Technology and the Theory of Vintage Aggregation

Michael J. Harper*
U.S. Bureau of Labor Statistics

For presentation at the

Conference on Hard-to-Measure Goods and Services in Honor of Zvi Griliches

September 19-20, 2003

* The author is Chief of the Division of Productivity Research and Program Development, Office of Productivity and Technology. The views expressed are those of the author, and do not necessarily reflect the policies of the U.S. Bureau of Labor Statistics or the views of other staff members. This paper merges and extends two earlier unpublished papers. The first of these made an initial attempt at defining concepts from the vintage model, and was presented at the NBER Summer Institute in 1999 where Zvi Griliches made helpful comments. I would also like to acknowledge Tony Barkume, Carol Corrado, Edwin Dean, Erwin Diewert, Murray Foss, Robert Gordon, Chuck Hulten, Marilyn Manser, Peter Meyer, Linda Moeller, Phyllis Otto, Sabrina Pabilonia, Matt Shapiro, Dan Sichel, Brian Sliker, Leo Sveikauskas, Jack Triplett, and Cindy Zoghi for helpful comments, suggestions and encouragement at various stages of this line of work. Any errors are entirely my own.

Technology and the Theory of Vintage Aggregation

Michael J. Harper, 9/19/03
Bureau of Labor Statistics

" I only want to urge that research not get tied to any one particular picture of the way the economy functions "
- Robert Solow [2001]

Abstract: Each machine's marginal product will be impacted by *exogenous cyclical and temporal changes in demand*. A capital stock measure holds the exogenous *cyclical effects* fixed, so they fall into its "shadow" price. But if technical change improves the productivity of labor using newer vintages, the stock will not be homogeneous and the cyclical impacts will be proportionally greater for older models. An alternative aggregate capital measure is devised that simply weights each machine by its contemporaneous marginal product. The new measure, which is a supplement to traditional capital measures rather than a replacement, varies flexibly with demand, and dramatically reduces the pro-cyclical volatility of Solow's [1957] residual. Quality adjustment assumes durable goods prices reflect marginal products, at odds with the neoclassical axiom that prices reflect *discounted streams of future rents*. The exogenous *temporal effects of obsolescence* will likely depress future rents of the oldest models fastest. This will be reflected in goods prices and may lead to an overstatement of quality change.

I. Introduction

Robert Solow [1957, 1959] proposed two different models of how technology and capital formation can explain growth in labor productivity. The first model assumed that technology was "disembodied" in that it raised productivity independently of the level of investment. Vintage investments were summarized in a "capital stock" measure that was used to separate the contributions of capital and of "residual" technology change to labor productivity growth. In the second model, technology was "embodied" in capital, and the contribution of each "vintage" of capital to labor productivity could be different. Solow described the dynamic allocation of labor in competitive neoclassical terms, without needing a vintage aggregate of capital. Dale Jorgenson [1966] observed that these two Solow models were indistinguishable if growth rates were exponential. In this vein, Robert Hall [1968] integrated the two capital measurement models by specifying capital "services" to be a function of time as well as of vintage.

Recognizing that the U.S. National Income and Product Accounts of the time did not account for the dramatic quality change occurring in computers, Jack Triplett [1989] proposed adjusting computer price indexes for quality change using hedonic price indexes. BEA and BLS now deflate high tech investments with these prices in estimating real final output and capital stock. As Chuck Hulten [1992] observed, this identifies quality change with embodied technical change, as Hall had defined it. Although some researchers were suspicious of the rapid rates of quality change estimated for computers, Ana Aizcorbe, Carol Corrado, and Mark Doms [2000] allayed many of these fears by showing that hedonic price indexes could be reproduced to a high degree of precision with matched models.

Computer quality reached the center stage of U.S. economic measurement issues when Steve Oliner and Dan Sichel [2000] recognized that high tech equipment, though small in nominal value, accounted for about two-thirds of the speedup in labor productivity in the late 1990s. Many U.S. researchers quickly reached the same conclusion. However, Peter Hill [2000] pointed out that quality adjustments made from vintage accounts of prices omitted obsolescence effects from measures of real depreciation and capital. Indeed, Jorgenson and Kevin Stiroh [2000] regard obsolescence to be accounted for in their capital "rental price", a treatment that Hill regards as running counter to a tradition in the national accounting literature.

This paper takes a fresh look at the underpinnings of capital measurement and the issues raised by technology and obsolescence. The approach conforms to a strictly neoclassical view of firm behavior, assuming perfect competition in both the short and long run. After a brief review of the literature and the issues (section II), this paper asserts that many of the key concepts have been either loosely defined or defined in a context where strong measurement assumptions have already been imposed. A "model of production with machines" is developed in section III, in which two key assumptions of the Solow vintage model are extended to individual machines. One assumption is that output is made by specific vintages (machines) among which labor is specifically allocated. The other assumption is that, in equilibrium, the marginal product of labor will be the same for each vintage (machine). This context permits precise characterization of how the use of capital is driven by rent-seeking behavior. Key concepts are defined including deterioration and embodied and disembodied technical change.

Section IV examines the machine model in nominal terms, develops nominal aggregates of property income, and examines the effects of various phenomena on the growth of aggregate rents. The notion of asset price is introduced, and its usual neoclassical association with discounted future rents is adopted. Nominal wealth and depreciation are defined.

Section V considers the idea of real capital input. The machine model is used to clarify what the marginal product of capital is --- the added output obtained from a collection of machines by adding one machine (not a machine hour) *and without adding any labor*. A new capital input measure is proposed that counts machines, weighting each by its marginal product. This measure evades the Leontief [1947] aggregation theorem essentially by conceding that capital will not be separable. This measure is compared to the standard measure used in multifactor productivity in order to provide perspective on what has been assumed by Jorgenson, in many studies, and by BLS [2001] for the past two decades. It is noted that traditional capital stock measures assume a fixed age/marginal product relationship, essentially by imposing separability of the vintage capital measure even though the Leontief conditions are seriously and systematically violated. The onerous restriction that this places on the characterization of technical change is demonstrated graphically. When calculated with the new capital input measures, the Solow residual has much less pro-cyclical variance than when calculated with standard capital measures. While the new measure may add information about the utilization of capital, it does not provide a clear indication of the amount of capital goods, so it is a supplement to traditional measures and not a replacement for them. Section VI considers the effects of obsolescence on observed capital goods prices. It is shown how the deflation of vintage investments with "constant quality" price indexes could lead to exaggerated estimates of the accumulation of real capital. Finally Section VII considers the measurement of the real output of durable goods.

II. Theory Conventionally Used to Adjust Capital Measures for Quality Change

The Solow Residual Model

Solow's [1957] residual model constructed an aggregate capital stock, K, and used it in a production function, f, of the form Y = f(L,K,t) (Y is a real value added output measure, L is labor hours, K is aggregate capital stock, and t is the time of observation), to parse out the contributions to labor productivity growth of capital and of shifts in the production function.[1] Solow showed that:

$$(y-l) = s_K (k-l) + a , \qquad (1)$$

where y, l, and k represent the growth rates of output, labor and capital stock, respectively, and where s_K is the share of capital or "property" income, Ψ , in the value of output. Property income is calculated using output prices, p, and wages, w, as the residual of labor compensation in nominal "value added" output: $\Psi = pY - wL$. The "Solow residual", a, is a measure of "disembodied technical change" in that it is presumed to contribute independently of the level of investment in capital. BLS [2001] produces "multifactor productivity" measures using this framework.

Time series estimates of "*a*" are pro-cyclical. This has been a troubling problem for neoclassical models because "*a*" is designed to measure technological progress which should not be highly sensitive to the cycle. Many ideas, such as labor hoarding and disequilibrium have been put forth in an effort to reconcile apparent short-run increasing marginal returns to labor with the neoclassical prediction of diminishing returns. Section V will show that the cyclical nature of the residual is partly a consequence of the rigidity with which capital is measured.

The Solow Vintage Model

"The controversies still rage(d) " when Harcourt [1969, p.369] wrote his account of a bitter debate in the literature over whether capital measurement is useful. An understanding of the issues had gradually emerged in the context of Leontief's [1947] aggregation theorem. To build a stock, capital had to be like jelly --- the ratios of marginal products of different investments could not vary as functions of output or other inputs in the production function. Empirically the Leontief conditions are rarely satisfied. For example, newer electric power plants are used continuously while older plants are reserved to meet peak demand. Another example is that one fast computer is not a perfect substitute for two slow machines for which the total cost is the same, because the latter are designed to work with two people.

---

[1] The derivation involves differentiating the production function with respect to time, assuming constant returns to scale and that inputs are paid the values of their marginal products.

In response to this type of problem, Solow [1959] had proposed a vintage capital model. In this model, each vintage, v, of capital has its own production function, $f_{t,v}$, where v is the time at which the capital was built. The function describes how much output could be made with any given amount of surviving capital, $K_{t,v}$, and labor, $L_{t,v}$, in any period subsequent to the year of an initial investment, I_v:

$$Y_{t,v} = f_{t,v}(K_{t,v},L_{t,v}) \ . \qquad (2)$$

This allowed for technical progress to be "embodied" in capital goods. To enforce the idea that vintage production functions are separate, Solow imposed the "ground rule" that firms must apply labor to specific vintages with no joint effects. Thus the observed totals for labor hours and output are the sums of vintage specific contributions:

$$L_t = \Sigma_v L_{t,v} \text{ and } Y_t = \Sigma_v Y_{t,v} \ . \qquad (3)$$

This structure accommodates heterogeneity among the production processes used by capital assets of different vintages. In some year, the capital measure required for the Solow residual model might count two slower computers as the same amount of capital as the one fast one. But relative valuations might change over time and so an aggregate counting them as the same would be ambiguous. In the vintage model, firms would operate each vintage so that the marginal product of labor would be the same for each. This model showed how neoclassical theory could describe micro level behavior even when conditions for aggregation failed.

The Hall Equation

It is now common to adjust capital quantity for quality change and use the resulting measure of "capital services" in calculating the Solow residual. The foundation for this builds on Franklin Fisher's [1965] equation describing a capital services aggregate,

$$J_t = \int_v z_{t,v} I_v \, dv \quad . \qquad (4)$$

The "J", which presumably stands for jelly, refers to the assumption that capital goods of different vintages had to be perfect substitutes in production. As such, they would be free of distinctive features. Earlier proofs establishing vintage aggregation conditions had shown that the efficiency function (z_v), must adjust the quantity measure for *differences in marginal product* while maintaining the independence of the capital measure from exogenous variables including variables affecting short-run production decisions such as output prices and wages. Many economists in the 1950s and 1960s felt that the likely failure of these conditions led to a dead end for capital measurement. However, Hall [1968] pointed out that we have a lot of latitude in defining jelly. He observed (p. 36) that:

"The basic theorem on capital aggregates makes no restriction on the behavior of the function z(v) over calendar time. From one year to the next, the pattern of efficiency as a function of vintage may change arbitrarily. … This formulation is so general as to be almost vacuous".

In proposing that the recipe for jelly could be changed from year to year, Hall recognized that this generalization was so vast as to obscure the capital-related phenomena addressed in previous literature. To reach an interpretation, he proposed a structural form for z involving a decomposition into three factors which he could loosely associate with important phenomena: functions of time (d_t, disembodied technical change), of age (ϕ_{t-v},deterioration), and of vintage (b_v, embodied technical change):

$$J_t = \int_v z_{t,v} I_v \, dv = d_t \int_v \phi_{t-v} \, b_v I_v \, dv \qquad (5)$$

Hall then pointed out the functions d_t, ϕ_{t-v}, and b_v reflect only two influences, time and vintage, and so the specification can be written in terms of two functions, eliminating the third by including its influence in re-specified versions of the other two. Thus equation (5) implies that there are really only two distinct phenomena. Hall considered which of the three functions we might eliminate, by holding it constant. Since deterioration was closely associated with observable phenomena, Hall suggested that we measure deterioration and collapse the two forms of technical change into one, leaving either embodied (d_t=1) or disembodied technical change (b_v=1) to be measured. With either "normalization", Hall suggested a further convention that the deterioration index be unity for a new asset, $\phi_0 = 1$. Thus with equation (5) imposed, Hall pulled the two Solow models together. This supported Jorgenson's [1966] idea that the two models were sometimes indistinguishable. We will refer to equation (5) as Hall's equation.

