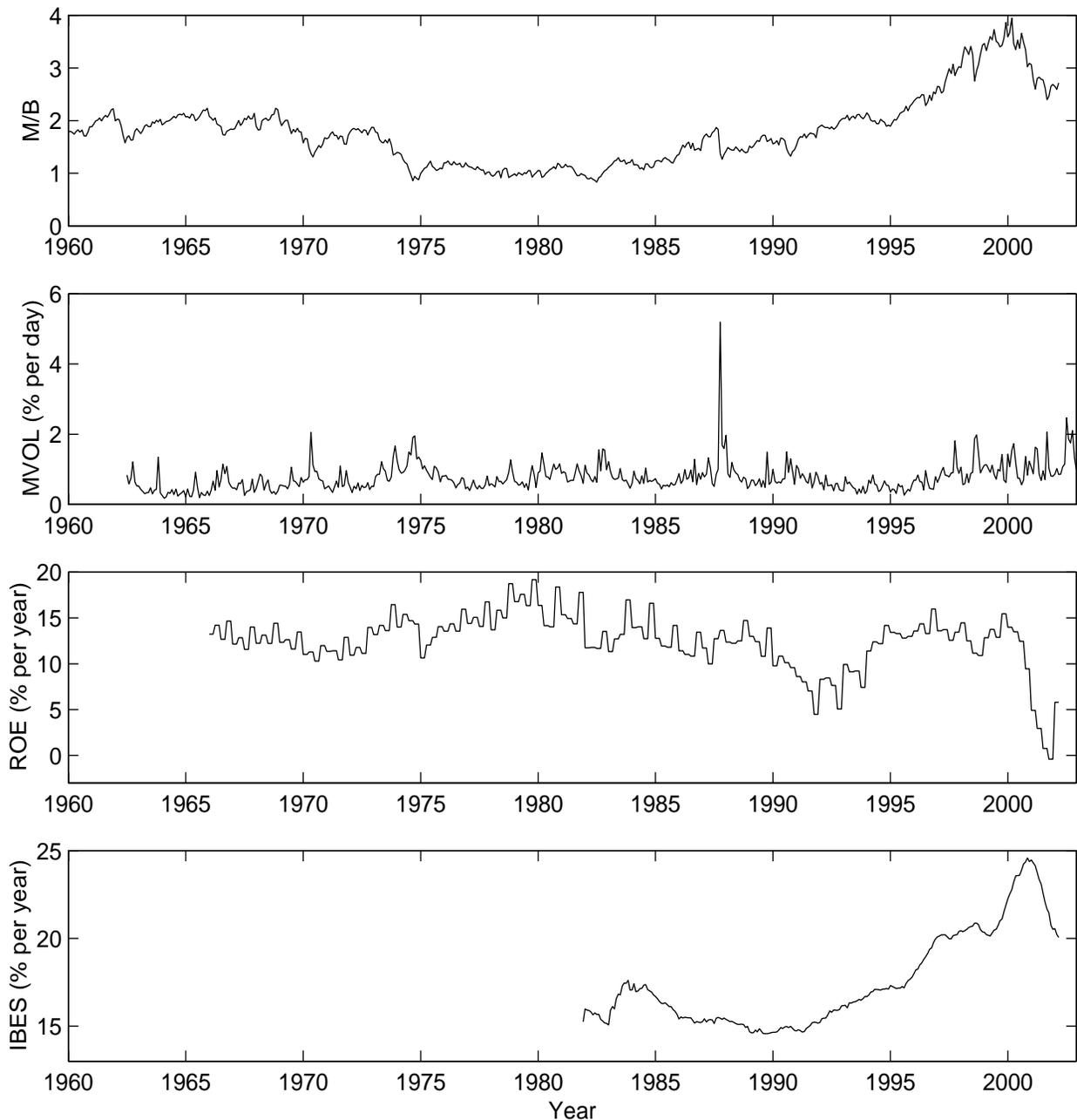
**Figure 1. IPO volume.** The figure plots the number of IPOs in each month between January 1960 and December 2002. The data is obtained from Jay Ritter's website.



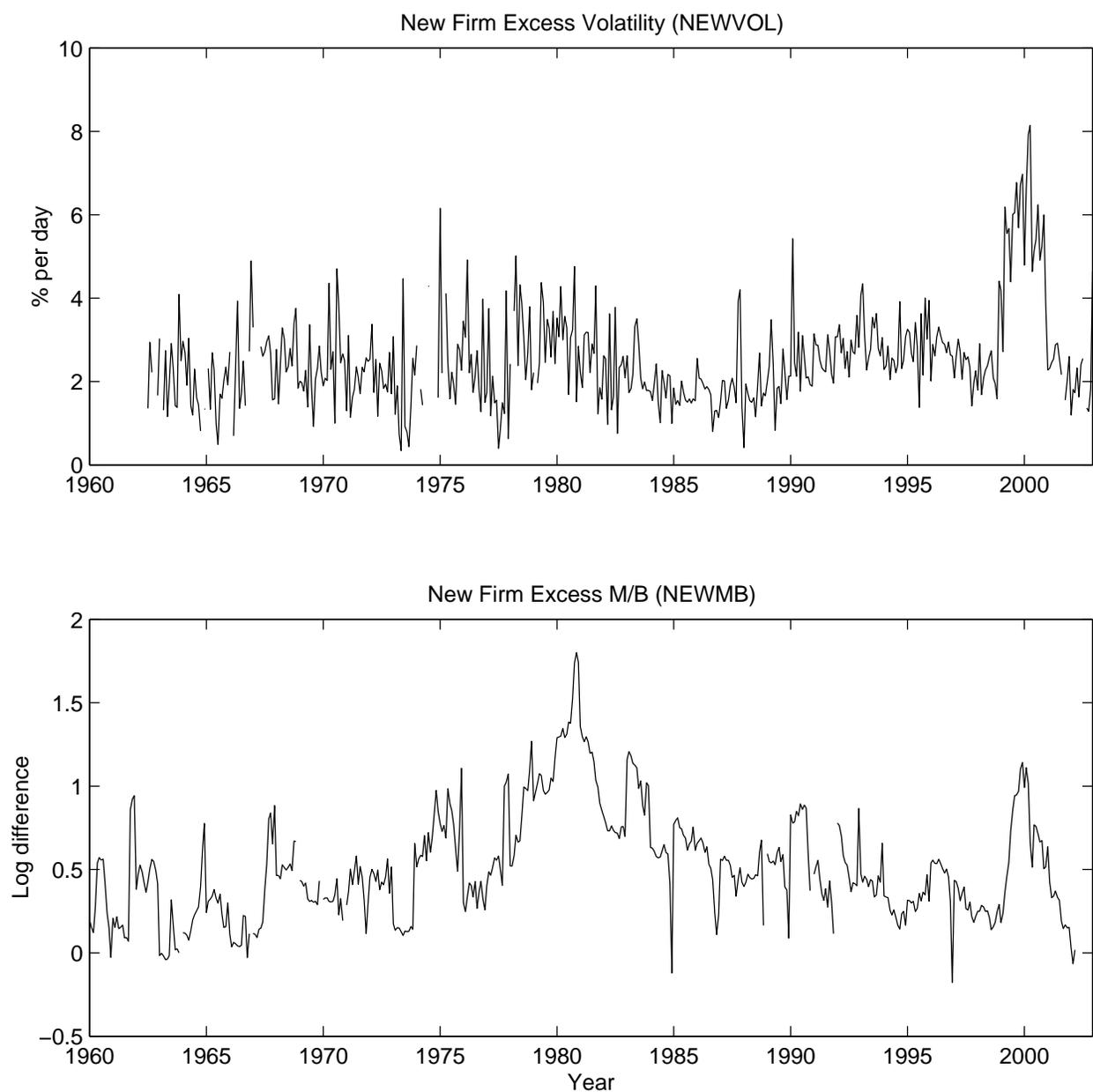
**Figure 2.** The timing of the events in our model.



