

On the Optimal Fares for Public Transport

Ian W.H. Parry

Resources for the Future
1616 P Street, N.W.
Washington, DC 20036
(202) 328-5151
parry@rff.org

Kenneth A. Small

Department of Economics
University of California
Irvine, CA 92797-5100
(949) 824-5658
ksmall@uci.edu

July 8, 2002

For presentation at the NBER summer workshop, environmental economics, July 2002.

This version is highly preliminary. A substantially revised paper will be available at the workshop.

The authors thank the University of California Energy Institute for financial support.

On the Optimal Fares for Public Transport

Ian W.H. Parry and Kenneth A. Small

Abstract

Estimates of optimal transport subsidies cover a wide range. Typically each covers one or two cities and includes some but not all of the following key factors: congestion externalities, other externalities from motor vehicles, scale economies due to the effect of service frequency on waiting time, revenue constraints, intermodal substitutability, and preexisting fuel taxes. Some results are from computational models whose inner workings are not transparent. This paper presents a single analytical model including nearly all the above factors. We derive an analytical formula for optimal transit prices under alternative scenarios about adjustment of transit supply. We then use it to compute optimal using parameters intended to represent Los Angeles and Washington. In future work we intend to add London and Brussels. The results are used to compare current and optimal transit subsidies, and in future will be used to explore variation across regions and to reconcile insofar as possible the previous disparate findings.

On the Optimal Fares for Public Transport

1. Introduction

Throughout the industrial world, passenger fares for public transportation are heavily subsidized. For the United States as a whole, fares covered only 56 percent of operating costs for mass transit systems 1997 (Winston 2000, pp. 407).

There are two main efficiency rationales for operating subsidies to mass transit (Kerin 1992). First, lower transit fares reduce automobile use and thereby reduce externality costs from automobiles such as traffic congestion, accidents, and air pollution. Subsidizing transit fares is a second-best response to these externalities but may be appropriate if more direct policies are infeasible. Second, there are scale economies in mass transit from fixed costs (for example, maintenance of track and stations) and from effect of increased service frequency on waiting times at transit stops (Mohring 1972, Jansson 1979). Counteracting the scale economies are crowding costs that arise because full transit vehicles are less comfortable, spend more time stopped while passengers board and alight, and cause delays when passengers cannot enter and have to wait for the next vehicle (Kraus 1991).

However, a number of factors complicate the calculation of optimal transit prices. The external cost of automobile travel and the supply cost of transit both vary dramatically by time of day and location. Furthermore, gasoline is taxed heavily in many countries implying that the external costs of driving in some time periods are already partly or fully internalized. And of course transit vehicles themselves contribute substantially to congestion, pollution, and accidents. Finally, cross-elasticities among modes and time periods require simultaneous optimization of many prices, as in Glaister and Lewis (1978).

Several previous studies have estimated optimal transit prices. For London, Glaister and Lewis estimated optimal rail and bus fares at about 50–60% of marginal costs (line 3b, Table 4). A study for the San Francisco Bay Area and for Pittsburgh by Viton (1983) found optimal fares to be virtually zero. Winston and Shirley (1998) find quite the opposite for the United States as a whole, with bus and rail fares covering 84% and 97% of marginal operating costs, respectively.¹ For a prototype representing Belgian cities in 1989, De Borger et al. (1996) estimate that optimal transit fares would be 114% of average agency costs if service frequency is adjusted proportionally to demand, and 50% if service frequency is held fixed.² For Brussels and London, Van Dender and Proost (2001) estimate that optimal

¹ These estimates assume optimized service frequency (pp. 50 and 58). Winston and Shirley put the annual welfare gains from implementing optimal transit prices and service frequency at \$9 billion.

² Based on optimal peak bus/tram prices from tables 4 and 6, divided by average variable private money costs (to the agency) in table 3.

bus and rail fares would be much higher than present fares if marginal-cost pricing of automobile is permitted, but if not optimal fares would be nearly zero in peak periods and about double present fares in off-peak periods.

It is difficult to reconcile these strikingly different results. The studies apply to different regions and to different years. They also differ with respect to the efficiency effects they take into account. For example, the Belgian studies incorporate pollution and accident externalities, pre-existing fuel taxes, and interactions with the broader fiscal system, while the other studies do not.³ Some studies but not others take account of waiting costs and the way they are affected by service frequency; or separate peak and off-peak periods; or interactions between rail and bus demand. Some of the empirical results are from simple stylized models and others from elaborate computational models; the former simplify the situation more, whereas the latter tend to obscure the relative contributions of underlying relationships and parameters.

This paper attempts to reconcile previous results using a single analytical model that incorporates most of the effects that past work have suggested may be empirically important. We derive explicit formulas for the optimal transit fares for both bus and rail, at both peak and off-peak periods. Our model takes into account scale economies, crowding costs, externalities from pollution, congestion, and accidents for all travel modes, pre-existing gasoline taxes, and (in future work not yet completed) optimized service frequency. We compute optimal fares using parameters intended to represent two very different US cities: Washington (DC) and Los Angeles. In subsequent work, we intend to add London and Brussels to obtain still wider variation and to permit comparison with Van Dender and Proost (2001). We also intend in future work to use parameters from previous studies in our model in order to see exactly where the differences come from.

The results are used to compare current and optimal transit subsidies, to explore variation across regions, and to make transparent the contribution of various effects and parameters to the optimal subsidy. In future versions we will also compute the welfare gains from changing current prices to their optimal levels, with and without changes in service frequency.

Several important issues are beyond the scope of the paper. We exclude distributional issues and the burden of transportation deficits on the broader fiscal system, though we discuss how these considerations might affect our results. We do not analyze the desirability of infrastructure investments. We do not consider the social value of providing urban mobility to elderly and low-income individuals. For the most part we refrain from analyzing the political difficulties of implementing optimal policies and

³ Winston and Shirley (1998) do discuss accident and pollution externalities (pp. 64-66) but argue that they have little effect on their results.

thus do not address, for example, the argument of Winston (2000) that transit privatization offers the only real hope of significant reform.

The rest of the paper is organized as follows. Section 2 describes the analytical model and derives optimal pricing formulas. Section 3 discusses parameter values. Section 4 presents the main quantitative results and sensitivity analysis. Section 5 offers conclusions.

