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On the cyclical behavior of employment, unemployment and labor force participation*

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VERY PRELIMINARY AND INCOMPLETE

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1. Introduction

In terms of labor market dynamics, standard real business cycle models (RBC) focus on explaining the cyclical behavior of employment and total hours worked. For this reason, RBC models typically lump together unemployment and out of the labor force into a single nonemployment state and analyze variations in employment and hours worked by either studying a work-leisure decision (e.g. Kydland and Prescott [8] and Hansen [7]) or a work-home production decision (e.g. Greenwood and Hercowitz [6] and Benhabib, Rogerson, and Wright [3]). While this strategy has led to important advances in real business cycle theory and considerable success in explaining employment and hours fluctuations, it abstracts from one of the main characteristics of labor markets that labor economists usually emphasize, this is, search frictions. Despite the preponderant role that search has played in the labor literature it is surprising how little attention it has received in the RBC literature. The few exceptions are Andolfatto [2] and Merz [10], [11], who studied versions of the Mortensen-Pissarides [12] matching framework, and Greenwood, Gomes and Rebelo [5], who analyzed a version of the Lucas-Prescott [9] islands framework. Despite using substantially different models all these papers reached a similar conclusion: they found that a RBC model that incorporates search decisions can account for the cyclical behavior of employment and unemployment observed in U.S. data. However, this result was obtained under two important assumptions. In all these papers the labor force was assumed to be fixed and agents were allowed to enjoy leisure while searching. These assumptions are not innocuous. To the extent that leisure plays an important role in generating unemployment fluctuations, it should have important implications for labor force participation. In particular, if the main reason why agents enter unemployment is to enjoy leisure, some of them may want to go all the way and leave the labor force in order to enjoy even more leisure. Thus, part of the flows from employment to unemployment captured in those models could end up being flows from employment to out of the labor force once a labor force participation margin is allowed for. Changes in labor force participation may also feed back into employment and unemployment fluctuations in important ways. These reasons suggest that a complete theory of employment and unemployment fluctuations must incorporate labor force participation.

The purpose of this paper is to analyze to what extent a RBC model that includes employment, unemployment and labor force participation decisions can account for the observed fluctuations in these variables. The model is a version of one used by Alvarez and Veracierto [1], which in turn is based on the Lucas and Prescott [9] equilibrium search model. Output, which can be consumed or invested, is produced by a large number of islands that use capital and labor as inputs into a decreasing returns to scale production technology. Contrary to the deterministic steady state analysis of Alvarez and Veracierto [1], the islands are subject both to idiosyncratic and aggregate productivity shocks. At the beginning of each period agents must decide whether to work in the island where they are currently located or search for a new employment opportunity. An important difference with Lucas and Prescott [9] is that agents have the choice of making their search directed or undirected. Another difference is that the model incorporates an out of the labor force margin and investment.

Parameter values are chosen so that the deterministic steady state of the model economy reproduces important observations from the National Income and Product Accounts (NIPA) and labor market statistics. Aggregate productivity shocks in turn are selected to match the empirical behavior of Solow residuals. I find that once a labor force participation decision is included, the RBC model fails to account for the behavior of employment, unemployment and labor force participation. The search and leisure decisions embodied in this version of the neoclassical growth model generate drastically counterfactual behavior: 1) employment fluctuates as much as the labor force while in the data it is three times more variable, 2) unemployment fluctuates as much as output while in the data it is six times more variable, and 3) unemployment is acyclical while in the data it is strongly countercyclical. Even though the model fails to account for the empirical observations, it fails in an informative way. The paper points out that in a RBC model the cyclical behavior of employment and unemployment is extremely sensitive to the incorporation of a labor force participation decision. In addition, the paper suggests that a successful model, whatever that may end up being, should give a much more important role to variations in search decisions than to fluctuations in leisure.

The paper is organized as follows: Section 2 describes the model economy. Section 3 describes a deterministic steady state equilibrium. Section 4 parameterizes the model, and

Section 5 presents the results. A detailed Appendix discusses the computational methodology.

2. The benchmark economy

The economy is populated by a representative household constituted by a large number of members with names in the unit interval. The household's preferences are given by:

$$E\sum_{t=0}^{\infty}\beta^t\left\{\ln c_t + A\left(\frac{h_t^{1-\phi}-1}{1-\phi}\right) - \psi D_t\right\} \tag{2.1}$$

where c_t is consumption of a market good, h_t is consumption of a home good, D_t is the number of agents doing directed search, $0 < \beta < 1$ is the subjective time discount factor, and $\phi > 0$. Notice that there is a disutility cost $\psi \geq 0$ incurred by each agent that performs directed search.

The market good, which can be consumed or invested, is produced in a continuum of islands. Each island has a production function given by

$$y_t = e^{(1-\varphi)a_t} z_t n_t^{\gamma} k_t^{\varphi}$$

where y_t is production, n_t is the labor input, k_t is the capital input, z_t is an idiosyncratic productivity shock, a_t is an aggregate productivity level common to all islands, $\gamma > 0$, $\varphi > 0$, and $\gamma + \varphi \leq 1$. The idiosyncratic productivity shock z_t follows a finite Markov process with transition matrix Q, where $Q(z, z')$ is the probability that $z_{t+1} = z'$ conditional on $z_t = z$. Realizations of z_t are assumed to be independent across islands.

The aggregate productivity level evolves according to the following AR(1) process:

$$a_{t+1} = \rho a_t + \varepsilon_{t+1}, \tag{2.2}$$

where $0 < \rho < 1$ and ε_{t+1} is i.i.d., normally distributed, with variance σ_ε^2 and zero mean.

Capital is assumed to be freely mobile across islands, but not labor. At the beginning of every period there is a given distribution of agents across islands. An island cannot employ more than the total number of agents x present in the island at the beginning of the period. If an agent stays in the island in which he is currently located, he produces market goods and starts the following period in the same location. Otherwise, the agent leaves the island and becomes nonemployed.

A nonemployed agent has three alternatives. The first alternative is to search for a new employment opportunity using an undirected search technology. If the agent uses this technology he gets zero production during the current period, but becomes randomly assigned to an island at the beginning of the following period. Since agents that perform undirected search have no control over which islands they will arrive to, they are assumed to arrive uniformly across all islands in the economy.

The second alternative for an nonemployed agent is to perform directed search. The directed search technology entails a disutility cost. However, agents in this technology learn the current idiosyncratic shocks of all islands in the economy and are allowed to pick which island to arrive to in the following period.

The third alternative is to leave the market sector in order to engage in home production. The home production technology is described by the following linear function:

$$h_t = 1 - \pi_U U_t - \pi_D D_t - \pi_N N_t \tag{2.3}$$

where U_t is the number of agents that do undirected search, D_t is the number of agents that do directed search, and N_t is the number of agents that are employed in the market sector. Each household member is endowed with one unit of time. As a consequence, π_U, π_D, π_N denote the amount of time required by the undirected search, the directed search and the production technologies, respectively. It is assumed that $0 < \pi_U \leq \pi_D \leq \pi_N \leq 1$. Observe that at any point in time an agent can be employed in the market sector, doing undirected search, doing directed search, or being out of the labor force, but cannot be in more than one alternative at the same time.

In order to describe the aggregate feasibility conditions for this economy, it will be important to index each island according to its individual state: the idiosyncratic productivity level of the island z and the number of agents available at the beginning of the period x.

Feasibility requires that the island's employment level $n_t(x, z)$ do not exceed the number of agents initially available:

$$n_t(x, z) \leq x. \tag{2.4}$$

The number of agents in the island at the beginning of the following period, is given by

$$x' = n_t(x, z) + U_t + d_t(x, z)$$

where U_t is total undirected search in the economy and $d_t(x, z)$ is the number of agent that direct their search to this particular island. Observe that this equation uses the fact that undirected searchers become uniformly distributed across all islands in the economy.

