The Cross-Section of Volatility and Expected Returns*

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Abstract

We examine how volatility risk, both at the aggregate market and individual stock level, is priced in the cross-section of expected stock returns. Stocks that have high sensitivities to innovations in aggregate volatility have low average returns and a cross-sectional factor capturing systematic volatility risk earns -0.87% per month. We find that stocks with high idiosyncratic volatility have abysmally low returns. The quintile portfolio composed of stocks with the highest idiosyncratic volatilities does not even earn an average positive total return. We find that the low returns earned by stocks with high exposure to systematic volatility risk and the low returns of stocks with high idiosyncratic volatility are not priced by the standard size, value or momentum factors and are not subsumed by liquidity or volume effects.

1 Introduction

The volatility of stock returns, both at the individual level and at the aggregate level, varies over time. While there has been extensive study of the relation between aggregate volatility and expected returns (see, among others, Campbell and Hentschel, 1992; Glosten, Jagannathan and Runkle, 1993; Scruggs, 1998; Goyal and Santa-Clara, 2003), the question of how volatility affects the cross-section of expected stock returns has received little attention.

We provide a systematic investigation of how stochastic volatility is priced in the cross-section of expected stock returns. Our goals are twofold. First, we examine the rewards for holding stocks that have high sensitivities to fluctuations in aggregate market volatility. If the volatility of the market return is a systematic risk factor, an APT or factor model indicates that aggregate volatility should also be priced in the cross-section of stocks. We find that innovations to aggregate volatility carry a statistically significant negative price of risk. Economically, a negative premium for systematic volatility risk implies that assets with positive exposures to aggregate volatility risk pay off in times of high market uncertainty when market returns tend to be low. Since equity prices react negatively to positive shocks in aggregate volatility, investors are willing to pay premiums to hold assets with high exposure to systematic volatility risk. Hence, assets with high sensitivities to fluctuations in aggregate volatility earn low average returns. Our findings are consistent with many option pricing studies that have also documented negative prices of aggregate volatility risk.[1] However, all of these option pricing studies estimate the price of aggregate volatility risk using, at most, only the time-series and a cross-section of options on an aggregate market index, in addition to returns on the market portfolio.

There are several advantages of using a cross-section of returns on stocks, rather than a cross-section of options on the market, to examine how investors price aggregate volatility risk. First, using the cross-section of returns allows us to create a useful hedging, or factor, portfolio for aggregate volatility risk. If the price of volatility risk is negative, the zero-cost hedge portfolio has average returns that are consistently negative. The portfolio is easy to construct and reflects exposure to innovations in aggregate volatility. The second reason for using the cross-section of stock returns is to gauge the strength of exposure to volatility risk in the pricing of individual stocks or portfolios. This approach creates a new set of assets with exposure to volatility risk that are not options. Finally, using the cross-section allows us to estimate the price of volatility risk controlling for other standard cross-sectional effects, such

[1] See, for example, Jackwerth and Rubinstein (1996), Bakshi, Cao and Chen (2000), Chernov and Ghysels (2000), Burashi and Jackwerth (2001), Coval and Shumway (2001), Benzoni (2002), Jones (2002), Pan (2002), Bakshi and Kapadia (2003) and Eraker, Johannes and Polson (2003).

as the size and value effects of Fama and French (1993), the momentum effect of Jegadeesh and Titman (1993), and the liquidity effect of Pástor and Stambaugh (2003). Estimating the reward for holding stocks with high exposure to volatility risk while controlling for other cross-sectional factors cannot be done using only a cross-section of options on the market portfolio.

We find strong evidence that systematic volatility risk is priced in the cross-section of stocks. The difference in average returns between the highest and lowest quintile portfolios sorted by exposure to market volatility innovations is -1.04% per month. This difference remains statistically significant at -0.83% per month after controlling for the Fama and French (1993) factors. The cross-sectional volatility risk effect is robust to liquidity effects, and it is not priced by momentum effects. We find that a factor created to represent exposure to systematic volatility risk is significantly priced in the cross-section of stock returns, earning, on average, -0.87% per month.

The second goal of this paper is to examine the relationship between idiosyncratic volatility as a characteristic and expected stock returns. Portfolios of stocks sorted by idiosyncratic volatility are a set of assets that we expect to be mispriced relative to standard models of systematic risk because these models do not account for market volatility risk. Recent studies focus only on the average level of firm-level volatility. For example, Campbell et al. (2001) and Xu and Malkiel (2001) document that idiosyncratic volatility, relative to the market or to the Fama-French (1993) three-factor model, has increased over time. Goyal and Santa-Clara (2003) demonstrate that idiosyncratic risk has positive predictive power for excess market returns. In contrast, we focus on how idiosyncratic risk is cross-sectionally reflected in expected returns.

Standard asset pricing models predict that idiosyncratic volatility is not priced and thus cannot influence cross-sectional average returns. However, recent economic theory indicates that idiosyncratic risk may be positively related to expected returns, if investors demand compensation for not being able to diversify risk (see Malkiel and Xu, 2002; Jones and Rhodes-Kropf, 2003). Merton (1987) suggests that in an information-segmented market, firms with larger firm-specific variance require higher returns to compensate for imperfect diversification. Recent behavioral models, like Barberis and Huang (2001), also predict that higher idiosyncratic volatility stocks should earn higher expected returns. Our results are directly opposite to these theories. We find that stocks with low idiosyncratic risk deliver high average returns. There is a strongly significant pattern of over -1.06% per month in the average return difference between quintile portfolios of lowest and highest idiosyncratic risk, computing idiosyncratic volatility relative to the Fama-French (1993) model.

Our findings are also the total opposite of Tinic and West (1986) and Malkiel and Xu (2002).

These authors find that portfolios with higher idiosyncratic risk have higher average returns. However, they do not directly sort stocks based on the measure of interest, idiosyncratic volatility, nor do they tabulate any significance levels for their idiosyncratic volatility premiums. Instead, Tinic and West (1986) work only with 20 portfolios sorted on market beta, while Malkiel and Xu work only with 100 portfolios sorted on market beta and size.[2] Hence, Tinic and West and Malkiel and Xu miss the strong negative relation between idiosyncratic volatility and expected returns.

One explanation for the pattern of very low average returns for high idiosyncratic volatility is that stocks with high idiosyncratic risk may have high exposure to aggregate volatility and this lowers average returns. We find that portfolios sorted by idiosyncratic volatility have some exposure to systematic volatility risk, but this accounts for only a small fraction of the low returns earned by stocks with high idiosyncratic volatility. Our results represent a puzzle. We outline some potential explanations and investigate if they can explain these puzzling results. Our results are robust to controlling for value, size, liquidity, volume, dispersion of analysts' forecasts, and momentum effects. In particular, we find that the effect is common to stocks of all sizes, but is strongest among middle-sized, not the smallest-sized, stocks. The effect persists in both bull and bear markets, recessions and expansions, and volatile and stable periods. Moreover, we find the effect robust to different formation periods for computing idiosyncratic volatility and for different holding periods.

The rest of this paper is organized as follows. In Section 2, we examine how systematic volatility is priced in the cross-section of stock returns. Section 3 documents that firms with high idiosyncratic volatility have very low average returns. Finally, Section 4 concludes.

2 Pricing Systematic Volatility in the Cross-Section

2.1 Theoretical Motivation

When there are time-varying investment opportunities, the multi-factor models of Merton (1973) and Ross (1976) suggest that there will be risk premia associated with innovations in the state variables that describe the time-variation of investment opportunities. Innovations in variables that forecast future market returns or future market variances must be priced cross-sectionally because they represent changes in the investment opportunity set.

[2] Malkiel and Xu (2002) do consider a cross-sectional regression on individual stocks, but instead of using a measure of an individual stock's idiosyncratic volatility, they assign a stock's residual standard deviation to be the idiosyncratic risk of one of the 100 beta/size portfolios to which that stock belongs each month.

For example, in the factor (APT) model of Chen, Roll and Ross (1986), changes in the investment opportunity set are summarized by innovations to macro-economic factors. Chen, Roll and Ross show how changes in interest rate spreads, inflation, economic growth and default rates are significantly priced in the cross-section of returns. Innovations of state variables, rather than their levels, are priced because agents assign risk premiums only to unanticipated changes.

A related multi-factor approach is taken by Campbell (1993 and 1996), who builds on the Intertemporal CAPM (I-CAPM) of Merton (1973) and shows that investors care about risk from the market return and changes in forecasts of future market returns. When the representative agent is more risk averse than log utility, assets that covary positively with good news about future returns on the market have higher mean returns. These assets command a risk premium because they reduce a consumer's ability to hedge against a deterioration in investment opportunities.

Chen (2002) extends Campbell's model to allow for time-varying covariances and stochastic market volatility. Chen shows that risk-averse investors also want to hedge against changes in future market volatility. An asset's return now depends on risk from the market return, changes in forecasts of future market returns and changes in forecasts of future market volatilities. For an investor more risk averse than log, Chen shows that assets that covary positively with changes in forecasts of future market volatilities have lower returns. This is because risk-averse investors reduce current consumption to increase precautionary savings in the presence of increased uncertainty. If an asset covaries positively with changes in the uncertainty of future returns, then such an asset is attractive and carries low expected returns because it pays off in times when uncertainty is high.

Motivated by these multi-factor approaches, we study how market volatility risk is priced in the cross-section of stock returns. We focus on a setting with two systematic factors: the market return and the volatility of the market return. A two-factor pricing kernel with the market return and stochastic volatility as factors is also the standard set-up commonly assumed by many stochastic option pricing studies (see, for example, Heston, 1993). Unanticipated changes in these variables are then priced cross-sectionally and command risk premiums. Since market returns are nearly serially uncorrelated, we treat market returns to be innovations of the market factor. We proxy unanticipated changes in aggregate volatility by using innovations of the systematic volatility factor. Hence, the factor specification that we consider is:

$$r_t^i = a + \beta_m^i r_t^m + \beta_v^i \Delta v_t + \varepsilon_t^i, \tag{1}$$

where r_t^i is the excess return on stock i, r_t^m is the excess market return, Δv_t is the innovation in aggregate volatility, β^i_m is the loading on the excess market return and β^i_v is the beta sensitivity to innovations in aggregate volatility.

Equation (1) is a standard linear two-factor model with the market return and innovation in market volatility as factors. This is a direct result of assuming shocks to the market return process and stochastic volatility are correlated with a stochastic discount factor. Previous empirical studies suggest that there are other cross-sectional factors that have explanatory power for the cross-section of returns, such as the size and value factors of Fama and French (1993). We do not directly model these effects in equation (1), but we are careful to ensure that we control for these and other cross-sectional factors in assessing how volatility risk is priced. Because volatility is a latent process, we use several proxies to measure innovations in volatility.

Equation (1) is in the standard form of an APT or factor model, so that stocks with different sensitivities β^i_v to systematic volatility risk have different expected returns. The goal in our empirical work is to only examine the cross-sectional pricing implications of equation (1). In contrast, an I-CAPM implies joint time-series as well as cross-sectional predictability. We do not examine time-series predictability of asset returns by systematic volatility because we do not take a stand on the utility function of a representative agent or parameterize the time-series process of volatility.[3]

2.2 Estimating Aggregate Volatility

We proxy systematic (market-wide) volatility using three estimators. The first estimator is the standard French, Schwert and Stambaugh (1987) and Schwert and Seguin (1990) measure, which is the sum of squared daily returns over the past N_t days, adjusted for first-order autocorrelations:[4]

$$\hat{\sigma}^2_t = \frac{1}{N_t}\left[\sum_{i=0}^{N_{t-1}} r^2_{t-i} + 2\sum_{i=1}^{N_t} r_t r_{t-i}\right], \tag{2}$$

where r_t is the return on the market portfolio. We denote the volatility measure $\hat{\sigma}_t$ in (2) by $SVOL$ (sample volatility) and compute $SVOL$ using daily returns on the market index from CRSP. We compute $SVOL$ at a daily frequency by using the last $N_t = 22$ trading days. The use of past daily data over the previous month to estimate volatility at time t means that the $SVOL$

[3] Time-varying volatility risk generates intertemporal hedging demands in partial equilibrium asset allocation problems. Liu (2001) and Chacko and Viceira (2003) examine how volatility risk affects the portfolio allocation of stocks and risk-free assets, while Liu and Pan (2003) show how investors can optimally exploit the variation in volatility with options. Guo and Whitelaw (2003) examine intertemporal components of time-varying systematic volatility in a Cambell (1993 and 1996) equilibrium I-CAPM.

[4] Our results are unchanged if we omit the autocorrelation terms, as in Schwert (1989).

estimates do not reflect the true market volatility at t, rather they represent an average of the daily volatility from month $t-1$ to month t. Nevertheless, $SVOL$ should pick up broad trends in true volatility movements.

