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How Are (Danish) Wages Determined?

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Abstract

Within the context of a search equilibrium approach, two alternative wage determination hypotheses are tested for consistency with Danish worker flow and wage distribution data found in a matched employer-employee data set. The monopsony hypothesis that wages are set to maximize the present value of the future profit flow attributable to recruiting activity is rejected in favor of a dynamic version of the monopoly union model. Given this model, both the marginal cost of recruiting workers and the distribution of labor productivity across employing firms are identified. The empirical inferences about both are plausible and interesting. For example, the model and the data imply that the cost of hiring a worker for a firm paying the median wage equals 40% of annual earnings and that the most productive 1% of Danish privately owned firms employ almost half of all workers in the sector.

JEL Classification: D58, D83, E24, J30, J40, J60
Key Words: monopoly, monopsony, search equilibrium, wage determination, wage dispersion.

1 Introduction

Empirical evidence suggests that there are "good" and "bad" jobs in the sense that similar people are paid differently.[1] Wage dispersion of this kind has allocative consequences. Specifically, workers have incentives to seek out and move to employers who offer higher pay and amenities. If differences in wage policies reflect differential productivity across firms, more productive profit maximizing employers want to recruit more workers as well as pay better. The resulting

[1] There is now a substantial empirical literature on this topic which I review in Mortensen (2002). The most convincing evidence is based on analyzes of matched employer-empoyee data. Examples include the papers by Abowd and his co-authors listed in the references.

job-to-job mobility induced by search and recruiting efforts improves the allocation of labor resources. However, the supply of labor to an individual firm is not perfectly elastic and competitive forces do not fully weed out inefficient employers as a consequence of imperfect mobility. The empirical questions posed by this view of the labor market are many.

Do employed workers in fact move from lower to higher paying employers in response to wage dispersion? If so, can one find evidence that workers invest search effort in the mobility process? Do differences in wage and recruiting policies reflect attempts by more productive employers to attract and retain workers? What is the distribution of productivity across employers that would support the wage dispersion observed in the data? How are wages set in labor markets characterized by imperfect job to job mobility? These are the questions raised in the paper.

Bontemps et al. (1999, 2000) suggest a semi-parametric method of structural empirical inference that can be used to suggest answers to many of these questions in the context of the Burdett and Mortensen (1998) model of wage determination in markets with search friction. The method is based on the fact that the structure of the wage choice problem in the Burdett-Mortensen model with productive heterogeneity is identified by quantitative information implicit in the observed flow of workers from job to job and the cross firm distribution of wages paid when the latter is interpreted as outcome of the market equilibrium solution. The observed market distribution of wages paid across employers is regarded as consistent with the theory, *admissible* in their terminology, if and only if each firm pays a wage that maximizes expected profit appropriately defined for some feasible but unobserved level of employer productivity. Given admissibility, they show that the empirical distribution of employer productivity that accounts for wage dispersion is identified by the observed wage distribution and parameters of the job separation process. In this paper, I apply their method to two variants of the Burdett-Mortensen model.

The data used are cross firm observations on wages paid by and worker flows into and out of Danish firms found in the Integrated Database for Labour Market Research (IDA).[2] For each privately owned firm in Denmark, observations of interest include the number of employees in November 1994, the number of these who were still employed the next November, the number of workers hired during the year and the prior employment status of each new hire, and the hourly wage earned by each employee during the year. From these observation, cross firm distributions of average wages offered, average wages paid and labor force size are constructed.

Using the IDA worker flows data, Christensen et al. (2001) estimate a model of job separation at the firm level that embeds investment in search effort as an

[2] The IDA is a matched Danish employee-employer data base constructed from data collected in an annual employment survey of all workplaces taken in November and from register data on individual workers. The data are organized as an annual panel of firms covering the period 1980-1995. The data set was created by Statistics Denmark and is maintained by Centre for Labor and Social Research (CLS) located in Aarhus Denmark. On line information about the data set can be found at www.cls.dk.

endogenous choice made by each employed worker. Their results clearly support the hypothesis that workers invest in job to job mobility in response to the wage dispersion. In this paper, I incorporate the Christensen et al. model of worker separation into variants of the Burdett and Mortensen (1998) model of wage determination. Wage distribution information from the Danish IDA and the Christensen et al. estimates of the parameters of their separation model are then used to answer the question posed in the title.

A principal finding is that the original Burdett-Mortensen monopsony model of wage determination with productive heterogeneity across firms is simply not consistent with the Danish IDA data in the sense that wages in left tail of the observed average hourly wage distribution cannot be profit maximizing. In addition, monopsony profit is implausibly large on much of the portion of the support of the distribution for which observed wages are admissible.[3] However, I also show that closely related dynamic version of the classic monopoly union model is consistent with the IDA data.

In the monopoly union variant of the general model, each local union sets the wage policy of the firm employing its members to maximize the rent accruing to the flow of workers hired under the assumption that the firm chooses its recruiting effort to maximize the employer's expected value of the hire flow taking the union wage as parametric. Furthermore, the local union takes the outside options of each worker and the worker's search behavior as given. Using the model estimated by Christensen et al. (2001) on the same data to model worker search behavior, the observed wage and size distributions are shown to be consistent with the local monopoly model of wage determination.

Under the assumption that the wage paid by each firm is determined by a local monopoly union, one can infer the cross firm distribution of productivity that supports the wage dispersion observed and the marginal cost of recruiting a worker. The productivity distribution is unimodal but skewed with a very long right Pareto-like tail. Indeed, the model implies that almost half of the employed private labor force work for the most productive 1% of all privately owned firms. This result contrasts with the fact that the distribution of average wages paid is well approximated by a log normal. This inference seems to support the view that the labor market allocates most workers to the relatively few employers who are significantly more productive than the typical firm even though search and recruiting friction permits considerable productive heterogeneity across firms and wages are set by unions to maximize the rent of the worker flow into each firm. Finally, the recruiting costs implied by the estimated model are plausible and consistent with admittedly limited evidence available. Namely, for a firm paying the median wage the estimated expected average recruiting cost per worker hired is approximately 40% of the firm's average annual wage bill per worker.

[3] P. Koning et al. (2000) come to a similar conclusion using IDA data for individual workers for a different time period. Specifically, they test and reject restrictions that the monopsony model imposes on a standard wage equation regression.

2 Worker Search Behavior

In much of the existing literature applying the Burdett and Mortensen (1998) approach to wage determination, workers receive offers at rates that are independent of their own effort.[4] The only worker decision of interest is to accept or not wage offers as they arrive in this case. Since the worker's incentive to seek a higher paying job is increasing in the difference between the worker's outside alternatives and current wage, this assumption is inappropriate. Recently, Christensen et al. (2001) have developed and estimated a model of endogenous search effort using Danish IDA data. I start by reviewing their model and parameter estimates.

2.1 Optimal Search Strategy

Suppose that offers arrive at a Poisson frequency which is proportional to search effort. Specifically, let λs represent the offer arrival rate where s is search effort and λ is a contact frequency parameter. Assume in addition that workers act to maximize expected wealth and live forever. In this case, a worker's maximal value of a match paying wage w solves

$$rW(w) = \max_{s \geq 0} \left\{ \begin{array}{c} w + \delta\,(U - W(w)) \\ +\lambda s \int [\max \langle W(x), W(w) \rangle - W(w)] dF(x) - c_w(s) \end{array} \right\} \quad (1)$$

where $c_w(s)$ denotes the cost of search effort. In addition, r represents the common discount rate, δ is an exogenous job turnover rate, and $F(w)$ denotes the fraction of vacancies that offer wage w or less.

Equation (1) is a continuous time Bellman equation that reflects the optimal search effort and separation choices that the worker makes while employed and the fact that a transition to unemployment may occur. Specifically, the worker quits to take an outside employment option if and only if it offers a higher present value of expected future income than the current job, the optimal search effort choice maximizes the difference between the expected gain in value attributable to search and the cost of search, and the worker realizes an expected future income equal to U after a transit to unemployment as a consequence of the destruction of the match. Similarly, the value of unemployment solves

$$rU = \max_{s \geq 0} \left\{ b + \lambda s \int [\max \langle W(w), U \rangle - U]\, dF(w) - c_w(s) \right\} \quad (2)$$

where b an income flow received when unemployed.

