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Individual Euler Equations Rather Than Household Euler Equations

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Preliminary

Abstract

Empirical evidence indicates that household Euler equations should be replaced by individual Euler equations, i.e. a set of equations for each household member. Individual Euler equations are defined in terms of individual consumption and labor supply. Unfortunately only total household consumption is observable. In this paper, I show that individual Euler equations are completely identified observing individual labor supplies and wages, i.e. with the limited information available in the CEX. Moreover, I estimate them by means of the identification procedure and GMM.

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1 Introduction

Most of the literature on household intertemporal optimization assigns a unique utility function to the entire household, derives household Euler equations and, with them, estimates intertemporal elasticities of substitution. In Mazzocco (2001) I find that household Euler equations should be replaced by individual Euler equations, i.e. a set of equations for each household member. Individual Euler equations are defined in terms of individual consumption and labor supply. Unfortunately only total household consumption is observable. This paper focuses on the identification and estimation of individual Euler equations when only total household consumption, individual labor supplies, wages and interest rates are observed. In particular, using the suggested approach it is possible to identify and estimate intertemporal elasticities of substitution for the wife and separately for the husband.

To give quantitative answers to several policy questions, it is crucial to determine which parameters govern household intertemporal behavior and have their estimates. In the past 20 years, a considerable amount of time has been devoted to the identification of two key parameters: the intertemporal elasticities of substitution. As discussed in Browning, Hansen and Heckman (1999), the estimation of those two parameters are almost exclusively based on household Euler equations. Browning and Lusardi (1996) survey the literature on intertemporal optimization and find that the majority of studies reject the household Euler equations. A remarkable exception is the paper by Attanasio and Weber (1995). In their insightful paper, they show that the intertemporal model of optimizing behavior for consumption is consistent with US micro data if household preferences are modeled so as to take into consideration changes in family composition and labor supply behavior over the life cycle. In Mazzocco (2001), I propose a structural explanation of their findings and more generally of the previous rejections of the theory. If households are characterized by several individual members with different utility functions, household preferences in general depend on the relative power of each spouse and on all the variables having an effect on it, namely labor supply and family composition. Consequently, standard Euler equations should be rejected for households with several members. In the same paper, I test the empirical relevance of this explanation, estimating Euler equations separately for singles and couples after controlling for self selection. I find that standard Euler equations are rejected for couples but not for singles.

Even if household Euler equations are affected by the suggested aggregation problem, individual Euler equations should be satisfied. Those equations depend on individual consumption as well as labor supply. The CEX, probably the most exhaustive micro dataset on consumption, collects information on household consumption, but not data on individual consumption.[1] The goal of this paper is to identify and estimate individual Euler equations with the information available in the CEX. In particular, I focus on the following two issues.

a) Suppose that husband and wife are characterized by individual preferences and cooperate. Assume that preferences are intertemporally separable. Finally, suppose that only total

[1] Clothing is the only exception.

consumption, individual labor supplies, wages and interest rates are observed. I show that individual Euler equations are identified, given the limited amount of information. Specifically, if both agents work, I find that Euler equations of husband and wife can be identified up to a constant. Consequently, the intertemporal elasticities of substitution for consumption and leisure can be determined for the wife and separately for the husband. If only one agent works, Euler equations for the spouse supplying labor can be identified up to a constant. For the spouse not working, only the consumption Euler equation can be identified.

b) The CEX contains all the data required to implement the identification procedure outlined in this work. The second part of the paper focuses on the estimation of individual Euler equations. In particular, I use the panel structure of the CEX survey and a flexible functional form for individual preferences, to estimate individual Euler equations of wife and husband and, more specifically, their intertemporal elasticities of substitution. Given the nonlinearity of individual Euler equations and endogeneity problems, the Generalized Method of Moments represents the most appropriate estimation procedure.

Some assumptions are implicit in the current identification results. In the theoretical section, I do not consider public goods. However, children and houses are likely to have considerable impact on consumption and labor supply dynamics. In the empirical section I control for the number of children in the household. Second, individual preferences depend on leisure and on a composite good representing total individual consumption. Consumption goods with heterogeneous characteristics are likely to have different elasticities of substitution. Therefore it is important to generalize the current approach to allow individual utility functions to depend on different types of goods.

Euler equations have been estimated for the past 20 years, as reported in the survey by Browning and Lusardi (1996). The identification and estimation approach that I propose is new, as I consider Euler equations for each household member and not for the entire household. I employ an intertemporal framework in which each spouse is represented by individual preferences, therefore generalizing the static collective model developed by Chiappori (1988, 1992). Chiappori (1988, 1992) shows that in a static framework individual preferences can be identified under some separability restrictions. Blundell, Chiappori, Magnac and Meghir (2001) extend Chiappori's results to allow for households in which only one spouse works. While this project is concerned with the identification of preferences, the focus is on household intertemporal optimization. Specifically, the goal is to identify and estimate individual Euler equations and, as a byproduct, the intertemporal elasticities of substitution for each household member.

The paper is organized as follows. In section 2, I derive individual Euler equations. Section 3 discusses how to bypass the lack of information on individual consumption using m-demand functions. Section 4 outlines the identification procedure for households with both agents working. Section 5 discusses identification for households with only one spouse working. Section 6 considers the assumptions implicit in the identification procedure. Section 7 outlines the empirical implementation. Section 8 concludes the paper.

2 Household and Individual Euler Equations

Consider a household composed by 2 members living for T periods. In each period $t \in \{0,...,T\}$ and state of the world $\omega \in \Omega$, member i consumes a private consumption good in quantity $c^i(t,\omega)$ and supplies labor in quantity $h^i(t,\omega)$. Member i is endowed with an exogenous stochastic income stream $\{y^i(t,\omega)\}_{t\in\mathbf{T},\omega\in\Omega}$. For any given (t,ω), the household can either consume or save. Two assets are traded in the economy, a risky and a risk-free asset. $s(t,\omega)$, $b(t,\omega)$, $R(t)$ and $\tilde{R}(t,\omega)$ denote respectively the amount of wealth invested in the risky and risk-free asset at (t,ω), the gross return on the risk-free asset and the gross return on the risky asset. Let $Y(t,\omega)=\sum_{i=1}^{2} y^i(t,\omega)$ and $C(t,\omega)=\sum_{i=1}^{2} c^i(t,\omega)$. Total time available to each household member is denoted with T.