The law of motion for the distribution μ_t of islands across idiosyncratic productivity levels and available agents is then given by

$$\mu_{t+1}(X', Z') = \int_{\{(x,z):n_t(x,z)+U_t+d_t(x,z)\in X'\}} Q(z, Z')\mu_t(dx, dz) \tag{2.5}$$

for all X' and Z'. This equation states that the total number of islands with a number of agents in the set X' and a productivity shock in the set Z' is given by the sum of all islands that transit from their current shocks to a shock in Z' and choose an employment level such that $x' = n_t(x, z) + U_t + d_t(x, z)$ is in X'.

Aggregate employment is then

$$N_t = \int n_t(x, z)\mu_t(dx, dz), \tag{2.6}$$

the number of agents that do directed search is

$$D_t = \int d_t(x, z)\mu_t(dx, dz), \tag{2.7}$$

and aggregate capital is

$$K_t = \int k_t(x, z)\mu_t(dx, dz). \tag{2.8}$$

In turn, aggregate feasibility for the market good is given by

$$c_t + K_{t+1} - (1-\delta)\,K_t \leq \int e^{(1-\varphi)a_t} z_t n_t(x,z)^{\gamma} k_t(x,z)^{\varphi} \mu_t(dx,dz). \tag{2.9}$$

Assuming complete markets, a competitive equilibrium can be obtained by solving the social planner's problem, which is given by maximizing (2.1) subject to equations (2.2), (2.3), (2.4), (2.5), (2.6), (2.7), (2.8), and (2.9).

3. Deterministic steady state

Solving for a competitive equilibrium of the above economy is a complicated task because the state space is highly dimensional (the distribution of islands across idiosyncratic shocks and available agents constitutes one of the state variables of the economy). To get an idea of the different economic margins underlying a competitive equilibrium this section describes the steady state of a deterministic version of the model where the aggregate productivity shock a_t is set to its unconditional mean of zero. Computing an equilibrium for the stochastic economy requires a linear quadratic approximation about this deterministic steady state. The appendix describes in detail the computational strategy used in the paper, as well as a formal derivation of the steady state conditions described in this section.

To start with observe that, since consumption is constant at steady state, the interest rate is given by $1/\beta$. Hence the rental rate of capital is given by

$$r = \frac{1}{\beta} - 1 + \delta. \tag{3.1}$$

In what follows it will be assumed that there are competitive labor markets within islands. Hence firms equate the marginal productivity of labor and capital to the wage rate w and the rental rate r, respectively:

$$w = z\gamma n^{\gamma-1} k^{\varphi} \tag{3.2}$$

$$r = z n^{\gamma} \varphi k^{\varphi-1} \tag{3.3}$$

Substituting (3.3) in (3.2), we get the following wage function:

$$w(n,z) = z\gamma n^{\gamma-1}\left(\frac{zn^{\gamma}\varphi}{r}\right)^{\frac{\varphi}{1-\varphi}}$$

where n is the employment level of the island.

Let consider the decision problem of an agent that begins a period in an island of type (x,z) and must decide whether to stay or leave the island taking as given the employment level of the island $n(x,z)$, the number of agents directing their search to the island $d(x,z)$ and the aggregate number of agents doing undirected search U. If the agent decides to stay, he earns the competitive wage rate $w(n(x,z),z)$ and begins the following period in the same island. If the agent decides to leave, he becomes non-employed and obtains a value of θ. His problem is then described by the following Bellman equation:

$$v(x,z) = \max\left\{\theta, w\left(n(x,z),z\right) + \beta\int v\left(n(x,z)+U+d(x,z),z'\right)Q\left(z,dz'\right)\right\}$$

where $v(x,z)$ is the expected value of beginning a period in an island of type (x,z).

At equilibrium, the employment rule $n(x,z)$ must be consistent with individual decisions. In particular, if the state of the island is such that $v(x,z) > \theta$ (agents are strictly better-off staying than leaving), then all agents stay, that is:

$$n(x,z) = x$$

On the other hand, if $v(x,z) = \theta$ (agents are indifferent between staying or leaving) then some agents must leave until

$$n(x,z) = \bar{n}\left(z\right),$$

where $\bar{n}\left(z\right)$ satisfies:

$$\theta = w(\bar{n}\left(z\right),z) + \beta\int v\left(\bar{n}\left(z\right)+U,z'\right)Q\left(z,dz'\right). \tag{3.4}$$

Observe that the employment rule is then given by

$$n(x,z) = \min\{x, \overline{n}(z)\}. \tag{3.5}$$

That is, if the number of agents in the island is less than $\overline{n}(z)$ then everybody stays. On the other hand, if the number of agents in the island exceeds $\overline{n}(z)$ then some agents leave until employment equals $\overline{n}(z)$.

If nobody directs its search to the island the next period number of agents available will be given by $n(x,z) + U$. However if the expected value of being in the island is sufficiently large, some agents will direct its search to it. In particular, there will be an upper bound σ on the expected value of an island. If the expected value of the island is less than σ nobody will direct his search to it. But the arrival of directed searchers will preclude the expected value of the island to exceed σ. Let $\underline{n}(z)$ be the number of agents such that $\underline{n}(z) + U$ makes directed searchers indifferent between arriving to the island or not:

$$\sigma = \int v(\underline{n}(z) + U, z')Q(z,z')dz'. \tag{3.6}$$

Then, the number of agents available in the island at the beginning of the following period is given by

$$p(x,z) = \max\{\underline{n}(z) + U, n(x,z) + U\} \tag{3.7}$$

and the number of agents directing their search to the island is given by

$$d(x,z) = p(x,z) - (n(x,z) - U). \tag{3.8}$$

That is, if employment $n(x,z)$ is less than $\underline{n}(z)$ the expected value of the island would exceed σ. This attracts directed searchers until the number of of agents in the island the following period becomes $\underline{n}(z) + U$.

Observe from the above discussion that we can write the equilibrium value v as the solution to the following Bellman equation:

$$v(x,z) = \max\left\{\theta,\ z\gamma x^{\gamma-1}\left[\frac{zx^{\gamma}\varphi}{r}\right]^{\frac{\varphi}{1-\varphi}} + \beta\min\left[\sigma, \int v(x+U,z')Q(z,z')dz'\right]\right\} \tag{3.9}$$

Given the law of motion $p(x,z)$, the invariant distribution of islands μ must satisfy the following recursion:

$$\mu(X',Z') = \int_{\{(x,z):p(x,z)\in X'\}} Q(z,Z')\mu(dx,dz). \tag{3.10}$$

The steady state aggregate capital, employment, directed search and consumption levels are then given by

$$K = \int k(x,z)\mu(dx,dz) \tag{3.11}$$

$$N = \int n(x,z)\mu(dx,dz) \tag{3.12}$$

$$D = \int d(x,z)\mu(dx,dz) \tag{3.13}$$

$$c = \int zn_t(x,z)^{\gamma}k(x,z)^{\varphi}\mu(dx,dz) - \delta K \tag{3.14}$$

respectively.

The last equilibrium conditions determine the labor force participation decision, the value of nonemployment θ and the upper bound on the expected value of an island σ. With respect to labor force participation decisions we have that

$$(1-\beta)\,\theta = \pi_N A\left(1-\pi_U U - \pi_D D - \pi_N N\right)^{-\phi} c \tag{3.15}$$

must hold. This condition states that an agent located in an island where some agents leave ($v(x,z)=\theta$) must be indifferent between staying in the island for one more period, receiving a flow value equal to $(1-\beta)\,\theta$, and working at home for one period. The value for this agent of working at home for one period is the marginal utility of the home good multiplied by the hours π_N freed from employment, divided by the marginal utility of consumption (to obtain a magnitude comparable with θ).