2. Analytical Model

We consider a static model of passenger travel in an urban area.⁴ Travel time, which depends on travel choices as well as on congestion and transit frequency, enters utility directly rather than through a time budget. Travel produces pollution and accident externalities, some of which are offset by fuel taxes (which here are equivalent to vehicle-mile taxes since fuel efficiency of vehicles is fixed). The government chooses transit supply and collects fuel taxes at an exogenous rate. We then describe household optimization and derive equations for transit prices under various scenarios.

A. Model Assumptions

(i) *Household utility and Travel.* The utility of the representative agent is:

$$(1) \quad U = u(C, M, T, \Psi) - Z$$

where C is the quantity of a numeraire consumption good, M is sub-utility from travel, T is time spent traveling, Ψ is disutility from crowding on mass transit, and Z is disutility from the external costs of pollution and traffic accidents. The function $u(\cdot)$ is quasi-concave in C and M , and declining in T and Ψ . The decline with respect to T is because greater travel time reduces time available for other pursuits; in the sensitivity analysis we also consider that travel on different modes could provide different amounts of disutility. Variables are expressed in per capita terms. Section 5 discusses heterogeneous agents and distributional issues.

Within the regional transport network, agents make trips by bus, rail and auto during a peak and off-peak period. Sub-utility from travel is:

$$(2) \quad M = M(\{M^{ij}, i = P, O; j = A, B, R\})$$

⁴ The model developed here shares some features of a theoretical model in section 3 of van Dender and Proost (2001). It differs from that model by incorporating more driving externalities, by disaggregating bus and rail, and by explicitly modeling service frequency and wait times. In addition, we compute empirical values directly from our optimal subsidy formula, whereas Van Dender and Proost use a computational model that is considerably more elaborate than the theoretical model they present.

where M^{ij} is *passenger* miles traveled during period i by mode j . The two time periods are $i = P$ (peak), and O (off-peak). The three modes are $j = A$ (auto), B (bus), and R (rail). The function $M(\cdot)$ is quasi-concave; thus, travel on different modes and at different times of day are imperfect substitutes. In what follows we interpret variations in M^{ij} as representing variations in trip-making by mode, rather than in trip length.

External costs and supply costs by the transit agency are determined by *vehicle*-miles (denoted V^{ij}) rather than passenger-miles. They are related as follows:

$$(3) \quad M^{ij} = V^{ij} \mathbf{a}^{ij}$$

where \mathbf{a}^{ij} is the average number of occupants of an automobile, bus or train. Automobile occupancy is assumed fixed.⁵ Transit occupancy may or may not vary, depending on how the government adjusts the availability of vehicle miles.

(ii) *Travel times and travel quality*. The average time to travel a mile in period i by mode j , excluding the costs of waiting for transit service, is denoted \mathbf{p}^{ij} . We assume that the street system and rail network are separate, and that congestion (caused by both autos and buses) affects only the former. Thus:

$$(4) \quad \mathbf{p}^{iA} = \mathbf{p}^{iA}(V^{iA}, V^{iB}); \quad \mathbf{p}^{iB} = \mathbf{p}^{iB}(V^{iA}, V^{iB}); \quad \mathbf{p}^{iR} = \bar{\mathbf{p}}^R$$

Each congestion function $\mathbf{p}^{ij}(\cdot)$ is quasi-convex with positive first derivatives. For given traffic volumes, $\mathbf{p}^{iA} > \mathbf{p}^{iB}$: that is, autos go faster than buses, not least because buses have to make stops. Also $\mathbf{p}_B^{ij} \equiv \partial \mathbf{p}^{ij} / \partial V^{iB} < \mathbf{p}_A^{ij} \equiv \partial \mathbf{p}^{ij} / \partial V^{iA}$: an extra vehicle-mile by a bus adds more to congestion than an extra vehicle-mile by car, as buses go slower and take up more room.

When agents decide on auto travel they take account of their average time cost, but not of their effect on congestion. The marginal externalities from an extra vehicle-mile by auto, bus and rail (in per capita time units) are:

$$(5) \quad \Pi^{ij} = M^{iA} \mathbf{p}_j^{iA} + M^{iB} \mathbf{p}_j^{iB}, j=A,B \quad \Pi^{iR} = 0$$

We assume that the Π^{ij} s are constant are the relevant range; thus, marginal congestion costs at the optimum transit prices are equal to estimated marginal congestion costs at observed (non-optimal prices). This is a reasonable simplification because transit is only a small share of total travel in the cities we study (both initially and in the optimum), so that changes in transit prices have negligible effects on marginal congestion costs.

⁵ This assumption ignores incentives to carpool, which would be affected by congestion taxes but very little by transit prices.

We define w^{iB} and w^{iR} as the average time spent waiting for bus and rail service per passenger mile in period i using the following constant elasticity functional form:

$$(6) \quad w^{ij}(V^{ij}) = w_0^{ij} \cdot (V^{ij})^{h_w^{ij}}, j=B,R \quad w^{iA} = 0$$

where $w_0^{ij} > 0$ and $h_w^{ij} < 0$ are parameters. $h_w^{ij} \equiv w_V^{ij} V^{ij} / w^{ij}$ is the elasticity of wait time with respect to service frequency; it is negative because wait time on a transit mode falls when service frequency is increased. There is no waiting time for auto.

Thus, the household's total time spent traveling is:

$$(7) \quad T = \sum_{i,j} (p^{ij} + w^{ij}) M^{ij}$$

Disutility from crowding on mass transit, for example having to stand (e.g. Kraus 1991), is a function of occupancy:

$$(8) \quad \Psi = \sum_{i,j} y^{ij} M^{ij}; \quad y^{ij}(a^{ij}) = y_0^{ij} (a^{ij} / a_0^{ij})^{h_y^{ij}}; \quad y^{iA} = 0$$

where a_0^{ij} is initial vehicle occupancy, $y_0^{ij} > 0$ is initial crowding cost, and $h_y^{ij} \equiv y_a^{ij} a^{ij} / y^{ij} > 0$ is the elasticity of crowding costs with respect to the load factor, assumed constant. There is no crowding on auto.