Most of the literature on household intertemporal optimization assumes that the household can be represented by a unique utility function independently of the number of individual members. Moreover, preferences are intertemporally separable and the one-period utility function $U(.)$ depends on total household consumption C and individual leisure $l^1=T-h^1, l^2=T-h^2$. According to the standard framework the optimal household allocation of resources over time and across households is obtained solving the following problem,

$$\max_{\left\{C_t,b_t,s_t,h_t^i\right\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,2}} E_0\left[\sum_{t=0}^{T}\beta^t U\left(C_t,T-h_t^1,T-h_t^2\right)\right] \tag{1}$$

$$s.t.\ C_t+b_t+s_t\leq\sum_{i=1}^{2}\left(y_t^i+w_t^ih_t^i\right)+R_tb_{t-1}+\tilde{R}_ts_{t-1}\ \ \forall(t,\omega)$$

$$b_T+s_T\geq 0\ \ \forall\omega.$$

The main testable implications of the standard intertemporal framework (1) are the household Euler equations,[2]

$$U_C\left(C_t,T-h_t^1,T-h_t^2\right) = \beta E_t\left[U_C\left(C_{t+1},T-h_{t+1}^1,T-h_{t+1}^2\right)R_{t+1}\right], \tag{2}$$

$$U_{l^i}\left(C_t,T-h_t^1,T-h_t^2\right) = \beta E_t\left[U_{l^i}\left(C_{t+1},T-h_{t+1}^1,T-h_{t+1}^2\right)\frac{R_{t+1}w_t^i}{w_{t+1}^i}\right]\ \ i=1,2, \tag{3}$$

$$U_{l^i}\left(C_t,T-h_t^1,T-h_t^2\right) = \beta E_t\left[U_C\left(C_{t+1},T-h_{t+1}^1,T-h_{t+1}^2\right)\frac{R_{t+1}}{w_{t+1}^i}\right]\ \ i=1,2, \tag{4}$$

where U_C and U_{l^i} are the marginal utilities of household consumption and member i's leisure. The majority of papers testing consumption intertemporal optimization by means of Euler equations reject the predictions of the standard model.[3] A remarkable exception is the finding of Attanasio and Weber (1995). They show that household intertemporal maximization is consistent

[2] In the theoretical part of the paper I only discuss Euler equations corresponding to the risk-free asset. A similar argument applies to Euler equations corresponding to the risky asset.

[3] For a survey of the literature on Euler equations see Browing and Lusardi (1996).

with US micro data if household preferences are modeled so as to depend on changes in family composition and labor supply behavior over the life cycle.

A possible explanation of Attanasio and Weber's findings and more generally of the previous rejections of the theory is that the assumption of a unique utility function for the household is excessively restrictive. Consider an intertemporal framework in which the household consists of 2 members, each being represented by individual preferences. Assume that the household decision process is characterized by efficiency.[4] Suppose that preferences are intertemporally separable and that member i's one-period utility function depends on individual consumption c^i and individual leisure $T - h^i$. Moreover, suppose that individual utility functions are twice continuously differentiable, strictly increasing and concave. Then, the household allocates resources over time and across members according to the following problem,

$$\max_{\left\{c_t^i,b_t,s_t,h_t^i\right\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,2}} \mu^1(\Theta)\,E_0\left[\sum_{t=0}^{T}\beta_1^t u^1(c_t^1,T-h_t^1)\right]+\mu^2(\Theta)\,E_0\left[\sum_{t=0}^{T}\beta_2^t u^2(c_t^2,T-h_t^2)\right] \quad (5)$$

$$\sum_{i=1}^{2} c_t^i + b_t + s_t \leq \sum_{i=1}^{2}\left(y_t^i + w_t^i h_t^i\right) + R_t b_{t-1} + \tilde{R}_t s_{t-1} \;\; \forall\,(t,\omega)$$

$$b_T + s_T \geq 0 \;\; \forall\omega, \in \Omega$$

for some pair of Pareto weights $(\mu^1(\Theta), \mu^2(\Theta))$, where Θ is the set of variables affecting the decision power of individual members.[5]

By means of this model, in Mazzocco (2001), I show that the household can be characterized by a unique utility function if and only if all individual members have identical discount factors and Harmonic Absolute Risk Aversion preferences with identical shape parameter.[6] If these restrictions are not satisfied, household preferences and therefore household Euler equations depend on individual decision power and on all the variables having an effect on it. To see this let h^{i*} be optimal labor supply for agent i. Note that it is always possible to construct the

[4]The idea that household members cooperate is well established in the literature, see for instance Becker (1973, 1974, 1991) and Chiappori (1992). Additionally, the general assumption of efficiency has the advantage of imposing no restriction on which point of the Pareto frontier will be chosen. Mazzocco (2001) and Attanasio and Mazzocco (2001) analyze the effect of limited commitment on household intertemporal behavior.

[5]It is useful to note that with this framework I am able to take into consideration the additive form of altruism,

$$U^i\left(c^1, 1-h^1, c^2, 1-h^2\right) = u^i(c^i, 1-h^i) + \delta_j u^j(c^j, 1-h^j).$$

To see this note that the collective formulation (5) is equivalent to a model with additive altruism in which μ^1, μ^2 are the sum of Pareto weights and altruism parameters. The important issues of public goods and household production are discussed later in the paper.

[6]Two agents have HARA preferences with identical shape parameter if $u^{i\prime}(x) = \left(a_i + \frac{x}{\gamma_i}\right)^{-\gamma_i}$, with $\gamma_1 = \gamma_2$.

representative agent corresponding to the household solving the following problem,

$$v\left(C,\left\{T-h^{i*}\right\}\left\{\mu^{i}\left(\Theta\right)\right\}\right)=\max_{\left\{c^{i},h^{i}\right\}_{i=1,2}}\mu_{1}\left(\Theta\right)u^{1}\left(c^{1},T-h^{1*}\right)+\mu_{2}\left(\Theta\right)u^{2}\left(c^{2},T-h^{2*}\right)$$

$$s.t. \sum_{i=1}^{2} c^{i}=C$$

However, the results in Mazzocco (2001) establish that the household preferences $v\left(.\right)$ always depend on the set of Pareto weights and indirectly on Θ. Therefore, for general preferences, household Euler equations are affected by the relative power of the two spouses and through it on Θ. For instance, the standard consumption Euler equation has the following characterization,

$$v_{C}\left(C_{t},\left\{T-h^{i*}\right\},\left\{\mu^{i}\left(\Theta\right)\right\}\right)=\beta E_{t}\left[v_{C}\left(C_{t+1},\left\{T-h^{i*}\right\},\left\{\mu^{i}\left(\Theta\right)\right\}\right)R_{t+1}\right], \quad (6)$$

where v_c is the partial derivative of v with respect to C. In Mazzocco (2001) I verify the empirical relevance of these results. The test is based on the following argument. If the standard model (1) is a complete characterization of household intertemporal optimization, the household Euler equations (2), (3) and (4) should be satisfied for all families independently of the number of decision-makers in the households. If the collective formulation (5) is correct, household Euler equations should be satisfied for households with one decision-maker, but rejected for households with several decision-makers. Using the PSID and controlling for self selection, I find that Euler equations are strongly rejected for couples, but cannot be rejected for singles.

The main testable implication of the Collective Intertemporal Model (5) is that individual Euler equations should always be satisfied independently of the preferences characterizing each individual member. Specifically, for each agent i,

$$\begin{aligned}
u_{c}^{i}\left(c_{t}^{i},T-h_{t}^{i}\right) &= \beta_{i}E_{t}\left[u_{c}^{i}\left(c_{t+1}^{i},T-h_{t+1}^{i}\right)R_{t+1}\right], & (7)\\
u_{l}^{i}\left(c_{t}^{i},T-h_{t}^{i}\right) &= \beta_{i}E_{t}\left[u_{l}^{i}\left(c_{t+1}^{i},T-h_{t+1}^{i}\right)\frac{R_{t+1}w_{t}^{i}}{w_{t+1}^{i}}\right], & (8)\\
u_{l}^{i}\left(c_{t}^{i},T-h_{t}^{i}\right) &= \beta_{i}E_{t}\left[u_{c}^{i}\left(c_{t+1}^{i},T-h_{t+1}^{i}\right)\frac{R_{t+1}}{w_{t+1}^{i}}\right]. & (9)
\end{aligned}$$

Consequently, if individual consumption and individual labor supply were observed, it would be possible to estimate the Euler equation coefficients using (7), (8) and (9). Moreover, it would be possible to construct tests of intertemporal optimization based on individual Euler equations. Unfortunately, individual consumption is not observed.

