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Life-Cycle Asset Allocation: A Model with Borrowing Constraints, Uninsurable Labor Income Risk and Stock-Market Participation Costs*

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Abstract

We study life-cycle asset allocation in the presence of liquidity constraints and undiversifiable labor income risk. The model includes three different assets (cash, long-term government bonds and stocks) and it takes into account the life-cycle profile of housing expenditures. With a modest correlation between stock returns and earnings innovations, the mean share of wealth invested in stocks never exceeds 45% during working-life. Moreover, the combination of uninsurable human capital and borrowing constraints rationalizes the asset allocation puzzle of Canner, Mankiw and Weil (1997). Nevertheless we argue that asset allocation models must match another important feature of the data: a low stock market participation rate. Along this dimension the model provides a very modest improvement, still predicting a counterfactually high participation rate. We show that this arises from the link between risk aversion and prudence, implying that explanations for the participation puzzle based on the role of background risk are unlikely to succeed.

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Key Words: Life-Cycle Asset Allocation, Liquidity Constraints, Stock Market Participation Costs, Uninsurable Labor Income Risk.

1 Introduction

In this paper we present a life-cycle asset allocation model with intermediate consumption and stochastic uninsurable labor income. A seminal reference addressing the problem of portfolio choice over the life-cycle is Samuelson (1969). Under the assumptions of independently and identically distributed returns, homothetic preferences, and the absence of labor income, he concludes that the optimal asset allocation should remain constant over time. However, in practice, labor income (or human capital) constitutes a crucial asset for most households. To the extent that the level and risk of the labor income stream change over the life-cycle, this can provide a rationale for age-varying investment strategies. If markets are complete so that labor income risk can be insured, its role is well understood from the seminal work by Merton (1971). However, the incomplete markets case remained unsolved for many years.

Following Deaton (1991) and Carroll (1992), a large successful literature on buffer stock saving with undiversifiable labor income risk has emerged.[1] More recently these models have been extended to include an asset allocation decision, both in an infinite-horizon setting (see Haliassos and Michaelides (2002), Viceira (2001), Michaelides (2001), Heaton and Lucas (2000, 1997 and 1996), Koo (1998), Lucas (1994) or Telmer (1993)) and in a finite-horizon life-cycle setting (see Yao and Zhang (2002), Davis, Kluber and Willen (2002), Gomes and Michaelides (2002), Campbell, Cocco, Gomes and Maenhout (2001), Hu (2001), Storesletten, Telmer and Yaron (2001), Dammon, Spatt and Zhang (2001), Cocco (2000), Cocco, Gomes and Maenhout (1999), Hochguertel (1999), or Gakidis (1998)).[2]

Consistent with the intuition in Jagannathan and Kocherlakota (1996), and with the recommendations offered by financial advisors (see for example, Malkiel, 1999), these models predict that the optimal share of wealth invested in equities is roughly decreasing during (working) life.[3] Nevertheless, as mentioned above, several important predictions of the model

[1]See, for instance Cagetti (2001), Gourinchas and Parker (2001), Ludvigson and Michaelides (2001), Carroll (1997), Carroll and Samwick (1997), or Hubbard, Skinner and Zeldes (1995).

[2]Constantinides, Donaldson and Mehra (2001) and Bertaut and Haliassos (1997) analyze three period models where each period amounts to 20 years.

[3]This result can also be obtained from Merton (1971), in a complete markets setting.

are still at odds with empirical regularities. First, the zero median stock holding puzzle (Mankiw and Zeldes (1991)) persists. The latest Survey of Consumer Finances reports that only around 49 percent of US households hold stocks either directly or indirectly (through pension funds, for instance). Yet, the frictionless portfolio choice model predicts that, given the equity premium, all households should participate in the stock market as soon as saving takes place. Second, households invest almost all of their wealth in stocks, and this result is particularly stronger for young households. This is contrary to both casual empirical observation and more formal evidence (see Poterba and Samwick (1999) or Ameriks and Zeldes (2001), for instance).

In this paper we develop a life-cycle asset allocation model that tries to address these puzzles. Our model has three key features. First, the household can invest in three financial assets: cash, stocks and long-term government bonds. Traditionally, life-cycle asset allocation models have ignored long-term bonds, and in this context future labor income (even though it is risky) acts as a closer substitute for cash rather than stock holdings. This generates a very strong demand for equities, particularly for young households. In a more general setting the degree of substitutability between multiple risky assets and human capital will determine the optimal portfolio decision. Second, we include a fixed entry cost for households which want to invest in any risky assets.[4] Empirical work by Vissing-Jorgensen (2002) and Paiella (1999) suggests that small entry costs can be consistent with the observed low stock market participation rates[5], and Attanasio, Banks and Tanner (2002) argue that the implications of the Consumption Capital Asset Pricing Model cannot be rejected for households predicted to hold both stocks and bonds ("shareholders").[6] Third, we take into account

[4]A large literature has concluded that some level of fixed costs seems to be necessary to improve the empirical performance of asset pricing models (see, among others, Brav, Constantinides, Geczy (2002), Polkovnichenko (2000), Basak and Cuoco (1998), Vayanos (1998), Luttmer (1996, 1999), Heaton and Lucas (1996), He and Modest (1995), Aiyagari and Gertler (1991) or Constantinides (1986)). In this paper we take a partial equilibrium approach but, the excessive demand for equities predicted by asset allocation models, is just the portfolio-demand manifestation of the equity premium puzzle.

[5]Degeorge, Jenter, Moel and Tufano (2002), estimate such a fixed-cost in the context of a specific even study and obtain relatively modest values.

[6]Campbell, Cocco, Gomes and Maenhout (2001) introduce such a fixed-cost in a life-cycle asset allocation

life-cycle expenditure patterns implied by housing consumption. In a life-cycle model with uninsurable labor income, total financial wealth is a crucial variable for determing both the participation decision and the optimal asset allocation. Given the existence of distortions between the housing sales market and its rental market (e.g. tax incentives), young households have a stronger incentive to buy a house and increase their spending early in life. We use data from the PSID to calibrate a typical life-cycle profile of housing expenditures and include it (exogenously) in the model.

Our main results can be listed as follows. First, immediately after paying the fixed cost, households invest almost all of their wealth in stocks. The desire to pay the cost is motivated by the willingness to hold stocks, not long-term bonds. Second, the fixed cost has a limited impact in postponing stock market participation. With a coefficient of relative risk aversion equal to 5, by age 26 the participation rate is already above 50%, and by age 30 almost all households have paid the entry cost. Third, a lower coefficient of relative risk aversion can actually lead to a decrease in the participation rate. This result occurs because of the link between prudence and risk aversion, which implies that a less risk-averse investor accumulates less wealth early in life, and this decreases the incentive to pay the fixed cost. Nevertheless, once retirement savings start, even the low-prudence households decide to invest in risky assets. With a coefficient of relative risk aversion of 2, by age 40 all households have already paid the fixed cost. These results have a very important implication: an increase in background risk (e.g. by introducing uncertainty about the parameters of the labor income process, consumption risk, or housing/mortgage risk) will not help to solve the participation puzzle. When faced with higher background risk the agent will build up a larger buffer stock of wealth thus creating a stronger incentive to enter the stock market. Fourth, for the households that have paid the fixed cost, long-term bonds are a stronger substitute for cash, rather than for stocks. Relative to the results from the existing two-

model, and find that it has a very small impact on the participation decision. On the other hand Haliassos and Michaelides (2002) argue that, in an infinite horizons model with impatient households (as defined in Deaton (1991)), a small one-time entry cost can generate much larger stock market non-participation as the consumer only accumulates a small buffer of assets, and therefore the benefit from investing in equities is much lower.

asset models the share of wealth invested in stocks is not strongly affected by the presence of the new asset. On the other hand, cash is almost completely replaced by long-term bonds with its share dropping to zero at age 35, and staying very close to that for most of the life-cycle. Fifth, even a moderate degree of correlation between stock returns and earnings shocks generates a very large negative hedging demand, with a significant crowding-out effect on stock holdings. In this case, during working-life the typical household never invests more than 45% of her wealth in the stock market, and immediately after paying the fixed cost she starts holdings long-term bonds in addition to stocks.

As mentioned before, for the households that have incurred the fixed cost, long-term bonds become a very good substitute for cash. As a result the share of financial wealth allocated to cash drops almost to zero. This result rationalizes the asset allocation puzzle identified by Canner, Mankiw and Weil (1997): popular financial advisors recommend that more risk-averse investors should allocate a higher fraction of their risky portfolio (stocks plus bonds) to bonds. This is inconsistent with the predictions of the static and frictionless Capital Asset Pricing Model which implies that all investors should hold the same combination of risky assets; risk aversion only determines the size of the investment to the risky assets as a whole. Brennan and Xia (2002 and 2000), Campbell, Chan and Viceira (2002) and Campbell and Viceira (2001) show how this result can be derived in the context of an intertemporal asset allocation model with time-varying expected returns. In this paper we show that the combination of borrowing constraints and undiversifiable human capital also rationalizes this puzzle. In a 2-asset model, without long-term bonds, the more risk-averse investors hold less stocks and more cash. In the 3-asset model, since bonds substitute for cash, they hold less stocks and more bonds.

The paper is organized as follows. Section 2 summarizes results from the existing empirical literature on life-cycle asset allocation. Section 3 lays out the consumption and portfolio choice model and outlines the numerical solution method, while the corresponding results are shown and discussed in Section 4. Section 5 provides some robustness/sensitivity analysis while in Section 6 we present an explanation for the Canner, Mankiw and Weil puzzle. Section 7 concludes.

2 Empirical Evidence on Life-Cycle Asset Allocation and Stock Market Participation

In most industrialized countries, stock market participation rates have increased substantially during the last decade.[7] Nevertheless, the majority of the population still does not own any stocks (either directly or indirectly through pension funds), and the participation decision occurs gradually over the life cycle. The empirical evidence on life-cycle asset allocation is significantly constrained by data limitations and important identification problems.[8] Figures 1.1 and 1.2 summarize evidence reported in Ameriks and Zeldes (2001). Figure 1.1 plots the average life-cycle equity holdings (as a share of total financial wealth), based on the 1989, 1992 and 1995 waves of the Survey of Consumer Finances (SCF). With respect to the life-cycle profile, the results are particularly sensitive to the inclusion of time dummies versus the inclusion of cohort dummies. In both cases, the average stock holdings are quite low. Figure 1.2 plots the corresponding stock market rate participation rate, based on the same data.[9] These results are less sensitive to the choice of time versus cohort dummies. As expected, a very large fraction of the population does not own equities. In both cases the participation rate gradually increases until approximately age 50. When including cohort dummies, the profile is flat after age 50, while with time dummies it is decreasing. Ameriks and Zeldes (2001) obtain exactly these same results using TIAA-CREF data from 1987-1996, and so do Poterba and Samwick (1999), using SCF data. The next two figures (1.3 and 1.4) report evidence from Guiso, Haliassos and Japelli (2001), using cross-sectional information for five different countries (U.S.A., U.K., Netherlands, Germany and Italy). Figure 1.3 plots equity holdings as a fraction of total financial wealth, conditional on stock market participation. We observe an increasing pattern for 4 countries (the U.K. is the exception), and again a very low

[7]Hong, Kubik and Stein (2001) and Calvet, Gonzalez-Eiras and Sodini (2001) present models of time-varying participation that provide two different rationales for this behavior.

[8]Ameriks and Zeldes (2001) provide a good discussion and illustration of the problems associated with identifying time, cohort and age effects in the context of life-cycle asset allocation.

[9]The profiles were obtained by running a Probit regression.

level of stock holdings.[10] Figure 1.4 plots the participation rates for the different countries. It shows an increasing participation rate until age 60: for all countries the participation rate is higher for the age bracket 50-60 than for the age bracket 20-30. After age 60, 4 out of 5 countries have a decreasing participation rate, which could be due to cohort effects.

Although several other papers have contributed to this research, our objective here is to briefly report the main results, rather than provide a detailed literature survey.[11] We can summarize the existing evidence as follows. First, at least 50% of the population does not own equities. Second, participation rates increase significantly during working life and there is some evidence suggesting that they might decrease during retirement although this might be due to cohort effects. Third, conditional on stock market participation, households invest a large fraction of their financial wealth in alternative assets but there is no clear pattern of equity holdings over the life-cycle.

3 The Model

3.1 Preferences

Time is discrete and t denotes adult age which following the typical convention in this literature, corresponds to effective age minus 19. Households have Epstein-Zin-Weil utility functions (Epstein and Zin (1989) and Weil (1989)) defined over one single non-durable consumption good:

$$U_t = \{(1-\beta)C_t^{1-1/\psi} + \beta(E_t[U_{t+1}^{1-\rho}])^{\frac{1-1/\psi}{1-\rho}}\}^{\frac{1}{1-1/\psi}} \tag{1}$$

where ρ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution, β is the discount factor and C_t is the consumption level at time t.[12]

[10] Note that, even ignoring time and cohort effects, Figures 1a and 1c are not directly comparable because Figure 1c also conditions on stock market participation, which explains the higher level of stockholdings.

[11] Other important papers on this topic include Heaton and Lucas (2000) and Guiso, Jappelli and Terlizzese (1996) (which focus mostly on the impact of background risk on asset allocation), Bertaut and Haliassos (1997) and King and Leape (1998).

[12] For $\rho = 1/\psi$ we have the standard, power (CRRA) utility specification.