Use of Hall's Embodied Technical Change Factor to Adjust Capital for Quality Improvements

The conventional quality adjustment made by U.S. statistical agencies involves comparing prices of new goods of the same vintage with differing characteristics. New improved models of high tech equipment that embody improvements are frequently introduced and marketed alongside older models. Prices can be observed at any time, for each vintage, and for each model, m, where m is *the time at which the* ***model*** *was first introduced*. These capital goods prices will be denoted using three subscripts, $p^K_{t,v,m}$. In principle, quality change is defined by comparing the prices of brand new goods (v=t) of the latest model (m=t) to brand new goods of a previous period's model. For example in matched model procedures, m=t-1, i.e. $b_m/b_{m-1}=p^K_{t,t,t}/p^K_{t,t,t-1}$. Hedonic models estimate b from wider sets of prices and characteristics. The rationale is sometimes described in terms of consumer theory: relative prices should reflect relative utilities. This "user-value" rationale is extended to the relative marginal products of durable goods (carelessly so, as will be demonstrated later). A rationale can also be derived from the point of view of the "resource costs" of producers of the durable goods. In practice, real investments, I_v, cannot be observed and so I and b cannot simply be plugged into Hall's equation (5). In practice capital is measured by starting with "nominal capital expenditures", E_t, by then "deflating" with a price index that tracks the price for a new good of a specific model from the period when it is introduced until the subsequent period:

$$J_t / J_{t-1} = (E_t / p^K_{t,t,t-1}) / (E_t / p^K_{t-1,t-1,t-1}) \qquad (6)$$

This is equivalent to deflating with a "quality adjusted price index", that is, $J_t/J_{t-1}=(E_t\, b_t/p^K_{t,t,t})/(E_t\, b_{t-1}/p^K_{t-1,t-1,t-1})$. This type of quality adjustment is widely regarded in the U.S. as the state of the art and as something that should be done for more kinds of capital. This paper will not be raising an issue about the hedonic price estimates. However, from capital theory it will be raising an issue about how these estimates relate to quality (section VI).[2]

After the late 1960s, the old capital debates died down without resolution. Economists in Cambridge, Massachusetts continued to measure capital, having addressed some issues raised by economists from Cambridge, England[3]. One issue left unresolved was the problematic nature of vintage aggregation. Little has been written recently about the problem, with one side regarding the issue as solved and the other side regarding it as unsolvable, as was evident in meetings of the "Canberra Group"[4]. In the U.S., measurement proceeds on the basis of some assumptions. Some Europeans remain skeptical of the foundations. Hopefully this paper will help reinvigorate discussion of these issues.

III. A Model of Production with Machines

[2] The author has participated in preparing the BLS [2001] multifactor productivity measures, which include quality adjustments for the high tech capital inputs and other durable goods based on current U.S. procedures.
[3] Some strong assumptions were relaxed, by means such as using rental prices to aggregate assets of different types.
[4] This international group led by Rob Edwards of the Australian Bureau of Statistics met three times and developed a manual on capital stock issues edited by Derek Blades, OECD[2001].

In this section, a model is developed describing production from individual "machines", which could be almost any type of asset such as computers, trucks, or buildings. The model will seem most natural for machines belonging to fairly specific classes of assets or for plants making fairly specific outputs. The model will first be developed to analyze events in the "short run".

The Machine Production Function

As in section II, three subscripts are used to denote time, vintage, and "model", where the model variable will be regarded as the time at which machines with specific physical characteristics were first introduced to the market. This third subscript will be used in the formulation of definitions as well as in the description of available data on capital goods prices. Solow's vintage model (equation 2) can be modified to describe output as a function of labor associated with specific models, m, as well as specific vintages, i.e., $Y_{t,v,m} = f_{t,v,m}(K_{t,v,m},L_{t,v,m})$. At any time the economy-wide "stocks" of each vintage and model, $K_{t,v,m}$, will be regarded as fixed by past investment decisions. A firm owning (and planning to keep) a machine, of type t,v,m, faces the following "short-run" production possibilities for generating output from labor:

$$Y_{t,v,m}/ K_{t,v,m} = g_{t,v,m}(L_{t,v,m}/K_{t,v,m}) \quad . \qquad (7)$$

Now the capital stock variable, $K_{t,v,m}$, already was measured by aggregation across machines in the set of all machines in existence at any time, $\mathbf{K}_{t,v,m.}$ Assume that all machines in each vintage-model category are used identically at each point in time. The vintage investments are not blobs of jelly. They are collections of machines, each with dynamic production options. Consider $\mathbf{K}_{t,v,m}$ to consist of a discrete number, $n_{t,v,m}$, of identical machines. Total output from all machines of each vintage-model combination will be $n_{t,v,m}$ times the output of each machine.

The *machine production function*, f, is then defined in terms of output per machine, by vintage and model:

$$(Y_{t,v,m} / n_{t,v,m}) = f_{t,v,m} (L_{t,v,m} / n_{t,v,m}) \quad \forall \; v, m \subset \mathbf{K}_{t,v,m} , \qquad (8)$$

Assume that the output coming from each machine is a smooth function, f, of labor and diminishing marginal returns to labor. Figure 1 depicts such a machine production function, f. If the firm chooses to operate at point A, the average product of labor (labor productivity) will be the slope of ray OA and the marginal product of labor will be the slope of the tangent to f at A.

Extending Solow's "ground rule" (equation 3) so that labor is allocated to specific machines to produce output:

$$L_t = \int_v\int_m L_{t,v,m} \, dm \, dv \quad \text{and} \quad Y_t = \int_v\int_m Y_{t,v,m} \, dm \, dv. \qquad (9)$$

As in Solow's vintage model, assume that output and labor can be measured and that they are homogeneous. Also, cost minimization implies that the marginal product of labor applied to each machine will be the same (and will equal the wage rate relative to the price of output):

$$\partial f_{t,v,m}/\partial L_{t,v,m} = w_t/p_t \qquad \forall \; t, v, m . \qquad (10)$$

In figure 2 three functions are depicted. Expression (10) implies they will be operated at points where tangents are parallel. It is also important to note that labor productivity differs by vintage (even though the marginal product of labor is the same). In this situation, if w_t/p_t changed, labor would be reallocated in such a way that the marginal product of labor would adjust proportionately across vintages, but the average product of labor would be affected *disproportionately*.

If this model were ever to be elaborated as thoroughly as Solow's residual model, issues such as the heterogeneity of labor and of output (composition or "quality" effects) and the relationships among different types of capital might be addressed. However this paper will describe a situation where one type of output is made with one type of labor using progressively advancing versions of one type of machine. This will facilitate the exposition of some difficult concepts.

Relationships Among Functions

Zvi Griliches [1963] made one of the most thorough efforts in the literature to define the key concepts of capital measurement, such as replacement, depreciation, deterioration, obsolescence, and capital services. Here and later this paper will define a similar family of concepts with essentially the same meanings, but with explicit reference to the machine model.

In order to facilitate compact mathematical definitions and analysis of phenomena associated with capital, assume that machine production functions, $f_{t,v,m}$, and related variables are continuous functions of time, vintage, and model. These processes are not really continuous, but the notions developed could be adapted to discrete situations.

As it ages, each machine's physical characteristics change, due to wear and tear. The *rate of deterioration of output,* $\chi^{f}_{t,v,m}$ *,* (the output decay rate) is defined as the rate at which the output produced by a given amount of labor with a given model varies by vintage:

$$\chi^{f}_{t,v,m} = \partial \ln f_{t,v,m}/\partial v = - \partial \ln f_{t,v,m}/\partial (t-v). \qquad (11)$$

As indicated, this will be the additive inverse of the rate at which output varies by age alone for a given model. Note that the deterioration rate can vary with time, vintage, or model. Newer models embody features that permit them to make more output with the same amount of labor.

The *rate at which functions differ due to embodied technical change,* $B^{f}_{t,v,m}$ *,* is defined in terms of models, or equivalently, model-age:

$$B^{f}_{t,v,m} = \partial \ln f_{t,v,m}/\partial m = - \partial \ln f_{t,v,m}/\partial(t\text{-}m). \qquad (12)$$

These are the only two types of shifts considered that are due solely to the machine's physical characteristics. However, as time passes, people may learn how to get more out of a given machine. *Disembodied technical change,* $D^{f}_{t,v,m}$, is the rate at which the function shifts over time for a specific model and age:

$$D^{f}_{t,v,m} = \partial \ln f_{t,v,m}/\partial t \quad . \qquad (13)$$

There was an identification problem with Hall's functions of time, vintage, and age (t-v). This is not the case with time, vintage and *model* because model is an independent variable controlling for the characteristics of a machine. In principle one could use empirical observations to identify the three phenomena separately. Identical brand new models made in different years could help identify disembodied technical change. Thus one could observe $f_{t+1,\, v+1,m}/f_{t,\, v,m}$ to measure D, $f_{t,v+1,m}/f_{t,v,m}$ to measure χ and $f_{t,v,m+1}/f_{t,v,m}$ to measure B.

IV. The Nominal Earnings and Valuations of Assets Described with the Machine Model

This section will use the machine model to analyze the earnings of assets under dynamic conditions, such as how they are influenced by technology and cyclical fluctuations in demand. This material will be helpful in tackling the issues in measuring real capital in Section V.

Extraction of Rents from Machines – The Structure of the Shadows

For each vintage and model, define the "rent" or "property income", $\Psi_{t,v,m}$, generated per machine as the difference between revenues and variable costs associated with the machine:

$$\Psi_{t,v,m}/ n_{t, v,m} = p_t Y_{t,v,m}/ n_{t, v,m} - w_t L_{t,v,m}/ n_{t, v,m} . \qquad (14)$$

As Berndt and Fuss [1986] assumed, in the short run firms can be expected to behave as if capital costs are fixed and sunk, and so they will go about the business of maximizing the rate at which they accrue property income, $\Psi_{t,v,m}$. The ex-post rents generated by the aggregate capital stock emerge as the "shadow price" of the capital stock. The machine model supports an explanation of how output prices and wages influence decisions on operating individual machines. This begins by assuming that, in the short run, each firm has a fixed collection of assets and is too small to influence wages and output prices. Assume that each firm will *extract as much rent as possible* from each machine it owns. With a given price, a given wage, and a given set of machines in place, the decision as to how much to run each machine can be represented in terms of values rather than in terms of input and output units. The following describes how much revenue can be earned from one machine as a function of expenditure on labor costs:

$$p_t Y_{t,v,m}/ n_{t,v,m} = f'_{t,v,m}(w_t L_{t,v,m}/ n_{t,v,m}) , \qquad (15)$$

where f' is a "revenue function" that is closely related to the corresponding machine production functions: $\partial f'_{t,v,m}/\partial L_{t,v,m} = w_t \partial f_{t,v,m}/\partial L_{t,v,m}$ $\forall$ t, v and m.

Given the assumed price taking behavior, at any time, t, one can relabel the axes of figure 1 as "revenues" and "labor costs" and construct the scale so that w_t=1 and p_t=1. Then the revenue function, f' will be in the same location as f, as depicted in figure 3. Ray OB has been added through points in the first quadrant for which revenues equal labor costs. A machine earns positive rents when operated at any point above ray OB. Rents will be at a maximum when expression (14) is satisfied, so the firm will operate at point A. The tangent to f' at A is parallel to OB. Line segment AB is a measure of the rents generated by the machine (revenue less cost).

Visualizing changes in output prices or wages

Fixed output prices and wages are "built in" to figure 3. The revenue function would move when prices or wages changed, while the ray, OB, would remain fixed. If the price of output declined, all points on the function would shift proportionally downward. Similarly, if wages rose, the function would shift rightward and would be “stretched” to the right. With a little imagination, figure 3 can be used for a different visualization of the consequences of changes in these variables. Rather than redrawing all of the curves, one can simply adjust the scales by re-normalizing wages and prices. Then the revenue function will stay in its original place and OB will appear to rotate counterclockwise (up) through the first quadrant, perhaps to position, OD, as depicted in figure 4. A wage increase would reduce the rents earned from f' from the length of segment AB to that of CD. (A price decrease would involve a change in the scale of the vertical axes, ruining the correspondence of vertical segment lengths to rents, so we will focus on the wage increase.) This illustrates how a wage increase drives down rents and creates pressure to economize on labor. Note that CD is to the left of AB. Faced with a wage increase, the firm will reduce the amount of labor and output slightly, raising average labor productivity, consistent with what Cooper and Haltiwanger [1993] noticed happening to plants as they aged. In the long run, technological improvements generally lead to investments in improved capital goods which, in turn, bid for scarce labor, driving a persistent upward rotation in the ray representing revenue equals cost. The effect of *obsolescence* is just the rent lost due to the persistent rise in wages relative to the price of output. This rise (or rotation in ray OB) is not necessarily a constant --- a cyclical downturn in the economy can accelerate the upward rotation of the ray, while a surge in demand can temporarily reverse the process, causing a downward rotation of ray OB and an increase in rents. A key point is that the rotation of OB reflects both temporal and cyclical influences. The two exogenous influences tend to get swept together in the standard approach to capital measurement (further elaboration is provided later). The temporal influence creates "obsolescence" while the cyclical influence is what underlies the Berndt and Fuss [1986] "temporary equilibria."