The remaining sections of the paper discuss the identification and estimation of individual Euler equations if total consumption, individual labor supplies, wages and interest rates are observable, but individual consumption is not.

3 Individual Euler Equations and M-consumption Functions

Consider a household characterized by an arbitrary pair of individual utility functions u^1, u^2. This paper has two goals. First, I propose to develop an approach to identify individual Euler equations given u^1, u^2. Second I plan to implement the identification procedure, estimating individual Euler equations by means of the CEX and the Generalized Method of Moments. To that end, it is crucial to answer the following question. Which variables do we observe? The most exhaustive micro datasets contain at best information on total household consumption, individual labor supplies and wages. However, with the exception of clothing in the CEX, no survey contains data on individual consumption.[7]

Individual consumption is not observed. Given that household members are expected utility maximizers, however, the marginal rate of substitution between labor and consumption must be related to the wage rate divided by the price of consumption.[8] Specifically, assuming an interior solution for c_t^i and h_t^i, the first order conditions of the Intertemporal Collective Model (5) imply that for each agent i the relation between c_t^i, h_t^i and w_t^i is given by the following equation,[9]

$$\frac{u_l^i\left(c_t^i, T-h_t^i\right)}{u_c^i\left(c_t^i, T-h_t^i\right)} = q^i\left(c_t^i, h_t^i\right) = w_t^i. \tag{10}$$

If the function $q^i(c,h)$ is invertible, it is possible to determine individual consumption as a function of individual labor supply and wage rate,

$$c_t^i = g^i\left(w_t^i, h_t^i\right). \tag{11}$$

The function $g^i\left(w_t^i, h_t^i\right)$ corresponds to the m-consumption function introduced by Browning (1999). The following theorem establishes the condition under which $q^i(c,h)$ is invertible.

Theorem 1 *The m-consumption function $g^i\left(w^i, h^i\right)$ is well-defined if*

$$u_{lc}^i\left(c^i, T-h^i\right) u_c^i\left(c^i, T-h^i\right) - u_{cc}^i\left(c^i, T-h^i\right) u_l^i\left(c^i, T-h^i\right) \neq 0 \tag{12}$$

for any c^i and h^i that satisfy (10) for some feasible w^i.

Proof. For any c^i, h^i, w^i satisfying (10) define,

$$d^i\left(c^i, h^i, w^i\right) = q^i\left(c_t^i, h_t^i\right) - w_t^i = 0.$$

By the implicit function theorem, $g^i\left(w^i, h^i\right)$ is well-defined if $\frac{\partial d^i}{\partial c^i} \neq 0$. Which implies condition (12). ∎

[7] I will discuss the use of individual expenditure on clothing for identification later on.

[8] This result does not depend on efficiency. An intertemporal framework with no cooperation will imply an identical result.

[9] The case of a corner solution will be considered in section 5.

Consequently, even if individual consumption is not observed, there exists a function that relates it to observable variables. Total household consumption C is also observed and this information has not been used so far. Given the nature of the problem,

$$c_t^2 = C_t - c_t^1 = C_t - g^1\left(w_t^1, h_t^1\right).$$

By means of these results, individual Euler equations of member 1 can be characterized as a function of her labor supply and wage rate, by substituting the m-consumption function (11) for c_t^1 and c_{t+1}^1,

$$\begin{aligned} v^1\left(w_t^1, h_t^1\right) &= \beta_1 E_t\left[v^1\left(w_{t+1}^1, h_{t+1}^1\right) R_{t+1}\right], \\ f^1\left(w_t^1, h_t^1\right) &= \beta_1 E_t\left[f^1\left(w_{t+1}^1, h_{t+1}^1\right)\frac{R_{t+1}w_t^1}{w_{t+1}^1}\right], \\ v^1\left(w_t^1, h_t^1\right) &= \beta_1 E_t\left[f^1\left(w_{t+1}^1, h_{t+1}^1\right)\frac{R_{t+1}}{w_{t+1}^1}\right] \end{aligned}$$

where,

$$v^1\left(w^1, h^1\right) = u_c^1\left(g^1\left(w^1, h^1\right), T - h^1\right), \tag{13}$$
$$f^1\left(w^1, h^1\right) = u_l^1\left(g^1\left(w^1, h^1\right), T - h^1\right). \tag{14}$$

Similarly, individual Euler equations of member 2 can be characterized as a function of total household consumption, his labor supply and the labor supply and wage rate of members 1, by substituting $C_t - g^1\left(w_t^1, h_t^1\right)$ and $C_{t+1} - g^1\left(w_{t+1}^1, h_{t+1}^1\right)$ for c_t^1 and c_{t+1}^1,

$$\begin{aligned} v^2\left(C_t, w_t^1, h_t^1, h_t^2\right) &= \beta_2 E_t\left[v^2\left(C_{t+1}, w_{t+1}^1, h_{t+1}^1, h_{t+1}^2\right) R_{t+1}\right], \\ f^2\left(C_t, w_t^1, h_t^1, h_t^2\right) &= \beta_2 E_t\left[f^2\left(C_{t+1}, w_{t+1}^1, h_{t+1}^1, h_{t+1}^2\right)\frac{R_{t+1}w_t^2}{w_{t+1}^2}\right], \\ v^2\left(C_t, w_t^1, h_t^1, h_t^2\right) &= \beta_2 E_t\left[f^2\left(C_{t+1}, w_{t+1}^1, h_{t+1}^1, h_{t+1}^2\right)\frac{R_{t+1}}{w_{t+1}^2}\right]. \end{aligned}$$

where,

$$v^2\left(C, w^1, h^1, h^2\right) = u_c^2\left(C - g^1\left(w^1, h^1\right), T - h^2\right), \tag{15}$$
$$f^2\left(C, w^1, h^1, h^2\right) = u_l^2\left(C - g^1\left(w^1, h^1\right), T - h^2\right). \tag{16}$$

I will refer to the new Euler equations as transformed Euler equations. Given that total household consumption, individual labor supplies and wage rates are observed, the functions v^1, f^1, v^2 and f^2 can be identified non-parametrically or parametrically. It is interesting to notice that, contrary to the standard method, this approach allows the identification of the individual discount factor of the two spouses. However, we are not interested in v^1, f^1, v^2 and f^2 but rather in u_c^i and u_l^i for $i = 1, 2$. The next sections discuss how to derive $u_c^i\left(c^i, T - h^i\right)$ and $u_l^i\left(c^i, T - h^i\right)$ given $v^1\left(w^1, h^1\right)$, $f^1\left(w^1, h^1\right)$, $v^2\left(C, w^1, h^1, h^2\right)$ and $f^2\left(C, w^1, h^1, h^2\right)$.

