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Directed Search with Incomplete Information

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Abstract

This study revisits the question what happens in a search equilibrium when many heterogeneous agents are looking for partners and search is directed. The model considered features incomplete information, a novel feature in a directed search model. Particular attention is paid to the fact that workers with different amount of uncertainty about their quality may behave differently in their search behavior, even if they have the same expected quality. Moreover, I endogenize search intensity by allowing workers to make multiple applications and thus I can study the effect of search costs on application behavior and on sorting outcomes.

I find that as the cost of multiple applications increases, the number of applicants increases while the number of applications declines. This leads to firms becoming less selective for high application costs and sorting of workers declines. I also find that as the amount of uncertainty about workers' quality declines, both the number of applications and the number of applicants decline, firms become more selective and sorting of workers increases.

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1 Introduction

This study revisits the question what happens in a search equilibrium when many heterogeneous agents are looking for partners and search is directed. The model considered features incomplete information, a novel feature in a directed search model. Particular attention is paid to the fact that workers with different amount of uncertainty about their quality may behave differently in their search behavior, even if they have the same expected quality. Moreover, I endogenize search intensity by allowing workers to make multiple applications and thus I can study the effect of search costs on application behavior and on sorting outcomes.

Most search models assume homogeneity and random search (see the search and matching literature reviewed in Pissarides (2000)), though recently a literature has developed that focuses on the effects of heterogeneity on search outcomes. While most papers in this literature still assume random search (for example, Burdett and Coles (1997)), there are a handful of papers that consider directed search with heterogeneous agents. The starting point for these papers and the literature on search with heterogeneous agents is Becker's famous result (Becker (1973)) in which he establishes that, given that traits of workers and firms are complements (i.e., given a supermodular production function), perfectly assortative matching (PAM) is the optimal matching pattern and it is achieved in equilibrium. Perfectly assortative matching means that, in the case of the labor market, the highest quality worker matches with the highest quality firm, the next highest quality worker matches with the next highest quality firm, and so on.

This very stark prediction of Becker's model is in contrast with empirical observations and intuition. There has been considerable effort in the literature to establish more general conditions under which Becker result holds or breaks down (for example, Shimer and Smith (2000)). A recent paper emphasizing the role of coordination frictions in breaking down PAM is Shimer (2001).

The role of incomplete information has received practically no attention in this discussion. The only other extension of Becker's model, to my knowledge, that considers an incomplete information framework is Anderson and Smith (2001). Their focus is on a different aspect of incomplete information, however. In particular, they show that in a dynamic setting, PAM may fail even with a supermodular production function when matches yield not only output but also information about agents' types. In order to be able to characterize the dynamic equilibrium of their economy, they assume a simple form of heterogeneity. In particular, they assume that workers are either high or low quality. Workers' beliefs about their quality is then binary, thereby the same parameter determines the expected quality of the worker and the variance of her belief about her quality. In their setup it is thus not possible to study the question how workers' search behavior changes as the uncertainty they face changes while keeping their expected quality the same. In order to study this question, I abstract from dynamics in my model and introduce a richer form of heterogeneity.

While the model considered is highly stylized, and makes strong assumptions in order to gain tractability, it allows me to highlight some important effects not considered so far in the search literature, such as the effect of incomplete information on search behavior, the effect of search costs on sorting, etc.

I consider a model where workers are heterogeneous in their belief about their own quality. They all have beliefs that are distributed normally, but their posterior means and variances differ. These workers then decide how many applications to make. Each application has a fixed cost, and gains the possibility for the worker of being hired by the firm she applied to. There are two firms, one is more desirable than the other for all workers. These firms collect applications, then screen the applicants to learn their actual quality, and finally they make offers that are either accepted or turned down by workers.

I find that as the cost of multiple applications increases, the number of applicants increases while the number of applications declines. This leads to firms becoming less selective for high application costs and sorting of workers declines. I also find that as the amount of uncertainty about workers' quality declines, both the number of applications and the number of applicants decline, firms become more selective and sorting of workers increases.

2 Model

The economy lasts for one period and is populated by workers and firms. The size of the workforce is of measure one. The number of firms is equal to two. Each worker has a distinct quality, μ, that is drawn from a normal distribution with mean 0 and variance 1.[2] The worker does not know her own quality. Instead, she gets a signal about her quality, $\hat{\mu}_i = \mu + \varepsilon_i$, where ε has a distribution $N(0, \sigma_{\varepsilon i}^2)$. The standard deviation σ_ε can take on S distinct values $\{\sigma_{\varepsilon 1}, ..., \sigma_{\varepsilon S}\}$, and each worker knows the value of σ_ε corresponding to the signal she receives. The measure of workers who get type i signal is γ_i, where $\sum_{i=1}^{S} \gamma_i = 1$. Once the worker receives this signal, she updates her belief about her quality, and her posterior belief will be normal with mean

$$\tilde{\mu}_i = \frac{1}{1 + \sigma_{\varepsilon i}^2}(\mu + \varepsilon_i),$$

and variance

$$\tilde{\sigma}_i^2 = \frac{\sigma_{\varepsilon i}^2}{1 + \sigma_{\varepsilon i}^2}.$$