(iii) *Household budget constraint.* The money costs per passenger mile are denoted p^{ij} . For bus and rail these represent fares.⁶ For auto $p^{iA} = \tilde{p}^{iA} + t^{iA}$, where \tilde{p}^{iA} includes all non-tax costs (pre-tax gasoline expenses, maintenance costs, parking fees, etc.) and t^{iA} is the tax paid on gasoline, expressed on a per passenger mile basis. We allow the tax per mile to vary between peak and off-peak periods, as fuel consumption per mile can differ significantly between congested and free flowing roads. We assume that fuel efficiency in a given period is given, which is reasonable because fuel taxes per gallon of gasoline are fixed, and changes in transit prices only have modest effects on congestion levels.⁷

The household budget constraint equates expenditures on consumption and transport to disposable income:

$$(9) \quad C + \sum_{i,j} p^{ij} M^{ij} = I + G$$

⁶ Agents may also incur parking fees when using transit. However in aggregate these costs are small relative to fares (e.g., WMATA 2001) so we ignore them.

⁷ In contrast, when analyzing the welfare effects of changes in gasoline taxes it is important to account for changes in fuel efficiency. Those changes weaken the link between reducing fuel and reducing congestion and accidents, implying a much lower optimal tax (see Parry and Small 2001).

where I is (exogenous) income, G is a lump-transfer from the government, and the price of the general consumption good is normalized to unity.

(iv) *Government and production.* Bus and rail transit are publicly provided. The agency cost functions are linear:

$$(10) \quad K^{iR}(V^{iR}) = H^{iR} + k^{iR}V^{iR}; \quad K^{iB}(V^{iB}) = k^{iB}V^{iB}$$

The fixed cost for rail, H^R , represents fixed operating costs, such as the cost of operating stations. (It does not include the costs of constructing rail infrastructure, which we consider in Section 5). Our assumption of no economies or diseconomies of scale for agency cost of providing bus vehicle-miles is consistent with the empirical evidence (Small 1992, p. 57). Parameters k^{iR} and k^{iB} are the constant costs per vehicle mile, such costs as drivers, electricity, and diesel fuel excluding tax.⁸ $k^{Pj} > k^{Oj}$ because we impute vehicle capital costs to the peak period and peak-only service does not conveniently fit an eight-hour workday for drivers and consequently labor costs are higher.

The government chooses the supply of vehicle miles for bus and rail. We take vehicle sizes and route lengths as given; therefore an increase in the supply of vehicle miles represents an increase in service frequency. The government budget constraint is then:

$$(11) \quad G = \sum_{i,j \neq A} p^{ij} M^{ij} + \sum_t t^{iA} M^{iA} - \sum_{i,j \neq A} K^j(V^{ij})$$

That is, the transfer to households (possibly negative) equals government revenues from transit fares and fuel taxes, minus the costs of providing transit services. Thus, the benefits of reducing the losses or increasing the profitability of transit are accounted for in higher disposable income $I+G$ for households.

We assume that gasoline, diesel fuel, and the consumption good are produced competitively under constant returns, so that producer prices are fixed.

(v) *Other externalities.* External damages from pollution and accidents are given by:

$$(12) \quad Z = \sum_{i,j} \mathbf{q}^{ij} V^{ij}$$

Parameters \mathbf{q}^{ij} are the external pollution and accident damages per vehicle-mile, assumed constant. The constancy of these parameters is discussed in Section 3 when we choose empirical estimates of them for our simulations. The part of accident costs that is internalized (e.g., own driver injury risk) is implicitly taken into account in the sub-utility function $M(\cdot)$ for travel.

⁸ Taxes on fuel used in public transport are simply a transfer within the government and are not distortionary if the government can pursue a stated objective as we assume here.

(vi) *Household optimization.* Households choose passenger miles and consumption to maximize utility (1) and (2) subject to their budget constraint (9), with travel time given by (7) and crowding disutility by (8).

The household regards as given the parameters \mathbf{p}^{ij} and w^{ij} for travel times, \mathbf{a}^{ij} for occupancies, and Z for total externalities. This yields the first order conditions:

$$(13) \quad u_{M^{ij}} / \mathbf{I} = p^{ij} + \mathbf{r}^T (\mathbf{p}^{ij} + w^{ij}) + \mathbf{r}^\Psi \mathbf{y}^{ij}; \quad \mathbf{I} \equiv u_c$$

where the Lagrange multiplier $\mathbf{I} \equiv u_c$ is the marginal utility of income/consumption and $\mathbf{r}^T \equiv -u_T / \mathbf{I}$ and $\mathbf{r}^\Psi \equiv -u_\Psi / \mathbf{I}$ are the (marginal) values of travel time and crowding. Agents equate the marginal benefit from travel (in dollars) with sum of money costs, travel costs and (for bus and rail) wait and crowding costs, per passenger mile of travel. Using (13), and the household constraints, we can obtain the demand for passenger miles, and consumption, as functions of the money, time, crowding, and income. Substituting these functions in (1) and (2) the indirect utility function can be expressed:

$$(14) \quad M^{ij} = M^{ij}(\mathbf{p}, \mathbf{p}, \mathbf{w}, \mathbf{y}, G); W = W(\mathbf{p}, \mathbf{p}, \mathbf{w}, \mathbf{y}, G) - Z;$$

where bold type indicates a vector with elements corresponding to different periods and modes.

B. Model Solution

We now compute the marginal welfare effects of higher transit prices and use them to derive optimal transit pricing formulas and the welfare gains from policy reform. We use alternate assumptions about adjustments in service frequency, represented by the quantity $V_M^{ij} \equiv dV^{ij} / dM^{ij}$ which describes the marginal supply response of the transit agency to marginal changes in demand.

For notational convenience we focus on peak-period rail, but the formulas are analogous for other transit modes.

(i) *Marginal welfare effects.* We differentiate indirect utility with respect to p^{PR} while taking account of induced changes in travel, their effects on waiting and crowding costs and other externalities, and the effect on household income of smaller transit deficits (through the government budget constraint). The result (see Appendix A) is:

$$(15) \quad \frac{1}{\mathbf{I}} \frac{dW}{dp^{PR}} = -\sum_{i,j} H^{ij} \frac{dM^{ij}}{dp^{PR}} - \mathbf{r}^T \sum_{i,j} M^{ij} w_V^{ij} V_M^{ij} \frac{dM^{ij}}{dp^{PR}} - \mathbf{r}^\Psi \sum_{i,j} M^{ij} \mathbf{y}_a^{ij} \frac{d\mathbf{a}^{ij}}{dM^{ij}} \frac{dM^{ij}}{dp^{PR}}$$

where

$$(16) \quad H^{iA} = E^{iA} - t^{iA}; \quad H^{ij} = E^{ij} - (p^{ij} - k^{ij} V_M^{ij}), j \neq A; \quad E^{ij} = (\mathbf{q}^{ij} / \mathbf{I} + \mathbf{r}^T \Pi^{ij}) V_M^{ij}$$

Assuming the demand for rail transit is downward-sloping, $dM^{PR} / dp^{PR} < 0$; and since all modes are substitutes for each other, $dM^{ij} / dp^{PR} > 0$ for $ij \neq PR$. The quantity $d\mathbf{a}^{ij} / dM^{ij} \equiv (1 - M^{ij}V_M^{ij} / V^{ij}) / V^{ij}$ is another facet of the supply response, showing how occupancy changes in response to increased demand; it is equal to one if vehicle-miles are held fixed for transit, zero if they are increased proportionally to passenger-miles, and zero in the case of auto.