4 Identification When Both Agents Work

Suppose that husband and wife both work. Then variations in labor supply and wages are observed for both spouses and v^1, f^1, v^2 and f^2 can be identified using the approach discussed in the previous section. From relations (15) and (16) we can deduce that,

$$v^2_{h^2} = u^2_{cl}, \;\; f^2_w = -u^2_{lc} g^1_w, \;\; f^2_{h^1} = -u^2_{lc} g^1_h.$$

Given that v^2 and f^2 are known functions, their derivatives are known as well and g^1_w and g^1_h can be identified,

$$g^1_w = -\frac{f^2_w}{v^2_{h^2}}, \;\; g^1_l = -\frac{f^2_{h^1}}{v^2_{h^2}} \tag{17}$$

where the result follows from $u^2_{lc} = u^2_{cl}$. Hence (17) provides a partial differential system, which can be integrated to give $g\left(w^1, h^1\right)$ up to the constant of integration. From relations (15) and (16) we can also deduce,

$$v^2_w = -u^2_{cc} g^1_w, \;\; f^2_{h^2} = u^2_{ll},$$

which imply that u^2_{cc}, u^2_{ll} and u^2_{cl} can be identified,

$$u^2_{cc} = \frac{v^2_w v^2_{h^2}}{f^2_w}, \;\; u^2_{ll} = f^2_{h^2}, \;\; u^2_{cl} = v^2_{h^2}. \tag{18}$$

The partial differential system can be solved to derive $u^2_c\left(c^2, T - h^2\right)$ and $u^2_l\left(c^2, T - h^2\right)$ up to a constant. From equations (13) and (14) we can obtain,

$$v^1_w = u^1_{cc} g^1_w, \;\; f^1_w = u^1_{lc} g^1_w, \;\; f^1_{h^1} = u^1_{lc} g^1_h + u^1_{ll},$$

which imply,

$$u^1_{cc} = -\frac{v^1_w v^2_{h^2}}{f^2_w}, \;\; u^1_{ll} = f^1_{h^1} - \frac{f^1_w f^2_{h^1}}{f^2_w}, \;\; u^1_{cl} = -\frac{f^1_w v^2_{h^2}}{f^2_w}. \tag{19}$$

Hence $u^1_c\left(c^1, T - h^1\right)$ and $u^1_l\left(c^1, T - h^1\right)$ can be identified up to a constant.

Proposition 2 *Let u^1 and u^2 be von Neumann-Morgenstern utility functions. Assume that both agents work and that either u^1 or u^2 satisfies the invertibility condition (12). Then individual Euler equations are identified up to an additive constant.*

Most investigators estimate household Euler equations to derive coefficients of intertemporal substitution for consumption and labor supply.[10] The identification of those coefficients for each household member is unaffected by the fact that individual Euler equations are identified up to an additive constant. Finally note that this method generates a set of overidentifying restrictions, which can be employed to test the model.

[10] See Browning, Hansen and Heckman (1999) for a survey of papers estimating the coefficients of intertemporal substitutions by means of Euler equations.

5 Identification When Only One Agent Works

Consider a household in which only one spouse works. Without loss of generality suppose that agent 1 supplies labor. The function $g^1(w^1, h^1)$ is still well-defined and the approach outlined in section 3 can be implemented setting $h^2 = 0$. However, given that no variations in member 2's labor supply is observed, we can identify $v^1(w^1, h^1)$, $f^1(w^1, h^1)$, $v^2(C, w^1, h^1, 0)$, but not $f^2(C, w^1, h^1, 0)$ as the labor Euler equation is replaced by an inequality. This section will discuss whether individual Euler equations are identified with the limited amount of information available.

From equations (15) we can deduce that,

$$v_C^2 = u_{cc}^2, \; v_w^2 = -u_{cc}^2 g_w^1, \; v_{h^1}^2 = -u_{cc}^2 g_h^1,$$

which imply that g_w^1 and g_h^1 can be identified even if only spouse 1 works,

$$g_w^1 = -\frac{v_w^2}{v_C^2}, \; g_h^1 = -\frac{v_{h^1}^2}{v_C^2}. \tag{20}$$

Hence, g^1 can be derived up to a constant solving the partial differential system (20). Moreover, relations (15) and (16) imply that,[11]

$$v_C^2 = u_{cc}^2.$$

Hence $u_c^2(c^2, T)$ can be identified by integration.[12] Finally, from equations (13) and (14) we can derive,

$$v_w^1 = u_{cc}^1 g_w^1, \; f_w^1 = u_{lc}^1 g_w^1, \; f_{h^1}^1 = u_{lc}^1 g_h^1 + u_{ll}^1,$$

which imply,

$$u_{cc}^1 = -\frac{v_w^1 v_C^2}{v_w^2}, \; u_{ll}^1 = f_{h^1}^1 - \frac{f_w^1 v_{h^1}^2}{v_w^2}, \; u_{cl}^1 = -\frac{f_w^1 v_C^2}{v_w^2},$$

and $u_c^1(c^1, T - h^1)$ and $u_l^1(c^1, T - h^1)$ can also be identified.

Proposition 3 *Let u^1 and u^2 be von Neumann-Morgenstern utility functions. Assume that only agent 1 works and that u^1 satisfies the invertibility condition (12). Then individual Euler equations of member 1 are identified up to an additive constant. Moreover, the consumption Euler equation of member 2 can be identified up to an additive constant.*

[11] Only the function $u_{cc}^2(c^2, T)$, and by integration $u_c^2(c^2, T)$, can be identified in this case.

[12] Given that no variations in member 2's labor supply is observed, only the variation in total consumption, member 1's leisure and wage rate can be used to deduce u_c^2. As a result, it is not possible to identify how u_c^2 varies with leisure.

It is interesting to note that if agent 2 does not work, c^2 can vary with w^1 and h^1 even if w^2 and h^2 do not change, since a variation in w^1 or h^1 changes the decision power within the household.[13]

6 Empirical Implementation

In this section I discuss the empirical estimation of individual Euler equations.