Our second proxy for market volatility is a range-based estimate, following Parkinson (1980) and Alizadeh, Brandt and Diebold (2002):

$$\hat{\sigma}_t = \log \left(\sup_{0<\tau\leq 1} S_\tau \Big/ \inf_{0<\tau\leq 1} S_\tau \right), \tag{3}$$

where S_τ is the level of the S&P500 index over day t. We denote this range-based estimate for aggregate volatility as $RVOL$. While easy to compute, $RVOL$ suffers from several drawbacks. First, $RVOL$ is biased downwards because the range on a discrete grid of prices is always less than the range of a true continuous sample path. Second, the use of equation (3) assumes that the volatility of the market is constant each day, but changes from day to day. Third, the log range estimator relies on the assumption that a log volatility process is a good approximation for the underlying true volatility process. Finally, even if the true volatility process follows a log process, Andersen and Bollerslev (1998) and Alizadeh, Brandt and Diebold (2002) show that the efficiency of $RVOL$ is similar to the efficiency of estimates which use intra-day realized volatility forecasts of 4-6 hour windows, which provide at most two observations per trading day. Since $SVOL$ uses only one observation per trading day, the $RVOL$ measure should be better than, but may not be a substantial improvement on, using $SVOL$.

Our last proxy for volatility is the VIX index from the Chicago Board Options Exchange (CBOE). The VIX index is constructed so that it represents the implied volatility on a synthetic at-the-money option contract on the S&P100 index that has a one month maturity. It is constructed from eight S&P100 index puts and calls and takes into account the American features of the option contracts, discrete cash dividends and microstructure frictions such as bid-ask spreads (see Whaley, 2000, for further details).[5]

At first glance, since VIX is representative of traded option securities whose prices directly reflect volatility risk, VIX might seem to be the most natural measure of changes in aggregate volatility. However, there are three main caveats with using VIX to represent observable

[5] On September 22, 2003, the CBOE implemented a new formula and methodology to construct its volatility index. The new index is based on the S&P500 (rather than the S&P100) and takes into account a broader range of strike prices rather than using only at-the-money option contracts. The CBOE now uses VIX to refer to this new index. We use the old index (denoted by the ticker VXO). We do not use the new index because it has been constructed by back-filling only to 1990, whereas the VXO is available in real-time from 1986. The CBOE continues to make both volatility indices available. The correlation between the new and the old CBOE volatility series is 98% from 1990-2000, but the series that we use has a slightly broader range.

market volatility. First, the VIX index from the Black-Scholes (1973) model, rather than representing the true unobservable volatility process. Nonetheless, we expect that Black-Scholes volatilities are highly correlated with the true volatility process.

The second caveat is that VIX may also reflect an interaction of a jump process and a diffusion process (see Eraker, Johannes and Polson, 2003). Bates (1991 and 2000) argues that implied volatilities computed taking into account jump risk are very close to Black-Scholes implied volatilities. Even if we cannot disentangle the effect of jumps from diffusion movements, jumps are also part of the total quadratic variation in the market return, so jumps should also be priced in the cross-section of stock returns. Hence, our market volatility innovations more broadly reflect innovations in total quadratic variation.

The third, but most serious, reservation about the VIX index is that VIX combines both stochastic volatility itself and the stochastic volatility risk premium. Only if the risk premium is zero or constant would ΔVIX represent only an innovation in volatility. Decomposing ΔVIX into the true innovation in volatility and the risk premium can only be done by writing down a formal model. The form of the risk premium depends on the parameterization of the price of volatility risk, the number of factors and the evolution of those factors. Each different model specification implies a different risk premium. For example, many stochastic volatility option pricing models parameterize the volatility risk premium to be a linear function of volatility (see, for example, Chernov and Ghysels, 2000; Benzoni, 2002; Jones, 2002; Pan, 2002). Rather than imposing a structural form, we use an unadulterated VIX series. This has the additional advantage that our analysis is simple to replicate.

Other common methods of estimating volatility include GARCH-based models and methods based on intra-day, or high frequency, data (see, for example, Andersen et al., 2003). We do not use a GARCH model because the parameters of the GARCH process must be estimated before computing the implied innovations in the variances. Hence, this method entails a look-ahead bias if the full sample is used. When we form portfolios, it is important that we form portfolios only using data available as of the formation date. If a rolling GARCH estimator is used to avoid look-ahead bias, the time-periods near the beginning of the sample suffer from very poor estimates of the GARCH process. While Andersen et al. (2003) formally justify the use of the realized sample volatility measured with intra-day data as a highly efficient volatility proxy, intra-day data on market returns are not readily available, making this estimation method hard to implement. In particular, intra-day data are collected only for individual stocks and the main source of these data, the TAQ database, starts only in 1993.

We concentrate on using the sample period from January 1986 to December 2000. This is because the data for the VIX series begins in January 1986, and we would like to compare all our series on a common sample period. Nevertheless, we also comment on the sample period July 1963 to December 2000 for $SVOL$ and $RVOL$.

Table 1 presents some summary statistics for $SVOL$, $RVOL$ and VIX at a daily frequency. The annualized mean of $SVOL$ ($RVOL$) is $0.0099 \times \sqrt{250} = 16\%$, ($0.0120 \times \sqrt{250} = 19\%$). The mean of VIX is higher than both these two measures, at 21%. The higher average of VIX volatility indicates that it is a biased forecast of realized future volatility. The bias may reflect a risk premium for stochastic volatility, a market inefficiency or a Peso-problem.[6] The annualized standard deviations of $SVOL$, and VIX are approximately equal, at 9% and 8%, but the annualized standard deviation of $RVOL$ is higher, at 14%. All three series are negatively correlated with the market return, with $RVOL$ (VIX) having a -23% (-18%) correlation. The correlation of $SVOL$ with the market is noticeably less, at only -4%. The low correlation of $SVOL$ is due to the fact that a large negative movement in returns has only a 1/22th weight in the computation of $SVOL$ from equation (2), whereas the increase in volatility is reflected more immediately by $RVOL$ and VIX.

We graph the three volatility measures in Figure 1, which annualizes each volatility measure so that they are comparable. Overall, all three measures share the same trends. In particular, each series has two noticeable spikes. The first spike shows the increase in implied volatilities after the 1987 crash, and the second spike occurs in 1998 during the Russian default, the emerging markets crises, and the bailout of Long Term Capital Management. Figure 1 shows that while $SVOL$ and VIX are fairly smooth series (autocorrelations of 98% and 94%, respectively at at a daily frequency), the $RVOL$ measure is much less smooth (the daily autocorrelation of $RVOL$ is 49%). The more volatile range-based measure also magnifies the movements in volatility measured by $SVOL$ and VIX.

To measure daily innovations in aggregate volatility, we compute daily changes in $SVOL$, daily changes in $RVOL$ or daily changes in VIX. We denote these measures as $\Delta SVOL$, $\Delta RVOL$ or ΔVIX, respectively. From equation (2) (and ignoring the autocorrelation term), $\Delta SVOL$ effectively takes the difference between the squared market return at t and the squared market return 22 trading days prior to time t. Hence, the time-series of daily $\Delta SVOL$ effectively measures monthly innovations in volatility at time t. In contrast, $\Delta RVOL$ and ΔVIX reflect daily changes in volatility movements and may be better estimates of changes in true market volatility. In particular, ΔVIX reflects a daily change in implied option volatilities.

[6] See, among many others, Day and Lewis (1992), Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Blair, Poon and Taylor (2001), Poteshman (2000) and Chernov (2002).

Nevertheless, $\Delta SVOL$, $\Delta RVOL$ and ΔVIX are all quite highly correlated with each other. For example, Table 1 reports that the correlation of ΔVIX with $\Delta SVOL$ ($\Delta RVOL$) is 45% (39%).

In Table 1, all the estimates for daily innovations in volatility have strong negative correlations with the market return. The correlations of the market with $\Delta SVOL$, $\Delta RVOL$ and ΔVIX are -25%, -26% and -64%, respectively. Hence, when a positive volatility shock arrives, the market excess return decreases. The best example of this effect is the increase in volatility over 1987 in Figure 1, coinciding with the large negative returns of the market over this period. Table 1 shows one source of discrepancy between $\Delta RVOL$ and the two other estimators $\Delta SVOL$ and ΔVIX. While $\Delta SVOL$ and ΔVIX have very low autocorrelations (7% and -7%, respectively), $\Delta RVOL$ has a strong negative autocorrelation of -43%. In fact, $RVOL$ and $\Delta RVOL$ also have almost the same standard deviation. This is due to the large movements in $RVOL$, which is shown pictorially in Figure 1.

2.3 Systematic Volatility Risk in the Cross-Section

Portfolios Sorted by Exposure to Systematic Volatility

Equation (1) predicts that firms with different sensitivities (measured by betas) to innovations in systematic volatility should have different expected excess returns. Based on this implication, we sort firms into portfolios according to their sensitivities to systematic volatility. If stochastic volatility risk is priced, the average returns on these volatility sensitivity-sorted portfolios should be different. Our first step is to check that firms with different sensitivities to market volatility innovations indeed have different average returns.

Equation (1) suggests estimating the sensitivities of stock i to systematic volatility in the following regressions:

$$\begin{aligned}
r_t^i &= \alpha^i + \beta_{MKT}^i \cdot MKT_t + \beta_{\Delta SVOL}^i \cdot \Delta SVOL_t + \varepsilon_t^i \\
r_t^i &= \alpha^i + \beta_{MKT}^i \cdot MKT_t + \beta_{\Delta RVOL}^i \cdot \Delta RVOL_t + \varepsilon_t^i \\
r_t^i &= \alpha^i + \beta_{MKT}^i \cdot MKT_t + \beta_{\Delta VIX}^i \cdot \Delta VIX_t + \varepsilon_t^i
\end{aligned} \tag{4}$$

where r_t^i is firm i's excess return and MKT is the market excess return. Equation (4) proxies the innovation in market volatility (Δv_t in equation (1)) by $\Delta SVOL$, $\Delta RVOL$, or ΔVIX. The coefficients $\beta_{\Delta SVOL}^i$, $\beta_{\Delta RVOL}^i$ and $\beta_{\Delta VIX}^i$ represent the sensitivity of firm i's returns to innovations in market volatility, measured by these proxies. Note that, as equation (1) suggests, we control for the effect of the market in computing the volatility betas in equation (4).

To form portfolios, we run regression (4) on daily excess returns over the previous month for each firm with more than 17 daily observations within that month on all stocks on AMEX, NASDAQ and the NYSE. At the end of each month, we sort the stocks into quintiles, based on the value of the $\beta_{\Delta SVOL}$, $\beta_{\Delta RVOL}$ or $\beta_{\Delta VIX}$ coefficients. Firms in quintile 1 (5) have the lowest (highest) coefficients. Within each quintile portfolio, we value-weight the stocks. If volatility risk is priced cross-sectionally, the average returns of these quintile portfolios should be different.

Table 2 reports various summary statistics for quintile portfolios sorted by exposure to aggregate volatility shocks. The first two columns report the mean and standard deviation of monthly total, not excess, simple returns. If the negative price of systematic volatility risk found by the option pricing studies is reflected in the cross-section, we should see lower average returns with higher coefficients of $\beta^i_{\Delta SVOL}$, $\beta^i_{\Delta RVOL}$ or $\beta^i_{\Delta VIX}$. We turn first to the portfolios sorted by $\beta_{\Delta SVOL}$. The $\beta_{\Delta SVOL}$ portfolios have little differences in spreads or alpha's, relative to the CAPM or to the Fama-French (1993) model (FF-3 hereafter), reported in the last two columns. This is not surprising since we know that $\Delta SVOL$ is potentially a poor measure for daily changes in stochastic volatility.

We next turn to the quintile portfolios sorted by $\beta_{\Delta RVOL}$. Quintiles 1-4 all have higher average returns than quintile 5, and the 5-1 spread in average returns between the quintile portfolios with the lowest and highest $\beta_{\Delta RVOL}$ values is -0.42% per month. When we control for the Fama-French factors, the 5-1 alpha is -0.39% per month. While the negative point estimates of the 5-1 spread in average returns or alpha's are consistent with a negative price of volatility risk, the spreads are statistically insignificant at the 5% marginal level of significance using robust Newey-West (1987) t-statistics. The sample period of Table 2 is from January 1986 to December 2000. If we use more data from July 1963 to December 2000, for more power, we still cannot reject that the 5-1 difference in alphas or expected returns for both $\beta_{\Delta SVOL}$ and $\beta_{\Delta RVOL}$ portfolios are equal to zero.