**Figure 3. Optimal IPO Timing.** Each panel plots the entry boundary, the set of pairs of expected market return (horizontal axis) and expected aggregate profitability (vertical axis) for which the inventor optimally decides to go public. The entry boundaries are reported for three levels of prior uncertainty  $\hat{\sigma}_t$  in percent per year (top left panel), firm-specific excess profitability  $\hat{\psi}$  in percent per year (top right panel), and time to the patent's expiration in years (the bottom panels). An IPO takes place in the parameter region north-west of each boundary. Unless noted otherwise, the parameter values used to compute the optimal IPO timing decision are given in Table 1.



**Figure 4. Monthly time series of selected aggregate variables.** The top panel plots aggregate M/B (M/B), the sum of market values of equity across all firms divided by the sum of the most recent book values of equity. The second panel plots market return volatility (MVOL), the standard deviation of daily market returns within the month, which is available since July 1962. The third panel plots aggregate profitability (ROE), the sum across stocks of earnings in the current quarter divided by the sum of book values of equity at the end of the previous quarter. The monthly ROE series is created from the quarterly series for 1966Q1 through 2002Q1 by intrapolation. The bottom panel plots the average analyst long-term earnings growth forecast (IBES). For each firm, forecasts of long-term earnings growth are averaged across all analysts covering the firm, and IBES is computed as the average of such averages across firms. The IBES series is available for November 1981 through March 2002.



**Figure 5. Monthly time series of proxies for prior uncertainty.** The top panel plots NEWVOL, the median return volatility (standard deviation of daily returns) across all newly listed firms in excess of market return volatility. NEWVOL is available between July 1962 and December 2002. The bottom panel plots NEWMB, the log median M/B across all newly listed firms in excess of the log median M/B across all firms. NEWMB is available between January 1960 and March 2002.

**Table 1**  
**Parameter Values in the Calibrated Model.**

The table reports the parameter values used to calibrate our model. The parameters of the processes for expected aggregate profitability and consumption growth are estimated from the consumption and aggregate profitability data using the Kalman filter.  $\sigma_{L,0}$  is restricted to zero to eliminate correlation across the three state variables ( $\bar{\rho}_t$ ,  $y_t$  and  $\hat{\sigma}_t$ ). The parameters of the individual profitability process are calibrated to the median firm in our sample. The utility parameters ( $\eta$  and  $\gamma$ ), the parameters defining the log surplus consumption ratio  $s(y) = a_0 + a_1 y_t + a_2 y_t^2$ , and those characterizing the state variable  $y_t$  are calibrated to match the observed levels of the equity premium, market volatility, aggregate  $M/B$ , and the interest rate. The transition probabilities  $\lambda_{i,i\pm 1}$  that characterize the uncertainty process  $\hat{\sigma}_t$  on the grid  $\mathcal{V} = \{0, .1, \dots, .12\}$  are chosen to obtain plausible dynamics for  $\hat{\sigma}_t$ .  $\lambda_b$  denotes the transition probability at the boundaries of the grid. All entries are annualized.

Aggregate Profitability				Consumption Growth			Individual Profitability		
$k_L$	$\bar{\rho}_L$	$\sigma_{LL}$	$\sigma_{L,0}$	$b_0$	$b_1$	$\sigma_c$	$\phi^i$	$\sigma_{i,0}$	$\sigma_{i,i}$
0.1412	12.16 %	0.64 %	0	1.40 %	0.0812	0.94 %	0.3968	4.79 %	6.82 %
Utility		Surplus Consumption Ratio					Uncertainty		
$\eta$	$\gamma$	$k_y$	$\bar{y}$	$\sigma_y$	$a_0$	$a_1$	$a_2$	$\lambda_{i,i\pm 1}$	$\lambda_b$
0.0475	3.70	0.073	-0.0017	0.5156	-2.8779	0.2132	-0.0198	10%	20%
Unconditional Moments from Calibration									
$E[R_t^{mkt}]$	$\sigma(R_t^{mkt})$	$E[r_{f,t}]$	$\sigma(r_{f,t})$	$E[M/B]$	$\sigma(M/B)$	$E[\hat{\sigma}_t]$	$\sigma(\hat{\sigma}_t)$	$E[\bar{\rho}_t]$	$\sigma(\bar{\rho}_t)$
6.8%	15%	3.3%	3.9%	1.7	.614	6.11%	3.5%	12.1%	1.2%

**Table 2**  
**Simulation Evidence Around IPO Waves.**

The table reports averages of selected variables and market returns around simulated IPO waves, which are defined in the text. “b” stands for the beginning of an IPO wave, more precisely the end of the last month before the wave begins. “e” stands for the end of the wave’s last month. “b(e) $\pm$ n” denotes  $n$  months before or after the beginning (end) of a wave. A pre-wave is defined as the period that begins at the end of month b-3 and ends at the end of month e-3. Expected excess and total returns are computed for the market portfolio, the value-weighted portfolio of all existing simulated firms. Expected profitability stands for  $\bar{p}$ , and prior uncertainty stands for  $\hat{\sigma}$ . MVOL denotes market return volatility, M/B is the aggregate M/B ratio, RF is the risk-free rate, NEWVOL is the difference between the return volatility of a new firm and market volatility, and NEWMB is the log difference between the M/B of a new firm and the M/B of the market. All variables except for M/B and NEWMB are expressed in percent per year.

Panel A. Averages of first-column variables.

	Avg change in pre-wave	Before wave			Wave		After wave		Outside wave
		b-6	b-3	b	b+1:e	e	e+3	e+6	
Expected total return	-0.99	10.50	10.03	9.23	8.75	9.27	9.31	9.33	9.93
Expected profitability	0.06	12.12	12.14	12.18	12.21	12.17	12.17	12.17	12.16
Prior uncertainty	0.33	5.10	5.12	5.41	5.52	5.41	5.44	5.43	5.73
Expected excess return	-0.46	7.45	7.26	6.88	6.66	6.90	6.90	6.90	6.79
RF	-0.53	3.05	2.77	2.34	2.08	2.37	2.40	2.43	3.15
M/B	0.11	1.59	1.64	1.73	1.78	1.73	1.72	1.72	1.76
MVOL	-0.47	16.22	16.06	15.67	15.45	15.69	15.68	15.66	15.22
NEWVOL	2.34	43.62	43.30	45.46	46.18	45.48	45.77	45.74	48.73
NEWMB	0.07	0.22	0.24	0.30	0.33	0.30	0.30	0.30	0.29

Panel B. Average realized market returns.

	Pre-wave	Outside	Before wave		Wave	After wave		Outside
	b-2:e-3	pre-wave	b-5:b-3	b-2:b	b+1:e	e+1:e+3	e+4:e+6	wave
Total return	40.30	7.05	21.99	32.38	8.99	9.10	9.59	10.27
Excess return	37.97	3.92	18.99	29.84	6.91	6.72	7.18	7.12

**Table 3**  
**Simulation Evidence: Regressions of IPO Volume on Selected Variables.**

Each column represents a quarterly regression of IPO volume on the variables listed in the first column. All variables are taken from a 10,000-year-long sample simulated from our calibrated model. No  $t$ -statistics are given because all reported numbers are highly significant. “ $\Delta$ ” denotes changes (first differences), and “ $-n$ ” (“ $+n$ ”) denotes quarterly lags (leads). ER denotes expected total market return, and all other variables are defined in Table 2. The units were chosen to ensure some significant digits for all coefficients in the table: IPO volume is measured as the number of firms that went public this quarter, MKT is measured in decimals per month, and all other variables except for the unitless M/B and NEWMB are in percent per year.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	2.25	2.21	2.22	2.03	2.22	2.23	0.69	2.22	2.22
$\Delta$ ER-2	-0.24								
$\Delta$ ER-1	-0.65								
$\Delta\bar{\rho}$ -2		0.05							
$\Delta\bar{\rho}$ -1		0.46							
$\Delta\hat{\sigma}$ -2			0.08						
$\Delta\hat{\sigma}$ -1			0.34						
MKT-2				4.30					
MKT-1				9.92					
MKT				-1.50					
MKT+1				-1.78					
MKT+2				-1.53					
$\Delta$ MVOL-2					-0.15				
$\Delta$ MVOL-1					-0.98				
$\Delta$ RF-2						-0.25			
$\Delta$ RF-1						-0.87			
M/B-1							0.93		
$\Delta$ NEWMB-2								0.07	
$\Delta$ NEWMB-1								6.54	
$\Delta$ NEWVOL-2									0.00
$\Delta$ NEWVOL-1									0.03
IPO( $t-1$ )	0.19	0.20	0.20	0.18	0.20	0.20	0.16	0.20	0.20
$T$	40000	40000	40000	40000	40000	40000	40000	40000	40000
$R^2$	0.11	0.04	0.05	0.12	0.08	0.10	0.07	0.08	0.04

**Table 4**  
**IPO Waves Observed in the Data**

The table reports some summary statistics for the 16 IPO waves observed in our sample period of January 1960 through December 2002. IPO waves and pre-waves are defined in the text. "Avg pre-wave market return" for a given wave denotes the average monthly total market return during the respective pre-wave. MVOL denotes market return volatility, M/B is the aggregate M/B ratio, RF is the risk-free rate, ROE is aggregate profitability (return on equity), IBES is the average analyst forecast of long-term earnings growth, NEWVOL is the difference between the median return volatility of new firms and market volatility, and NEWMB is the log difference between the median M/B of new firms and the median M/B across all firms. All variables except for M/B and NEWMB are expressed in percent per year. n/a denotes values that are not available due to missing data.