Assuming that there is undirected search at equilibrium, the value of being nonemployed θ must satisfy

$$\theta\,(1-\beta)\,\frac{\pi_U}{\pi_N}+\beta\theta=\beta\int v(x,z)\mu(dx,dz). \tag{3.16}$$

The right hand side of this equation gives the expected value of doing undirected search: it is the expected value under the invariant distribution (since undirected searchers arrive uniformly across all islands) discounted by the factor β (since the arrival takes place the following period). The left hand side is the value of spending one period doing home production and becoming nonemployed the following period.

Assuming that directed search also takes place at equilibrium, unemployed agents must be indifferent between both alternatives. This requires that

$$\beta\sigma-\psi c=\theta\,(1-\beta)\,\frac{\pi_D}{\pi_N}+\beta\theta \tag{3.17}$$

The right hand side is the value of spending one period doing home production and becoming nonemployed the following period. The left hand side is the expected value of doing directed search σ discounted by the factor β minus the disutility of doing directed search expressed in consumption units.

4. Parameterization

This section describes the steady state observations used to select the parameters of the model. The parameters to be chosen are β, A, ϕ, ψ, γ, φ, π_U, π_D, π_N, δ, the values for the idiosyncratic productivity shock z, the transition matrix Q, and the parameters determining the driving process for the aggregate productivity shock: ρ and σ_ε^2. The time period selected for the model is one month. A short time period is called for in order to match the relatively short average duration of unemployment observed in U.S. data.

The curvature of home production in the utility function ϕ and the disutility from directed search ψ will be taken as free parameters in the experiments below. As a consequence I will postpone discussing their values until the next section. The calibration of other parameters are conditional on the choices for ϕ and ψ. As a consequence in this section I discuss the observations used to calibrate them, but I will provide actual values only after ϕ and ψ are determined.

The stock of capital in the market sector K is identified with business capital, that is, with plant, equipment and inventories. As a result, investment in business capital I is associated in the National Income and Product Accounts with fixed private non-residential investment plus changes in business inventories. Given that the depreciation rate is related to steady state I and K according to

$$\delta = \frac{I}{K},$$

averaging the ratios I/K over the period 1967:Q1 to 1999:Q4 gives a monthly depreciation rate $\delta = 0.00659$.

In turn, consumption c is identified with consumption of non-durable goods and services (excluding housing services). Output is then defined as the sum of our measures of consumption and investment. The average monthly capital-output ratio K/Y corresponding to the period 1967:Q1 to 1999:Q4 is 25.8.

The interest rate in the model economy is given by

$$1 + i = \frac{1}{\beta}$$

As a consequence $\beta = 0.9967$ is chosen to reproduce an annual interest rate of 4 percent, roughly the average between the return on equity and the return on treasury bills in the U.S. economy.

The Cobb-Douglas production function and the competitive behavior assumption implies that φ equals the share of capital in market output. That is,

$$\left(\frac{1}{\beta} - 1 + \delta\right)\frac{K}{Y} = \varphi.$$

Given the previous values for β, δ, and $\frac{K}{Y}$, it follows that $\varphi = 0.2554$. On the other hand, $\gamma = 0.64$ is selected to reproduce the labor share in National Income.

The idiosyncratic productivity levels z and the transition matrix Q are chosen to approximate (by quadrature methods) the following AR(1) process:

$$\log z_{t+1} = \rho_z \ln z_t + \varepsilon^z_{t+1}$$

where ε^z_{t+1} is i.i.d., normally distributed, with zero mean and variance σ^2_z.[1] Since the stochastic process for the idiosyncratic productivity shocks is a crucial determinant of the unemployment rate and the average duration of unemployment, ρ_z and σ^2_z will be selected later on to reproduce an unemployment rate of 6.2 percent and an average duration of unemployment equal to one quarter, which correspond to U.S. observations. The weight for the utility of home production A in turn will be selected to reproduce a labor force participation equal to 74 percent (the average ratio between the labor force and the size of the population between 16 and 65 years old) The actual values for ρ_z, σ^2_z and A will depend on the values for ϕ and ψ, which are taken as free parameters.

The rest of the parameters to calibrate are the time requirements for undirected search π_U, for directed search π_D and for employment π_N. Hereon, I will assume that $\pi_U = \pi_D$, that is, that both types of search take the same amount of hours. The combination of directed-undirected search will be strictly determined by the disutility of directed search parameter ψ (this will improve the ability of the model to generate search fluctuations). Let π_S be the time requirement of search. Given that there is a lot of uncertainty about how large π_S should be relative to π_N, I will report results under different assumptions (in particular, I will consider values for π_S/π_N equal to 1, 0.5, and 0.1) . The only constraint that I imposed is that total hours spent in market activities ($\pi_S \times (U + D) + \pi_N \times N$) must be equal to 0.33, which is consistent with the evidence provided by Ghez and Becker [4], and is the magnitude commonly used in the RBC literature.

Finally, values for ρ and σ^2_ε must be determined. The strategy for selecting these parameters is to choose them so that measured Solow residuals in the model economy replicate the behavior of measured Solow residuals in the data. Proportionate changes in measured

[1] Only three values for z will be allowed for in the computations. While this may not seem a large number, it leads to a considerable amount of heterogeneity: the support of the invariant distribution will have about two thousand (x, z) pairs in the experiments reported.

Solow residual are defined as the proportionate change in aggregate output Y minus the sum of the proportionate change in aggregate employment N times the labor share γ, minus the sum of the proportionate change in aggregate capital K times $(1-\gamma)$. Using the measure of output described above and a labor share of 0.64, measured Solow residuals are found to be as highly persistent but a bit more variable than in Prescott [13]: the standard deviation of quarterly technology changes is 0.009 instead of 0.0076. As a consequence, $\rho = 0.98$ and $\sigma_\varepsilon^2 = 0.009^2/3$ are chosen here.

5. Results

In order to evaluate the behavior of the model economy, Table 1 reports U.S. business cycle statistics. Before any statistics were computed, all time series were logged and detrended using the Hodrick-Prescott filter. The empirical measures for output Y, consumption c, investment I and capital K reported in the table correspond to the measures described in the previous section, and cover the period between 1967:Q1 and 1999:Q4. The table shows some well known facts about U.S. business cycle dynamics: that consumption and capital are less variable than output while investment is much more volatile, and that consumption and investment are strongly procyclical while capital is acyclical. The variability of labor relative to output (0.57) is lower than usual among other things because it refers to employment instead of total hours worked. What is important in Table 1 is the variability of unemployment, which is 6.25 times the variability of output, and the variability of the labor force, which is only 0.19 times the variability of output. While employment is strongly procyclical, labor force participation is only weakly procyclical. On the contrary, unemployment is strongly countercyclical: its correlation with output is -0.83. Note that even though unemployment is a small fraction of the labor force, its behavior is key in generating a much larger variability in employment than in labor force participation.

Before reporting the results for the benchmark economy I will proceed to analyze the business cycles of different environments. The analysis will help relate the paper to the previous literature and better understand the results for the benchmark economy.

5.1. Inelastic labor force and utility from search

As was pointed out in the introduction, all previous RBC models have assumed a fixed labor force participation. To facilitate comparisons with the literature, in this section I analyze a version of the economy where the labor force is fixed at 0.74 (same magnitude as the benchmark economy) and preferences are given by:

$$E\sum_{t=0}^{\infty}\beta^t\left\{\ln c_t + B\times U_t\right\}$$

where $B > 0$. These preferences correspond to the employment lotteries model analyzed by Hansen [7]. For the time being I abstract from directed search, the only type of search allowed is undirected (that is, the disutility of directed search ψ is assumed to be large).

