The Academic Effects of Patentable Research

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April 11, 2003

Abstract

We examine concerns as to whether recent changes in patent policy that increased both the ability and incentives for US universities to patent inventions have been detrimental to academic research and education. We analyze a model where a researcher allocates time between applied and basic research given administration choices of salary and teaching load. Her choice depends on a "composite" marginal rate of substitution of applied for basic research which takes into account both the utility of each type of research and its productivity in generating income and prestige. In equilibrium, if her license income is the same whether she remains in the university or not, an increase in her share leads to either no change in her optimal teaching load or salary, or a change in opposite directions. A decrease in the portion of patentable knowledge that can be used in education either has no effect on the optimal teaching load or her salary, or causes them to move in opposite directions. Results on the quality of education are typically ambiguous because changes in the teaching load induce changes in research that have an opposite effect on educational quality.

*We thank Tom Gresik for several helpful discussions and suggestions. We also thank Paul David, Irwin Feller, Josh Lerner, Richard Nelson, Paula Stephan, and seminar participants at the 2003 American Economic Assocation Meetings, the Midwest Economic Theory Meetings at the University of Wisconsin, the Conference on Innovation and Intellectual Property at the University of Brussels, CORE, and the EARIE meetings at the University of Juan Carlos III in Madrid. Thursby thanks the Alan and Mildred Peterson Foundation and the National Science Foundation (SES-0094573) for financial support.

1 Introduction

A sequence of related policy changes in the 1980's increased both the ability and incentive for U.S. universities to patent and license the results from faculty research. The Bayh-Dole Act of 1980 gave universities the right to own, patent, and license results of federally-funded research, and the Supreme Court approved patentability of genetically engineered bacteria and software, thereby expanding patentability of inventions in biological and computer sciences.[1] Since then there has been a dramatic increase in patent activity in U.S. universities. For example, the 86 universities responding to the Association of University Technology Managers (AUTM) *Survey* in 1991 and 1998 reported an increase in patent applications of 176 percent and licenses executed of 131 percent. While university administrators cite such figures as evidence of the increasing role of universities in economic development, others see the dramatic increase in patent licensing as problematic, potentially compromising the traditional role of universities in research and education.[2]

Two issues of concern are the nature of faculty research and the way results are disseminated. The conventional wisdom is that faculty have a comparative advantage and taste for basic research. Publication of this research creates a nonexcludable public good which serves as a basis for further research in industry as well as academia. Thus to the extent that financial incentives associated with licensing divert faculty from basic research and/or limit the dissemination of their work, it puts both current and future innovation at risk (Griliches 1988, Osano 1992).

Those concerned about the impact on education raise several issues. While they recognize that faculty involvement in patent licensing has the potential to improve curricula, they express concern that faculty may spend less time in education as they pursue entrepreneurial activities associated with licensing. Critics, in fact, worry that university administrators may

[1] Other patent reforms in the 1980s, such as the creation of the Federal Circuit Court and the lengthening of patent life for pharmaceuticals, also strengthened patent protection for businesses as well as universities. Here we focus on those changes that were particularly relevant for university inventions. See Gallini (2002) for a discussion of the implications of these reforms for businesses.

[2] These issues are sufficiently controversial to be the subject of a recent cover story of *Atlantic Monthly* titled "The Kept University" (Press and Washburn 2000). Moreover, Congress, the Science, Technology and Economics Policy (STEP) board of the National Research Council, and the President's Commission on Science and Technology have all undertaken review of university patenting.in the context of Bayh-Dole.

encourage such activity in the interest of capturing license income.[3] The most serious concern, however, is that the patent process may prevent or delay the use of research results in the lab or classroom, diminishing the quality of education (Stephan 2001, and Stephan and Levin 1996).

We construct a model of university research and education that allows us to examine these issues. The university administration determines faculty salaries and teaching loads, taking into account the productivity of faculty in research and education and the resulting income from licensing and tuition. Given the salary and teaching load, the faculty choose whether to work in the university or exit for their next best alternative and, if they remain, the allocation of their research time between basic and applied research. Basic research is freely published, adding to the scientific knowledge base and earning prestige for the faculty and university. Applied research is patentable, earning license income as well as prestige. Prestige and income provide utility for both the faculty and administration. Faculty, however, also receive utility simply from the time spent on research. Administration income depends on the quality of education, which in turn depends on the research output that is available for use in the classroom and the time the faculty spends on education.

Given the teaching load and salary, the researcher allocates her remaining time between applied and basic research. Whether she specializes in basic or applied research, or spends some time on each, depends on a "composite" marginal rate of substitution of applied for basic research which takes into account both the utility associated with each type of research and the productivity of this research in generating income and prestige. Consider, for example, someone who specializes in basic research. We show that the income incentives associated with patent licensing will induce this person to reallocate time to applied research only if this change increases the stock of patentable knowledge she produces. However, this is not sufficient because this reallocation reduces her production of scientific knowledge and its associated prestige, and it may reduce her utility from research *per se*. In fact, it is not clear that this substitution will increase the stock of patentable knowledge for faculty whose basic research yields both patentable and scientific knowledge (i.e., those whose work falls in the much publicized Pasteur's Quadrant (Stokes 1997)).

In an interior solution, an increase in her teaching load decreases time in

[3] Cash constrained universities in the United Kingdom explicitly encourage faculty to engage in income generating activities, such as applied research and/or consultancy, and tax the resulting income to increase university resources (see Beath *et al.* 2002).

both basic and applied research. A change in her salary, her share of license income, and either stock of knowledge results in a reallocation of time to applied research if and only if the resulting change increases the composite marginal rate of substitution of applied for basic research. In the case of quasi-linear utility, we show that a change in her salary has no effect on her allocation of time in applied and basic research, and an increase in her share of license income increases her allocation of time to applied research if and only if it is more productive in the creation of patentable knowledge than her time in basic research.

In equilibrium, the salary and teaching load are set so that the faculty researcher's participation constraint binds and the administration's willingness to pay her to teach an additional hour equals her composite marginal rate of substitution between research and income. Her participation constraint is positively sloped, so she must be paid a higher salary to carry a greater teaching load. Although we focus on interior solutions, one corner solution is noteworthy. It may be optimal for the administration to allow star scientists to specialize completely in research. In this case, offering them a higher salary to induce them to teach more reduces their production of both knowledge stocks and thus may lower the quality of education and reduce tuition revenue.

Comparative statics reveal that the equilibrium teaching load and salary chosen by the administration may or may not change in response to a change in the researcher's share of license income. If preferences are homothetic, we show that the researcher's share of license income has no effect on her teaching load, and thus no effect on her allocation of time between basic and applied research. In general, if a change in her share affects her utility within the university the same as her reservation utility, which is likely as she can continue to earn royalties after she exits, then there is no change in her participation constraint. In this case, the change in her share either has no effect on the optimal teaching load or her salary (as in the case of homothetic preferences), or causes them to move in opposite directions. Note, however, that a change in the teaching load does not affect the researcher's allocation of time between basic and applied research unless it changes the composite marginal rate of substitution between the two types of research.

We also examine the effect of changes in the researcher's initial stocks of scientific and patentable knowledge on the equilibrium teaching load and salary. If the prestige associated with scientific knowledge is greater within the university, then an increase in its initial stock typically decreases the salary and increases the teaching load, but in general cannot both increase the salary and decrease the teaching load. If the prestige associated with patentable knowledge is greater outside the university, then an increase in its initial stock typically increases the salary and decreases the teaching load, but in general cannot both decrease the salary and increase the teaching load. In the case of homothetic preferences, the salary will increase and the teaching load will decrease. Because the researcher's utility does not depend on the portion of patentable knowledge that can be disseminated in education, a change in this portion has no effect on her participation constraint. Thus a decrease in this portion either has no effect on the teaching load or her salary (as in the case of homothetic preferences), or causes them to move in opposite directions.

Thus, to the extent that the court's extension of patent protection to research in biological and computer sciences increased opportunities for faculty outside the university, it could have led to increased university salaries and lower teaching loads. Bayh-Dole would have reinforced this effect as administrators would be willing to pay more for less time spent in education because they could substitute license income for tuition. It is important to note that this need not have led to a decrease in the quality of education. The increase in patentable knowledge tended to increase the quality of education, *ceteris paribus*. There is evidence, however, of communication restrictions associated with commercialization which could have resulted in a decrease in the amount of this knowledge actually used in education. Blumenthal et al. (1997) and Louis et al. (2001) find that faculty in the life sciences who are involved with commercialization are more likely to withhold information from colleagues than other faculty. In Thursby and Thursby's (2002) survey of businesses who license from universities, 50 percent of the respondents reported that their contracts specified delays of publication and rights to delete information. To the extent that this is the case, our results show that the critics have legitimate concerns because the university administration's incentive to reduce teaching loads would have to be offset by increases in the amount of patentable knowledge used in education in order to avoid a decrease in the quality of education.

These results contribute to the growing literature on university patenting, which with few exceptions has abstracted from the implications for basic research and education.[4] Much of the literature focuses on the role of patent

[4] The related literature on university-industry collaboration has examined the effects on research, but not education. Cohen *et.al* (1998) provide a useful review. Cohen *et.al's* (1994) survey of university-industry research centers (UIRCs) provides evidence of the countervailing effects of industry collaboration on faculty productivity, with so-called commercial outputs of research increasing and publications decreasing (except in biotechnology). On the other hand, Mansfield's (1995) study of 321 academic researchers shows a

citations and patent licensing in the transfer of research to industry (see, for example, Henderson et al. 1998, Jaffe et al. 1993, Jensen and Thursby 2001, Mowery *et al.* 2001 a, b). Thursby and Thursby (2002) provide evidence to support the view that entrepreneurial university administrations and faculty research orientation have both been factors in increased university patenting, but they do not address issues related to education. With the exception of recent work by Beath et al. (2000), studies of the tension between basic and applied research have focused on the behavior of profit maximizing firms (Osano 1992, Cockburn, Henderson, and Stern 1999, and Stern 1999). Beath et al. (2000) show that universities may want to allow faculty to conduct applied research on a consulting basis in order to ease the university's budget constraint, however, they do not examine the implications of such work for the quality of university education.