We now turn to our last volatility proxy, ΔVIX, which directly reflects the volatility of the market portfolio priced in option contracts. The average returns of the quintile portfolios are monotonically decreasing from 1.64% per month for low $\beta_{\Delta VIX}$ stocks to 0.60% per month for high $\beta_{\Delta VIX}$ stocks. The 5-1 spread in average returns between the quintile portfolios with the highest and lowest $\beta_{\Delta VIX}$ coefficients is -1.04% per month. This dramatic spread in average returns does not seem to be due to patterns in size or book-to-market characteristics. In the two last columns of Table 2, we compute alpha's relative to the CAPM and FF-3. Controlling for the MKT factor only exacerbates the 5-1 spread (from -1.04% to -1.15% per month), while controlling for the FF-3 model decreases the 5-1 spread to -0.83% per month. Both the CAPM and FF-3 alpha's are significant at the 1% level using robust t-statistics. The results of the sorts on $\beta_{\Delta VIX}$ confirm the negative price of volatility risk estimated by option pricing studies. The higher the $\beta_{\Delta VIX}$ coefficient, the higher is the exposure of a stock to systematic volatility risk. Since stocks with higher $\beta_{\Delta VIX}$ loadings have lower expected returns, this is consistent with stochastic volatility carrying a negative risk premium.

One curious pattern about the average returns and the alpha's for the $\beta_{\Delta VIX}$ quintile portfolios is that the average returns and alpha's for quintiles 1-4 are approximately the same, with a slight downward trend. However, there is a dramatic fall in the average return and alpha for quintile 5. This implies that while there is a monotonic relation between increasing $\beta_{\Delta VIX}$ loadings and decreasing average returns and alpha's, the biggest effect is for stocks with the highest values of $\beta_{\Delta VIX}$, which have extremely low returns or alpha's. This category of stocks is not a small proportion of the market: the percentage market capitalization of quintile 5 is 7.4%. Quintile 5 portfolio's turnover is also not substantially higher than the other portfolios; its average turnover is 73%, of the same order of magnitude as decile portfolios sorted on book-to-market ratios.

Robustness

Because VIX is a stationary series, using ΔVIX as the innovation in VIX may over-difference, causing the slight negative serial correlation reported in Table 1. The finding of the low average returns on stocks with high volatility risk loadings $\beta_{\Delta VIX}$ is robust to measuring volatility innovations by specifying various models for the conditional mean of VIX. If we fit an AR(1) model to VIX and measure innovations relative to the AR(1) specification, we find that the results of Table 2 are almost unchanged. Specifically, the mean return of the difference between the first and fifth $\beta_{\Delta VIX}$ portfolios is -1.08% per month, and the FF-3 alpha of the 5-1 difference is -0.90%, both highly statistically significant. Using an optimal BIC choice for the number of AR lags (which is 11) produces a similar result. The mean (FF-3 alpha) of the 5-1 difference is -0.81% (-0.66%), both significant at the 5% level.[7]

Another concern is the time window we use to estimate the factor loadings $\beta_{\Delta VIX}$ to volatility risk. In Table 2, we use a 1-month window of daily VIX innovations. The results in Table 2 become weaker if we extend the formation period of the portfolios. Although the point

[7] In these exercises, we estimate the AR coefficients only using data up to time t to compute the innovation for $t + 1$, so that no forward-looking information is used. We initially estimate the AR models using one year of daily data. However, the optimal BIC lag length is chosen using the whole sample.

estimates of the $\beta_{\Delta VIX}$ portfolios have the same qualitative patterns as Table 2, statistical significance drops. For example, if we use the past 3-months of VIX innovations to compute volatility betas, the mean return of the 5th quintile portfolio with the highest $\beta_{\Delta VIX}$ stocks is 0.79%, compared with 0.60% with a 1-month formation period. The FF-3 alpha on the 5th quintile portfolio decreases in magnitude to -0.37%, with a robust t-statistic of -1.62, with a 3-month window, compared to -0.53%, with a t-statistic of -2.93, with a 1-month formation period. If we use the past 12-months of VIX innovations, the 5th quintile portfolio mean (FF-3 alpha) increases to 0.97% (-0.33%, with a t-statistic of -1.04). The weakening of the effect is due to two reasons. First, the turnover in the monthly $\beta_{\Delta VIX}$ portfolios is fairly high (above 70%) and using longer formation periods causes less turnover. Second, the $\beta_{\Delta VIX}$ loadings vary over time. Suppose we use information over the past year prior to time t to estimate a factor loading with regressions. Instead of obtaining a time t estimate of the factor loading, the estimate is actually an average factor loading that applies to time $t - 6$ months ago. By using only information over the past month, we obtain an estimate of the factor loading much closer to time t.

Table 2 shows that the low $\beta_{\Delta VIX}$ stocks tend to be relatively small stocks with high book-to-market ratios. Small growth firms are precisely those firms with option value that would be expected to do well when aggregate volatility increases. The portfolio of small growth firms is also one of the hardest of the Fama-French (1993) 25 portfolios sorted on size and book-to-market that is hardest to price by standard factor models. Could small growth stocks be stocks with high $\beta_{\Delta VIX}$ loadings?

The results are mixed. If we exclude the portfolio of small growth firms, and repeat the quintile portfolio sorts in the last panel of Table 2, we find that the 5-1 mean difference reduces in magnitude to -0.63% per month (compared to -1.04% with all firms), with a t-statistic of -3.30. Excluding small growth firms, the FF-3 5-1 alpha reduces to -0.44% per month, and is no longer significant at the 5% level, compared to -0.83% per month with all firms. Hence, the factor controls suggest that small growth stocks may play a role.

However, a more thorough characteristic-matching procedure suggests that size or value characteristics are not driving the results. Table 3 reports mean returns of the $\beta_{\Delta VIX}$ portfolios characteristic-matched by size and book-to-market ratios. Following Daniel et al. (1997), every month, each stock is matched with one of the Fama and French (1993) 25 size and book-to-market portfolios by appropriate size and book-to-market characteristics. The table reports value-weighted simple returns in excess of the characteristic-matched returns. Table 3 shows that controlling for size and book-to-market decreases the raw mean 5-1 difference of -1.04%

in Table 2 to -0.90%. If we exclude firms that are members of the smallest growth portfolio of the Fama-French 25 size-value portfolios, the mean 5-1 difference decreases to -0.64% per month. However, the characteristic-controlled differences are still highly significant. Hence, the low returns to high $\beta_{\Delta VIX}$ stocks do not seem to be completely driven by a disproportionate concentration among small growth stocks.

Other cross-sectional effects other than value or size effects may have more explanatory power to explain the low returns of stocks with high systematic volatility risk. Periods of very high volatility tend to coincide with periods of market illiquidity. Chordia et al. (2001), Jones (2002) and Pástor and Stambaugh (2003) all comment that such periods often coincide with market downturns. For example, during the 1987 crash and the 1998 Russian debt and subsequent emerging markets crises, realized market returns and liquidity were low. Pástor and Stambaugh demonstrate that stocks with high liquidity betas have high expected returns. We now check that the spread in average returns reflecting sensitivities to volatility risk is not due to liquidity effects. Panel A of Table 4 reports the results.

To control for liquidity, we first sort stocks into five quintiles based on their historical liquidity betas, β^L, computed following Pástor and Stambaugh (2003). Then within each quintile, we sort stocks into five quintiles based on their $\beta_{\Delta VIX}$ coefficient loadings. These portfolios are rebalanced monthly and are value-weighted. After forming the 5×5 liquidity beta and $\beta_{\Delta VIX}$ portfolios, we average the returns of each $\beta_{\Delta VIX}$ quintile over the five liquidity beta portfolios. Thus, these quintile $\beta_{\Delta VIX}$ portfolios control for differences in liquidity.

Table 4, Panel A shows that controlling for liquidity reduces the 5-1 difference in average returns from -1.04% per month in Table 2 to -0.68% per month. In particular, after controlling for liquidity, we still observe the monotonically decreasing pattern of average returns of the $\beta_{\Delta VIX}$ quintile portfolios. The liquidity control also does not remove the sharp decrease in the average return of the fifth $\beta_{\Delta VIX}$ quintile. When we control for the CAPM (FF-3 model), the alpha becomes -0.73% (-0.55%) per month. Both these alpha's are significant at the 5% level. We also observe the same pattern of very low returns for the highest $\beta_{\Delta VIX}$ stocks within each liquidity beta quintile, before averaging across the liquidity beta portfolios, but do not report these results to save on space. Hence, liquidity effects cannot account for the spread in returns resulting from sensitivity to aggregate volatility risk.

Panel B reports the same exercise except we control for volume effects rather than liquidity. Gervais, Kaniel and Mingelgrin (2001) find that stocks with high trading volume earn higher average returns than stocks with low trading volume. It could be that the low average returns (and alpha's) we find for stocks with high $\beta_{\Delta VIX}$ loadings are just stocks with low volume.

Panel B shows that this is not the case. In Panel B, we control for volume the same way that we control for liquidity in Panel A, except we first sort stocks into quintiles based on their trading volume (rather than Pástor-Stambaugh liquidity betas). Controlling for volume, the FF-3 alpha of the 5-1 difference remains significant at the 5% level at -0.58% per month. Finally, in Panel C, we control for momentum characteristics, measured by past 6-month returns. Controlling for momentum reduces the raw -1.04% per month difference between stocks with low and high $\beta_{\Delta VIX}$ loadings to -0.83%, but remains highly significant. The CAPM and FF-3 alpha's controlling for momentum are also significant at the 1% level. Hence, liquidity, volume and momentum cannot account for the low returns of stocks with high sensitivities to aggregate volatility risk.

2.4 A Cross-Sectional Volatility Factor

Constructing the VOL Factor

The linear factor model (1) implies risk premiums for the market and volatility risk of the form:

$$\mathrm{E}(r_t^i) = \beta_{MKT}^i \cdot \lambda_{MKT} + \beta_{\Delta VIX}^i \cdot \lambda_{VOL}, \tag{5}$$

where $\mathrm{E}(r_t^i)$ is the expected excess return of stock i, λ_{MKT} is the market risk premium and λ_{VOL} is a risk premium for a tradeable aggregate volatility risk factor. In this section, we build a factor capturing exposure to systematic volatility, allowing us to cross-sectionally estimate its price of risk. The advantage of constructing a factor representing risk from aggregate volatility innovations, which we call VOL, rather than just using ΔVIX is that ΔVIX does not represent the realized return on a tradable asset. By creating a tradeable factor, we can interpret alpha's from standard time-series factor regressions as well as directly estimate the volatility innovation risk premium cross-sectionally.

Our volatility factor, VOL, reflects the spread in returns from the different $\beta_{\Delta VIX}$ loadings and is formed as follows. Each month, we rank stocks based on their $\beta_{\Delta VIX}$ coefficients into three groups: low, medium and high $\beta_{\Delta VIX}$ groups with 33.3% and 66.7% cutoffs. We calculate monthly value-weighted returns for each of these three portfolios. The VOL factor is formed as the return difference between the high $\beta_{\Delta VIX}$ group and the low $\beta_{\Delta VIX}$ group. Hence, the VOL factor goes long stocks with high volatility innovation sensitivities, which have low expected returns, and shorts stocks with low sensitivities, which have high expected returns.

Note that VOL is not a mimicking factor, or tradeable tracking portfolio, for systematic volatility in the sense of Lamont (2001). That is, VOL is not the traded factor most highly correlated with the non-traded VIX index. Rather, VOL reflects the premium investors are willing to pay to hold stocks that covary positively with aggregate volatility shocks. The price of risk of VOL is a measure of the difference in the premiums between stocks that have low and high sensitivities to innovations in systematic volatility.

Table 5 lists some summary statistics for the VOL factor. The VOL factor has a monthly mean return of -0.58% per month, and the mean is statistically significant at the 1% marginal level of significance. Table 5 also lists the correlation of VOL with the excess market return MKT, the Fama and French (1993) size and value factors SMB and HML, and UMD, a momentum factor constructed by Kenneth French. The momentum factor UMD is constructed in a similar way to Carhart (1996)'s momentum factor, which goes long stocks with past high returns and shorts stocks with past low returns. The correlation of VOL with the MKT is 16%, which is smaller in magnitude than the respective correlations of SMB, HML and UMD with MKT over our sample period. The low correlation results from controlling for the MKT factor in our initial computation of $\beta_{\Delta VIX}$ in the regression (4). However, our VOL factor is relatively highly correlated with SMB, at 48%, and HML at -40%. This is consistent with the results in Table 2, where the alpha's from the FF-3 model for the quintile $\beta_{\Delta VIX}$ portfolios are slightly smaller than the raw average returns.