3.2 Adjusting for life-cycle patterns in expenditures

Our utility function ignores certain factors which might generate life-cycle patterns in household expenditures. First, the presence of durable goods, and in particular housing, can provide an incentive for higher spending early in life (for example if there are frictions in the corresponding rental markets). Second, household composition often changes over the life cycle and an increase in the number of children will typically be associated with higher household consumption. Modelling these decisions directly is beyond the scope of the paper, but nevertheless we will take into account these potential patterns in life-cycle expenditures.[13] More specially, for each age (t) we use the estimate the percentage of household income that is dedicated to housing expenditures (he_t) and we subtract it from the disposable income.[14]

3.3 Retirement and Bequests

The agent lives for a maximum of T periods, and retirement occurs at time K, $K < T$. For simplicity K is assumed to be exogenous and deterministic. We allow for uncertainty in T in the manner of Hubbard, Skinner and Zeldes (1995). The probability that a consumer/investor is alive at time $(t+1)$ conditional on being alive at time t is denoted by p_t ($p_0 = 1$). Although the numerical solution can easily accommodate a bequest motive, we set it to zero in this model.

3.4 Labor Income Process

The labor income process is the same as the one used by Gourinchas and Parker (2001), or Cocco, Gomes and Maenhout (1999). Before retirement, the exogenous stochastic process for individual labor income is given by

$$Y_{it} = P_{it} U_{it} \tag{2}$$

$$P_{it} = \exp(f(t, Z_{it})) P_{it-1} N_{it} \tag{3}$$

[13] A similar approach is taken by Flavin and Yamashita (02) in a model without labor income.

[14] The details on the estimation, and the law of motion for wealth are described in detail below.

where $f(t, Z_{it})$ is a deterministic function of age and household characteristics Z_{it}, P_{it} is a "permanent" component, and U_{it} a transitory component. We assume that the $\ln U_{it}$, and $\ln N_{it}$ are each independent and identically distributed with mean $\{-.5 * \sigma_u^2, -.5 * \sigma_n^2\}$[15],and variances σ_u^2, and σ_n^2, respectively. The log of P_{it}, evolves as a random walk with a deterministic drift, $f(t, Z_{it})$.

Given these assumptions, the growth in individual labor income follows

$$\Delta \ln Y_{it} = f(t, Z_{it}) + \ln N_{it} + \ln U_{it} - \ln U_{it-1}, \tag{4}$$

and its unconditional variance equals $(\sigma_n^2 + 2\sigma_u^2)$. This process has a single Wold representation that is equivalent to the MA(1) process for individual earnings growth estimated using household level data (MaCurdy (1982), Abowd and Card (1989), and Pischke (1995)).[16]

Earnings in retirement ($t > K$) follow: $Y_{it} = \lambda P_{iK}$, where λ is the replacement ratio (a scalar between zero and one). Although oversimplified, this specification considerably facilitates the solution of the model, as it does not require the introduction of an additional state variable (see subsection 3.6).

3.5 Assets and wealth accumulation

The investment opportunity set is constant and there are three financial assets, one riskless asset (treasury bills or cash) and two risky assets, stocks and long-term (government) bonds. The riskless asset yields a constant gross after tax real return, R^f, while the risky assets returns are denoted by R_t^S and R_t^B.The returns on the risky assets are given by

$$R_{t+1}^S - R^f = \mu^S + \varepsilon_{t+1}^S \tag{5}$$

$$R_{t+1}^{LB} - R^f = \mu^{LB} + \varepsilon_{t+1}^{LB} \tag{6}$$

where $\varepsilon_t^S \sim N(0, \sigma_{\varepsilon^S}^2)$ and $\varepsilon_t^B \sim N(0, \sigma_{\varepsilon^B}^2)$.[17]

[15] With this specification the mean of the level of the log random variables equals 1.

[16] Although these studies generally suggest that individual income changes follow a MA(2), the MA(1) is found to be a close approximation.

[17] With this specification we abstract from term structure considerations (or correlation between stock returns and bond returns) since we are not interested in market timing issues (see, among others, Campbell,

We allow for positive correlation between stock returns and earnings shocks, as a number of recent theoretical papers argue for the importance of positive correlation between stock returns and (permanent) earnings shocks for portfolio choice decisions.[18,19]. More formally, we let the permanent earnings innovation follow:

$$\ln N_{it} = (\phi \frac{\varepsilon_t^S}{\sigma_\varepsilon^S} + (1-\phi^2)^{1/2} \ln N_{it}^*)\sigma_n \tag{7}$$

where $\ln N_{it}^*$ follows a standard normal, and ϕ is the correlation coefficient between $\ln N_{it}$ and $\frac{\varepsilon_t}{\sigma_\varepsilon}$.

The household must allocate her cash-on-hand (X_{it}) between consumption expenditures (C_{it}) and savings, so that

$$X_{it} = C_{it} + S_{it} + LB_{it} + B_{it} \tag{8}$$

where S_{it}, LB_{it} and B_{it} denote respectively stock holdings, holdings of long-term bonds and riskless asset holdings (cash) at time t.

An agent who has never participated in the stock market prior to date t enters that period with wealth invested in riskless assets (B_{it-1}) and receives Y_{it} of labor income. The investor then decides whether to pay a fixed lump sum to invest in any of the risky assets. This entry fee represents both the explicit transaction cost from opening a brokerage account and, more importantly, the (opportunity) cost of acquiring information about the stock market.[20] We define a dummy variable I_P which is equal to one when the cost is incurred for the first time

Chan and Viceira (2002), Brennan and Xia (2002), Campbell and Viceira (2001), or Brennan, Schwartz and Lagnado (1997)).

[18]Viceira (2001), Michaelides (2001), Heaton and Lucas (2000), Campbell, Cocco, Gomes and Maenhout (2000), Cocco, Gomes and Maenhout (1999), Haliassos and Michaelides (2002) conclude that when stock returns are positively correlated with labor income shocks, the implied negative hedging demands can be quite large, thus significantly crowding-out stock holdings, without a first-order impact on the consumption decision. Large positive correlation between wages and stock returns is also generated endogenously in general equilibrium models with a Cobb-Douglas production technology (Storesletten, Telmer, Yaron (2001)) and substantially affects the optimal asset allocation over the life-cycle.

[19]The empirical evidence on the magnitude of this correlation is mixed (see discussion below).

[20]One could argue that the fixed cost is smaller for government bonds than for stocks. Naturally this would only help the model as it would decrease stock market participation early in life. However, since the cost is already hard to calibrate, we decided not to pursue this possibility further at this stage.

and zero otherwise. The fixed cost (F) is scaled by the level of permanent component of labor income (P_{it}) both because this will (significantly) simplify the solution of the model, but this assumption is also motivated by the opportunity cost interpretation for the entry fee.

Following Deaton (1991) we denote cash on hand as the liquid resources available for consumption and saving. When the stock market entry fee has not been paid yet, cash on hand is given by

$$X_{it+1} = S_{it}R^S_{t+1} + LB_{it}R^B_{t+1} + B_{it}R^f + (1 - he_t)Y_{it+1} - FI_P P_{it+1} \tag{9}$$

where he_t is the fraction of income dedicated to housing-related expenditures. If the fixed cost was already paid in the past, we have

$$X_{it+1} = S_{it}\widetilde{R}_{t+1} + LB_{it}R^B_{t+1} + B_{it}R_f + (1 - he_t)Y_{it+1} \tag{10}$$

Following Deaton (1991), we prevent households from borrowing against their future labor income. More specifically we impose the following restrictions:

$$B_{it} \geq 0 \tag{11}$$

$$LB_{it} \geq 0 \tag{12}$$

$$S_{it} \geq 0 \tag{13}$$

3.6 The optimization problem

The complete optimization problem can now be written as

$$MAX_{\{S_{it},LB_{it},B_{it}\}_{t=1}^{T}} E \sum_{t=1}^{T} \beta^{t-1}\{\Pi_{j=0}^{t-1}p_j\}U_t, \tag{14}$$

where E is the expectation conditional on information available at time 1 and

$$U_t = \{(1-\beta)C_t^{1-1/\psi} + \beta(E_t[U_{t+1}^{1-\rho}])^{\frac{1-1/\psi}{1-\rho}}\}^{\frac{1}{1-1/\psi}} \tag{15}$$

subject to the constraints given by equations (5) to (13), and to the stochastic labor income process given by

$$Y_{it} = P_{it}U_{it} \tag{16}$$

$$P_{it} = \exp(f(t, Z_{it}))P_{it-1}N_{it} \tag{17}$$

if $t \leqslant K$, and

$$Y_{it} = \lambda P_{iK} \tag{18}$$

if $t > K$. The share of liquid wealth invested in the stock market (long-term bonds) at age t is denoted by α_t^S (α_t^{LB}).

3.7 Solution Method

Analytical solutions to this problem do not exist. We therefore use a numerical solution method based on the maximization of the value function to derive optimal policy functions for total savings and the share of wealth invested in the stock market. The details are given in appendix A, and here we just present the main idea.

We first simplify the solution by exploiting the scale-independence of the maximization problem and rewriting all variables as ratios to the permanent component of labor income (P_{it}). The equations of motion and the value function can then be rewritten as normalized variables and we use lower case letters to denote them (for instance, $x_{it} \equiv \frac{X_{it}}{P_{it}}$). This allows us to reduce the number of state variables to three: age (t), normalized cash-on-hand (x_{it}) and participation status (whether the fixed cost has already been paid or not). In the last period the policy functions are trivial (the agent consumes all available wealth) and the value function corresponds to the indirect utility function. We can use this value function to compute the policy rules for the previous period and given these, obtain the corresponding value function. This procedure is then iterated backwards. We optimize over the different choices using grid search.

3.8 Computing Transition Distributions

It is common practice for researchers to simulate a model over the life-cycle for a large number of individuals (say 10000) to compute the statistics of interest (mean wealth holdings, for instance) for any given age. We propose an alternative method of computing various statistics that is based on the explicit calculation of the transition distribution of cash on hand from one age to the next. There are a number of important advantages in using the transition distributions rather than simulation in this analysis. First, there is no need to use an interpolation procedure at any stage (see appendix B), thereby avoiding potential numerical approximation errors that might arise from interpolation. Second, the distributions provide an explicit metric of inequality for different variables over time and therefore can potentially be more suitable in matching the model to the data. The computational details are delegated to Appendix B. Once we have the transition densities we can easily obtain all the relevant population means at each age. The unconditional mean consumption for age t can then be computed as[21]

$$\overline{c}_t = \theta_t \left\{ \sum_{j=1}^{J} \pi^I_{t,j} * c^I(x_j, t) \right\} + (1 - \theta_t) \left\{ \sum_{j=1}^{J} \pi^O_{t,j} * c^O(x_j, t) \right\} \tag{19}$$

where J is the number of grid points used in the discretization of normalized cash on hand, and $\pi^I_{t,j}$ and $\pi^O_{t,j}$ are the probability masses associated with each grid point, respectively for stockholders and non-stockholders. The participation rate at age t (θ_t) is given by

$$\theta_t = \theta_{t-1} + (1 - \theta_{t-1}) * \sum_{x_j > x^*} \pi^O_{t,j} \tag{20}$$

where x^* is the trigger point that causes participation, which is determined endogenously through the participation decision rule. For a given age t, the unconditional portfolio allocation are computed as:

$$\overline{\alpha}^S_t = \frac{\theta_t * \{\sum_{j=1}^{J} \pi^I_{t,j} * \alpha^S(x_j, t) * (x_j - c^I(x_j, t))\}}{\theta_t * \sum_{j=1}^{J}[\pi^I_{t,j} * (x_j - c^I(x_j, t))] + (1 - \theta_t) * \sum_{j=1}^{J}[\pi^O_{t,j} * (x_j - c^O(x_j, t))]} \tag{21}$$

$$\overline{\alpha}^{LB}_t = \frac{\theta_t * \{\sum_{j=1}^{J} \pi^I_{t,j} * \alpha^{LB}(x_j, t) * (x_j - c^I(x_j, t))\}}{\theta_t * \sum_{j=1}^{J}[\pi^I_{t,j} * (x_j - c^I(x_j, t))] + (1 - \theta_t) * \sum_{j=1}^{J}[\pi^O_{t,j} * (x_j - c^O(x_j, t))]} \tag{22}$$

and

$$\overline{\alpha}^B_t = 1 - \overline{\alpha}^S_t - \overline{\alpha}^{LB}_t \tag{23}$$

[21] Superscript I denotes a variable for households participating in the stock market while superscript O denotes households out of the stock market.

3.9 Parameter Calibration

We begin by solving the model under a set of "baseline" parameter assumptions at an annual frequency but we also investigate how our results vary when these parameter values are changed.

3.9.1 Preference Parameters

For most of the paper we will use a coefficient of relative risk aversion (ρ) equal to 5 and an elasticity of intertemporal substitution (ψ) equal to 0.2. With these preference parameters the Epstein-Zin-Weil utility function becomes the standard power utility. However we will also report results for other combinations, including cases with $\rho \neq 1/\psi$. In all cases we set the discount factor (β) equal to 0.95.

3.9.2 Labor Income Process

Carroll (1992) estimates the variances of the idiosyncratic shocks using data from the *Panel Study of Income Dynamics*, and our baseline simulations use values close to those: 0.1 percent per year for σ_u and 0.08 percent per year for σ_n. The deterministic labor income profile reflects the hump shape of earnings over the life-cycle, and the corresponding parameter values, just like the retirement transfers (λ), are taken from Cocco, Gomes and Maenhout (1999). It is common practice to estimate different labor income profiles for different education groups (college graduates, high-school graduates, households without a high-school degree). In the paper we only report the results obtained with the parameters estimated for the sub-sample of high-school graduates as the results for the other two groups are very similar.