Our results also contribute to the literature on teaching, research, and educational quality. Recent work by del Ray (2001) examines how different schemes for financing undergraduate education affect the quality of education. In this, as well as earlier work (see Barooah 1991), the quality of education is determined by the quality of students admitted and the resources devoted to teaching. By contrast, we model quality as a function of the knowledge that can be shared in the classroom as well as the amount of time devoted to teaching.

2 Model

We consider a problem with a university that produces three outputs: scientific knowledge k, which results from basic research; patentable knowledge p, which results from applied research; and education of quality q. We assume that advances in scientific knowledge can be disseminated immediately, either through publication or use in education. In contrast, advances in patentable knowledge are not disseminated until the patent application is filed. As a result, only a portion, $\theta \in (0, 1)$, of the stock of patentable knowledge is used in education. Moreover, its commercial use is restricted to firms who acquire a license from the university

Production of the three outputs requires use of a university researcher, F, who is a member of the faculty. In any period, the researcher can work for the university or exit to her next best alternative. That is, we assume

complementarity between consulting and basic research. Similarly, Zucker *et. al.*(1994, 1998) found that the most productive scientists in biotechnology tend to capitalize on commercial applications of their basic research.

a principal-agent relationship in which the university administration is the principal and the researcher is the agent.[5] If she works for the university, she allocates her time between education and the two types of research. We normalize time available in each period to one, so that a is the fraction of time devoted to applied research in the university, b is the fraction of time devoted to basic research, and $e = 1 - a - b$ is her teaching load, the fraction of time spent in education. As is standard in science and engineering departments, the teaching load is defined by total student contact hours, which includes not only classroom instruction, but also other activities such as advising dissertations or other lab research. If she exits, she receives her reservation utility. In this context, however, her reservation utility is not an exogenously fixed value because her scientific and patentable knowledge affects the utility of her next best alternative.

The timing of decision-making unfolds as follows. The university administration moves first, determining a salary and teaching load for the researcher. Given this information, the researcher decides whether to remain in the university or exit to her next best alternative. If she remains in the university, she decides how to allocate her remaining time between basic and applied research. The production of knowledge and education then occurs. Her time in each type of research and the initial knowledge stocks determine the increments to those knowledge stocks. Because production of research and education occurs concurrently, those parts of the increments to knowledge stocks that can be disseminated affect educational quality. Thus, her time in education and the final knowledge stocks disseminated determine educational quality, and so the university's income from tuition. The final stock of patentable knowledge determines license income, which is split between the faculty and the university. All payoffs are received at the end of the period. We assume that inputs and outputs are perfectly observable by the administration, while inputs and outputs that can be disseminated are perfectly observable by students, so that we abstract from any moral hazard issues in the provision of research or education.[6] Under this assumption, students can accurately predict the increments to the stocks of knowledge disseminated, and so we could equivalently assume that both tuition and salary are paid up front. This is not to say that moral hazard issues are not

[5]This implies that the university administration assumes it can hire a replacement researcher who can take full advantage of the existing stocks of scientific and patentable knowledge. Empirical evidence supporting this principal-agent relationship can be found in Jensen *et al.* (2003).

[6]For a seminal analysis of contract design when agents perform multiple tasks with asymmetric information, see Holmstrom and Milgrom (1991).

important in the provision of both research and education. Rather, because there are no theoretical studies of the tensions among basic research, applied research, and educational quality in a university setting, we construct a tractable model that is no more complex than necessary to analyze the problem.

2.1 Preferences

Researchers have preferences that depend on both pecuniary and nonpecuniary sources: income, time in research, and the prestige resulting from successful research. In particular, we assume the researcher's utility is given by $U_F(a, b; Y_F, p, k)$ where Y_F is her income. The assumption that utility is a function of time spent in research is consistent with the notion that researchers who do basic research may have a taste for it (see Stern (1999)). We take this idea one step further by assuming that some researchers may also have a taste for applied research. The faculty we consider get utility from simply working on research problems. That is, we focus on faculty who view research as a "...puzzle-solving operation in which the solution of the puzzle is its own reward" (Hagstrom (1965, p. 16); see also Kuhn (1970)). There is substantial evidence that such researchers also get utility from the prestige associated with the successes of their research (see Stephan (1996) for a survey). We use the stocks of knowledge as measures of this prestige, so that past success in either basic or applied research generates additional utility, independently of whether it generates additional income. By not including time spent teaching in the utility function, we assume that research provides greater utility than teaching. Finally, we assume that $U_F(a, b; Y_F, p, k)$ has nonnegative marginal utilities and is strictly concave.

As a faculty member, the researcher earns income from her university salary and her share of the university's license income from its patentable knowledge. License income depends on the stock of patentable knowledge, $L = L(p)$. We denote the salary by s, and the share of license income for the researcher by ϕ, so current income for a faculty researcher is

$$Y_F(s, \phi, p) = s + \phi L(p). \tag{1}$$

Naturally we assume that $L(0) = 0$ and $L'(p) > 0 > L''(p)$ for $p \geq 0$. The assumption that payoffs are received at the end of the period, after production, implies that research affects current utility directly through increases in the knowledge stocks and indirectly through the effect of the increases in patentable knowledge on license income.

From the administration's perspective, all three outputs bring prestige and income to the university. The administration is assumed to maximize utility subject to its net income constraint and the researcher's participation constraint. The administration's utility is given by $U_A(Y_A; p, k)$, where Y_A is the university's net income. Again, we assume that $U_A(Y_A; p, k)$ has nonnegative marginal utilities and is strictly concave. In each period the university earns enrollment income, T, which depends on the quality of education, q. For simplicity we assume the faculty researcher or replacement is the only variable input, so the university's net income is

$$Y_A(s, \phi, p, q) = T(q) + (1 - \phi)L(p) - s. \tag{2}$$

2.2 Production

Production of knowledge depends upon both types of research and the current knowledge base, (ρ, κ), the initial stocks of patentable and scientific knowledge. We assume the production of patentable knowledge is given by $p = P(a, b; \rho, \kappa)$ and the production of scientific knowledge is given by $k = K(a, b; \rho, \kappa)$, where p and k are the final stocks (i.e., the increments are $p - \rho$ and $k - \kappa$). The production functions P and K are assumed to be increasing and jointly concave in their arguments. We therefore allow for research in "Pasteur's Quadrant," where a researcher's basic research produces both patentable and scientific knowledge, as well as the type of research characterized by Mansfield in which applied research has a positive impact on the researcher's basic research agenda (Mansfield 1995 and Stokes 1997).

The production of educational quality depends on the time the researcher spends in education as well as the stocks of knowledge that can be freely disseminated. One provision of the Bayh-Dole Act is that in order to claim title to federally funded inventions, universities must file patent applications. Thus patentable knowledge may not be used in education until the patent application is filed or perhaps even granted. To capture this, we assume the patentable knowledge that can be disseminated currently is $d = \rho + \theta(p - \rho)$. We then assume that the production of educational quality is given by $q = Q(e; d, k)$, which we assume is increasing and jointly concave in its arguments.

This is one way to capture the controversy surrounding Bayh-Dole and related changes in patent policy regarding educational quality. There would be no incentive to delay dissemination of patentable knowledge if the university did not own the patent rights, so $\theta = 1$ and educational quality would be $q = Q(e; p, k)$. Moreover, critics have argued that this legislation can lead to a substitution of applied research for education. This approach also allows us to highlight the fact that there is a trade-off between reduced time in education and increases in knowledge that result from increased research on the quality of education.

3 Equilibrium

Given the assumed principal-agent relationship, we begin with an analysis of the researcher's optimal allocation of time between applied and basic research for a given salary and teaching load chosen by the administration. We then analyze the administration's optimal choice of salary and teaching load, and finally consider comparative statics of the equilibrium.

3.1 The Allocation of Time Between Basic and Applied Research

The researcher's problem if she remains with the university is to allocate her time between basic and applied research to maximize utility subject to the time constraint $e = 1 - a - b$, the definition of income (1), and production relations $p = P(a, b; \rho, \kappa)$ and $k = K(a, b; \rho, \kappa)$. Note that the time the researcher must spend in education and the salary, as well as the initial knowledge stocks, are given when she makes her choices. Substituting the production functions, her utility function can be rewritten as

$$W_F(a, b; s, \phi, \rho, \kappa) = U_F(a, b, Y_F(s, \phi, P(a, b; \rho, \kappa)), P(a, b; \rho, \kappa), K(a, b; \rho, \kappa)) \tag{3}$$

and her problem restated as

$$\max_{(a,b)} W_F(a, b; s, \phi, \rho, \kappa) \text{ subject to } a + b = 1 - e. \tag{4}$$

This approach allows us to work with a "composite" utility function that embeds the production activities of the researcher. We assume that this function is strictly quasi-concave in (a, b). The corresponding composite marginal rate of substitution of applied for basic research is $M(a, b; s, \phi, \rho, \kappa) = \frac{\frac{\partial W_F(a,b;s,\phi,\rho,\kappa)}{\partial a}}{\frac{\partial W_F(a,b;s,\phi,\rho,\kappa)}{\partial b}}$, which we assume is decreasing in a and increasing in b. We thus obtain the following result.