Negative rents, if permanent, will induce asset retirement

Negative rents would occur if wages rose enough so that a revenue function fell entirely below the revenue/cost ray, as is the case with g′ and ray OD in figure 4. Negative rents can occur if the revenue function has a fixed labor requirement. No output (revenue) is produced unless this requirement is met, but once it is met, the function rises rapidly. If OD is high enough, and if diminishing marginal returns set in soon enough, revenues may never cover costs. Any attempt to operate the machine will result in a loss. If this is the situation, *assume that the machine is shut down* rather than operated at a loss. Note that this gives rise to the possibility of an abrupt shutdown of a machine (or a plant!) --- as OB rises (and before it reaches OD), rents would transition from positive to negative. So the model posits that all labor will be suddenly withdrawn from the asset, leaving it idle.

This situation is all too realistic --- the ability of the machine model to explain the abrupt removal of a computer from service, or an abrupt plant closing, is a consequence of the chosen specification with the fixed labor requirement. This cannot happen with the Cobb-Douglas specification[5] of the vintage production function that Solow [1959] used in an empirical exercise. As depicted in figure 5, Solow's functions would start at the origin and move out into the first quadrant, with newer vintages above older. The slope of each curve would gradually diminish reflecting diminishing marginal returns to labor. But there would be diminishing average returns to labor throughout each curve. If the ray OB gradually rotated upward squeezing rents, the firm would continue to operate the machine using less and less labor until labor reached zero. Falling rents would not lead to abrupt shutdowns, and instead old machines would just gradually fade away. Solow allowed for distinctions between vintages, but with this specification, the structure within each vintage of investment would be amorphous, each a unique kind of jelly. Structural phenomena, such as shutdowns, would fall below the radar screen. It is not necessary to assume a fixed labor requirement to allow for sudden asset retirements. There just needs to be a region where average returns to labor (labor productivity) are rising. Such a region is likely to exist. Few assets produce efficiently with scant labor. A worker has to pick up a shovel before digging with it. The firm will never operate in that region. The phenomena of plant shutdowns and of placing old computers that still work in the attic are evidence that the figures are drawn correctly, that is that average returns to labor on their production functions rise before falling. At some point the required marginal returns to labor exceeded the highest possible average returns and the assets were suddenly withdrawn from service. An interesting follow up that has been suggested would be to use this type of model and micro data to investigate plant closing behavior, but this paper will focus on measuring capital.

Shocks of investments or retirements are necessary to explain pro-cyclical productivity

[5] Output per unit of capital, with the Solow's vintage Cobb-Douglas function, is given by $(Y_{t,v}/K_{t,v})=Be^{\lambda v}(L_{t,v}/K_{t,v})^{\alpha}$.

It is interesting that on any function, f, that has a region where labor productivity is rising, labor productivity will reach a maximum at the point at which the curve is tangent to a ray from the origin. As a consequence of diminishing marginal returns, a machine will never operate to the left of this point. It follows that the labor productivity associated with a machine is at its maximum when the machine is marginal. Machines will operate to the right of this point, where *the response of labor productivity to an exogenous change in demand will be slightly counter-cyclical*. Pro-cyclical productivity must come from changes in the investment mix. As Foster, Haltiwanger, and Crizan [2001] showed, the biggest differences in labor productivity are associated with embodied technological differences. The machine model would predict that the productivity of individual machines will be countercyclical, so the pro-cyclical tendency of observed productivity is likely the result of changes in the age distribution of capital. For example, surges in technology rich investment --- or surges in retirements of the least efficient assets --- would tend to boost labor productivity.

The Components of Rent

While regrettably tedious, identifying the contributions of changes in technology, prices, wages and deterioration to aggregate rents will prove useful to untangling the issues in capital measurement. The total flow of rents at any time is just the sum of rents from machines, i.e.:

$$\Psi_t = \int_v \int_m \Psi_{t,v,m} \, dm \, dv \quad . \tag{16}$$

In the machine model, total rents earned by each vintage and model are fully determined by the machine production functions, by the numbers of machines of each model and vintage, and by prices and wages. It will facilitate interpretation to specify wages relative to prices, i.e. $\omega_t = w_t/p_t$, so that rents are given by $\Psi_{t,v,m} = p_t (Y_{t,v,m} - \omega_t L_{t,v,m})$. So total rents are $\Psi_t(f_{t,v,m}, n_{t,v,m}, p_t, \omega_t)$, and their time path is given by totally differentiating this unconstrained function with respect to time:

$$d\ln\Psi_t /dt = (1/\Psi_t)\Big\{ \int_v\int_m (\partial\Psi_{t,v,m}/\partial f_{t,v,m})(df_{t,v,m}/dt)dmdv + \int_v\int_m (\partial\Psi_{t,v,m}/\partial n_{t,v,m})(dn_{t,v,m} /dt)dmdv + $$
$$+ \int_v\int_m (\partial\Psi_{t,v,m}/\partial\rho_t)(d\rho_t /dt)dm\,dv + \int_v\int_m [\partial\Psi_{t,v,m}/\partial(\omega_t)]\,[d(\omega_t) /dt]dmdv \Big\}, \qquad (17)$$

where a derivative with respect to ρ_t represents the effect of a proportional change in p_t and w_t. The next few subsections consider each of the four right hand side terms, and in the process define the effects of various phenomena on rent.

The first term of equation (17), $(1/\Psi_t) \int_v\int_m (\partial\Psi_{t,v,m}/\partial f_{t,v,m})(df_{t,v,m}/dt)dm\,dv$, describes the effects of various shifts in the functions. The partial derivative, $(\partial\Psi_{t,v,m}/\partial f_{t,v,m})$, is the extra rent a firm would get from a slight shift in a function, i.e. $p_t\, n_{t,v,m}.z_{t,v,m} = c_{t,v,m}.n_{t,v,m}$. In turn, the machine function shifts, $df_{t,v,m}/dt$, can be further associated with deterioration and embodied and disembodied technical change (expressions 11, 12, and 13):

$$df_{t,v,m}/dt = (1/Y_{t,v,m})\{- [\partial f_{t,v,m}/\partial(t-v)]\, [d(t-v)/dt] - [\partial f_{t,v,m}/\partial(t-m)]\, [d(t-m)/dt] + \partial f_{t,v,m}/\partial t\} =$$
$$= \quad \chi^f_{t,v,m} + B^f_{t,v,m} + D^f_{t,v,m} \qquad . \qquad (18)$$

Therefore the first term can be written: $(1/\Psi_t)\int_v\int_m c_{t,v,m}.n_{t,v,m}(\chi^f_{t,v,m}+B^f_{t,v,m}+D^f_{t,v,m})dmdv$**.**

The second term of expression (17), $(1/\Psi_t) \int_v\int_m (\partial\Psi_{t,v,m}/\partial n_{t,v,m})(dn_{t,v,m}/dt)dm\,dv$, integrates the product of two components across vintages and models. The first, $\partial\Psi_{t,v,m}/\partial n_{t,v,m}$, is just the rental price of a machine, call it $c_{t,v,m}$. The second is the rate of change in the number of machines of a given vintage and model over time. (Later in this section, we will consider the determinants of investment.) Once a machine is made, assume it exists indefinitely (although it becomes irrelevant once its rental value is exhausted), so $dn_{t,v,m}/dt=0$ for $v>t$. What remains of this component, $dn_{t,t,m}/dt$, is new investment at time t in machines of each model m. The second term is therefore: $(1/\Psi_t) \int_v \int_m c_{t,t,m}(dn_{t,t,m}/dt)dv$.

An increase in the price of output, with relative wages held constant ($\omega_t = 1$), will not cause any reallocation of labor in the short run and so will translate into a proportional increase in rents. The third term reduces to the rate of price increase:

$$(1/\Psi_t) \int_v \int_m (\partial\Psi_{t,v,m}/\partial\rho_t)(d\rho_t /dt)dm\, dv \quad = \quad d\ln p_t/dt. \qquad (19)$$

Also, as time goes by, wages change relative to output prices (as figure 4 depicts in the form of a rotation of ray OB). Any increase in wages relative to prices will squeeze down rents, and conversely, an increase in prices that exceeds wage increases will enhance profitability. Variations in relative prices hence cause short-term variations in rents. While this phenomenon involves capital markets being temporarily out of long-run equilibrium, the resulting rents should be consistent with competitive "temporary" equilibrium that Berndt and Fuss [1986] described. As mentioned earlier, the machine model can be thought of as an elaboration of the dynamics underlying temporary equilibrium where the variations in rent are recognized to differ by vintage and model. These variations are driven by exogenous cyclical fluctuations in w_t/p_t.

In addition, even if machines do not deteriorate, a persistent increase in wages relative to output prices will *cause* their rents to decline. This temporal effect will be driven by investments in new technology that make labor more efficient. It will be specific to the type of capital, and for high tech assets it may actually drive down the cost of specific outputs, such as the cost of processing a grocery order, resulting in an increase in wage relative to product price. The effects on rents of the temporal component of w_t/p_t are the effects of obsolescence. The *rate at which the wage/price ratio affects the rents on an asset* will be defined as:

$$\theta_{t,v,m} = [\partial\Psi_{t,v,m}/\partial(\omega_t)]\ [d(\omega_t)/dt] \quad , \tag{20}$$

so that the fourth term of expression (17) can be written: $(1/\Psi_t) \int_v\int_m \theta_{t,v,m}\, dm\, dv$. An essential point is that this term reflects two effects: the cyclical effects of capital markets being temporarily out of equilibrium and the persistent effects of obsolescence.

To summarize, the growth rate of total rents emerges from the shadows:

$$d\ln\Psi_t/dt = d\ln p_t/dt + (1/\Psi_t) \int_m \{c_{t,t,m}(dn_{t,t,m}/dt) + \int_v [c_{t,v,m}\, n_{t,v,m}\, (\chi^f_{t,v,m} + B^f_{t,v,m} + D^f_{t,v,m})] + \theta_{t,v,m}]dv\}\, dm. \tag{21}$$

As might have been expected, rental growth depends on growth in output prices *and a weighted average*, across vintages and models, of the effects of investments, deterioration, technical change, cyclical variations in capital markets, and obsolescence. It will be necessary to classify these determinants as either price or quantity effects in order to measure real capital (Section V).

The Price of a Capital Asset, and Nominal Investment, Wealth and Depreciation

Neoclassical theory assumes that investment decisions are made by competitive producers with perfect foresight. Each producer will select the amount of *investment* such that the *price of an asset*, $p^K_{t,v,m}$, will equal the discounted value of future rents, that is: $p^K_{t,v,m} = \int_{\tau=t}^{\infty} \Psi_{\tau,v,m} e^{-r\tau}\, d\tau$. The stock of nominal wealth at any time will simply be the sum of asset values over all assets. Nominal investment is the increment to wealth at time, t, from acquiring new assets, $\int_m n_{t,t,m}\, p^K_{t,t,m}$. This corresponds to the output of capital goods. Nominal economic depreciation will be the declines in assets' values due to age, that is, the actual declines plus any revaluation in the prices of new goods, $n_{t,v,m}(p^K_{t,v,m} - p^K_{t-1,v,m})$. These are similar to the standard definitions, but they do bring in the link to future rents and expectations, which are themselves complex. So, for example, depreciation will be strongly influenced by deterioration and obsolescence, as Wykoff [2003] shows, but could also be influenced by changing interest rates, evolving expectations, or the like.

V. Measuring Real Capital

In the spirit of Solow's [1959] vintage paper, the machine model is a neoclassical description of how an asset is used that does not require vintage aggregation. It is now possible to consider what measurement units and weights would be suitable for the aggregation of "real capital inputs", what assumptions are necessary, and the implications of these assumptions.

Measurement Units and the Aggregation of Machines

The machine model can be used to devise an aggregate capital measure that reflects many of the factors affecting capital vintages. This perspective will help to identify how these factors are being treated in recent studies of capital and productivity. The model includes a unit, the "number of machines", $n_{t,v,m}$, that can be used to add up identical assets. This is a less constrained starting point for vintage aggregation than the usual "real value of investment". However, each category of machine is different. Expression (4) indicates that machine counts need to be ***weighted by marginal products*** in order to pass the Leontief aggregation conditions. The intuition is that investments must be adjusted for how much work they do. Recently, William Nordhaus [2002] has explored the possibility of measuring "computing power" using engineering characteristics. The idea is to build a natural unit that reflects *temporal changes* in computers, but further attention is needed to valuing the characteristics. Sound application of this approach to a significant portion of the capital stock would be a daunting empirical project.