6.1 Parametric Identification and Estimation

For the parametric identification and estimation I assume that the one-period utility function can be written in the form,[14]

$$u^i\left(c^i, T-h^i\right) = \frac{\left[\left(c^i\right)^{\alpha_i}\left(T-h^i\right)^{\sigma_i}\right]^{1-\rho_i}}{\alpha_i\left(1-\rho_i\right)},$$

where $\alpha_i, \sigma_i, \rho_i > 0$. For this specification of preferences, the intra-period condition (10) becomes,

$$q^i\left(c^i, h^i\right) = \frac{\sigma_i}{\alpha_i}\frac{c^i}{T-h^i} = w^i.$$

The invertibility condition (12) is clearly satisfied. Specifically,

$$\sigma_i\left(c^i\right)^{2\alpha_i(1-\rho_i)-2}\left(T-h^i\right)^{2\sigma_i(1-\rho_i)-1} > 0$$

for every $\alpha_i, \sigma_i > 0$.[15] Hence, it is possible to obtain the m-consumption function for agent 1,

$$c^1 = g^1\left(w^1, h^1\right) = \frac{\alpha_1}{\sigma_1}w^1\left(T-h^1\right).$$

The functions v^1, f^1, v^2 and f^2 have the following representation,

$$\begin{aligned}
v^1\left(w^1, h^1\right) &= \delta_1\left(w^1\right)^{\theta_1}\left(T-h^1\right)^{\lambda_1}, \\
f^1\left(w^1, h^1\right) &= \delta_1\left(w^1\right)^{\theta_1+1}\left(T-h^1\right)^{\lambda_1}, \\
v^2\left(C, w^1, h^1, h^2\right) &= \left(C-\phi_1 w^1\left(T-h^1\right)\right)^{\theta_2}\left(T-h^2\right)^{\lambda_2} \\
f^2\left(C, w^1, h^1, h^2\right) &= \phi_2\left(C-\phi_1 w^1\left(T-h^1\right)\right)^{\theta_2+1}\left(T-h^2\right)^{\lambda_2-1}
\end{aligned}$$

[13]This point is stressed by Blundell, Chiappori, Magnac and Meghir (2001). Given that I am considering a full efficiency model of household intertemporal behavior, only the decision power at time 0 can affect the allocation of resources over time and across households. Therefore a change in w^1 or h^1 can have an impact on c^2 only insofar it was predicted at time 0.

[14]I divide by α_i to normalize the multiplicative constant of u_l^i to 1.

[15]The utility function satisfies the Inada condition. Therefore, individual consumption and leisure must be strictly positive.

where $\theta_1 = \alpha_1(1-\rho_1) - 1$, $\lambda_1 = (\sigma_1+\alpha_1)(1-\rho_1) - 1$, $\theta_2 = \alpha_2(1-\rho_2) - 1$, $\lambda_2 = \sigma_2(1-\rho_2)$, $\phi_1 = \frac{\alpha_1}{\sigma_1}$, $\phi_2 = \frac{\sigma_2}{\alpha_2}$ and $\delta_1 = \phi_1^{\theta_1}$. Substituting these functions in the individual Euler equations,

$$\begin{aligned}
1 &= \beta_1 E_t \left[\left(\frac{w_{t+1}^1}{w_t^1} \right)^{\theta_1} \left(\frac{T-h_{t+1}^1}{T-h_t^1} \right)^{\lambda_1} R_{t+1} \right], \\
1 &= \beta_2 E_t \left[\left(\frac{C_{t+1} - \phi_1 w_{t+1}^1 \left(T-h_{t+1}^1\right)}{C_t - \phi_1 w_t^1 \left(T-h_t^1\right)} \right)^{\theta_2} \left(\frac{T-h_{t+1}^2}{T-h_t^2} \right)^{\lambda_2} R_{t+1} \right], \\
1 &= \beta_2 E_t \left[\left(\frac{C_{t+1} - \phi_1 w_{t+1}^1 \left(T-h_{t+1}^1\right)}{C_t - \phi_1 w_t^1 \left(T-h_t^1\right)} \right)^{\theta_2+1} \left(\frac{T-h_{t+1}^2}{T-h_t^2} \right)^{\lambda_2-1} \frac{R_{t+1} w_t^2}{w_{t+1}^2} \left| h^2 > 0 \right. \right], \\
1 &= \beta_2 \phi_2 E_t \left[\frac{\left(C_{t+1} - \phi_1 w_{t+1}^1 \left(T-h_{t+1}^1\right)\right)^{\theta_2+1}}{\left(C_t - \phi_1 w_t^1 \left(T-h_t^1\right)\right)^{\theta_2}} \frac{\left(T-h_{t+1}^2\right)^{\lambda_2-1}}{\left(T-h_t^2\right)^{\lambda_2}} \frac{R_{t+1}}{w_{t+1}^2} \left| h^2 > 0 \right. \right].
\end{aligned}$$

where $E_t\left[.\left|h^i > 0\right.\right]$ indicates that Euler equations containing the marginal utility of leisure are satisfied only if the spouse works.[16] The coefficients θ_i, λ_i and ϕ_i are estimated using the Generalized Method of Moments. This implies that $\delta_1 = \phi_1^{\theta_1+1}$ is identified as well. A detailed description of the implementation of the GMM is given in the last section.

Before analyzing the data and the GMM implementation, I will discuss how the individual Euler equations are identified knowing θ_i, λ_i, ϕ_i and δ_1. Assume that both agent are working. By (17),[17]

$$\begin{aligned}
g_w^1 &= \phi_1 \left(T-h^1\right), \\
g_l^1 &= \phi_1 w^1,
\end{aligned}$$

which give,

$$g^1\left(w^1, h^1\right) = \phi_1 w^1 \left(T-h^1\right) + K.$$

Since $\phi_1 = \frac{\alpha_1}{\sigma_1}$, the function that we obtain with the suggested approach is equivalent to the

[16] The second and third equation for spouse 1 are collinear with the first equation.

[17] Note that $\frac{\theta_2+1}{\lambda_2} = \frac{1}{\phi_2}$.

function we started with. The partial differential system (18) implies,[18]

$$\begin{aligned} u^2_{cc} &= \theta_2 \left(C - \phi_1 w^1 \left(T - h^1\right)\right)^{\theta_2 - 1} \left(T - h^2\right)^{\lambda_2}, \\ u^2_{cl} &= \lambda_2 \left(C - \phi_1 w^1 \left(T - h^1\right)\right)^{\theta_2} \left(T - h^2\right)^{\lambda_2 - 1} \\ &= \phi_2 \left(\theta_2 + 1\right) \left(C - \phi_1 w^1 \left(T - h^1\right)\right)^{\theta_2} \left(T - h^2\right)^{\lambda_2 - 1}, \\ u^2_{ll} &= \phi_2 \left(\lambda_2 - 1\right) \left(C - \phi_1 w^1 \left(T - h^1\right)\right)^{\theta_2 + 1} \left(T - h^2\right)^{\lambda_2 - 2}, \end{aligned}$$

which imply,

$$\begin{aligned} u^2_c &= \left(c^2\right)^{\theta_2} \left(T - h^2\right)^{\lambda_2} + K, \\ u^2_l &= \phi_2 \left(c^2\right)^{\theta_2 + 1} \left(T - h^2\right)^{\lambda_2 - 1} + K. \end{aligned}$$

Finally, by (19),[19]

$$\begin{aligned} u^1_{cc} &= \frac{\delta_1 \theta_1}{\phi_1} \left(w^1\right)^{\theta_1 - 1} \left(T - h^1\right)^{\lambda_1 - 1} \\ &= \phi_1 \theta_1 \left(c^1\right)^{\theta_1 - 1} \left(T - h^1\right)^{\lambda_1 - \theta_1}, \\ u^1_{cl} &= \frac{\delta_1 \left(\theta_1 + 1\right)}{\phi_1} \left(w^1\right)^{\theta_1} \left(T - h^1\right)^{\lambda_1 - 1} = \\ &= \phi_1 \left(\lambda_1 - \theta_1\right) \left(c^1\right)^{\theta_1} \left(T - h^1\right)^{\lambda_1 - \theta_1 - 1}, \\ u^1_{ll} &= \delta_1 \left(\lambda_1 - \theta_1 - 1\right) \left(w^1\right)^{\theta_1 + 1} \left(T - h^1\right)^{\lambda_1 - 1} \\ &= \left(\lambda_1 - \theta_1 - 1\right) \left(c^1\right)^{\theta_1 + 1} \left(T - h^1\right)^{\lambda_1 - \theta_1 - 2}. \end{aligned}$$

Consequently,

$$\begin{aligned} u^1_c &= \phi_1 \left(c^1\right)^{\theta_1} \left(T - h^1\right)^{\lambda_1 - \theta_1} + K, \\ u^1_l &= \left(c^1\right)^{\theta_1 + 1} \left(T - h^1\right)^{\lambda_1 - \theta_1 - 1} + K. \end{aligned}$$

By similar argument, the individual Euler equations for households in which only one agent is working can be identified using the corresponding relations.