The solution to (1) is increasing in w. Indeed, because

$$W'(w) = \frac{1}{r + \delta + \lambda s(w)[1 - F(w)]} > 0 \quad (3)$$

[4] Examples include Bontemps (1999, 2000) and Rosholm and Svarer (2000). See van den Berg (1999) for a recent review of the literature.

by the envelope theorem, an employed worker quits to take an alternative job offer if and only if it pays a higher wage. Hence, the rate at which workers quit an employer paying w to take a job elsewhere is $\lambda s(w)[1-F(w)]$ where optimal search effort is

$$s(w) = \arg\max_{s\geq 0} \left\{ \lambda s \int_{w}^{\overline{w}} [W(x) - W(w)]\, dF(x) - c_w(s) \right\}. \tag{4}$$

When unemployed, the typical worker accepts an offer only if it is no smaller than the reservation wage, denoted as R. As the reservation wage equates the value of employment to the value of unemployed search, denoted by U, acceptance requires that $w \geq R$ where

$$U = W(R). \tag{5}$$

As a consequence, equations (1) and (2) together imply that the search intensity of an unemployed worker is the same as that of a worker employed at the reservation wage provided that the reservation wage is the unemployment benefit,

$$s_0 = s(R), \tag{6}$$

and the reservation wage is equal to the unemployment benefit,

$$R = b. \tag{7}$$

3 The IDA Distributions of Wages

As part of their project, Christensen et al. (2001) use Danish wage and worker flow data from the Integrated Database for Labor Market Research (IDA) for the year between November 1994 and November 1995 to construct an employment weighted distribution of average wages paid by privately owned Danish firms. First, the average hourly wage paid during the year by each of the 113,325 employers identified in the data is calculated. Second, the average wage paid and the number of workers employed by each firm are used to construct the employment weighted distribution of wages paid across employers. The empirical counterpart of the wage density obtained, $g(w) = G'(w)$, is plotted in Figure 1. Obviously, the distribution is skewed right but the log normal density with same log wage mean and variance, represented in the figure by the dashed curve, approximates the general shape of the wage density quite well. This result contrasts with the shapes found for distributions of earning which typically have longer and fatter right tails, one that is better approximated by the Pareto distribution.[5]

Christensen et al. (2001) also construct the distribution of wages offered during the November 1994 to November 1995 year by weighting the average wage of each firm by the number of workers hired by the firm from non-employment during the year. Workers hired from employment are excluded in the construction

[5] See Neal and Rosen (2000) for documentation of this fact.

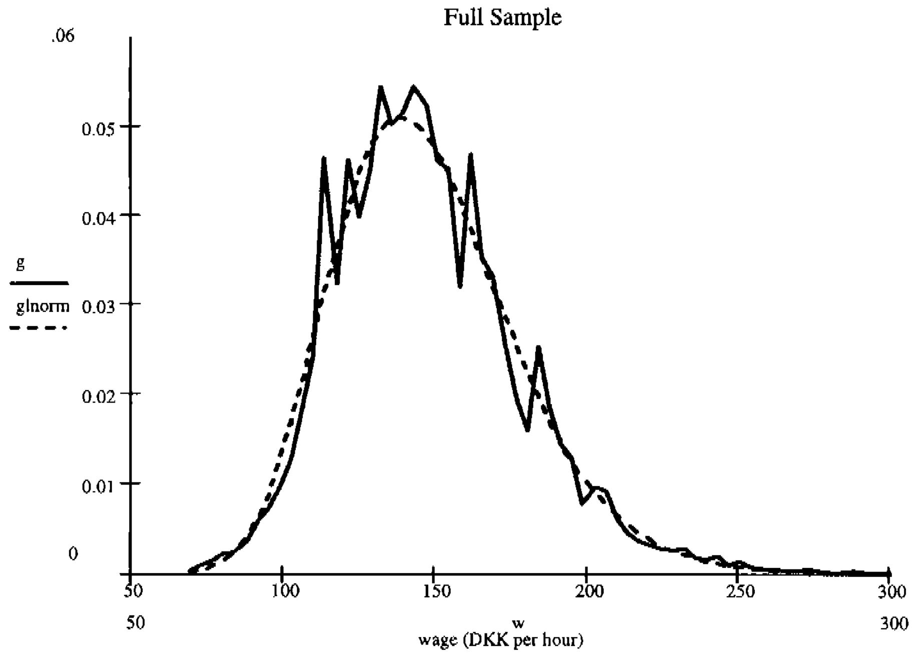

Figure 1: Actual (g) and Log Normal ($g \ln norm$) Wage Densities

because this sub sample is biased upward to the extent that the employed only move from lower to higher paying firms as predicted by the Burdett-Mortensen model. The resulting wage offer distribution density function, $f(w)$, is illustrated by the dotted curve Figure 2. Again, the wage density, $g(w)$, is plotted as a solid curve. Although a smoothed version of the empirical offer distribution is also unimodal, it is more skewed-right than the wage density.

If all workers were identical, as in our hypothetical environment, the observed variation in average wages paid would reflect only wage dispersion induced by differences in employer pay policies. Of course, workers are not equally able in reality. Now, it is conceivable that this fact alone provides an explanation for the observed skew in the distribution of wages paid in Denmark. However, if worker heterogeneity and employer heterogeneity are orthogonal, then the distribution of differences in average pay represent the distribution of employer component plus noise that vanishes with firm size.

Formally, suppose that the natural log of the wage earned by worker i in firm j is

$$\ln w_{ij} = x_i + y_j$$

where x_i and y_j represent worker and employer fixed effects respectively. Of

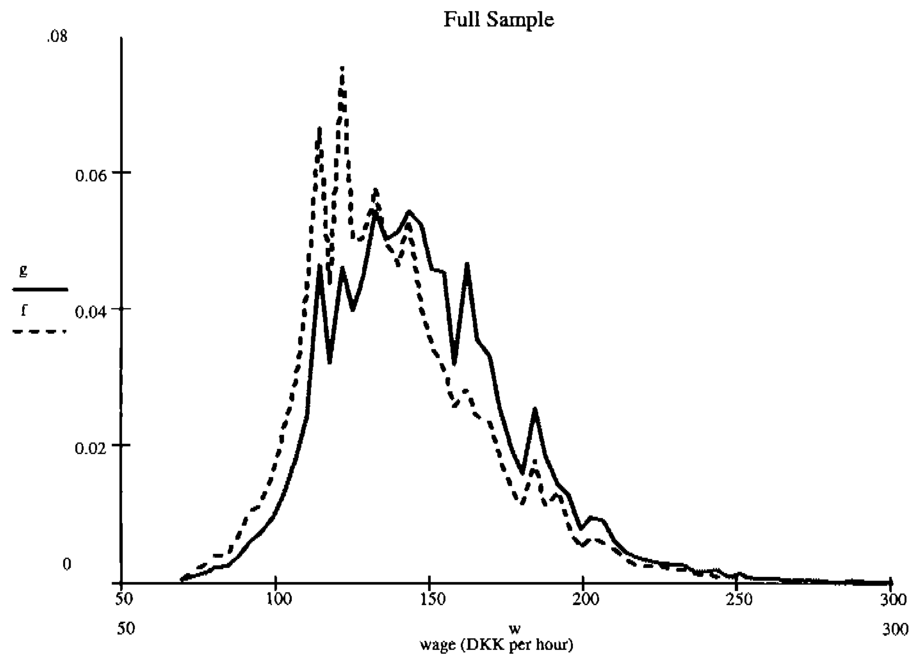

Figure 2: Offer (f) and Wage (g) Densities

course, the average wage paid by employer j is

$$\overline{\ln w}_j = \frac{1}{n_j} \sum_{i \in I_j} \ln w_{ij} = y_j + \overline{x}_j,\ \overline{x}_j = \frac{1}{n_j} \sum_{i \in I_j} x_j$$

where I_j is the set of workers employed by firm j, n_j is the size of the set, and $\overline{x}_j$ is the average worker effect. If the set of employees is a random sample of workers, then the variance of the average wage paid by firms of the same size is the variance in the employer fixed effect plus the variance of the average worker effect which is proportional to the inverse of firm size, i.e.,

$$Var(\overline{\ln w}_j | n_j = n\} = \sigma_y^2 + \frac{\sigma_x^2}{n}.$$

In other words, the dispersions of the distributions of wage offered and paid represented in Figure (2) reflect dispersion in employer fixed effects plus sampling noise which vanishes with firm size if employer and worker effects are independent in the sample. As I show elsewhere in Mortensen (2002), the matching model presented below implies no correlation between worker and employer effects across firms. This property of the model is a crucial source of identification.

3.1 Parameter Estimates

Christensen et al. (2001) use observations on worker separations at the firm level in the IDA to estimate the turnover parameters given a cost of search effort function of the power form

$$c_w(s) = \frac{c_0 s^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}. \tag{8}$$

In this case, the first order condition, equation (4), can be rewritten as

$$\lambda s(w) = \left(\frac{\lambda^2}{c_0} \int_w^{\overline{w}} \frac{[1-F(x)]dx}{r+\delta+\lambda s(x)[1-F(x)]} \right)^{\gamma}. \tag{9}$$

In other words, for any vector of parameters $(c_0, \delta, \lambda, \gamma)$, the product of the arrival rate and the optimal search effort function, $\lambda s(w)$, is the unique solution to this functional equation.