Table 5 also reports the results of regressing VOL onto various factors in a time-series regression. Controlling for the MKT factor decreases the α from -0.58% per month to -0.68% per month. The FF-3 model reduces this magnitude to -0.46% per month. Nevertheless, the alpha is still significant at the 5% level. When we add the UMD momentum factor, the loading on UMD is zero, and the point estimate of the alpha is almost unchanged, decreasing by only 1 basis point to -0.47% per month. However, the extra noise added by UMD causes the VOL alpha to be borderline significant at the 5% level.

In Figure 2, we plot the level of the VIX index, together with cumulative returns of the VOL factor from January 1986 to December 2000. Over the sample, no particular time period drives the significantly negative mean (-0.58% per month) of the VOL factor. The VOL factor should do well when volatility spikes up, by construction. There are two episodes of large volatility spikes in our sample coinciding with large negative moves of the market: October 1987 and August 1998. In 1987, VIX volatility jumped from 22% at the beginning of October to 61% at the end of October. At the end of August 1998, the level of VIX reached 48%. These two episodes are also the largest two negative movements of the market, in the second half of the 20th century (see Ang and Chen, 2002). During periods of simultaneous dramatic increases in market volatility and large falls of the market, we expect VOL to perform well, on average.

During the October 1987 crash, the VOL factor returned 6.6%, while the market crashed -23%. Hence, the VOL factor provided a hedge during this period. Although all stocks fell during this month, high $\beta_{\Delta VIX}$ stocks fell less than low $\beta_{\Delta VIX}$ stocks, enabling VOL to earn a positive return. During August 1998, the market fell -16%, but VOL returned -8.1%. While VOL did not earn a positive return, it did not fall as much as the market. We would also expect that VOL would earn negative, or close to zero returns, when the market does extremely well. This is indeed the case. The two largest moves of the market in our sample are January 1987, where the market climbed 12%, and December 1991, where the market returned 11%. In January 1987, the VOL factor returned -2.4%, while in December 1991, the VOL factor was flat at 0.1%.

The periods of high spikes in volatility and large negative jumps in market returns are infrequent. The low frequency of these events may suggest a potential Peso problem story for the low return on stocks with high ΔVIX sensitivities. The argument is that we have had too few crashes relative to what was expected. Suppose we focus only on the largest market decline (October 1987), where VOL returned 6.6%. How many October 1987-like crashes would be necessary in our 180 month sample to make the average VOL return equal to zero? Assuming that the VOL factor earns, on average, 6.6% during each crash, we would require 14 jumps of the magnitude of the October 1987 crash in order to eliminate the abnormally low return on volatility risk sensitive stocks. In our sample, we observe only two large market crashes. Using Gabaix et al. (2003)'s power law distribution for extreme events, we would expect to observe three large market crashes below three standard deviations during this period. Given the large number of crashes required to eliminate the low returns of high $\beta_{\Delta VIX}$ stocks, compared to the low number of actual or expected crash periods, a potential peso explanation for the negative risk premium of VOL is unlikely.

Fama-MacBeth (1973) Estimates of the Price of Volatility Risk

Equations (1) and (5) of the linear factor model imply a standard cross-sectional regression. If excess returns of assets are regressed on the $\beta_{\Delta VIX}$ coefficients of those assets, then there should be a significant coefficient on the $\beta_{\Delta VIX}$ loadings. This coefficient, λ_{VOL}, is the cross-sectional premium for systematic volatility risk innovations. To estimate λ_{VOL}, equations (1) and (5) suggest the need to create a set of assets whose market betas and ΔVIX betas are sufficiently disperse. We construct 25 portfolios sorted by β_{MKT} and $\beta_{\Delta VIX}$ as follows. At the end of each month, we sort stocks based on β_{MKT}, computed by a univariate regression of excess stock returns on excess market returns over the past month using daily data. We compute the $\beta_{\Delta VIX}$ loadings using the bivariate regression (4) also using daily data over the past month. Stocks are ranked first into quintiles based on β_{MKT} and then within each β_{MKT} quintile into $\beta_{\Delta VIX}$ quintiles.

Panel A of Table 6 reports FF-3 alpha's of these 25 $\beta_{MKT} \times \beta_{\Delta VIX}$ set of portfolios. There is some heterogeneity in the alpha's, but the 5-1 difference in the $\beta_{\Delta VIX}$ quintiles is always negative. For the larger β_{MKT} quintiles 4 and 5, the alpha's are almost monotonic.[8] Across each β_{MKT} quintile, it is always the fifth $\beta_{\Delta VIX}$ quintile that has the steepest drop in returns. Hence, this finer sort of stocks based on β_{MKT} and $\beta_{\Delta VIX}$ coefficients has the same qualitative pattern of alpha's as the quintile $\beta_{\Delta VIX}$ portfolios in Table 2, which do not control for the market beta.

We use the base assets of Panel A to estimate factor premiums in Panel B, following the two-step procedure of Fama-MacBeth (1973). In addition to the standard FF-3 and UMD factors, we include the Pástor-Stambaugh (2003) liquidity factor, LIQ. Although the LIQ factor is non-traded, we can still include it in the cross-sectional regression and examine the statistical significance of its premium. Panel B shows that the premiums of the standard factors (MKT, SMB, HML and UMD) are estimated very imprecisely with this set of base assets. The premium on SMB is consistently estimated to be negative because the size strategy has performed poorly from the 1980's onwards. The low and insignificant premiums of UMD and LIQ illustrate that the spreads in expected returns of the $\beta_{\Delta VIX}$ portfolios are not related to momentum or liquidity effects, confirming the characteristic controls in Table 4.

When VOL is included in the cross-sectional regressions, it is the only factor estimated to have a significant loading. Its premium of around -0.83% per month is of the same order of magnitude as the time-series mean of VOL (-0.58% per month). The VOL premium is significant in all the various specifications of including different factors. The cross-sectional R^2's also increase significantly once VOL is included. For example, the R^2 of the FF-3 specification is 50%, and it increases to 67% when the VOL factor is included. Hence, the VOL premium is robust to size, value, momentum and liquidity effects. We consider the VOL factor to be a new cross-sectional factor representing systematic volatility risk.

The VOL Factor and Option Returns

The VOL factor reflects exposure to systematic volatility risk and is constructed using the cross-section of stock returns. An alternative way to construct a traded asset reflecting volatility risk is to consider option returns. To construct a mimicking factor for systematic volatility risk

[8] The pattern in the means of raw returns is qualitatively similar to the the pattern of FF-3 alpha's.

from cross-sectional options is infeasible because of low liquidity and large bid-ask spreads. However, it is possible to construct a zero-delta straddle position in options on the aggregate market (S&P 100 options) which has zero market exposure but provides exposure to systematic volatility. This is precisely what Coval and Shumway (2001) do. They approximate daily at-the-money straddle returns by taking a weighted average of the zero-delta straddle returns corresponding to strike prices immediately above and below each day's opening level of the S&P 100 and cumulate these daily returns each month. We denote this factor as STR (for "straddle returns").

It is reassuring that over the 1986 to 1995 sample period used by Coval and Shumway, the STR and VOL factors have a positive correlation of 19%. Since STR and VOL both measure systematic volatility exposure, a time-series regression of STR on VOL, or vice versa, should yield significant loadings. Unfortunately, Table 7 shows that while STR and VOL load on each other positively, the coefficient loadings are insignificant. The adjusted R^2's of the regressions are also only 3%. The reason for the poor correspondence is that the STR returns are extremely volatile, compared with the low volatility of VOL. The volatility of STR is 35.48% per month (122.9% per annum), whereas Table 5 shows that the volatility of VOL is only 3.29% per month (11.40% per annum).

Table 7 shows that when VOL is regressed onto STR, the constant (-0.28%) is insignificant. This is what we would expect if VOL and STR are able to price each other. However, a regression of STR onto VOL only reduces the magnitude of the raw STR average return of -11.02% per month to -9.96% per month, which is still significant at the 5% level. Since the zero-beta straddle positions are only approximately delta-neutral, because the approximations rely on a Black-Scholes (1973) formula to compute the weights in the option positions, it is likely that the STR returns still incorporate some residual MKT exposure. When the MKT factor is added, the alpha becomes insignificant and the adjusted R^2 increases to 15%.

While STR has a very impressive negative return, its large volatility means a person selling straddles can easily take enormous losses, which would have happened during the 1987 crash where the monthly return on STR over October 1987 was 285%. In contrast, the low volatility of VOL makes it a less risky trading strategy. Another advantage of VOL over STR is that taking straddle positions requires daily or weekly rebalancing (done by Coval and Shumway, 2001), whereas VOL is re-balanced at a monthly frequency. Finally, the VOL factor is easy to construct as the VIX index is publicly available. The main source of option data, the Berkeley Option Database has reliable data only from the late 1980's and stops in 1995. The database is also no longer made available for research purposes.

3 Pricing Idiosyncratic Volatility in the Cross-Section

So far, we have examined how systematic volatility risk affects cross-sectional average returns by focusing on portfolios of stocks sorted by their sensitivity to innovations in aggregate volatility. In this section, we investigate a second set of assets sorted by idiosyncratic volatility, as a stock characteristic. Naturally, if the factors driving systematic risk are correctly specified, we should see no reward for bearing idiosyncratic risk. However, we expect that standard models of systematic risk, such as the Fama-French (1993) model, would mis-price portfolios sorted by idiosyncratic volatility, because the standard set of linear factor models do not include factors measuring exposure to market volatility risk. We specifically examine how any patterns in the cross-section of expected returns from idiosyncratic volatility can be potentially explained by systematic volatility exposure.

3.1 Estimating Idiosyncratic Volatility

To measure idiosyncratic volatility for an individual stock, we run a Fama-French (1993) regression:

$$r_t^i = \alpha^i + \beta^i_{MKT} MKT_t + \beta^i_{SMB} SMB_t + \beta^i_{HML} HML_t + \varepsilon^i_t. \tag{6}$$

Given the failure of the CAPM to explain cross-sectional returns and the ubiquity of FF-3 in empirical financial applications, we concentrate on idiosyncratic volatility measured relative to the Fama-French model ($\sqrt{\text{var}(\varepsilon^i_t)}$ in equation (6)). We also comment on the results using total volatility (without decomposing total volatility into systematic and idiosyncratic components) and measuring idiosyncratic volatility relative to the traditional CAPM.

To examine trading strategies based on idiosyncratic volatility, we describe trading strategies based on a formation period of L months, a waiting period of M months and then a holding period of N months. We can describe an $L/M/N$ strategy as follows. At month t, we compute idiosyncratic volatilities from the regression (6) on daily data over an L month period from month $t - L - M$ to month $t - M$. At time t, we construct value-weighted portfolios based on these idiosyncratic volatilities and hold these portfolios for N months. We concentrate our analysis on the $1/0/1$ strategy, but we examine robustness to various choices of L, M and N. For the $1/0/1$ strategy, we simply sort stocks into quintile portfolios based on their level of idiosyncratic volatility computed using daily returns over the past month, and hold these value-weighted portfolios for 1 month. The portfolios are rebalanced each month.

The construction of the $L/M/N$ portfolios for $L > 1$ and $N > 1$ is similar to Jegadeesh and Titman (1993), except our portfolios are value-weighted. For example, to construct $12/1/6$

quintile portfolios, each month we construct a value-weighted portfolio based on idiosyncratic volatility computed on 12-months of returns ending one month prior. Similarly, we form a value-weighted portfolio based on 12-months of returns ending two months prior, three months prior, and so on up to six months prior. Each of these portfolios is value-weighted. We then take the simple average of these six portfolios. Hence, each quintile portfolio changes 1/6th of its composition each month, where each 1/6th part of the portfolio consists of a value-weighted portfolio. The first (fifth) quintile portfolio consists of 1/6th of the lowest value-weighted (highest) idiosyncratic stocks from one month ago, 1/6th of the value-weighted lowest (highest) idiosyncratic stocks two months ago, etc.

3.2 Patterns in Average Returns for Volatility Risk

Table 8 reports average returns of total and idiosyncratic volatility sorted portfolios, using a $1/0/1$ strategy. We turn first to the portfolios sorted by total volatility, without any control for systematic risk. Table 8 shows that average returns increase from 1.06% per month going from quintile 1 (low total volatility stocks) to 1.22% per month for quintile 3. Then, average returns drop. Quintile 5, which comprises stocks with the highest total volatility, experiences a dramatic decrease in average total returns (only 0.09% per month). A FF-3 alpha, reported in the last column, for quintile 5 is -1.16% per month, and while highly significant, it is the only portfolio that has a significant alpha. The large spread in average returns between quintiles 1 and 5 (-0.97% per month) may just be due to inappropriate controls for systematic risk.