The quantity E^{ij} denotes the (marginal) external cost of a passenger-miles in time period i by mode j . These consist of per-vehicle pollution and accident costs $\mathbf{q}^{ij} / \mathbf{l}$ and, in the case of auto and bus, per-vehicle congestion costs $\mathbf{r}^T \Pi^{ij}$, all multiplied by the effect on vehicle miles from a marginal increase in passenger miles, V_M^{ij} . The quantity H^{ij} denotes the *net* marginal external cost; that is, external cost net of taxes and government pricing policy. In the case of auto, the net marginal external cost is reduced by the fuel tax. In the case of rail and bus, it is reduced to the extent that the transit price is above the supply cost of an extra passenger mile ($k^{ij}V_M^{ij}$). The first term on the right in (15) shows that the marginal welfare effect of higher peak rail prices depends on the (negative of these) net marginal external costs, each multiplied by the change in corresponding passenger miles and aggregated over all time periods and modes.

The second term in (15) is the marginal welfare effect from the change in wait times, summed across bus and rail travel in both periods. For a given time period and mode this equals passenger miles times the change in wait costs per mile from any adjustment to vehicle miles (or service frequency) in response to changes in the demand for passenger miles. If supply responses V_M^{ij} are positive, then these wait costs will go up for peak rail, but down for other time periods and modes, as the price of peak rail rises. Similarly, the third term in (15) reflects the change in crowding costs aggregated across bus and rail modes, from induced changes in occupancy rates. As people shift from peak rail, crowding costs fall for peak rail but rise for other times and transit modes.

Suppose that the agency adjusts vehicle-miles in proportion to changes in the demand for passenger miles, keeping load factors constant. Then there is no change in crowding costs, $d\mathbf{a}^{ij} / dM^{ij} = 0$, and the third term in (15) drops out. At the other extreme, if vehicle miles or service frequency are not adjusted, then $V_M^{iB} = V_M^{iR} = 0$ so that the second term drops out; in this case welfare effects arise only from changes in crowding costs and from indirect changes in auto use.

(ii) *Optimal transit price: proportional service adjustment.* First, we assume that vehicle miles change in proportion to passenger miles for each time period and transit mode; that is, $V_M^{ij} = 1 / \mathbf{a}^{ij}$ and

$d\mathbf{a}^{ij} / dM^{ij} = 0$. Using this, substituting H^{PR} from (16) into (15), setting (15) to zero, and dividing through by dM^{PR} / dp^{PR} we can obtain the following formula for the optimal peak rail fare:

$$(17) \quad \hat{p}^{PR} = (k^{PR} / \mathbf{a}^{PR} + E^{PR}) - \sum_{ij \neq PR} m_{PR}^{ij} H^{ij} - \mathbf{r}^T \sum_{ij} m_{PR}^{ij} w^{ij} \mathbf{h}_w^{ij}$$

$$m_{PR}^{ij} = -\frac{dM^{ij} / dp^{PR}}{dM^{PR} / dp^{PR}};$$

Here $m_{PR}^{PR} = -1$, while for $ij \neq PR$, $m_{PR}^{ij} \geq 0$ is the fraction of the incremental reduction in peak rail passenger miles that is due to agents substituting into mode j in period i . Not all of the reduction in peak rail mileage is due to substitution into other modes—a portion is due to reduced overall travel demand. Thus $\sum_{ij \neq PR} m_{PR}^{ij} < 1$. To simplify computation, we assume that the m_{PR}^{ij} s are constant.

The optimal transit price as expressed in (17) consists of three components. First, the marginal social cost of providing peak rail passenger miles, equal to the marginal external cost plus the marginal supply cost. Second, a negative adjustment due to the exacerbation of net external costs from substitution onto other modes. Third, the change in waiting costs summed across all transit modes: wait costs increase for peak rail travel from lower service frequency, and decrease on bus and off-peak rail as agents shift to them and suppliers respond by increasing service.

(iii) *Optimal transit price: no service adjustment.* In this scenario, vehicle-miles of service does not change, that is, $V_M^{ij} = 0$ and $d\mathbf{a}^{ij} / dM^{ij} = 1/V^{ij}$. Following the same derivation as above gives:

$$(18) \quad \tilde{p}^{PR} = -\sum_{ij \neq PR} H^{ij} m_{PR}^{ij} - \mathbf{r}^\Psi \sum_{ij} \mathbf{y}^{ij} \mathbf{h}_y^{ij} m^{ij}; \quad H^{ij} = -p^{ij}, j \neq A$$

When vehicle miles are held fixed for all transit modes, changes in the fare charged to passengers have no effect on the external costs or supply costs of transit; that is, the social cost of supplying additional passenger miles is zero. Thus the first component of the optimal fare in (17) is absent from (18). In addition, the substitution into other transit does not affect externalities: welfare effects depend on the transit price, since this is the wedge between the marginal benefit and the (zero) marginal social costs of additional passenger miles. Since vehicle miles are fixed there is no change in wait costs in (18) analogous to the last term in (17). However, changes in occupancy rates give rise to changes in crowding costs on transit. Crowding costs fall on peak rail, and increase on other transit modes.