Estimates of the parameters are obtained by finding those values that maximize the likelihood of the observed number of separations experienced by each firm during the year beginning in November 1994. Since the duration of an job spell in a firm paying wage w is exponential with parameter equal to the separation rate

$$d(w) = \delta + \lambda s(w)[1-F(w)], \tag{10}$$

the number of workers who stay with the firm is binomially distributed with "sample size" equal to firm size n and "probability of success" equal to $e^{-d(w)}$. Under the assumption that the parameters are identical across firms, the maximum likelihood estimates conditional on the interest rate r and offer distribution F are

$$(c_0, \delta, \lambda, \gamma) = \arg\max \sum_i \left[(n_i - x_i) \ln(1 - e^{-d(w_i)}) - d(w_i) x_i \right] \tag{11}$$

where $d(w)$ is the function specified in equation (10), w_i represents the wage, n_i the size, and x_i the number of stayers for firm i. Since c_0 and λ^2 are not separately identified, search effort at the lowest wage $s(\underline{w})$ is normalized to unity. Given the offer cdf $F(w)$ observed in the data and an annual interest rate r equal to 4.9% per annum, the estimates of the remaining parameters are $\delta = 0.287, \lambda = 0.593$ and $\gamma = 1.105$.[6] (Because the sample size is very large, the precision of the estimates is virtually exact to the third significant digit.)

Given the cost function specified in equation (8), the fact that the estimate of the elasticity of search effort with respect to the return to search, γ, is close to unity suggests that the cost of search function is reasonably approximated by a quadratic. The arrival rate and job turnover rate parameter estimates

[6] These estimates are slightly different from those reported in Christiansen et al. (2001) because they were obtained using a different bin width for the construction of the wage and offer cdfs, F and G. Otherwise, the estimates where obtained using the same procedure.

reflect relatively high turnover in the Danish labor market. Obviously, the Christensen et al. value of δ is too large to be interpreted as the layoff or job destruction rate. Rosholm and Svarer (2000) derive an estimate of the employment to employment transition rate equal to 0.099 per annum using panel data on Danish worker labor market event histories for the 1980s. In fact, the Christensen et al. estimate should be interpreted as the intercept of the separation function because the worker's destination state was not used in the estimation procedure. This reasoning suggests that $\delta = \delta_0 + \delta_1$ where $\delta_0 = 0.099$ represents the transition rate to unemployment and δ_1 is that part of the job-to-job transition rate which is not related to cross firm wage differences.

3.2 Steady State Conditions

The separation theory relates the distribution of offers, F, to the cross firm distribution of wages paid, G. Because "experienced" employed workers are more likely to have "moved up the job ladder" than newly hired workers, the distribution of average wages paid stochastically dominates the offer distribution. The curves of the two density functions, f and g, reported in Figure 2 are consistent with this prediction. Since the actual distribution of wages paid observed in the data was not used in the estimation, its availability provides an out-of-sample test of how well the theory explains the difference between the two distributions functions.

According to the theory introduced above, δ_0 represents the transition rate from employment to unemployment and $\lambda s(R)$ is transition rate from unemployment to employment. Hence, the steady state unemployment rate, which balances the flows into and out of the stock, is

$$u = \frac{\delta_0}{\delta_0 + \lambda s(R)}. \tag{12}$$

Under the assumption that workers who move between jobs for non-wage reasons earn a random wage offer at the destination job and all wage offers are acceptable, the flow of workers to jobs that pay w or less is

$$\lambda s(R)F(w)u + \delta_1 F(w)(1-u),$$

where the first term is the inflow from unemployment and the second term is the inflow from employment. The flow out of this state is the sum of the job destruction flow and the flow of quits to employers who pay more than w. Equating the inflow to the outflow yields the following steady state relationship between the offer and wage distribution functions

$$\begin{aligned} & \delta G(w) + \lambda[1 - F(w)] \int_{\underline{w}}^{w} s(x) dG(x) \qquad (13) \\ = \; & \frac{s(R)\lambda F(w)u + \delta_1 F(w)(1-u)}{1-u} \\ = \; & (\delta_0 + \delta_1)F(w) = \delta F(w). \end{aligned}$$

Given the offer distribution function $F(w)$, parameters δ and λ, and the search effort function $s(w)$ reported by Christensen et al., one can compute the unique solution for the steady state wage distribution implied by equation (13).

The density of the steady state distribution of wages implied by the observed IDA offer distribution and the Christensen et al. estimates of the separation model parameters and the observed actual density are plotted in Figure 3. The actual density, $g(w)$, is represented by the solid lines and the predicted steady state density, $gss(w)$, is portrayed by the dashed lines in the figure. Obviously, the two are quite close. Indeed, even the location of the observed spikes in the actual density are anticipated by the theory.

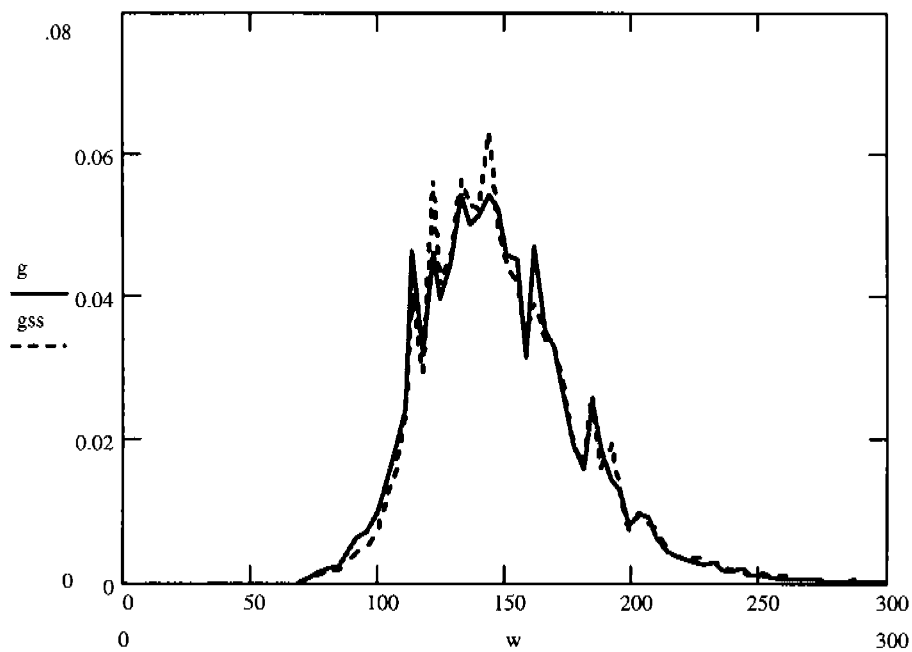

Figure 3: Actual $g(w)$ and Predicted Steady State $gss(w)$ Wage Densities

4 Is the Danish Labor Market Monopsonistic?

At one time, the Danish wages in all industries were determined by a national bargaining agreement. More recently wage setting institutions have been decentralized. Although national agreements still exist, these only determine wage minimums by industry and occupation. The actual wages paid are set at the local level with the national minimums serving as a lower bound on what firms can pay. As already documented, there is considerable variation in the average wage paid across firms. Do these differences arise because firms are heterogenous with respect to labor productivity and because firms set wages to maximize profit? In spite of the large unionization rate in Denmark, 80%-90%, the hypothesis is worth testing.

4.1 Monopsony Wage and Recruiting Policy

Let p represent a firm's marginal product of labor. This parameter, assumed to be independent of employer size, differs across firms. The firm's problem is one of choosing its wage and recruiting policy to maximize the expected value of its future profit given its type as represented by p. The recruiting problem is modeled as one of simply contacting a chosen number of workers v at random subject to an increasing convex cost of recruiting $c_f(v)$. The optimal monopsony wage and recruiting policy of a type p firm maximizes the employer's expected profit flow in the sense that

$$(w(p), v(p)) = \arg \max_{(w,v)\geq(R,0)} \{\pi(p,w)v - c_f(v)\}. \tag{14}$$

where w represents the wage and $\pi(p,w)$ denotes the expected profit per worker contacted expressed as a function of the firm's productivity and wage. The expected profit per worker contacted is

$$\pi(p,w) = h(w)J(p,w) = \frac{h(w)\,(p-w)}{r+d(w)} \tag{15}$$

where $h(w)$ is the probability that a worker contacted at random will accept the wage w, and $J(p,w)$ is the expected present value of employing a worker to a firm of productivity p paying wage w, and $d(w)$ denotes the separation rate as defined by equation (10). The acceptance probability $h(w)$ is simply the fraction of a representative sample of the flow of workers who meet employers per period willing to accept a wage w or less. Because the aggregate flow of meetings represented by workers who are unemployed is $\lambda s(R)u$, who are employed but moving a new job for exogenous reasons is $\delta_1(1-u)$, and who are employed, currently paid wage x, and seeking a higher wage is $\lambda s(x)g(x)$ and because the first two groups accept any wage while each member of the remainder accepts only if the offer exceeds her current wage,

$$h(w) = \frac{u\lambda s(R) + (1-u)\left[\delta_1 + \lambda \int_{\underline{w}}^{w} s(x)dG(x)\right]}{u\lambda s(R) + (1-u)\left[\delta_1 + \lambda \int_{\underline{w}}^{\overline{w}} s(x)dG(x)\right]} = \frac{\delta + \lambda s(R)\int_{\underline{w}}^{w} s(x)dG(x)}{\delta + \lambda s(R)\int_{\underline{w}}^{\overline{w}} s(x)dG(x)} \tag{16}$$

where the second equality follows from the steady state equation (12) and the definition $\delta = \delta_0 + \delta_1$.