Hence, after each worker receives her signal and updates her belief, there are S distinct posterior variances amongst the population of workers and there is a continuum of posterior means that depend on the realization of μ and ε. Since there is a one-to-one correspondence between $\sigma_{\varepsilon i}^2$ and $\tilde{\sigma}_i^2$, I can treat $\{\tilde{\sigma}_1, ..., \tilde{\sigma}_S\}$ as the primitives of the model, where $0 \leq \tilde{\sigma}_i = \sqrt{\frac{\sigma_{\varepsilon i}^2}{1+\sigma_{\varepsilon i}^2}} \leq 1$, $i = 1, ..., S$. In terms of $\tilde{\sigma}_i$, the variance of ε_i and the posterior mean $\tilde{\mu}_i$ can be expressed as

$$\sigma_{\varepsilon i}^2 = \frac{\tilde{\sigma}_i^2}{1 - \tilde{\sigma}_i^2}, \tag{1}$$

and

$$\tilde{\mu}_i = (1 - \tilde{\sigma}_i^2)(\mu + \varepsilon). \tag{2}$$

Note that $\tilde{\mu}_i$ also has a normal distribution with mean 0 and variance $1 - \tilde{\sigma}_i^2$.

[2]Normalizing the quality distribution to be standard normal is without loss of generality.

Once the worker updates her belief about her quality, she decides — simultaneously with all the other workers in the economy — whether to apply for jobs at any of the two firms in the economy. When making this decision, the worker is maximizing her expected payoff. The cost of the first application is $c_1 \geq 0$, while the cost of the second application is $c_2 \geq 0$. Since there are two firms in the economy, the worker submits at most two applications. (There is no benefit from submitting multiple applications to the same firm.) The worker then gets offers or rejections from firms to which she submitted applications. Finally, the worker decides which one of the offers to accept (at most one offer can be accepted), and if the worker accepts the offer of firm j, then she gets a payoff of $\eta_j > 0$. If the worker accepts no offers, her payoff is 0.

The two firms differ in terms of the payoff workers derive from working there, i.e., their desirability (which I will call *reputation* from now on). The high reputation firm has reputation η_h, while the low reputation firm has reputation $\eta_l < \eta_h$. The firms collect applications from workers. After collecting applications, they screen each worker costlessly and observe the applicants' quality. Then they make a decision about each applicant, whether to make her an offer or not. The maximum workforce of firm j is κ_j, $j = l, h$, therefore this is the maximum measure of offers the firm can make. Any firm's payoff from hiring a worker of quality μ is μ.

3 Equilibrium

I look for a Nash equilibrium in this economy. (Its definition is straightforward and is hence omitted.) In this setup firms always prefer higher quality workers. The optimal policy of the two firms is then to establish a cutoff quality, and make offers to workers above this cutoff policy to fill their workforce. Let the cutoff of the high reputation firm be $\underline{\mu}_h$ and the cutoff of the low reputation firm be $\underline{\mu}_l$. The solution strategy is to characterize the optimal application policy of the workers with different beliefs for any cutoff policies $\underline{\mu}_l < \underline{\mu}_h$, then to find the optimal cutoff policies as the solution of a two-dimensional fixed-point problem.[3]

3.1 Optimal application policies

Consider the application policy of the workers given some cutoff policies $\underline{\mu}_l < \underline{\mu}_h$ of the two firms. A worker with posterior belief $(\tilde{\mu}, \tilde{\sigma})$ has four options: she can choose not to apply to any firm, she can apply to the low reputation firm, to the high reputation firm, or to both firms. The optimal policy of a worker with any belief can be described by comparing the above alternatives, two at a time, and establishing the values of $(\tilde{\mu}, \tilde{\sigma})$ for which each is preferred.

3.1.1 Pairwise comparison of alternative policies

Applying to a single firm versus not applying If the worker with belief $(\tilde{\mu}, \tilde{\sigma})$ applies to the low reputation firm, then her expected payoff is

$$\bar{F}\left(\frac{\mu_l - \tilde{\mu}}{\tilde{\sigma}}\right)\eta_l - c_1, \tag{3}$$

where $\bar{F}$ is the survival function of the standard normal distribution, i.e., $\bar{F} = 1 - F$, where F is the cumulative density function of the standard normal distribution. One necessary condition for this expected payoff to be higher than 0, the payoff from not applying, is that $c_1 < \eta_l$. This assumption is maintained throughout, since it simply means that the cost of applying to the low reputation firm is outweighed by the payoff from being hired by that firm. Then, given this assumption, the expected payoff is higher than 0 if and only if

$$\tilde{\mu} \geq \mu_l - F^{-1}\left(1 - \frac{c_1}{\eta_l}\right)\tilde{\sigma}.$$

This relationship defines a curve in $(\tilde{\mu}, \tilde{\sigma})$ space that separates the region where it is better to apply to the low reputation firm from the one where it is better to not apply. This curve is either downward- or upward-sloping, depending on the sign of $F^{-1}\left(1 - \frac{c_1}{\eta_l}\right)$. This curve is denoted m_{nl} in Figure 2.

[3]It is straightforward to see that in equilibrium $\underline{\mu}_l < \underline{\mu}_h$, since otherwise no worker would apply to the low reputation firm.

Similarly, if the worker with belief $(\tilde{\mu}, \tilde{\sigma})$ applies to the high reputation firm, then her expected payoff is

$$\bar{F}\left(\frac{\mu_h - \tilde{\mu}}{\tilde{\sigma}}\right)\eta_h - c_1. \tag{4}$$

Given that $c_1 < \eta_l < \eta_h$, this expected payoff is higher than 0 if and only if

$$\tilde{\mu} \geq \mu_h - F^{-1}\left(1 - \frac{c_1}{\eta_h}\right)\tilde{\sigma}.$$

Similarly to the m_{nl} curve, this relationship defines the m_{nh} curve in $(\tilde{\mu}, \tilde{\sigma})$ space, as demonstrated in Figure 2. The slope of the m_{nh} curve is lower than the slope of the m_{nl} curve, since $\eta_h > \eta_l$.