Theorem 1 *Let $a^*(e, s, \phi, \rho, \kappa)$ and $b^*(e, s, \phi, \rho, \kappa)$ solve the researcher's optimization problem. Then the researcher specializes in basic research, $a^* = 0$*

and $b^* = 1 - e$, *if*

$$\frac{\partial W_F(0, 1-e; s, \phi, \rho, \kappa)}{\partial b} \geq \frac{\partial W_F(0, 1-e; s, \phi, \rho, \kappa)}{\partial a}, \tag{5}$$

and specializes in applied research, $a^* = 1 - e$ *and* $b^* = 0$, *if*

$$\frac{\partial W_F(1-e, 0; s, \phi, \rho, \kappa)}{\partial a} \geq \frac{\partial W_F(1-e, 0; s, \phi, \rho, \kappa)}{\partial b}. \tag{6}$$

Otherwise, if (5) and (6) do not hold, then the researcher devotes some time to each type of research, $a^* > 0$, $b^* > 0$, *and* $a^* + b^* = 1 - e$, *where*

$$\frac{\partial W_F(a^*, b^*; s, \phi, \rho, \kappa)}{\partial a} = \frac{\partial W_F(a^*, b^*; s, \phi, \rho, \kappa)}{\partial b}. \tag{7}$$

Although there are a variety of ways to characterize and interpret these results, the one that seems most natural uses the composite marginal utilities,

$$\frac{\partial W_F}{\partial a} = \frac{\partial U_F}{\partial a} + \frac{\partial U_F}{\partial Y_F}\phi L'(P)\frac{\partial P}{\partial a} + \frac{\partial U_F}{\partial p}\frac{\partial P}{\partial a} + \frac{\partial U_F}{\partial k}\frac{\partial K}{\partial a} \tag{8}$$

and

$$\frac{\partial W_F}{\partial b} = \frac{\partial U_F}{\partial b} + \frac{\partial U_F}{\partial Y_F}\phi L'(P)\frac{\partial P}{\partial b} + \frac{\partial U_F}{\partial p}\frac{\partial P}{\partial b} + \frac{\partial U_F}{\partial k}\frac{\partial K}{\partial b}. \tag{9}$$

The first term on the right hand side of (8) and (9) reflects the researcher's love for research and is a pure utility effect, while the others reflect not only her utility but also her productivity in generating knowledge. The second term reflects her ability to earn license income from producing patentable research and can be thought of as her "love of money." The last two are pure "ego" effects showing the reputation effects associated with producing patentable and scientific knowledge.

This result also allows us to show the complexity of the response of the researcher to the Bayh-Dole Act and other similar policies. For example, suppose she specializes in basic research, so $\frac{\partial W_F}{\partial a} - \frac{\partial W_F}{\partial b} \leq 0$, or

$$\left[\frac{\partial U_F}{\partial a} - \frac{\partial U_F}{\partial b}\right] + \left[\frac{\partial U_F}{\partial Y_F}\phi L'(P) + \frac{\partial U_F}{\partial p}\right]\left[\frac{\partial P}{\partial a} - \frac{\partial P}{\partial b}\right] + \frac{\partial U_F}{\partial k}\left[\frac{\partial K}{\partial a} - \frac{\partial K}{\partial b}\right] \leq 0 \tag{10}$$

at $(a^*, b^*) = (0, 1-e)$. Although the direct effect of an increase in ϕ is to increase her utility by increasing income, this will not increase $\frac{\partial W_F}{\partial a} - \frac{\partial W_F}{\partial b}$, and thereby increase the likelihood that she will begin to allocate time to applied research, unless $\frac{\partial P}{\partial a} > \frac{\partial P}{\partial b}$ at $(a^*, b^*) = (0, 1-e)$. Because $a + b$

is fixed, the condition $\frac{\partial P}{\partial a} > \frac{\partial P}{\partial b}$ says that the first hour that is reallocated from basic to applied research must increase the production of patentable knowledge. This may seem, *a priori*, quite likely. If so, however, this reallocation is similarly likely to decrease the production of scientific knowledge, $\frac{\partial K}{\partial a} < \frac{\partial K}{\partial b}$. And, of course, if this researcher likes basic research more, then it is possible that $\frac{\partial U}{\partial a} < \frac{\partial U}{\partial b}$ as well. Thus, even if this reallocation would increase the production of patentable knowledge, and so her income and prestige, then it also could decrease the production of scientific knowledge, her prestige, and her love of research. It is worth noting that $\frac{\partial P}{\partial a} - \frac{\partial P}{\partial b}$ is lower the greater is the extent to which her research fits in the so-called "Pasteur's Quadrant," in which case a reallocation of time to applied research is less likely to increase patentable knowledge. It is, therefore, not obvious that a faculty member who specializes in basic research will change her research agenda in response to Bayh-Dole licensing.

We can also convert (10) into an expression in the usual marginal rates of substitution. For example, define $m_{xb} = \frac{\frac{\partial U_F}{\partial x}}{\frac{\partial U_F}{\partial b}}$ as the marginal rate of substitution between x and time in basic research for $x = a, Y_F, p, k$, and $m_p = \frac{\frac{\partial P}{\partial a}}{\frac{\partial P}{\partial b}}$ and $m_k = \frac{\frac{\partial K}{\partial x}}{\frac{\partial K}{\partial b}}$ as the marginal rates of technical substitution in the production of patentable and scientific knowledge. Then specialization in basic research occurs if $[m_{ab} - 1] + [m_{Yb}\phi L'(P) + m_{pb}][m_p - 1] + m_{kb}[m_k - 1] \leq 0$ at $(a^*, b^*) = (0, 1-e)$, and an increase in ϕ now will not make it more likely that she begins to do applied research unless $m_p > 1$ at $(a^*, b^*) = (0, 1-e)$.

Although the preceding discussion has focused on a researcher who specializes in basic research, it is also possible for her to specialize in applied research or to do some of each. In what follows we shall focus on the case of an interior solution, in which case the comparative statics have the expected properties.

Theorem 2 *In an interior solution to* (4)*, an increase in time spent in education decreases time in both basic and applied research. A change in the salary, the share of license income, and either stock of knowledge implies an increased allocation of time to applied research if and only if the resulting change in the composite marginal utility of time in applied research is greater than the resulting change in the composite marginal utility of time in basic research (i.e., if the change increases the composite marginal rate of substitution of applied for basic research).*

Setting $P^*(e,s,\phi,\rho,\kappa) = P(a^*,b^*;\rho,\kappa)$ and $K^*(e,s,\phi,\rho,\kappa) = K(a^*,b^*;\rho,\kappa)$, the researcher's indirect utility function is then

$$V_F(e,s,\phi,\rho,\kappa) = U_F(a^*,b^*;s+\phi L(P^*),P^*,K^*) = W_F(a^*,b^*;s,\phi,\rho,\kappa). \tag{11}$$

For any given level of utility, v, the equation $V_F(e,s,\phi,\rho,\kappa) = v$ implicitly defines an indifference curve in education-salary space for the inventor, $I_F(e,\phi,\rho,\kappa,v)$. The properties of this indirect utility function and its indifference curve map are crucial to understanding the university administration's choices.

Theorem 3 *The researcher's indirect utility function is increasing in the salary, her share of license income, and both knowledge stocks, but decreasing in her time spent in education:* $\frac{\partial V_F}{\partial s} > 0$, $\frac{\partial V_F}{\partial \phi} > 0$, $\frac{\partial V_F}{\partial \rho} > 0$, $\frac{\partial V_F}{\partial \kappa} > 0$, *and* $\frac{\partial V_F}{\partial e} < 0$. *Thus, a researcher's typical indifference curve in* (e,s) *space is positively sloped and decreasing in her share of license income and both knowledge stocks.*

Each indifference curve is positively sloped because she must be paid a higher salary to accept a heavier teaching load. It is worth noting that its slope is

$$\frac{\partial I_F(e,\phi,\rho,\kappa,v)}{\partial e} = \frac{\frac{\partial W_F}{\partial a}}{\frac{\partial U_F}{\partial Y_F}} = \frac{\frac{\partial W_F}{\partial b}}{\frac{\partial U_F}{\partial Y_F}}, \tag{12}$$

which is just the marginal rate of substitution between time in applied (or basic) research and income. An increase in her share of license income or either knowledge stock shifts each indifference curve down since the utility associated with any point in (e,s) space increases.

She also has the option of exiting the university for other opportunities. If she does, then we assume her reservation utility is $\bar{U}_F(\phi,\rho,\kappa)$. Her best external option must depend on the initial stock of patentable knowledge and her share of the revenue from it because she does not forfeit her license income when she departs. It also depends on her initial stock of scientific knowledge because her external options may depend upon the prestige associated with that stock, which she is free to use in any alternative employment opportunity. For any (ϕ,ρ,κ) she remains in the university only if her participation constraint,

$$V_F(e,s,\phi,\rho,\kappa) \geq \bar{U}_F(\phi,\rho,\kappa), \tag{13}$$

is satisfied. There is a specific indifference curve for which (10) binds, which is characterized in the following result.

Theorem 4 *Assume there exist s_0 and s_1 such that $s_1 > s_0 > 0$,*

$$V_F(0, s_0, \phi, \rho, \kappa) = \bar{U}_F(\phi, \rho, \kappa), \tag{14}$$

and

$$V_F(1, s_1, \phi, \rho, \kappa) = \bar{U}_F(\phi, \rho, \kappa). \tag{15}$$

Then $V_F(e, s, \phi, \rho, \kappa) = \bar{U}_F(\phi, \rho, \kappa)$ defines a function $S(e, \phi, \rho, \kappa)$ where S maps $[0, 1]$ onto $[s_0, s_1]$ such that the participation constraint binds for all $(e, s) = (e, S(e, \phi, \rho, \kappa))$ and:
(1) S is increasing in the time spent in education, $\frac{\partial S}{\partial e} > 0$.
(2) S is increasing in the inventor's share or either knowledge stock if and only if a change in that parameter has a greater impact on her reservation utility than her utility within the university, $\frac{\partial S}{\partial x} > 0$ if and only if $\frac{\partial \bar{U}_F}{\partial x} > \frac{\partial V_F}{\partial x}$ for $x = \phi, \rho, \kappa$.

That is, $S(e, \phi, \rho, \kappa) = I_F(e, \phi, \rho, \kappa, \bar{U}_F(\phi, \rho, \kappa))$ is the indifference curve where the participation constraint (13) binds. The conditions in (14) and (15), together with the results of Theorem 3, state that she will not work for the university, even if she does not have to teach, unless she is paid a salary of at least s_0. Similarly, she must be paid a greater salary of at least s_1 if the university wants her to teach full time. This indifference curve must be positively sloped, of course, $\frac{\partial S}{\partial e} > 0$. An example is depicted in Figure 1, where her utility increases to the northwest. For any point (e, s) on $S(e, \phi, \rho, \kappa)$, the participation constraint binds and she is indifferent between remaining in the university and exiting. For any point above $S(e, \phi, \rho, \kappa)$, she remains in the university because she earns rents, $V_F(e, s, \phi, \rho, \kappa) > \bar{U}_F(\phi, \rho, \kappa)$, but for any point below she exits because $V_F(e, s, \phi, \rho, \kappa) < \bar{U}_F(\phi, \rho, \kappa)$.