As a corollary of (14),

$$w(p) = \arg\max_{w\geq R} \pi(p,w) \tag{17}$$

and

$$c_f'(v(p)) = \max_{w \geq R} \pi(p, w). \tag{18}$$

In other words, the optimal wage choice maximized the expected profit per worker contacted while the number of workers contacted equates the cost of contacting the marginal worker with the expected profit attributable to contacting that worker. Because $\pi(p, w)$ and $\pi_w(p, w)$ both increase with p, the optimal wage and contact frequency both increase with productivity. In other words, more productive employers pay more and hire more workers than less productive firms. As a consequence of this fact and the fact that the quit rate declines with the relative wage paid, the average long run size of a firm, given by

$$n(p) = \frac{h(w(p))v(p)}{d(w(p))}, \tag{19}$$

increases with productivity.

4.2 Admissibility

Following Bontemps et al. (2000), the observed wage distribution is *admissible* under the monopsony hypothesis if and only if every element of its support could be profit maximizing for some employer. Equivalently, the relationship between the wage and unobserved firm productivity implied by the observed wage distribution and the worker turnover behavior must be monotone increasing. This relationship can be derived using the following first order condition for the monopsony wage:

$$\frac{\pi_w(p, w)}{\pi(p, w)} = \frac{h'(w)}{h(w)} - \frac{d'(w)}{r + d(w)} - \frac{1}{p - w} = 0.$$

Because the implied inverse of the wage policy function can be written as

$$p(w) = w \left[1 + \frac{1}{\frac{wh'(w)}{h(w)} - \frac{wd'(w)}{r + d(w)}} \right], \tag{20}$$

the wage policy function $w(p)$ can be computed using the empirical acceptance and separation functions, $h(w)$ and $d(w)$, implied by the Christensen et al. (2001) estimates.

To compute $h(w)$ and $d(w)$, I use the previously reported estimates of $\delta = 0.287, \lambda = 0.593$ and $\gamma = 1.105$ which embody the nomalization $s(R) = 1$, a smooth approximation to the distribution of average wages paid, and the associated offer distribution implied by the steady state equation (13). In other words, the Christensen estimates and the equivalent of the log normal approximation to $G(w)$ are used to compute $h(w)$ as specified in equation (16). Similarly, $d(w)$ is then computed using equation (10) where

$$F(w) = \frac{\delta G(w) + \lambda \int_{\underline{w}}^{w} s(x) dG(x)}{\delta + \lambda \int_{\underline{w}}^{\overline{w}} s(x) dG(x)} \tag{21}$$

from the steady state condition. Because the Christensen estimates and the observed offer and wage densities satisfy the steady state condition, this procedure simply provides a smooth estimate of both $G(w)$ and $F(w)$. The associated approximate probability density functions are illustrated in Figure 4. The implied acceptance probability and separation rate functions are plotted in Figure 5.

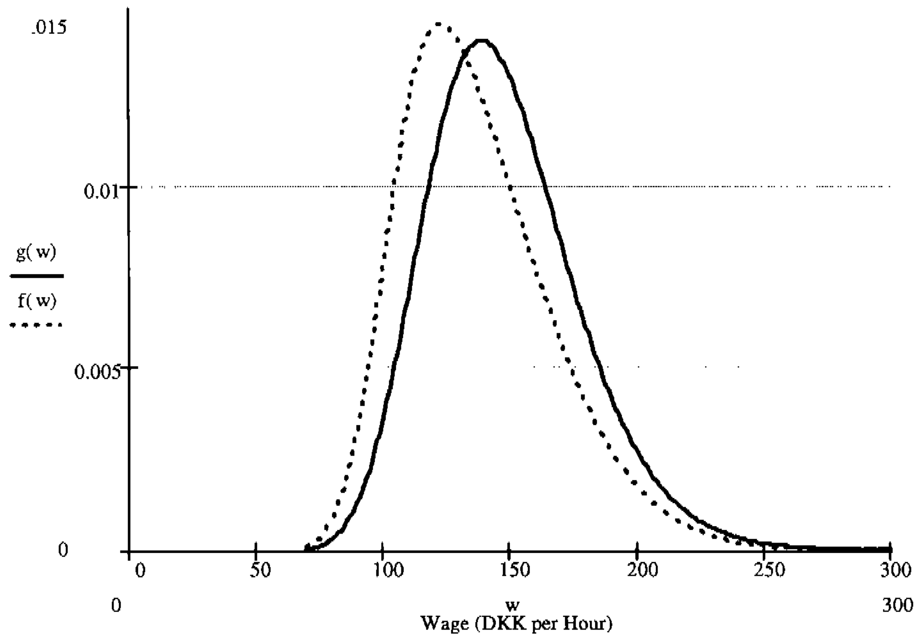

Figure 4: Approximate Wage $g(w)$ and Offer $f(w)$ Probability Density Functions

The wage policy function, $w(p)$, implied by equation (20) and the Christensen et al. parameter estimates is illustrated in Figure 6. The fact that the curve is negatively sloped for wage rates below 105 DKK per hour implies that two values of the wage exist that satisfy the first order condition for optimality. As the second order condition requires that the optimal policy function $w(p)$ is increasing, the wage rates on the negatively sloped portion of the curve minimize rather than maximize profits. In other words, there is no level of productivity for which the wage rates observed in this region are profit maximizing. This fact implies that the observed wage distribution cannot be an equilibrium outcome of the model. Approximately, 13% of all offers made are in the inadmissible region and 6% of employed workers earn such a wage.

Over the remaining range, the wage increases rapidly with productivity initially but then the rate of increase falls dramatically. As a consequence, the implied ratios of productivity to wage are implausibly large over much of the wage support. For example, at the median wage earned, equal to 144 DKK per hour,

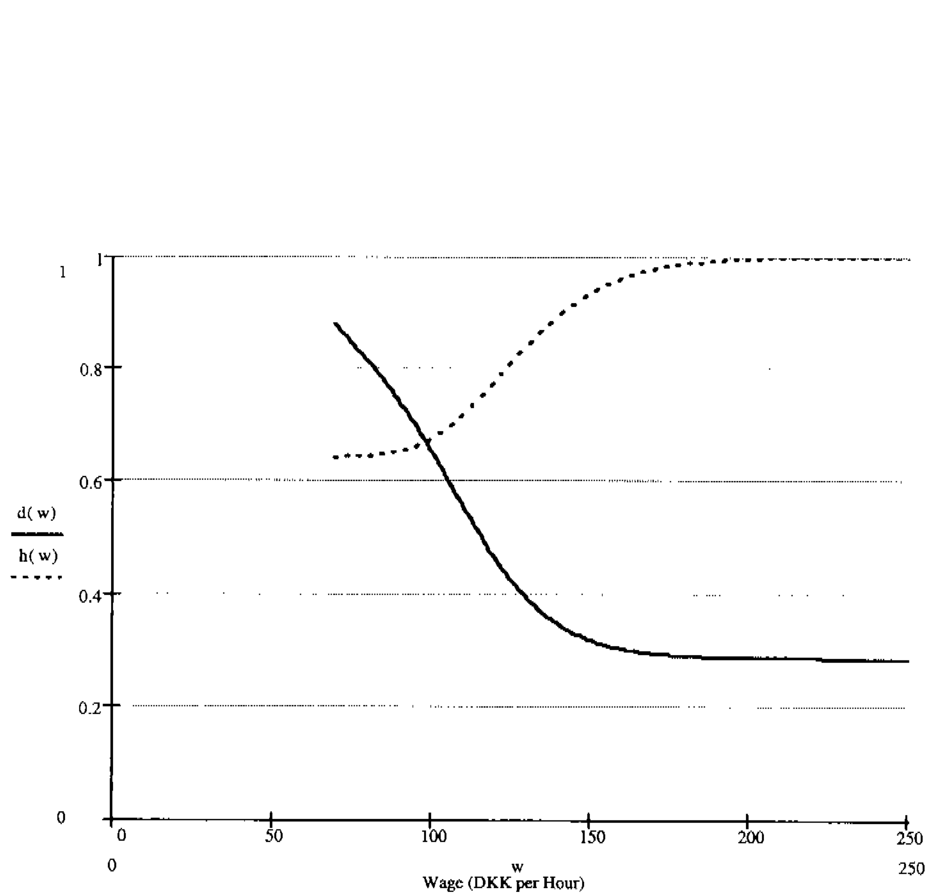

Figure 5: The Separation Rate $d(w)$ and Acceptance Probability $h(w)$ Functions

monopsony power as reflected in the ratio of rent to the wage, $(p - w(p))/w(p)$ is equal to 60%. At the 90th percentile, $w = 186$ DKK per hour, the implied value of p is 2,720 DKK per hour which implies that marginal productivity is almost 6.5 times the wage while at the 95th percentile, $w = 200$ DKK per hour, the ratio is equal to 12. These inferences are hardly plausible even if the wage distribution were admissible.