3.9.3 Asset Returns

The constant net real interest rate ($R^f - 1$) is set at 2 percent. The bond return process is calibrated using the historical mean and standard deviation of twenty-year US government bonds, so $\mu^B = 2\%$, and $\sigma_{\varepsilon^B} = 8\%$.[22] For the stock return process we consider a mean equity

[22] The values are not particularly sensitive to the choice of the maturity date.

premium (μ^S) equal to 4 percent and a standard deviation (σ_{ε^S}) of 18 percent. Considering an equity premium of 4% (as opposed to the historical 6%) is a fairly common choice in this literature (e.g. Yao and Zhang (2002), Cocco (2001) or Campbell et al. (2001)) and this is motivated by two different concerns. First, Campbell et al. (2001) argue that this is actually a better measure of a forward-looking equity premium. Second, even after having paid the fixed entry cost, small investors typical face non-trivial transacion costs. This adjustment can be seen as a short-cut representation for these costs, since the dimensionality of the problem prevents us from modelling them explicitly (as in Heaton and Lucas (1996), for example).

The empirical evidence on the correlation between labor income shocks and stock returns is mixed. Davis and Willen (2001) find evidence for positive correlation but Campbell et al. (2001) and Heaton and Lucas (2000) only find such results when considering small subgroups of the population (e.g. self-employed households or households with private businesses). However it has been argued that these estimations using micro-data suffer from a small sample bias (as the time-series dimension is too short) and in fact estimations using macro data and longer time horizons usually yield more significant correlations (see, for example Jermann (1999)). Therefore we start by considering a case with zero correlation ($\phi = 0$) and later on we will relax this assumption.

With respect to the fixed cost of participation, for most of the paper we will consider two cases: one where the cost is zero, and one where it equals 0.1 (10% of the household's expected annual income, gross of housing expenditures).

3.9.4 Housing expenditures

We measure housing expenditures using data from the Panel Study of Income Dynamics from 1976 until 1993.[23] For each household, in each year, we compute the ratio of annual mortgage payments and rent payments (housing related expenditures - HE) relative to annual labor

[23]Before 1976 there is no information on mortgage expenditures, and 1993 is the last year available on final release from the PSID.

income (Y):

$$he_{it} \equiv \frac{HE_{it}}{Y_{it}} \tag{24}$$

We combine mortgage payments and rent together since we are not modelling the housing decision explicitly.[24] Consistent with the previous literature, and with our definition below, the measure of annual labor income also includes transfers from relatives and workers compensation. We then identify the age effects by running the following regression on the full panel:

$$he_{it} = A + B_1 * age + B_2 * age^2 + B_3 * age^3 + time\ dummies + \zeta_{it} \tag{25}$$

where age is defined as the age of the head of the household. We eliminate all observations with age greater than 75.[25] The estimation results are reported in Table 1. In the model we use

$$he_t = Max(A + B_1 * age + B_2 * age^2 + B_3 * age^3, 0) \tag{26}$$

which, given our parameter estimates, truncates he_t at zero for $age \geqslant 80$.

4 Benchmark Results

4.1 Results without Fixed Participation Cost

4.1.1 Consumption and Wealth

Figures 2.1 and 2.2 plot some of the distributions of normalized cash on hand (x_{it}) for different ages during working life and during retirement, respectively. The distributions can best be thought of as the distributions of cash on hand that would result if a large number of individuals were born at the same time (that is, the evolution of the distribution for cash on hand for a particular cohort). Figure 2.1 shows how wealth accumulation affects the

[24] We do not care whether the household owns or rents the house where she lives, we just assume that she derives the same utility from it, in both cases.

[25] There are several reasons for eliminating these households. First , there are very few observations within each age group beyond age 75. Second, for most of these households the values of he_{it} are equal to zero. Third this is consistent with the estimation procedure used for the labor income process (see Cocco et al. (99)).

distribution of wealth. As households age and wealth accumulation increases, there is a shift of mean cash on hand to a higher level while at the same time the full distribution widens. Figure 2.2 illustrates the wealth decumulation that should occur during retirement. Mean cash on hand now is reduced towards the mean pension level as the individual ages, and the distribution is progressively collapsing to that single point.

Mean normalized consumption ($\overline{c}_t$), mean normalized wealth ($\overline{w}_t$) and mean normalized income net of housing expenditures ($(1 - he_t) * y_t$) are plotted in Figure 2.3. The results are qualitatively the same as the one obtained for the two-asset case (e.g. Cocco et al. (1999)). Early in life the household is liquidity constrained and saves only a small buffer stock of wealth. From age 30 onwards she starts saving for retirement and quickly accumulates significant wealth that allows her to smooth consumption after age 65. During the retirement period, consumption decreases at a very fast pace, as a result of the very high effective discount rate (high mortality risk).[26]

4.1.2 Asset Allocation

Figure 2.4 graphs the unconditional mean asset allocation in equities ($\overline{\alpha}_t^S$), long-term bonds ($\overline{\alpha}_t^{LB}$) and cash ($\overline{\alpha}_t^B$). In the two asset case, even though earnings risk is uninsurable, future labor income is found to be a closer substitute for holding of safe assets rather than risky assets (e.g. Heaton and Lucas (1996) or Viceira (2001)). We find that, by adding long-term bonds to the model this result remains unchanged. Early in life households invest most of their wealth in stocks, since their future labor income (human capital) already provides them with significant implicit holdings of safer assets.[27] With respect to the long-term bond itself, we find that it acts as another close substitute for labor income. Households only invest in these bonds as they get older, and their human capital becomes a progressively smaller fraction of their total wealth. In fact, when retirement savings is at its peak, more than 50% of total wealth is now being invested in long-term bonds and only approximately

[26]Net income increases during the first years of retirement because the housing expenditures (he_t) are positive and decreasing towards zero.

[27]During the very first years of adult life households still hold a small fraction of their wealth in cash since the present value of future labor income is actually still increasing (as shown by Cocco et al. (1999)).

40% is being invested in the stock market. Once retired, the investors increase the share of wealth allocated to stocks because labor income uncertainty has been eliminated. During the retirement stage both future labor income and wealth are falling, and the asset allocation is determined by the relative speed at which these two decrease.[28]

4.2 Results with Fixed Participation Cost

4.2.1 Consumption, Wealth and Participation Decision

We first plot the evolution for the distributions of cash on hand for the two types of agents: participants and non-participants in the stock market. Figure 3.1 illustrates some of the results by plotting the distributions of normalized cash on hand for individuals aged 30 who have incurred the fixed cost and those who have not. These are conditional distributions and the participation rate can be used as a probability weight to generate the unconditional distribution of cash on hand in the cohort. There is a pronounced spike at around the normalized cash on hand level of 0.92; beyond that level of cash on hand, stock market participation becomes optimal and the two distributions overlap for a small interval mostly representing the incurrence of the fixed entry cost. The distribution is now made up almost completely of rich stockholders and poor non-stockholders. Figure 3.2 plots the distributions of cash on hand for ages 50 for both types of agents. One recurring feature of the distribution of wealth figures conditional on age is that the distribution of cash on hand for stock holders has a higher variance than the wealth distribution for the households which have not participated in the stock market.

Figure 3.3 plots the normalized mean wealth, income (net of housing expenditures) and consumption over the life-cycle, while Figure 3.4 shows the participation rate. The wealth accumulation profile is very similar to the one obtained without the fixed cost (Figure 2.3). Wealth accumulation is slightly reduced as most households do not invest in the stock market in the first years of their adult lives. However, as we can see from Figure 3.4, by age 26

[28] Since these models typically include very crude representations of retirement behavior it is both advisible and usual practice not to draw many conclusions or implications from the asset allocations predicted for this period.

the participation rate is already above 50%, and by age 30 almost all households have accumulated enough wealth to warrant payment of the fixed cost. This explains why the wealth and consumption profiles are not significantly affected.

4.2.2 Asset Allocation

The main conclusions from Figure 3.5 (which plots the asset allocation decision) are the following. First, immediately after paying the fixed cost, households invest almost all of their wealth in stocks. The desire to pay the cost is motivated by the willingness to hold stocks, not long-term bonds. One implication of this result is that the stock market participation rate should be virtually the same as in the corresponding 2-asset model without long-term bonds.[29] Second, from age 35 onwards, the mean profiles are almost identical to the ones in Figure 2.4. Since by age 35 all households have already paid the fixed cost, and since we have just concluded that the wealth accumulation is almost the same in both cases, this result was perfectly expected. Third, since during their first adult years the households which have not yet paid the fixed cost must invest all of their wealth in cash, the average cash balances are initially quite high and then decrease very rapidly towards zero. Fourth, for the households that have paid the fixed cost, long-term bonds are a stronger substitute for cash, than for stocks. Relative to the results from the existing two-asset models the share of wealth invested in stocks is not strongly affected by the presence of the new asset. On the other hand cash is almost completely replaced by long-term bonds with its share dropping to zero at age 35 and staying very close to that for most of the life-cycle.

4.2.3 Allowing for Correlation Between Labor Income Shocks and Stock Returns

So far we have not allowed for any correlation between labor income shocks and stock returns ($\phi = 0$). As argued before the empirical evidence is relatively inconclusive, although it seems clear that we can reject very large values of ϕ (see the discussion on the calibration section).

In this subsection we relax this assumption and consider a correlation coefficient (ϕ)

[29] This is indeed the case, and the results are available upon request.

equal to 0.2. With respect to the mean asset allocations (shown in Figure 3.6) we obtain the following results. First, even a moderate degree of correlation generates a very large negative hedging demand, with a significant crowding-out effect on stock holdings. During working-life the typical household never invests more than 45% of her wealth in the stock market. Second, contrary to the results with $\phi = 0$, immediately after paying the fixed cost, households start holding long-term bonds in addition to stocks.[30]

The consumption rules remain essentially the same and therefore so does total savings. As a result, the stock market participation decision is very similar to the one obtained previously, with all households participating in the stock market by age 35.

5 Sensitivity Analysis

5.1 Changing the preference parameters

5.1.1 CRRA utility

We start by decreasing the coefficient of relative risk aversion (ρ) to 2 while reducing the elasticity of intertemporal substitution (ψ) to 0.5, thus maintaining the CRRA assumption. A lower ρ generates less risk aversion which leads to an increase in the optimal share of wealth invested in equities. However, is also generates less prudence which reduces buffer stock wealth accumulation and this decreases the incentive to pay the fixed cost. The balance between these effects will determine the optimal time at which the fixed entry cost is paid.

Figure 4.1 plots the wealth accumulation profile, while figure 4.2 plots the corresponding participation rate. For comparison purposes we also report the results for the benchmark case. We find that wealth accumulation is substantially lower for $\rho = 2$ than for $\rho = 5$. In fact, before age 30, the low-prudence households ($\rho = 2$) save virtually nothing and as a result the participation rate is equal to zero. In contrast, by age 30 almost all high-prudence households have already paid the fixed cost. Nevertheless, once retirement savings start, even the low-prudence households decide to invest in risky assets. Figure 4.2 shows that by age 40 they all have paid the fixed cost as well.

[30] Stockholdings increase signficantly at age 66 since the crowding-out effect disappears with retirement..

These results have a very important implication: an increase in background risk (e.g. by introducing uncertainty about the parameters of the labor income process, consumption risk, or housing/mortgage risk) will not help to solve the participation puzzle. When faced with more background risk the agent will build up a larger buffer stock of wealth thus creating a stronger incentive to enter the stock market.[31]

Figure 4.3 plots the life-cycle asset allocation for both $\rho = 2$ and $\rho = 5$.[32] The less risk-averse investor allocates a higher fraction of her portfolio to stocks and a lower fraction to long-term bonds but, once the fixed cost is paid by all households, the profiles are qualitatively the same in both cases. We will discuss these results again in the next section.

5.1.2 Non-CRRA utility

Motivated by the previous results we now deviate from the CRRA framework. By increasing risk aversion and decreasing the EIS we can reduce the willingness to hold stocks without increasing saving. Our goal is to keep the coefficient of relative risk aversion at our benchmark level of $\rho = 5$ (thereby generating a modest share of wealth allocated to stocks), while simultaneously reducing the elasticity of intertemporal substitution so as to decrease the level of savings (from the weaker motive to smooth consumption intertemporally), and therefore reduce the participation rate. It is important to remember however that with a CRRA utility function, a higher risk aversion coefficient already implies a low value of the EIS. In particular, for $\rho = 5$ we already have $\psi = 0.2$, and since ψ must be non-negative we cannot reduce it much more. We will set $\psi = 0.1$ and consider values of ρ equal to 5 and 4.[33] The corresponding participation rates are shown in figure 4.4 and for comparison purposes we also report the previous results for the two CRRA cases. As expected, a reduction in the

[31] We have in fact consider two different experiments in which we have increased the investor's background risk. In the first one we consider a higher variance for the (permanent and transitory) labor income shocks, and in the second we add a positive probability of an fat tail negative labor income draw. The results, which are available upon request, are as predicted: the investor increases her buffer stock of wealth and as a result she pays the entry cost earlier in life.

[32] The mean allocation to cash is just the residual of the two and therefore we will omit it from now on.

[33] Given our previous results we know that, if we want to decrease the participation rate, we have to consider low values of ρ.

elasticity of intertemporal substitution postpones the participation decision, but the overall impact is relatively small.