A change in any of the parameters ϕ, p, or k has an uncertain effect on the location of $S(e, \phi, \rho, \kappa)$ because an increase in any of these parameters not only increases the utility she associates with any (e, s) if she remains in the university, but also increases her reservation utility if she exits. That is, the sign of $\frac{\partial S}{\partial x}$ is given by the sign of $\frac{\partial \bar{U}_F}{\partial x} - \frac{\partial V_F}{\partial x}$ for $x = \phi, \rho, \kappa$. If the effect on her reservation utility is greater, then $S(e, \phi, \rho, \kappa)$ shifts up in (e, s) space. Conversely, if the effect on her utility within the university is greater (as we might expect, for example, if the knowledge was "firm-specific"), then $S(e, \phi, \rho, \kappa)$ shifts down in (e, s) space.

3.2 The Administration's Choice of Researcher Salary and Teaching Load

Because the university administration's net income and prestige depend on the researcher's optimal choices, which in turn depend on time spent in education and her salary, his utility can also be expressed as a function her time in education and her salary (as well as the parameters of the model). That is, setting $Q^*(e, s; \phi, \theta, \rho, \kappa) = Q(e; \rho + \theta(P^* - \rho), K^*)$, using (2), and substituting P^*, K^*, and Q^* gives the administration's "reduced-form" utility function

$$V_A(e, s; \phi, \theta, \rho, \kappa) = U_A(Y_A(s, \phi, P^*, Q^*)), P^*, K^*). \tag{16}$$

For a given level of utility, v, the equation $V_A(e, s; \phi, \theta, \rho, \kappa) = v$ implicitly defines an indifference curve in education-salary space for the administration, $I_A(e; \phi, \theta, \rho, \kappa, v)$. The properties of this utility function and its associated indifference curves are important in understanding the administration's choices. However, because it depends on the researcher's optimal behavior, it is not readily apparent how $V_A(e, s; \phi, \theta, \rho, \kappa)$ varies as a function of its arguments under the assumptions made thus far. This is not uncommon, of course, for principal-agent problems. Nevertheless, we can draw the following conclusions.

Theorem 5 *Administration utility in* (16) *is increasing in the portion of patentable knowledge disseminated in education,* $\frac{\partial V_A}{\partial \theta} > 0$*, but ambiguously related to its other arguments. However, absent current effects on the knowledge stocks, administration utility is increasing in the time spent by the researcher in education and both knowledge stocks, but decreasing in her salary and her share of license income:* $\frac{\partial V_A}{\partial e} > 0$*,* $\frac{\partial V_A}{\partial k} > 0$*, and* $\frac{\partial V_A}{\partial p} > 0$*, but* $\frac{\partial V_A}{\partial s} < 0$ *and* $\frac{\partial V_A}{\partial \phi} < 0$*. In this case, a typical indifference curve in* (e, s) *space is positively sloped, increasing in both knowledge stocks and the portion of patentable knowledge disseminated, but decreasing in her share of license income.*

The only general result we find is that administration utility is increasing in the portion of patentable knowledge disseminated in education. The reason this result is unambiguous is that such a change does not cause the researcher to change her allocation of time between applied and basic research, but instead merely causes a change the quality of education and tuition revenue. Thus, if recent legislation reduces the dissemination of patentable knowledge, then *ceteris paribus* educational quality and administration utility decrease.

The difficulty in obtaining unambiguous results for how the administration's utility varies with his choice variables when the researcher's optimal behavior is taken into account stems from the obvious sources. For example, suppose the administration increases her teaching load e. The direct effect is to increase the quality of education, as $\frac{\partial Q}{\partial e} > 0$, and thus increase tuition revenue and his utility. However, from Theorem 2, this also decreases the time she spends in both applied and basic research and thus the production of both types of knowledge. This in turn tends to decrease his utility by decreasing tuition revenue, licensing income, and the prestige of those knowledge stocks. These "second-order" effects could dominate and lead to lower administration utility. Similarly, an increase in her salary s has the direct effect of reducing his net income and utility, but also may induce a change in her research behavior. This might lead her to allocate more time to basic research, thus increasing the production of scientific knowledge, but decreasing the production of patentable knowledge and the license income for it. Conceivably the latter losses could dominate.

To confirm our intuition, we consider the case where research does not affect the current stocks of knowledge. This can be interpreted as the case where production simply takes time, so increments on the knowledge stocks are unavailable until the next period; i.e., $P^* = \rho$, $K^* = \kappa$, and $d = \theta\rho$. However, it can also be interpreted as the case of a faculty member who is no longer productive in research. In this case $V_A(e, s; \phi, \theta, \rho, \kappa)$ is increasing in e and decreasing in s, so his indifference curve in (e, s) space is positively sloped, $\frac{\partial I_A(e;\phi,\theta,\rho,\kappa,v)}{\partial e} = T'(q)\frac{\partial Q^*(e,s;\phi,\theta,\rho,\kappa)}{\partial e} > 0$. The slope of his indifference curve is the marginal effect of her teaching load on educational quality and tuition income, or her marginal revenue product in education. This represents the additional salary he is willing to pay to induce her to spend more time in education. Notice that this slope depends only on the trade-off in his net income between salary and tuition revenue.

In general, however, the slope of his indifference curve is

$$\frac{\partial I_A(e; \phi, \theta, \rho, \kappa, v)}{\partial e} = -\frac{\frac{\partial V_A(e,s;\phi,\theta,\rho,\kappa)}{\partial e}}{\frac{\partial V_A(e,s;\phi,\theta,\rho,\kappa)}{\partial s}}, \tag{17}$$

which has an ambiguous sign. As noted above, the marginal contributions of her teaching load and salary to his utility depend not just on their effects on his net income, but also their effects on the increments to the stocks of knowledge, and thus his license income and prestige. In what follows, we shall assume that administration utility is increasing in the researcher's time in education and decreasing in her salary over the relevant range, $\frac{\partial V_A}{\partial e} < 0$

and $\frac{\partial V_A}{\partial s} > 0$ for $e \in [0,1]$ and $s \in [s_0, s_1]$, so his indifference curves are also positively sloped. That is, we assume that the first-order effects of a change in her teaching load on educational quality and his tuition revenue and a change in her salary on his net income predominate the second-order effects of changes in her time allocation between applied and basic research on current production of incremental knowledge.

It then follows from the preceding theorem that her participation constraint (13) must bind if she is hired by the university. If not, then a small increase in e and/or decrease in s will increase his utility without inducing her to exit the university. The administration's problem thus becomes

$$\max_{e\in[0,1],s\in[s_0,s_1]} V_A(e,s;\phi,\theta,\rho,\kappa) \text{ subject to } V_F(e,s,\phi,\rho,\kappa) = \bar{U}_F(\phi,\rho,\kappa), \tag{18}$$

or choose the point $(e, S(e,\phi,\rho,\kappa))$ on the researcher's participation constraint that maximizes his utility.

Theorem 6 *Assume that* (14) *and* (15) *hold, that* $V_F(e,s,\phi,\rho,\kappa)$ *is strictly quasi-concave in* (e,s)*, and that* $V_A(e,s;\phi,\theta,\rho,\kappa)$ *is increasing in* e *and decreasing in* s *for* $e \in [0,1]$ *and* $s \in [s_0,s_1]$*. Also assume that, for all* $(\phi,\theta,\rho,\kappa,v)$, $\frac{\partial I_A(0;\phi,\theta,\rho,\kappa,v)}{\partial e} > \frac{\partial S(0;\phi,\rho,\kappa)}{\partial e}$, $\frac{\partial I_A(1;\phi,\theta,\rho,\kappa,v)}{\partial e} < \frac{\partial S(1;\phi,\rho,\kappa)}{\partial e}$*, and* $\frac{\partial^2 I_A(e;\phi,\theta,\rho,\kappa,v)}{\partial e^2} > \frac{\partial^2 S(e;\phi,\rho,\kappa)}{\partial e^2}$ *for all* $e \in [0,1]$*. Then the solution to* (18) *is interior,* $e^*(\phi,\theta,\rho,\kappa) \in (0,1)$ *and* $s^*(\phi,\theta,\rho,\kappa) \in (s_0,s_1)$*, and is characterized by*

$$\frac{\partial S(e^*,\phi,\rho,\kappa)}{\partial e} = \frac{\partial I_A(e^*;\phi,\theta,\rho,\kappa,v^*)}{\partial e} \tag{19}$$

where $v^* = V_A(e^*,s^*;\phi,\theta,\rho,\kappa)$.

An interior solution occurs at the point on the researcher's participation constraint $S(e,\phi,\rho,\kappa)$ where the administration reaches his "highest" indifference curve. This is depicted in Figure 1, where his utility increases as the outcome moves southeast, and his indifference curves are strictly concave. However, $I_A(e;\phi,\theta,\rho,\kappa,v)$ need not be concave. The second-order sufficient conditions are satisfied as long as his indifference curve is concave, or at least no more convex than her participation constraint.

The assumption that his indifference curve at $(0,s_0)$ is steeper than her participation constraint there implies that his utility increases if he pays her the higher salary required to induce her to teach. Because this implies she does less research, his utility increases only if: (1) there is an increase in tuition revenue that is large enough to offset the higher salary and lower license income, so that his net income increases; and (2) the marginal utility associated with this net income increase exceeds that associated with the decrease in prestige resulting from the production of smaller knowledge stocks. As he continues to pay her the additional salary required to induce her to give up time in research and teach more, his gains in net income from higher tuition decline and his losses in prestige from lower production of knowledge increase. Equilibrium occurs when her salary and teaching load have increased enough that these two effects offset each other, or where the administration's willingness to pay her to teach an additional hour equals her composite marginal rate of substitution between research and income. Alternatively, in equilibrium he chooses a teaching load on her participation constraint where the minimum additional salary required to induce her to spend the last incremental unit of time in education is the same as the maximal additional salary he is willing to pay to induce her to do so.

3.3 Comparative Statics

Unsurprisingly, comparative statics results for the equilibrium teaching load and salary are difficult to obtain without specific utility functions. Nevertheless, several general results are straightforward.