In productivity measurement, BLS adjusts employment by average hours. Many authors have considered an analogous treatment of capital, i.e. adjusting the number of machines, $n_{t,v,m}$, for the intensity of their use. The idea is to adjust for the cyclical changes in marginal product. Jorgenson and Griliches [1967] originally made "capacity utilization" adjustments, but they later decided to avoid measuring capital in terms of other variables in the production function, like labor hours or energy use. These adjustments seemed to undermine the notion of an independent capital measure. Present day neoclassical capital measurement studies, such as Jorgenson and Stiroh [2001] and the BLS [2001] measures, do not make explicit capacity adjustments.

Even machine hours, $h^K_{t,v,m}$, fail to represent the marginal product of capital. Workers lose utility by giving up leisure time to work --- and so they are (usually) compensated by the hour. But idle capital has no utility and an asset's owner is (usually) not compensated by how many hours per day it is used. In the machine model, an asset, once acquired, is used for as many hours per day as necessary to maximize the difference between revenues and labor costs. So in the "temporary equilibrium" described by Berndt and Fuss [1986], the marginal value of running a machine one more hour per day will be zero, $\partial\Psi_{t,v,m}/\partial h^K_{t,v,m}=0$. That is, one machine hour is not equivalent to another, and the contribution of the last hour is marginal. Therefore the total hours of each type of machine *is not necessarily the appropriate weight* for use in the aggregation of machines.

The Marginal Product and Rental Price of a Machine

While Fisher showed that vintages needed to be aggregated in terms of marginal product (expression 4), the literature lacks a careful discussion of what the marginal product of capital is. Present measurement conventions regard "a spade to be a spade" (the “Gertrude Stein dictum”, as Harcourt [1969] put it) so a brand new machine of a given model is assumed to represent the "same amount" of capital (to have the same marginal product) in each time period. Thus the quantity unit for capital is tied exclusively to the inherent characteristics of the machine.

The machine model leads to a very different conclusion. When a machine is added to the economy, while total labor is held constant, the output of the new machine will be gained but some output from other machines will be lost because labor must be re-deployed to the new machine.[6] Since labor is always re-deployed at labor's marginal product, the new machine will boost output by the difference of the average product of labor on this machine, and the marginal product of labor (which will be the same on the new machine as on all other machines). The *marginal product of machines*, $z_{t,v,m} = \partial Y_t / \partial n_{t,v,m}$, is hence:

$$z_{t,v,m} = Y_{t,v,m}/n_{t,v,m} - L_{t,v,m}/n_{t,v,m}\, \partial Y_{t,v,m}/\partial L_{t,v,m} = \{(Y_{t,v,m}/L_{t,v,m}) - \partial Y_{t,v,m}/\partial L_{t,v,m}\} L_{t,v,m}/n_{t,v,m}. \quad (22)$$

The marginal product of the machine is determined by the machine's own production function *and* the marginal product of labor, which in turn is determined by the ratio of exogenous

[6] An example may help. Suppose there are 50 identical machines in the economy, each machine using 10 workers to make 100 units of output (500 workers and 5,000 units of output in all). If one more machine is added to the economy *and 10 more workers* 100 more units of output will be produced. But to compute the marginal product of capital, total labor must be held fixed, so 10 of the 51 machines now must be operated with only 9 workers. If these 10 machines now produce only 94 units of output each, the net gain from adding the 51st machine to the economy would be only 40 units (100-10*(100-94)). This marginal product will depend on how scarce labor is. Had we started instead by operating the 50 machines with only 7 laborers making 80 units of output each, the introduction of the 51st machine would require 7 machines to be operated with 6 workers each, the 7 machines producing perhaps only 70 units each. Then the marginal product of capital would be only 10 units (80-7*(80-70)). The usefulness of another machine is lower when labor is a relatively scarce resource.

functions of time, w_t/p_t. Marginal product closely corresponds to rent. Rent per machine, or the "machine rental price", $c_{t,v,m}$, is just the price of output times the machine's marginal product:

$$c_{t,v,m} = \Psi_{t,v,m} / n_{t,v,m} = p_t \ z_{t,v,m} \qquad (23)$$

Note that, $c_{t,v,m}$ reflects the marginal product of capital in that differences in marginal products between machines, z, will show up as differences in the rental prices, c.

It is possible to picture how a machine's marginal product changes by projecting figure 4 back into the output/labor-hours plane of figure 1. Figure 6 depicts the rays OB and OD representing the two given wage/price ratios, projected into output/labor space. Marginal products are proportional to the vertical distances between each operating point and the relevant ray. As the wage/price ratio changes, the vertical distances associated with different vintages will clearly change. If figure 6 depicted several functions like figure 2, it would be clear that the marginal products of machines are affected disproportionately by variations in the exogenous price of output and wage rate. In particular, rents and *the marginal products of older assets are affected proportionally more by cyclical effects and obsolescence* than are the rents and marginal products of newer more productive assets. The Leontief aggregation conditions require ratios of marginal products among machines to be independent of exogenous variables. Capital stock measures impose this, at odds with how assets with differences in productivity will behave.

Define *capital quantity measured in terms of marginal product*, $M_{t,v,m}$, for each vintage and model, and a *capital aggregate measured in terms of marginal product*, M_t, as:

$$M_{t,v,m} = z_{t,v,m} \, n_{t,v,m} \quad \text{and} \quad M_t = \int_v \int_m M_{t,v,m} \, dm \, dv \qquad (24)$$

This is essentially Fisher's (4) aggregation formula applied to counts of machines instead of to "real investment." It is tempting to call M_t "capital services." But this term is already used to describe measures emerging from Hall's equation (5). In order to consider what happens if the Leontief conditions (which underlie Hall's equation) are not imposed, and in order to avoid more confusion about terminology in the capital literature[7], this paper will refer to M_t as "capital measures in terms of marginal product" or "M-capital."

The Price and Components of M-Capital

Expression (17) defined a machine's rental price, $c_{t,v,m}$. Now define the *price of a unit of M-capital*, $p^M_{t,v,m}$, as rents per unit of capital. This price corresponds to the price of output:

$$p^M_{t,v,m} = \Psi_t / M_{t,v,m} = p_t \quad . \qquad (25)$$

There is no independent price for a measure of capital that is based purely on marginal product. The necessary dependency of neoclassical capital measures on measures of labor and output was one theme of Robinson's [1954] criticism of capital measurement. Robinson tried to explain that capital must be measured as either the amount of labor used to make the capital or the amount of output that the capital would produce in the future. Robinson and her many contemporaneous neoclassical authors failed to properly define the marginal product of capital and to recognize the dependency of the amount of capital services on the relationship of output and labor in short-run equilibrium. The neoclassicals did not want to acknowledge that capital services are inseparable part of the short run production decision, and so this is probably why no one faced up to this simple solution to the vintage aggregation problem.

And simple it is. Besides bringing different vintages and models together, expression (25) will apply to any type of capital (e.g. trucks, personal computers, and office buildings). It follows that the computation of aggregate M-capital across all types of assets boils down to the

[7] Jack Triplett [1998] has documented how many disagreements in the capital literature are the result of differences

deflation of a sector's total rents (property income) with the sector's own output price: $M_t=\Psi_t/p_t$. This may seem a trivial measure of capital inputs. If labor too were measured by deflating labor cost with the price of output, then the entire Solow residual would be zero. But a few decades ago most experts, including Moses Abramovitz [1956], actually believed that, correctly measured, the Solow residual *should be zero*. Call this important notion the "Abramovitz principle". This point was implicit in the early analysis of Jorgenson and Griliches [1967] and was acknowledged by Solow [1959, p.90]:

"…the notion of time shifts in the (aggregate production) function is a confession of ignorance rather than a claim to knowledge; they ought to be analyzed further into such components as improvements in the skill and quality of the labor force, returns to investment in research and development, improvements in techniques within industries, and changes in the industrial composition of input and output, etc."

While conceding to this on capital, this paper will not embrace the Abramovitz principle on the labor side. While upgrades in the skills of the labor force should be assigned to labor input, and not to the Solow residual, this section (V) will show later why one should not expect the residual to be zero even if labor is homogeneous. The balance of this section will also investigate the properties of M-capital, compare it to traditional capital, and describe its usefulness and limitations as a supplemental capital measure.

The properties can be illuminated by recalling that output price separated neatly out of the decomposition of changes in rents in expression (21). Thus in the machine model the growth rate of M-capital would be determined by investment, deterioration, embodied and disembodied technical change, and obsolescence:

$$d\ln M_t/dt = (1/\Psi_t)\int_m\left\{c_{t,t,m}(dn_{t,t,m}/dt)+\int_v\left[c_{t,v,m}\,n_{t,v,m}\,(\chi^f_{t,v,m}+B^f_{t,v,m}+D^f_{t,v,m})+\theta_{t,v,m}\right]dv\right\}dm\,. \quad (26)$$

in people's terminology.

Thus the new measure, M_t represents a host of components (reminiscent of Solow's list). Aside from output price, if it affects rents, its in there.

J-Capital

Equation (26) parallels Hall's equation (5), which leads economists to base capital stocks, J_t, on investment, deterioration, and embodied and disembodied technical change. However, expression (26) is more complex than Hall's equation, being presented in growth rate form and reflecting a more meticulous accounting of the sources of rents. Hall's equation can be thought of as a "specification" or a "structural model" of this more general form. Any difference between J_t and M_t will reflect components affecting the marginal product which are left out of J_t. These components are left out by default or by design, depending on one's perspective. In accounting for property income with J-capital, these components fall into the "shadow price" or "rental price", Ψ_t/J_t.

K-Capital

This paper will refer (somewhat loosely) to measures emerging from the methodology originated by Jorgenson and Griliches [1967] as "K-capital". This approach is currently used by Jorgenson and Stiroh [2001], and by BLS [2001], and has been adopted by the Australian Bureau of Statistics and by Statistics Canada.[8] Building K-capital involves a two-stage process. First, vintage aggregate capital stocks ($J_{t,A}$) are computed using Hall's equation (5) for detailed types of

[8] Recent analysis of the Australian accounting structure numbers was presented by Edwards, Comisari, and Johnson, [2002]. Canadian results are presented by Harchaoui, Armstrong, Tarkhani, and Jackson [2002].

asset, A. Fixed age/efficiency schedules (deterioration) are applied to real investments (capital expenditures deflated by price). Second, these stocks are aggregated across types using estimates of each stock's "rental price". These rental prices measure the rent associated with one "real dollar" of the respective stock. The rental prices are computed by plugging estimates of prices and depreciation (the same ones used in vintage aggregation), tax effects and a rate of return (driven by property income) into a structural formula. The formula, in turn, emerges from the neoclassical axiom that the price of an asset will equal its discounted future rents. The K-capital rental prices differ from asset to asset. These differences reflect differences in the marginal products of the stocks. In putting together K-capital, a wealth of detail on service lives, aging patterns, types of asset, and other structural features of capital must be assembled.

Homogeneous Machines

One possible arrangement of machine functions is of special significance. A group of machines, $\mathbf{G}_{t,v,m}$, is defined to be *homogeneous* if, for any two functions, $f_{i,j,k}$ and $f_{t,v,m} \subset \mathbf{G}_{t,v,m}$ there exists an $\alpha_{i,j,k}$ such that:

$$f_{i,j,k}\,(\alpha_{i,j,k}\,L) = \alpha_{\,i,j,k}\,f_{t,v,m}\,(L) \quad \forall\, L. \qquad (27)$$

Fisher [1965] and Hall [1967] used different proofs to show that vintages must be homogeneous in order for the vintage aggregate, J, to exist. Figure 7 illustrates the similarity of machine functions for a homogeneous group of machines. For any given p_t/w_t ratio, all machines will operate with the same proportions of output and labor, that is, the same labor productivity. One function is never "strictly above" another like in figure 2, i.e. one machine will never produce more output than another with the same labor. It is clear that two machines are not homogeneous if one of them embodies an improvement that enhances labor productivity. Nor can an older machine, whose profile has deteriorated, be part of the same homogeneous class as a new machine.

Within a homogeneous group, machines can produce different amounts of output with proportionally different amounts of labor input. Call the value of $\alpha_{i,j,k}$ that satisfies expression (27) the *size* of machine i,j,k compared to machine t,v,m. A new machine can be "bigger" but not "better". Imposing a homogeneity assumption seems like a pretty bad idea if one is interested in measuring high technology capital and characterizing the sources of growth. Yet this is the astonishing assumption made in capital stock measurement.