Applying to the low reputation firm versus applying to both firms If the worker with belief $(\tilde{\mu}, \tilde{\sigma})$ applies to both firms, then her expected payoff is

$$\bar{F}\left(\frac{\mu_h - \tilde{\mu}}{\tilde{\sigma}}\right)\eta_h + \left[\bar{F}\left(\frac{\mu_l - \tilde{\mu}}{\tilde{\sigma}}\right) - \bar{F}\left(\frac{\mu_h - \tilde{\mu}}{\tilde{\sigma}}\right)\right]\eta_l - c_1 - c_2. \tag{5}$$

Comparing this with the payoff in Equation (3) from applying to just the low reputation firm, we can see that it is worth applying to both firms instead of applying to just the low reputation firm if and only if

$$\bar{F}\left(\frac{\mu_h - \tilde{\mu}}{\tilde{\sigma}}\right)(\eta_h - \eta_l) - c_2 \geq 0.$$

This holds if $c_2 < \eta_h - \eta_l$ and

$$\tilde{\mu} \geq \mu_h - F^{-1}\left(1 - \frac{c_2}{\eta_h - \eta_l}\right)\tilde{\sigma}.$$

This relationship defines the m_{bl} curve in $(\tilde{\mu}, \tilde{\sigma})$ space. Its slope is higher than that of m_{nh} (as in Figure 2) if $\frac{c_1}{\eta_h} \leq \frac{c_2}{\eta_h - \eta_l}$, otherwise it is lower.

Applying to the high reputation firm versus applying to both firms Comparing the payoff from applying to both firms in Equation (5) to that from applying to the high reputation firm in Equation (4), we can see that it is worth applying to both firms instead of applying to just the high reputation firm if

$$\bar{F}\left(\frac{\mu_l - \tilde{\mu}}{\tilde{\sigma}}\right) - \bar{F}\left(\frac{\mu_h - \tilde{\mu}}{\tilde{\sigma}}\right) \geq \frac{c_2}{\eta_l}.$$

This relationship defines the m_{bh} curve in $(\tilde{\mu}, \tilde{\sigma})$ space in Figure 2.

Applying to the high reputation versus the low reputation firm Comparing the payoff from applying to the high reputation firm in Equation (4) versus that from applying to the low reputation firm in Equation (3), we can see that it is worth applying to the high reputation firm instead of applying to the low reputation firm if and only if

$$\frac{\bar{F}\left(\frac{\mu_h-\tilde{\mu}}{\tilde{\sigma}}\right)}{\bar{F}\left(\frac{\mu_l-\tilde{\mu}}{\tilde{\sigma}}\right)} \geq \frac{\eta_l}{\eta_h}.$$

This relationship defines the m_{lh} curve in $(\tilde{\mu}, \tilde{\sigma})$ space in Figure 2.

Applying to both firms versus not applying Finally, for some parameter configurations, it is necessary to compare directly the payoff from applying to both firms and that from not applying. Applying to both firms is preferred to not applying if and only if

$$\bar{F}\left(\frac{\mu_h-\tilde{\mu}}{\tilde{\sigma}}\right)\eta_h + \left[\bar{F}\left(\frac{\mu_l-\tilde{\mu}}{\tilde{\sigma}}\right) - \bar{F}\left(\frac{\mu_h-\tilde{\mu}}{\tilde{\sigma}}\right)\right]\eta_l \geq c_1 + c_2.$$

This relationship defines the m_{nb} curve in $(\tilde{\mu}, \tilde{\sigma})$ space. This curve does not appear in Figure 2, since that represents a parameter configuration such that making this comparison explicitly is not necessary. This curve will become relevant in only one of the cases discussed below, where I will return to its properties.

3.1.2 The role of the application cost

Based on the value of the second application cost, c_2, we can distinguish three cases based on the value of the model's parameters (notably, not based on the value of $\underline{\mu}_l$ and $\underline{\mu}_h$.) First, if $(\eta_h - \eta_l)\frac{\eta_l}{\eta_h} \leq c_2$, then the m_{bl}, m_{bh}, and m_{lh} curves do not intersect (since that would imply $\bar{F}\left(\frac{\mu_l-\tilde{\mu}}{\tilde{\sigma}}\right) \geq 1$, which is not possible for $\tilde{\sigma} > 0$.) This case is represented in Figure 1. Note that, when $\eta_h - \eta_l \leq c_2$, then there is no part of the $(\tilde{\mu}, \tilde{\sigma})$ space in which applying to both firms is preferred to applying to just the low reputation firm (i.e., there is no m_{bl} curve), but this is qualitatively the same as when there is an m_{bl} curve that does not intersect the m_{bh} curve. The empty circles represent $(\tilde{\mu}, \tilde{\sigma})$ combinations for which it is preferable to apply to the low reputation firm, while the small dots represent posterior beliefs for which it is preferable to apply to the high reputation firm. (Circles with dots in them represent posterior beliefs for which it is preferable to apply to both firms, though there are no such beliefs in this case.)