6.2 The Data

To implement the identification procedure, the dataset employed in the estimation must have the following two characteristics. First, information on total household consumption, individual labor supplies, wages and interest rates must be available. Second, the dataset should have

[18]Note that $\phi_2 = \dfrac{\lambda_2}{\theta_2 + 1}, \dfrac{\lambda_2}{\phi_2} = \theta_2 + 1$ and $c^2 = C - \phi w^1 \left(T - h^1\right)$.

[19]Note that $\delta_1 = \phi_1^{\theta_1 + 1}$ and $\left(\theta_1 + 1\right) = \phi_1 \left(\lambda_1 - \theta_1\right)$

a panel structure to determine consumption, labor supply and wage dynamics for each household. The CEX survey satisfies these requirements.[20] Since 1980, the CEX survey has been collecting data on household consumption, labor supply, wages and demographics. The survey is a rotating panel organized by the Bureau of Labor Statistics (BLS). Each quarter about 4500 households, representative of the US population, are interviewed: 80% are reinterviewed the following quarter, while the remaining 20% are replaced by a new randomly selected group. Each household is interviewed at most for four quarters and detailed information are elicited in regard to expenditures, labor supply and demographics for each of the three months preceding the interview.[21]

The data I use cover the period 1982-1998. The first two years are dropped, since the data were collected with a different methodology. As in Meghir and Weber (1996) and Attanasio and Mazzocco (2001), I use the rotating feature of the panel, i.e. I employ household level data for the four quarters available. Consequently, I drop all households that are not in the survey for all four interviews. I exclude from the sample rural households, households living in student housing, household in which the head is younger than 21 and older than 65 and households with incomplete income responses. The identification method is useful to identify individual preferences of couples. Therefore, I concentrate on married households. Singles will be used as a benchmark to evaluate the performance of the method. To implement the identification procedure, at least one household member must be employed. Hence, I exclude all households in which the husband is not working. The husband claims to be employed in all four available interviews for more than 90 percent of the households in the sample. Finally I drop households experiencing a change in marital status.

The CEX dataset contains monthly data on consumption. However, the labor supply variables are available only every quarter. Consequently in the estimation quarterly variables are employed. Total consumption is computed as the sum of food at home, food out, tobacco, alcohol, other nondurable goods and services such as heating fuel, public and private transportation, personal care and semidurable goods which include clothing and shoes. In particular, from the definition of total consumption I exclude consumer durables, housing, education and health expenditure. Total consumption is deflated using the Consumer Price Indices published by the BLS. Specifically, the price index for the composite good is calculated as a weighted average of individual price indices, with weights equal to the expenditure share for the particular consumption good. The gross hourly wage rate is computed using three variables: the amount of the last gross pay; the time period of the last gross pay covered; the number of hours usually worked per week in the corresponding period. Since the wage rate is not directly observed, the measure used in this paper might be affected by endogeneity. In particular, the amount of the last gross pay is likely to be affected by the number of hours of work in a given period. However, this criticism applies to any work unless wage rates are directly observed. Moreover, even in

[20]The Federal Reserve publishes detailed data on interest rates for different assets.

[21]Each household is interviewed for 5 quarter, but the first interview is used to make contact and no information is publicly available.

this case the wage rate will depend on hours of work through the investment in human capital. To calculate the after tax wage rate, federal effective tax rates are generated using the NBER's TAXSIM model. Finally, the real after tax wage rate is determined using the individual price indices. Total time available to each household member is computed as 12 hours per day times 7 days per week times 13 weeks per quarter. This implies that $T = 1092$. Quarterly individual labor supply is obtained multiplying by 14 the number of hours usually worked per week in the corresponding period. The interest rate is the quarterly average of the 3-month Treasury bill rate. The real after tax interest rate is calculated by using the output of the TAXSIM model and the household price indices. Finally to account for preference shocks, the one-period utility function of each spouse is augmented to include a function of the following demographic variables: number of children and family size.

The CEX collects very detailed information on consumption. Unfortunately, the quality of the labor supply data is not comparable. In particular, individual labor income data are collected at the first and last interviews unless a member of the household reports changing his or her employment. Moreover, 27 percent of respondents reporting to be employed do not have data on gross pay.

After all these selections have been carried out, the sample is reduced to 8033 couples spread over 68 quarters of the sample period.

6.3 Econometric Issues

In the theoretical section I do not model public goods. However, at least one type of public good affects individual consumption and household savings, namely children. In the empirical section I allow for public goods in two different ways. Following most of the literature on Euler equations, the utility function of each household member is multiplied by a heterogeneity term,

$$u^i\left(c^i, T-h^i, z\right) = \frac{\left[\left(c^i\right)^{\alpha_i}\left(T-h^i\right)^{\sigma_i}\right]^{1-\rho_i}}{\sigma_i\left(1-\rho_i\right)} \prod_{j=1}^{m}\left(z^j\right)^{\xi_j},$$

where z is a vector of demographics including number of children and family size. With this generalization of preferences, the Euler equation will depend on the demographics. However, for a given level of household savings, the allocation of available resources between the two agents will not depend on demographics. The intuition is straightforward. Under the assumption of separability over time and across state of the world, the intertemporal household problem can be divided in two steps. In the first step the household decides how much to save in each period. Since agents are characterized by von Neumann-Morgenstern utility functions, in the augmented utility function $u^i\left(c^i, T-h^i, z^j\right)$, the vector of demographics z is not separable from consumption and leisure. Therefore it will affect the Euler equation and through it the saving decision. Given savings at each t, in the second step for each period and state of the world the household will allocate total consumption between the two agents. In each possible period and state of the world the vector z is separable from consumption and leisure. Hence it has no impact on the intra-household allocation of resources. To allow the intra-household allocation of resources to depend on demographics, the function $g^1(w^1, h^1)$ defining consumption of member 1 is augmented to include a heterogeneity term,

$$g^1(w^1, h^1, q) = g^1(w^1, h^1) + \gamma_k q^k,$$

where the vector q includes number of children and family size.