(iv) *Transit service optimization.* [To be completed]

(v) *Computational considerations.* Equations (17) and (18) are not explicit formulas for the optimal transit price, for two reasons. First, the magnitude of H^{ij} on the right hand side depends on the prices of other transit, which are optimized simultaneously; and second, the wait and crowding costs per mile vary with service frequency and occupancy, which in turn depend on transit demands. To solve the model we need to specify functional forms for the transit demands. For now we adopt the following CES function of prices only:

$$(19) \quad M^{ij} = \mathbf{g}^{ij} \cdot \prod_{\substack{k=P,O; \\ l=R,B}} (p^{kl})^{h_{kl}^{ij}}, \quad j \neq A$$

Here \mathbf{g}^{ij} is a parameter and the \mathbf{h} s are own and cross price elasticities, assumed constant here despite this being inconsistent with constant m 's in (17). (Since p^{iA} is fixed, it is not explicitly included in these functions.) Initial attempts to incorporate waiting-time and crowding costs into the demand functions created difficulty in obtaining convergence, but we hope to do this in future work. We obtain the elasticities from our assumed values of m_{ij} using the following formula computed at initial modal shares:

$$(20) \quad \mathbf{h}_{PR}^{ij} = -m_{PR}^{ij} \mathbf{h}_{PR}^{PR} M^{PR} / M^{ij}, \quad ij \neq PR$$

and so on.

We solve the model simultaneously for all travel demands, transit prices, and waiting and/or crowding costs, using (6), (8), (17)–(19), and the parameter values described in the next section. We do this using a spreadsheet and iterating manually.

(vi) *Welfare gains.* [To be completed – based on rewriting (15) in a form that can easily be integrated.]

3. Parameter Values

Key parameter values used are shown in Table 1. We outline below our methods for obtaining these values. These should be regarded as preliminary.

Automobile trips: We take automobile vehicle-miles from data used in the series of “urban mobility” reports by the Texas Transportation Institute (Shrank and Lomax 2002). We estimate vehicle occupancy by starting with average occupancy by metropolitan area, from the journey-to-work data from the 1990 census. We then estimate ratios for peak to off-peak occupancies from the National Personal Transportation Survey (NPTS).⁹ Although we know from the NPTS that automobile occupancy

⁹ Average vehicle occupancies for work trips are from US FHWA (1994a). Average vehicle occupancies for all trip purposes are estimated from the percentages of single- and multiple-occupant trips in very large metropolitan areas,

nationwide has declined since 1990, ride-sharing policies and construction of carpool lanes have kept this decline to a minimum in the Washington and Los Angeles areas so we think the occupancies are still accurate for 2000.

Transit system characteristics, and system costs: Transit system characteristics, including vehicle-miles, passenger-miles, unlinked passenger trips (from which we derive average trip length for each modal trip segment), and cost data are taken from the web sites of the local transit authorities. We do not include infrastructure cost, i.e. roads, track, stations, and so forth. We do include vehicle capital costs, annualized using a capital recovery factor based on lifetimes of 25 and 12 years for rail and bus, and real interest rate of 7 percent. Vehicle capital costs are allocated entirely to the peak period, on the assumption that any increase in vehicle-miles in that period requires purchasing more vehicles, whereas an increase in the off-peak period does not. Vehicle purchase costs are taken from nationwide figures (American Public Transportation Association 2002, Table 60) except for rail costs for Los Angeles, which were available from the local agency web site.

We assume that operating costs during the peak period (excluding vehicle capital) are 50 percent more than off-peak, due to the difficulties in scheduling labor for split shifts with down time between morning and afternoon peaks.

Los Angeles operates a light-rail system that carries three times as many vehicle-miles as its one heavy-rail line, known as the red line. In other cities (particularly Brussels) light rail operates much like bus in mixed automobile traffic, so we plan to aggregate light rail and bus. In Los Angeles, however, most light rail is grade-separated, yet has very different characteristics from heavy rail. For this reason we did not want to aggregate light rail with either transit mode in Los Angeles, so have omitted it. This is one reason the modal share for rail transit is so tiny (only 0.06 percent).

Own-price elasticities of transit demands, \mathbf{h}_{ij}^{ij} : Lago et al. (1981) provide three “stylized facts” that appear to still hold, based on evidence from more recent reviews such as Pratt et al. (2000). First, the overall own-price elasticity for bus transit (over the entire day) based on roughly one-year changes from actual experiments is about -0.3 . Second, the own-price elasticity for rail transit is about half that for bus transit. Third, off-peak elasticities are two to three times as large as those for peak periods.

However, Lago et al. also find that elasticities from cross-sectional studies tend to be somewhat higher, perhaps -0.5 . These presumably measure a longer-run elasticity, which is what we want, but also

from the 1990 National Personal Transportation Survey (NPTS) (US FHWA 1994b). From the 1995 NPTS we find that work trips are 38 percent of all vehicle trips.

provide less consistent results across studies. This accords with the summary by Goodwin (1992), who finds short- and long-run elasticities for urban bus travel of -0.4 and -0.65 . From this evidence, we take the bus elasticity to be $e=-0.5$, and rail half as large.¹⁰ Given that about three-fourths of transit travel in both US cities is during the peak, this leads to the following values:

- Peak bus: $0.7e$
- Off-peak bus: $1.4e$
- Peak rail: $0.35e$
- Off-peak rail: $0.7e$

Modal diversion ratios, m_{im}^{ij} . Pratt et al. (2000, pp. 12-41 ff.) provide a number of estimates of the proportion of incremental transit trips that come from (or are diverted to) other modes and/or time periods, based mainly on surveys of riders who switch to a transit mode when it is made more attractive. Some typical numbers are 64% and 80% from car in Atlanta and Los Angeles, respectively. We assume the latter figure represents the average of m_{PA}^{PT} and m_{OA}^{OT} for Los Angeles (where T stands for either transit mode), with progressively lower values for Washington, Brussels, and London. In each city we assume the peak value (s_{PA}^{PT}) is 0.05 higher than this average and the off-peak value (s_{OA}^{OT}) is 0.05 lower than this average.

In cities with good coverage by both bus and rail transit systems, such as London and Chicago, the few studies of cross-elasticities between bus and rail transit often find them to be about half the direct elasticities (Gilbert and Jalilian 1991, Table 3b; Talvitie 1973). Assuming equal travel volume by mode, this would imply $m_{iR}^{iB} = m_{iB}^{iR} \approx 0.5$ for $i=P,O$. We expect the substitutability between modes to decrease as one expands beyond the city to the metropolitan area, and to decrease more as one considers cities with less and less well-developed rail networks. We also expect them to have declined considerably from the 1970s or 1980s to the year 2000 due to increasing competition from the automobile. Finally, in the newer US rail systems the bus lines are typically reconfigured to serve as feeders to the rail system, with

¹⁰ There is contradictory evidence from Petite (2001) for the Washington metropolitan area. He finds that when non-core-oriented trips are included, the rail fare-elasticity is considerably higher than suggested by the other studies, with a best estimate of -0.54 (Table 4). This is from a fairly complex specification and we take it to be as yet unverified. There is also some evidence that price-elasticities from “large multinucleated cities” such as Los Angeles are slightly smaller than those from “large core-concentrated cities” such as Washington (Chan and Ou 1978, Tables 2, 4); but we ignore this distinction partly because Washington has become much less core-concentrated since the times of the studies reviewed by Chan and Ou.

competitive routes discontinued. Therefore we assume the cross-mode diversion ratios to be 0.15 for London, 0.10 for Brussels, and just 0.05 for the two US cities.