This is the case when the cost of the second application is prohibitively expensive so that a second application is never submitted. While this case features no multiple applications, it demonstrates several interesting results. First, for a given posterior variance, the PAM result of Becker (1973) holds in the sense that there is PAM along the posterior means of the agents, those with high posterior means (good signals) match with the high reputation firm, while those with low posterior means (bad signals) match with the low reputation firm. This result holds for any posterior variance if there is just one possible posterior variance in the population (the informativeness of the signals is the same). This follows from the fact that with $S = 1$ (one possible posterior variance) the equilibrium must feature applications to both types of firms, therefore it must be in a region of Figure 1 such that both types of applications are submitted.

More interestingly, however, we can see from Figure 1 that workers with the same posterior mean but different posterior variance might follow different application policies. For very high values of the posterior variance, workers will "take long shots" and apply to the high reputation firm or not apply, without considering to apply to the low reputation firm for any value of the posterior mean. For lower values of the posterior variance, workers with intermediate values of the posterior mean will "play it safe" and apply to the low reputation firm. This means that some workers with the same posterior mean will follow different application policies and will potentially be hired by different firms. Thus Becker's result no longer holds in the simple sense of PAM on posterior means.

The second case, when $(\eta_h - \eta_l)\frac{c_1}{\eta_h} \leq c_2 < (\eta_h - \eta_l)\frac{\eta_l}{\eta_h}$, is displayed in Figure 2. In this case the m_{bl}, m_{bh}, and m_{lh} curves intersect and do so at a higher $\tilde{\mu}$ than that corresponding to the intersection of the m_{nh}, m_{nl}, and m_{lh} curves. Once again, the empty circles represent $(\tilde{\mu}, \tilde{\sigma})$ combinations for which it is preferable to apply to the low reputation firm, the small dots represent posterior beliefs for which it is preferable to apply to the high reputation firm and the circles with dots in them represent posterior beliefs for which it is preferable to apply to both firms.

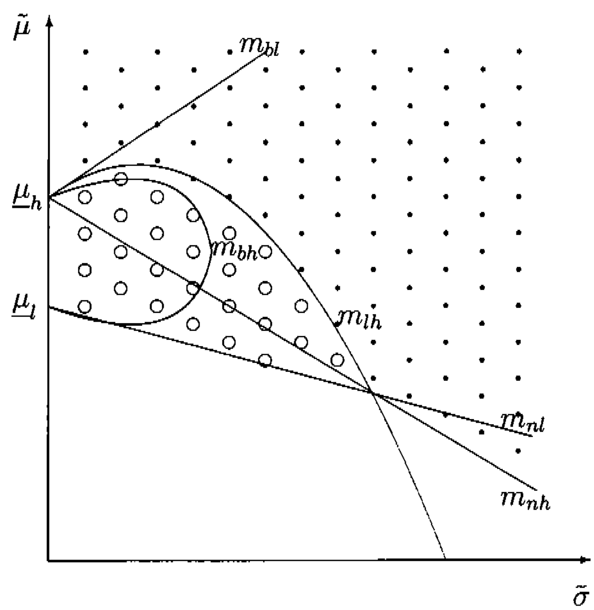

Figure 1: Optimal application policy of workers as a function of posterior mean and variance in the case when $(\eta_h - \eta_l)\frac{\eta_l}{\eta_h} \leq c_2$.

This case is more interesting than the previous one in the sense that it features multiple applications for some posterior beliefs. The $(\tilde{\mu}, \tilde{\sigma})$ combinations for which workers will choose to apply to both firms are relatively low values of the posterior variance coupled with posterior means around the cutoff of the high reputation firm. These workers have a quite precise idea about their own quality and realize that being so close to the cutoff of the high reputation firm, they run the risk of "just not making it", i.e., ending up with a quality that is just below the cutoff.

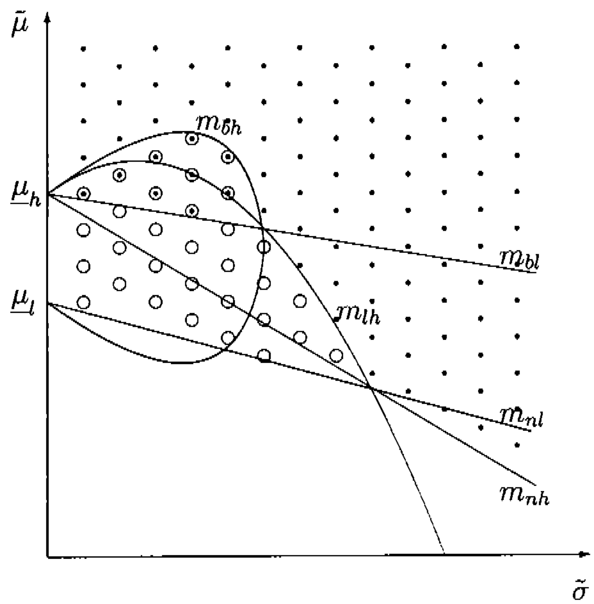

Figure 2: Optimal application policy of workers as a function of posterior mean and variance in the case when $(\eta_h - \eta_l)\frac{c_1}{\eta_h} \leq c_2 < (\eta_h - \eta_l)\frac{\eta_l}{\eta_h}$.