To identify individual preference parameters I will employ the consumption Euler equations of both members and the Euler equations of member 2 relating leisure today with consumption tomorrow. Indeed, the Inada condition guarantees that the consumption Euler equations are always satisfied. The individual leisure Euler equations are satisfied only if the corresponding agent supplies a positive amount of hours on the labor market in period and state of the world. This restriction is satisfied by member 1. However, the identification method implies that member 1's leisure Euler equation is identical to his consumption Euler equation. Member 2 can either work or not. Observing the data it is possible to determine whether she is working at time t and at time $t+1$, but it is not possible to verify whether she would have worked at time $t+1$ in a different state of the world. Consequently, it is not possible to establish if member 2's leisure Euler equation is satisfied by simply looking at the data. On the other hand, if member 2 is employed at time t, her Euler equations relating leisure today with consumption tomorrow will be satisfied.

The estimation of Euler equations using data on labor supply generates a problem of self-selection. The assumption that the husband is always employed is not very restrictive, since more than 90 percent of males between the ages of 21 and 65 in the sample is working in all four quarters of the survey. The use of member 2's Euler equations relating leisure today with consumption tomorrow is more problematic. These equations are satisfied only for agents working at time t. Therefore the estimates will be consistent only if the expectational error of these equations are not correlated with the decision of supplying labor. The labor force participation decision might have important effects on household intertemporal behavior, but this issue is left for future research.

The identification of individual preferences requires the estimation of non linear equations. I estimate them using the Generalized Method of Moments (GMM). Define,

$$\begin{aligned}
gc^1(w^1, h^1; \xi) &= \beta_1 \left(\frac{w_{t+1}^1}{w_t^1}\right)^{\theta_1} \left(\frac{T - h_{t+1}^1}{T - h_t^1}\right)^{\lambda_1} \prod_{j=1}^{m} \left(\frac{z_{t+1}^j}{z_t^j}\right)^{\xi_j} R_{t+1}\text{-}1, \\
gc^2(w^2, h^2; \xi) &= \beta_2 \left(\frac{C_{t+1}\text{-}\phi_1 w_{t+1}^1 (T\text{-}h_{t+1}^1)\text{-}\gamma_k q_{t+1}^k}{C_t\text{-}\phi_1 w_t^1 (T\text{-}h_t^1)\text{-}\gamma_k q_t^k}\right)^{\theta_2} \left(\frac{T\text{-}h_{t+1}^2}{T\text{-}h_t^2}\right)^{\lambda_2} \prod_{j=1}^{m} \left(\frac{z_{t+1}^j}{z_t^j}\right)^{\xi_j} R_{t+1}\text{-}1, \\
glc^2(w^2, h^2; \xi) &= \beta_2 \frac{\left(C_{t+1}\text{-}\phi_1 w_{t+1}^1 (T\text{-}h_{t+1}^1)\text{-}\gamma_k q_{t+1}^k\right)^{\theta_2}}{\left(C_t\text{-}\phi_1 w_t^1 (T\text{-}h_t^1)\text{-}\gamma_k q_t^k\right)^{\theta_2+1}} \frac{(T\text{-}h_{t+1}^2)^{\lambda_2}}{(T\text{-}h_t^2)^{\lambda_2-1}} \prod_{j=1}^{m} \left(\frac{z_{t+1}^j}{z_t^j}\right)^{\xi_j} \frac{w_t^2 R_{t+1}}{\phi_2}\text{-}1,
\end{aligned}$$

where ξ is the vector of parameters to estimate. Let q be a vector of variables known at time t and $1_{\{h_t^2>0\}}$ an indicator function equal to one if member 1 works in period t. Then, under the assumption of rational expectations, the following moment conditions are satisfied,

$$\begin{aligned} E\left[gc^1\left(w^1,h^1;\xi\right)\right] &= 0,\ E\left[gc^1\left(w^1,h^1;\xi\right)q\right]=0, \\ E\left[gc^2\left(w^2,h^2;\xi\right)\right] &= 0,\ E\left[gc^2\left(w^2,h^2;\xi\right)q\right]=0, \\ E\left[1_{\{h_t^2>0\}}glc^2\left(w^2,h^2;\xi\right)\right] &= 0,\ E\left[1_{\{h_t^2>0\}}glc^2\left(w^2,h^2;\xi\right)q\right]=0. \end{aligned}$$

I substitute the expectations with the corresponding sample moments to obtain estimates of ξ. The vector of instruments used in the estimation are the second, third and fourth lag of the nominal interest rate and age of the husband. In all estimations, the model is overidentified. Therefore a weighting matrix must be employed to aggregate the moment conditions. The GMM estimator is obtained using a standard method. Consistent estimates are first obtained using the identity matrix. By means of them, it is possible to compute the efficient weighting matrix as derived in Hansen (1982). The final estimates can then be computed.

The estimation of Euler equations with GMM is not free of problems. As for any estimator, the GMM estimator is consistent only if data are not characterized by measurement errors. With GMM the measurement error problem is exacerbated because of the non-linearities. The quality of the consumption data in the CEX is very good. However, a good look at the labor supply data suggests that they may be affected by measurement errors. Using a Montecarlo simulation, in his insightful paper, Carroll (2001) finds that measurement errors should bias the estimates of intertemporal substitution downward. Consequently, the estimates reported in this paper represent at least a lower bound of the preference parameters.

The GMM estimation has also an important advantage: it does not requires the log-linearization of the Euler equations. Carroll (2001) and Ludvigson and Paxon (forthcoming) find that the approximation method may introduce a substantial bias in the estimation of the preference parameter. On the other hand, Attanasio and Low (2000) show that using long panels it is possible to estimate consistently log-linearized Euler equations. The results reported in this paper are not affected by a possible approximation bias.

In the paper I abstract from liquidity constraints. If household members are restricted in their ability to borrow, Euler equations are replaced by inequalities as shown in Zeldes (1989). The introduction of liquidity constraints are left for future research.

6.4 Empirical results

Consider first the standard household Euler equation under the assumption of separability,

$$U'\left(C_t,z_t\right)=\beta E_t\left[U'\left(C_{t+1},z_{t+1}\right)R_{t+1}\right]. \tag{21}$$

Table 1: Log-linearized Euler Equation.

	Entire Sample	
	Coef.	**Std. Err.**
Real After Tax Int. Rate	0.031	0.026
log(kids$_{t+1}$/kids$_t$)	-0.10	0.02
log(fam.size$_{t+1}$/fam.size$_t$)	0.21	0.015
constant	0.013	0.015
Number of Households	13727	

Table 2: Estimation of Household Euler Equation by GMM.

	$\beta = 0.93$		$\beta = 0.95$		$\beta = 0.97$	
	GMM	**Std. Err.**	**GMM**	**Std. Err.**	**GMM**	**Std. Err.**
(cons$_{t+1}$/cons$_t$)	0.99	0.07	0.82	0.05	0.60	0.04
log(kids$_{t+1}$/kids$_t$)	0.59	0.57	0.43	0.49	0.28	0.37
log(fam.size$_{t+1}$/fam.size$_t$)	-0.35	0.93	-0.31	0.78	-0.27	0.58
Chi	2.11		2.35		2.75	
Prob.	0.35		0.31		0.25	
Number of Households	13727					

Suppose that the household utility function $U(.)$ can be written in the form,

$$U(C, z) = \frac{C^{1-\gamma}}{1-\gamma} \prod_{j=1}^{m} \left(\frac{z_{t+1}^{j}}{z_t^{j}} \right)^{\xi_j}. \tag{22}$$

It is straightforward to show that the consumption Euler equation can be log-linearized and it can be written as follows:

$$ln\left(C_{t+1}/C_t\right) = k + \frac{1}{\gamma}\ln\left(R_{t+1}\right) + \sum_{j=1}^{m} \frac{\xi_j}{\gamma} \ln\left(R_{t+1}\right) + \eta_t.$$

where η_t is an error term containing the expectational error and unobservables. In absence of measurement errors and log-linearization errors, the estimation of (21) by GMM and the estimation of (22) should generate identical estimates. The results are reported in tables 1 and 2.