5.2 Changing the fixed cost

We now investigate the sensitivity of the results to the size of the fixed entry cost, as this is clearly one of the hardest parameters to calibrate. Figure 4.5 shows the participation rate implied by our benchmark value of F (0.1), and by setting $F = 0.2$, while the mean asset allocation are plotted in Figure 4.6.[34] As expected a higher fixed costs postpones the participation decision until later in life, but the impact of doubling F is actually very small: by age 35 almost all households are already investing stocks. These results indicate that, in the context of the model, the magnitude of fixed cost required to generate significant non-participation is extremely high. The asset allocation decisions are almost identical from age 35 onwards, when all households have paid the entry cost.

6 An explanation for the Canner, Mankiw and Weil puzzle

The results in the paper help to rationalize the asset allocation puzzle identified by Canner, Mankiw and Weil (1997): popular financial advisors recommend that more risk-averse investors should allocate a higher fraction of their risky portfolio (stocks plus bonds) to bonds. This advice is inconsistent with the predictions of static and frictionless Capital Asset Pricing Model which implies that all investors should hold the same combination of risky assets. Their risk aversion should only determine the size of their investment to the risky assets as a whole. Brennan and Xia (2002 and 2000), Campbell, Chan and Viceira (2002) and Campbell and Viceira (2001) show how this result can be derived in the context of an intertemporal asset allocation model with time-varying expected returns. In this paper we show that the combination of borrowing constraints and undiversifiable human capital also rationalizes this

[34]The results are plotted for the case with a correlation coefficient of 0.2, so they should be compared with the ones in Figure 3.6.

puzzle, even with a constant investment opportunity set.

In a 2-asset model, without long-term bonds, the more risk-averse investors hold less stocks and more cash. In the 3-asset model, since bonds substitute for cash, they hold less stocks and more bonds (see Figure 4.3). Canner et. al. (1997) also restate this puzzle as implying that the ratio of bonds to stocks should increase when the ratio of stocks to total financial wealth falls. In the context of our model we do not need heterogeneity in risk aversion to generate this prediction. As shown in Figure 2.4, the ratio of stocks to total financial wealth falls at mid-life, as the ratio of human capital to financial wealth falls and the investor is less willing to take risks. The decrease in stock holdings is compensated by higher bond holdings, thus leading to an increase in the ratio of bonds to stocks, as recommended by financial consultants.

7 Conclusion

This paper studies optimal consumption and portfolio choice over the life-cycle in the presence of non-diversifiable idiosyncratic labor income risk, borrowing and short sale constraints and fixed, one-time, transaction costs associated with the investment in risky assets.

The model is quite successful in matching asset allocation decisions. A moderate degree of correlation between stock returns and earnings shocks generates a very large negative hedging demand, with a significant crowding-out effect on stock holdings. During working-life the typical household never invests more than 45% of her wealth in the stock market. For the households that have paid the fixed cost, long-term bonds are a stronger substitute for cash, than for stocks, and this result rationalizes the asset allocation puzzle identified by Canner, Mankiw and Weil (1997).

Nevertheless we argue that asset allocation models must match another important feature of the data: a low stock market participation rate. Along this dimension the model provides a very modest improvement and it still predicts a counterfactually high participation rate. With a coefficient of relative risk aversion of 5, by age 26 the participation rate is already above 50%, and by age 30 almost all households have paid the entry cost. A lower coefficient of relative risk aversion actually leads to a decrease in the participation rate. This result occurs because of the link between prudence and risk aversion, which implies that a less risk-averse investor accumulates less wealth early in life, and this decreases the incentive to pay the fixed cost. Nevertheless, once retirement savings start, even the low-prudence households decide to invest in risky assets. With a coefficient of relative risk aversion of 2, by age 40 all households have already paid the fixed cost. These results have a very important implication: an increase in background risk (e.g. by introducing uncertainty about the parameters of the labor income process, consumption risk, or housing/mortgage risk) will not help to solve the participation puzzle either. When faced with more background risk the agent will build up a larger buffer stock of wealth thus creating a stronger incentive to enter the stock market.

Appendix A: Numerical Solution Method

We first simplify the solution by exploiting the scale-independence of the maximization problem and rewriting all variables as ratios to the permanent component of labor income (P_{it}). The equations of motion and the value function can then be rewritten as normalized variables and we use lower case letters to denote them (for instance, $x_{it} \equiv \frac{X_{it}}{P_{it}}$). This allows us to reduce the number of state variables to three; one continuous state variable (cash on hand) and two discrete state variables (age and participation status). The relevant policy functions can then be computed as functions of the three state variables: age (t), normalized cash-on-hand (x_{it}), and participation status (whether the fixed cost has already been paid or not). In the last period the policy functions are trivial (the agent consumes all available wealth) and the value function corresponds to the indirect utility function. We can use this value function to compute the policy rules for the previous period and given these, obtain the corresponding value function. This procedure is then iterated backwards.

We discretize the state-space (cash-on-hand) and use Gaussian quadrature to approximate the distributions of the innovations to the labor income process and risky asset returns (Tauchen and Hussey (1991)). In every period t prior to T, and for each admissible combination of the state variables, we compute the value associated with each level of consumption, the decision to pay the fixed cost, and the share of liquid wealth invested in both stocks and long-term bonds. This value is equal to current utility plus the expected discounted continuation value. To compute this continuation value for points which do not lie on the grid for the state space we use a univariate cubic spline interpolation procedure along the cash on hand dimension. The participation decision is obtained by comparing the value function conditional on having paid the fixed cost (adjusting for the payment of the entry cost itself) with the value function conditional on non-payment, for each point of the state space (t and x_i). We optimize over the different choices using grid search.

Appendix B: Computing the Transition Distributions

To find the distribution of cash on hand, we first compute the relevant optimal policy rules; bond and stock policy functions for stock market participants and non-participants and the $\{0,1\}$ participation rule as a function of cash on hand. Let $b^I(x)$, $Lb^I(x)$ and $s^I(x)$ denote respectively the cash, (long-term) bonds and stock policy rules for individuals participating in the stock market, and let $b^O(x)$ be the savings decision for the individual out of the stock market. We assume that households start their working life with zero liquid assets. During working life, for the households that have not paid the fixed cost, the evolution of normalized cash on hand is given by[35]

$$\begin{aligned} x_{t+1} &= [b^O(x_t)R_f]\left\{\frac{P_t}{P_{t+1}}\frac{\exp(f(t,Z_t))}{\exp(f(t+1,Z_{t+1}))}\right\}+(1-he_{t+1})U_{t+1} \\ &= w\left(x_t|\frac{P_t}{P_{t+1}},\frac{\exp(f(t,Z_t))}{\exp(f(t+1,Z_{t+1}))}\right)+(1-he_{t+1})U_{t+1} \end{aligned} \tag{27}$$

where $w(x)$ is defined by the last equality and is conditional on $\{\frac{P_t}{P_{t+1}}\}$ and the deterministically evolving $\frac{\exp(f(t,Z_t))}{\exp(f(t+1,Z_{t+1}))}$. Denote the transition matrix of moving from x_j to x_k,[36] conditional on not having paid the fixed cost as T^O_{kj}. Let Δ denote the distance between the equally spaced discrete points of cash on hand. The random permanent shock $\frac{P_t}{P_{t+1}}$ is discretized using Gaussian quadrature with H points: $\frac{P_t}{P_{t+1}}=\{N_m\}_{m=1}^{m=H}$. $T^O_{kj}=\Pr(x_{t+1=k}|x_{t=j})$ is found using[37]

$$\sum_{m=1}^{m=H}\Pr\left(x_{t+1}|x_t,\frac{P_t}{P_{t+1}}=N_m\right)*\Pr\left(\frac{P_t}{P_{t+1}}=N_m\right) \tag{28}$$

Numerically, this probability is calculated using

$$T^O_{kjm}=\Pr\left(x_k+\frac{\Delta}{2}\geqslant x_{t+1}\geqslant x_k-\frac{\Delta}{2}|x_t=x_j,\frac{P_t}{P_{t+1}}=N_m\right) \tag{29}$$

Making use the approximation that for small values of σ_u^2, $U\sim N(\exp(\mu_u+.5*\sigma_u^2),(\exp(2*\mu_u+(\sigma_u^2))*(\exp(\sigma_u^2)-1)))$, and denoting the mean of $(1-he_t)U$ by $\overline{U}$ and its standard

[35] To avoid cumbersome notation, the subscript i that denotes a particular individual is omitted in what follows.

[36] The normalized grid is discretized between $(x\min, x\max)$ where $x\min$ denotes the minimum point on the equally spaced grid and $x\max$ the maximum point.

[37] The dependence on the determinastically evolving $\frac{\exp(f(t,Z_t))}{\exp(f(t+1,Z_{t+1}))}$ is implied and is omitted from what follows for expositional clarity.

deviation by σ, the transition probability conditional on N_m equals

$$T^O_{kjm} = \Phi\left(\frac{x_k + \frac{\Delta}{2} - w(x_t|N_m) - \overline{U}}{\sigma}\right) - \Phi\left(\frac{x_k - \frac{\Delta}{2} - w(x_t|N_m) - \overline{U}}{\sigma}\right) \tag{30}$$

where Φ is the cumulative distribution function for the standard normal. The unconditional probability from x_j to x_k is then given by

$$T^O_{kj} = \sum_{m=1}^{m=H} T^O_{kjm} \Pr(N_m) \tag{31}$$

Given the transition matrix $\mathbf{T}^O$ (letting the number of cash on hand grid points equal to J, this is a J by J matrix; T^O_{kj} represents the {kth,jth} element), the next period probabilities of each of the cash on hand states can be found using

$$\pi^O_{kt} = \sum_j T^O_{kj} * \pi^O_{jt-1} \tag{32}$$

We next use the vector $\mathbf{\Pi}^O_t$ (this is a J by 1 vector representing the mass of the population out of the stock market at each grid point; π^O_{kt} represents the {kth} element at time t) and the participation policy rule to determine the percentage of households that optimally choose to incur the fixed cost and invest in risky assets. This is found by computing the sum of the probabilities in $\mathbf{\Pi}^O_t$ for which $x > x^*$, x^* being the trigger point that causes participation (x^* is determined endogenously through the participation decision rule). These probabilities are then deleted from $\mathbf{\Pi}^O_t$ and are added to $\mathbf{\Pi}^I_t$, appropriately renormalizing both $\{\mathbf{\Pi}^O_t, \mathbf{\Pi}^I_t\}$ to sum to one. The participation rate (θ_t) can be computed at this stage as

$$\theta_t = \theta_{t-1} + (1 - \theta_{t-1}) * \sum_{x_j > x^*} \pi^O_{t,j} \tag{33}$$

The same methodology (but with more algebra and computations) can then be used to derive the transition distribution for cash on hand conditional on having paid the fixed cost, $\mathbf{T}^I_t$. The corresponding normalized cash on hand evolution equation is

$$\begin{aligned} x_{t+1} &= [b(x_t)R^f + Lb(x_t)R^B_{t+1} + s(x_t)R^S_{t+1}]\left\{\frac{P_t}{P_{t+1}}\frac{\exp(f(t,Z_t))}{\exp(f(t+1,Z_{t+1}))}\right\} + (1 - he_{t+1})U_{t+1} \\ &= w\left(x_t|R^S_{t+1}, R^B_{t+1}, \frac{P_t}{P_{t+1}}\right) + (1 - he_{t+1})U_{t+1} \end{aligned} \tag{34}$$

where $w(x)$ is now conditional on $\{R^S_{t+1}, R^B_{t+1}, \frac{P_t}{P_{t+1}}\}$[38]. The random processes R^S_{t+1}, R^B_{t+1} and $\frac{P_t}{P_{t+1}}$ are discretized using Gaussian quadrature with H points: $R^S_{t+1} = \{R^S_l\}_{l=1}^{l=H}$, $R^B_{t+1} = \{R^B_m\}_{m=1}^{m=H}$ and $\frac{P_t}{P_{t+1}} = \{N_n\}_{n=1}^{n=H}$. $T^I_{kj} = \Pr(x_{t+1=k}|x_{t=j})$ is obtained from

$$\sum_{l=1}^{l=H}\sum_{m=1}^{m=H}\sum_{n=1}^{n=H} \Pr\left(x_{t+1}|x_t, R^S_{t+1} = R^S_l, R^B_{t+1} = R^B_m, \frac{P_t}{P_{t+1}} = N_n\right) * \Pr(R^S_l) * \Pr(R^B_m) * \Pr(N_n) \tag{35}$$

where $\Pr(R^S_l)$, $\Pr(R^B_m)$ and $\Pr(N_n)$ stand respectively for $\Pr(R^S_{t+1} = R^S_l)$, $\Pr(R^B_{t+1} = R^B_m)$ and $\Pr\left(\frac{P_t}{P_{t+1}} = N_n\right)$, and where the independence between $\frac{P_t}{P_{t+1}}$, R^S_{t+1} and $R^B_{t+1}$was used.[39] Numerically, this probability is calculated using

$$T^I_{kjlmn} = \Pr\left(x_k + \frac{\Delta}{2} \geqslant x_{t+1} \geqslant x_k - \frac{\Delta}{2}|x_t = x_j, \frac{P_{it}}{P_{it+1}} = N_n, R^S_{t+1} = R^S_l, R^B_{t+1} = R^B_m\right) \tag{36}$$

The transition probability conditional on N_n, R^S_l and R^B_m equals

$$T^I_{kjlmn} = \Phi\left(\frac{x_k + \frac{\Delta}{2} - w(x_t|N_n, R^S_l, R^B_m) - \overline{U}}{\sigma}\right) - \Phi\left(\frac{x_k - \frac{\Delta}{2} - w(x_t|N_n, R^S_l, R^B_m) - \overline{U}}{\sigma}\right) \tag{37}$$

The unconditional probability from x_j to x_k is then given by

$$T^I_{kj} = \sum_{l=1}^{l=H}\sum_{m=1}^{m=H}\sum_{n=1}^{n=H} T^I_{kjlmn} \Pr(R^S_l)\Pr(R^B_m)\Pr(N_n) \tag{38}$$

Given the matrix $\mathbf{T}^I$, the probabilities of each of the states are updated by

$$\pi^I_{kt+1} = \sum_j T^I_{kj} * \pi^I_{jt} \tag{39}$$

[38]The dependence on the non-random earnings component is omitted to simplify notation.