Theorem 7 *In equilibrium, an increase in either initial knowledge stock or the researcher's share of license income increases her utility. A decrease in the portion of patentable knowledge disseminated has no effect on her utility, but decreases the quality of education and the administration's utility.*

Because $\bar{U}_F(\phi, \rho, \kappa)$ is increasing in ϕ, ρ, and κ, an increase in any of them must increase her equilibrium utility whether or not she remains in the university. Because her utility inside or outside the university does not depend on θ, a change in it has no effect on her equilibrium utility. However, the administration's utility $V_A(e, s; \phi, \theta, \rho, \kappa)$ is increasing in θ by Theorem 5, so a decrease in it reduces his equilibrium utility.

Some additional, reasonable assumptions on her utility lead to more precise results. For example, the initial knowledge stocks enter her preferences as measures of the prestige associated with past research. Thus, for an increase in κ to increase her utility more outside the university, it must be the case that this increase in her initial stock would provide more prestige outside the university. *A priori*, this seems unlikely. If she does decide to exit, then her options would typically be another university, an existing firm, or a start-up. One typically thinks that the private sector values patentable knowledge more highly, or at least no less highly. Thus, it is reasonable to assume that an increase in her initial stock of scientific knowledge increases her utility inside the university at least as much as that outside, $\frac{\partial V_F}{\partial \kappa} \geq \frac{\partial \bar{U}_F}{\partial \kappa}$, so $\frac{\partial S}{\partial \kappa} \leq 0$ by Theorem 4. Similarly, it is reasonable to assume that an increase in her initial stock of patentable knowledge increases her utility at least as much outside the university, $\frac{\partial V_F}{\partial \rho} \leq \frac{\partial \bar{U}_F}{\partial \rho}$, so $\frac{\partial S}{\partial \rho} \geq 0$ by Theorem 4. Finally, because she earns her share of license income wherever she works, a change in ϕ should not change her utility within the university viz a viz that outside, $\frac{\partial V_F}{\partial \phi} = \frac{\partial \bar{U}_F}{\partial \phi}$, so $\frac{\partial S}{\partial \phi} = 0$ by Theorem 4. The next result follows immediately.

Theorem 8 *In equilibrium:*
(i) If the researcher's prestige from scientific knowledge is greater in the university, then an increase in her initial stock either decreases her salary or increases her teaching load or both. If the prestige from patentable knowledge is greater outside the university, then an increase in her initial stock either increases her salary or decreases her teaching load or both.
(ii) If the researcher's share of license income is the same wherever she works, then an increase in her share either changes her salary and teaching load in opposite directions or has no effect on them. A decrease in the portion of patentable knowledge disseminated either changes her salary and teaching load in opposite directions or has no effect on them.

The effects on her salary and teaching load can be easily seen in Figure 1. If her prestige from scientific knowledge is greater in the university, then an increase in κ shifts $S(e, \phi, \rho, \kappa)$ down, and the administration can pay her a lower salary and/or require a higher teaching load without inducing her to exit. Typically, we would expect both a lower salary and higher teaching load. The administration has an incentive to pay a lower salary, and might pay a salary so much lower that he must require a lower teaching load as well.

If her prestige from patentable knowledge is greater outside the university, then an increase in ρ shifts $S(e, \phi, \rho, \kappa)$ up, and the administration must pay her a higher salary and/or require a lower teaching load to prevent her from exiting. Typically, we expect both a higher salary and a lower teaching load. A lower salary is possible, but only if her teaching load is also much lower. A higher teaching load is possible, in which case the quality of education is higher, but only if her salary is also much higher.

If license income is the same inside or outside the university, then an increase in ϕ shifts both indifference maps, but the participation constraint

(13) still binds at the original equilibrium point (i.e., the new $S(e;\phi,\rho,\kappa)$ passes through the old equilibrium point). However, because the indifference curves through that point are associated with different levels of utility (her utility is higher and his is lower), their curvatures need not be the same as those of the indifference curves through that point before the change in ϕ. Nevertheless, because the new $S(e;\phi,\rho,\kappa)$ must also be positively sloped, the salary and teaching loads must move in opposite directions, if they change at all.

While θ does not affect the indifference curve associated with the participation constraint, it does affect the quality of education and therefore administration utility. As with a change in ϕ, the slopes of the indifference curves imply that if the teaching load and salary change, they must move in opposite directions.

The results in one special case are noteworthy. Suppose $V_F(e,s,\phi,\rho,\kappa)$ and $V_A(e,s;\phi,\theta,\rho,\kappa)$ are homothetic with respect to (e,s), so the indifference curves $I_F(e,\phi,\rho,\kappa,v)$ and $I_A(e,\phi,\rho,\kappa,v)$ have the same slope at any (e,s) on a negatively sloped ray emanating from the point $(e,s)=(1,0)$. Then an increase in κ decreases the salary and increases the teaching load, an increase in ρ increases the salary and decreases the teaching load, and an increase in ϕ or θ has no effect on the salary and teaching load.

Finally, it is also difficult to obtain general results for the effects of parametric changes on the quality of education. However, as a benchmark, we return to the case where research does not affect the current stocks of knowledge.

Theorem 9 *Absent current effects on the knowledge stocks, the equilibrium quality of education is directly related to the researcher's teaching load, so any parametric change that increases (decreases) her teaching load also increases (decreases) educational quality. However, the equilibrium quality of education is unaffected by changes in her salary.*

In general, $\frac{\partial Q^*}{\partial e}=T'(\frac{\partial Q}{\partial e}+\frac{\partial Q}{\partial d}\theta\frac{\partial P^*}{\partial e}+\frac{\partial Q}{\partial k}\frac{\partial K^*}{\partial e})$, and $\frac{\partial Q^*}{\partial s}=\frac{\partial Q}{\partial d}\theta\frac{\partial P^*}{\partial s}+\frac{\partial Q}{\partial k}\frac{\partial K^*}{\partial s}$, so absent current effects on the stocks, $\frac{\partial Q^*}{\partial e}=T'\frac{\partial Q}{\partial e}>0$ and $\frac{\partial Q^*}{\partial s}=0$. In this situation, perhaps the most useful interpretation is that this is the case of a faculty member who is no longer productive in research. For such faculty, any parametric change that results in a higher teaching load definitely increases the quality of education, because there is no accompanying cost of foregone research.

However, it is possible that an increase in the teaching load of an extremely productive researcher (i.e., a "star scientist") may decrease educational quality by decreasing her research productivity and thus the knowledge stocks she uses in education, because $\frac{\partial P^*}{\partial e} = \frac{\partial P}{\partial a}\frac{\partial a^*}{\partial e} + \frac{\partial P}{\partial b}\frac{\partial b^*}{\partial e} < 0$ and $\frac{\partial K^*}{\partial e} = \frac{\partial K}{\partial a}\frac{\partial a^*}{\partial e} + \frac{\partial K}{\partial b}\frac{\partial b^*}{\partial e} < 0$. In this event, an expansion of her teaching load decreases tuition revenue, so that inducing her to teach more necessarily decreases administration utility. In fact, for a star scientist the equilibrium may be at the corner $(0, s_0)$, where she completely specializes in research. Nevertheless, this is not a foregone conclusion. An increase in the teaching load of a star scientist need not decrease educational quality, particularly if she works in a prestigious private institution where tuition rates are exorbitant, so an increase in contact hours may generate substantial additional tuition revenue.

4 The Special Case of Quasi-linear Utility

In this section we make the assumption, common in principal-agent models, that inventor utility is quasi-linear,

$$U_F(a, b; Y_F, \rho, \kappa) = f(a, b; p, k) + s + \phi L(p). \tag{20}$$

Her problem is to choose (a, b) to maximize utility $W_F(a, b; s, \phi, \rho, \kappa) = f(a, b; P, K) + \phi L(P) + s$ subject to her time constraint. The necessary condition for an interior solution remains her composite marginal utilities of applied and basic research be equal, or her composite marginal rate of substitution between them is one. Two noteworthy differences from the general case follow.

Theorem 10 *If the researcher's utility is quasi-linear, then:*
(i) A change in her salary has no effect on her allocation of time between basic and applied research.
(ii) An increase in her share of license income increases the time she allocates to applied research if and only if her marginal productivity in creating patentable knowledge from applied research exceeds that from basic research, $\frac{\partial a^*}{\partial \phi} > 0 > \frac{\partial b^*}{\partial \phi}$ *if and only if* $\frac{\partial P}{\partial a} > \frac{\partial P}{\partial b}$.

Because her utility is linear in income, one difference is that her optimal research allocation does not depend on her salary, $a^* = a^*(e; \phi, \rho, \kappa)$ and $b^* = b^*(e; \phi, \rho, \kappa)$. This implication of quasi-linear faculty utility explains why the comparative statics results below differ from those in the preceding section, even for the case of homothetic preferences. Another is that the effect of a change in her share affects her composite marginal rate of substitution between applied and basic research only through its effect on her license income, and thus on her production of patentable knowledge. As a result, the inventor adjusts by allocating more time to the type of research that has the largest marginal effect on the production of patentable knowledge. In this case, legislation such as the Bayh-Dole Act induces a shift into applied research if and only if the researcher's time in applied research is more productive than that in basic research in generating patentable knowledge.

Her indirect utility is $V_F(e, s, \phi, \rho, \kappa) = W_F(a^*, b^*; s, \phi, \rho, \kappa)$, where now her corresponding indifference curves $I_F(e, \phi, \rho, \kappa, v)$ have the same slope for any given value of $e \in [0, 1]$. The administration again chooses the teaching load and salary to maximize his utility $U_A(Y_A; p, k)$ subject to the faculty's participation constraint $V_F(e, s, \phi, \rho, \kappa) = f(a^*, b^*; P^*, K^*) + s + \phi L(P^*) \geq \bar{U}_F(\phi, \rho, \kappa)$. Assuming this constraint binds, it can be solved for $\phi L + s$ so that administration net income can be written

$$\hat{Y}_A = T(Q^*) + L(P^*) + f(a^*, b^*; P^*, K^*) - \bar{U}_F(\phi, \rho, \kappa). \tag{21}$$

His problem can therefore be restated as

$$\max_{e \in [0,1]} V_A(e; \phi, \theta, \rho, \kappa) = U_A(\hat{Y}_A; P^*, K^*). \tag{22}$$

That is, when faculty utility is quasi-linear, the administration merely chooses her teaching load to maximize his utility and then adjusts her salary so that the participation constraint binds.