In this case there are three parameters (the utility of search B, the persistence of the idiosyncratic shocks ρ_z and the variance of the idiosyncratic shocks σ_z^2) to determine two observations: the unemployment rate and the average duration of unemployment. Later on I will be show that these parameters are key determinants of the cyclical behavior of employment. In this section I report business cycle fluctuations under the most favorable specification for these parameters (in terms of matching the relative variability of employment). Table 2 reports results under $B = 1.09$, $\rho_z = 0.63$, and $\sigma_z^2 = 0.0004$. The statistics correspond to averages across 100 simulations of 408 periods each (corresponding to the 136 quarters of data). Before computing these statistics, the monthly data generated by the model was aggregated to a quarterly time period and then logged and detrended using the Hodrick-Prescott filter. Comparing the business cycles generated by the model with those of the U.S. economy summarized in Table 1 we see that consumption fluctuates less than output in both economies, however it is considerably smoother in the model than in the U.S: 0.32 instead of 0.57. Investment is about 4.5 times as variable as output in both economies, and it is strongly procyclical in both. Employment fluctuates the same amount in the model as in the data (parameters were selected to generate this result) and is strongly procyclical both in the model and the data. Given the fixed labor force, the behavior of employment leads to a highly variable and countercyclical unemployment, actually a bit too variable and countercyclical compared to the data (8.75 versus 6.25 and -0.96 versus -0.83, respectively).

Overall, these results indicate that this version of the model in principle can be reconciled with salient features of business cycle data data.

In this economy there are two channels that generate employment fluctuations. One is the standard channel: when there is a good aggregate shock it is a bad time to enjoy leisure, so agents substitute leisure intertemporally and supply more employment. The second channel arise from the search decisions: when there is a good aggregate shock it is a bad time to search for a good idiosyncratic productivity shock, so agents accept employment more easily and leave the islands less frequently. An important question is which channel is the most important for generating the relatively large employment fluctuations reported here. The following section addresses this particular question.

5.2. Inelastic labor force and no utility from search

In order to evaluate the importance of intertemporal leisure substitution in the previous section, this section analyzes an identical environment except that search is assumed to provide no leisure ($B = 0$). In order for the model to reproduce the unemployment rate and average duration observed in the data when unemployment does not provide utility, the persistence and variance of the idiosyncratic shocks must be made much larger than before ($\rho_z = 0.795$, and $\sigma_z^2 = 0.0221$), that is agents must face stronger "idiosyncratic" reasons to search.

Table 3 reports the results. We see that the variability and persistence of consumption, capital and investment are not very different from Table 2. The big difference is with the labor dynamics. Instead of fluctuating 56% of output, employment fluctuates only 3% when utility does not provide leisure. In the same vein, instead of fluctuating 8.75 times more than output, unemployment fluctuates only 41% when utility has no direct value. In both cases the correlations with output are significantly reduced. These results show that the assumption that agents enjoyed leisure while unemployed was key to generating the large fluctuations in employment reported in the previous section. It is true that when the economy receives a good aggregate shock it is a bad time to search and employment increases, but this is found to be a weak channel. When agents do not derive utility from unemployment the idiosyncratic productivity shocks have to be so persistent and their innovations so large, that when an agent decides to leave or stay in an island it is due to strong idiosyncratic reasons (it is from comparing large and persistent differences in idiosyncratic productivities across islands). In this context the aggregate shocks can do very little to affect the reallocation of labor.

A reason for this result could be that the undirected search decisions are too rigid: agents have no control over which islands they can arrive to. The following section explores this possibility by analyzing how the results change when search can be made directed.

5.3. Inelastic labor force, no utility from search and directed search margin

I extend the version in the previous section by introducing the directed search margin. Preferences are now given by

$$E\sum_{t=0}^{\infty}\beta^t \{\ln c_t - \psi D_t\}$$

where $\psi > 0$ (the model in the previous section assumed ψ to be a large number).

There is a good reason to believe that this version of the model can give rise to larger fluctuations in employment than in the previous version. Suppose that a good aggregate productivity shock hits the economy. It is still true that agents tend to accept employment more easily and leave less frequently because it is a good time to produce instead of searching. However, we saw in the previous section that this margin is not very responsive. The additional margin introduced here is that, after a good aggregate shock hits the economy, some of the agents that were doing undirected search are willing to pay the disutility cost and do directed search in order to become employed faster. Since the disutility cost enters linearly, there will be a high willingness to substitute the effort intertemporally and the shifts from undirected search to directed could be large. Since variations in directed search have an immediate impact on employment, this could lead to higher employment variability.

Observe that the disutility parameter ψ is an important determinant of the steady state ratio of directed searchers to undirected searchers (D/U). Since it is not clear what an empirically relevant magnitude for D/U should be, I here treat ψ as a free parameter. In what follows I report results under the most favorable case that I found in the experiments.

This is the case where ψ generates a D/U ratio equal to 0.20.[2] Table 4 shows the results. We see that when the directed search margin is introduced both employment and unemployment become twice as variable (the variability of employment increases from 0.03 to 0.06 while the variability of unemployment increases from 0.41 to 0.98), verifying the intuition provided above. However, the employment and unemployment fluctuations generated by the search decisions are still small.[3] The next section will show that for this reason the benchmark economy will fail in reproducing the cyclical behavior of employment, unemployment and labor force participation once a labor force participation decision is introduced.

5.4. The benchmark economy

In this section I report results for the benchmark economy with elastic labor force participation. Similarly to the previous section, directed search is allowed for. Preferences are the ones described in section 2, they are given by

$$E\sum_{t=0}^{\infty}\beta^t\left\{\ln c_t + A\left[\frac{(1-\pi_S[U_t+D_t]-\pi_N N_t)^{1-\phi}-1}{1-\phi}\right]-\psi D_t\right\}.$$

The assumption that $\pi_S \leq \pi_N$ implies that unemployment provides more leisure than employment, but as long as $0 < \pi_S$ the maximum amount of leisure is obtained by staying out of the labor force. As was mentioned in section 4, the ratio π_S/π_N will be treated as a free parameter. For simplicity, I only report the case $\pi_S/\pi_N = 0.5$ here. The other cases analyzed ($\pi_S/\pi_N = 1.0$ and $\pi_S/\pi_N = 0.1$) give qualitatively similar results.

The disutility of directed search is chosen to reproduce the same directed to undirected search ratio D/U as in the previous section (0.20). While the parameter A is picked to reproduce a labor force participation equal to 74 percent, the curvature parameter ϕ is not

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[2]The values for ψ, ρ_z and σ_z^2 used in this section are 5.95, 0.833 and 0.020, respectively.

[3]Lowering ψ further does not continue to improve the ability of the model in generating higher fluctuations in employment. When ψ is lowered, there are more unemployed agents doing directed search, which lowers the average duration of unemployment and the unemployment rate. To match the unemployment rate and average duration of unemployment in the data, the persistence and variance of the idiosyncratic shocks must be increased substantially. This reduces the responsiveness of search decisions to aggregate productivity shocks.

pinned down from steady state observations. Since it is an important determinant of the variability of the labor force I choose it to reproduce the labor force variability observed in the U.S. economy.[4] The key question will be how well the model can reproduce the cyclical behavior of the rest of the variables, specially the behavior of employment and unemployment.