[39]The methodology can be applied for an arbitrary correlation structure between the stock market and permanent shock innovation.

References

[1] Abowd, John and David Card. 1989. "On the Covariance Structure of Earnings and Hours Changes." *Econometrica* 57: 411-45.

[2] Aiyagari, Rao, S. and Gertler, Mark. 1991. "Asset Returns With Transactions Costs and Uninsured Individual Risk." *Journal of Monetary Economics*, 27(3), pp. 311-31.

[3] Ameriks, John and Zeldes, Stephen. 2001. "How Do Household Portfolio Shares Vary With Age?" *Working Paper,* Columbia Business School.

[4] Attanasio, Orazio; Banks, James; and Tanner, Sarah. 2002. "Asset Holding and Consumption Volatility" *Journal of Political Economy*, forthcoming.

[5] Basak, Suleyman and Cuoco, Domenico. 1998. "An Equilibrium Model with Restricted Stock Market Participation." *Review of Financial Studies*, 11(2), pp. 309-41.

[6] Bertaut, C. and Michael Haliassos. 1997. "Precautionary Portfolio Behavior from a Life-Cycle Perspective." *Journal of Economic Dynamics and Control*, 21, 1511-1542.

[7] Brav, Alon, Constantinides, George M. and Geczy, Christopher C. 2002. "Asset Pricing with Heterogeneous Consumers and Limited Participation: Empirical Evidence." *Journal of Political Economy*, forthcoming.

[8] Brennan, Michael and Yihong Xia. 2002. "Dynamic Asset Allocation Under Inflation." *Journal of Finance*, Vol. LVII, No. 3, pp. 1201-1238.

[9] Brennan, Michael and Yihong Xia, 2000, "Stochastic Interest Rates and Bond-Stock Mix." *European Finance Review*, 4, 197-210.

[10] Brennan, M., E. Schwartz and R. Lagnado. 1997. "Strategic Asset Allocation." *Journal of Economic Dynamics and Control* 21, 1377-1403.

[11] Cagetti, Marco. 2001. "Wealth Accumulation over the life cycle and Precautionary Savings" UVA working paper.

[12] Calvet, Laurent, Martin Gonzalez-Eiras and Paolo Sodini. 2001. "Financial Innovation, Market Participation and Asset Prices." *Working Paper*, Harvard University.

[13] Campbell, John, Y. Lewis Chan and Luis Viceira. 2002. "A Multivariate Model of Strategic Asset Allocation." *Journal of Financial Economics*, forthcoming.

[14] Campbell, John, Joao Cocco, Francisco Gomes and Pascal Maenhout. 2001. "Investing Retirement Wealth: A Life-Cycle Model." In "Risk Aspects of Social Security Reform." *University of Chicago Press.*

[15] Campbell, John and Luis Viceira. 2001 "Who Should Buy Long-Term Bonds?" *American Economic Review*, 91, 99-127.

[16] Canner, Niko, N. Gregory Mankiw and David N. Weil, "An Asset Allocation Puzzle," *American Economic Review*, 1997, 87, 181-191.

[17] Carroll, Christopher D.. 1992. "The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence." *Brookings Papers on Economic Activity* no. 2: 61-156.

[18] Carroll, Christopher D.. 1997. "Buffer Stock Saving and the Life-Cycle / Permanent Income Hypothesis." *Quarterly Journal of Economics* CXII no. 1: 3-55.

[19] Carroll, Christopher and Andrew Samwick. 1997. "The Nature of Precautionary Wealth." *Journal of Monetary Economics*, 40(1).

[20] Chamberlain, G. and K. Hirano. 1999. "Predictive Distributions Based on Longitudinal Earnings Data." *Annals of Economics and Statistics* Sep-Dec, 211-242.

[21] Cocco, Joao. 2000. "Portfolio Choice in the Presence of Housing." *Working Paper.* London Business School.

[22] Cocco, J., F. Gomes and P. Maenhout. 1999. "Portfolio Choice Over The Life-Cycle." *Working Paper.* Harvard University.

[23] Constantinides, George. 1986. "Capital Market Equilibrium with Transaction Costs." *Journal of Political Economy*; 94, pages 842-62.

[24] Constantinides, George, Donaldson John and Mehra Rajnish. 2001. "Junior Can't Borrow: A New Perspective on the Equity Premium Puzzle." *Quarterly Journal of Economics.*

[25] Dammon, Robert, Chester Spatt and Harold Zhang. 2001. "Optimal Consumption and Investment with Capital Tax Gains" *Review of Financial Studies*, forthcoming.

[26] Davis, Steven J. and Paul Willen. 2001. "Occupation-level Income Shocks and Asset Returns: Their Covariance and Implications for Portfolio Choice." *Working Paper*, University of Chicago.

[27] Davis, Steven J., Felix Kluber and Paul Willen. 2002. "Borrowing Costs and the Demand for Equity over the Life Cycle." *Working Paper*, University of Chicago and Stanford University.

[28] Deaton, Angus. 1991. "Saving and Liquidity Constraints." *Econometrica* 59(5): 1221-48.

[29] Degeorge, Francois, Dirk Jenter, Alberto Moel and Peter Tufano. 2002. "Selling Company Shares to Reluctant Employees: France Telecom's Experience." *Working Paper*, HEC and Harvard University.

[30] Epstein, Lawrence and Stanley Zin. 1989. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theorethical Framework." *Econometrica*, 57, 937-969.

[31] Flavin, Marjorie and Takashi Yamashita. 2002. "Owner-Occupied Housing and the Composition of the Household Portfolio." *The American Economic Review*, 92(1), pp. 345-362.

[32] Gakidis, Haralabos. 1998. "Stocks for the Old? Earnings Uncertainty and Life-Cycle Portfolio Choice." MIT doctoral thesis.

[33] Gomes, Francisco and Alexander Michaelides. 2002. "Portfolio Choice with Habit Formation: A Life-Cycle Model with Uninsurable Labor Income Risk." *Working Paper*, LBS and LSE.

[34] Gourinchas Pierre-Olivier and Jonathan Parker. 2002. "Consumption over the Life-Cycle." *Econometrica*, Vol 70, 1, 47-89.

[35] Guiso, Luigi, Tullio Jappelli and Daniele Terlizzese. 1996. "Income Risk, Borrowing Constraints and Portfolio Choice." *American Economic Review* vol. 86, pp. 158-172.

[36] Guiso, Luigi, Michael Haliassos and Tullio Japelli. 2001. "Household Portfolios: An International Comparison." in *Household Portfolios*, eds. Luigi Guiso, Michael Haliassos and Tullio Japelli, MIT Press.

[37] Haliassos, Michael and Alexander Michaelides. 2002. "Portfolio Choice and Liquidity Constraints." *International Economic Review*, forthcoming.

[38] He, Hua and Modest, David, M. 1995. "Market Frictions and Consumption-Based Asset Pricing." *Journal of Political Economy*, 103(1), pp. 94-117.

[39] Heaton John, and Deborah Lucas. 1996. "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing." *Journal of Political Economy*, 104(3), pp. 443-87.

[40] Heaton John, and Deborah Lucas. 1997. "Market Frictions, Savings Behavior, and Portfolio Choice." *Macroeconomic Dynamics*, 1(1): 76-101.

[41] Heaton John, and Deborah Lucas. 2000. "Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk." *Journal of Finance,* 55: 1163-1198.

[42] Hong, Harrison, Jeffrey Kubik, and Jeremy Stein. 2001. "Social Interaction and Stock Market Participation." *Working Paper*, Stanford University, Syracuse University and Harvard University.

[43] Hochguertel, Stefan. 1999. "A Buffer-Stock Model with Portfolio Choice: Implications of Income Risk and Liquidity Constraints." *Working-Paper*, Tilburg University.

[44] Hu, Xiaoqing. 2001. "Portfolio Choices for Home Owners." Working Paper. Northwestern University.

[45] Hubbard, Glenn, Jonathan Skinner, and Stephen Zeldes. 1995. "Precautionary Saving and Social Insurance." *Journal of Political Economy*, 103: 360-399.

[46] Jagannathan, R. and N. R. Kocherlakota. 1996. "Why Should Older People Invest Less in Stocks Than Younger People?" Federal Reserve Bank of Minneapolis, *Quarterly Review*, 11-23.

[47] Koo, H. K. 1998. "Consumption and Portfolio Selection with Labor Income: A Continuous Time Approach." *Mathematical Finance*, 8, 49-65.

[48] King, Mervyn and Jonathan Leape. 1998. "Wealth and Portfolio Composition: Theory and Evidence." *Journal of Public Economics*, 69, 155-193.

[49] Jermann, Urban. 1999. "Social Security And Institutions for Intergenerational, Intragenerational and International Risk-Sharing: A Comment." Carnegie-Rochester Conference Series on Public Policy, 50, pp. 205-212.

[50] Lucas, Deborah. 1994. "Asset Pricing with Undiversifiable Risk and Short Sales Constraints: Deepening the Equity Premium Puzzle", *Journal of Monetary Economics*, 34, 325-41.

[51] Ludvigson, Sydney and Alexander Michaelides. 2001. "Does Buffer Stock Saving Explain the Smoothness and Excess Sensitivity of Conumption?" *American Economic Review*, 91(3), pp. 631-647.

[52] Luttmer, Erzo, G, J. 1996. "Asset Pricing in Economies with Frictions." *Econometrica*, 64(6), pp. 1439-67.

[53] Luttmer, Erzo, G, J. 1999. "What Level of Fixed Costs Can Reconcile Consumption and Stock Returns?" *Journal of Political Economy*, 107(5), pp. 969-97.

[54] MaCurdy, T. E.. 1981. "The use of time series processes to model the error structure of earnings in longitudinal data analysis." *Journal of Econometrics*, 18: 83-114.

[55] Malkiel, G., Burton. 1999. "A Random Walk Down Wall Street." 7th edition, *New York: Norton.*

[56] Mankiw, N. Gregory and Stephen Zeldes. 1991. "The Consumption of Stockholders and Non-Stockholders." *Journal of Financial Economics*, 29, 97-112.

[57] Merton, Robert C. 1971. "Optimum Consumption and Portfolio Rules in a Continuous-Time Model." *Journal of Economic Theory*, 3, 373-413.

[58] Michaelides, Alexander. 2001. "Portfolio Choice, Liquidity Constraints and Stock Market Mean Reversion." CEPR Discussion Paper No. 2823.

[59] Paiella, Monica. 2000. "Transaction Costs and Limited Stock Market Participation to Reconcile Asset Prices and Consumption Choices." *Working Paper.*

[60] Pischke Jörn-Steffen. 1995. "Individual Income, Incomplete Information and Aggregate Consumption," *Econometrica.* Vol.63, No.4, 805-840.

[61] Polkovnichenko, Valery. 2000. "Heterogeneous Labor Income and Preferences: Implications for Stock Market Participation." University of Minnesota Working Paper.

[62] Poterba, James and Andrew Samwick. 1999. "Household Portfolio Allocation over the Life-Cycle." NBER Working Paper 6185.

[63] Samuelson, P. A. 1969. "Lifetime Portfolio Selection by Dynamic Stochastic Programming." *Review of Economics and Statistics*, 51, 239-46.

[64] Storesletten, Kjetil, Telmer Chris and Yaron Amir. 2001. "Persistent Idiosyncratic Shocks and Incomplete Markets." Working paper, Carnegie Mellon University, Stockholm School of Economics and The Wharton School.

[65] Tauchen, G. and R. Hussey. 1991. "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models." *Econometrica*, 59, 371-396.

[66] Telmer, Chris . 1993. "Asset-Pricing Puzzles and Incomplete Markets." *Journal of Finance*, 48, 1803-32.

[67] Vayanos. Dimitri. 1998. "Transaction Costs and Asset Prices: A Dynamic Equilibrium Model." *Review of Financial Studies*, 11(1), pp. 1-58.

[68] Viceira, Luis. 2001. "Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income." *Journal of Finance*, 55, 1163-98.

[69] Vissing-Jorgensen, Annette. 2002. "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution." *Journal of Political Economy*, Forthcoming.

[70] Weil, Phillipe. 1989. "The Equity Premium Puzzle and the Risk-Free Rate Puzzle." *Journal of Monetary Economics*, 24, 401-421.

[71] Yao, Rui and Harold Zhang. 2002. "Optimal Consumption and Portfolio Choices with Risky Housing and Stochastic Labor Income." *Working Paper*, University of North Carolina.