The Consequences of Imposing Homogeneity in Constructing Capital Measures

The vintage aggregation step in assembling K-capital assumes a fixed age/efficiency function for each type of asset. This is a fixed schedule intended to describe how the marginal product of an asset declines as it ages, relative to the marginal product of a new asset. Suppose an exogenous shock to w_t/p_t comes along and affects marginal products through $\theta_{t,v,m}$ in expression (26). The fixed age/efficiency schedule prevents *relative* marginal products from varying by vintage. The fixed schedule imposes *homogeneity*, as defined by equation (27) on the vintage machine production functions. All vintages are assumed to be affected proportionally by the shock. In effect, vintage machines are assumed to differ only in *size*. To the extent that assets *actually* differ in labor productivity, like in figure 2, any exogenous shock should actually affect the marginal products of the oldest and least efficient vintages proportionally more then those of the newer ones. Use of capital stocks can complicate short run analysis of technology, as Brynjolfsson and Hitt [2002] found. Neoclassical theory is not flawed, capital stock measurement just imposes an unrealistic specification of what happens in the short-run.

Homogeneity Bias from Temporal and Cyclical Influences

This paper has emphasized that both temporal and cyclical factors affect asset rents and measures of M-capital. The age/efficiency function, and hence measures of J-capital and K-capital, can be adjusted to compensate for any steady and persistent temporal influences. Consider a steady state growth economy where new models of machines bring higher labor productivity. Suppose each machine function shifts steadily down with age, due to deterioration, while the wage/price ray rotates steadily upward. The least productive machines are constantly being retired while new improved models are popping up. By assuming that all relevant processes occur at constant rates, a modified age/efficiency function could be distilled from expression 26 that described the behavior of marginal products as machines age while accounting for deterioration and also correcting the capital stock measures for homogeneity bias related to temporal influences of obsolescence. On this issue, Frank Wykoff [2003] has showed how to empirically account for the effects of obsolescence by adjusting the age/efficiency function. However it is clear that an age/efficiency function will be unable to correct for cyclical influences as long as it is time-invariant. The idea of a capacity utilization adjustment is to correct a fixed capital stock for cyclical variations in its service flow. However a single annual utilization ratio will not consistently account for variations in vintage-specific marginal products unless the vintage assets are homogeneous.

Functional Separability of the Capital Measure and Uses of K- and M-Capital

Unless the machine production functions are homogeneous, the Leontief aggregation conditions will be violated and measures of multifactor productivity will contain a cyclical component related to variations in capital service flows. Faced with this, is it best to abandon the quest for a single neoclassical capital measure (as critics of the 1950s and 1960s suggested)? Or is it better to impose the Leontief conditions, and proceed to enforce these conditions in measurement (as K-capital does)? Or does M-capital resolve all of the issues?

Each of these solutions is extreme. Capital plays a crucial role in production, and the idea of measuring it is useful. A significant body of literature on separability, aimed at inputs in general and not capital in particular, has established that measurement remains useful, though with limitations, when the Leontief conditions fail.[9]

M-capital slips past the Leontief conditions by doing what they require --- weighting machines with their actual contemporaneous marginal products. However, there is a problem. M-capital is *mushy* in that it responds to cyclical influences. It is so mushy that a superlative index formula is not even required to aggregate it --- all machines have the same price (the price of output). M-capital does not measure "the amount of goods" in any useful sense, i.e. itdoes not

[9] This literature began to appear just as the capital debates wore down. Berndt and Christensen [1973] tested for “weak” separability and related that to aggregation with more flexible formulas than the explicit addition allowed under the Leontief conditions. Under these weaker conditions, input aggregates can be devised that will serve well for small or “local” changes. In this vein, Hulten [1973] showed that a Divisia index (a “chain” index is a discrete analog to this), will depend on the time “paths” of the prices of items being aggregated. If inputs are only weakly separable, then their aggregate is somewhat ambiguous. Diewert [1976] showed that "superlative" formulas permitted aggregation under more general circumstances than those inherent in the Leontief conditions. Then Caves, Christensen, and Diewert [1982] showed a sense in which separability will not impede productivity measurement. By applying a superlative index formula to prices and quantities of outputs and inputs at two remote points in time, we can avoid the path dependency problem of the Divisia. One can even take advantage of the path dependency of aggregates to estimate cross-price elasticities between inputs and the effects of prices on technical change, as demonstrated by Jorgenson, Frank Gollop, and Barbara Fraumeni [1986]. Erwin Diewert and Denis Lawrence [1999] may be the only study that has attacked the problem the Leontief conditions pose for *vintage* aggregation. They created superlative aggregates of real capital from data on vintage investments. However, this work imposed various fixed age-efficiency functions on the data and thereby precluded the dramatic differences in vintage-specific rents that the machine model predicts.

summarize the amount of goods separately from the amount of services flowing. J-capital and K-capital do measure the amount of goods by imposing homogeneity, i.e. by holding fixed the component of marginal product that reflects exogenous cyclical influences. K-capital is as an indicator of the existing amount of capital, which economists need for modeling demand for investment and for other purposes. The construction of K-capital also provided a valuable detailed accounting of the goods.

While K-capital freezes capital services in the short run across the vintage dimension, it does reflect differences in capital services across different types of assets, by aggregating their J-capital stocks using rental price weights. These reflect differences in marginal product (expression 23). In principle M-capital will reflect asset-type components of marginal product and also the exogenous cyclical and temporal components. Thus the ratio M_t/K_t will reflect capacity utilization and also temporal influences. Capacity utilization adjustments fell into disfavor with neoclassical practitioners because they involved adjusting the capital stock with other inputs. However, it must be conceded that technology destroys homogeneity across vintages and along with it any possibility for a separable measure of capital services. Even though simple to measure, the observed M-capital should reflect the details of the real world while allowing the cyclical influences that K-capital is formulated to omit. Therefore M-capital may be a useful and complementary measure in assessing how intensively capital goods are used. While K-capital is bad and M-capital is ugly, both may be useful.

Two Alternative Solow Residuals based on M-capital

The Solow residual (expression 1) will be computed using M-capital (MFP_M). This will shortly be compared to the multifactor measure published by BLS (MFP_K). M-capital is computed as the simple ratio of property income to the price of output using data from the BLS MFP calculations. A third MFP measure, MFP_{M*}, is also formulated by noting two things: a) under perfect competition, MFP is equivalently measured as the difference between share weighted input prices and output price, and b) from expression (25) the price of M-capital is just the price of output:

$$MFP_{M*} = s_K\, p_t + s_L\, w_t - p_t \;\; = s_L \;(w_t - p_t) \quad , \qquad (28)$$

where growth rates are italicized. Notwithstanding the Abramovitz principle and homogeneous labor, this residual will rise in step with real wages.

Expression (28) is a Solow residual (MFP) that sheds a great deal of light on how M-capital and K-capital differ. Note that $(w_t - p_t)$ is just the upward rotation of ray OB in figure 4. Expression (28), formulated from the point of view of the dual, takes advantage of the fact that w_t and p_t are endogenously determined in a closed and competitive economy. What happens when an improved asset with technology B hits the market? First a few firms in the using industry will adopt it --- and there will be a premium associated with the labor costs that technology B saves them compared to the old technology A. This premium likely will be pocketed by the firm in the using industry or extracted by an upstream firm making the capital goods. In any event, the labor market will minimize any wage premium for workers who happen to be using technology B. At this *point in time* the new technology assets represent more capital input than the older ones, and so the premium is scored as M-capital and does not fall through to productivity. However, *as time passes* and more firms adopt technology B and as more competing technologies come along, the premium starts to erode (obsolescence). Since the firms must compete for workers, the workers will ultimately command all of the benefits associated with technology B. So the Solow residual of expression (28) rises to the extent that the benefits of technology B and other technologies are reflected in real wages --- while the benefits stay in M-capital to the extent that wages do not rise. *As the benefits obsolesce out of M-capital they shift to the residual*. By adhering to the Gertrude Stein dictum, traditional measures block the obsolescence component of real capital input. They effectively integrate expression (26) over time while omitting the obsolescence effects. Technological advances accrue and compound inside the capital measure without an offset. One measure cannot be considered correct and the other wrong, but it is important to understand how measurement conventions govern what is in each.

A Description of the BLS Data Used for an Empirical Example

Measures were computed using data associated with the BLS [2001] private nonfarm business multifactor productivity report. The MFP_K measures come straight from the report and reflect the structural approach used in the construction of K. The MFP_M measures will use property income derived for the report from the National Income and Product Accounts and reflect the scope of this measure. Notably, this measure may be influenced by taxes or by returns to intangible capital not included in K. The MFP_{M*} measure is relatively "pure" in that its movement mainly reflects movements in w_t and p_t. Any errors in w_t and p_t are problems for all three measures (indeed there may be an exaggeration in the quality-adjusted price of output for high tech assets affecting all three measures), while MFP_{M*} is free of the other limitations of MFP_K and MFP_M.

Empirical Example

Table 1 compares these MFP measures. The business-cycle length trends are at the bottom. The trends from 1949 to 1999 of all three are *within one-tenth of a percentage point* of one another, as are the trends for the two main sub-periods, 1949-73 and 1973-99. Evidently K_t, M_t , and M* approximate one another closely over the long run. This close conformance lends mutual support to the measures. MFP_M provides an easy short-cut to compute the long-run Solow residual. At the same time, if M-capital is regarded to be the correct summary measure of capital services, then it confirms the complex structural calculation of MFP_K.

The long-term MFP_M trends are not zero as the Abramovitz [1956] principle would suggest. During most intervals after 1973, MFP_M and MFP_{M*} were a few tenths higher, except for the 1990-1995 period when MFP_K was higher. The reasons for the post-1973 pattern of differences are unclear. The slightly higher MFP_M from 1995-1999 indicates that K-capital was growing several tenths of a percent per year faster than M. In this recent period, high tech equipment began to dominate investment, and so the result may reflect the effects of obsolescence. As we noted, obsolescence is accounted for on the quantity side in M but not K. These effects may have become big enough to affect aggregate statistics by the late 1990s. All three MFP measures sped up after 1995. Oliner and Sichel [2001] showed that 70 percent of this speedup is attributable to high tech assets.

The year-to-year differences in the measures are often significant. The BLS [2001] has consistently urged customers of the MFP data to focus on business-cycle length periods, in recognition of the short-term limitations of the Solow residual. It is striking to note that the cyclical "misbehavior" of MFP_K is much greater than that of MFP_M and MFP_{M*} --- the latter have about 60 percent less variance and 40 percent less correlation with output growth than MFP_K. Since the K-capital quantity measure is designed to exclude the short-run influences of demand, the MFP_K measure will reflect these influences in the form of pro-cyclical volatility. The effects of under-utilization of capital do not necessarily belong in the Solow residual, because these undermine its interpretation as a measure of technological change. The lower variance is a consequence of how M-capital reflects the incorporation of variations in marginal product into the measure of quantity rather than allowing the variations to fall into price. Thus, by affording flexibility to the capital measure during the business cycle, MFP_M appears to mitigate the theoretically puzzling cyclical behavior of MFP_K. It does so within the context of a purely competitive model. So MFP_M is a cleaner indication of shifts in the aggregate production function of the Solow residual than is MFP_K. Growth accountants may want to calculate the Solow residual both ways, with the cyclical effects both above and below the "line", so as to sort out the sources of fluctuations in labor productivity.

Note that the MFP_{M*} measure can be extended into 2000 and 2001 using the most recent data on wages and prices by assuming that the share of labor has been constant at its 1999 level. Any plausible change in this share would have only very small effects. It takes about a year before the BLS receives the data needed to do the detailed structural calculations involved in K-capital. The ease of constructing MFP_{M*} would allow measures to become available more quickly.

Table 2 presents the indexes of capital themselves, i.e. K-capital and M-capital. An index of the ratio of M-capital to K-capital is also presented, which should be an indicator of capacity utilization. However, there is a recent steady decline in this ratio since the mid-1960s. This temporal pattern might be partly a result of accelerating importance of assets subject to obsolescence. M-capital is empirically driven by property income, and so its sluggish growth might also reflect increased competitiveness among producers.

VI. Measuring Quality Change

In the literature on quality adjustment of consumer goods, it is axiomatic that relative prices reflect relative utilities. In measuring inputs associated with durable capital goods, the usual assumption is that relative goods prices measure relative marginal products. My point of departure is that they do not.