Table 5 reports the results. Similarly to the previous cases considered, the benchmark economy does a relatively good job in reproducing the business cycle behavior of consumption, capital and investment observed in U.S. data. However, the model fails badly in terms of the labor market dynamics that it generates. Table 5 shows three main problems: 1) that employment fluctuates as much as the labor force while employment is three times more variable than the labor force in the U.S. economy, 2) that unemployment is only slightly more variable than output while it is six times more volatile than output in the data, and 3) that unemployment is acyclical while it is strongly countercyclical in the U.S. The previous cases analyzed provide some intuition for these results. The fact that employment varies as much as the labor force should not be surprising. Section 5.3 showed that the search decisions alone do not generate sizable fluctuations in employment: the idiosyncratic shocks are so variable and persistent that the productivity thresholds for accepting employment do not respond to aggregate productivity shocks in a significant way.[5] This suggests that most of the variation in employment must come from variations in labor force participation. To understand the lack of cyclicality of the labor force it is important to observe that agents that enter the labor force must go through unemployment first. Thus increases in labor force will be initially accompanied by increases in unemployment. This will tend to make unemployment move procyclically. On the other hand, when a positive aggregate shock hits the economy, unemployed agents accept jobs more easily and are more willing to do directed search, which tends to make unemployment move countercyclically. When both effects are considered unemployment behaves acyclically. Finally, the relatively small variation in un-

[4]Everything considered, I end with the following parameters: $\pi_S = 0.23$, $\pi_\eta = 0.46$, $A = 0.30$, $\phi = 5$, $\psi = 3.58$, $\rho_z = 0.83$ and $\sigma_z^2 = 0.012$.

[5]In the benchmark economy unemployment provides some leisure, so the idiosyncratic shocks do not have to be as persistent and variable as in section 5.3, but they are still very persistent and variable.

employment can be understood by the fact that at times when agents are more willing to accept low productivity jobs (that is, when a positive aggregate shock hits the economy), the flows from out of the labor force into unemployment increase, dampening the decrease in unemployment.

6. Conclusions

In this paper I analyzed the behavior of a RBC model that incorporates employment, unemployment and labor force participation. Fluctuations in these variables are the result of search and leisure decisions. I found that the model has serious difficulties in reproducing the labor market dynamics observed in U.S. data. The model delivers three counterfactual results: 1) employment fluctuates as much as the labor force while in the data it is three times more variable, 2) unemployment fluctuates as much as output while in the data it is six times more variable, and 3) unemployment is acyclical while in the data it is strongly countercyclical. The main reason for the poor empirical performance of the model is the preponderant role of intertemporal substitution in leisure for generating employment fluctuations. Given the large persistence and variability of the idiosyncratic shocks needed to match the unemployment rate and average duration of unemployment observed in the U.S. economy, search decisions end up responding too little to aggregate shocks.

Despite the empirical failure of the model, the paper delivers two important results. First, it points out that the predicitions of RBC models can be quite sensitive to the introduction of labor force participation. Second, it shows that a successful model, whatever that may end up being, will have to give a much more important role to search decisions than to leisure in generating fluctuations in employment. Finding such a model promises to be a challenging area of research.

Within the realms of the model described in this paper there are two extensions that could improve its empirical performance. The first extension is to make the cost of directed search in terms of goods instead of disutility. The current version of the model relies on intertemporal substitution of effort to generate large shifts between directed and undirected search, but that proved to be a weak channel. The alternative version could generate much larger swings. When an aggregate productivity shock hits the economy, the resources needed to do directed search would become more plentiful. Since this coincides with a moment when more employment is desired (in order to smooth consumption over time) it will lead to a large shift of undirected search into directed search, producing a substantial effect on employment. The second extension is to introduce an unemployment insurance system. Since unemployment insurance provides agents incentives to remain unemployed, the model will not have to rely on a large persistence and variability of the idiosyncratic shocks to generate the observed unemployment rate and average duration of unemployment. Reducing the importance of idiosyncratic shocks in the model could lead to more elastic search decisions. Exploring these alternatives will be the focus of future research.

A. Appendix

A.1. Deterministic planning problem

In this section I characterize the solution to the planning problem corresponding to the deterministic version of the economy, i.e. where the aggregate productivity shock a_t is set to its unconditional mean of zero. The aggregate shock will be reintroduced later on.

Instead of indexing islands according to their current number of agents available and their current productivity levels, it will be convenient to index them according to their history of shocks z^t and the number of agents available in the island at date 0. Let $q_t(z^t, x_0)$ be the time t distribution of islands across productivity histories z^t and number of agents available at date 0. For simplicity, I will assume that q_0 has a finite support over pairs (z_0, x_0). Later on I will show that there is no loss of generality in this assumption. The social planner's problem is then given by

$$MAX \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + A \left[\frac{(1 - \pi_U U_t - \pi_D D - \pi_N N_t)^{1-\phi} - 1}{1-\phi} \right] - \psi D_t \right\}$$

subject to

$$\begin{aligned}
c_t + K_{t+1} - (1-\delta) K_t &\leq \sum z_t n_t(z^t, x_0)^{\gamma} k_t(z^t, x_0)^{\varphi} q_t\left(z^t, x_0\right) & \text{(A.1)} \\
n_t(z^t, x_0) &\leq n_{t-1}(z^{t-1}, x_0) + U_{t-1} + d_{t-1}\left(z^{t-1}, x_0\right) \\
N_t &\geq \sum n_t(z^t, x_0) q_t\left(z^t, x_0\right) & \text{(A.2)} \\
D_t &\geq \sum d_t(z^t, x_0) q_t\left(z^t, x_0\right) & \text{(A.3)} \\
K_t &\geq \sum k_t(z^t, x_0) q_t\left(z^t, x_0\right) & \text{(A.4)} \\
n_0(z_0, x_0) &\leq x_0 \\
& q_0,\ K_0 \text{ given} & \text{(A.5)}
\end{aligned}$$

where the sums in equations (A.1), (A.2), (A.3) and (A.4) are over histories z^t and initial number of agents x_0.

Let the Lagrange multipliers for the above restrictions be $\beta^t \alpha_t$, $\beta^t \alpha_t \xi_t(z^t, x_0) q_t\left(z^t, x_0\right)$, $\beta^t \alpha_t \omega_t$, $\beta^t \alpha_t s_t$, $\beta^t \alpha_t r_t$ and $\alpha_0 \xi_0(z_0, x_0) q_0\left(z_0, x_0\right)$, respectively. Then, the first order conditions for c_t, K_{t+1}, $n_t(z^t, x_0)$, U_t, $d_t(z^t, x_0)$, N_t, D_t and $k_t(z^t, x_0)$ are the following:

$$c_t^{-1} = \alpha_t$$

$$\alpha_t = \beta \alpha_{t+1}\left[r_{t+1} + 1 - \delta\right]$$

$$\xi_t(z^t, x_0) + \omega_t = z_t \gamma n_t(z^t, x_0)^{\gamma-1} k_t(z^t, x_0)^{\varphi} + \beta \frac{\alpha_{t+1}}{\alpha_t} \sum_{z_{t+1}} \xi_{t+1}((z^t, z_{t+1}), x_0) Q(z_t, z_{t+1})$$

$$\pi_U A\left(1 - \pi_U U_t - \pi_D D_t - \pi_N N_t\right)^{-\phi} = \beta \alpha_{t+1} \sum_{z^{t+1}, x_0} \xi_{t+1}(z^{t+1}, x_0) q_{t+1}\left(z^{t+1}, x_0\right)$$

$$s_t \geq \beta \frac{\alpha_{t+1}}{\alpha_t} \sum_{z_{t+1}} \xi_{t+1}((z^t, z_{t+1}), x_0) Q(z_t, z_{t+1}) \quad (= \text{ if } d_t(z^t; x_0) > 0)$$

$$\pi_N A\left(1 - \pi_U U_t - \pi_D D - \pi_N N_t\right)^{-\phi} = \alpha_t \omega_t$$

$$\pi_D A\left(1 - \pi_U U_t - \pi_D D - \pi_N N_t\right)^{-\phi} = \alpha_t s_t - \psi$$

$$r_t = z_t n_t(z^t, x_0)^{\gamma} \varphi k_t(z^t, x_0)^{\varphi - 1}.$$

Define

$$\begin{aligned}\theta_t &= \frac{\omega_t}{1-\beta} \\ \widetilde{v}_t(z^t, x_0) &= \xi_t(z^t, x_0) + \theta_t\end{aligned}$$