	Beginning of wave	End of wave	Number of IPOs	Avg pre-wave market return	Pre-wave change in						
					MVOL	M/B	RF	ROE	IBES	NEWVOL	NEWMB
1	196108	196205	480	10.56	n/a	0.02	0.22	n/a	n/a	n/a	0.33
2	196810	197002	1061	0.07	-1.33	-0.26	-0.37	0.34	n/a	11.90	-0.07
3	197110	197207	488	13.47	-4.21	0.11	0.64	0.36	n/a	3.40	-0.15
4	197209	197209	32	-2.70	-4.09	-0.04	0.10	0.00	n/a	24.32	-0.12
5	197211	197211	40	19.02	-1.19	0.06	-0.08	0.00	n/a	-7.98	-0.02
6	198103	198107	245	16.16	-7.72	-0.06	2.81	-3.45	n/a	31.90	-0.51
7	198302	198407	1223	22.72	-17.14	0.15	1.45	0.48	1.71	0.38	-0.18
8	198507	198511	249	12.78	-1.39	0.01	-0.15	-0.96	-0.29	-0.82	-0.19
9	198601	198709	1517	27.69	-0.58	0.57	-2.22	-1.82	-0.75	6.11	-0.13
10	199111	199112	93	25.04	-2.65	0.03	-0.42	0.00	0.26	9.88	0.05
11	199202	199205	206	23.86	-0.30	0.17	0.74	3.83	0.19	22.81	0.46
12	199304	199406	828	6.85	3.88	-0.02	-1.02	6.33	0.79	0.64	-0.54
13	199507	199507	50	31.20	-1.99	-0.01	0.79	-0.12	-0.05	-3.66	0.01
14	199510	199612	1068	22.15	-0.80	0.34	-0.09	-0.01	2.21	-3.06	0.23
15	199710	199711	145	33.76	1.52	0.03	-0.31	-1.17	-0.11	-1.93	-0.14
16	199906	199907	122	19.60	-4.16	0.26	0.11	0.91	-0.08	54.14	0.21

**Table 5**  
**Empirical Evidence Around IPO Waves**

The table reports averages of selected variables and market returns around IPO waves, which are defined in the text. “b” stands for the beginning of an IPO wave, more precisely the end of the last month before the wave begins. “e” stands for the end of the wave’s last month. “b(e) $\pm n$ ” denotes  $n$  months before or after the beginning (end) of a wave. A pre-wave is defined as the period that begins at the end of month b-3 and ends at the end of month e-3. MVOL denotes market return volatility, M/B is the aggregate M/B ratio, RF is the risk-free rate, ROE is aggregate profitability (return on equity), IBES is the average analyst forecast of long-term earnings growth, NEWVOL is the difference between the median return volatility of new firms and market volatility, and NEWMB is the log difference between the median M/B of new firms and the median M/B across all firms. All variables except for M/B and NEWMB are expressed in percent per year. The  $t$ -statistics, reported in parentheses, assess the significance of the difference between the variable’s averages in the given period and outside that period.

Panel A. Averages of selected variables.

	Avg change in pre-wave	Before wave			Wave		After wave		Outside wave
		b-6	b-3	b	b+1:e	e	e+3	e+6	
MVOL	-2.81 (-2.27)	12.36 (-1.14)	13.66 (-0.48)	12.70 (-0.97)	12.67 (-3.18)	12.73 (-0.96)	15.63 (0.54)	15.00 (0.21)	15.24 (3.18)
M/B	0.08 (1.79)	1.78 (-0.06)	1.87 (0.50)	1.94 (0.96)	1.81 (0.50)	1.93 (0.89)	1.90 (0.72)	1.97 (1.15)	1.78 (-0.50)
RF	0.14 (0.50)	1.46 (0.05)	1.55 (0.22)	1.77 (0.65)	2.22 (5.13)	2.37 (1.84)	1.95 (1.02)	2.06 (1.23)	1.17 (-5.13)
ROE	0.32 (0.54)	11.69 (-1.02)	11.59 (-1.15)	11.71 (-1.00)	12.24 (-1.01)	12.11 (-0.48)	13.10 (0.81)	12.02 (-0.60)	12.57 (1.01)
IBES	0.39 (1.32)	16.93 (-0.66)	16.88 (-0.72)	16.86 (-0.74)	16.81 (-2.90)	17.45 (-0.03)	17.58 (0.12)	17.57 (0.12)	17.83 (2.90)
NEWVOL	9.87 (2.27)	46.92 (-0.24)	42.26 (-1.06)	47.65 (-0.11)	44.82 (-1.97)	47.19 (-0.19)	50.12 (0.34)	49.87 (0.28)	49.46 (1.97)
NEWMB	-0.05 (-0.69)	0.44 (-0.91)	0.56 (0.50)	0.55 (0.41)	0.55 (1.18)	0.46 (-0.67)	0.49 (-0.31)	0.46 (-0.74)	0.51 (-1.18)

Panel B. Average realized market returns.

	Pre-wave	Outside	Before wave		Wave	After wave		Outside
	b-2:e-3	pre-wave	b-5:b-3	b-2:b	b+1:e	e+1:e+3	e+4:e+6	wave
Total return	15.20 (1.11)	9.13 (-1.11)	31.17 (2.77)	21.45 (1.45)	9.51 (-0.28)	0.49 (-1.35)	18.86 (1.09)	11.03 (0.28)
Excess return	9.54 (1.12)	3.44 (-1.12)	25.83 (2.81)	15.74 (1.44)	3.83 (-0.28)	-5.35 (-1.36)	13.42 (1.12)	5.35 (0.28)

**Table 6a**  
**Empirical Evidence: Regressions of IPO Volume on Selected Variables**

Each column represents a quarterly regression of IPO volume on the variables listed in the first column. “ $\Delta$ ” denotes changes (first differences), and “ $-n$ ” (“ $+n$ ”) denotes quarterly lags (leads). All variables are defined in Table 5. Their units were chosen to ensure some significant digits for all coefficients in the table: Scaled IPO volume is measured in percent per month, MKT in decimals per month, MVOL in percent per day, and RF in percent per month. The  $t$ -statistics, given in parentheses, are computed using standard errors that are robust to heteroskedasticity and serial correlation of residuals (Newey-West with five lags).