Table 1: Regression of the ratio of housing expenditures to labor income (he_{it}), on age polynomials and time dummies

	coefficient	t-stat
Constant	0.703998	5.47
Age	-0.0352276	-3.70
Age^2	0.0007205	3.17
Age^3	-0.0000049	-2.84
adj. R^2	0.025	

Figure 1.1 - Equity Holdings as a Fraction of Total Financial Wealth (Ameriks and Zeldes, 2001 - OLS Regressions)

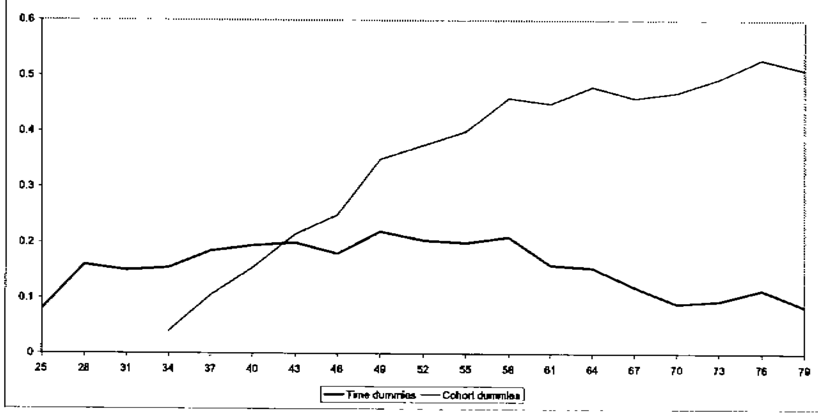


Figure 1.2 - Stock Market Participation (Ameriks and Zeldes, 2001 - Probit Regressions)

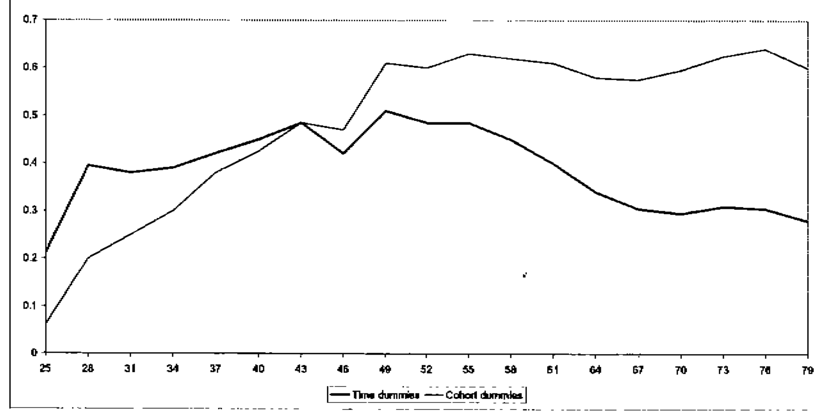


Figure 1.3 - Equity Holdings as a Fraction of Total Financial Wealth, Conditional on Participation (Guiso, Haliassos and Japelli, 2001)

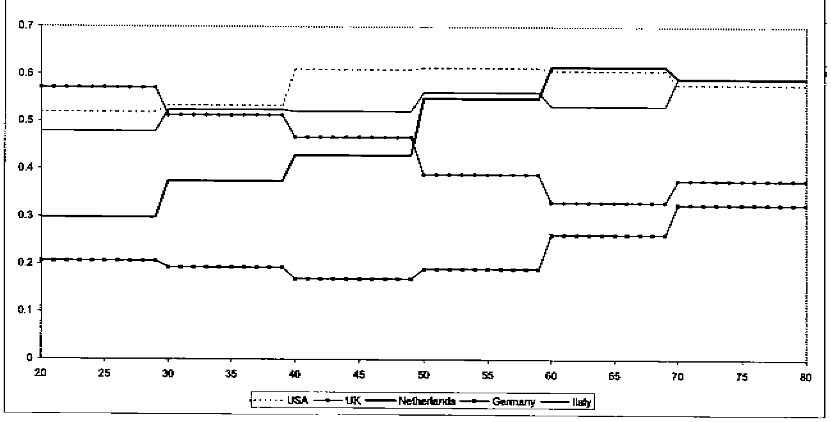
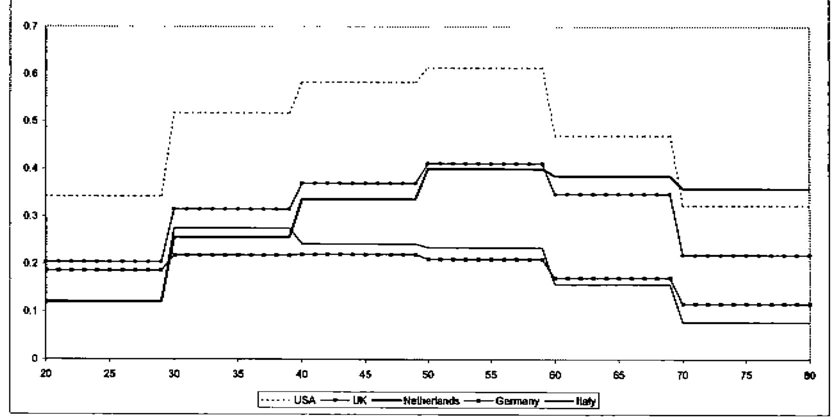


Figure 1.4 - Stock Market Participation (Guiso, Haliassos and Japelli, 2001)



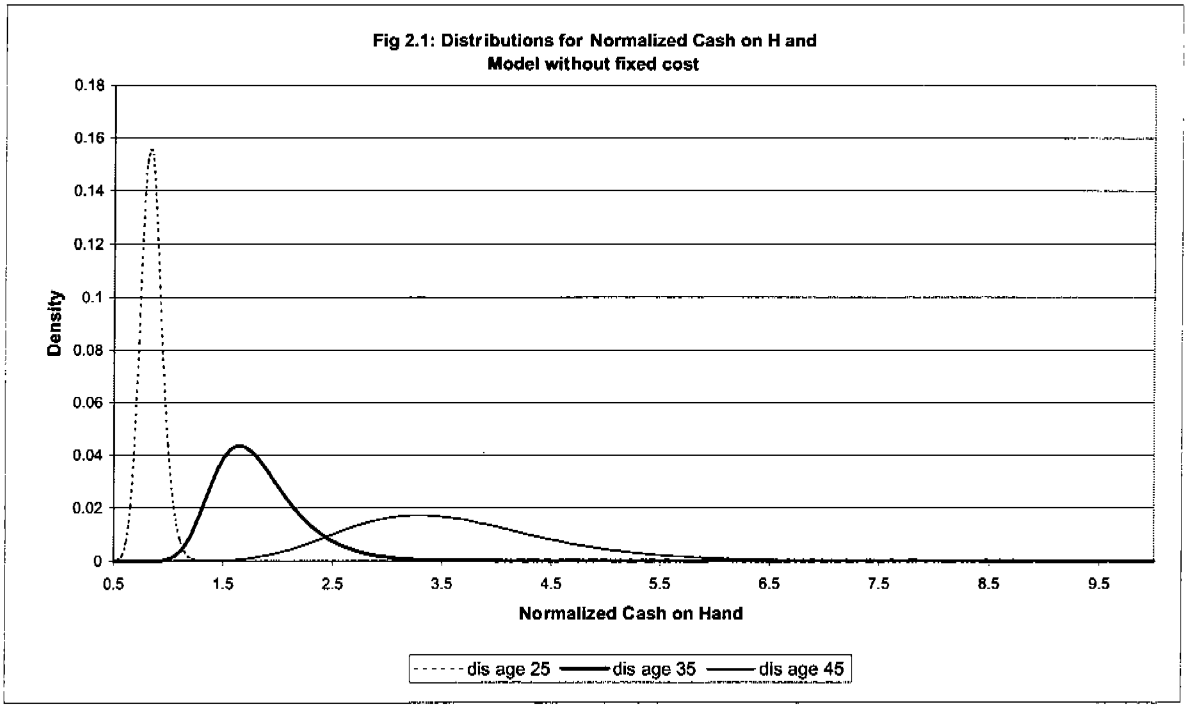


Fig 2.1: Distributions for Normalized Cash on H and
Model without fixed cost
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Density
0.5
1.5
2.5
3.5
4.5
5.5
6.5
7.5
8.5
9.5
Normalized Cash on Hand
dis age 25
dis age 35
dis age 45

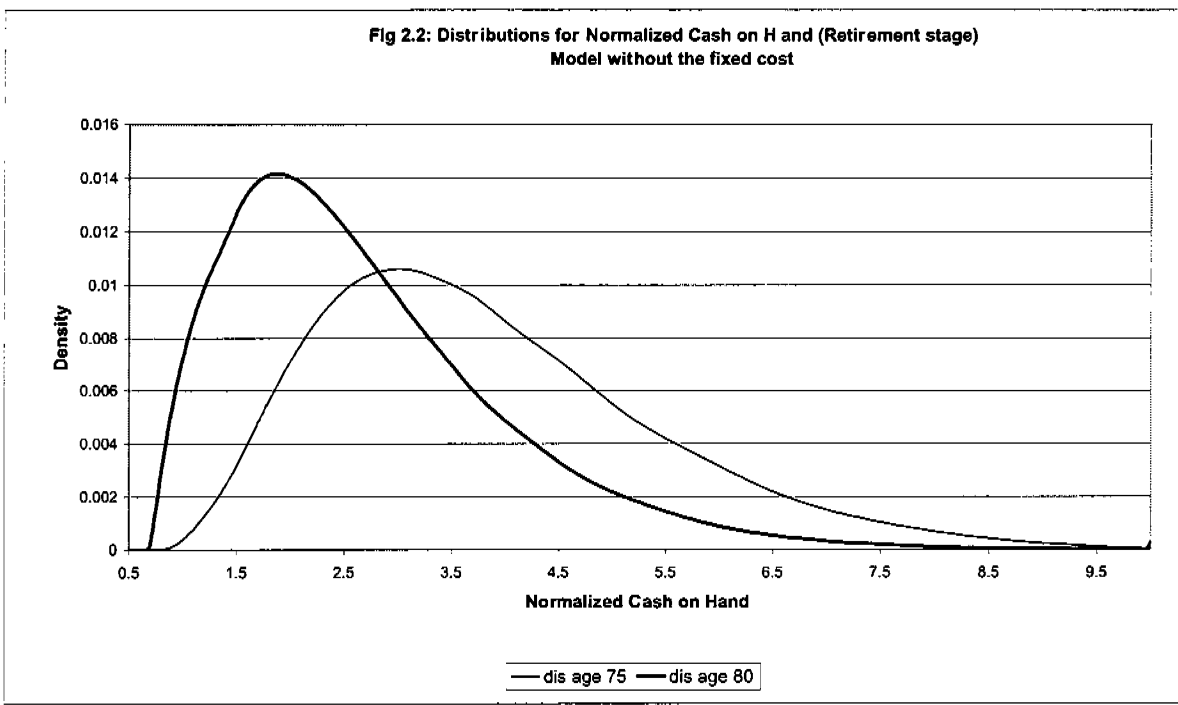


Fig 2.2: Distributions for Normalized Cash on H and (Retirement stage)
Model without the fixed cost
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
Density
0.5
1.5
2.5
3.5
4.5
5.5
6.5
7.5
8.5
9.5
Normalized Cash on Hand
dis age 75
dis age 80

Figure 2.3 - Consumption, Wealth and Income
(benchmark model with F=0.0)

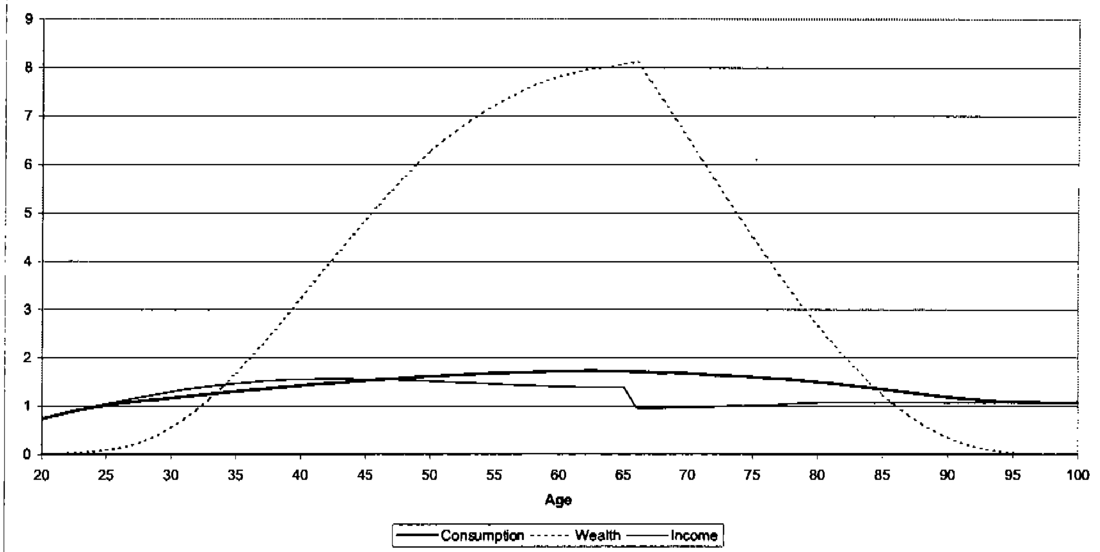
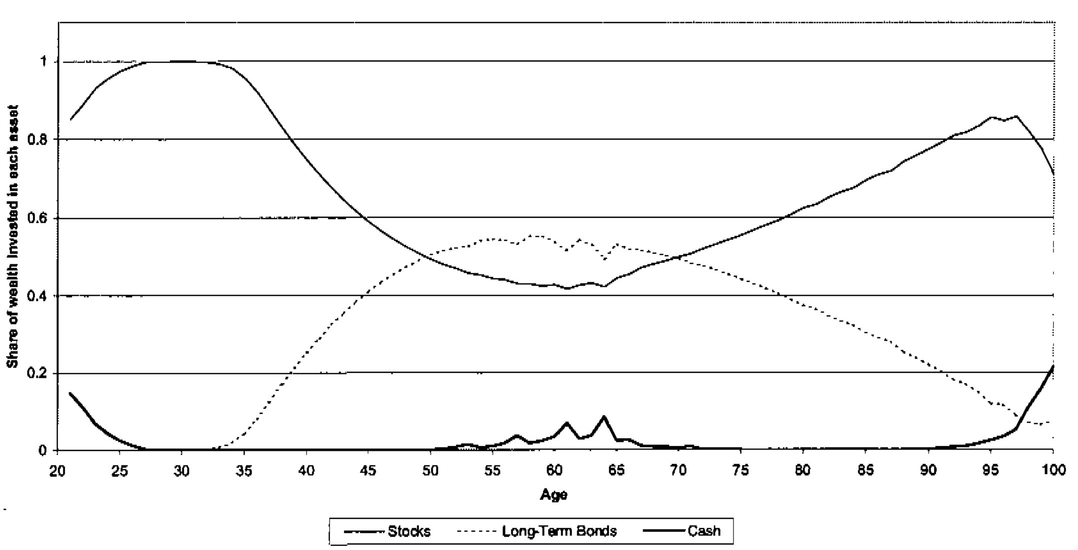