From the first order condition for $n_t(z^t, x_0)$ we obtain

$$\begin{aligned}\widetilde{v}_t(z^t, x_0) &= z_t \gamma n_t(z^t, x_0)^{\gamma-1} k_t(z^t, x_0)^{\varphi} \\ &\quad + \beta \frac{\alpha_{t+1}}{\alpha_t} \sum_{z_{t+1}} \widetilde{v}_{t+1}((z^t, z_{t+1}), x_0) Q(z_t, z_{t+1}) + \beta \left(\theta_t - \frac{\alpha_{t+1}}{\alpha_t} \theta_{t+1} \right)\end{aligned}$$

Observe that

$$n_t(z^t, x_0) < n_{t-1}(z^{t-1}, x_0) + U_{t-1} + d_{t-1}\left(z^{t-1}, x_0\right)$$

implies that

$$\widetilde{v}_t(z^t, x_0) = \theta_t$$

Also from first order condition for $d_t(z^t, x_0)$

$$s_t + \beta \frac{\alpha_{t+1}}{\alpha_t} \theta_{t+1} \geq \beta \frac{\alpha_{t+1}}{\alpha_t} \sum_{z_{t+1}} \widetilde{v}_{t+1}((z^t, z_{t+1}), x_0) Q(z_t, z_{t+1})$$

with equality if $d_t(z^t, x_0) > 0$.

Defining

$$\begin{aligned}x_t\left(z^t, x_0\right) &= n_{t-1}(z^{t-1}, x_0) + U_{t-1} + d_{t-1}\left(z^{t-1}, x_0\right) \\ v_t\left(x_t\left(z^t, x_0\right), z_t\right) &= \widetilde{v}_t(z^t, x_0)\end{aligned}$$

we have that v_t and x_t satisfy

$$v_t(x_t(z^t, x_0), z_t) = \max \left\{ \begin{array}{c} \theta_t, \\ z_t \gamma x_t(z^t, x_0)^{\gamma-1} \left[\frac{z_t x_t(z^t, x_0)^\gamma \varphi}{r_t}\right]^{\frac{\varphi}{1-\varphi}} \\ +\min \left[\begin{array}{c} s_t + \beta \frac{\alpha_{t+1}}{\alpha_t} \theta_{t+1}, \\ \beta \frac{\alpha_{t+1}}{\alpha_t} \sum_{z_{t+1}} v_{t+1}(x_t(z^t, x_0) + U_t, z_{t+1}) Q(z_t, z_{t+1}) \end{array} \right] \\ +\beta \left(\theta_t - \frac{\alpha_{t+1}}{\alpha_t} \theta_{t+1}\right) \end{array} \right\}$$

A.2. Deterministic steady state

A steady state is an initial state (q_0, K) such that the solution to the social planner problem corresponding to that initial state has the following characteristics:

$$\begin{aligned} c_t &= c \\ K_t &= K \\ U_t &= U \\ N_t &= N \\ D_t &= D \end{aligned}$$

for every t. Also there is a measure μ and three functions v, n, and d, such that

$$\begin{aligned} q_t(z^t, x_0) &= \mu(x_t(z^t, x_0), z_t) \\ \widetilde{v}_t(z^t, x_0) &= v(x_t(z^t, x_0), z_t) \\ n_t(z^t, x_0) &= n(x_t(z^t, x_0), z_t) \\ d_t(z^t, x_0) &= d(x_t(z^t, x_0), z_t) \end{aligned}$$

for all t, z^t, and x_0. In steady state there is a constant distribution of islands across available workers x and productivity levels z, employment in an island depends only on (x, z), and the number of agents doing directed search in an island also depends only on (x, z).

In steady state we then have that

$$r = \frac{1}{\beta} - 1 + \delta$$

$$v(x,z) = \max \left\{ \theta,\ z\gamma x^{\gamma-1} \left[\frac{z x^{\gamma} \varphi}{r} \right]^{\frac{\varphi}{1-\varphi}} + \min \left[s + \beta\theta, \beta \sum_{z'} v(x+U, z') Q(z,z') \right] \right\}$$

$$\pi_U A \left(1 - \pi_U U - \pi_D D - \pi_N N\right)^{-\phi} c = \beta \int v(x,z) \mu(dx,dz) - \beta\theta$$

$$\pi_N A \left(1 - \pi_U U - \pi_D D - \pi_N N\right)^{-\phi} c = \omega$$

$$\pi_D A \left(1 - \pi_U U - \pi_D D - \pi_N N\right)^{-\phi} c = s - \psi c$$

$$r = z n(x,z)^{\gamma} \varphi k(x,z)^{\varphi - 1}$$

Observe that

$$\begin{aligned}
\omega &= \pi_N A \left(1 - \pi_U U - \pi_D D - \pi_N N\right)^{-\phi} c \\
s - \psi c &= \omega \frac{\pi_D}{\pi_N} \\
\omega \frac{\pi_U}{\pi_N} &= \beta \int v(x,z) \mu(dx,dz) - \beta\theta \\
\omega &= \theta \left(1 - \beta\right)
\end{aligned}$$

Define σ such that

$$\beta\sigma = s + \beta\theta$$

Then,

$$\begin{aligned}
\beta\sigma &= \theta \left(1 - \beta\right) \frac{\pi_D}{\pi_N} + \psi c + \beta\theta \\
\theta \left(1 - \beta\right) \frac{\pi_U}{\pi_N} + \beta\theta &= \beta \int v(x,z) \mu(dx,dz) \\
\theta &= \frac{\pi_N A \left(1 - \pi_U U - \pi_D D - \pi_N N\right)^{-\phi} c}{\left(1 - \beta\right)}
\end{aligned}$$

The steady state conditions are then the following

$$r = \frac{1}{\beta} - 1 + \delta \tag{A.6}$$

$$v(x,z) = \max\left\{\theta,\ z\gamma x^{\gamma-1}\left[\frac{zx^{\gamma}\varphi}{r}\right]^{\frac{\varphi}{1-\varphi}} + \beta\min\left[\sigma, \sum_{z'} v(x+U,z')Q(z,z')\right]\right\} \tag{A.7}$$

$$\theta = \ z\gamma\overline{n}\,(z)^{\gamma-1}\left[\frac{z\overline{n}\,(z)^{\gamma}\,\varphi}{r}\right]^{\frac{\varphi}{1-\varphi}} + \beta\sum_{z'} v(\overline{n}\,(z)+U,z')Q(z,z') \tag{A.8}$$

$$\sigma = \sum_{z'} v(\underline{n}\,(z)+U,z')Q(z,z') \tag{A.9}$$

$$n(x,z) = \min\left\{x, \overline{n}\,(z)\right\} \tag{A.10}$$

$$p(x,z) = \max\left\{\underline{n}\,(z)\,, n(x,z)\right\} + U \tag{A.11}$$

$$d(x,z) = p(x,z) - U - n(x,z) \tag{A.12}$$

$$\mu\left(X',Z'\right) = \int_{\{(x,z):p(x,z)\in X'\}} Q(z,Z')\mu(dx,dz) \tag{A.13}$$