The next two panels of Table 8 report average returns of stocks sorted by idiosyncratic volatility measured relative to the CAPM and FF-3 model, respectively.[9] An interesting pattern is that there is a reward in raw average returns for increasing idiosyncratic volatility, but this does not hold for stocks with the highest idiosyncratic volatilities in quintiles 4 and 5. In both the CAPM and FF-3 cases, the low average returns of quintiles 4 and 5 are exacerbated, compared to the sorts on total volatility, and their alpha's are highly statistically significant. In particular, the average returns of quintile 5 are -1 basis point (-2 basis points) for idiosyncratic volatility relative to the CAPM (FF-3). Stocks with high idiosyncratic risk have abysmally low average returns.

Let us focus attention on sorts by idiosyncratic volatility relative to FF-3, which is reported in the last panel of Table 8. The difference in raw average returns between quintile 1 and 5 is a

[9] If we compute idiosyncratic risk relative to a factor model with MKT and VOL factors, the patterns in average returns and alpha's in Table 8 are qualitatively preserved. The 5-1 alpha's are smaller in magnitude than for the case of idiosyncratic volatility relative to the CAPM or FF-3 models, but the alpha's are still significant.

very large -1.06% per month. Controlling for the CAPM (FF-3) model increases the difference in magnitude to -1.38% (-1.31%) per month. Clearly, the FF-3 model is unable to price these portfolios. This puzzle may be due to missing risk factors, not controlling for cross-sectional characteristic effects, market inefficiency or peso problems.

Table 8 shows distinct patterns in the size and book-to-market ratios of the FF-3 idiosyncratic volatility portfolios. Stocks with low (high) idiosyncratic volatility are generally large (small) stocks with low (high) book-to-market ratios. The FF-3 model predicts that quintile 5 stocks should have high, not low, average returns. These findings are quite provocative, but there are several concerns raised by the anomalously low returns of quintile 5. First, although quintile 5 contains 20% of the stocks sorted by idiosyncratic volatility, quintile 5 are only a small proportion of the market (only 1.9% on average). Are these patterns repeated if we only consider large stocks, or only stocks traded on the NYSE? Second, because illiquidity distortions are strongest among small stocks, we should control for liquidity. Third, are these patterns robust to different formation and holding periods? The next two sections evaluates the robustness of our findings.

3.3 Robustness

Using Only NYSE Stocks

Table 9 examines robustness of our $1/0/1$ portfolio formation strategy for FF-3 idiosyncratic volatility portfolio sorts, controlling for various effects. The table reports FF-3 alpha's, and the difference in FF-3 alpha's between the quintile portfolios with the lowest and highest idiosyncratic risks. First, we rank stocks based on idiosyncratic volatility using only NYSE stocks. Excluding NASDAQ and AMEX has little effect on our results. The highest quintile of idiosyncratic volatility stocks has a FF-3 alpha of -0.60% per month and the 5-1 difference is still high, at -0.66% per month, which is significant at the 1% level.

Controlling for Size

We control for size by first forming quintile portfolios ranked on size and then within each size quintile, we sort stocks based into quintile portfolios ranked on FF-3 idiosyncratic volatility. Within each size quintile, quintile 5 contains the stocks with the highest idiosyncratic volatility, and in each case this quintile has a dramatically lower FF-3 alpha than the other quintiles. The effect is not most pronounced among the smallest stocks. Rather, quintiles 2-4 have the largest

5-1 differences in FF-3 alpha's, at -1.91%, -1.61% and -0.86% per month, respectively. The average market capitalization of quintiles 2-4 is, on average, approximately 21% of the market. The t-statistics of these alpha's are all above 4.5 in absolute magnitude. The 5-1 alpha's for the smallest and largest quintiles are actually statistically insignificant at the 5% level. Hence, it is definitely not small stocks that are driving these results. We can control for size by averaging the returns of the quintile idiosyncratic volatility portfolios over the five size portfolios. Controlling for size, the 5-1 difference in FF-3 alpha's is still -1.04% per month.

The remainder of Table 9 repeats the explicit double-sort characteristic controls for book-to-market ratios, leverage liquidity, volume, dispersion in analysts' forecasts, and momentum. In each case, we first sort stocks into quintiles based on the characteristic and then, within each quintile we sort stocks based on FF-3 idiosyncratic volatility. To control for the characteristic, we average the returns over each of the five characteristic portfolios.

Controlling for Book-to-Market

We turn next to the row labelled "Controlling for Book-to-Market." Perhaps our idiosyncratic volatility portfolios are primarily composed of growth stocks, with lower average returns than value stocks. Table 8 shows that this is not the case. When we control for the book-to-market effect, stocks with the highest idiosyncratic volatility still have very low FF-3 alpha's, and the 5-1 difference in alpha's is -80% per month, and highly significant.

Controlling for Leverage

In the next line of Table 8, we examine the returns to high idiosyncratic volatility stocks controlling for leverage. Firms with high idiosyncratic volatility may have high asset volatility, not permitting them to be highly levered. This gives them a relatively lower average equity volatility (with corresponding relatively lower average returns) to other firms that have lower asset volatility with higher leverage. Controlling for leverage, the difference between the 5-1 alpha's is still -1.23% per month, so leverage cannot be an explanation for low returns to high idiosyncratic volatility.

Controlling for Liquidity Effects

We use the historical liquidity betas of Pástor and Stambaugh (2003) to proxy for liquidity. Controlling for liquidity does not remove the low average returns of high idiosyncratic volatility stocks. Quintile 5 still has very low average returns, with a FF-3 alpha of -1.01% per month. The 5-1 difference in alpha's is -1.08% per month, only slightly less in magnitude than the 5-1 difference in alpha's without the liquidity control in Table 8 (-1.31% per month). We control for volume because Lee and Swaminathan (2000) argue that high volume proxies for differences in opinion, which predicts lower returns. When we control for volume, the 5-1 difference in alpha's remains significant at the 1% level at -1.22% per month. Hence, the low returns on high idiosyncratic risk stocks are robust to controlling for liquidity and volume.

Controlling for Dispersion in Analysts' Forecasts

Diether, Malloy and Scherbina (2002) provide evidence that stocks with higher dispersion in analysts' earnings forecasts have lower average returns than stocks with low dispersion of analysts' forecasts. They argue that dispersion in analysts' forecasts is a proxy for differences of opinion among investors. When there is a large difference in stock valuations, equity prices tend to reflect the view of the more optimistic agents, leading to low future returns for stocks with large dispersion in analyst' forecasts.

If stocks with high dispersion in analysts' forecasts tend to be more volatile stocks, then we may be finding a similar anomaly to Diether, Malloy and Scherbina. If we use the sample period 1983-2000, similar to Diether, Malloy and Scherbina, we can test this hypothesis by performing a characteristic control for the dispersion of analysts' forecasts. We take the quintile portfolios of stocks sorted on increasing dispersion of analysts' forecasts (Table VI of Diether, Malloy and Scherbina, 2002, p2128) and within each quintile sort stocks on idiosyncratic volatility. Note that this universe of stocks contains mostly large firms, where the idiosyncratic volatility effect is weaker, because multiple analysts usually do not make forecasts for small firms.

The line labelled "Controlling for Dispersion of Analysts' Forecasts" in Table 8 presents the results for averaging the idiosyncratic volatility portfolios across the forecast dispersion quintiles. The 5-1 difference in alpha's is still -0.39% per month, with a robust t-statistic of -2.09. While the shorter sample period may reduce power, the dispersion of analysts' forecasts reduces the non-controlled 5-1 alpha considerably (from -1.31% per month). However, dispersion in analysts' forecasts cannot account for all of the low returns to stocks with high idiosyncratic risk.

We can also turn the question around and ask if the low average returns of stocks with high dispersion of analysts' forecasts is due to the low returns of stocks with high idiosyncratic risk. We first sort stocks into quintiles on the basis of idiosyncratic volatility, and then within each quintile sort stocks into portfolios ranked by forecast dispersion, using the set of firms used by Diether, Malloy and Scherbina. We compute the difference in FF-3 alpha's for stocks with high and low forecast dispersion, controlling for idiosyncratic volatility by averaging stocks over the idiosyncratic volatility quintiles. The 5-1 FF-3 alpha for forecast dispersion, controlling for idiosyncratic volatility, is -0.36% per month, which is insignificant at the 5% level (the robust t-statistic is -1.47).

Controlling for Momentum

Jegadeesh and Titman (1993) document that stocks with past low (high) returns continue to be losers (winners). One possibility that could explain the low returns of high idiosyncratic risk stocks is this momentum effect. Perhaps stocks with very low returns have very high volatility. Of course, stocks that are past winners also have very high volatility, but loser stocks could be over-represented in the high idiosyncratic risk quintile. Hong, Lim and Stein (2000) also argue that momentum is asymmetric and has a stronger negative effect on declining stocks than a positive effect on rising stocks.

The last row of Table 8 shows that momentum is not driving the results. Controlling for returns over the past month does not remove the very low FF-3 alpha of quintile 5 (-0.59% per month), and the 5-1 difference in alpha's is still -0.66% per month, which is statistically significant at the 1% level. As we control for past 6-month and past 12-month returns, the difference in FF-13 alpha's between portfolios 1 and 5 increase to -1.10% and -1.22% for the 6-month and 12-month momentum, respectively, and both alpha's are highly significant. What is surprising is that even the 5-1 difference in the raw average returns is very large in magnitude for momentum controls over all horizons. For 1-month momentum, the 5-1 difference is -0.84%, while for 6-month (12-month) momentum it is -0.90% (-0.99%), all significant at the 1% level. Clearly, momentum cannot account for these patterns.

3.4 Can we Explain the Negative Premium for Idiosyncratic Risk?

A possible explanation for the large negative returns of high idiosyncratic volatility stocks is that stocks with large idiosyncratic risk relative to FF-3 have larger exposure to movements in systematic volatility. This is not unreasonable, since if market volatility increases, average individual stock volatility also increases. Table 10 tries to price the FF-3 idiosyncratic volatility quintiles with various factor models.

Panel A shows that including the VOL factor into the standard linear factor specifications reduces the difference in alpha's between portfolios with the highest and lowest idiosyncratic risk. For example, for the FF-3 model, the 5-1 alpha is -1.43%. If we add the UMD factor, the 5-1 alpha becomes -1.36% per month. Adding the VOL factor to the FF-3 model reduces the magnitude of the 5-1 alpha slightly to -1.34% per month. If we add VOL to the FF-3 model augmented with UMD, the 5-1 alpha reduces in magnitude to -1.26% per month. These reductions are small and amount to approximately 10 basis points per month. All of the factor specifications fail to pass a Gibbons Ross and Shanken (1989) test that the alpha's are jointly equal to zero, with p-values of less than 0.3%.

Panel B of Table 10 reports alpha's and factor loadings for a factor model specification with MKT, SMB, HML, a winner factor WIN, a loser factor LOS and the VOL factor. The LOS (WIN) factor is the excess return on a value-weighted decile portfolio of past loser (winner) stocks formed on the basis of past 12-month returns. The idea of separating winner and loser effects is to capture the potential effect of asymmetries in momentum. However, the factor loadings on WIN and LOS are approximately the same, indicating that any momentum asymmetry is small. The momentum loadings also do not display a monotonic pattern.

Panel B shows that only HML and VOL have factor loadings that go in the correct direction from quintile 1 (high returns) to 5 (low returns). The spread in HML factor loadings is $0.17 - (-0.43) = 0.60$. However, the average return of HML over the 1986-2000 sample period is -4 basis points per month, so HML's contribution to explaining the large negative 5-1 spread is negligible. While the mean of VOL is -0.58% per month (see Table 5), the spread in the factor loadings of VOL is only $-0.13 - .07 = -0.20$, so VOL only reduces the large negative alpha by $-0.20 \times -0.58\% = 0.12\%$ per month. Hence, aggregate volatility risk accounts for some, but cannot remove, the anomalous low returns of stocks with high idiosyncratic risk.

If standard factor models cannot price idiosyncratic volatility risk and exposure to systematic volatility also cannot explain the low returns to high idiosyncratic risk stocks, are there other explanations? To help disentangle various stories, Table 11 reports FF-3 alpha's of other $L/M/N$ strategies, with an L-month formation period that ended M months ago prior to time t and is held for N months. First, we can rule out possible contemporaneous measurement errors through forming the portfolios using data ending one-month prior ($M = 1$). In the $1/1/1$ strategy, the 5-1 difference in FF-3 alpha's is still -0.82% per month and is highly significant.

One possible behavioral explanation for our results is that higher idiosyncratic volatility does earn higher returns, but short-term over-reaction forces returns to be low in the next month. If we hold high idiosyncratic risk stocks for a long horizon ($N = 12$ months), we might see a positive relation between idiosyncratic risk and average returns. The second row of Table 11 shows that this is not the case. For the $1/1/12$ strategy, we still see very low FF-3 alpha's for quintile 5, and the 5-1 difference in alpha's is still -0.67% per month and highly significant.