Theorem 11 *If* (14) *and* (15) *hold,* $\frac{\partial V_A(0;\phi,\theta,\rho,\kappa)}{\partial e} > 0 > \frac{\partial V_A(1;\phi,\theta,\rho,\kappa)}{\partial e}$, *and* $V_A(e; \phi, \theta, \rho, \kappa)$ *is strictly concave in e, then the solution to* (22) *is interior,* $e^*(\phi, \theta, \rho, \kappa) \in (0, 1)$ *and* $s^*(\phi, \theta, \rho, \kappa) \in (s_0, s_1)$, *and is characterized by* (19). *Absent current effects on the knowledge stocks, this interior solution is characterized by*

$$T'(q)\frac{\partial Q^*(e; \theta, \rho, \kappa)}{\partial e} = \frac{\partial f(a^*, b^*; \rho, \kappa)}{\partial a} = \frac{\partial f(a^*, b^*; \rho, \kappa)}{\partial b} \tag{23}$$

and:
(i) An increase in the researcher's share of license income has no effect on her optimal teaching load, her salary, or the quality of education.
(ii) A decrease in the portion of patentable knowledge disseminated decreases her optimal teaching load, her salary, and the quality of education.

If we again consider the case where research does not affect the current knowledge stocks, then $P^* = \rho$, $K^* = \kappa$, and $d = \theta\rho$ implies that her optimal research allocation also does not depend on her share of license revenue, $a^* = a^*(e; p, k)$ and $b^* = b^*(e; p, k)$, and that the quality of education depends only on her teaching load, her initial knowledge stocks, and the portion of patentable knowledge disseminated, $Q^* = Q^*(e; \theta\rho, \kappa)$. A change in her share therefore has no effect on her optimal teaching load or on the quality of education. Moreover, because a change in her share has the same effect on her utility inside and outside the university, her participation constraint still binds and there is no need to adjust her salary. Lastly, a decrease in the portion of patentable knowledge disseminated decreases her marginal revenue product in education, so the administration decreases her teaching load. The combined effect is a decrease in the quality of education and tuition revenue. Because the lower teaching load increases her time in research and her utility, the administration adjusts by decreasing her salary until her participation constraint binds.

5 Conclusion

This paper has sought to explain whether policy-makers should be concerned about whether patent policy changes that created incentives for commercialization of university research also might be detrimental to basic science and education. Recall that in our model these policy changes are equivalent to an increase in the researcher's share of license income, an increase in the initial stock of patentable knowledge, and a decrease in the fraction of it that can be used in education. One concern is that faculty research agendas may have changed. In our model, faculty decisions regarding applied and basic research depend on a composite marginal rate of substitution that incorporates not only their utility from conducting the research but also their productivity in producing knowledge. It is not obvious how these policy changes would affect a researcher's composite marginal rate of substitution between applied and basic research. However, in the case of quasi-linear utility, we found that an increase in the researcher's share of license income would lead her to reallocate time from basic to applied research if and only if this would increase her production of patentable knowledge.

Other concerns focus on the effects of these policy changes on the quality of education. We show that in equilibrium the effects of these policy changes on the quality of education depend not only on how they influence the teaching load, but also on how they influence the stocks of knowledge used in education. The effect of these policies on the amount of patentable knowledge used in education is itself ambiguous because even if the stock of patentable knowledge increases, the fraction of it that is used in education decreases. Generally, changes in the initial stock of patentable knowledge, the researcher's share of license income, and the fraction of patentable knowledge that can be used in education have an ambiguous effect on the equilibrium teaching load. Finally, to the extent that these policies have reduced the teaching loads of star scientists, if only by allowing them to buy off time with grants, the equilibrium quality of education may have increased. Although the lower teaching load tends to decrease the quality of education, it also allows the star scientist to spend more time in research, increasing the knowledge stocks they can use in education. The quality of education increases if the latter effect dominates.

Because this analysis abstracts from uncertainty, there are several interesting questions that could not be addressed. For example, if time in education is unobservable then there is a moral hazard problem which the administration must deal with in designing the researcher's contract. In fact, if faculty behavior is unobservable, then there may be moral hazard problems in research that the administration must consider. An additional class of moral hazard issues arise if the administration cannot observe research output. This may be particularly important in the case of patentable knowledge since the researcher may have an incentive to "cherry pick" and not disclose their most valuable inventions. A more detailed analysis of the researcher's outside options might also be of interest, particularly because universities are increasingly allowing faculty to take sabbaticals to develop their inventions in start up ventures. In the latter case, administrations must be careful in designing contracts that take into account potential conflicts of interest and commitment.

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7 Appendix

7.1 Proofs of Theorems 1, 2, 3, and 4

To prove Theorem 1, let $N_F(b; s, \phi, \rho, \kappa) = W_F(1-e-b, b; s, \phi, \rho, \kappa)$. Then because $\frac{\partial N_F}{\partial b} = -\frac{\partial W_F}{\partial a} + \frac{\partial W_F}{\partial b}$, and $\frac{\partial^2 N_F}{\partial b^2} = \frac{\partial^2 W_F}{\partial a^2} + \frac{\partial^2 W_F}{\partial b^2} - 2\frac{\partial^2 W_F}{\partial a \partial b} < 0$ by assumption, it follows that N_F is maximized at $b = 0$ if $\frac{\partial N_F(0)}{\partial b} = -\frac{\partial W_F(1-e,0;s,\phi,\rho,\kappa)}{\partial a} + \frac{\partial W_F(1-e,0;s,\phi,\rho,\kappa)}{\partial b} \leq 0$, and at $b = 1-e$ if $\frac{\partial N_F(1-e)}{\partial b} = -\frac{\partial W_F(0,1-e;s,\phi,\rho,\kappa)}{\partial a} + \frac{\partial W_F(0,1-e;s,\phi,\rho,\kappa)}{\partial b} \geq 0$. Otherwise, the solution is interior with $\frac{\partial N_F(b^*)}{\partial b} = -\frac{\partial W_F(1-b^*-e,b^*;s,\phi,\rho,\kappa)}{\partial a} + \frac{\partial W_F(1-b^*-e,b^*;s,\phi,\rho,\kappa)}{\partial b} = 0$.

Assuming an interior solution, the comparative statics results in Theorem 2 are given by $\frac{\partial b^*}{\partial x} = -\frac{\frac{\partial^2 N_F(b^*)}{\partial b \partial x}}{\frac{\partial^2 N_F(b^*)}{\partial b^2}}$ for any $x = e, s, \phi, k, p$. Hence, $\frac{\partial b^*}{\partial x}$ has the sign of $\frac{\partial^2 N_F(b^*)}{\partial b \partial x}$ for any $x = e, s, \phi, k, p$, and $a^* = 1 - e - b^*$ implies $\frac{\partial a^*}{\partial x}$ has the sign of $-\frac{\partial b^*}{\partial x}$ for $x = s, \phi, \rho, \kappa$, but $\frac{\partial a^*}{\partial e}$ has the sign of $-1 - \frac{\partial b^*}{\partial e}$. The result for e then follows from $\frac{\partial^2 N_F(b^*)}{\partial b \partial e} = \frac{\partial^2 W_F}{\partial a^2} - \frac{\partial^2 W_F}{\partial a \partial b} < 0$. Next, setting $\Delta P = \frac{\partial P}{\partial b} - \frac{\partial P}{\partial a}$, $\Delta K = \frac{\partial K}{\partial b} - \frac{\partial K}{\partial a}$, and $\Upsilon = \frac{\partial P}{\partial \rho} + \frac{\partial K}{\partial \rho}$ for notational convenience, $\frac{\partial^2 N_F(b^*)}{\partial b \partial s} = -\frac{\partial^2 W_F}{\partial a \partial s} + \frac{\partial^2 W_F}{\partial b \partial s} = \frac{\partial^2 U_F}{\partial b \partial Y_F} - \frac{\partial^2 U_F}{\partial a \partial Y_F} + \frac{\partial^2 U_F}{\partial Y_F^2}\phi L' \Delta P + \frac{\partial^2 U_F}{\partial p \partial Y_F}\Delta P + \frac{\partial^2 U_F}{\partial k \partial Y_F}\Delta K$, $\frac{\partial^2 N_F(b^*)}{\partial b \partial \phi} = -\frac{\partial^2 W_F}{\partial a \partial \phi} + \frac{\partial^2 W_F}{\partial b \partial \phi} = (\frac{\partial^2 U_F}{\partial b \partial Y_F} - \frac{\partial^2 U_F}{\partial a \partial Y_F})L + (\frac{\partial^2 U_F}{\partial Y_F^2}\phi L + \frac{\partial U_F}{\partial Y_F} L')\Delta P + \frac{\partial^2 U_F}{\partial p \partial Y_F} L \Delta P + \frac{\partial^2 U_F}{\partial k \partial Y_F} L \Delta K$, $\frac{\partial^2 N_F(b^*)}{\partial b \partial \rho} = -\frac{\partial^2 W_F}{\partial a \partial \rho} + \frac{\partial^2 W_F}{\partial b \partial \rho} = \Upsilon(\frac{\partial^2 U_F}{\partial b \partial p} - \frac{\partial^2 U_F}{\partial a \partial p}) + (\frac{\partial^2 U_F}{\partial Y_F \partial p}\phi L' + \frac{\partial U_F}{\partial Y_F}\phi L'')\Upsilon \Delta P + (\frac{\partial^2 U_F}{\partial p^2} + \frac{\partial^2 U_F}{\partial p \partial k})\Upsilon \Delta P + (\frac{\partial^2 U_F}{\partial p \partial k} + \frac{\partial^2 U_F}{\partial k^2})\Upsilon \Delta K + (\frac{\partial U_F}{\partial Y_F}\phi L' + \frac{\partial U_F}{\partial p})\frac{\partial \Delta P}{\partial \rho} + \frac{\partial U_F}{\partial k}\frac{\partial \Delta K}{\partial \rho}$, and $\frac{\partial^2 N_F(b^*)}{\partial b \partial \kappa} = -\frac{\partial^2 W_F}{\partial a \partial \kappa} + \frac{\partial^2 W_F}{\partial b \partial \kappa}$ is symmetric to $\frac{\partial^2 N_F(b^*)}{\partial b \partial \rho}$.