The third case, when $c_2 < (\eta_h - \eta_l)\frac{c_1}{\eta_h}$, is displayed in Figure 3. In this case the m_{bl}, m_{bh}, and m_{lh} curves intersect at a lower $\tilde{\mu}$ than that corresponding to the intersection of the m_{nh}, m_{nl}, and m_{lh} curves. In this case, in the area bordered by the m_{nh}, m_{nl}, m_{bh}, and m_{bl} curves, these curves are not sufficient to determine the optimal policy of the worker, since this is the case when applying to both firms and not applying are both preferred to applying to just a single firm. Thus this is the case when the direct comparison of the payoff from applying to both firms and that from not applying, the determination of the m_{nb} curve, is necessary. It is easy to see, that this curve has to pass inside the above area, and has to go through the intersection of the m_{nh} and m_{bh} curves and that of the m_{nl} and m_{bl} curves. Only the relevant section of the m_{nb} curve is drawn in Figure 3. Beyond the need for the determination of the m_{nb} curve, this case is similar to the second case. It features multiple applications for some posterior beliefs in a manner much like above. Of course, as the cost of the second application is declining, the $(\tilde{\mu}, \tilde{\sigma})$ combinations for which workers will choose to apply to both firms expands until, at $c_2 = 0$, all workers who apply do so at both firms.

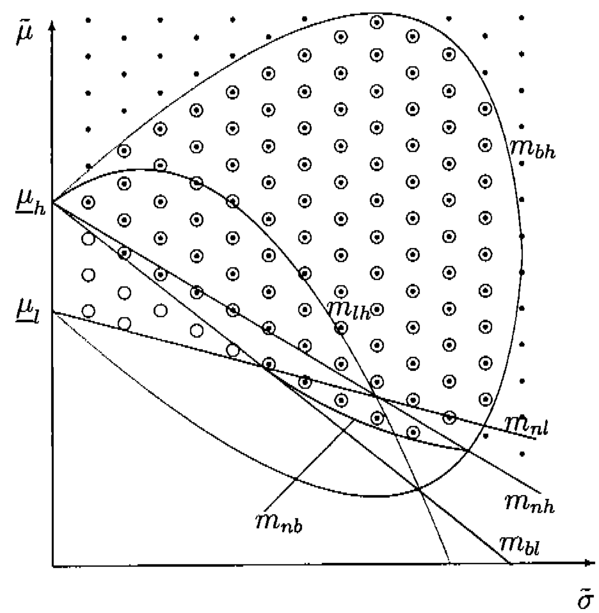

Figure 3: Optimal application policy of workers as a function of posterior mean and variance in the case when $c_2 < (\eta_h - \eta_l)\frac{c_1}{\eta_h}$.

3.2 Optimal hiring policies

The high reputation firm knows that any worker that it decides to make an offer to will accept this offer. By setting a cutoff $\underline{\mu}_h$, the high reputation firm attracts and hires those workers that have a high enough posterior belief to apply to the high reputation firm and that have a quality higher than $\underline{\mu}_h$. The measure of such workers is

$$\sum_{i=1}^{S} \gamma_i P(\mu > \underline{\mu}_h \cup \tilde{\mu}_i > m_{bi}). \tag{6}$$

Here m_{bi} is the boundary above which a type i worker applies to the high reputation firm (either exclusively, or together with the low reputation firm). As can be seen from Figures 1 through 3, this boundary can be determined as

$$m_{bi} = \begin{cases} \max(m_i^{lh}, m_i^{nh}) & \text{if } (\eta_h - \eta_l)\frac{\eta_l}{\eta_h} \leq c_2 \\ \max(\min(m_i^{bl}, m_i^{lh}), m_i^{nh}) & \text{if } (\eta_h - \eta_l)\frac{c_1}{\eta_h} \leq c_2 < (\eta_h - \eta_l)\frac{\eta_l}{\eta_h} \\ \min(\max(m_i^{bl}, m_i^{nb}), m_i^{nh}) & \text{if } c_2 < (\eta_h - \eta_l)\frac{c_1}{\eta_h}. \end{cases} \tag{7}$$

Given the expression for the posterior mean, $\tilde{\mu}_i$, in Equation (2) and that for the variance of ε_i in Equation (1), the above probability can be written as

$$\begin{aligned} P(\mu > \underline{\mu}_h \ \cup \ \tilde{\mu}_i > m_{bi}) &= P\left(\mu > \underline{\mu}_h \ \cup \ \varepsilon_i > \frac{m_{bi}}{1-\tilde{\sigma}_i^2} - \mu\right) = \\ &= \int_{\underline{\mu}_h}^{\infty} \left(1 - F\left(\frac{\frac{m_{bi}}{1-\tilde{\sigma}_i^2} - \mu}{\tilde{\sigma}_i}\sqrt{1-\tilde{\sigma}_i^2}\right)\right) f(\mu)d\mu, \end{aligned}$$

where f is the density function and F is the cumulative density function of the standard normal distribution.

In equilibrium, the high reputation firm will choose $\underline{\mu}_h$ so as to set equal its required workforce, κ_h, to the measure of hired workers in Equation (6), thereby giving an implicit equilibrium condition for $\underline{\mu}_h$.