	(1)	(2)	(3)	(4)	(5)
Intercept	0.23 (3.03)	0.31 (4.48)	0.33 (4.76)	0.33 (2.51)	0.28 (2.02)
MKT-2	1.67 (3.25)				1.68 (3.23)
MKT-1	2.09 (3.34)				2.11 (3.27)
MKT	2.06 (4.51)				2.03 (4.50)
MKT+1	-0.95 (-2.23)				-0.98 (-2.25)
MKT+2	-0.49 (-0.74)				-0.52 (-0.77)
$\Delta$ MVOL-2		-0.31 (-1.91)			
$\Delta$ MVOL-1		-0.63 (-3.59)			
$\Delta$ MVOL		-0.60 (-4.41)			
$\Delta$ RF-2			1.10 (3.10)		
$\Delta$ RF-1			0.28 (1.21)		
$\Delta$ RF			0.91 (2.53)		
M/B-1				0.01 (0.12)	-0.03 (-0.44)
IPO(t-1)	0.84 (23.09)	0.87 (21.47)	0.84 (20.37)	0.84 (19.21)	0.84 (22.75)
Q1 Dummy	-0.48 (-4.92)	-0.42 (-4.52)	-0.38 (-4.04)	-0.42 (-4.91)	-0.43 (-4.28)
$T$	169	159	171	169	157
$R^2$	0.78	0.75	0.73	0.72	0.79

**Table 6b**  
**Empirical Evidence: Regressions of IPO Volume on Selected Variables**

Each column represents a quarterly regression of IPO volume on the variables listed in the first column. “ $\Delta$ ” denotes changes (first differences), and “ $-n$ ” (“ $+n$ ”) denotes quarterly lags (leads). All variables are defined in Table 5. Their units were chosen to ensure some significant digits for all coefficients in the table: Scaled IPO volume is measured in percent per month, ROE in percent per month, IBES in percent per year, and NEWVOL in percent per day. The  $t$ -statistics, given in parentheses, are computed using standard errors that are robust to heteroskedasticity and serial correlation of residuals (Newey-West with five lags).

	(6)	(7)	(8)	(9)	(10)
Intercept	0.34 (3.92)	0.57 (5.04)	0.31 (4.33)	0.37 (4.67)	0.37 (3.77)
$\Delta$ ROE	0.90 (2.50)				0.19 (0.46)
$\Delta$ ROE+1	0.55 (1.43)				0.31 (0.76)
$\Delta$ ROE+2	0.64 (1.90)				1.00 (2.40)
$\Delta$ IBES-2		-0.16 (-0.87)			
$\Delta$ IBES-1		-0.43 (-1.37)			
$\Delta$ IBES		0.78 (5.07)			
$\Delta$ NEWMB-2			0.46 (2.35)		0.48 (2.28)
$\Delta$ NEWMB-1			0.52 (3.18)		0.62 (2.78)
$\Delta$ NEWMB			0.11 (0.59)		-0.11 (-0.49)
$\Delta$ NEWVOL-2				0.12 (2.23)	0.12 (2.87)
$\Delta$ NEWVOL-1				0.03 (0.57)	-0.00 (-0.09)
$\Delta$ NEWVOL				0.01 (0.19)	-0.04 (-0.94)
IPO( $t-1$ )	0.84 (18.30)	0.79 (11.16)	0.87 (19.79)	0.84 (18.75)	0.86 (16.56)
Q1 Dummy	-0.26 (-2.22)	-0.67 (-4.59)	-0.51 (-5.40)	-0.43 (-4.42)	-0.46 (-3.26)
$T$	142	79	136	144	105
$R^2$	0.72	0.70	0.76	0.71	0.77

## 8. Appendix

### (A) Data

Aggregate consumption data is obtained quarterly from NIPA. Consumption is defined as real per capita consumption expenditures on non-durables plus services, seasonally adjusted. The series is deflated by the personal consumption expenditure deflator (PCE), also taken from NIPA.

The following data is obtained from CRSP and Compustat. Quarterly aggregate profitability (ROE) is computed as the sum across stocks of earnings in the current quarter divided by the sum of book values of equity at the end of the previous quarter. Quarterly earnings, which are generally available from 1966Q1, denote income before extraordinary items available for common (Compustat item 25) plus deferred taxes from the income account (item 35, if available). If either value is indicated as .A (annual) or .S (semi-annual) in the quarterly file, these values are divided by four (if .A) or two (if .S). When quarterly book equity is missing, it is replaced by the most recent annual book equity. Following Fama and French (1993), annual book equity is constructed as stockholders' equity plus balance sheet deferred taxes and investment tax credit (item 35) minus the book value of preferred stock. Depending on availability, stockholder's equity is computed as Compustat item 216, or 60+130, or 6-181, in that order, and preferred stock is computed as item 56, or 10, or 130, in that order. Quarterly book equity, which is generally available from 1972Q1, is constructed analogously. Stockholders' equity is item 60, or 59+55, or 44-54, preferred stock is item 55, and deferred taxes and tax credit is item 52. If the quarterly values are indicated as .A (annual) or .S (semi-annual) in the SAS datafile, the respective annual or semiannual values are used. Monthly ROE values are intrapolated from quarterly values. Market equity is computed monthly by multiplying the common stock price by common shares outstanding, both obtained from CRSP. M/B ratio is computed as market equity divided by book equity from the most recent quarter. We eliminate the values of market equity and book equity smaller than \$1 million, as well as M/B ratios smaller than 0.01 and larger than 100. All variables that require Compustat data (e.g. ROE, M/B) are constructed through the end of 2002Q1.

### (B) Preferences and the Stochastic Discount Factor

This appendix describes the properties of the process of log surplus consumption

$$\log(S_t) \equiv s_t \equiv s(y_t) = a_0 + a_1 y_t + a_2 y_t^2 \quad (12)$$

The process for  $y_t$  implies a normal unconditional distribution for  $y_t$  with mean  $\bar{y}$  and variance  $\sigma_{y,0}^2/2k_y$ . Let  $y_D = \bar{y} - 4\sigma_y/\sqrt{2k_y}$  and  $y_U = \bar{y} + 4\sigma_y/\sqrt{2k_y}$  be the boundaries between which  $y_t$  lies 99.9% of the time. To ensure that log surplus  $s_t$  conforms to the economic intuition of a habit formation model, we impose the following parametric restrictions:  $a_2 < 0$ ,  $a_1 > -2a_2 y_U$  and  $a_0 < 1/4(a_1^2/a_2)$ . These restrictions ensure that for all  $t$ ,  $s_t < 0$ , and thus  $S_t \in (0, 1)$ , and that  $s(y)$  is increasing in  $y$  for all  $y \in [y_D, y_U]$ . The process for log surplus is given by

$$ds_t = \mu_s(y) dt + \sigma_s(y) dW_{0,t} \quad (13)$$

with

$$\begin{aligned} \mu_s(y) &= k_y(\bar{y} - y_t)(a_1 + 2a_2 y) + a_2 \sigma_y^2 \\ \sigma_s(y) &= (a_1 + 2a_2 y) \sigma_y \end{aligned}$$

The restrictions above imply that  $\sigma_s(y)$  is positive and decreasing in  $y$ , for all  $y \in [y_D, y_U]$ . Since  $s$  increases with  $y$  in the relevant range, surplus is perfectly correlated with innovations to aggregate consumption, and its volatility is higher for low surplus levels.

Given the dynamics of consumption in (7) and log surplus in (13), the process for the stochastic discount factor  $\pi_t = U_C(C_t, X_t, t) = e^{-\eta t} (C_t S_t)^{-\gamma} = e^{-\eta t - \gamma(c_t + s_t)}$  is given by

$$d\pi_t = -r_t \pi_t dt - \pi_t \sigma_{\pi,t} dW_{0,t}$$

where

$$r_t = R_0 + R_1 \bar{\rho}_t + R_2 y_t + R_3 y_t^2$$

with

$$\begin{aligned} R_0 &= \eta + \gamma b_0 + \gamma a_1 k_y \bar{y} - \frac{1}{2} \gamma^2 \sigma_c^2 + (\gamma a_2 - \frac{1}{2} \gamma^2 a_1^2) \sigma_y \sigma'_y - \gamma^2 a_1 \sigma_c \sigma'_y \\ R_1 &= \gamma b_1 \\ R_2 &= \gamma (2a_2 k_y \bar{y} - a_1 k_y - \gamma a_2 (2\sigma_c \sigma'_y + a_1 2\sigma_y \sigma'_y)) \\ R_3 &= 2a_2 \gamma (-k_y - \gamma a_2 \sigma_y \sigma'_y) \end{aligned}$$

and

$$\sigma_{\pi,t} = \gamma (\sigma_c + (a_1 + 2a_2 y_t) \sigma_y)$$

The parameter restrictions imposed earlier imply that  $\sigma_{\pi,t}$  decreases as  $y_t$  (and hence also the surplus  $S_t$ ) increases. As a result, expected returns and return volatility are low when  $y_t$  is high.