Alternatively, because the interior solution is also defined by $M^*(b^*; s, \phi, \rho, \kappa) = M(1-e-b^*, b^*; s, \phi, \rho, \kappa) = 1$, $\frac{\partial b^*}{\partial x} = -\frac{\frac{\partial M^*}{\partial x}}{\frac{\partial M^*}{\partial b}}$ where $\frac{\partial M^*}{\partial b} = \frac{\partial M}{\partial a}(-1) + \frac{\partial M}{\partial b} > 0$ by assumption, so $\frac{\partial b^*}{\partial x}$ has the sign of $-\frac{\partial M^*}{\partial x}$, and $\frac{\partial a^*}{\partial x}$ has the sign of $\frac{\partial M^*}{\partial x}$. Hence, $\frac{\partial b^*}{\partial e} = \frac{-\frac{\partial M}{\partial a}}{(\frac{\partial M}{\partial a} - \frac{\partial M}{\partial b})} < 0$, whence $\frac{\partial a^*}{\partial e} = -1 - \frac{\partial b^*}{\partial e} = \frac{\frac{\partial M}{\partial b}}{(\frac{\partial M}{\partial a} - \frac{\partial M}{\partial b})} < 0$. Similarly, $\frac{\partial b^*}{\partial x} = \frac{\frac{\partial M}{\partial x}}{(\frac{\partial M}{\partial a} - \frac{\partial M}{\partial b})}$ has the sign of $-\frac{\partial M}{\partial x}$ and $\frac{\partial a^*}{\partial x}$ has the sign of $\frac{\partial M}{\partial x}$ for $x = s, \phi, \rho, \kappa$.

To prove Theorem 3, note from (9) that $\frac{\partial V_F}{\partial s} = \frac{\partial W_F}{\partial a}\frac{\partial a^*}{\partial s} + \frac{\partial W_F}{\partial b}\frac{\partial b^*}{\partial s} + \frac{\partial W_F}{\partial s}$. Because $\frac{\partial W_F}{\partial a} = \frac{\partial W_F}{\partial b}$ at an interior solution, $\frac{\partial V_F}{\partial s} = \frac{\partial W_F}{\partial a}\left(\frac{\partial a^*}{\partial s} + \frac{\partial b^*}{\partial s}\right) + \frac{\partial W_F}{\partial s}$. But $a^* + b^* = 1 - e$ implies that $\frac{\partial a^*}{\partial s} + \frac{\partial b^*}{\partial s} = 0$, so $\frac{\partial V_F}{\partial s} = \frac{\partial W_F}{\partial s} = \frac{\partial U_F}{\partial Y_F} > 0$. Similarly, because $\frac{\partial a^*}{\partial x} + \frac{\partial b^*}{\partial x} = 0$ for $x = \phi, \rho, \kappa$, $\frac{\partial V_F}{\partial x} = \frac{\partial W_F}{\partial a}(\frac{\partial a^*}{\partial x} +$

$\frac{\partial b^*}{\partial x}) + \frac{\partial W_F}{\partial x} = \frac{\partial W_F}{\partial x} > 0$ for $x = \phi, \rho, \kappa$. Finally, because $\frac{\partial a^*}{\partial e} + \frac{\partial b^*}{\partial e} = -1$, $\frac{\partial V_F}{\partial e} = \frac{\partial W_F}{\partial a}\frac{\partial a^*}{\partial e} + \frac{\partial W_F}{\partial b}\frac{\partial b^*}{\partial e} = \frac{\partial W_F}{\partial a}(\frac{\partial a^*}{\partial e} + \frac{\partial b^*}{\partial e}) = -\frac{\partial W_F}{\partial a} < 0$. The results for indifference curves then follow from the fact that $\frac{\partial I_F(e;\phi,\rho,\kappa,v)}{\partial x} = -\frac{\frac{\partial V_F}{\partial x}}{\frac{\partial V_F}{\partial s}}$ for $x = e, \phi, \rho, \kappa$.

If there exists a $s_0 > 0$ such that $V_F(0, s_0; \phi, \rho, \kappa) = \bar{U}_F(\phi, \rho, \kappa)$, then because $\frac{\partial V_F}{\partial s} > 0 > \frac{\partial V_F}{\partial e}$ by Theorem 3, there exists a unique $s_1 > s_0$ such that $V_F(1, s_1; \phi, \rho, \kappa) = \bar{U}_F(\phi, \rho, \kappa)$. It then follows from the implicit function theorem that there exists a $s = S(e; \phi, \rho, \kappa)$ such that $V_F(S(e; \phi, \rho, \kappa), e; \phi, \rho, \kappa) = \bar{U}_F(\phi, \rho, \kappa)$ where, for given (ϕ, ρ, κ), S maps $[0,1]$ onto $[s_0, s_1]$ and is increasing in e. Standard comparative statics show that $\frac{\partial S}{\partial e} = -\frac{\frac{\partial V_F}{\partial e}}{\frac{\partial V_F}{\partial s}} > 0$, $\frac{\partial S}{\partial \phi} = -\frac{\left(\frac{\partial V_F}{\partial \phi} - \frac{\partial \bar{U}_F}{\partial \phi}\right)}{\frac{\partial V_F}{\partial s}}$, $\frac{\partial S}{\partial k} = -\frac{\left(\frac{\partial V_F}{\partial k} - \frac{\partial \bar{U}_F}{\partial k}\right)}{\frac{\partial V_F}{\partial s}}$, and $\frac{\partial S}{\partial p} = -\frac{\left(\frac{\partial V_F}{\partial p} - \frac{\partial \bar{U}_F}{\partial p}\right)}{\frac{\partial V_F}{\partial s}}$, which proves Theorem 4.

7.2 Proofs of Theorems 5 and 6

From (16), $\frac{\partial V_A}{\partial s} = \frac{\partial U_A}{\partial Y_A}\frac{\partial Y_A}{\partial s} + \frac{\partial U_A}{\partial p}\frac{\partial P^*}{\partial s} + \frac{\partial U_A}{\partial k}\frac{\partial K^*}{\partial s}$ where $\frac{\partial Y_A}{\partial s} = T'\frac{\partial Q^*}{\partial s} + (1-\phi)L'\frac{\partial P^*}{\partial s} - 1$, $\frac{\partial P^*}{\partial s} = \frac{\partial P}{\partial a}\frac{\partial a^*}{\partial s} + \frac{\partial P}{\partial b}\frac{\partial b^*}{\partial s} = \Delta P\frac{\partial b^*}{\partial s}$, $\frac{\partial K^*}{\partial s} = \frac{\partial K}{\partial a}\frac{\partial a^*}{\partial s} + \frac{\partial K}{\partial b}\frac{\partial b^*}{\partial s} = \Delta K\frac{\partial b^*}{\partial s}$, and $\frac{\partial Q^*}{\partial s} = [\frac{\partial Q}{\partial d}\theta\Delta P + \frac{\partial Q}{\partial k}\Delta K]\frac{\partial b^*}{\partial s}$, and so $\frac{\partial V_A}{\partial s} = \frac{\partial U_A}{\partial Y_A}[T'(\frac{\partial Q}{\partial d}\theta\Delta P + \frac{\partial Q}{\partial k}\Delta K) + (1-\phi)L'\Delta P]\frac{\partial b^*}{\partial s} - \frac{\partial U_A}{\partial Y_A} + (\frac{\partial U_A}{\partial p}\Delta P + \frac{\partial U_A}{\partial k}\Delta K)\frac{\partial b^*}{\partial s}$. Next, $\frac{\partial V_A}{\partial e} = \frac{\partial U_A}{\partial Y_A}[T'(\frac{\partial Q}{\partial e} + \frac{\partial Q}{\partial d}\theta\frac{\partial P^*}{\partial e} + \frac{\partial Q}{\partial k}\frac{\partial K^*}{\partial e}) + (1-\phi)L'\frac{\partial P^*}{\partial e}] + \frac{\partial U_A}{\partial p}\frac{\partial P^*}{\partial e} + \frac{\partial U_A}{\partial k}\frac{\partial K^*}{\partial e}$, where now $\frac{\partial a^*}{\partial e} + \frac{\partial b^*}{\partial e} = -1$ implies $\frac{\partial P^*}{\partial e} = \Delta P\frac{\partial b^*}{\partial e} - \frac{\partial P}{\partial a}$ and $\frac{\partial K^*}{\partial e} = \Delta K\frac{\partial b^*}{\partial e} - \frac{\partial K}{\partial a}$. Similarly, $\frac{\partial V_A}{\partial \phi} = \frac{\partial U_A}{\partial Y_A}[T'(\frac{\partial Q}{\partial d}\theta\Delta P + \frac{\partial Q}{\partial k}\Delta K) + (1-\phi)L'\Delta P]\frac{\partial b^*}{\partial \phi} - \frac{\partial U_A}{\partial Y_A}L + (\frac{\partial U_A}{\partial p}\Delta P + \frac{\partial U_A}{\partial k}\Delta K)\frac{\partial b^*}{\partial \phi}$. For $x = \rho, \kappa$, $\frac{\partial V_A}{\partial x} = \frac{\partial U_A}{\partial Y_A}[T'(\frac{\partial Q}{\partial d}\theta\Delta P + \frac{\partial Q}{\partial k}\Delta K) + (1-\phi)L'\Delta P]\frac{\partial b^*}{\partial x} + (\frac{\partial U_A}{\partial p}\Delta P + \frac{\partial U_A}{\partial k}\Delta K)\frac{\partial b^*}{\partial x} + [\frac{\partial U_A}{\partial Y_A}(T'\frac{\partial Q}{\partial d}\theta + (1-\phi)L') + \frac{\partial U_A}{\partial p}]\frac{\partial P}{\partial x} + (\frac{\partial U_A}{\partial Y_A}T'\frac{\partial Q}{\partial k} + \frac{\partial U_A}{\partial p})\frac{\partial K}{\partial x}$. Finally, $\frac{\partial V_A}{\partial \theta} = \frac{\partial U_A}{\partial Y_A}T'\frac{\partial Q}{\partial d}(P^* - \rho) > 0$.