Figure 2.4 - Asset Allocation
(benchmark model with F=0.0)



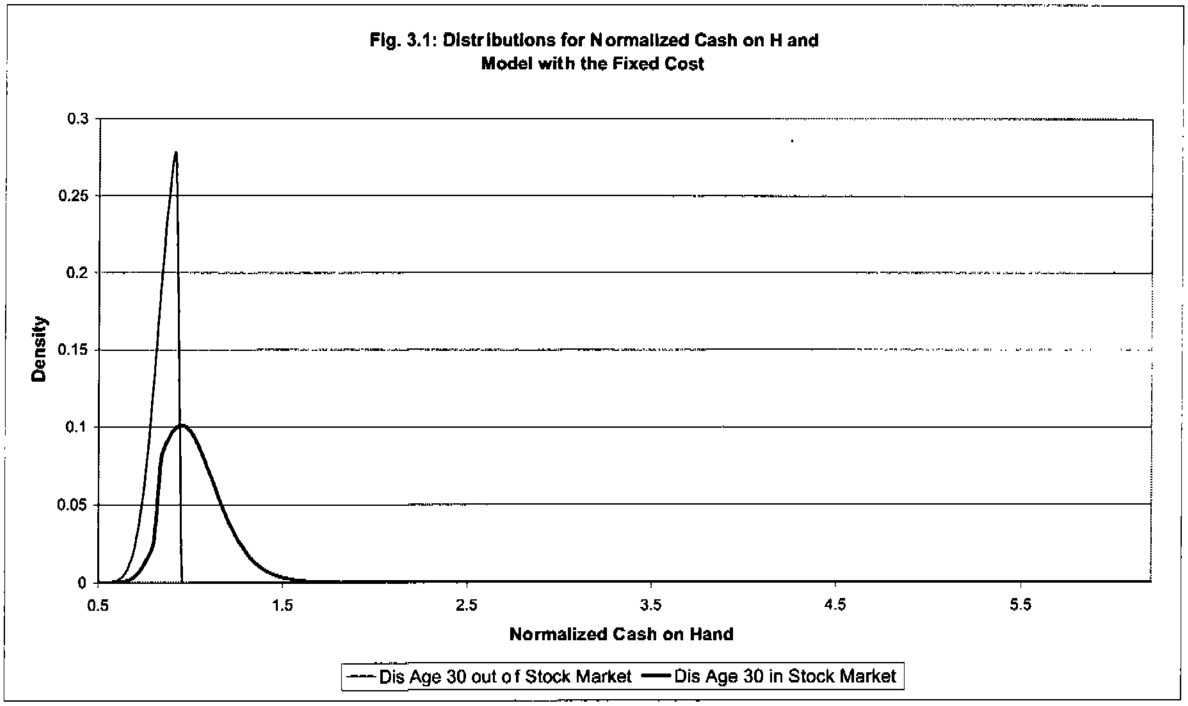


Fig. 3.1: Distributions for Normalized Cash on Hand
Model with the Fixed Cost
Density
0.3
0.25
0.2
0.15
0.1
0.05
0
0.5
1.5
2.5
3.5
4.5
5.5
Normalized Cash on Hand
Dis Age 30 out of Stock Market
Dis Age 30 in Stock Market

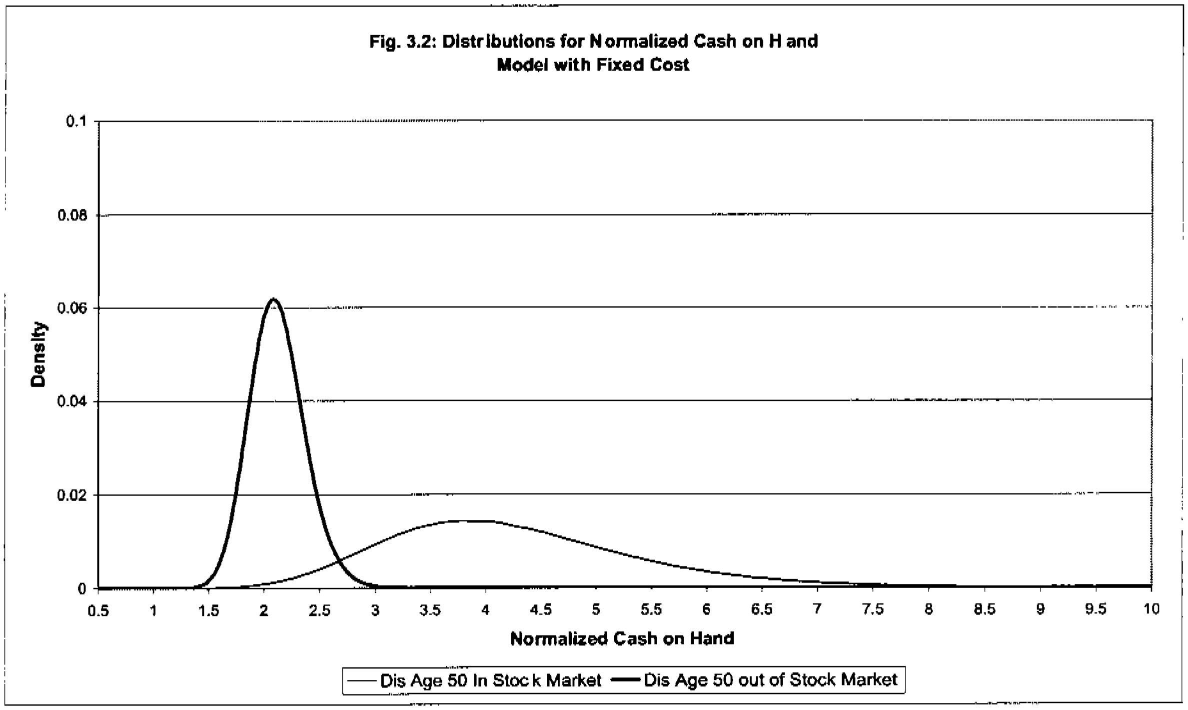


Fig. 3.2: Distributions for Normalized Cash on Hand
Model with Fixed Cost
Density
0.1
0.08
0.06
0.04
0.02
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
Normalized Cash on Hand
Dis Age 50 In Stock Market
Dis Age 50 out of Stock Market

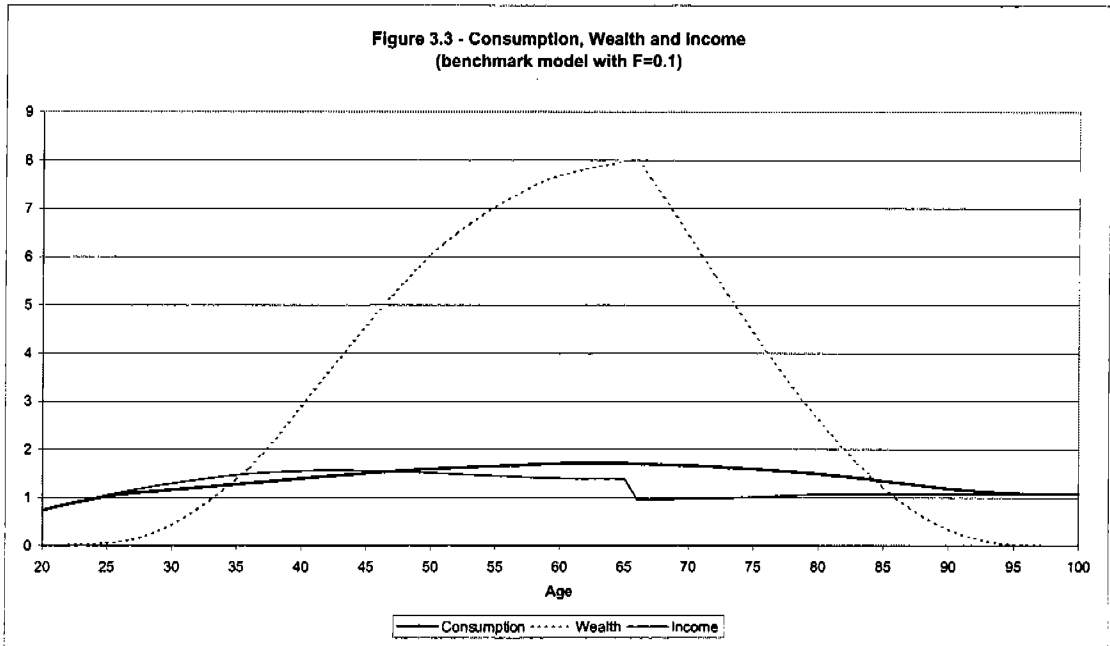


Figure 3.3 - Consumption, Wealth and Income
(benchmark model with F=0.1)
9
8
7
6
5
4
3
2
1
0
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
Age
Consumption
Wealth
Income

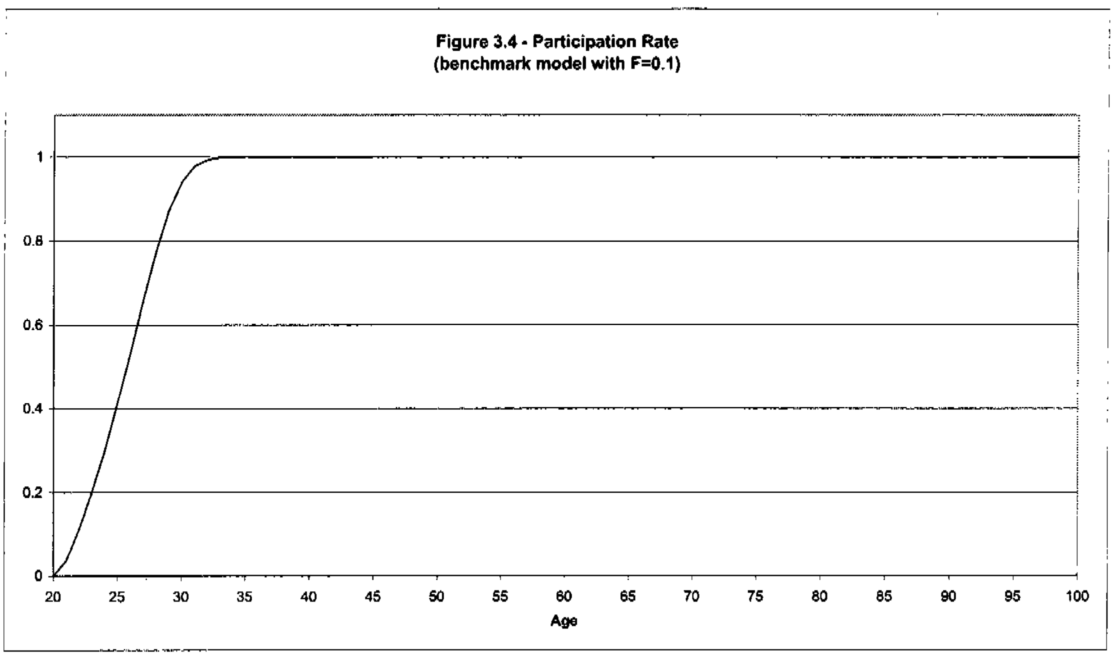


Figure 3.4 - Participation Rate
(benchmark model with F=0.1)
1
0.8
0.6
0.4
0.2
0
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
Age

Figure 3.5 - Asset Allocation
(benchmark model with F=0.1)