### (C) Learning

**Lemma 1.** Suppose the prior of  $\bar{\psi}^i$  at time  $t_0$  is normal,  $\bar{\psi}^i \sim \mathcal{N}(\hat{\psi}_{t_0}^i, \hat{\sigma}_{t_0}^2)$ , and the priors are uncorrelated across firms. Let  $\mathcal{I}_t$  denote the set of firms that are alive at time  $t$ . Then the posterior of  $\bar{\psi}^i$  at any time  $t > t_0$  conditional on  $\mathcal{F}_t = \left\{ (\rho_s^j, c_s, \bar{\rho}_s) : t_0 \leq s \leq t, j \in \mathcal{I}_t \right\}$  is also normal,  $\bar{\psi}^i | \mathcal{F}_t \sim \mathcal{N}(\hat{\psi}_t^i, \hat{\sigma}_{i,t}^2)$ , where

(a) The mean squared error  $\hat{\sigma}_{i,t}^2 = E \left[ (\bar{\psi}^i - \hat{\psi}_t^i)^2 | \mathcal{F}_t \right]$  is non-stochastic and given by

$$\hat{\sigma}_{i,t}^2 = \frac{1}{\frac{1}{\hat{\sigma}_{t_0}^2} + \frac{(\phi^i)^2}{\sigma_{i,i}^2} (t - t_0)} \quad (14)$$

Note that the uncertainty about  $\bar{\psi}^i$  declines deterministically over time due to learning.

(b) The conditional mean  $\hat{\psi}_t^i = E[\bar{\psi}^i | \mathcal{F}_t]$  evolves according to the process

$$d\hat{\psi}_t^i = \hat{\sigma}_{i,t}^2 \frac{\phi^i}{\sigma_{i,i}} d\widetilde{W}_{i,t}, \quad (15)$$

where  $\widetilde{W}_{i,t}$  is the idiosyncratic component of the Wiener process capturing the agents' perceived expectation errors (see equation (16) below).

*Proof:* Consider the vector  $\mathbf{Z}_t = (c_t, \bar{\rho}_t, \rho_t^1, \dots, \rho_t^n)$  of signals to identify the unobservable variables, stacked in another vector  $\bar{\psi} = (\bar{\psi}^1, \dots, \bar{\psi}^n)'$ . The assumptions in the text imply

$$d\mathbf{Z}_t = (\mathbf{A} + \mathbf{B}\mathbf{Z}_t + \mathbf{C}\bar{\psi}) dt + \mathbf{b}d\mathbf{W}_t$$

where  $\mathbf{W}_t = (W_{0,t}, W_{L,t}, W_{1,t}, \dots, W_{n,t})$  and

$$\mathbf{A} = \begin{pmatrix} b_0 \\ k_L \bar{\rho}_L \\ \vdots \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 & b_1 & 0 & 0 & 0 \\ 0 & -k_L & 0 & 0 & 0 \\ 0 & \phi^1 & -\phi^1 & 0 & 0 \\ 0 & \vdots & & \ddots & 0 \\ 0 & \phi^n & 0 & 0 & -\phi^n \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ \phi^1 & & \\ & \ddots & \\ & & \phi^n \end{pmatrix}, \mathbf{b} = \begin{pmatrix} \sigma_c & 0 & \cdots & \cdots & \cdots & 0 \\ \sigma_{L,0} & \sigma_{L,L} & 0 & \cdots & \cdots & 0 \\ \sigma_{0,1} & 0 & \sigma_{1,1} & & & \\ \sigma_{0,2} & \vdots & & \sigma_{2,2} & & \\ \vdots & \vdots & & & \ddots & \\ \sigma_{0,n} & 0 & & & & \sigma_{n,n} \end{pmatrix}$$

From Liptser and Shirayev (1977, Ch. 10.3), the posterior of  $\bar{\psi}$  is given by  $\bar{\psi} \sim \mathcal{N}(\hat{\psi}_t, \hat{\Sigma}_t)$ , where  $d\hat{\psi}_t = \tilde{\Sigma}_t d\tilde{\mathbf{W}}_t$  with  $\tilde{\Sigma}_t = \hat{\Sigma}_t \mathbf{C}'(\mathbf{b}')^{-1}$ ,  $\frac{d\tilde{\Sigma}_t}{dt} = -\tilde{\Sigma}_t \tilde{\Sigma}_t'$ , and

$$d\tilde{\mathbf{W}}_t = \mathbf{b}^{-1} \{d\mathbf{Z}_t - E[d\mathbf{Z}_t | \mathcal{F}_t]\} = \mathbf{b}^{-1} \{d\mathbf{Z}_t - [\mathbf{A} + \mathbf{B}\mathbf{Z}_t + \mathbf{C}\hat{\psi}_t] dt\} \quad (16)$$

is a Brownian motion with respect to  $\mathcal{F}_t$ . The claim is proved using the fact that  $\tilde{\Sigma}_t$  is diagonal. See Pástor and Veronesi (2003b). Note that this lemma is similar to Lemma 1 in PV. ■

## (D) Pricing

**Lemma 2:** Let  $\tilde{b}_t$  follow the process

$$d\tilde{b}_t = (\zeta_0 \bar{\rho}_t + \zeta_1 \rho_t^i - \zeta_2) dt,$$

where  $\rho_t^i$  and  $\bar{\rho}_t$  follow the processes in equations (1) and (2), and  $\zeta_i$  are constants. Define  $\mathbf{Y}_t = (v\tilde{b}_t - \gamma c_t, y_t, \bar{\rho}_t, \rho_t^i, \hat{\psi}_t^i)'$  and  $g(\mathbf{Y}_T) = e^{Y_{1,T} - \gamma a_1 Y_{2,T} - \gamma a_2 Y_{2,T}^2}$ , where  $v$  is a constant,  $Y_{i,t}$  denotes the  $i$ -th element of  $\mathbf{Y}_t$ , and  $\gamma$ ,  $a_1$ , and  $a_2$  are taken from equations (3) and (4). Then

$$E_t [e^{-\eta(T-t)} g(\mathbf{Y}_T)] \equiv H(\mathbf{Y}_t, t) = e^{K_0(t;T) + \mathbf{K}(t;T)' \cdot \mathbf{Y}_t + K_6(t;T) Y_{2,t}^2} \quad (17)$$

where  $K_0(t;T)$ ,  $\mathbf{K}(t;T) = (K_1(t;T), \dots, K_5(t;T))'$ , and  $K_6(t;T)$  satisfy a system of ordinary differential equations (ODE)

$$\frac{dK_6(t;T)}{dt} = -2K_6^2(t;T) \sigma_y^2 + 2K_6(t;T) k_y \quad (18)$$

$$\left( \frac{d\mathbf{K}(t;T)}{dt} \right)' = -\mathbf{K}(t;T)' \cdot [\mathbf{B}_Y + 2K_6(t;T) [\Sigma_{Y,t} \Sigma_{Y,t}' ]_2 \mathbf{e}_2] - 2K_6(t;T) k_y \bar{y} \mathbf{e}_2 \quad (19)$$

$$\frac{dK_0(t;T)}{dt} = \eta - \mathbf{K}(t;T)' \cdot \mathbf{A}_Y - \frac{1}{2} \mathbf{K}(t;T)' \Sigma_{Y,t} \Sigma_{Y,t}' \mathbf{K}(t;T) - K_6(t;T) \sigma_y^2 \quad (20)$$

subject to the final condition  $K_6(T; T) = -\gamma a_2$ ,  $\mathbf{K}(T; T) = (1, -\gamma a_1, 0, 0, 0)$ , and  $K_0(T; T) = 0$ . In the above,  $\mathbf{e}_2 = (0, 1, 0, \dots, 0)$  and

$$\mathbf{A}_Y = \begin{pmatrix} -\gamma b_0 - v\zeta_2 \\ k_y \bar{y} \\ k_L \bar{\rho}_L \\ 0 \\ 0 \end{pmatrix}; \mathbf{B}_Y = \begin{pmatrix} 0 & 0 & -\gamma b_1 + v\zeta_0 & v\zeta_1 & 0 \\ 0 & -k_y & 0 & 0 & 0 \\ 0 & 0 & -k_L & 0 & 0 \\ 0 & 0 & \phi^i & -\phi^i & \phi^i \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}; \Sigma_{Y,t} = \begin{pmatrix} -\gamma\sigma_c & 0 & 0 \\ \sigma_y & 0 & 0 \\ \sigma_{L,0} & \sigma_{L,L} & 0 \\ \sigma_{i,0} & 0 & \sigma_{i,i} \\ 0 & 0 & \frac{\phi^i}{\sigma_{i,i}} \widehat{\sigma}_{i,t}^2 \end{pmatrix}.$$