Triplett [1989] recognized that rental prices rather than purchase prices should be used to compare marginal products. In neoclassical theory the purchase price of an asset presumably equals the discounted value of its future rents. The ratio of purchase prices of two assets is therefore proportional to the ratio of their discounted streams of future rents, and not necessarily to the ratio of marginal products. At first blush, it seems modest to assume the rental streams will be proportional to marginal products, ensuring that the purchase prices are in step with the rental prices. After all, this proportionality will occur if age/efficiency functions are geometric.

But if new machines embody technical change, that is, if the labor productivity associated with newer models is higher, and if the machine functions contain regions of increasing returns to labor, then obsolescence will push down rents of older models proportionately faster than rents of newer models. Because of this, the ratio of the price of a more productive model to that of a less productive model will overstate the ratio of marginal products. As Hulten [1992] noted, the notion of capital quality is grounded in Hall's [1968] embodiment factor, which itself is grounded in aggregation with marginal products.

Obsolescence, the Functional Form of Marginal Product with Age, and Quality Change

The observed purchase prices, whether determined with hedonic or matched model techniques, will still be required to measure quality. But information on the functional pattern by which obsolescence affects marginal products as models age will also be required. For example, assume that models are impacted as they age by obsolescence, but not by deterioration. (Oliner and Sichel [2001] contend that obsolescence dominates deterioration in contributing to a high tech asset's demise.) Further, assume that w_t/p_t rotates upward at a steady rate without cyclical disturbances. Processes, such as quality improvements, are then presumed to occur at fixed rates, so that the quality of a new model, in any year t, *relative* to one introduced one year earlier will be $B_t=z_{t,t,t}/z_{t,t,t-1}$. Let γ_τ be the age(τ)/efficiency function, i.e. $\gamma_\tau = z_{t,t,t-\tau} /z_{t,t,t}$. Under the assumptions, $B_t=1/\gamma_1$. From the neoclassical axiom that the price of an asset equals the discounted future rents, one can determine the price of a new model relative to last year's model:

$$p^K{}_{t,t,t}/p^K{}_{t,t,t-1} = \int_{u=0}^{\infty} p_{t+u}\,\gamma_u\, e^{-ru}du \;/\; \int_{u=0}^{\infty} p_{t+u}\,\gamma_{u+1}e^{-ru}du =$$

$$= B_t \int_{u=0}^{\infty} p_{t+u}\,\gamma_u\, e^{-ru}du \;/\; \int_{u=0}^{\infty} p_{t+u}\,(\gamma_{u+1}/\gamma_1)\, e^{-ru}du \;. \qquad (29)$$

If quality raises the labor productivity of newer models, (which it must do if there is a technological improvement as distinct from an increase in size), rents, $p\gamma$, will be forced down by obsolescence proportionally faster with age, i.e. $d^2\ln\gamma_\tau/d\tau^2>0$, and the machine will eventually be retired, i.e. $\gamma_\tau=0$ for $\tau>L$. Under these conditions the ratio of integrals on the r.h.s. of expression (29) will be greater than one. The ratio of model prices will exceed relative quality.

While awaiting evidence on this ratio of integrals, it should be recognized that that the bias in the existing durable goods quality adjustments is likely to be substantial. Figure 8 plots the marginal products of goods. For example, computers could be depicted by straight-line age/efficiency functions with short lives. The age/efficiency functions decline because of the temporal effects of a steady increase in w_t/p_t. Newer models embody technical improvements. The relative marginal product of a newer model to that of an older one at any time would be proportional to the ratio of heights of the lines in left-hand-side portion of figure 8. The relative asset prices will be proportional to a ratio involving areas. Thus the area under each line, from a given time through the rest of the life of the asset, will represent the asset's (non-discounted) future rents. If the discount rate is zero and the effects of obsolescence are straight-line, as in figure 8, ***quality change would be overstated by a factor of about two***. For an age/marginal product relationship that declined slowly at first and then faster in absolute level with age, as the BLS assumes, the factor would be even higher. The problem could be exacerbated by compounding when index numbers are created using overstated quality change estimates.

A geometric age/efficiency specification appears to escape the problem, i.e. $\gamma_{u+1}/\gamma_1=\gamma_u$ for all u, reducing the r.h.s. of (29) to B_t. The age/efficiency profiles of all models will fade away proportionately and, therefore, corresponding goods prices will take on the same proportions. Before taking any comfort in this, note that a geometric age/efficiency function cannot describe a situation where obsolescence erodes the marginal products of assets with older designs proportionately more than newer ones, as is likely to happen when the assets embody different technologies. Capital goods prices will be proportional to marginal products only if the geometric model really describes events at the micro level, that is, only if the older and newer assets belong to a homogeneous family. This will not happen when labor productivity in newer models is higher as a result of embodied quality improvements. Obsolescence forces many older vintages out of service. This is an observational fact, not a theory. As the level of an asset's marginal product approaches zero, it will decline faster and faster in percentage terms. The geometric model can accommodate neither the fact nor the math. The theory leading to expression (29) permits a more realistic interpretation of the observed model price differentials.

VII. Measuring the Output of Durable Goods

So far this paper has discussed vintage aggregation of durable goods as a problem in measuring capital *input*. What are the implications, if any, of obsolescence for measures of the real *output* of durable goods? Statistical agencies treat durable goods symmetrically with consumer goods in that they are measured in line with Gertrude Stein dictum. An item with given physical characteristics is counted the same at any point in time. We do not penalize for obsolescence. Here we will briefly consider the case for doing so.

As a practical matter, output is measured by dividing expenditures on these goods (nominal investment) by price index numbers. This procedure places the burden of quality adjustment on these price series. Use of the “dual” or price approach to quality adjustment rests formally on some assessment of relative asset values to either or both of the two agents involved in the investment transaction: the producer of the goods (the resource cost point of view), and the buyer (the user value point of view). The resource cost rationale for quality adjustment assumes that the price ratio between successive models at any point in time reflects the relative marginal costs of producing the models. This is grounded in assuming competitive equilibrium among producers of durable goods. Ana Aizcorbe [2002] argues that producers of computer chips have market power and she measures the erosion of their price markups, concluding that part of the decline in prices is due to changes in market power rather than quality.

The user value rationale for quality adjustment regards the price ratio as reflective of the ratio of marginal products. This rationale is applicable to material inputs, but the present paper has demonstrated that this does not necessarily carry over to durable goods, because they earn their returns from capital services over multiple time periods. The price ratio does reflect a ratio of "future user values". The future user value would be the discounted future contributions to real output (an area in the first panel of figure 8). Thus the ratio reflects the relative real wealth embodied in the two models. What economists do is identify the relative quality of two models, as appropriate to output and real wealth measurement, with this ratio. A buyer pays more for the superior model not only because it will produce more now but also because it will be productive longer. Shouldn't quality measure both? Yes! But this logic ties the definition of real wealth to the sum of future marginal outputs. Given that, real wealth should *also reflect the decline in these future marginal outputs over time due to obsolescence*. The capital measure may overstate growth in "real wealth" because it compounds quality differences measured from cross sections, one on top of another over time, without also adjusting the amount of capital represented by each specific model downward over time, as its future possibilities obsolesce.

In national accounting, the featured measure of output is gross domestic product (GDP), which includes investment in capital goods. This is also the preferred measure of output for computing the Solow [1957] residual for an aggregate economy. In GDP only "final" outputs are included. Intermediate inputs are not counted so as to avoid counting products at successive stages of production. This treatment leaves the output measure invariant to the amount of intermediate activity, regarding this activity as an internal detail of the aggregate economy that should not be counted. However capital is counted in GDP. It is granted special status to allow a period-by-period description of growth --- once created, capital will be around to contribute in future periods. For the Solow residual, individual capital goods are counted on the output side when they are made, and, as this paper describes, they are counted later on the input side as they contribute to production.

But in granting this special status, keep in mind that capital, like intermediate input, is not an end in itself. Consumers cannot eat computers, they can only consume their services. Similarly firms can only use computers. The "real wealth" or "final product" represented by a newly produced computer is the (discounted) sum of future contributions it will make and is not the computer itself. To ensure consistent concepts, a measure of these contributions is needed. This measure might reflect the role of durables as capital goods rather than treating them like apples. Present-day quality adjustments impose the Gertrude Stein dictum and define the unit for capital in terms of the characteristics of the good itself and thus place a priori restrictions on how market prices reflect future marginal product. Obsolescence is treated as a factor affecting the price of capital services rather than the quantity.

Real wealth measures in line with the machine model would not correspond to M-capital (capital services), but the two would be closely related. The real wealth represented by an asset, $W_{t,v,m}$, is the real analogue to the value of the asset (as given by the various integrals in equation 29). Real wealth will be the discounted total of future marginal products of the asset, $\int_{u=0}^{\infty} z_{t+u,v,m} e^{-ru} du$. These marginal products would be denominated in the units of the other, non-durable goods and services, which the durable asset will help to produce. In neoclassical equilibrium the discounted value of future additions to consumption will equal foregone consumption in the current period. So we could obtain $W_{t,v,m}$ by deflating the nominal wealth measure for fixed assets by a contemporaneous price index for all nondurable goods and services. Similarly, the real output represented by new investments could be obtained by deflating capital expenditures with the same deflator. The use of such a measure in national accounting and in computing the Solow residual would address issues raised by Ed Denison [1993] and T.K. Rymes [1993] about deflation of computers with quality adjusted price indexes. They objected to technological change being counted in capital rather than being allowed to fall into productivity. Their conclusion was that current methods ascribe an exaggerated role in U.S. economic growth to the production of and investment in computers, and ascribe too little productivity growth to computer-using industries. More recently, Hill [2000] identified obsolescence as the root of the conflict in using quality adjusted durable goods prices in national accounting. Adjusting durable goods for quality but not for obsolescence can disrupt the identity between real savings and real investment. In comparing the real values of investments made at different points in time, the identity implies that they should reflect the increments to future consumption. If a brand new durable good with given characteristics is counted as the same amount of real investment every year (the Gertrude Stein dictum), but obsolescence is eroding its future earnings potential, then it will be overstating the amount of savings represented by goods produced later, relative to goods produced earlier.

VIII. Conclusions

From the viewpoint of owners of older assets, investment by others in technologically improved models is an exogenous force leading to the obsolescence of the older assets. Hill's

[2000] contention that obsolescence is neglected in quality adjustments has been brushed aside by neoclassical practitioners on the grounds that standard K-capital measures account for obsolescence in rental prices. But the literature has not adequately explained why obsolescence is best treated as a price effect instead of a quantity effect. This paper has suggested that quantity measures be adjusted for obsolescence. This would fulfill a neoclassical agenda of grounding capital measures in marginal product units. If the exogenous influence of obsolescence depresses the marginal products of assets, the corresponding quantity of capital should be reduced. The convention of treating obsolescence as a price effect appears to stem from the Leontief [1947] aggregation theorem. In order to ensure consistent aggregation, vintage quantities must be independent of exogenous influences. However, the requirements of the theorem are not met when technologically improved assets raise labor productivity. The issue is how to proceed with measurement.

Robert Solow [1959] proposed the vintage model to show that the choices involving labor and capital could be analyzed with neoclassical models even without capital measures. Joan Robinson [1954] had proposed that capital must be measured in terms of either the labor used to produce it or in terms of the output it would produce in the future. In this paper a capital measure has been proposed that dynamically adjusts the quantity of capital for the exogenous influences. The related measure of real wealth would reflect the discounted contributions to future real output. While reminiscent of Robinson's preference to denominate capital in terms of future output, this prescription is more specific and is grounded in a neoclassical style model, developed by extending the Solow vintage model to machines. This capital measure could supplement, but never replace, detailed vintage accounting exercises like those of Jorgenson and Stiroh [2001].

The quality adjustments economists make to durable goods are predicated on treating capital goods as if they were consumed at purchase, neglecting the possibility that the relative usefulness of various models changes as they age. The marginal products or rents of older models are likely to be affected by obsolescence proportionately more than those of newer ones. These effects are reflected in empirical data on durable goods prices. In measuring "real" capital inputs and the real output of durable goods, it may not be sufficient to deflate capital expenditures with price indexes for goods with constant characteristics. It is also necessary to consider the systematic effects of obsolescence.

References

Abramovitz, Moses [1956] "Resource and Output Trends in the United States Since 1870", American Economic Review, Papers and Proceedings, May, p.11.

Aizcorbe, Ana [2002], "Why are Semiconductor Prices Falling So Fast?: Industry Estimates and Implications for Productivity Measurement," presented at the NBER/CRIW Summer Institute.

Aizcorbe, Ana, Carol Corrado, and Mark Doms [2000], "Constructing Price and Quantity Indexes for High Technology Goods", presented at the CRIW Workshop on Price Measurement at the National Bureau of Economic Research Summer Institute.