The estimates obtained log-linearizing the Euler equation are obtained using as instruments for the interest rate the second, third and fourth lag of the nominal interest rate and age. They are consistent with most of the literature. The coefficient of intertemporal substitution is positive, small and not statistically different from zero. The coefficient of family size is positive as

Table 3: Estimation of the Household Euler Equation with Labor Supply by GMM, Entire Sample

	$\beta = 0.93$		$\beta = 0.95$		$\beta = 0.97$	
	GMM	**Std. Err.**	**GMM**	**Std. Err.**	**GMM**	**Std. Err.**
$(\text{cons}_{t+1}/\text{cons}_t)$	0.53	0.084	0.53	0.073	0.46	0.056
$(\text{leis.husb}_{t+1}/\text{leis.husb}_t)$	0.50	0.051	0.54	0.045	0.56	0.040
$(\text{leis. wife}_{t+1}/\text{leis. wife}_t)$	0.87	0.093	0.84	0.088	0.76	0.084
$\log(\text{kids}_{t+1}/\text{kids}_t)$	-0.55	0.419	-0.47	0.351	-0.015	0.586
$\log(\text{fam.size}_{t+1}/\text{fam.size}_t)$	2.05	0.179	1.41	0.187	0.15	0.564
Chi	19.85		19.10		25.38	
Prob	0.03		0.04		0.005	
Number of Households	13727					

expected and strongly significant. The coefficient on children is negative and significant. The Euler equation (21) is estimated using GMM by fixing the discount factor at 3 different levels, $\beta = 0.93$, $\beta = 0.95$ and $\beta = 0.97$. In all three cases, the coefficient of relative risk aversion is positive and estimated very precisely. As expected the estimate of γ declines if β is increased. On the other hand, the coefficients on children and family size are poorly measured. Moreover, the overidentifying restrictions are not rejected. Note that the coefficient of intertemporal substitution is equal to the inverse of the coefficient of relative risk aversion. Hence, the two methods provide very different answers: either the estimate of γ obtained using GMM is too small due to measurement errors or the log-linearization biases upward the estimate of γ.

Suppose now that the assumption of separability between consumption and leisure is not satisfied. Then, the household Euler equations can be written in the form,

$$\begin{aligned} U_C\left(C_t, T-h_t^1, T-h_t^2\right) &= \beta E_t\left[U_C\left(C_{t+1}, T-h_{t+1}^1, T-h_{t+1}^2\right) R_{t+1}\right], \\ U_{l^i}\left(C_t, T-h_t^1, T-h_t^2\right) &= \beta E_t\left[U_C\left(C_{t+1}, T-h_{t+1}^1, T-h_{t+1}^2\right) \frac{R_{t+1}}{w_{t+1}^i}\right] \quad i=1,2. \end{aligned}$$

I do not attempt to estimate these equations using the instrumental variable method, since I do no have good instruments for labor supply.[22] The GMM results for the entire sample are reported in table 3.[23] The estimates when the sample is restricted to couples are described in table 4. The coefficient on consumption is about half as large as the one obtained under the assumption of separability. The coefficients on leisure are both positive, smaller than one and precisely estimated. Moreover, the standard errors of the demographic variables are much smaller. The overidentifying restrictions are rejected for the entire sample, but not for couples.

The estimation of individual Euler equations presents more challenges. According to the

[22]Each household is observed for only four quarters. Therefore, I cannot use household specific lagged variables

Table 4: Estimation of the Household Euler Equation with Labor Supply by GMM, Couples

	$\beta = 0.93$		$\beta = 0.95$		$\beta = 0.97$	
	GMM	**Std. Err.**	**GMM**	**Std. Err.**	**GMM**	**Std. Err.**
$(\text{cons}_{t+1}/\text{cons}_t)$	0.50	0.102	0.48	0.090	0.44	0.078
$(\text{leis.husb}_{t+1}/\text{leis.husb}_t)$	0.57	0.039	0.57	0.038	0.59	0.036
$(\text{leis. wife}_{t+1}/\text{leis. wife}_t)$	0.71	0.086	0.70	0.082	0.69	0.078
$\log(\text{kids}_{t+1}/\text{kids}_t)$	-0.48	0.702	-0.45	0.257	0.08	0.515
$\log(\text{fam.size}_{t+1}/\text{fam.size}_t)$	2.69	0.894	1.80	0.246	0.01	0.678
Chi2	10.554		10.867		13.884	
Prob.	0.39		0.37		0.18	
Number of Households	8033					

model, the husband consumption can be computed as follows:

$$c^1 = \phi_1 w^1 \left(T - h^1\right).$$

Setting $T = 1092$, in the data the husband's value of leisure, $w^1 \left(T - h^1\right)$, ranges between 1 dollar and 91200 dollars. Deflated household consumption goes form 269 dollars to 19660 dollars. Conditioning on the husband working full time, the value of leisure ranges between 1 and 54366 dollars, whereas household consumption is between 287 and 19660 dollars. According to the model, the wife's consumption can be calculated as follows:

$$c^2 = C - \phi_1 w^1 \left(T - h^1\right).$$

Therefore, even conditioning on the husband working full time, in the data in some cases the wife consumption is negative even for ϕ_1 very small. The introduction of children and family size reduces the incidence of the problem, but it does not solve it since the value of leisure is extremely large relative to consumption. I also experiment assuming that each agent has 60 hours per week to divide between leisure and labor. In this case, T drop to 780 and, conditioning on the husband working full time, member 1's value of leisure ranges between 1 and 25708 dollars, whereas consumption is still between 287 and 19660 dollars. The high value of leisure relative to consumption introduces a constraint in the estimation of individual Euler equations, because ϕ_1 is implicitly bounded above by the lowest values that makes $C_{t+1} - \phi_1 w^1_{t+1} \left(T - h^1_{t+1}\right) / C_t - \phi_1 w^1_t \left(T - h^1_t\right)$ negative for at least one household.

ESTIMATES FOR INDIVIDUAL EULER EQUATIONS TO BE ADDED

as instruments.

[23]According to the model, the Euler equations of singles are identical to Euler equations of couples in which the wife never works.

7 Conclusions

In this paper I discuss the identification and estimation of individual Euler equations. I show that individual Euler equations can be identified parametrically and non-parametrically observing only data on household total consumption, individual labor supply and wages, i.e. with the limited information available in the CEX. Moreover, assuming a specific utility function for each household member, I estimate them by means of the identification procedure developed in this paper and GMM.

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