*Proof:* From the definition of the vector  $\mathbf{Y}_t$ , we have

$$d\mathbf{Y}_t = (\mathbf{A}_Y + \mathbf{B}_Y \mathbf{Y}_t) dt + \Sigma_{Y,t} d\widetilde{\mathbf{W}}_t$$

The Feynman-Kac theorem implies that  $H(\mathbf{Y}_t, t)$  from (17) solves the partial differential equation

$$\frac{\partial H}{\partial t} + \sum_{i=1}^5 \left( \frac{\partial H}{\partial Y_i} \right) [\mathbf{A}_Y + \mathbf{B}_Y \mathbf{Y}_t]_i + \frac{1}{2} \sum_{i=1}^5 \sum_{j=1}^5 \frac{\partial^2 H}{\partial Y_i \partial Y_j} [\Sigma_{Y,t} \Sigma'_{Y,t}]_{ij} = \eta H \quad (21)$$

subject to the boundary condition

$$H(\mathbf{Y}_T, T) = g(\mathbf{Y}_T) \quad (22)$$

It can be easily verified that the exponential quadratic function (17) indeed satisfies (21) subject to (22), as long as  $K_0(t; T)$ ,  $\mathbf{K}(t; T)$ , and  $K_6(t; T)$  are the solutions to the system of ODEs in (18) through (20) under the final conditions presented in the claim of the Lemma. ■

**Proposition 1.** Let  $h_i = T_i - t$  be the time to expiration of the patent of public firm  $i$ . Then

(a) The firm's ratio of market value of equity to book value of equity is given by

$$\frac{M_t^i}{B_t^i} = Z^i \left( y_t, \bar{\rho}_t, \rho_t^i, \widehat{\psi}_t^i, h_i \right) = e^{Q_0(h_i; \widehat{\sigma}_{i,t}^2) + \mathbf{Q}(h_i)' \cdot \mathbf{N}_t + Q_5(h_i) y_t^2}, \quad (23)$$

where  $\mathbf{N}_t = \left( y_t, \bar{\rho}_t, \rho_t^i, \widehat{\psi}_t^i \right)$  is the vector of state variables characterizing firm  $i$ ,  $Q_0(h_i; \widehat{\sigma}_{i,t}^2) = K_0(T_i - h_i; T_i)$ ,  $Q_1(h_i) = K_2(T_i - h_i; T_i) + \gamma a_1$ ,  $Q_i(h_i) = K_{i+1}(T_i - h_i; T_i)$  for  $i = 2, \dots, 4$ ,  $Q_5(h_i) = K_6(T_i - h_i; T_i) + \gamma a_2$ , and  $K_i(\cdot; T_i)'$  are given in Lemma 2 for the parameterization  $\zeta_0 = \zeta_2 = 0$  and  $\zeta_1 = v = 1$ . Analytical, although rather complicated, formulas for these functions are available in Pástor and Veronesi (2003b).

(b) The firm's excess stock returns follow the process

$$dR_t^i = \mu_{R,t}^i(y_t, h_i) dt + \sigma_{R,0}^i(y_t, h_i) d\widetilde{W}_{0,t} + \sigma_{R,L}^i(h_i) d\widetilde{W}_{L,t} + \sigma_{R,i}^i(\widehat{\sigma}_{i,t}, h_i) d\widetilde{W}_{i,t}, \quad (24)$$

where  $d\widetilde{W}_{j,t}$ 's are the Wiener processes given in equation (16), and

$$\mu_{R,t}^i(y_t, h_i) = \sigma_{R,0}^i(y_t, h_i) \sigma_{\pi,t} \quad (25)$$

$$\sigma_{R,0}^i(y_t, h_i) = Q_3(h_i) \sigma_{i,0} + Q_2(h_i) \sigma_{L,0} + (Q_1(h_i) + 2Q_5(h_i) y_t) \sigma_y \quad (26)$$

$$\sigma_{R,L}^i(h_i) = Q_2(h_i) \sigma_{L,L} \quad (27)$$

$$\sigma_{R,i}^i(\widehat{\sigma}_{i,t}, h_i) = Q_3(h_i) \sigma_{i,i} + Q_5(h_i) \frac{\phi^i}{\sigma_{i,i}} \widehat{\sigma}_{i,t}^2 \quad (28)$$

*Note:* It can be shown that the firm's M/B is increasing in  $\hat{\sigma}_{i,t}^2$ , consistent with the main result of PV. Inspection of the ODEs in equations (18) through (20) reveals that  $\hat{\sigma}_{i,t}^2$  enters only the ODE defining  $K_0$  and hence only  $Q_0$  and no other  $Q_i$ 's.  $Q_0$  is increasing in  $\hat{\sigma}_{i,t}^2$ , so that the firm's M/B is increasing in  $\hat{\sigma}_{i,t}^2$  as well. This fact is shown more explicitly in Pástor and Veronesi (2003b).

*Proof:* From the pricing formula and Lemma 2:

$$M_t^i = \pi_t^{-1} E_t [\pi_{T_i} B_{T_i}^i] = e^{\gamma c_t + \gamma a_1 y_t + \gamma a_2 y_t^2} E_t \left[ e^{-\eta(T_i - t)} e^{b_{T_i}^i - \gamma c_{T_i} - \gamma a_1 y_{T_i} - \gamma a_2 y_{T_i}^2} \right] = e^{\gamma c_t + \gamma a_1 y_t + \gamma a_2 y_t^2} H(\mathbf{Y}_t, t)$$

Since  $\mathbf{B}_Y$  has only zeros in its first column, we have  $[\mathbf{K}(t; T_i)' \cdot \mathbf{B}_Y]_1 = 0$  in equation (19). This implies  $\frac{dK_1(t; T_i)}{dt} = 0$  and thus  $K_1(t; T_i) = 1$  for  $t \leq T_i$ . By substituting in  $H(\mathbf{Y}_t, t)$ , we obtain

$$M_t^i = e^{\gamma c_t + \gamma a_1 y_t + \gamma a_2 y_t^2} \times H(\mathbf{Y}_t, t) = B_t^i \times e^{\gamma a_1 y_t + \gamma a_2 y_t^2} \times e^{K_0(t; T_i) + \sum_{i=2}^5 K_i(t; T_i) Y_{i,t} + K_6(t; T_i) Y_{2,t}^2}$$

This expression leads immediately to claim (a) upon redefinition of variables. The proof of claim (b) follows from an application of Ito's lemma to  $M_t^i$ , and the equilibrium condition  $\mu_R = -cov\left(\frac{dM_t^i}{M_t^i}, \frac{d\pi_t}{\pi_t}\right)$ . See Pástor and Veronesi (2003b) for details. ■