The low reputation firm knows that a worker who applies to both firms will only accept its offer if the worker's quality is below the cutoff of the high reputation firm. On the other hand, any worker making a single application to the low reputation firm will accept that firm's offer. Hence the measure of workers who apply to the low reputation firm and receive and accept its offer is

$$\sum_{i=1}^{S} \gamma_i \left[P(\underline{\mu}_h > \mu > \underline{\mu}_l \cup m_{hi} > \tilde{\mu}_i > m_{bi}) + P(\mu > \underline{\mu}_l \cup m_{bi} > \tilde{\mu}_i > m_{li}) \right]. \tag{8}$$

Here m_{hi} is the boundary above which a worker only applies to the high reputation firm, m_{bi} is the boundary above which the worker applies to the both firms, and m_{li} is the boundary above which the worker applies to the low reputation firm. As can be seen from Figures 1 through 3, m_{hi} and m_{li} can be determined as

$$m_{hi} = \begin{cases} \max(m_i^{lh}, m_i^{nh}) & \text{if } (\eta_h - \eta_l)\frac{\eta_l}{\eta_h} \leq c_2 \\ \max(m_i^{bh}, m_i^{lh}, m_i^{nh}) & \text{if } c_2 < (\eta_h - \eta_l)\frac{\eta_l}{\eta_h} \end{cases} \tag{9}$$

and

$$m_{li} = \begin{cases} \min(m_i^{nl}, m_i^{nh}) & \text{if } (\eta_h - \eta_l)\frac{c_1}{\eta_h} \leq c_2 \\ \min(m_i^{nl}, m_i^{nb}, m_i^{nh}) & \text{if } c_2 < (\eta_h - \eta_l)\frac{c_1}{\eta_h}. \end{cases} \tag{10}$$

(These expressions assume that the intersection of $\max(m_i^{nh}, m_i^{bl})$ with the m_i^{bh} curve is above $\frac{\underline{\mu}_h + \underline{\mu}_l}{2}$. Otherwise, similar expressions can be derived for the case when, for some i, the interval of posterior means in which workers apply for both jobs is interior to the interval of posterior means in which workers apply for high jobs as, for example, in Figure 3 for very high posterior variances.)

Given the expression for the posterior mean, $\tilde{\mu}_i$, in Equation (2) and that for the variance of ε_i in Equation (1), the above probabilities can be written as

$$\begin{aligned} P(\underline{\mu}_h > \mu > \underline{\mu}_l \cup m_{hi} > \tilde{\mu}_i > m_{bi}) &= P\left(\underline{\mu}_h > \mu > \underline{\mu}_l \;\cup\; \frac{m_{hi}}{1-\tilde{\sigma}_i^2} - \mu > \varepsilon_i > \frac{m_{bi}}{1-\tilde{\sigma}_i^2} - \mu\right) = \\ &= \int_{\underline{\mu}_l}^{\underline{\mu}_h} \left(F\left(\frac{\frac{m_{hi}}{1-\tilde{\sigma}_i^2} - \mu}{\tilde{\sigma}_i} \sqrt{1-\tilde{\sigma}_i^2} \right) - F\left(\frac{\frac{m_{bi}}{1-\tilde{\sigma}_i^2} - \mu}{\tilde{\sigma}_i} \sqrt{1-\tilde{\sigma}_i^2} \right) \right) f(\mu) d\mu \end{aligned}$$

and

$$\begin{aligned} P(\mu > \underline{\mu}_l \;\cup\; m_{bi} > \tilde{\mu}_i > m_{li}) &= P\left(\mu > \underline{\mu}_l \;\cup\; \frac{m_{bi}}{1-\tilde{\sigma}_i^2} - \mu > \varepsilon_i > \frac{m_{li}}{1-\tilde{\sigma}_i^2} - \mu\right) = \\ &= \int_{\underline{\mu}_l}^{\infty} \left(F\left(\frac{\frac{m_{bi}}{1-\tilde{\sigma}_i^2} - \mu}{\tilde{\sigma}_i} \sqrt{1-\tilde{\sigma}_i^2} \right) - F\left(\frac{\frac{m_{li}}{1-\tilde{\sigma}_i^2} - \mu}{\tilde{\sigma}_i} \sqrt{1-\tilde{\sigma}_i^2} \right) \right) f(\mu) d\mu. \end{aligned}$$

In equilibrium, the low reputation firm will choose $\underline{\mu}_l$ so as to set equal its required workforce, κ_l, to the measure of hired workers in Equation (8), thereby giving an implicit equilibrium condition for $\underline{\mu}_l$ as a function of the parameters of the model and the decisions of the other agents.

Beyond these equilibrium conditions and the fact that $\underline{\mu}_l < \underline{\mu}_h$, it is not possible to give further restrictions on $\underline{\mu}_l$ and $\underline{\mu}_h$. This is because for any combination of $\underline{\mu}_l < \underline{\mu}_h$, we can find parameter values for which these cutoffs constitute an equilibrium. Simply, calculate the optimal policies of the workers given these cutoffs, calculate the measure of workers hired by the two firms for these two cutoffs using Equations (6) and (8), and set κ_h and κ_l equal to these measures.

Since the equilibrium cannot be characterized in more detail analytically, I next turn to simulations to describe what happens in equilibrium and what the effect of changing different parameters is on equilibrium outcomes.

4 Simulations

In this section I study the equilibrium of this model as a function of the underlying parameters. I consider the simplest case, when $S = 2$, i.e., there are two types of workers, those with a low posterior variance and those with a high posterior variance. I focus on the effect of two parameters, the cost of the second application, c_2, and the fraction of workers with a low posterior variance.

The other parameters of the model are chosen as follows. η_h is normalized to be equal to 1. $\eta_l = 0.7$, implying that the low reputation firm provides a 30% lower payoff to workers than the high reputation firm. $c_1 = 0.2$, a moderate application cost compared to the payoff from being hired. $\kappa_h = \kappa_l = 0.25$, thus in equilibrium half of the workers are hired. Finally, $\tilde{\sigma}_1 = 0.3$ and $\tilde{\sigma}_1 = 0.7$ implying a considerable difference in the uncertainty faced by the two types of workers about their quality.