Berndt, Ernst.R. and Laurits.R. Christensen [1973], "The Internal Structure of Functional Relationships: Separability, Substitution, and Aggregation," Review of Economic Studies, Vol. 40, pp. 403-410.

Berndt, Ernst R., and Melvyn A. Fuss [1986], "Productivity Measurement with Adjustments for Variations in Capacity Utilization and Other Forms of Temporary Equilibrium," Journal of Econometrics, Vol. 33, No. 1/2, October/November, pp. 7-29.

Brynjolfsson, Erik and Lorin M. Hitt [2001], "Computing Productivity: Firm-Level Evidence," Sloan Working Paper #4210-01, Massachusetts Institute of Technology.

Bureau of Labor Statistics [2001], "Multifactor Productivity Trends, 1999", Washington, D.C. 20212: U.S. Department of Labor News Release No. 01-125, May 3, 2001.

Caves, D.W., L.R. Christensen, and W.E. Diewert [1982], "The Economic Theory of Index Numbers and the Measurement of Input, Output and Productivity," Econometrica, Vol. 50, pp. 1393-1414.

Cooper, R and J. Haltiwanger [1993], "The Aggregate Implications of Machine Replacement: Theory and Evidence,"American Economic Review, Vol. 83, pp. 360-382.

Denison, Edward F. [1993], "Implications of BEA's Treatment of Computer Prices and Productivity Measurement," statement in Murray F. Foss, Marilyn E. Manser, and Allan H Young, eds., Price Measurements and Their Uses, Chicago: University of Chicago Press, pp. 351-355.

Diewert, W. Erwin [1976], "Exact and Superlative Index Numbers," Journal of Econometrics, Vol. 4, No. 4, pp. 115-145.

Diewert, W. E. and Denis A. Lawrence [1999], "Progress in Measuring the Price and Quantity of Capital", presented at the third meeting of the Canberra Group on Capital Measurement, held at the Bureau of Labor Statistics, Washington, DC.

Edwards, Rob; Peter Comisari, Tony Johnson, [2002], "Beyond 1993: The System of National Accounts and the New Economy", Australian Bureau of Statistics, Australia.

Fisher, Franklin. M. [1965], "Embodied Technical Change and the Existence of an Aggregate Capital Stock," Review of Economic Studies, Vol. 32, pp. 263-288.

Foster, Lucia, John Haltiwanger and C.J. Crizan [2001], "Aggregate Productivity Growth: Lessons from Microeconomic Evidence," ," in Hulten, Charles R., Edwin R. Dean and Michael J. Harper, eds., New Developments in Productivity Analysis, Chicago, University of Chicago Press, pp. 303-363.

Griliches, Zvi [1963], "Capital Stock in Investment Functions: Some Problems of Concept and Measurement," in Carl Christ, ed., Measurement in Economics, Stanford, CA: Stanford University Press, reprinted in Zvi Griliches, ed. Technology, Education and Productivity (1988), New York: Basil Blackwell, Inc. pp. 123-143.

Hall, Robert.E. [1968], “Technical Change and Capital from the Point of View of the Dual, ”Review of Economic Studies, January, pp. 35-46.

Harchaoui Tarek M., Philip Armstrong, Faouzi Tarkhani, and Chris Jackson [2002], "A Comparison of Canada-U.S. Productivity Growth in the Information Age, 1981-2000," draft manuscript, Statistics Canada.

Harcourt, G.C. [1969], "Some Cambridge Controversies in the Theory of Capital," Journal of Economic Literature, Volume VII, No. 2, June, pp. 369-405.

Hill, Peter [2000], "Economic Depreciation and the SNA," paper presented at the 26th Conference of the International Association for Research in Income and Wealth, Cracow.

Hulten, Charles R. [1973], "Divisia Index Numbers," Econometrica, Vol. 41, pp. 1017-1025.

Hulten, Charles R. [1992], "Growth Accounting When Technical Change is Embodied in Capital," American Economic Review, Vol. 82, No. 4, pp. 964-979.

Jorgenson, Dale W. [1966], "The Embodiment Hypothesis," Journal of Political Economy, Vol. 74, pp. 1-17.

Jorgenson, Frank M. Gollop and Barbara M. Fraumeni [1987], Productivity and U.S. Economic Growth, Harvard University Press, Cambridge, Mass.

Jorgenson, and Kevin J. Stiroh [2001], "Raising the Speed Limit: U.S. Economic Growth in the Information Age", Brookings Papers on Economic Activity, No. 1, pp. 125-235.

Jorgenson, and Z. Griliches [1967], “The Explanation of Productivity Change,” Review of Economic Studies, Vol. 34 (3), No. 99, July, pp. 249-282.

Leontief, W.W. [1947], "Introduction to a Theory of the Internal Structure of Functional Relationships," Econometrica, Vol. 15, pp. 361-373.

Nordhaus, William [2002], "The Progress of Computing," presented at a Workshop on Economic Measurement, Brookings Institution, Washington.

Oliner, Stephen D. and Daniel E. Sichel [2000], "The Resurgence of Growth in the Late 1990s: Is Information Technology the Story?" Journal of Economic Perspectives, Vol. 14, No. 4, Fall, pp. 3-22.

Organization for Economic Co-operation and Development [2001], Measuring Capital.

Robinson, Joan [1954], "The Production Function and the Theory of Capital," Review of Economic Studies, vol. 21, pp. 81-103.

Rymes, T.K. [1993], “Implications of BEA’s Treatment of Computer Prices and Productivity Measurement,” statement in Murray F. Foss, Marilyn E. Manser, and Allan H Young, eds., Price Measurements and Their Uses, Chicago: University of Chicago Press, pp. 362-366.

Solow, Robert M. [1957], “Technical Change and the Aggregate Production Function,” Review of Economics and Statistics, Vol. 39, No. 3, pp. 312-320.

Solow, Robert M. [1959], "Investment and Technical Progress," in K. Arrow, S. Karlin and P. Suppes, eds. Mathematical Methods in the Social Sciences, Stanford University Press.

Solow, R.M. [2001], "After 'Technical Progress and the Aggregate Production Function," in Hulten, Charles R., Edwin R. Dean and Michael J. Harper, eds., New Developments in Productivity Analysis, Chicago, University of Chicago Press, pp. 173-178.

Triplett, Jack E. [1989], "Price and Technological Change in a Capital Good: A Survey of Research on Computers," in Jorgenson, Dale W. and Ralph Landau, eds., Technology and Capital Formation, Cambridge, Mass., The MIT Press, pp. 127-213.

Triplett, Jack E. [1998], "A Dictionary of Usage for Capital Measurement Issues," presented to the Expert Group on Capital Stock Measurement, hosted by the Australian Bureau of Statistics, Canberra.

Wykoff, Frank C. [2003], "Obsolescence in Economic Depreciation from the Point of View of the Revaluation Term", presented at the NBER Summer Institute's CRIW seminar, Cambridge, Mass.

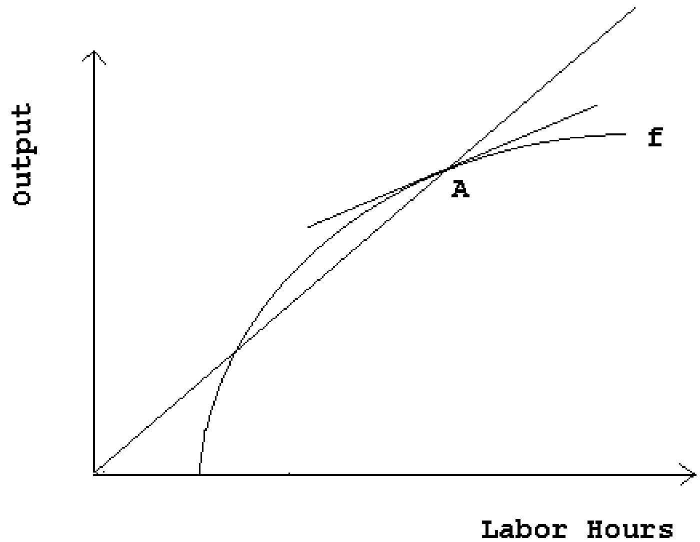

Figure 1. A Machine Production Function

The firm can choose where to operate along f, and chooses A. The slope of ray OA is the average product of labor (labor productivity). The slope of the tangent to f at A is the marginal product of labor.

Figure 2. A Family of Machine Functions

Several machines may operate simultaneously. Labor productivity may differ, even though the marginal product is the same (f, g, and h are tangent to parallel lines). A machine's function may shift down with age due to deterioration, or up with time due to disembodied technical change (or both). New machines tend to appear "higher" in the picture, meaning they allow higher labor productivity --- this is embodied technical change.

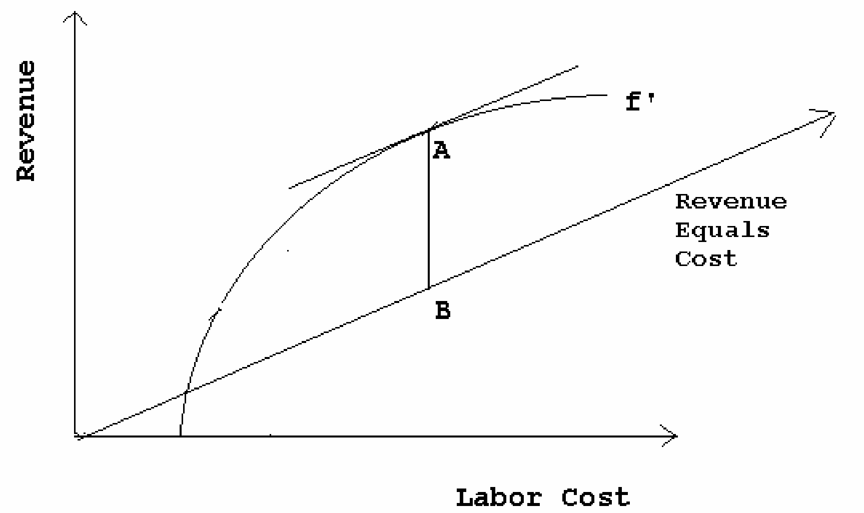

Figure 3. Revenue Function

The machine owner is a price taker for both wages and product price (these are exogenous). For a given wage and price, the machine function, f, can be projected into a revenue/cost plane. The revenue function, f', will look exactly like f if the revenue and cost axes are suitably normalized (w=1, p=1). Ray OB delineates where revenue equals labor cost. The owner will choose operating at point A, where the tangent to f' is parallel to OB. Then segment AB will measure rents (gross profits measured by revenue less variable cost).

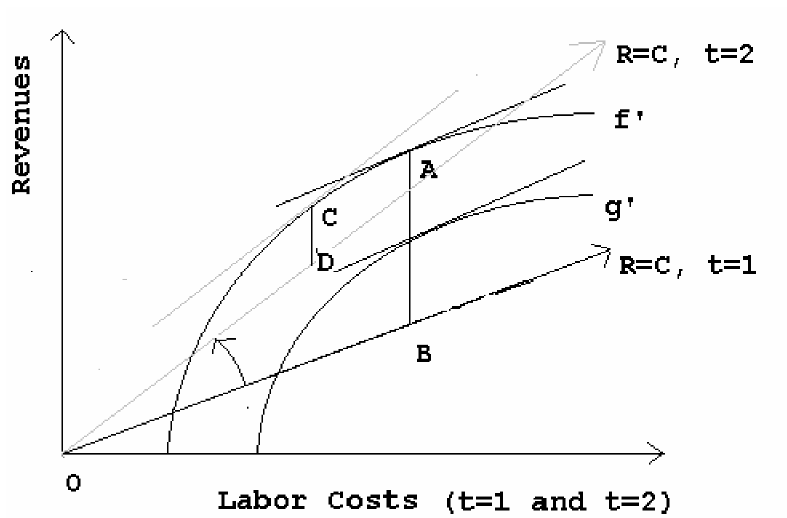

Figure 4. The Dynamics of a Wage Increase

If wages rise relative to prices, the revenue = cost ray, OB, would remain fixed and the functions, f' and g', would shift and elongate rightward. However, it is possible to re-normalize the axes in the plane as wages rise so that the functions stay put. The revenue = cost ray would then appear to rotate upward, to the position of ray OD. The rents from f' will be driven down from the length of segment AB to that of segment CD. Note that labor productivity rises slightly (the slope of OC is greater than that of OA). Machine g' meets a different fate: rents become negative after the wage increase, and so it is shut off abruptly to avoid an operating loss.