By restricting the formation period to $L = 1$ month, our previous results may just be capturing various short-term events that affect idiosyncratic volatility. For example, the portfolio of stocks with high idiosyncratic volatility may be largely composed of stocks that have just made, or are just about to make, earnings announcements. To ensure that we are not capturing specific short-term corporate events, we extend our formation period to $L = 12$ months. The third row of Table 11 reports FF-3 alpha's for a $12/1/1$ strategy. Using one entire year of data to compute idiosyncratic volatility does not remove the anomalous high idiosyncratic risk-low average return pattern: the 5-1 difference in alpha's is -1.12% per month. Similarly, the patterns are robust for the $12/1/12$ strategy, which has a 5-1 alpha of -0.77% per month.

While the low returns to high idiosyncratic risk stocks are amazingly robust to different formation and holding periods, a further possible explanation is asymmetry across business cycles or bull and bear markets. Because volatility is asymmetric (and larger with downward moves), high idiosyncratic risk stocks may have normal average returns during normal or bull markets, but their low returns may be driven largely by bear market periods. We check this hypothesis by examining the returns of high idiosyncratic volatility stocks conditioning on observations which have the lowest 20% of market returns and comparing them to bull markets that have the highest 20% of market returns. For the periods of lowest (highest) 20% of market returns, the FF-3 alpha of quintile 5 is -2.83% (-2.98%) per month, both highly significant at the 1% level. Hence, stocks with high idiosyncratic risk earn low returns in both bull and bear markets.

We also find that the low returns of quintile 5 are robust over NBER recessions and expansions. During NBER expansions (recessions), the FF-3 alpha of quintile 5 is -1.19% (-1.88%). Both the expansion and recession FF-3 alpha's are significant at the 1% level. There are more negative returns to high idiosyncratic volatility stocks during recessions, but the fact that the t-statistic in NBER expansions is -7.07 shows that the low returns from high idiosyncratic risk also thrives during expansions. A final possibility is that this effect is concentrated during the most volatile periods in the market. To test for this possibility, we compute FF-3 alpha's of quintile 5 conditioning on periods with the lowest or highest 20% of absolute moves of the market return. These are ex-post periods of low or high market volatility. During stable (volatile) periods, the FF-3 alpha of quintile 5 is -1.70% (-0.89%) per month, with both alpha's significant at the 5% level. Hence, the most negative returns of the high idiosyncratic risk strategy are earned during periods when the market is stable. These results indicate that it is not bull or bear market periods, asymmetries across the business cycle, or the clustering of periods of volatility that is driving the low returns to high idiosyncratic risk exposure.

4 Conclusion

Consistent with a factor or APT model, innovations in market volatility are priced as a risk factor in the cross-section of stock returns. Stocks with high exposure to innovations in systematic aggregate market volatility earn low returns. We proxy for innovations in aggregate volatility by using changes in the VIX index constructed by the Chicago Board Options Exchange. The low returns earned by stocks with high coefficient loadings to changes in the VIX index are consistent with a negative price of risk of systematic volatility. We estimate that a factor capturing the spread of average returns due to stocks having different sensitivities to innovations in market volatility has a significant mean of -0.87% per month in the cross-section and is robust to controlling for size, value, momentum and liquidity effects. Hence, the cross-sectional pattern of returns reflects systematic volatility risk.

On the other hand, according to the standard asset pricing framework, idiosyncratic volatility should not be priced. However, standard models of systematic risk do not incorporate factors measuring exposure to volatility risk. When market volatility increases, average stock volatility also increases. Recent theories also predict that stocks with high idiosyncratic volatility may earn high expected returns to compensate for imperfect diversification. A very robust result that we uncover is that stocks with high idiosyncratic volatility have abysmally low returns. In particular, using the Fama-French (1993) model to adjust for systematic risk, a quintile portfolio of stocks with the highest idiosyncratic risk earns total returns of just -0.02% per month. The results are amazingly robust to controlling for size, value, liquidity, volume and momentum effects and the effect persists in bull and bear markets, NBER recessions and expansions, and volatile and stable periods. Accounting for exposure to aggregate volatility helps to price, but cannot remove, the anomalous low returns of stocks with high idiosyncratic risk.

We also find that the low returns of high idiosyncratic volatility stocks persist for different formation and holding periods as long as one year. This rules out stories of short-term overreaction or reaction to short-term corporate events. It is also unlikely that economic agents actually prefer stocks with high idiosyncratic risk, leading to their low returns. Hence, our results on the cross-sectional expected return patterns to idiosyncratic volatility present something of a puzzle.

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Table 1: Daily Sample Moments

	Mean	Stdev	Auto	Correlations MKT	$SVOL$	$\Delta SVOL$	$RVOL$	$\Delta RVOL$	VIX
MKT	0.0004	0.0097	0.08						
$SVOL$	0.0099	0.0056	0.98	-0.04					
$\Delta SVOL$	0.0001	0.0148	0.07	-0.25	0.08				
$RVOL$	0.0120	0.0087	0.49	-0.23	0.54	0.43			
$\Delta RVOL$	-0.0000	0.0088	-0.43	-0.26	-0.00	0.40	0.50		
VIX	0.2052	0.0785	0.94	-0.18	0.79	0.15	0.68	0.03	
ΔVIX	0.0000	0.0265	-0.07	-0.64	0.00	0.45	0.29	0.39	0.16

We report daily sample moments of the excess market return MKT, sample volatility $SVOL$, range-based volatility measure $RVOL$, and the daily volatility VIX index from the CBOE. $\Delta SVOL$, $\Delta RVOL$ and ΔVIX refer to daily changes in $SVOL$, $RVOL$ and VIX, respectively. 'Auto' denotes daily autocorrelation. There are 3784 daily observations from January 1986 to December 2000.

Table 2: Portfolios Sorted by Exposure to Aggregate Volatility Shocks

Rank	Mean	Std Dev	Turn-over	% Mkt Share	Size	B/M	CAPM Alpha	FF-3 Alpha
Portfolios Sorted by $\beta_{\Delta SVOL}$								
1	1.11	5.65	0.74	7.9%	3.42	0.92	-0.27	-0.16
							[-1.44]	[-0.82]
2	1.16	4.58	0.78	25.5%	4.24	0.84	-0.10	-0.17
							[-0.89]	[-1.48]
3	1.41	4.31	0.72	30.5%	4.20	0.88	0.20	0.15
							[2.27]	[2.06]
4	1.34	4.57	0.77	27.6%	4.28	0.84	0.09	0.06
							[1.37]	[0.90]
5	1.12	6.13	0.75	8.4%	3.44	0.94	-0.32	-0.07
							[-1.48]	[-0.43]
5-1	0.01						-0.04	0.09
	[0.02]						[-0.15]	[0.30]
Portfolios Sorted by $\beta_{\Delta RVOL}$								
1	1.31	6.62	0.74	7.0%	3.75	0.96	-0.16	0.05
							[-0.68]	[0.27]
2	1.18	4.61	0.78	24.2%	4.81	0.75	-0.08	-0.11
							[-0.83]	[-1.06]
3	1.28	4.37	0.73	31.4%	4.90	0.77	0.06	-0.03
							[0.64]	[-0.46]
4	1.54	4.50	0.78	28.9%	4.90	0.75	0.29	0.28
							[4.04]	[3.69]
5	0.89	5.60	0.75	8.5%	3.77	0.98	-0.48	-0.34
							[-2.64]	[-2.04]
5-1	-0.42						-0.32	-0.39
	[-1.45]						[-1.07]	[-1.31]
Portfolios Sorted by $\beta_{\Delta VIX}$								
1	1.64	5.53	0.74	9.4%	3.70	0.89	0.27	0.30
							[1.66]	[1.77]
2	1.39	4.43	0.78	28.7%	4.77	0.73	0.18	0.09
							[1.82]	[1.18]
3	1.36	4.40	0.72	30.4%	4.77	0.76	0.13	0.08
							[1.32]	[1.00]
4	1.21	4.79	0.78	24.0%	4.76	0.73	-0.08	-0.06
							[-0.87]	[-0.65]
5	0.60	6.55	0.73	7.4%	3.73	0.89	-0.88	-0.53
							[-3.42]	[-2.88]
5-1	-1.04						-1.15	-0.83
	[-3.90]						[-3.54]	[-2.93]

We form value-weighted quintile portfolios every month from regressing excess stock returns of individual stocks on $\Delta SVOL$, $\Delta RVOL$ or ΔVIX, controlling for the MKT factor, as in equation (4). The regression is run on daily excess returns using data over the previous month. Stocks are sorted into quintiles based on the regression coefficients $\beta_{\Delta SVOL}$, $\beta_{\Delta RVOL}$ or $\beta_{\Delta SVOL}$ from lowest (quintile 1) to highest (quintile 5). The statistics in the columns labelled Mean and Std Dev are measured in monthly percentage terms and apply to total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The numbers in the Turnover column list the average proportion of firms that leave the quintile portfolio each month. The row 5-1 refers to the difference in monthly returns between portfolio 5 and portfolio 1. The Alpha columns report Jensen's alpha with respect to the CAPM or Fama-French (1993) three-factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

Table 3: Portfolios Sorted on $\beta_{\Delta VIX}$ with Characteristic Controls

	All Firms		Excluding Small, Growth Firms	
Rank	Mean	Std Dev	Mean	Std Dev
1	0.32	2.11	0.36	1.90
2	0.04	1.25	0.02	0.94
3	0.04	0.94	0.05	0.89
4	-0.11	1.04	-0.10	1.02
5	-0.58	3.39	-0.29	2.17
5-1	-0.90		-0.64	
	[-3.59]		[-3.75]	

The table reports mean and standard deviations of the $\beta_{\Delta VIX}$ quintile portfolios characteristic-matched by size and book-to-market ratios. Each month, each stock is matched with one of the Fama and French (1993) 25 size and book-to-market portfolios by appropriate size and book-to-market characteristics. The table reports value-weighted simple returns in excess of the characteristic-matched returns. The columns labelled 'Excluding Small, Growth Firms" exclude the Fama-French portfolio with the smallest stocks and the highest book-to-market ratio firms. The row 5-1 refers to the difference in monthly returns between portfolio 5 and portfolio 1. The sample period is from January 1986 to December 2000.

Table 4: Portfolios Sorted on $\beta_{\Delta VIX}$ Controlling for Liquidity, Volume and Momentum

Rank	Mean	Std Dev	CAPM Alpha	FF-3 Alpha
Panel A: Controlling for Liquidity				
1	1.57	5.47	0.21	0.19
			[1.31]	[1.34]
2	1.48	4.48	0.27	0.15
			[2.25]	[1.68]
3	1.40	4.54	0.15	0.09
			[1.59]	[0.97]
4	1.30	4.74	0.02	-0.02
			[0.21]	[-0.17]
5	0.89	5.84	-0.52	-0.36
			[-2.87]	[-2.09]
5-1	-0.68		-0.73	-0.55
	[-3.04]		[-2.99]	[-2.15]
Panel B: Controlling for Volume				
1	1.10	4.73	-0.11	-0.13
			[-0.58]	[-1.34]
2	1.18	4.01	0.08	-0.08
			[0.46]	[-0.92]
3	1.18	3.78	0.10	-0.04
			[0.66]	[-0.50]
4	0.98	4.18	-0.17	-0.23
			[-1.06]	[2.16]
5	0.38	5.31	-0.90	-0.71
			[-3.86]	[-4.84]
5-1	-0.72		-0.79	-0.58
	[-3.49]		[-3.22]	[-3.03]
Panel C: Controlling for Past 6 Month Returns				
1	1.14	5.49	-0.22	-0.22
			[-1.23]	[-1.45]
2	1.12	4.84	-0.15	-0.25
			[-1.17]	[-2.20]
3	1.21	4.83	-0.07	-0.13
			[-0.65]	[-1.14]
4	1.07	4.97	-0.24	-0.27
			[-2.00]	[-2.32]
5	0.31	6.06	-1.13	-0.93
			[-5.02]	[-5.53]
5-1	-0.83		-0.91	-0.71
	[-3.98]		[-3.69]	[-3.32]