Absent effects on the knowledge stocks, $P^* = \rho$, $K^* = \kappa$, and $d = \theta\rho$, so $\frac{\partial V_A}{\partial s} = -\frac{\partial U_A}{\partial Y_A} < 0$, $\frac{\partial V_A}{\partial e} = \frac{\partial U_A}{\partial Y_A}T'\frac{\partial q}{\partial e} > 0$, $\frac{\partial V_A}{\partial \phi} = \frac{\partial U_A}{\partial Y_A}\frac{\partial Y_A}{\partial \phi} = -\frac{\partial U_A}{\partial Y_A} - L < 0$, $\frac{\partial V_A}{\partial \rho} = \frac{\partial U_A}{\partial Y_A}\frac{\partial Y_A}{\partial \rho} + \frac{\partial U_A}{\partial \rho} = \frac{\partial U_A}{\partial Y_A}[T'\frac{\partial q}{\partial d}\theta + (1-\phi)L'] + \frac{\partial U_A}{\partial \rho} > 0$, and $\frac{\partial V_A}{\partial \kappa} = \frac{\partial U_A}{\partial Y_A}\frac{\partial Y_A}{\partial \kappa} + \frac{\partial U_A}{\partial \kappa} = \frac{\partial U_A}{\partial Y_A}T'\frac{\partial q}{\partial \kappa} + \frac{\partial U_A}{\partial \kappa} > 0$.

To solve (18), form the Lagrangian $\pounds(e, s, \lambda) = V_A(e, s; \phi, \theta, \rho, \kappa) + \lambda[V_F(e, s, \phi, \rho, \kappa) - \bar{U}_F(\phi, \rho, \kappa)]$. Using ordinary techniques, the first order conditions for an interior solution can be summarized as $\lambda = -\frac{\frac{\partial V_A}{\partial e}}{\frac{\partial V_F}{\partial e}} = -\frac{\frac{\partial V_A}{\partial s}}{\frac{\partial V_F}{\partial s}}$ and $V_F(e, s, \phi, \rho, \kappa) = \bar{U}_F(\phi, \rho, \kappa)$. The former implies $-\frac{\frac{\partial V_A}{\partial e}}{\frac{\partial V_A}{\partial s}} = -\frac{\frac{\partial V_F}{\partial e}}{\frac{\partial V_F}{\partial s}}$, or

$\frac{\partial I_A}{\partial e} = \frac{\partial I_F}{\partial e}$, which with the participation constraint proves (19). Again using standard techniques, the second order sufficient condition for a constrained maximum is $2\frac{\partial^2 V_A}{\partial e \partial s}\frac{\partial V_F}{\partial w}\frac{\partial V_F}{\partial s} - \frac{\partial^2 V_A}{\partial e^2}(\frac{\partial V_F}{\partial s})^2 - \frac{\partial^2 V_A}{\partial s^2}(\frac{\partial V_F}{\partial e})^2 + 2\lambda\frac{\partial^2 V_F}{\partial e \partial s}\frac{\partial V_F}{\partial e}\frac{\partial V_F}{\partial s} - \lambda\frac{\partial^2 V_F}{\partial e^2}(\frac{\partial V_F}{\partial s})^2 - \lambda\frac{\partial^2 V_F}{\partial s^2}(\frac{\partial V_F}{\partial e})^2 \geq 0$. Substituting for λ from the first order conditions, using the participation constraint, and rearranging terms, this can be rewritten as $\lambda^{-2}(\frac{\partial V_A}{\partial s})^3[\frac{\partial^2 I_A}{\partial e^2} - \frac{\partial^2 S}{\partial e^2}] \geq 0$. Because $\frac{\partial V_A}{\partial s} < 0$, this is equivalent to $\frac{\partial^2 I_A}{\partial e^2} \leq \frac{\partial^2 S}{\partial e^2}$. Thus, because the researcher's indifference curves are increasing and strictly convex, this condition merely requires the administration's (increasing) indifference curves to be less convex, or even concave. The conditions $\frac{\partial I_A(0;\phi,\theta,\rho,\kappa,v)}{\partial e} > \frac{\partial S(0;\phi,\rho,\kappa)}{\partial e}$ and $\frac{\partial I_A(1;\phi,\theta,\rho,\kappa,v)}{\partial e} < \frac{\partial S(1;\phi,\rho,\kappa)}{\partial e}$ for all $(\phi, \theta, \rho, \kappa, v)$ guarantee that this tangency must be at an $e^* \in (0, 1)$. That the solution also satisfies $s^* = S(e^*, \phi, \rho, \kappa) \in (s_0, s_1)$ then follows from Theorem 4.

7.3 Proofs of Theorems 10 and 11

Because the utility function in (20) is a special case of $U_F(a, b, Y_F(s, \phi, p), p, k)$, the results of Theorems 1 and 2 must hold for it. But there are two differences in this case. First, because $\frac{\partial W_F}{\partial a} = \frac{\partial f}{\partial a} + \frac{\partial f}{\partial p}\frac{\partial P}{\partial a} + \frac{\partial f}{\partial k}\frac{\partial K}{\partial a} + \phi L'\frac{\partial P}{\partial a}$ and $\frac{\partial W_F}{\partial b} = \frac{\partial f}{\partial b} + \frac{\partial f}{\partial p}\frac{\partial P}{\partial b} + \frac{\partial f}{\partial k}\frac{\partial K}{\partial b} + \phi L'\frac{\partial P}{\partial b}$, the optimal research allocations $a^*(e; \phi, \rho, \kappa)$ and $b^*(e; \phi, \rho, \kappa)$ do not depend on the salary. Second, again $\frac{\partial a^*}{\partial \phi}$ has the sign of $\frac{\partial M^*}{\partial \phi}$ and $\frac{\partial b^*}{\partial \phi}$ has the sign of $-\frac{\partial M^*}{\partial \phi}$, where now $\frac{\partial M^*}{\partial \phi} = \frac{L'\frac{\partial P}{\partial a}\frac{\partial W_F}{\partial b} - \frac{\partial W_F}{\partial a}L'\frac{\partial P}{\partial b}}{(\frac{\partial W_F}{\partial b})^2} = \frac{L'(\frac{\partial P}{\partial a} - \frac{\partial P}{\partial b})}{\frac{\partial W_F}{\partial b}}$ because $\frac{\partial W_F}{\partial a} = \frac{\partial W_F}{\partial b}$. Thus, $\frac{\partial a^*}{\partial \phi} \gtrless 0$ if and only if $\frac{\partial P}{\partial a} \gtrless \frac{\partial P}{\partial b}$, and conversely for $\frac{\partial b^*}{\partial \phi}$. The first order condition for the administration is $\frac{\partial V_A}{\partial e} = \frac{\partial U_A}{\partial Y_A}[T'\frac{\partial Q^*}{\partial e} + L'\frac{\partial P^*}{\partial e} + \frac{\partial f}{\partial a}\frac{\partial a^*}{\partial e} + \frac{\partial f}{\partial b}\frac{\partial b^*}{\partial e} + \frac{\partial f}{\partial p}\frac{\partial P^*}{\partial e} + \frac{\partial f}{\partial k}\frac{\partial K^*}{\partial e}] + \frac{\partial U_A}{\partial p}\frac{\partial P^*}{\partial e} + \frac{\partial U_A}{\partial k}\frac{\partial K^*}{\partial e} = 0$. Under the assumptions that $\frac{\partial V_A(0;\phi,\theta,\rho,\kappa)}{\partial e} > 0 > \frac{\partial V_A(1;\phi,\theta,\rho,\kappa)}{\partial e}$ and $V_A(e; \phi, \theta, \rho, \kappa)$ is strictly concave, $\frac{\partial^2 V_A}{\partial e^2} < 0$, the maximum must be interior, $e^*(\phi, \theta, \rho, \kappa) \in (0, 1)$. That $s^*(\phi, \theta, \rho, \kappa) = S(e^*, \phi, \rho, \kappa) \in (s_0, s_1)$ follows from Theorem 4.

Absent effects on the knowledge stocks, $P^* = \rho$, $K^* = \kappa$, and $d = \theta\rho$, so $Q = Q(e; \theta\rho, \kappa)$. Thus, from (21) and (22), $\frac{\partial V_A}{\partial e} = \frac{\partial U_A}{\partial Y_A}[T'\frac{\partial Q(e;\theta\rho,\kappa)}{\partial e} + \frac{\partial f}{\partial a}\frac{\partial a^*}{\partial e} + \frac{\partial f}{\partial b}\frac{\partial b^*}{\partial e}] = 0$. Recalling that $\frac{\partial a^*}{\partial e} = -1 - \frac{\partial b^*}{\partial e}$ when a^* and b^* are interior and that the first order conditions for the researcher are $\frac{\partial f}{\partial a} = \frac{\partial f}{\partial b}$ in this case, this condition is equivalent to $Z(e; \theta\rho, \kappa) = T'\frac{\partial Q(e;\theta\rho,\kappa)}{\partial e} - \frac{\partial f(a^*,b^*;\rho,\kappa)}{\partial a} = 0$. Differentiating this with respect to e and x gives $\frac{\partial e^*}{\partial x} = -\frac{\frac{\partial^2 Z}{\partial e \partial x}}{\frac{\partial^2 Z}{\partial e^2}}$ for $x = s, \phi, \theta$.

Because $\frac{\partial^2 Z}{\partial e^2} < 0$ by assumption, the sign of $\frac{\partial e^*}{\partial x}$ is given by the sign of $\frac{\partial^2 Z}{\partial e \partial x}$. Because a^* and b^* do not depend on ϕ, a change in it has no effect on $Z(e; \theta\rho, \kappa)$, and thus no effect on her teaching load. However, an increase in ϕ increases her license income and utility in the university as well as that outside the university, and these increases are the same by assumption, so her participation constraint continues to bind (and there is no need to adjust her salary). A decrease in θ decreases her teaching load because $\frac{\partial^2 Z}{\partial e \partial \theta} = \frac{\partial^2 T}{\partial e \partial \theta} > 0$ and the quality of education because $\frac{\partial Q}{\partial e} > 0$ and $\frac{\partial Q}{\partial \theta} = \frac{\partial Q}{\partial d}\rho > 0$. The decreased teaching load increases her time in research, which increases her utility and induces him to cut her salary to return her to her participation constraint.

Figure 1