Little information is available about shifts across time periods. We assume that in each case, 10 percent of the change in transit ridership represents such shifts, and that the shifts occur entirely to the same mode.

These assumptions lead to the values shown in Table 2. The fraction of trips diverted into non-trip-making or non-vehicle modes is a residual, equal to 10 percent in all cases. This is reasonably consistent with the review by Pratt et al. which suggests that 26% and 10% of new transit trips in Atlanta and Los Angeles, respectively, represent some combination of changes in walking, trip frequency, and destination; while noting that destination changes do not necessarily reduce passenger-miles.

Values of time: Our starting point is the value of in-vehicle time, which following Small (1992, p. 44) we take to be one-half the wage rate. We also adopt standard convention by assuming that waiting time at a transit stop is valued at twice that of in-vehicle time, i.e. equal to the wage rate. This hourly wage rate, averaged over all occupations in private industry and state and local government, is \$19.57 and \$17.14 for the Washington and Los Angeles Primary Metropolitan Statistical Areas (PMSAs) in April 2001 (US Bureau of Labor Statistics 2002). We round the figures to 32 and 28 cents per minute, and assume they apply during peak periods when many travelers are on work trips. We assume off-peak travel time is valued at 75 percent of these amounts.

Waiting Costs, w^{ij} , and waiting-cost elasticity, \mathbf{h}_w^{ij} . We distinguish between cases of random and planned times of arrival. These depend on headway H , defined as the times between transit vehicles on a route and inversely proportional to total vehicle-miles V .

When headways are small, it is reasonable and common to assume that travelers come randomly to a transit stop and wait half the headway, for an expected waiting cost of $w^{ij} = \mathbf{r}^T H / 2$ and elasticity $\mathbf{h}_w^{ij} = -1$. When headways are large, travelers can be expected instead to use a transit schedule. A full analysis of this decision and its consequences for costs is very complicated (Pisato 1998), but a reasonable simplification is that using a schedule entails three costs. We scale all costs by the value of waiting time, \mathbf{r}^T . First is a fixed planning cost, which we take to be the value of one minute. Second is a precautionary waiting time required because the exact arrival time of the bus is uncertain; we take this to be 5 minutes. Third is the expected value of early arrival at the destination arising assuming that the traveler chooses to ensure there will not be a late arrival and therefore chooses a transit vehicle arriving prior to the desired time; the expected value of this third component is $\mathbf{b}H / 2$ where \mathbf{b} is the per-minute

cost of early arrival at the destination. Following Arnott, de Palma and Lindsey (1993), we use choose \mathbf{b} to be 0.4 times the value of in-vehicle travel time, which in turn we take to be $0.5 \mathbf{r}^T$ (based on evidence that in-vehicle time is valued at about half as much as out-of-vehicle time).

These assumptions imply that the user will choose to arrive randomly when $H < 15$ and to follow a schedule when $H > 15$, with resulting cost per trip:

$$(21) \quad Lw^{ij} = H/2; \quad H < 15$$

$$\mathbf{r}^T [6 + (H/10)]; \quad H > 15$$

where L is trip length (in miles). The elasticity \mathbf{h}_w^{ij} is -1 for $H < 15$, and varies between -0.20 and -0.5 as H varies from 15 to 60 minutes. Aggregate waiting cost across many routes will involve a weighted average between these values. To simplify, we take $\mathbf{h}_w^{ij} = -0.5$ throughout, but calculate the initial waiting cost from the initial average headway using (21).

Crowding cost, \mathbf{y}^{ij} , and its elasticity, \mathbf{h}_y^{ij} . Kraus (1991) models crowding on a given bus line as a function of boarding times, probability of standing, and where along the line the passenger enters. Crowding costs should be non-linear in occupancy because they are very low until most seats are filled, then rise rapidly as the chance of getting a seat declines and the chance of missing a connection increases. For lack of information we assume the elasticity is $\mathbf{h}_y^{ij} = 3$.

For the initial crowding costs in Washington during peak operations, we use Kraus's simulated results representing Boston peak rail and bus operations, using his figures for someone boarding half-way along a line and averaging the inbound and outbound results.¹¹ We assume that Kraus's figures are representative of peak operations in Washington (which is an older city like Boston); for all other cases we adjust for the fraction of seats occupied using the assumed elasticity. In each case we multiply this crowding cost by the ratio of our assumed value of in-vehicle time (\mathbf{r}^T) to his assumed value (8.33 cents/min.). We then divide by trip length to put cost in cents per passenger-mile.

External congestion costs, \mathbf{P}^{ij} . We take average congestion cost (in time units) from the estimates by the Texas Transportation Institute (Schrank and Lomax 2002) of total person-hours of delay in 2000 for the Washington and Los Angeles urban areas. We assume that 90 percent of this occurs during the peak periods, which extend for a total of 8 hours per day. These assumptions imply that average peak delay (in

¹¹ See Kraus 1991, Tables 2-3. We use stop #15 for bus, and the average of stops #5 and 6 for rail.

minutes per passenger-mile) is 0.45 in Washington, and 0.85 in Los Angeles. (For example, the figure in Washington could result from congestion reducing average peak speed from 45 to 33.6 miles per hour).

In order to determine marginal delay, we apply the relationships fit by Small (1992, pp. 70-71) to travel-time data from simulated arterial travel times in Toronto and from express roads in Boston. These results suggest that delay is well approximated by a power function of traffic volume, with power 4.1 in Toronto and 3.3 in Boston. Taking the average of 3.7, this implies that the marginal external delay is 3.7 times the average delay. Applying the value of in-vehicle time already described yields the external congestion costs per vehicle-mile shown in Table 1, which for peak periods are 26.5 cents for Washington and 50.4 cents for Los Angeles. These figures are not out of line with the US average value of 5.0 cents/mile used in Parry and Small (2002), derived from US FHWA (1997).