$$\beta\sigma = \theta\left(1-\beta\right)\frac{\pi_D}{\pi_N} + \psi c + \beta\theta \tag{A.14}$$

$$\theta\left(1-\beta\right)\frac{\pi_U}{\pi_N} + \beta\theta = \beta\int v(x,z)\mu(dx,dz) \tag{A.15}$$

$$\theta = \frac{\pi_N A\left(1-\pi_U U - \pi_D D - \pi_N N\right)^{-\phi} c}{(1-\beta)} \tag{A.16}$$

$$r = zn(x,z)^{\gamma}\varphi k(x,z)^{\varphi-1} \tag{A.17}$$

$$c + \delta K = \int zn_t(x,z)^{\gamma} k(x,z)^{\varphi}\mu(dx,dz) \tag{A.18}$$

$$K = \int k(x,z)\mu(dx,dz) \tag{A.19}$$

$$N = \int n(x,z)\mu(dx,dz) \tag{A.20}$$

$$D = \int d(x,z)\mu(dx,dz) \tag{A.21}$$

Equations (A.6) through (A.21) correspond to equations (3.1) through (3.17) in the main text.

A.3. Algorithm for computing a steady state equilibrium

The algorithm to compute a steady state exploits the homogeneity property described in the following proposition.

Lemma A.1. *Suppose that equations (A.6), (A.7), (A.8), (A.9), (A.10), (A.11), (A.12), (A.13), (A.15), (A.17), (A.18), (A.19), (A.20), and (A.21) are satisfied. Consider changing U by the factor ζ. Then these same equations are satisfied under the following values.*

$$
\begin{aligned}
U(\zeta) &= \zeta U \\
N(\zeta) &= \zeta N \\
D(\zeta) &= \zeta D \\
K(\zeta) &= \zeta^{\frac{\gamma}{1-\varphi}} K \\
\theta(\zeta) &= \zeta^{\frac{\gamma+\varphi-1}{1-\varphi}} \theta \\
\sigma(\zeta) &= \zeta^{\frac{\gamma+\varphi-1}{1-\varphi}} \sigma \\
c(\zeta) &= \zeta^{\frac{\gamma}{1-\varphi}} c
\end{aligned}
$$

Proof: a straightforward verification.

The computational algorithm is then given by the following steps:

1) set r according to (A.6),

2) fix $\frac{\sigma}{\theta}$,

3) fix θ at some arbitrary value,

4) set $\sigma = \left(\frac{\sigma}{\theta}\right)\theta$

5) find, by root finding method, value of U that satisfies equation (A.15),

6) Find, by root finding method, value of ζ that satisfies

$$\theta(\zeta) = \frac{\pi_N A \left(1 - \pi_U U(\zeta) - \pi_D D(\zeta) - \pi_N N(\zeta)\right)^{-\phi} c(\zeta)}{(1-\beta)},$$

where the functions of ζ are given in Lemma A.1,

7) rescale all variables according to ζ as described in Lemma A.1,

8) evaluate equation (A.14), goto step 2) with new value for $\frac{\sigma}{\theta}$,

9) iterate steps 2) through 8) until a root is found for equation (A.14).

B. Stochastic equilibrium

This section describes how to compute the equilibrium business cycle dynamics of the economy subject to stochastic shocks. The method has close similarities with the one described in Veracierto [14].

From equation (2.9) we have that

$$c_t = \int e^{(1-\varphi)a_t} z_t n_t(x,z)^\gamma k_t(x,z)^\varphi \mu_t(dx,dz) - K_{t+1} + (1-\delta) K_t \tag{B.1}$$

Substituting equations (2.6) and (2.7) in (2.3) gives

$$h_t = 1 - \pi_U U_t - \pi_D \int d_t(x,z)\mu_t(dx,dz) - \pi_N \int n_t(x,z)\mu_t(dx,dz) \tag{B.2}$$

Observe that conditional on a_t, n_t, K_t, and μ_t, the optimal allocation of capital across islands k_t is obtained as a solution to the following static problem:

$$\max_{k_t} \left\{ \int e^{(1-\varphi)a_t} z_t n_t(x,z)^\gamma k_t(x,z)^\varphi \mu_t(dx,dz) \right\} \tag{B.3}$$

subject to

$$K_t = \int k_t(x,z)\mu_t(dx,dz)$$

Substituting the solution k_t to this problem into equation (B.1) and then substituting the resulting expression together with equations (B.2) and (2.7) into the one-period return function

$$R = \ln c_t + A\left(\frac{h_t^{1-\phi}-1}{1-\phi}\right) - \psi D_t$$

allows me to write the return function R as a function of $(a_t, \mu_t, K_t, n_t, d_t, U_t, K_{t+1})$.

The social planner's problem can then be written as

$$V(a_t, \mu_t, K_t) = \max_{\{n_t, d_t, U_t, K_{t+1}\}} \left\{ R\left(a_t, \mu_t, K_t, n_t, d_t, U_t, K_{t+1}\right) + \beta EV(a_{t+1}, \mu_{t+1}, K_{t+1}) \right\} \quad \text{(B.4)}$$

subject to

$$\mu_{t+1}\left(X', Z'\right) = \int_{\{(x,z): n_t(x,z)+d_t(x,z)+U_t \in X'\}} Q(z, Z')\mu_t(dx, dz) \quad \text{(B.5)}$$

$$n_t(x, z) \leq x$$

$$a_{t+1} = \rho a_t + \varepsilon_{t+1} \quad \text{(B.6)}$$

The high dimensionality of the state space seems to preclude any possibility of computing a solution to this problem. However, the (S,s) nature of the employment adjustment rule at the islands level will allow me to substitute the above problem by one that has linear constraints and is a good approximation to the original problem in a neighborhood of the deterministic steady state. The property that the approximation is good only in a neighborhood of the deterministic steady state is not very restrictive because the aggregate productivity shock a_t has a low variance in the experiments performed in the paper. The property that the problem can be written with linear constraints is crucial because it will allow me to compute the solution using standard linear-quadratic methods.

Before proceeding to the transformed problem it will be useful to establish the following Lemma. Hereon, any variable superscripted with a star $(*)$ will refer to its deterministic steady state value.

Lemma B.1. *The deterministic steady state distribution* μ^* *has a finite support given by the vector*

$$\mathbf{x}^* = \left(\bar{n}^*(z) + m \times U^*\right)_{\substack{m=1,\ldots,\overline{M}(z) \\ z=1,\ldots,z_{\max}}} \cup \left(\underline{n}^*(z) + m \times U^*\right)_{\substack{m=1,\ldots,\underline{M}(z) \\ z=1,\ldots,z_{\max}}}$$

where $\overline{M}(z)$ *is the lowest natural number that satisfies*

$$\bar{n}^*(z) + \overline{M}(z) \times U^* > \bar{n}^*(z_{\max})$$

and $\underline{M}(z)$ is the lowest natural number that satisfies

$$\underline{n}^*(z) + \underline{M}(z) \times U^* > \bar{n}^*(z_{\max})$$

Proof: *It follows from the fact that every time that an island of type (x, z) has a next period number of agents different from $x' = x + U^*$, it must be either $x' = \bar{n}^*(z) + U^*$ or $x' = \underline{n}^*(z) + U^*$.*