In Panel A, we first sort stocks into five quintiles based on their historical liquidity beta, following Pástor and Stambaugh (2003). Then, within each quintile, we sort stocks based on their $\beta_{\Delta VIX}$ coefficient loadings into five portfolios. All portfolios are rebalanced monthly and value-weighted. The five portfolios sorted on $\beta_{\Delta VIX}$ are then averaged over each of the five liquidity beta portfolios. Hence, they are $\beta_{\Delta VIX}$ quintile portfolios controlling for liquidity. In Panels B and C, the same approach is used except we control for past trading volume and past 6-month returns, respectively. The statistics in the columns labelled Mean and Std Dev are measured in monthly percentage terms and apply to total, not excess, simple returns. The table also reports alphas from a CAPM and Fama-French (1993) regression. The row 5-1 refers to the difference in monthly returns between portfolio 5 and portfolio 1. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

Table 5: A Cross-Sectional Volatility Factor

	Mean	Std Dev	Auto
VOL	-0.58	3.29	-0.15

Correlation of Factors

	MKT	SMB	HML	UMD
SMB	0.17			
HML	-0.50	-0.50		
UMD	0.22	0.32	-0.46	
VOL	0.16	0.48	-0.40	0.22

Regressing VOL onto Various Factors

const	MKT	SMB	HML	UMD	Adj R^2
-0.68	0.11				0.02
[-2.74]	[1.03]				
-0.46	-0.01	0.34	-0.21		0.25
[-2.12]	[-0.09]	[3.07]	[-0.21]		
-0.47	-0.01	0.34	-0.21	0.01	0.25
[-1.94]	[-0.09]	[3.24]	[-1.36]	[0.08]	

The factor VOL is formed by sorting all stocks into three portfolios based on $\beta_{\Delta VIX}$ from the regression (4) run at a daily frequency using data over the previous month and the portfolios are rebalanced every month. The three value-weighted portfolios have breakpoints set at one-third and two-thirds of the $\beta_{\Delta VIX}$ coefficients for all stocks. We take the return difference between the top third and bottom third portfolios to form VOL. We report monthly summary statistics of VOL and correlations of VOL with respect to other factors SMB, HML, the size and value factors of Fama and French (1993), and the momentum factor UMD from Kenneth French's web site. All returns are expressed as simple percentage returns per month. We also report regressions of VOL onto various combinations of MKT, SMB, HML and UMD. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

Table 6: Estimating the Price of Volatility by Fama-MacBeth (1973)

Panel A: Test Portfolio Fama-French (1993) Alpha's

		Ranking on $\beta_{\Delta VIX}$				
		1 low	2	3	4	5 high
Ranking on β_{MKT}	1 low	-0.54	0.13	-0.25	0.10	-1.04
		[-1.06]	[0.63]	[-1.13]	[0.51]	[-3.76]
	2	-0.22	0.02	-0.39	-0.17	-0.57
		[-1.24]	[0.14]	[-2.20]	[-1.30]	[-2.41]
	3	-0.20	-0.06	-0.00	-0.39	-0.28
		[-1.08]	[-0.31]	[-0.01]	[-2.38]	[-1.56]
	4	0.26	-0.04	-0.03	-0.11	-0.61
		[1.04]	[-0.33]	[0.19]	[-0.99]	[-2.61]
	5 high	0.31	0.30	0.10	-0.10	-0.66
		[0.95]	[1.42]	[0.48]	[-0.41]	[-1.92]

Panel B: Fama-MacBeth (1973) Factor Premiums

MKT	SMB	HML	UMD	LIQ	VOL	Adj R^2
0.22						-0.02
[0.34]						
1.06	-0.80	0.13				0.50
[1.59]	[-1.82]	[0.26]				
1.25	-0.69	0.31	0.32			0.48
[1.76]	[-1.47]	[0.56]	[0.41]			
1.27	-0.77	0.60	0.11	-0.03		0.51
[1.67]	[-1.48]	[1.00]	[0.13]	[-1.41]		
1.18					-0.87	0.56
[1.78]					[-2.46]	
0.52	-0.82	-0.33			-0.83	0.67
[0.77]	[-1.85]	[-0.65]			[-2.37]	
0.59	-0.78	-0.26	0.22		-0.83	0.65
[0.86]	[-1.70]	[-0.49]	[0.29]		[-2.37]	
0.59	-0.87	0.04	-0.01	-0.03	-0.87	0.70
[0.79]	[-1.71]	[0.07]	[-0.02]	[-1.86]	[-2.26]	

In Panel A, we report Fama-French (1993) alpha's for the 25 portfolios sorted first on β_{MKT} and then on $\beta_{\Delta VIX}$. These 25 portfolios are used as test assets in estimating the factor premiums using Fama-MacBeth (1973) in Panel B. MKT is the excess return on the market portfolio, SMB and HML are the Fama-French (1993) size and value factors, UMD is the momentum factor from Kenneth French's website and LIQ is the aggregate liquidity measure from Pástor and Stambaugh (2003). VOL is the volatility factor. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

Table 7: Zero-Beta Straddle STR Returns and VOL Factor Regressions

	const	MKT	VOL	STR	Adj R^2
VOL Regressions	-0.28			0.01	0.03
	[-1.46]			[1.19]	
	-0.27	-0.04		0.01	0.03
	[-0.22]	[-0.74]		[1.15]	
STR Regressions	-9.96		3.73		0.03
	[-2.21]		[0.78]		
	-8.23	-2.95	2.64		0.15
	[-1.77]	[-1.42]	[0.89]		

We regress STR, the monthly returns of zero-beta straddle positions constructed by Coval and Shumway (2001), onto MKT and VOL and regress VOL onto MKT and STR. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 1995.

Table 8: Portfolios Sorted by Volatility

Rank	Mean	Std Dev	Turn-over	% Mkt Share	Size	B/M	CAPM Alpha	FF-3 Alpha
Portfolios Sorted by Total Volatility								
1	1.06	3.71	0.40	41.7%	4.66	0.88	0.14	0.03
							[1.84]	[0.53]
2	1.15	4.48	0.63	33.7%	4.70	0.81	0.13	0.08
							[2.14]	[1.41]
3	1.22	5.63	0.67	15.5%	4.10	0.82	0.07	0.12
							[0.72]	[1.55]
4	0.99	7.15	0.64	6.7%	3.47	0.86	-0.28	-0.17
							[-1.73]	[-1.42]
5	0.09	8.30	0.41	2.4%	2.57	1.08	-1.21	-1.16
							[-5.07]	[-6.85]
5-1	-0.97						-1.35	-1.19
	[-2.86]						[-4.62]	[-5.92]
Portfolios Sorted by Idiosyncratic Volatility Relative to the CAPM								
1	1.10	3.87	0.41	51.6%	4.82	0.86	0.15	0.09
							[2.17]	[1.88]
2	1.13	4.70	0.63	28.5%	4.71	0.80	0.07	0.05
							[1.38]	[0.78]
3	1.23	5.86	0.67	12.4%	4.06	0.82	0.05	0.09
							[0.54]	[1.11]
4	0.94	7.07	0.64	5.5%	3.41	0.87	-0.32	-0.25
							[-2.03]	[-2.45]
5	-0.01	8.19	0.41	2.1%	2.51	1.10	-1.29	-1.28
							[-5.32]	[-7.84]
5-1	-1.11						-1.44	-1.37
	[-3.38]						[-4.88]	[-7.25]
Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3								
1	1.04	3.83	0.41	53.5%	4.86	0.85	0.11	0.04
							[1.57]	[0.99]
2	1.16	4.74	0.64	27.4%	4.72	0.80	0.11	0.09
							[1.98]	[1.51]
3	1.20	5.85	0.68	11.9%	4.07	0.82	0.04	0.08
							[0.37]	[1.04]
4	0.87	7.13	0.65	5.2%	3.42	0.87	-0.38	-0.32
							[-2.32]	[-3.15]
5	-0.02	8.16	0.42	1.9%	2.52	1.10	-1.27	-1.27
							[-5.09]	[-7.68]
5-1	-1.06						-1.38	-1.31
	[-3.10]						[-4.56]	[-7.00]

We form value-weighted quintile portfolios every month by sorting stocks based on total volatility, idiosyncratic volatility relative to the CAPM and idiosyncratic volatility relative to the Fama-French (1993) model. Portfolios are formed every month, based on volatility computed using daily data over the previous month. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labelled Mean and Std Dev are measured in monthly percentage terms and apply to total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The numbers in the Turnover column list the average proportion of firms that leave the quintile portfolio each month. The row '5-1' refers to the difference in monthly returns between portfolio 5 and portfolio 1. The Alpha columns report Jensen's alpha with respect to the CAPM or Fama-French (1993) three-factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is July 1963 to December 2000.

Table 9: Alphas of Portfolios Sorted on Idiosyncratic Volatility (FF-3) Controlling for Various Effects

		Ranking on Idiosyncratic Volatility					
		1 low	2	3	4	5 high	5-1
NYSE Stocks Only		0.06	0.04	0.02	-0.04	-0.60	-0.66
		[1.20]	[0.75]	[0.30]	[-0.40]	[-5.14]	[-4.85]
Size Quintiles	1 small	0.11	0.26	0.31	0.06	-0.43	-0.55
		[0.72]	[1.56]	[1.76]	[0.29]	[-1.54]	[-1.84]
	2	0.19	0.20	-0.07	-0.65	-1.73	-1.91
		[1.49]	[1.74]	[-0.67]	[-5.19]	[-8.14]	[-7.69]
	3	0.12	0.21	0.03	-0.27	-1.49	-1.61
		[1.23]	[2.40]	[0.38]	[-3.36]	[-10.1]	[-7.65]
	4	0.03	0.22	0.17	-0.03	-0.82	-0.86
		[0.37]	[2.57]	[2.47]	[-0.45]	[-6.61]	[-4.63]
	5 large	0.09	0.04	0.03	0.14	-0.17	-0.26
		[1.62]	[0.72]	[0.51]	[1.84]	[-1.40]	[-1.74]
Controlling for Size		0.11	0.18	0.09	-0.15	-0.93	-1.04
		[1.30]	[2.49]	[1.35]	[-1.99]	[-6.81]	[-5.69]
Controlling for Book-to-Market		0.61	0.69	0.71	0.50	-0.19	-0.80
		[3.02]	[2.80]	[2.49]	[1.47]	[-0.48]	[-2.90]
Controlling for Leverage		0.11	0.11	0.08	-0.24	-1.12	-1.23
		[2.48]	[2.20]	[1.19]	[-2.45]	[-7.81]	[-7.61]
Controlling for Liquidity		0.08	0.09	-0.01	-0.16	-1.01	-1.08
		[1.71]	[1.53]	[-0.09]	[-1.62]	[-8.61]	[-7.98]
Controlling for Volume		-0.03	0.02	-0.01	-0.39	-1.25	-1.22
		[-0.49]	[0.39]	[-0.32]	[-7.11]	[-10.9]	[-8.04]
Controlling for Dispersion		0.12	-0.07	0.11	0.01	-0.27	-0.39
in Analysts' Forecasts		[1.57]	[-0.76]	[1.12]	[0.09]	[-1.76]	[-2.09]
Controlling for Momentum	Past 1-month	0.07	0.08	0.09	-0.05	-0.59	-0.66
		[0.43]	[0.94]	[1.26]	[-0.47]	[-3.60]	[-2.71]
	Past 6-month	-0.01	-0.12	-0.28	-0.45	-1.11	-1.10
		[-0.20]	[-1.86]	[-3.60]	[-5.20]	[-9.35]	[-7.18]
	Past 12-month	0.01	-0.05	-0.28	-0.64	-1.21	-1.22
		[0.15]	[-0.76]	[-3.56]	[-6.95]	[-11.5]	[-9.20]

Note to Table 9

The table reports Fama and French (1993) alpha's, with robust Newey-West (1987) t-statistics in square brackets. All the strategies are $1/0/1$ strategies, but control for various effects. The column '5-1' refers to the difference in FF-3 alpha's between portfolio 5 and portfolio 1. In the panel labelled 'NYSE Stocks Only', we sort stocks into quintile portfolios based on their idiosyncratic volatility, relative to the FF-3 model, using only NYSE stocks. We use daily data over the previous month and rebalance monthly. In the panel labelled 'Size Quintiles', each month we first sort stocks into five quintiles on the basis of size. Then, within each size quintile, we sort stocks into five portfolios sorted by idiosyncratic volatility. In the panels controlling for size, liquidity volume and momentum, we perform a double sort. Each month, we first sort stocks based on the first characteristic (size, book-to-market, leverage, liquidity, volume, dispersion of analysts' forecasts, or momentum) and then, within each quintile we sort stocks based on idiosyncratic volatility, relative to the FF-3 model. The five idiosyncratic volatility portfolios are then averaged over each of the five characteristic portfolios. Hence, they represent idiosyncratic volatility quintile portfolios controlling for the characteristic. Liquidity represents the Pástor and Stambaugh (2003) historical liquidity beta, leverage is defined as the ratio of total book value of assets to book value of equity and momentum represents past 1-month, 6-month or 12-month returns. The sample period is July 1963 to December 2000 for all controls with the exceptions of liquidity (February 1968 to December 2000) and the dispersion of analysts' forecasts (February 1983 to December 2000). All portfolios are value-weighted.