In Figure 4, I plot the measure of workers — both of the high and low uncertainty types — that apply to firms as a function of the cost of the second application. The first observation is that the total measure of applicants across both types is significantly higher for all values of c_2 than the measure of available jobs ($\kappa_h + \kappa_l = 0.5$). We can also see that, as the cost of the second application rises, the measure of workers submitting two applications declines, as expected, and the measure of workers submitting a single application rises. Somewhat surprisingly, the total measure of workers submitting applications also rises as the cost of a second application rises (of course, this is not true of the measure of applications). This means that the low cost of multiple applications encourages some workers to submit multiple applications (as is the case for all high uncertainty applicants for very low cost of second application), but it also discourages some other workers from even trying to land a job by submitting an application. This is because, as can be seen from Figure 5, as the cost of a second application declines, firms become more selective, i.e., their cutoff rises). (Note that the cutoff of the high reputation firm in the absence of incomplete information would be 0.6745, while the cutoff the low reputation firm would be 0.)

Figure 6 demonstrates that as the cost of a second application declines, sorting improves in the sense that the average quality of workers working for the high reputation firm improves, while the average quality of workers working for the low reputation firm declines. (The average quality of workers at the high reputation firm in the absence of incomplete information would be 1.2709 for the given parameter values, while the average quality of those at the low reputation firm in the absence of incomplete information would be −0.4237.)

Figure 7 plots the measure of workers that apply to firms as a function of the fraction of workers with a low posterior variance. Since fewer low uncertainty workers submit applications, as their fraction in the population increases, the total number of applicants declines (without incomplete information it would be 0.5, the measure of available jobs). Also, the measure of workers submitting multiple applications increases , since — as can be seen from the optimal policies — low uncertainty workers are more likely to submit multiple applications. This implies that while the number of applications also declines, it does not decline by as much as the number of applicants.

As the amount of uncertainty declines with the fraction of low uncertainty types increasing, the cutoff policies of firms become more selective as can be seen from 8. Finally, Figure

9 demonstrates that as uncertainty about the quality of the workforce declines, sorting improves in the sense that the average quality of workers working for the high reputation firm improves, while the average quality of workers working for the low reputation firm declines.

5 Conclusion

To be completed.

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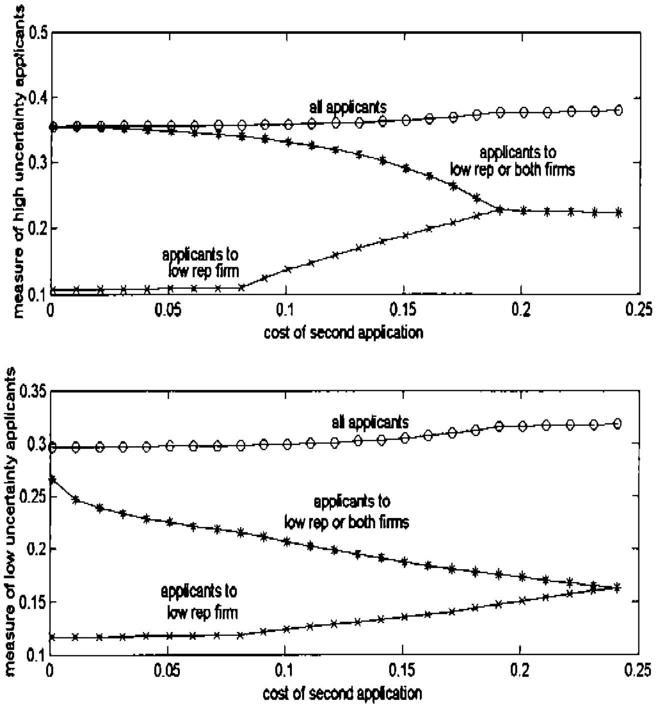

Figure 4: Measure of applicants to the low reputation firm, to both firms (difference between the middle line and the lowest line) and to the high reputation firm (difference between the highest line and the middle line) as a function of the cost of the second application.

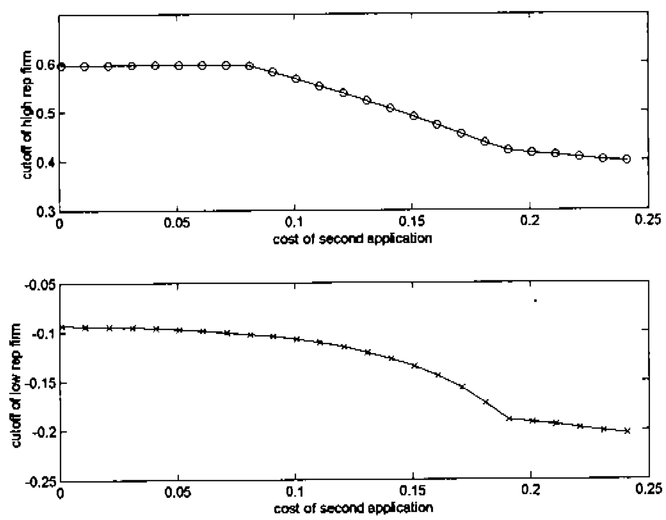

Figure 5: Cutoff policy of the high reputation firm and the low reputation firm as a function of the cost of the second application.