The reason inferred monopsony rents are large and increase so dramatically with productivity is reflected in the first order condition for an optimal wage choice, equation (20), and the shapes of the acceptance probability, $h(w)$, and separation rate, $d(w)$, illustrated in Figure 5. The first order condition for an optimal choice, implies that the monopsony rent measured at any level of productivity is approximately equal to the inverse of the sum of the elasticities of the separation rate function and the acceptance probability function. Both functions are relatively elastic at low wages but converge to constants as the wage tends to the upper support. In other words, a small wage differential in the lower range of the support has a substantial impact on an employer's ability to both attract and retain workers but the same differential in the upper reaches has little effect on either. Clearly, the reason for the differences in the response of the separation rate function at different wage levels is that well paid workers have less incentive to invest in search than do low paid workers

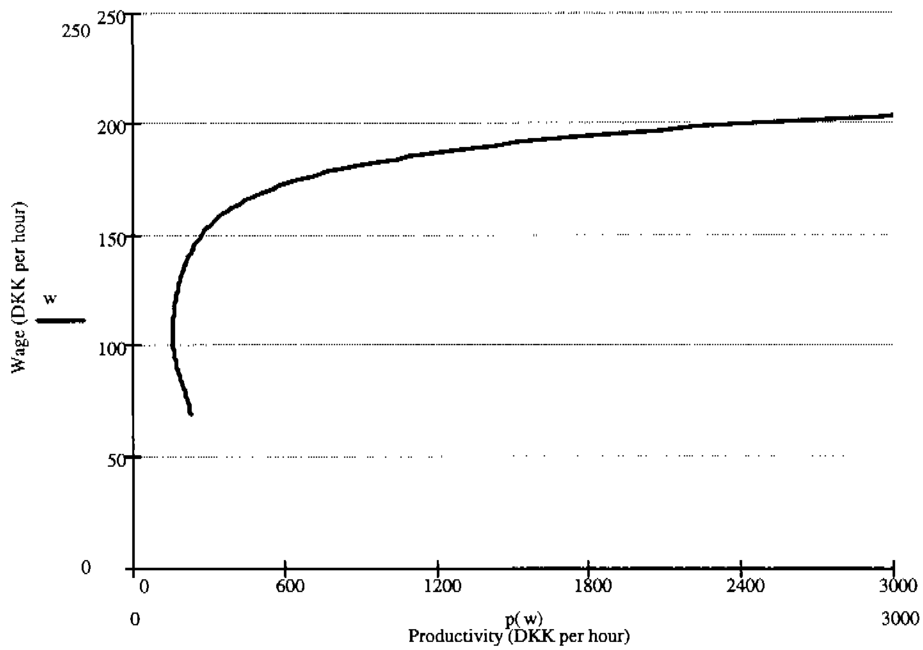

Figure 6: Inferred Monopsony Wage Policy Function $w(p)$

and workers employed by firms paying wages near the upper support have no incentive at all. For the same reason, an employer is more likely to contact a lower than a higher paid worker given the assumption that contact rates reflect search intensity. This fact plus the concentration of employment in jobs paying intermediate wages combine to explain the observed variation in the elasticity of the acceptance probability.

5 Is the Danish Labor Market Monopolistic?

Although monopsony is inconsistent with the IDA data, other forms of decentralized wage setting mechanisms are not ruled out. We know that the Danish labor market is highly unionized but that central negotiations only determine minimums as already noted. Hence, the extent to which a firm's wage may be the outcome of a bargain struck between the firm and its local union taking the outcomes of all similar bargains made both other firms and local unions as given. In this section, we consider the extreme case in which the union sets the wage and the employer determines the flow of workers hired in response, a dynamic version of the so-call union monopoly model.

5.1 Monopoly Wage and Recruiting Policy

In the standard static model of the monopoly union, the union trades lower employment for a higher wage. The simplest version of the model supposes that each firm choose a profit maximizing labor force size given the union wage and that the union with full knowledge of the firm's demand schedule sets the wage to maximize the wage bill net of the worker's earning were he paid the reservation wage. The equilibrium wage-employment combination is the point on the firm's demand curve characterized by a tangency of the curve with the highest attainable union indifference curve. Here we propose the natural dynamic generalization of this model.

Formally, we assume that the union chooses a wage to maximize the surplus value of employment to the flow of workers hired by the firm. Specifically, the wage policy function defined by

$$w(p) = \arg\max_{w \geq b} \{h(w)v(p,w)[W(w) - U]\} \tag{22}$$

where $W(w)$ and U represents the values of employment at wage w and unemployment respectively as defined by equations (1) and (2). In other words, the monopoly union wage maximizes the product of the expected hire flow, equal to hv, and the difference between the expected future income of a worker employed at wage w and the same worker's expected future income if unemployed.

Some may argue that the union criterion function embodied in (22) slights the power that 'insiders' have to set the prevailing wage policy at the firm level. Because the surplus value of employment to an insider, $W(w) - U$, is strictly increasing in the wage earned, a union dominated by insiders would choose the largest wage consistent with continued employment, which is the firm's marginal product of labor p. But, if $w(p) = p$ were the wage policy, the firm has no incentive to hire at all. That outcome would seem less than acceptable to any local union leader with future aspirations. Of course, intermediate ground exists on which the insider preferences are weighted more heavily than those of new hires in the union criterion function. But, their interests are already represented in the sense that applicant become 'insiders' once hired. Consequently, there is no qualitative difference associated with such a generalization although admittedly it would imply a smaller difference between productivity and wage than that derived below.

Given the common wage chosen by the union, the firm selects the frequency with which it contacts potential employees to maximize the expected present value to the employer of the flow of hires as in the monopsony model. In other words,

$$v(p,w) = \max_{v \geq 0} \{\pi(p,w)v - c_f(v)\}. \tag{23}$$

where $\pi(p,w)$ is expected future profit per worker contacted as defined by equation (15). One can easily show that the optimal choice of recruiting effort increases with p given w but decreases with w. Hence, the first and second order conditions for an interior solution to the optimal wage choice problem as specified in equation (22) requires that the wage paid increases with firm productivity provided of course that workers prefer employment to unemployment, $W(w) \geq U$, a condition that must hold in any equilibrium. In other words, more productive employers also pay more when the wage is set by a monopoly union.

Given (23), the first order condition for an optimal monopoly union wage choice as defined in (22) can be written as

$$\frac{c_f''(v)v}{c_f'(v)}\left(\frac{W'(w)}{W(w)-U}+\frac{h'(w)}{h(w)}\right)+\frac{\pi_w(w,p)}{\pi(w,p)}=0 \tag{24}$$

Obviously, the monopoly and monopsony solutions are related in the sense that the elasticities of both the acceptance and separation functions are determinants of the wage selected as well as the elasticity of the acceptance probability and the surplus value of being employed because the union takes account of the fact that employer profit per worker contacted determines the size of the hire flow. Indeed, if the recruiting cost function were linear, i.e., $c_f''(v) \equiv 0$, the monopsony and monopoly wage choices are the same. In the case of a strictly convex cost, a condition required for an interior choice of recruiting effort, the fact that both the net value of employment and the acceptance probability increase with the wage implies that both profit per contact and, consequently, the optimal contact frequency decrease with the w at the monopoly wage, i.e., $\pi_w(w(p),p) < 0$ and $v_w(w(p),p) < 0$. Since $\pi_w(w,p) = 0$ at the monopsony wage, the monopoly union wage generally exceeds the monopsony wage.