Congestion from buses is assumed to cause five times the external cost as from autos, per vehicle-mile.

External accident and pollution costs, q^{ij} : We start with the values used by Parry and Small (2002) for external costs from automobiles: namely (cents per vehicle-mile) 2.0 for tropospheric pollution, 0.3 for global warming, and 3.0 for accidents. We then double the US average figure for tropospheric pollution in Washington and triple it in Los Angeles to reflect the fact that these are large metropolitan areas instead of averages over all travel in the nation, and that the topography of Los Angeles causes pollutants to disperse especially slowly. We do not increase accident costs because studies suggest that, if anything, the external portion of accident costs is lower in crowded cities because higher traffic moves more slowly and therefore reduces the severity of accidents.

External accidents and pollution are assumed negligible for rail transit. For bus transit, accidents costs per vehicle-mile are taken to be the same as for auto because buses move more slowly and are driven by professionals; while pollution is taken to be triple that for automobiles.

4. Results

Our preliminary results are shown in Table 3.

Under proportional adjustment, optimal fares are well above current fares in the off-peak period but are substantially below current fares in the peak periods except for peak bus service in Washington. This is broadly consistent with the findings of Van Dender and Proost (2001). Nevertheless, in every case the optimal fare is less than the marginal agency supply cost and less than half the average operating cost.

The dominant fare component is marginal supply cost, but it is greatly reduced in all cases by one or both of the two primary effects mentioned in the introduction. The first is the “traffic diversion” effect: the reduction in external costs of automobiles caused by diversion to transit. This is large for peak service, especially in Los Angeles where the congestion cost of automobiles is especially high, but not for off-peak service. The second is the “Mohring effect:” the favorable effect of increased transit service on waiting times. It is large for all but Washington peak rail where wait times are already quite small, and is generally larger in off-peak periods. Thus as a broad generalization, the externalities of automobiles are more important in peak periods (where cars impose greater congestion costs) while waiting-cost scale economies are more important in off-peak periods (where longer headways make them more significant).

In the case of no service adjustment, most of the optimal fares are negative. Because (19) is undefined with negative or zero fares, we imposed a minimum fare of 10 cents/mile in computing (19) for this scenario. We are not very confident in this scenario at this point. For one thing, if optimal fares are really negative it implies that there is excessive service frequency in the initial situation. It may be that we have underestimated crowding costs, our least well-documented parameter. Also, in this scenario there is substantial influence of cross-elasticities, which may not be accurately captured by our constant-share assumption.

5. Conclusion

To the extent that transit subsidies are financed by higher taxes on labor and capital income they exacerbate efficiency costs of tax distortions in these markets. However, there is an offsetting effect to the extent they lower the costs of transportation, and increase real factor returns. According to recent literature, if travel were an average substitute for leisure the net impact of these two effects would reduce the optimal subsidy by around 10-25%.¹²

[To be completed – will discuss distributional issues, fixed infrastructure costs, interactions between transit subsidies and the broader fiscal system.]

¹² See for example Bovenberg and Goulder (2002) and Parry (1998) on how to adjust optimal externality taxes and subsidies to account for interactions with taxes on labor income. The assumption that travel is an average leisure substitute may be a reasonable approximation for peak-period commuting, as travel costs are effectively a tax on labor force participation. On the other hand, off-peak travel may be relatively complementary to leisure, implying a larger downward adjustment in the optimal transit subsidy.

References

- American Public Transportation Association, 2002. *Public Transportation Fact Book*. Washington, DC.
- Arnott, Richard, André de Palma, and Robin Lindsey, 1993. "A Structural Model of Peak-Period Congestion: A Traffic Bottleneck with Elastic Demand." *American Economic Review* 83:161-179.
- Bovenberg, A. Lans, and Lawrence H. Goulder, 2002. "Environmental Taxation." In A. Auerbach and M. Feldstein, eds., *Handbook of Public Economics* (second edition). New York: North-Holland, forthcoming.
- Chan, Y., and F.L. Ou, 1978. "Tabulating Demand Elasticities for Urban Travel Forecasting." *Transportation Research Record* 673: 40-46.
- De Borger, Bruno, Inge Mayeres, Stef Proost, and Sandra Wouters, 1996. "Optimal Pricing of Urban Passenger Transport: A Simulation Exercise for Belgium." *Journal of Transport Economics and Policy* 30: 31-54.
- Goodwin, P.B., 1992. "A Review of New Demand Elasticities with Special Reference to Short and Long" *Journal of Transport Economics and Policy*: 26(2): 155-169.
- Jansson, Jan Owen, 1979. "Marginal Cost Pricing of Scheduled Transport Services." *Journal of Transport Economics and Policy* 13: 268-294.
- Kerin, Paul D. 1992. "Efficient Bus Fares." *Transport Reviews*, 12: 33-47.
- Kraus, Marvin, 1991. "Discomfort Externalities and Marginal Cost Transit Fares." *Journal of Urban Economics* 29: 249-259.
- Lago, Armando M., Patrick D. Mayworm, and J. Matthew McEnroe, 1981. "Further Evidence on Aggregate and Disaggregate Transit Fare Elasticities." *Transportation Research Record* 799: 42-47.
- Mohring, H., 1972. "Optimization and Scale Economies in Urban Bus Transportation." *American Economic Review* 62: 591-604.
- Parry, Ian W.H., 1998. "A Second Best Analysis of Environmental Subsidies." *International Tax and Public Finance* 5: 157-74.
- Parry, Ian W.H., and Kenneth A. Small, 2002. "Does Britain or the United States Have the Right Gasoline Tax?" Working paper, Resources for the Future, Washington, DC.
- Pratt, Richard H., Texas Transportation Institute, Cambridge Systematics, Parsons Brinkerhoff Quade & Douglas, SG Associates, and McCollom Management Consulting, 2000. *Traveler Response to Transportation System Changes: Interim Handbook*. Transportation Research Board, Transit Cooperative Research Program Web Document 12 (March).
- Small, Kenneth A., 1992. *Urban Transportation Economics*. Fundamentals of Pure and Applied Economics, Volume 51, Harwood Academic Press, Chur, Switzerland.
- Tisato, Peter, 1998. "Optimal Bus Subsidy and Cross Subsidy with a Logit Choice Model." *Journal of Transport Economics and Policy* 32: 331-350.