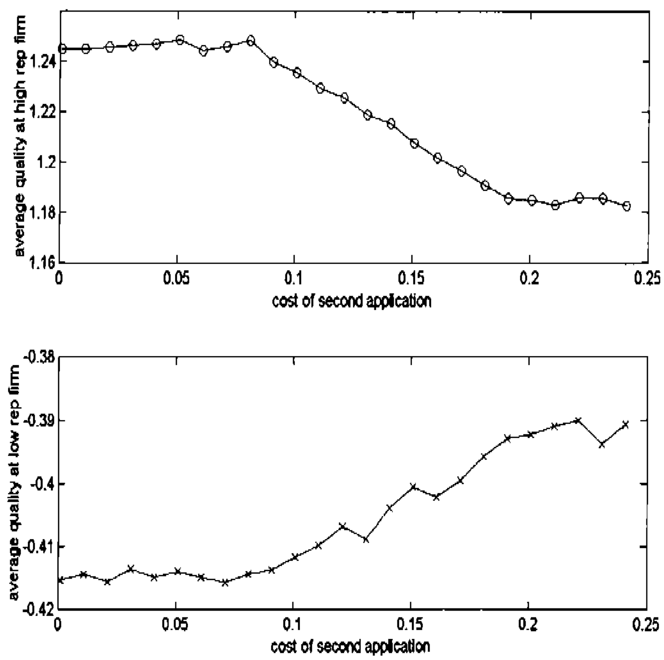

Figure 6: Average quality of workers at the high reputation firm and at the low reputation firm as a function of the cost of the second application.

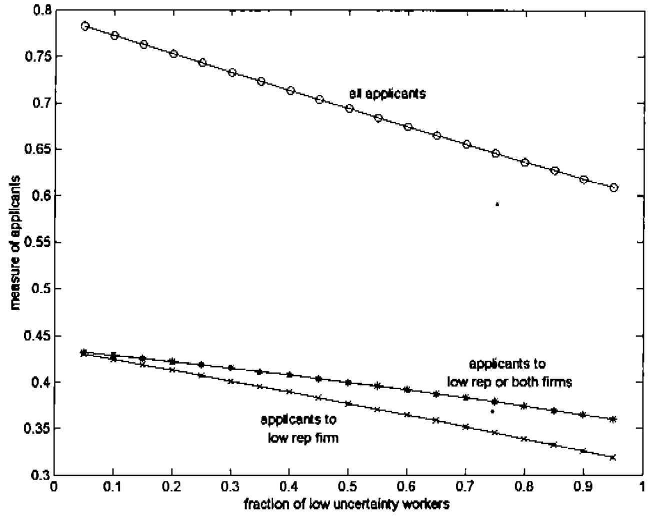

Figure 7: Measure of applicants to the low reputation firm, to both firms (difference between the middle line and the lowest line) and to the high reputation firm (difference between the highest line and the middle line) as a function of the fraction of low uncertainty workers.

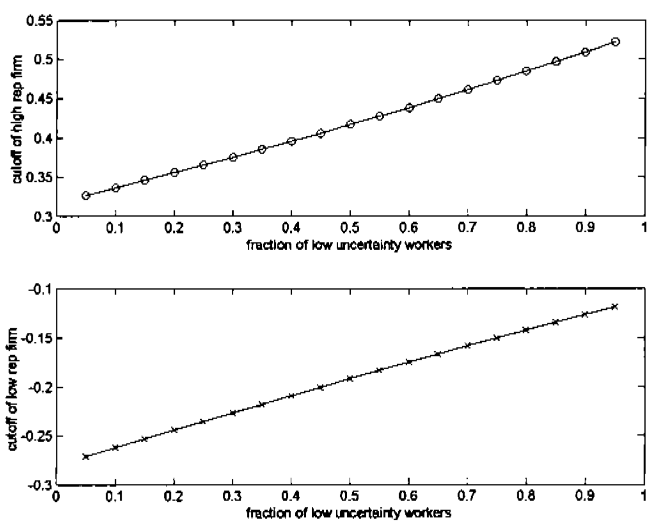

Figure 8: Cutoff policy of the high reputation firm and the low reputation firm as a function of the fraction of low uncertainty workers.

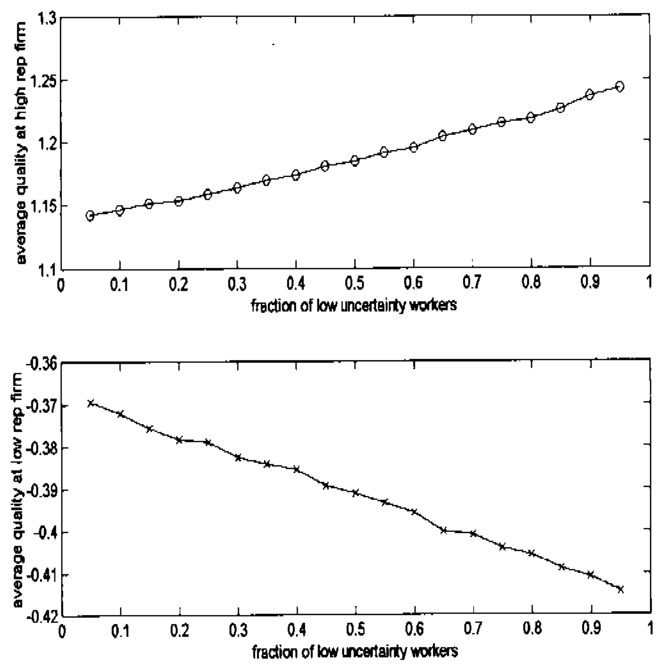

Figure 9: Average quality of workers at the high reputation firm and at the low reputation firm as a function of the fraction of low uncertainty workers.