5.2 Empirical Inference

Rewrite the first order condition for optimal recruiting effort, equation (23), as

$$c_f'\left(V(w)\right)=\pi(p(w),w) \tag{25}$$

where $p(w)$ represents the inverse of the monopoly union wage policy function and $V(w) = v(p(w), w)$ is the associated optimal contact rate of a firm paying wage w. By differentiating this expression with respect to w, it follows that

$$\frac{c_f''(v)v}{c_f'(v)}\left(\frac{V'(w)}{V(w)}\right)=\frac{\pi_w(p,w)+\pi_p(p,w)p'(w)}{\pi(p,w)}.$$

After using the first order condition for an optimal wage choice, equation (24), and the definition of $\pi(p,w)$, equation (15), to substitute out the elasticity of the recruiting cost function, one obtains the following ordinary differential equation representation of the inverse of the wage function $w(p)$:

$$\begin{aligned} p'(w) &= \left(1+\frac{\frac{wV'(w)}{V(w)}}{\frac{wh'(w)}{h(w)}+\frac{wW'(w)}{W(w)-U}}\right) \\ &\quad \times\left[1+\left(\frac{wh'(w)}{h(w)}-\frac{wd'(w)}{r+d(w)}\right)\left(\frac{w-p(w)}{w}\right)\right]. \end{aligned} \tag{26}$$

To compute the solution to this ODE for the desired inverse policy function, $p(w)$, implied by the Christensen et al. estimates and the observed offer distribution, one needs the empirical counter parts of surplus value of employment, $W(w)-U$, and recruiting effort, $V(w)$, in addition to the acceptance probability, $h(w)$, and the separation rate, $d(w)$, for all wages in the support of the wage distribution. The surplus value function can be inferred from the search cost function specified and parameters estimated by Christensen et al. To obtain an empirical relationship between recruiting effort and the wage, $V(w)$, requires more information. Below I show that this function as well as the size-wage relationship, denoted by $N(w)$, can be inferred from the observed distributions of offers, wages, and firm size under the identifying assumption that firm size increases with the firm productivity.

Equations (1)-(4) imply that the surplus value of employment can be written as

$$\begin{aligned} S(w) &= W(w) - U \qquad (27) \\ &= \frac{(w + s(w)c_w'(s(w)) - c_w(s(w)) - [R + s(R)c_w'(s(R)) - c_w(s(R))])}{r+\delta}. \end{aligned}$$

The one required parameter not provided by the Christensen et al. estimates is the reservation wage R. Under the assumption that the marginal cost of recruiting is zero at the origin and that there are no fixed cost, i.e., $c_f'(0) = c_f(0) = 0$, an employer participates by recruiting workers if and only if its productivity is greater than the reservation wage. Hence, free entry would require that both the lowest wage paid by and the lowest productivity of the marginal participating firm equal the reservation wage, i.e.,

$$\underline{w} = \underline{p} = R. \qquad (28)$$

This condition is imposed as the boundary condition need to compute the particular solution of interest to (26).

Expected firm size at each wage paid is related to recruiting effort by

$$n(p(w)) = N(w) = \frac{h(w)V(w)}{d(w)} \qquad (29)$$

from equation (19). Hence, one can compute $V(w)$ with using the functions $h(w)$ and $d(w)$ is the size wage relation $N(w)$ were known. Since the monopoly union wage is a strictly increasing function of productivity, the fraction of workers employed at wage no greater than $w = w(p)$ at any point is time is the fraction of workers who are employed by firms with productivity no greater than p. Formally,

$$G(w(p)) = \frac{\int_{\underline{p}}^{p} n(x)d\Gamma(x)}{\int_{\underline{p}}^{\overline{p}} n(x)d\Gamma(x)} \qquad (30)$$

where $n(p)$ is the average size of a firm with productivity p and $\Gamma(p)$ represents the measure of firms with productivity p or less. Under the identifying assumption that labor force size increases with firm productivity,[7] the fraction of all firms of size less than or equal to $n = n(p)$, denoted $Q(n)$, is equal to the fraction with productivity no greater than p, i.e.,

$$Q(n(p)) = \frac{\int_{\underline{p}}^{p} d\Gamma(x)}{\int_{\underline{p}}^{\overline{p}} d\Gamma(x)}. \tag{31}$$

Given the differential form of these two equation,

$$G'(w(p))w'(p) = \frac{n(p)\Gamma'(p)}{\int_{\underline{p}}^{\overline{p}} n(x)d\Gamma(x)}$$

and

$$Q'(n(p))n'(p) = \frac{\Gamma'(p)}{\int_{\underline{p}}^{\overline{p}} d\Gamma(x)},$$

it follow that

$$G'(w(p))w'(p)En = n(p)Q'(n(p))n'(p) \tag{32}$$

where

$$En = \frac{\int_{\underline{p}}^{\overline{p}} n(x)d\Gamma(x)}{\int_{\underline{p}}^{p} d\Gamma(x)} = \int_{\underline{n}}^{\overline{n}} ndQ(n) \tag{33}$$

is the average size of a firm in the population. Finally, because $p(w)$ is the inverse of $w(p)$ and $N(w) = n(p(w))$ imply $w'(p) = 1/p'(w)$ and $N'(w) = n'(p(w))p'(w)$ respectively, equation (32) can be rewritten as the ordinary differential equation

$$N'(w) = \frac{G'(w)En}{Q'(N(w))N(w)}. \tag{34}$$

Hence, the particular solution determined by the boundary condition

$$N(\underline{w}) = \underline{n} \tag{35}$$

where $\underline{n}$ is the lower support of the size distribution provides an estimate of $N(w)$.

A continuous approximation to the firm size density actually observed in the IDA is illustrated in Figure 7. Note that almost all firms are quite small. Indeed, 26.5% employ a single wage earner and 62% have 4 or fewer employees. Only, 9% of the firms are composed of more than 20 employees and only 3% have more than 50 workers. Still, there are a few immense firms. The obvious right skew in the distribution is reflected in the fact that the median size lies between 2 and 3 employees while the average is 13.2.

[7] Although not necessarily implied by the monopoly union model, the assumption is consistent with the findings of Albaek and Madsen (1996) that wages and firm size are positively associated in the IDA data.

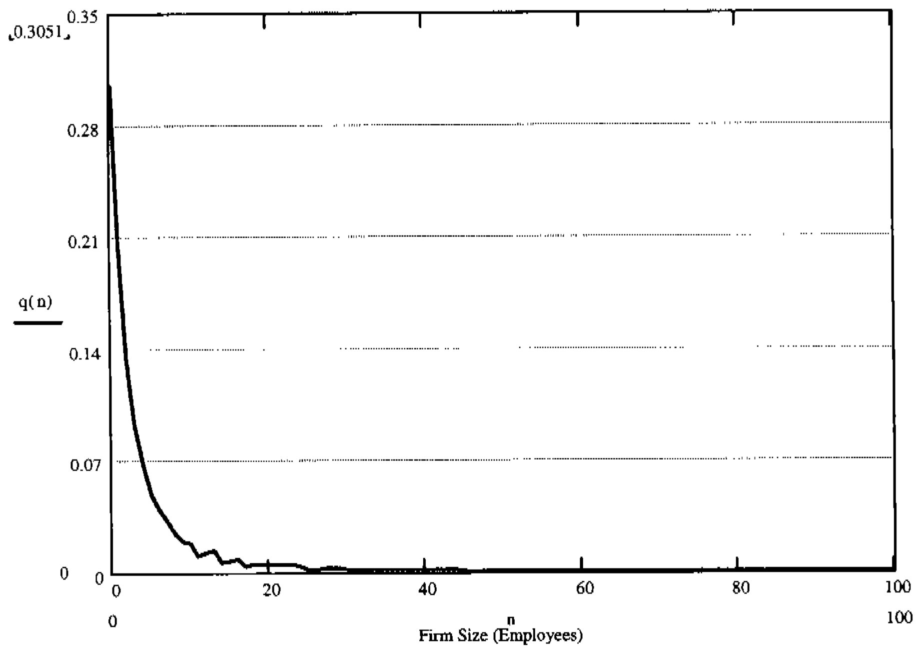

Figure 7: IDA Firm Size Probabiliy Density Function

Finally, the cross firm productivity density that supports the observed wage distribution can be recovered with knowledge of $p(w)$ and $N(w)$. Namely, from equation (??) and $w'(p(w)) = 1/p'(w)$, the cross firm productivity density is

$$\gamma(p(w)) = \frac{\Gamma'(p(w))}{\int_{\underline{p}}^{\overline{p}} d\Gamma(x)} = \frac{G'(w)En}{p'(w)N(w)}. \tag{36}$$

5.3 Results

The surplus value of employment, $S(w) = W(w) - U$, expressed as a function of the hourly wage earned implied by the Christensen et al. estimates of the separation and cost function parameters and the observed wage offer distribution is illustrated in Figure 8. As it should be, the function is increasing and convex in the wage earned.