### The Long-Lived Firm:

Let  $B_t^m$  denote the long-lived firm's book value, and  $D_t^m$  its dividends at time  $t$ . The firm's dividend yield,  $c^m = D_t^m / B_t^m$ , is constant, and its instantaneous profitability is  $\bar{\rho}_t$ . The firm's market value is  $M_t^m = E_t \left[ \int_t^\infty \pi_s / \pi_t D_s^m ds \right]$ . Since  $E_t [\pi_s D_s^m] = c^m E_t [\pi_s B_s^m]$ , Fubini's theorem, Lemma 2, and the same argument as in the proof of Proposition 1 yield the pricing formula:

$$\frac{M_t^m}{B_t^m} \equiv c^m \int_0^\infty Z^m(s, \bar{\rho}_t, y_t) ds, \quad (29)$$

where  $Z^m(s, \bar{\rho}_t, y_t) = e^{Q_0^m(s) + Q_1^m(s)y_t + Q_2^m(s)\bar{\rho}_t + Q_3^m(s)y_t^2}$ , and  $Q_0^m(s) = K_0(0; s)$ ,  $Q_1^m(s) = K_2(0; s)$ ,  $Q_2^m(s) = K_3(0; s)$ , and  $Q_3^m(s) = K_6(0; s)$ . Here,  $K_i(0; s)$ 's are as in Lemma 2 for the parametrization  $\zeta_0 = \zeta_2 = v = 1$ , and  $\zeta_1 = 0$ . Pástor and Veronesi (2003b) provide analytical, although complicated, formulas for these coefficients. Excess returns of the long-lived firm follow

$$dR_t^m = \mu_R^m(y_t, \bar{\rho}_t) dt + \sigma_{R,0}^m(y_t, \bar{\rho}_t) d\widetilde{W}_{0,t} + \sigma_{R,L}^m(y_t, \bar{\rho}_t) d\widetilde{W}_{L,t}, \quad (30)$$

where

$$\begin{aligned} \mu_R^m(y_t, \bar{\rho}_t) &= \sigma_{R,0}^m(y_t, \bar{\rho}_t) \sigma_{\pi,t} \\ \sigma_{R,0}^m(y_t, \bar{\rho}_t) &= F_{\bar{\rho}}^m(t) \sigma_{L,0} + (F_{y,1}^m(t) + F_{y,2}^m(t) y_t) \sigma_y \\ \sigma_{R,L}^m(y_t, \bar{\rho}_t) &= F_{\bar{\rho}}^m(t) \sigma_{L,L} \end{aligned}$$

In the above,

$$\begin{aligned} F_{\bar{\rho}}^m(t) &= \frac{\int_0^\infty Q_2^m(s) Z^m(s, \bar{\rho}_t, y_t) ds}{\int_0^\infty Z^m(s, \bar{\rho}_t, y_t) ds} \\ F_{y,1}^m(t) &= \frac{\int_0^\infty Q_1^m(s) Z^m(s, \bar{\rho}_t, y_t) ds}{\int_0^\infty Z^m(s, \bar{\rho}_t, y_t) ds} \\ F_{y,2}^m(t) &= 2 \frac{\int_0^\infty Q_3^m(s) Z^m(s, \bar{\rho}_t, y_t) ds}{\int_0^\infty Z^m(s, \bar{\rho}_t, y_t) ds} \end{aligned}$$

**(E) Payoff computation:** At the IPO at time  $\tau$ , the expected payoff at time  $\tau + \ell$  is

$$EPay_{\tau, \tau+\ell}^i = E_\tau \left( \frac{\pi_{\tau+\ell}}{\pi_\tau} (M_{\tau+\ell}^i (1-f) - B^{t_i}) \right) = B^{t_i} \left\{ (1-f) E_\tau \left( \frac{\pi_{\tau+\ell}}{\pi_\tau} \frac{M_{\tau+\ell}^i}{B^{t_i}} \right) - E_\tau \left( \frac{\pi_{\tau+\ell}}{\pi_\tau} \right) \right\} \quad (31)$$

Using (23) with  $\tilde{h} = T - (\tau + \ell)$ , we have

$$M_{\tau+\ell}^i = B^{t_i} e^{Q_0(\tilde{h}; \hat{\sigma}_{\tau+\ell}^2) + \mathbf{Q}(\tilde{h}_i)' \cdot \mathbf{N}_{\tau+\ell} + Q_5(\tilde{h}_i) y_{\tau+\ell}^2}$$

The initial profitability at the time of the IPO is unknown at  $\tau$ , so we assume it equal to its unconditional expectation  $\rho_{\tau+\ell}^i = \bar{\rho}_{\tau+\ell} + \hat{\psi}_{\tau+\ell}^i$ . Then

$$\begin{aligned} E_\tau \left( \frac{\pi_{\tau+\ell}}{\pi_\tau} \frac{M_{\tau+\ell}^i}{B^{t_i}} \right) &= e^{(Q_3(\tilde{h}) + Q_4(\tilde{h})) \hat{\psi}_{\tau+\ell}^i} \times e^{-\eta\tau - \gamma a_0} \times E_\tau \left[ e^{Q_0(\tilde{h}; \hat{\sigma}_{\tau+\ell}^2)} \right] \\ &\times E_\tau \left( e^{-\eta\ell} e^{-\gamma c_{\tau+\ell} + (Q_2(\tilde{h}) + Q_3(\tilde{h})) \bar{\rho}_{\tau+\ell} + (Q_1(\tilde{h}) - \gamma a_1) y_{\tau+\ell} + (Q_5(\tilde{h}) - \gamma a_2) y_{\tau+\ell}^2} \right) \end{aligned} \quad (32)$$

Note that the term  $e^{(Q_3(\tilde{h}) + Q_4(\tilde{h})) \hat{\psi}_{\tau+\ell}^i}$  can be taken out of the expectation as agents are assumed to know their prior mean  $\hat{\psi}_{\tau+\ell}^i$  at time  $\tau$ . Prior uncertainty is stochastic between  $\tau$  and  $\tau + \ell$  but it is independent of everything else, so  $E_\tau \left[ e^{Q_0(\tilde{h}; \hat{\sigma}_{\tau+\ell}^2)} \right]$  can be computed separately. Since  $\hat{\sigma}_t^2$  follows a continuous time Markov Chain process, we have  $E_\tau \left[ e^{Q_0(\tilde{h}; \hat{\sigma}_{\tau+\ell}^2)} | \hat{\sigma}_\tau^2 = v_j^2 \right] = [\mathbf{\Lambda}(\ell)]_j \mathbf{E}(v)$ , where  $\mathbf{\Lambda}(\ell) = \mathbf{W}^{-1} \text{diag}(e^{\omega_j \ell}) \mathbf{W}$ ,  $[\mathbf{E}(v)]_i = e^{Q_0(\tilde{h}; v_i^2)}$ ,  $\omega_j$  are the eigenvalues of the infinitesimal transition matrix  $\mathbf{\Lambda}$ , and  $\mathbf{W}$  is the matrix of corresponding eigenvectors.

The last term in (32) can be written as  $E_\tau (e^{-\eta\ell} \tilde{g}(\mathbf{Y}_{\tau+\ell}))$  with

$$\tilde{g}(\mathbf{Y}_{\tau+\ell}) = e^{Y_{1, \tau+\ell} + (Q_1(\tilde{h}) - \gamma a_1) Y_{2, \tau+\ell} + (Q_2(\tilde{h}) + Q_3(\tilde{h})) Y_{3, \tau+\ell} + (Q_5(\tilde{h}) - \gamma a_2) Y_{2, \tau+\ell}^2}$$

and  $v = 0$  in Lemma 2. Thus, Lemma 2 provides us with a solution of this expectation, with the only difference that the final conditions of the functions  $K_i$  are given by  $K_6(\tau + \ell; \tau + \ell) = (Q_5(\tilde{h}) - \gamma a_2)$ ,  $\mathbf{K}(\tau + \ell; \tau + \ell) = (1, Q_1(\tilde{h}) - \gamma a_1, Q_2(\tilde{h}) + Q_3(\tilde{h}), 0, 0)$ ,  $K_0(\tau + \ell; \tau + \ell) = 0$ . Pástor and Veronesi (2003b) report analytical, although complex, formulas for these functions.