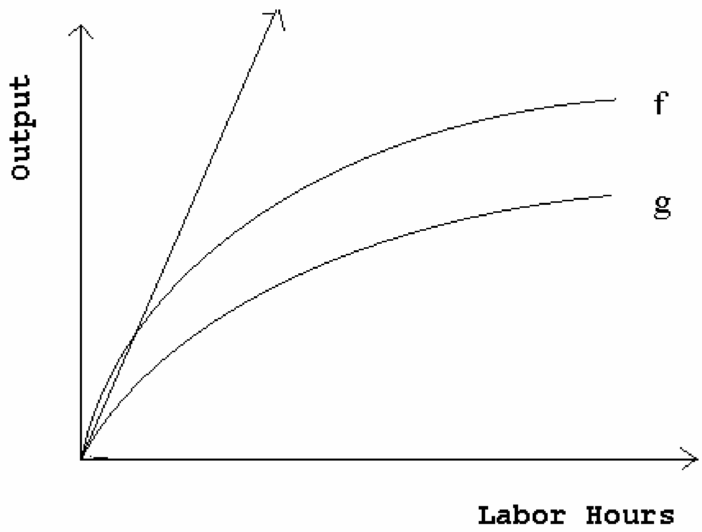

Figure 5. Cobb-Douglas Specification

Solow mentioned one possible specification for his vintage production function, and here it is graphed for two machines. Because there is no part of the domain where there are increasing average returns to labor, rents will only approach zero as wages become very high. Negative rents are impossible, and there is not be an abrupt transition from operating with a lot of labor to being shut down. Old machines do not die, they just gradually fade away.

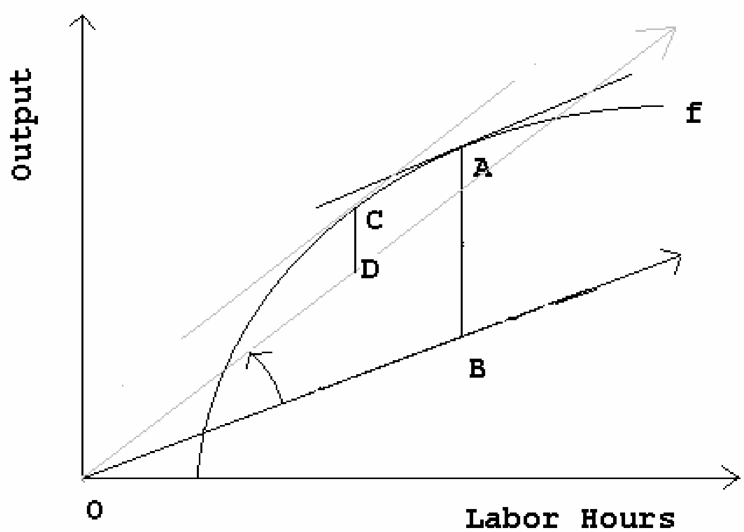

Figure 6. The Marginal Product of a Machine

Rays OB and OD in figure 4 can be projected back into output/labor-hours plane of figure 1. Segments AB and CD then represent the marginal product of the machine before and after the wage increase. Even though the machine is exactly the same, the marginal product of the machine is driven down by an increase in wages because the opportunity cost of labor (the output the worker could make with some other machine) has risen. In the long run, technical change and investments in efficient assets drive a steady upward rotation of the ray.

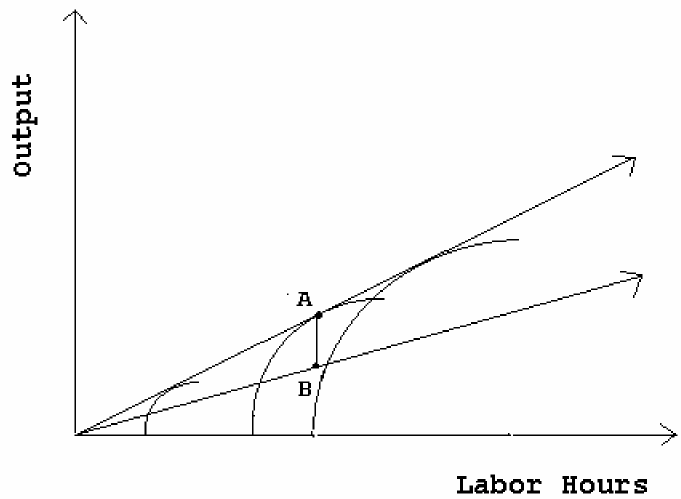

Figure 7. A Family of Homogeneous Machine Functions

The Leontief aggregation conditions fail unless the relative marginal products of machines are unaffected by the rotation of ray OB. Machines with higher potential labor productivity than others (like in figure 2) are ruled out of a homogeneous group. Machines can be bigger but not better. This is unsuitable for studying technologically improved equipment, but it is what economists assume in measuring capital stock.

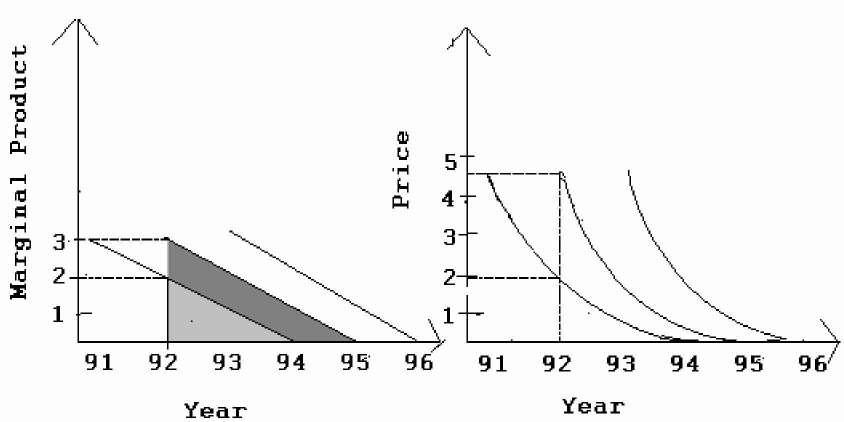

Figure 8. Tracing Vintage Marginal Products and Prices

Obsolescence causes the marginal products of machines of any model to fall over time. For example, suppose the m.p. of a new model in 1992 is 3 units of output and that of a new 1991 model (in 1992) is 2 units. Assuming interest rates are negligible, the price of each asset will reflect the remaining area under its m.p. curve. This is the light shaded area for the 1991 model and the total of the light and dark areas for the 1992 model. The ratio of prices (in units of output) will tend to exceed the ratio of marginal products. In the example the older model has 2/3 the marginal product but only 4/9 of the price of the newer one. If the m.p.s decline along parallel straight-lines and interest is negligible, the price ratio will be the product of the effects of m.p. (2/3) and of future obsolescence (2/3). Thus for small price differentials, "quality" will be overstated by a factor of about two.

Table 1. Three Measures of Multifactor Productivity
(annualized percentage changes)

	MFP_K (BLS)	MFP_M	MFP_{M*}
1949	**2.0**	**3.1**	**2.5**
1950	**6.0**	**2.8**	**3.2**
1951	**2.7**	**2.2**	**2.8**
1952	**0.0**	**2.0**	**1.6**
1953	**1.2**	**2.2**	**2.1**
1954	**0.0**	**1.0**	**1.1**
1955	**4.7**	**0.9**	**1.2**
1956	**-0.9**	**1.1**	**1.0**
1957	**1.0**	**1.5**	**1.7**
1958	**0.3**	**1.8**	**1.6**
1959	**3.5**	**0.6**	**0.9**
1960	**0.5**	**2.2**	**1.8**
1961	**1.6**	**1.4**	**1.4**
1962	**3.5**	**1.7**	**1.9**
1963	**2.7**	**1.7**	**1.8**
1964	**3.8**	**3.0**	**2.9**
1965	**2.8**	**1.4**	**1.6**
1966	**2.9**	**3.1**	**3.4**
1967	**-0.1**	**2.0**	**2.0**
1968	**2.6**	**2.6**	**2.5**
1969	**-0.9**	**1.1**	**1.0**
1970	**-0.6**	**2.1**	**1.9**
1971	**3.0**	**2.2**	**2.1**
1972	**3.0**	**1.9**	**2.4**
1973	**2.8**	**3.0**	**3.4**
1974	**-3.6**	**-0.8**	**-0.8**
1975	**0.2**	**-0.7**	**-0.2**
1976	**4.1**	**2.4**	**3.1**
1977	**1.4**	**0.8**	**0.6**
1978	**1.4**	**1.5**	**1.5**
1979	**-0.7**	**0.9**	**1.1**
1980	**-2.3**	**-0.1**	**-0.3**

	MFP_K (BLS)	MFP_M	MFP_{M*}
1981	**-0.5**	**-0.7**	**-1.1**
1982	**-3.3**	**-1.6**	**-0.8**
1983	**3.6**	**2.3**	**1.7**
1984	**2.6**	**0.5**	**0.8**
1985	**0.5**	**1.3**	**1.2**
1986	**1.5**	**3.0**	**2.9**
1987	**0.1**	**-0.7**	**-0.4**
1988	**0.7**	**-0.9**	**-0.2**
1989	**0.3**	**0.6**	**0.4**
1990	**0.0**	**0.7**	**1.0**
1991	**-1.0**	**0.5**	**-0.3**
1992	**2.0**	**2.1**	**1.9**
1993	**0.5**	**0.2**	**-0.1**
1994	**1.0**	**-0.7**	**-0.5**
1995	**0.5**	**0.3**	**0.1**
1996	**1.4**	**0.8**	**0.9**
1997	**1.0**	**0.9**	**0.3**
1998	**1.4**	**2.6**	**2.4**
1999	**0.6**	**1.0**	**1.4**
2000*			**1.5**
2001:1*			**0.7**
2001:2*			**1.1**
2001:3*			**0.8**
1948-1999	**1.2**	**1.3**	**1.3**
1948-1973	**1.9**	**1.9**	**2.0**
1973-1999	**0.5**	**0.6**	**0.6**
1973-1979	**0.4**	**0.7**	**0.9**
1979-1990	**0.3**	**0.4**	**0.5**
1990-1995	**0.6**	**0.5**	**0.2**
1995-1999	**1.1**	**1.3**	**1.3**
Variance, 1949-1999:	**3.66**	**1.42**	**1.39**
Correlation with Output:	**0.83**	**0.41**	**0.49**

(12/17/2001, Mike Harper)

Table 2. Two Measures of Capital and Their Ratio

(indexes, 1948=100)

	K-capital (BLS)	M-capital	M/K
1948	100.0	100.0	100.0
1949	102.7	99.4	96.8
1950	107.0	113.9	106.4
1951	112.9	122.1	108.2
1952	116.7	118.5	101.5
1953	120.4	118.6	98.5
1954	123.0	117.3	95.3
1955	127.4	136.6	107.2
1956	132.4	133.3	100.7
1957	136.5	134.8	98.8
1958	139.2	130.8	93.9
1959	142.7	146.8	102.9
1960	147.1	143.0	97.2
1961	150.8	147.2	97.6
1962	155.6	161.3	103.6
1963	161.1	172.8	107.2
1964	167.4	184.7	110.3
1965	175.7	202.8	115.4
1966	186.5	214.3	114.9
1967	198.0	213.5	107.8
1968	207.3	223.9	108.0
1969	218.5	220.8	101.0
1970	229.5	211.1	92.0
1971	239.6	227.3	94.9
1972	250.8	247.8	98.8
1973	265.6	261.7	98.5
1974	280.8	250.4	89.2
1975	292.3	266.9	91.3
1976	302.2	291.2	96.4
1977	314.6	310.2	98.6
1978	329.1	323.8	98.4
1979	346.2	323.5	93.4
1980	364.5	315.7	86.6
1981	384.2	334.7	87.1
1982	401.5	330.7	82.4
1983	415.9	358.1	86.1
1984	435.9	400.0	91.8
1985	422.0	338.9	80.3
1986	441.4	339.3	76.9
1987	458.1	360.3	78.6
1988	473.3	390.6	82.5
1989	487.9	400.3	82.0
1990	502.1	401.9	80.0
1991	513.6	392.3	76.4
1992	524.9	400.5	76.3
1993	538.6	415.7	77.2
1994	555.8	451.5	81.2
1995	577.5	471.8	81.7
1996	602.3	501.1	83.2
1997	633.0	528.6	83.5
1998	671.0	542.9	80.9
1999	715.3	571.7	79.9
1948-1999	4.1	4.0	-0.1
1948-1973	4	4.1	0.1
1973-1999	4.2	3.9	-0.3
1973-1979	4.5	3.7	-0.8
1979-1990	4.2	3.9	-0.3
1990-1995	2.9	3.3	-0.4
1995-1999	5.5	4.9	-0.6