As the wage and size densities are observed in the Danish IDA, the size-wage function $N(w)$ can be computed as the appropriate solution to the ODE (34). Given this solution, the Christensen et al. parameter estimates, and the observed wage and offer densities, one can compute the relationship between annual contact frequency and the wage, $V(w)$, using equation (29). The results of these calculation, are plotted over the relevant range of wage rates in Figure

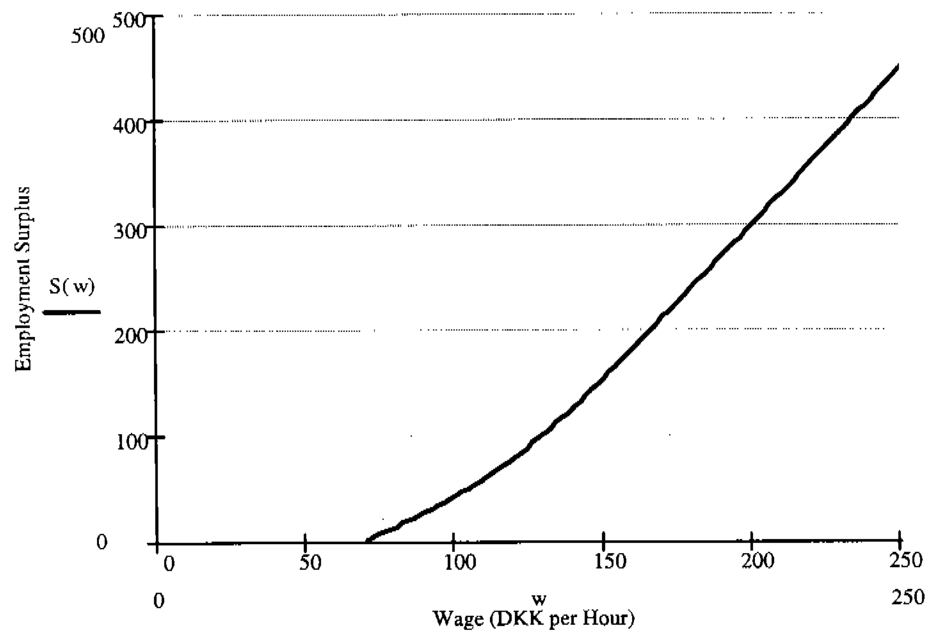

Figure 8: Surplus Value of Employment Function $S(w) = W(w) - U$

9. Both relationships have a logistic shape. Initially, firm size and contact frequency increase at an increasing rate with the wage paid but then both reverse curvature and flatten out at high wage levels.

The monopoly union wage policy solution to equation (26) implied by the derived empirical functions $h(w)$, $d(w), S(w)$, and $V(w)$ is illustrated in Figure 10 as the solid curve. The 45 degree ray is included for reference as the dotted line. Obviously, the function is very different from that illustrated in Figure 6 for the hypothetical monopsony wage case. First, it is admissible for all observed wage rates, i.e., a level of firm productivity exists such that every observed wage is an optimal choice of the monopoly union. Second, the differences between productivity and the wage are much more plausible at all wage rates. Specifically, the difference, $p - w(p)$, expressed as a percentage of the wage, w, is 15% at the 10th percentile of the wage distribution, increases to 30% at the median and equals 28% at the 90th percentile. In sum, I find the inferred wage policy both consistent with the monopoly union wage hypothesis and plausible.

The marginal cost per worker contacted by a firm paying wage w can be computed using the first order condition for an optimal choice of the contact frequency.

$$MC(w) \equiv c_f'(V(w)) = \frac{h(w)\,(p(w) - w)}{r + d(w)}. \tag{37}$$

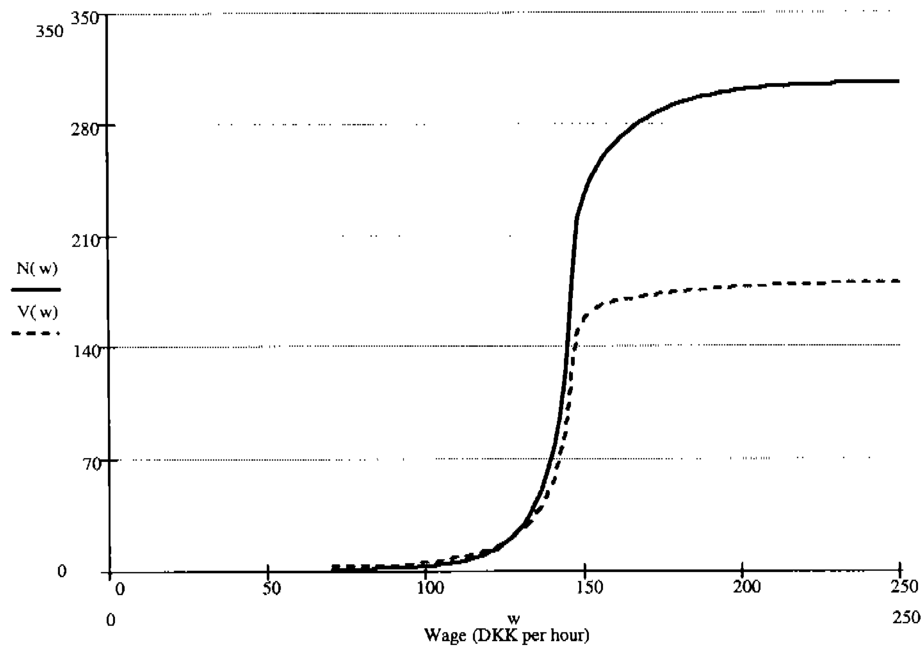

Figure 9: Size $N(w)$ and Contact Frequency $V(w)$ Functions

The computed values of this function can be plotted against the corresponding values of the inferred contact frequency $V(w)$ to determine whether the cost of recruiting increases at the margin as required by the theory. Although the number of hours worked per annum is needed to compute the right side of (37) on an annual basis, this parameter is simply a scale factor which does not affect the shape of the inferred marginal cost curve. Hence, a plot of the marginal cost per hour worked against the annual contact rate is equally informative. This plot is illustrated in Figure 11.

The inferred annual contact rate $V(w)$, illustrated in Figure 9, increases from 9 workers per year at the 10th percentile of the distribution of wages paid to 176 workers per year at the 90th. The inferred contract frequency of the employer paying the media wage, 144 DKK per hour, is 87 workers per year. Over the same range of wages paid, the marginal cost of recruiting expressed as a percentage of the annual wage bill per worker increases from 19.5% to 83% although at the median wage, the inferred marginal cost of recruiting is equal to 76% of the annual wage. Because the acceptance probability, $h(w)$, is 0.94 at the median wage and the average hiring cost per worker expressed as a fraction of the worker's annual earnings is about half the marginal cost, the implied average cost of hiring a worker at the median wage is 40% of the worker's annual wage bill. Although there is a dearth of direct evidence on

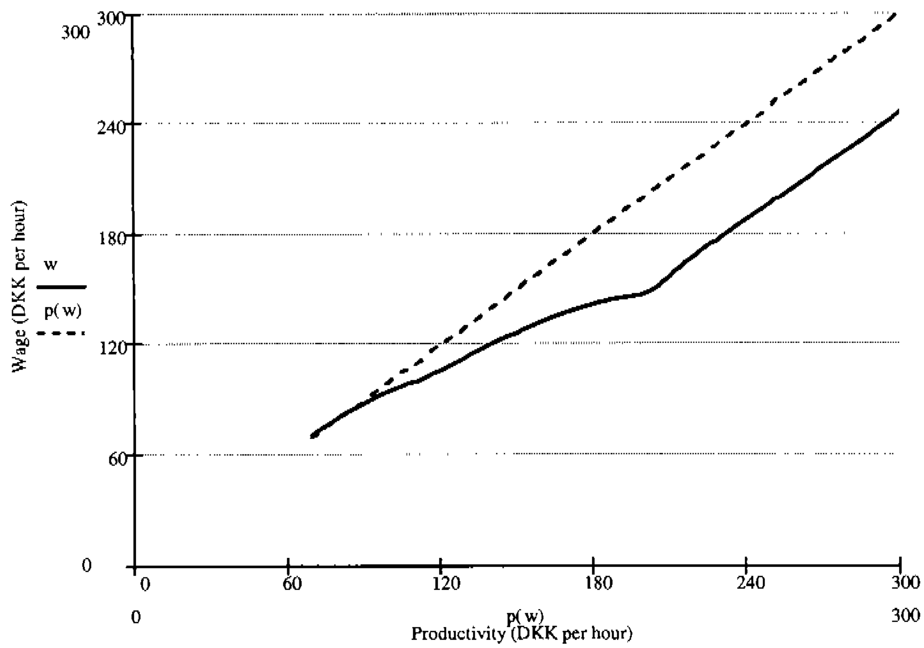

Figure 10: Monopoly Union Wage Function $w(p)$

recruiting costs, this indirect estimate is well within the range of survey results reported by Hamermesh (1993, p. 208). As Hamermesh notes, the size of the hiring investment can be considerable.