Finally, we can compute  $E_\tau \left( \frac{\pi_{\tau+\ell}}{\pi_\tau} \right)$  immediately from Lemma 2, under the assumption  $\nu = 0$ . ■

## REFERENCES

- Abel, Andrew B., Avinash K. Dixit, Janice C. Eberly, and Robert S. Pindyck, 1996, "Options, the value of capital, and investment," *Quarterly Journal of Economics* 111, 753–777.
- Abel, Andrew B., and Janice C. Eberly, 1996, "Optimal investment with costly reversibility," *Review of Economic Studies* 63, 581–593.
- Alti, Aydogan, 2003, "IPO market timing", working paper, University of Texas.
- Baker, Malcolm, and Jeffrey Wurgler, 2000, "The equity share in new issues and aggregate stock returns," *Journal of Finance* 55, 2219–2257.
- Baker, Malcolm, Jeremy C. Stein, and Jeffrey Wurgler, 2003, "When does the market matter? Stock prices and the investment of equity-dependent firms," *Quarterly Journal of Economics* 118, 969–1106.
- Barro, Robert J., 1990, "The Stock Market and Investment," *Review of Financial Studies* 3, 115–132.
- Bayless, Mark, and Susan Chaplinsky, 1996, "Is there a window of opportunity for seasoned equity issuance?", *Journal of Finance* 51, 253–278.
- Benninga, Simon, Mark Helmantel, and Oded Sarig, 2003, "The timing of initial public offerings", *Journal of Financial Economics*, forthcoming.
- Benveniste, Lawrence M., Walid Y. Busaba, and William J. Wilhelm, Jr., 2002, *Journal of Financial Intermediation* 11, 87–121.
- Berk, Jonathan B., 1999, "A Simple Approach for Deciding When to Invest," *American Economic Review* 89, 1319–1326.
- Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1553–1607.
- Bernanke, Ben S., 1983, "Irreversibility, uncertainty, and cyclical investment," *Quarterly Journal of Economics* 98, 85–106.
- Brainard, William, and James Tobin, 1968, "Pitfalls in Financial Model Building," *American Economic Review* 58, 99–122.
- Brennan, Michael J. and Eduardo S. Schwartz, 1985, "Evaluating natural resource investments," *Journal of Business* 58, 135–137.
- Campbell, John Y., 1987, "Stock returns and the term structure," *Journal of Financial Economics* 18, 373–399.
- Campbell, John Y., 1991, A variance decomposition for stock returns, *The Economic Journal* 101, 157–179.
- Chemmanur, Thomas J., and Paolo Fulghieri, 1999, "A theory of the going-public decision", *Review of Financial Studies* 12, 249–279.
- Chen, Hsuan-Chi, and Jay R. Ritter, 2000, "The seven percent solution", *Journal of Finance* 55, 1105–1131.
- Choe, Hyuk, Ronald W. Masulis, and Vikram Nanda, 1993, "Common stock offerings across the business cycle: Theory and evidence", *Journal of Empirical Finance* 1, 3–31.
- Cox, John C., and Chi-Fu Huang, 1989, "Optimal consumption and portfolio policies when asset prices follow a diffusion process", *Journal of Economic Theory* 49, 33–83.

- Cukierman, Alex, 1980, "The effects of uncertainty on investment under risk neutrality with endogenous information," *Journal of Political Economy* 88, 462–475.
- Dixit, Avinash K., 1989, "Entry and exit decisions under uncertainty," *Journal of Political Economy* 97, 620–638.
- Fama, Eugene F., and Kenneth R. French, 1988, "Permanent and temporary components of stock prices," *Journal of Political Economy* 96, 246–273.
- Fama, Eugene F., and Kenneth R. French, 1993, "Common risk factors in the returns on stocks and bonds", *Journal of Financial Economics* 33, 3–56.
- Fama, Eugene F., and Kenneth R. French, 2000, Forecasting profitability and earnings, *Journal of Business* 73, 161–175.
- Fama, Eugene F., and Kenneth R. French, 2003, "New lists: Fundamentals and survival rates", *Journal of Financial Economics*, forthcoming.
- Gale, Douglas, 1996, "Delay and Cycles", *Review of Economic Studies* 63, 169–198.
- Gomes, Joao, Leonid Kogan, and Lu Zhang, 2001, "Equilibrium cross-section of returns," *Journal of Political Economy*, forthcoming.
- Helwege, Jean, and Nellie Liang, 2003, "Initial public offerings in hot and cold markets", *Journal of Financial and Quantitative Analysis*, forthcoming.
- Hoffmann-Burchardi, Ulrike, 2001, "Clustering of initial public offerings, information revelation, and underpricing", *European Economic Review* 45, 353–383.
- Ibbotson, Roger G., and Jeffrey F. Jaffe, 1975, "Hot issue markets", *Journal of Finance* 30, 1027–1042.
- Ingersoll, Jonathan E., Jr., and Stephen A. Ross, 1992, "Waiting to invest: Investment and uncertainty", *Journal of Business* 65, 1–29.
- Jovanovic, Boyan, and Peter L. Rousseau, 2001, "Why wait? A century of life before IPO", *American Economic Review*, 336–341.
- Jovanovic, Boyan, and Peter L. Rousseau, 2002, "The Q-theory of mergers", *American Economic Review*, 198–204.
- Jovanovic, Boyan, and Peter L. Rousseau, 2003, "Mergers as reallocation", working paper, NYU.
- Keim, Donald B., and Robert F. Stambaugh, 1986, "Predicting returns in the stock and bond markets," *Journal of Financial Economics* 17, 357–390.
- Lamont, Owen A., 2002, "Evaluating value weighting: Corporate events and market timing", NBER working paper No. 9049.
- Liptser, Robert S., and Albert N. Shiriyayev, 1977, *Statistics of Random Processes: I, II*, Springer-Verlag, New York.
- Loughran, Tim, Jay R. Ritter, and Kristian Rydqvist, 1994, "Initial Public Offerings: International Insights," *Pacific-Basin Finance Journal* 2, 165–199.
- Loughran, Tim, and Jay R. Ritter, 1995, "The new issues puzzle," *Journal of Finance* 50, 23–51.
- Lowry, Michelle, 2003, "Why does IPO volume fluctuate so much?", *Journal of Financial Economics* 67, 3–40.
- Lowry, Michelle, and G. William Schwert, 2002, "IPO market cycles: Bubbles or sequential learning?", *Journal of Finance* 57, 1171–1198.

- Lucas, Deborah J., and Robert L. McDonald, 1990, "Equity issues and stock price dynamics", *Journal of Finance* 45, 1019–1043.
- McDonald, Robert L., and Daniel Siegel, 1986, "The value of waiting to invest," *Quarterly Journal of Economics* 101, 707–728.
- Myers, Stewart C., and Nicholas Majluf, 1984, "Corporate financing and investment decisions when firms have information the investors do not have", *Journal of Financial Economics* 13, 187–221.
- Novy-Marx, Robert, 2003, "An equilibrium model of investment under uncertainty," working paper, University of California, Berkeley.
- Pagano, Marco, Fabio Panetta, and Luigi Zingales, 1998, "Why do companies go public? An empirical analysis", *Journal of Finance* 53, 27–64.
- Pástor, Ľuboš, and Pietro Veronesi, 2003a, "Stock valuation and learning about profitability", *Journal of Finance* 58, 1749–1789.
- Pástor, Ľuboš, and Pietro Veronesi, 2003b, "Stock prices and IPO waves", NBER working paper #9858.
- Persons, John C., and Vincent A. Warther, 1997, "Boom and bust patterns in the adoption of financial innovations", *Review of Financial Studies* 10, 939–967.
- Rajan, Raghuram, and Henri Servaes, 1997, Analyst following of initial public offerings, *Journal of Finance* 52, 507–529.
- Rajan, Raghu, and Henri Servaes, 2003, "The effect of market conditions on initial public offerings", in *Venture Capital Contracting and the Valuation of High-Tech Firms*, J. McCahery and L. Rennebog (eds.), Oxford University Press, Oxford, forthcoming.
- Ritter, Jay R., 1991, "The long-run performance of initial public offerings", *Journal of Finance* 46, 3–27.
- Ritter, Jay R., and Ivo Welch, 2002, A review of IPO activity, pricing, and allocations, *Journal of Finance* 57, 1795–1828.
- Schultz, Paul, 2003, "Pseudo Market Timing and the Long-Run Underperformance of IPOs", *Journal of Finance* 58, 483–517.
- Schwartz, Eduardo S., 2001, "Patents and R&D as real options", working paper, UCLA.
- Shleifer, Andrei, 1986, "Implementation Cycles," *Journal of Political Economy* 94, 1163–1190.
- Stoughton, Neal M., Kit Pong Wong, and Josef Zechner, 2001, "IPOs and product quality", *Journal of Business* 74, 375–408.
- Subrahmanyam, Avanidhar, and Sheridan Titman, "The going public decision and the development of financial markets", *Journal of Finance* 54, 1045–1082.
- Veronesi, Pietro, 2000, "How does information quality affect stock returns?", *Journal of Finance* 55, 807–837.
- Vuolteenaho, Tuomo, 2002, "What drives firm-level stock returns?", *Journal of Finance* 57, 233–264.
- Zingales, Luigi, 1995, "Insider ownership and the decision to go public", *Review of Economic Studies* 62, 425–448.