Turvey, R. and H. Mohring, 1975. "Optimal Bus Fares." *Journal of Transport Economics and Policy* 9: 280-286.

US Bureau of Labor Statistics, 2002. "2000 Metropolitan Area Occupational Employment and Wage <http://www.bls.gov/oes/2000/oessrcma.htm>.

US FHWA, 1994a. Journey-To-Work Trends in the United States and its Major Metropolitan Areas 1960-1990. US Federal Highway Administration, Department of Transportation, Washington, D.C.

US FHWA, 1994b. National Personal Transportation Survey 1990. US Federal Highway Administration, Department of Transportation, Washington, D.C.

US FHWA, 1997. *1997 Federal Highway Cost Allocation Study*. US Federal Highway Administration, Department of Transportation, Washington, D.C.

Van Dender, Kurt, and Stef Proost, 2001. "Optimal urban transport pricing with congestion and economies of density." Working paper, Department of Economics, Katholiek Universiteit Leuven.

Viton, Philip A., 1983. "Pareto-Optimal Urban Transportation Equilibria." In Theodore E. Keeler, ed., *Research in Transportation Economics, Volume 1*. Greenwich, Connecticut: JAI Press, pp. 75-101.

WMATA 2001. *Fiscal 2002 Budget*. Washington Metropolitan Area Transit Authority. Available at: www.wmata.com/about/board_gm/01report/section14-operating.htm.

Winston, Clifford, 2000. "Government Failure in Urban Transportation." *Fiscal Studies* 21: 403-425.

Winston, Clifford and Chad Shirley, 1998. *Alternative Route: Toward Efficient Urban Transportation*. Brookings, Institution, Washington, DC.

Appendix A

Deriving Eq. (15)

Using (1), (7), (8) and (12), the agent's indirect utility function is defined by:

$$(A1) \quad W = W(\mathbf{p}, \mathbf{p}, \mathbf{w}, \mathbf{y}, G) - Z = \text{Max } u(C, M, \Sigma(\mathbf{p}^{ij} + w^{ij})M^{ij}, \Sigma\mathbf{y}^{ij}M^{ij}) - \Sigma\mathbf{q}^{ij}V^{ij} \\ + \mathbf{I}[I + G - C - \Sigma p^{ij}M^{ij}]$$

Differentiating gives:

$$(A2) \quad W_{p^{ij}} \equiv -\mathbf{I}M^{ij}; \quad W_{p^{ij}} = u_T M^{ij}; \quad W_{w^{ij}} = u_T M^{ij}; \quad W_{y^{ij}} = u_\Psi M^{ij}; \quad W_G = \mathbf{I}$$

When the agency increases p^{PR} , the welfare effect is given by differentiating the agent's indirect utility function taking into account changes in $\mathbf{p}, \mathbf{p}, \mathbf{w}, \mathbf{y}, G$ and Z . This gives:

$$(A3) \quad \frac{dW}{dp^{PR}} = W_{p^{PR}} + \Sigma \left\{ W_{p^{ij}} \frac{dp^{ij}}{dp^{PR}} + W_{w^{ij}} \frac{dw^{ij}}{dp^{PR}} + W_{y^{ij}} \frac{dy^{ij}}{dp^{PR}} \right\} + W_G \frac{dG}{dp^{PR}} - \Sigma \mathbf{q}^{ij} \frac{dV^{ij}}{dp^{PR}}$$

From (A2) and (A3), and using the definitions of \mathbf{r}^T and \mathbf{r}^Y we can obtain:

$$(A4) \quad \frac{1}{\mathbf{I}} \frac{dW}{dp^{PR}} = -M^{PR} - \Sigma M^{ij} \left\{ \mathbf{r}^T \frac{dp^{ij}}{dp^{PR}} + \mathbf{r}^T \frac{dw^{ij}}{dp^{PR}} + \mathbf{r}^\Psi \frac{dy^{ij}}{dp^{PR}} \right\} + \frac{dG}{dp^{PR}} - \frac{1}{\mathbf{I}} \Sigma \mathbf{q}^{ij} V_M^{ij} \frac{dM^{ij}}{dp^{PR}}$$

From (4):

$$(A5) \quad \frac{dp^{iA}}{dp^{PR}} = \mathbf{p}_A^{iA} V_M^{iA} \frac{dM^{iA}}{dp^{PR}} + \mathbf{p}_B^{iA} V_M^{iB} \frac{dM^{iB}}{dp^{PR}}$$

and the same for dp^{iB} / dp^{PR} . From (A5) and (4) we can obtain:

$$(A6) \quad \mathbf{r}^T \Sigma M^{ij} \frac{dp^{ij}}{dp^{PR}} = \mathbf{r}^T \Sigma \Pi^{ij} V_M^{ij} \frac{dM^{ij}}{dp^{PR}}$$

From (6):

$$(A7) \quad \frac{dw^{ij}}{dp^{PR}} = w_V^{ij} V_M^{ij} \frac{dM^{ij}}{dp^{PR}}$$

From (8):

$$(A8) \quad \frac{dy^{ij}}{dp^{PR}} = \mathbf{y}_a^{ij} \frac{d\mathbf{a}^{ij}}{dM^{ij}} \frac{dM^{ij}}{dp^{PR}}$$

From (3), (10), and (11):

$$(A9) \quad \frac{dG}{dp^{PR}} = \Sigma_{\neq A} p^{ij} \frac{dM^{ij}}{dp^{PR}} + M^{PR} + \Sigma t^{iA} \frac{dM^{iA}}{dp^{PR}} - \Sigma_{\neq A} k^{ij} V_M^{ij} \frac{dM^{ij}}{dp^{PR}}$$

Substituting (A6)–(A9) in (A4), using (16), and collecting terms, we can obtain (15).

Table 2. Modal and Inter-temporal Diversion Ratios

City	Time Period	Fraction Diverted To or From:			
		Automobile	Other transit mode, same time period	Other time period, same mode	Walk, bicycle, no trip
Los Angeles	P	0.85	0.05	0.10	0.00
	O	0.75	0.05	0.10	0.10
Washington	P	0.80	0.05	0.10	0.05
	O	0.70	0.05	0.10	0.15
Brussels	P	0.70	0.10	0.10	0.10
	O	0.60	0.10	0.10	0.20
London	P	0.60	0.15	0.10	0.15
	O	0.50	0.15	0.10	0.25