Finally, the curve of the productivity density function implied by the size density, the size-wage relationship, and the wage policy function expressed in equation (36) is plotted in Figure ??. The resulting probability density is unimodal but with a rather long right tail. The firms with productivity less than the mode employ only 10% of all workers in the private sector. The productivity of the firm paying the median wage, 144 DKK per hour, is 188 DKK per hour. Obviously, this firm is in the right tail of the distribution of productivity. Indeed, given that the inferred labor force size is 122 workers, the observed size distribution implies that only 1.1% of all firms are larger. In sum, the data and model imply that almost half of all Danish worker in private sector are employed by the largest and most productive 1% of the firms.

6 Summary

The following definitions of labor market equilibrium are implicit in the paper.

Definition 1: A *monopsony labor market equilibrium* is an aggregate contact

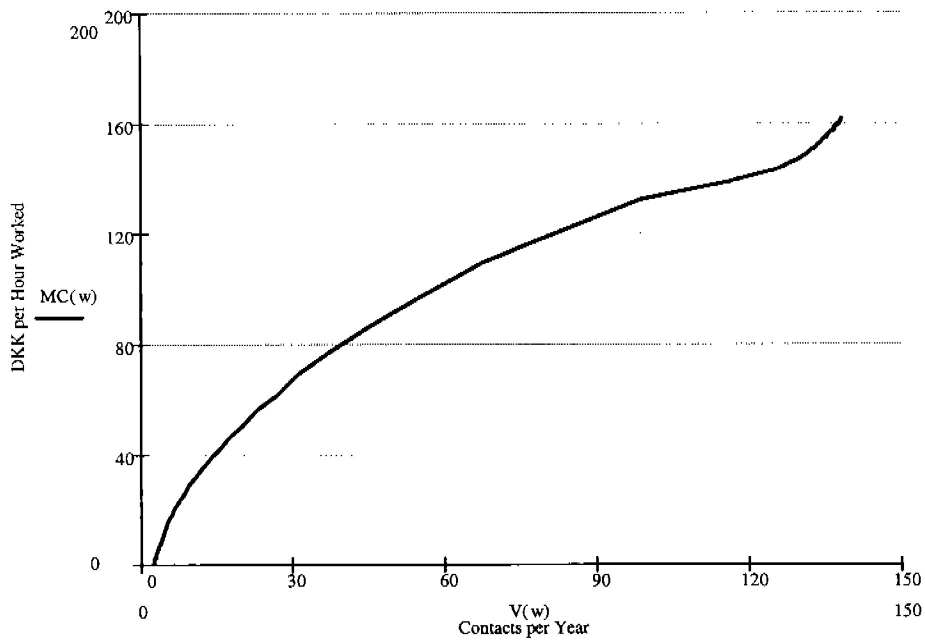

Figure 11: Marginal Recruiting Cost Curve $MC = c_f'(v)$

rate per worker λ, a wage offer distribution $F : [\underline{w}, \overline{w}] \rightarrow [0,1]$, and a distribution of wages earned $G : [\underline{w}, \overline{w}] \rightarrow [0,1]$ that are the solutions to equations (35), (??) and (21) respectively given that the typical worker's search effort strategy $s(w)$ and every employer's recruiting effort $v(p,w)$ are optimal in the sense that they respectively satisfy equations (4) and (18) given the wage policy $w(p)$ defined by (17).

Definition 2: A *monopoly union labor market equilibrium* is an aggregate contact rate per worker λ, a wage offer distribution $F : [\underline{w}, \overline{w}] \rightarrow [0,1]$, and a distribution of wages earned $G : [\underline{w}, \overline{w}] \rightarrow [0,1]$ that are the solutions to equations (35), (??) and (21) respectively given that the typical worker's search effort strategy $s(w)$ and every employer's recruiting effort $v(p,w)$ are optimal in the sense that they respectively satisfy equations (4) and (23) given the monopoly union wage policy $w(p)$ defined by (22).

In other words, an equilibrium is a map from the space of search effort, recruiting effort and wage policy functions induced by the optimality conditions and the assumption of rational expectations about the prevailing wage and offer distributions to itself. In each case, an equilibrium non-cooperative solution to the market game implicit in the specification detailed above.

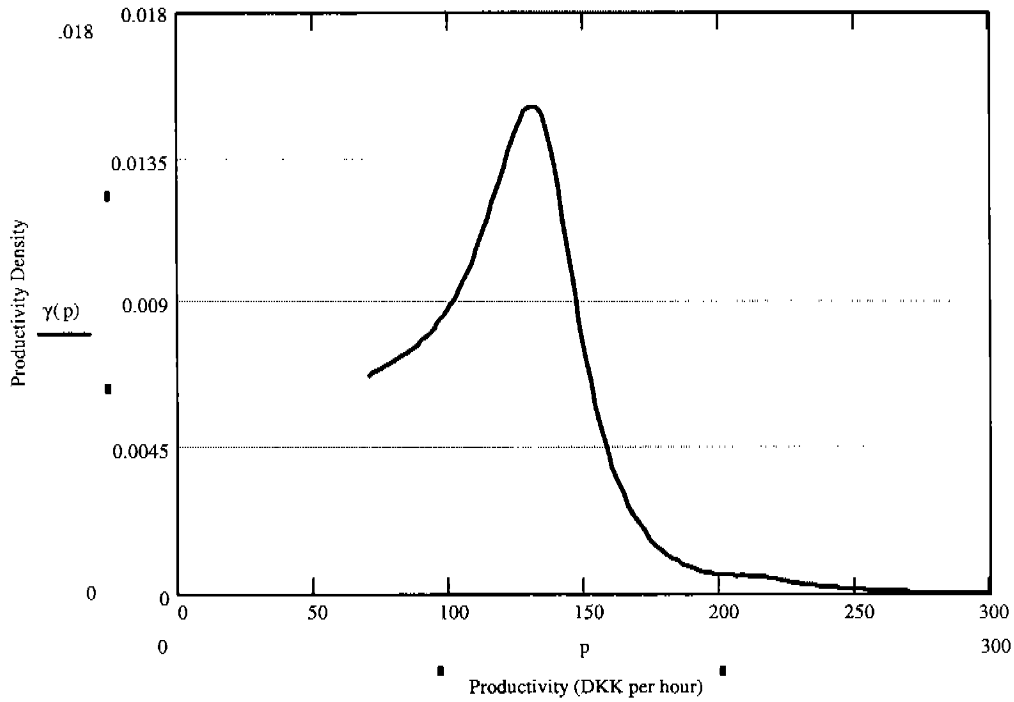

Figure 12: Inferred Productivity Density, $\gamma(p)$

Rather than provide conditions for general existence of either of these solution concepts, the approach in the paper is inductive. I demonstrate that the observed Danish wage distribution data and the job separations model estimated by Christensen et al. (2001) are inconsistent with the monopsony hypothesis in the sense that some of the observed hourly wage rates actually paid can not maximize the firms expected profit, the value of the flow of workers hired by any employer. However, the distribution of wages paid and firm size observed in the Danish IDA can be interpreted as solution outcomes of a monopoly union labor market equilibrium in the sense that the wage of each employer can be viewed as the choice that maximizes the net value of employment to the flow of workers hired and the employer's value of hiring them respectively conditional on the firms' productivity. Furthermore, the complete structure underlying the equilibrium, which includes the search effort function $c_w(s)$, the recruiting cost function $c_f(v)$, and the productivity density $\gamma(p)$, are all identified by the job separation flows and the wage and size distributions observed under the assumption that more productive firms are larger on average.

The analysis in the paper builds on the work of Christensen et al. (2001) who show that cross firm separation behavior observed in the IDA supports the proposition that employed workers search at intensities that reflect the expected gain in future income attributable to search. Furthermore, the estimated separation function and the observed wage offer distribution imply a steady state distribution of wages earned which is virtually identical to that observed in the data. Given their estimates of the turnover parameters and search cost function, I show that the monopoly union wage policy function implicit in the wage distribution data is strictly increasing in labor productivity as the theory requires. I also show that the wage and size distribution data are consistent with the requirement that the marginal cost of recruiting effort increases with the frequency with which workers are contacted by an employer. Indeed, the inferred marginal cost function is approximately linear. Furthermore, the recruiting cost magnitudes are significant; the average cost per worker hired by the firm paying the median wage is approximately equal to 40% of the workers annual earnings. Although the model and the data imply that productivity across employers is unimodal, it is also highly right skewed with the most productive 1% employing almost half of the workers in the Danish private sector.

To conclude, the analysis suggests that the Danish labor market is monopolistic rather than monopsonistic. However, because the union must provide an incentive for employers to hire workers, many of the qualitative implications of the model are the same as those of the original Burdett-Mortensen monopsony model. For example, more productive firm's pay a higher wage and invest more in recruiting effort. Wage dispersion reflects differences in employer productivity and workers move from low to high productivity firms as a consequence. These facts suggests that the Danish labor market, though imperfect in several senses, may allocate workers among firms reasonably efficiently.

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