Table 10: Pricing Portfolios Sorted on Volatility

Panel A: Pricing Idiosyncratic Volatility (Relative to FF-3) Quintile Portfolios Using Various Factor Models

Model					GRS Test p-value	Portfolio Alpha's 1	2	3	4	5	5-1
MKT					0.000	0.30	-0.01	-0.17	-0.93	-1.92	-2.22
						[2.16]	[-0.13]	[-0.83]	[-3.02]	[-3.98]	[-3.73]
MKT	SMB	HML			0.000	0.13	-0.83	0.05	-0.48	-1.31	-1.43
						[1.64]	[-0.06]	[0.38]	[-2.82]	[-4.04]	[-3.91]
MKT	SMB	HML	UMD		0.000	0.12	-0.11	0.03	-0.48	-1.28	-1.36
						[1.63]	[-0.86]	[0.24]	[-2.74]	[-4.09]	[-3.37]
MKT				VOL	0.001	0.11	-0.10	-0.01	-0.56	-1.35	-1.47
						[1.28]	[-0.83]	[-0.04]	[-1.99]	[-3.45]	[-3.23]
MKT	SMB	HML		VOL	0.000	0.07	-0.11	0.03	-0.48	-1.28	-1.34
						[1.04]	[-0.86]	[0.24]	[-2.74]	[-4.09]	[-3.90]
MKT	SMB	HML	UMD	VOL	0.003	0.06	0.06	0.14	-0.40	-1.21	-1.26
						[0.98]	[0.63]	[1.13]	[-2.22]	[-3.49]	[-3.41]

Panel B: Factor Loadings and Alpha's

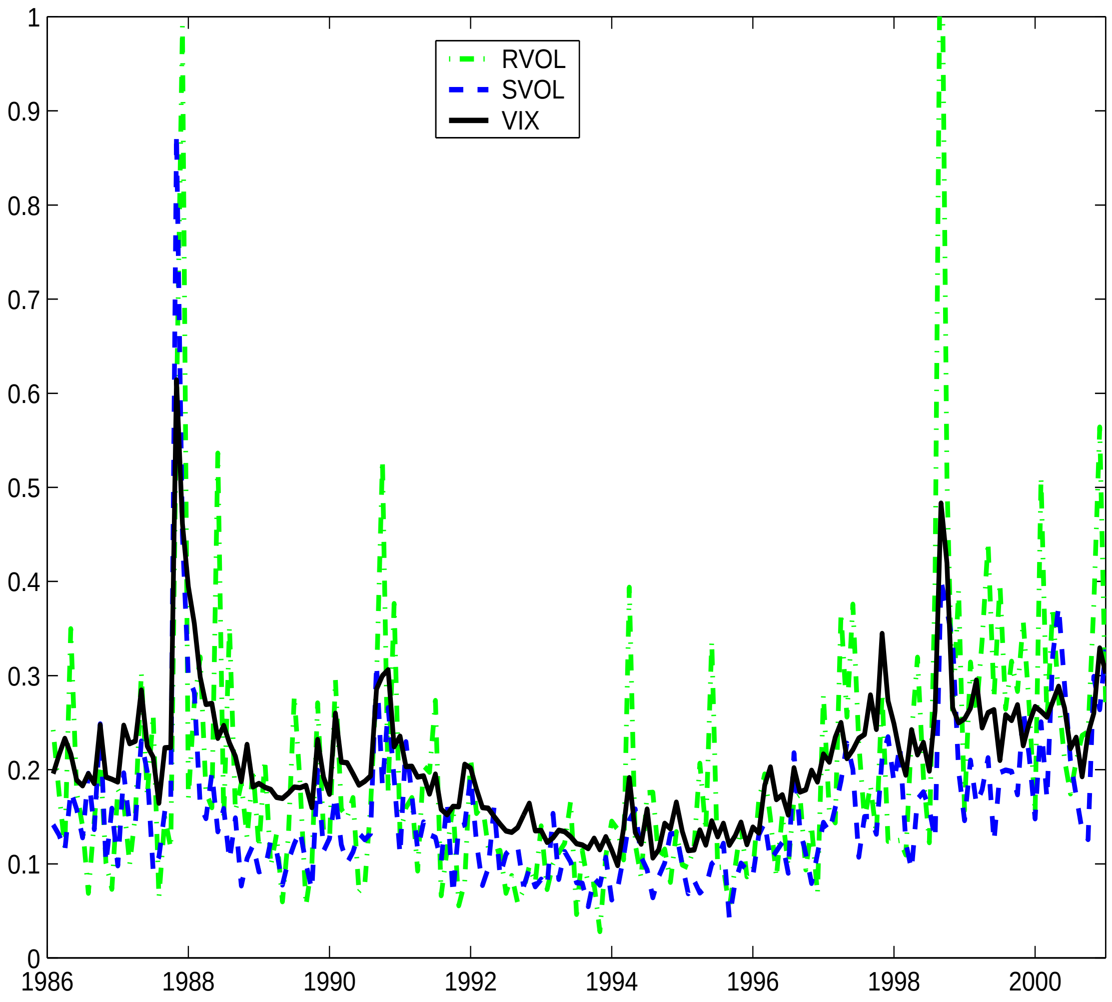
	Alpha	MKT	SMB	HML	WIN	LOS	VOL
1	0.05	1.02	-0.18	0.17	-0.03	-0.02	-0.13
	[0.85]	[17.6]	[-4.33]	[3.11]	[-0.85]	[-1.23]	[-2.42]
2	-0.04	0.89	-0.04	-0.01	0.07	0.09	-0.12
	[-0.37]	[11.3]	[-0.60]	[-0.28]	[1.48]	[3.47]	[-2.36]
3	0.06	0.83	0.38	-0.20	0.19	0.10	-0.04
	[0.52]	[11.8]	[6.07]	[-2.96]	[3.51]	[3.65]	[-0.87]
4	-0.39	0.97	0.81	-0.40	0.10	0.11	-0.01
	[-2.45]	[7.09]	[9.15]	[-3.60]	[1.35]	[2.60]	[-0.14]
5	-1.18	0.93	1.20	-0.43	0.05	0.09	0.07
	[-3.56]	[3.96]	[8.99]	[-2.03]	[0.29]	[1.18]	[0.35]
5-1	-1.23						
	[-3.47]						

In Panel A, we report a Gibbons-Ross-Shanken (1989) (GRS) test for pricing the quintile portfolios sorted by idiosyncratic volatility relative to FF-3, reported in the last panel of Table 8, for various factor models using combinations of the factors MKT, SMB, HML, UMD and VOL. UMD is the momentum factor from Kenneth French's website and VOL is the volatility factor. The columns labelled '1' through '5' report portfolio alpha's from each linear factor model. The column labelled '5-1' refers to the difference in monthly returns between portfolio 5 and portfolio 1. Panel B reports alpha's and factor loadings from a factor model with MKT, SMB, HML, a winner factor WIN, a loser factor LOS and the VOL factor. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

Table 11: Quintile Portfolios of FF-3 Idiosyncratic Volatility of $L/M/N$ Strategies

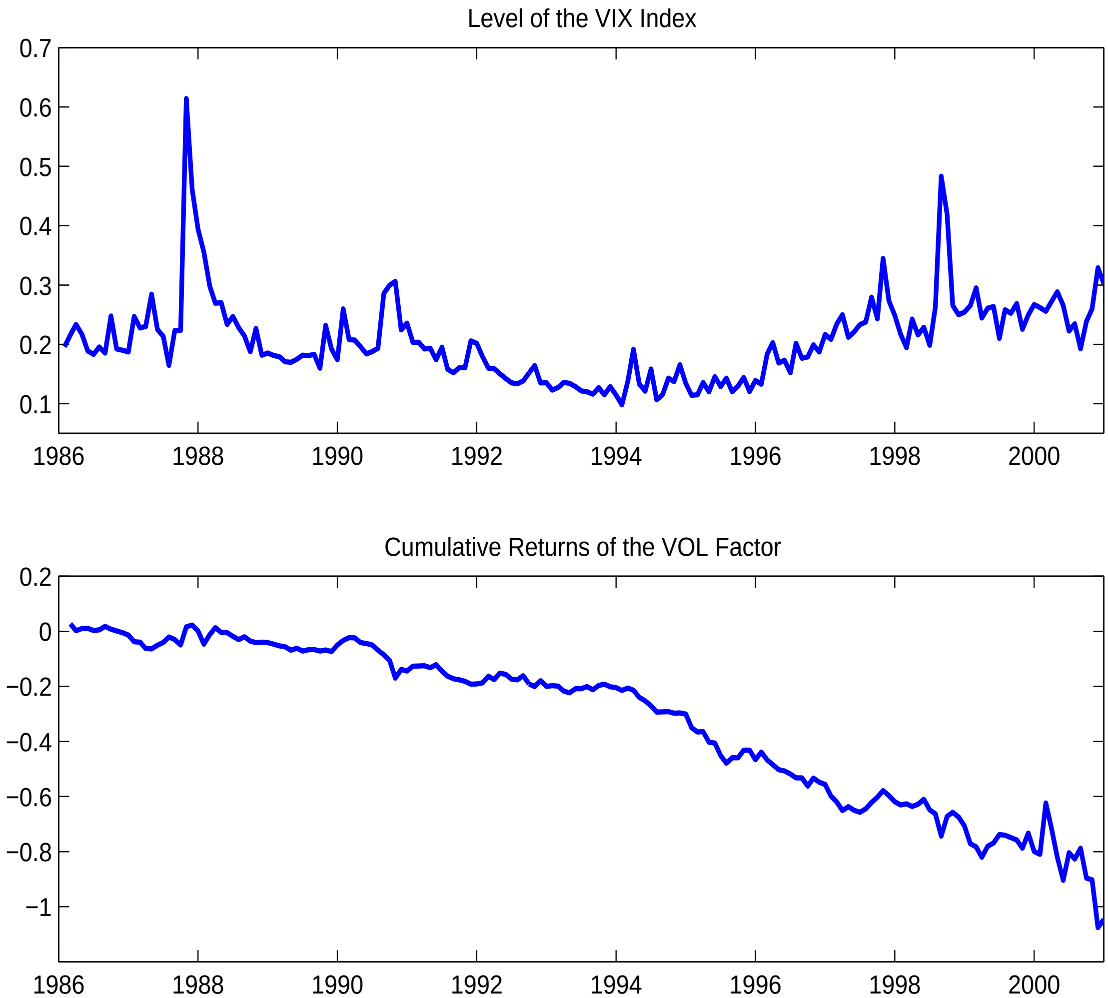
	Ranking on Idiosyncratic Volatility					
Strategy	1 low	2	3	4	5 high	5-1
$1/1/1$	0.06	0.04	0.09	-0.18	-0.82	-0.88
	[1.47]	[0.77]	[1.15]	[-1.78]	[-4.88]	[-4.63]
$1/1/12$	0.03	0.02	-0.02	-0.17	-0.64	-0.67
	[0.91]	[0.43]	[-0.37]	[-1.79]	[-5.27]	[-4.71]
$12/1/1$	0.04	0.08	-0.01	-0.29	-1.08	-1.12
	[1.15]	[1.32]	[-0.08]	[-2.02]	[-5.36]	[-5.13]
$12/1/12$	0.04	0.04	-0.02	-0.35	-0.73	-0.77
	[1.10]	[0.54]	[-0.23]	[-2.80]	[-4.71]	[-4.34]

The table reports Fama and French (1993) alpha's, with robust Newey-West (1987) t-statistics in square brackets. The column '5-1' refers to the difference in FF-3 alpha's between portfolio 5 and portfolio 1. We rank stocks into quintile portfolios of idiosyncratic volatility, relative to FF-3, using $L/M/N$ strategies. At month t, we compute idiosyncratic volatilities from the regression (6) on daily data over an L month period from months $t - L - M$ to month $t - M$. At time t, we construct value-weighted portfolios based on these idiosyncratic volatilities and hold these portfolios for N months, following Jegadeesh and Titman (1993), except our portfolios are value-weighted. The sample period is July 1963 to December 2000.



The figure shows $SVOL$, $RVOL$ and VIX, plotted at a monthly frequency. We annualize $SVOL$ and $RVOL$ by multiplying the daily series by $\sqrt{250}$. The sample period is January 1986 to December 2000.

Figure 1: $SVOL$ and VIX



The figure shows the level of the VIX index (top panel) and cumulative returns of the VOL factor (bottom panel). The sample period is January 1986 to December 2000.

Figure 2: Cumulative Returns of the VOL Factor