Hereon, I will refer to $\mathbf{x}^*(j)$ as the jth element of $\mathbf{x}^*$. The total number of elements in $\mathbf{x}^*$ will be denoted by J. It will be useful to classify the elements of $\mathbf{x}^*$ into three sets: 1) those that correspond to islands that in the previous period let some agents go (set $\mathcal{L}^*$), 2) those that correspond to islands that in the previous period were targeted by directed searchers (set $\mathcal{G}^*$), and 3) the rest (set $\mathcal{I}^*$). That is, for $j = 1, ..., J$:

$$\begin{aligned} j &\in \mathcal{L}^*, \text{ if } \mathbf{x}^*(j) = \bar{n}^*(z) + U^* \text{ for some } z \\ j &\in \mathcal{G}^*, \text{ if } \mathbf{x}^*(j) = \underline{n}^*(z) + U^* \text{ for some } z \\ j &\in \mathcal{I}^*, \text{ otherwise} \end{aligned}$$

Given that the steady state distribution has finite support and that the law of motion (B.5) maps distributions with finite support into distributions with finite support (recall that z takes a finite number of values), there is no loss of generality in assuming that the state variable μ_t always has a finite support. Moreover, if $\boldsymbol{\mu}_{t-1}$, n_{t-1}, d_{t-1} and U_{t-1} were close to their steady state values $\boldsymbol{\mu}^*$, n^*, d^* and U^*, by continuity of the operator (B.5) it follows that μ_t has a finite support $\mathbf{x}_t$ close to $\mathbf{x}^*$ and:

$$\mu_t(\mathbf{x}_t(j), z) = \mu^*(\mathbf{x}^*(j), z), \text{ for every } z \text{ and every } j = 1, ..., J. \tag{B.7}$$

Assuming the optimal plan is stable in a neighborhood of the deterministic steady state (a property that will have to be verified numerically), the distribution μ_t will always have a finite support $\mathbf{x}_t$ close to $\mathbf{x}^*$and (B.7) will be satisfied.

Given the (S,s) nature of the adjustment rules at the island level, there is no loss of generality in constraining the social planner to choose an employment function n_t of the following form:

$$n_t(\mathbf{x}_t(j), z) = \min\{\bar{n}_t(z), \mathbf{x}_t(j)\}$$

If $\bar{n}_t(z)$ and $\mathbf{x}_t$ are close to their steady state values $\bar{n}^*(z)$ and $\mathbf{x}^*$, it follows that

$$\begin{aligned} n_t(\mathbf{x}_t(j), z) &= \bar{n}_t(z), \text{ if } \bar{n}^*(z) < \mathbf{x}^*(j) \\ &= \mathbf{x}_t(j), \text{ otherwise} \end{aligned} \tag{B.8}$$

From equation (3.8) it follows that the directed search decisions are given by:

$$d_t(\mathbf{x}_t(j), z) = \max\{\underline{n}_t(z), n_t(\mathbf{x}_t(j), z)\} - n_t(\mathbf{x}_t(j), z) \tag{B.9}$$

where $n_t(\mathbf{x}_t(j), z)$ is given by (B.8).

If $\bar{n}$, $\underline{n}$ and $\mathbf{x}_t$ are close to their steady state values $\bar{n}^*$, $\underline{n}^*$ and $\mathbf{x}^*$, the next period finite support $\mathbf{x}_{t+1}$ is then obtained as follows:

$$\mathbf{x}_{t+1}(j) = \left\{ \begin{array}{c} \mathbf{x}_t(j-1) + U_t, \text{ if } j \in \mathcal{I}^* \\ \bar{n}_t(z) + U_t, \text{ if } j \in \mathcal{L}^*, \\ \underline{n}_t(z) + U_t, \text{ if } j \in \mathcal{G}^* \end{array} \right\}, \text{ for } j = 1, ..., J \tag{B.10}$$

where z in the second line of the equation satisfies that $\mathbf{x}^*(j) = \bar{n}^*(z) + U^*$, and z in the third line satisfies that $\mathbf{x}^*(j) = \underline{n}^*(z) + U^*$

The discussion provided so far suggests substituting the original social planner's problem in equation (B.4) for the following transformed problem:

$$V(a_t, \mathbf{x}_t, K_t) = \max_{\{\bar{n}_t, \underline{n}_t, U_t, K_{t+1}\}} \left\{ \widetilde{R}(a_t, \mathbf{x}_t, K_t, \bar{n}_t, U_t, K_{t+1}) + \beta E V(a_{t+1}, \mathbf{x}_{t+1}, K_{t+1}) \right\} \tag{B.11}$$

subject to (B.6) and (B.10). The return function $\widetilde{R}$ is given by the value of the return function R in (B.4) that corresponds to the discrete distribution μ_t defined by the finite support $\mathbf{x}_t$ and (B.7), the employment rule n_t defined in (B.8), and the directed search decisions d_t defined in (B.9). The advantage of working with the transformed problem (B.11) instead of the original problem (B.4) is that its constraints describe linear laws of motion. Since all the endogenous arguments of $\widetilde{R}$ take positive values in the deterministic steady state, a second order Taylor expansion around the deterministic steady state can be performed to obtain a quadratic return function. This leaves a standard linear-quadratic structure that can be solved using standard techniques.

Table 1

U.S. business cycle statistics

variable	relative std. dev. (σ_x/σ_Y)	correlation $(\rho_{x,Y})$
output (Y)	1.00	1.00
consumption (c)	0.57	0.80
investment (I)	4.28	0.91
capital (K)	0.43	0.05
employment (N)	0.58	0.81
unemployment $(U+D)$	6.25	-0.83
labor force $(N+U+D)$	0.19	0.39
unemployment rate	6.35	-0.83
labor productivity (Y/N)	0.63	0.84

Table 2

Inelastic labor force and utility from search

variable	relative std. dev. (σ_x/σ_Y)	correlation ($\rho_{x,Y}$)
output (Y)	1.00	1.00
consumption (c)	0.32	0.89
investment (I)	4.60	0.99
capital (K)	0.32	0.17
employment (N)	0.56	0.98
unemployment ($U + D$)	8.75	-0.96
labor force ($N + U + D$)	0.0	0.0
unemployment rate	8.75	-0.96
labor productivity (Y/N)	0.47	0.97

Table 3

Inelastic labor force and no utility from search

variable	relative std. dev. (σ_x/σ_Y)	correlation ($\rho_{x,Y}$)
output (Y)	1.00	1.00
consumption (c)	0.35	0.92
investment (I)	4.37	0.99
capital (K)	0.31	0.18
employment (N)	0.03	0.76
unemployment ($U+D$)	0.41	-0.76
labor force ($N+U+D$)	0.0	0.0
unemployment rate	0.41	-0.76
labor productivity (Y/N)	0.98	1.00

Table 4

Inelastic labor force, no utility from search and directed search margin

variable	relative std. dev. (σ_x/σ_Y)	correlation $(\rho_{x,Y})$
output (Y)	1.00	1.00
consumption (c)	0.36	0.93
investment (I)	4.36	0.99
capital (K)	0.31	0.17
employment (N)	0.06	0.88
unemployment $(U + D)$	0.98	-0.88
labor force $(N + U + D)$	0.0	0.0
unemployment rate	0.98	-0.88
labor productivity (Y/N)	0.94	1.00

Table 5

Benchmark economy

variable	relative std. dev. (σ_x/σ_Y)	correlation ($\rho_{x,Y}$)
output (Y)	1.00	1.00
consumption (c)	0.34	0.92
investment (I)	4.42	0.99
capital (K)	0.31	0.17
employment (N)	0.20	0.98
unemployment ($U+D$)	1.38	0.07
labor force ($N+U+D$)	0.20	0.92
unemployment rate	1.31	-0.07
labor productivity (Y/N)	0.81	1.00

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