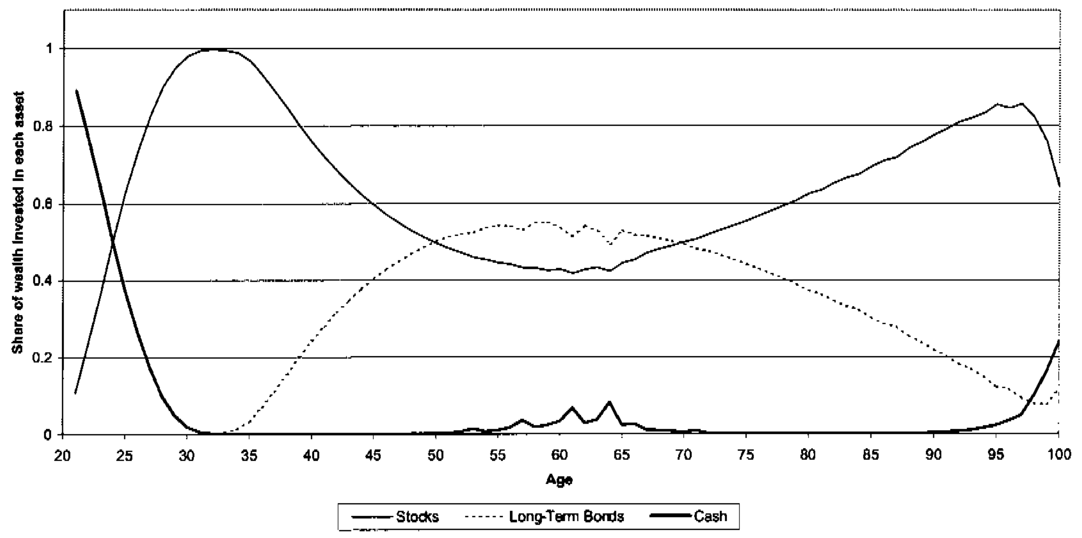
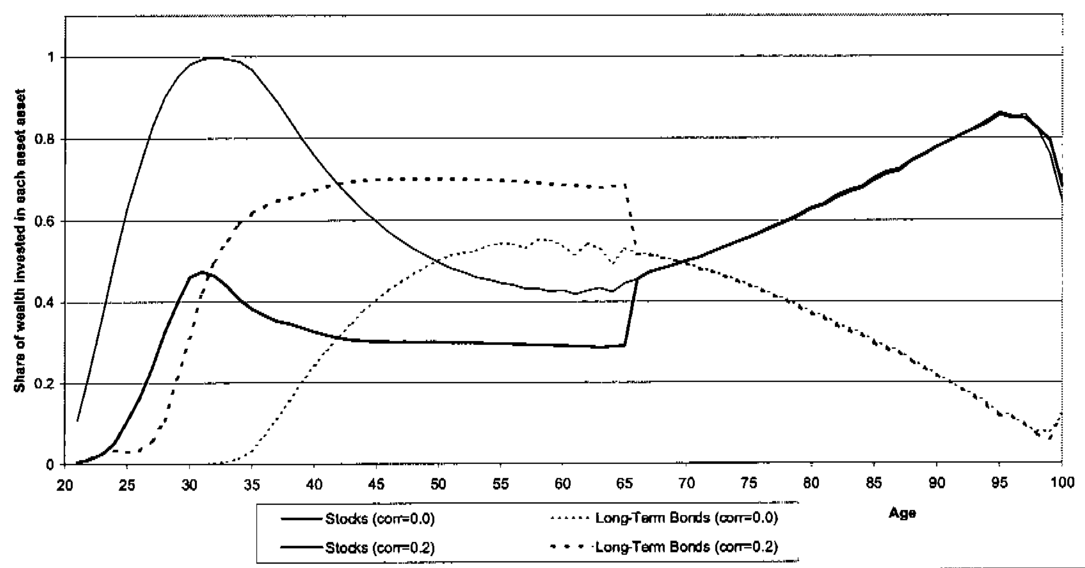


Figure 3.6 - Asset Allocation
(benchmark model, different degrees of correlation between stock returns and earnings shocks)



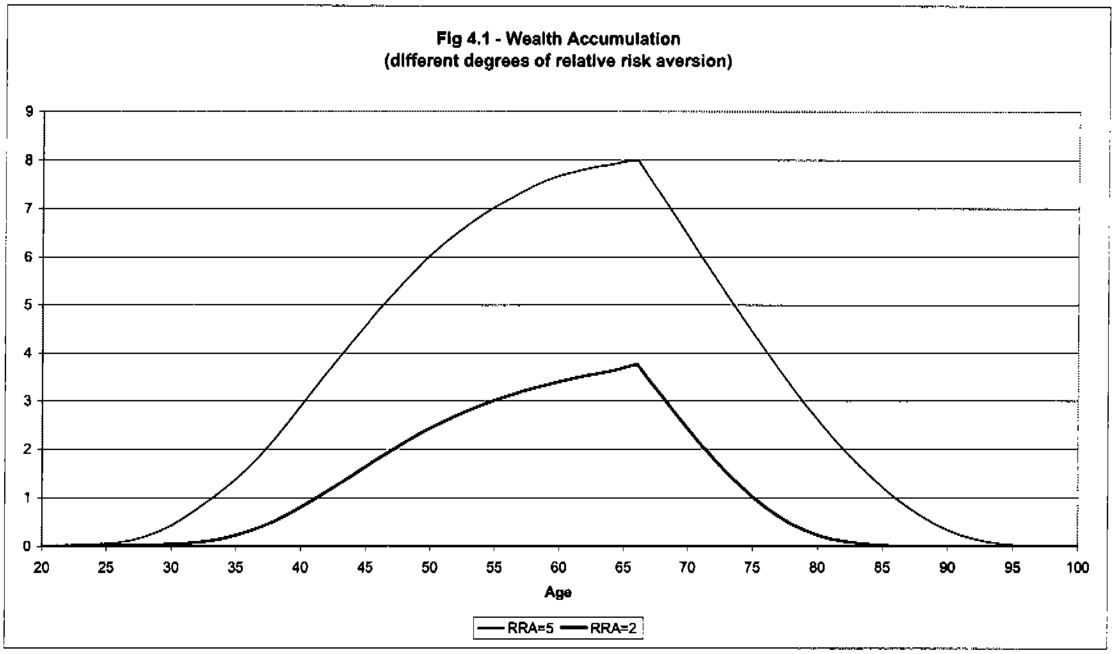


Fig 4.1 - Wealth Accumulation
(different degrees of relative risk aversion)
0
1
2
3
4
5
6
7
8
9
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
Age
RRA=5
RRA=2

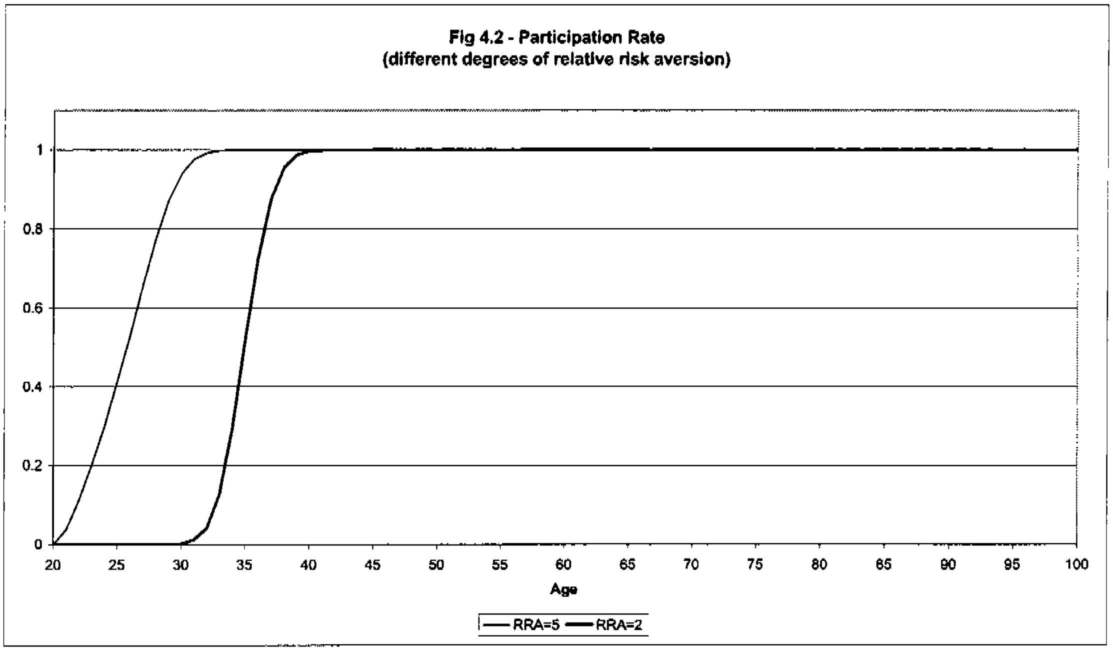


Fig 4.2 - Participation Rate
(different degrees of relative risk aversion)
0
0.2
0.4
0.6
0.8
1
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
Age
RRA=5
RRA=2

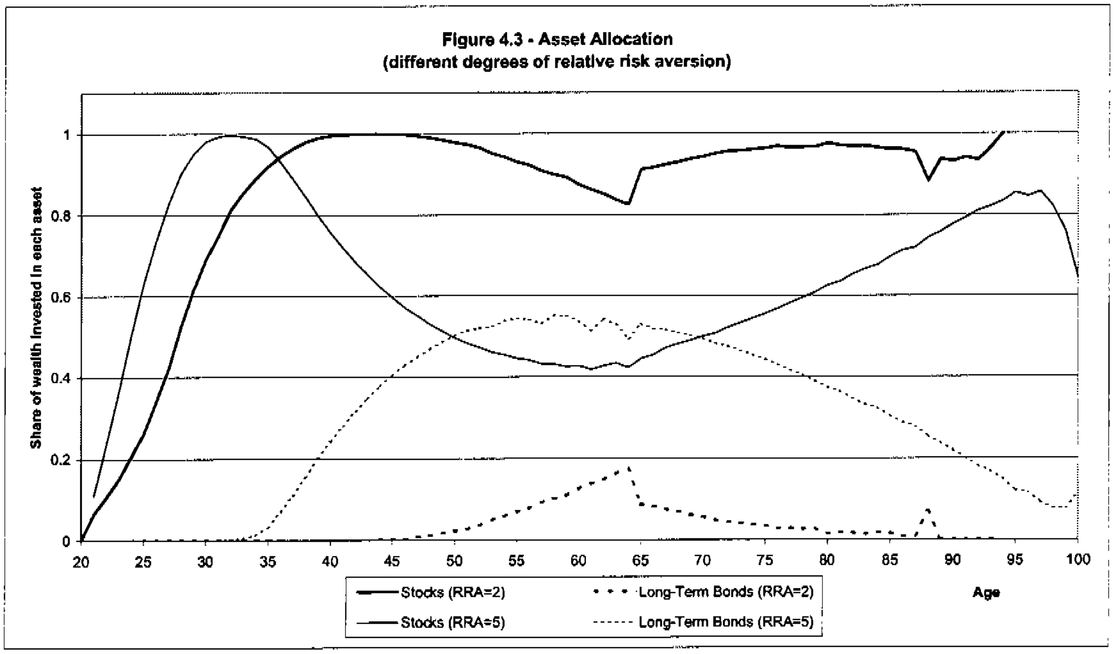


Figure 4.3 - Asset Allocation
(different degrees of relative risk aversion)
Share of wealth invested in each asset
0
0.2
0.4
0.6
0.8
1
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
Age
Stocks (RRA=2)
Long-Term Bonds (RRA=2)
Stocks (RRA=5)
Long-Term Bonds (RRA=5)

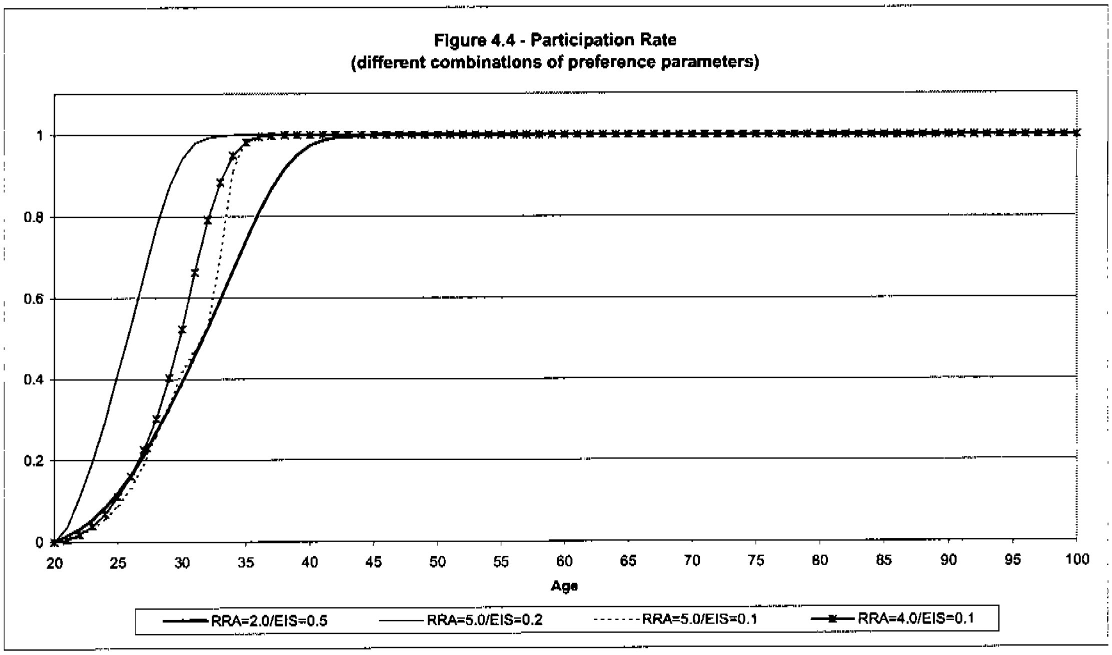


Figure 4.4 - Participation Rate
(different combinations of preference parameters)
0
0.2
0.4
0.6
0.8
1
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
Age
RRA=2.0/EIS=0.5
RRA=5.0/EIS=0.2
RRA=5.0/EIS=0.1
RRA=4.0/EIS=0.1

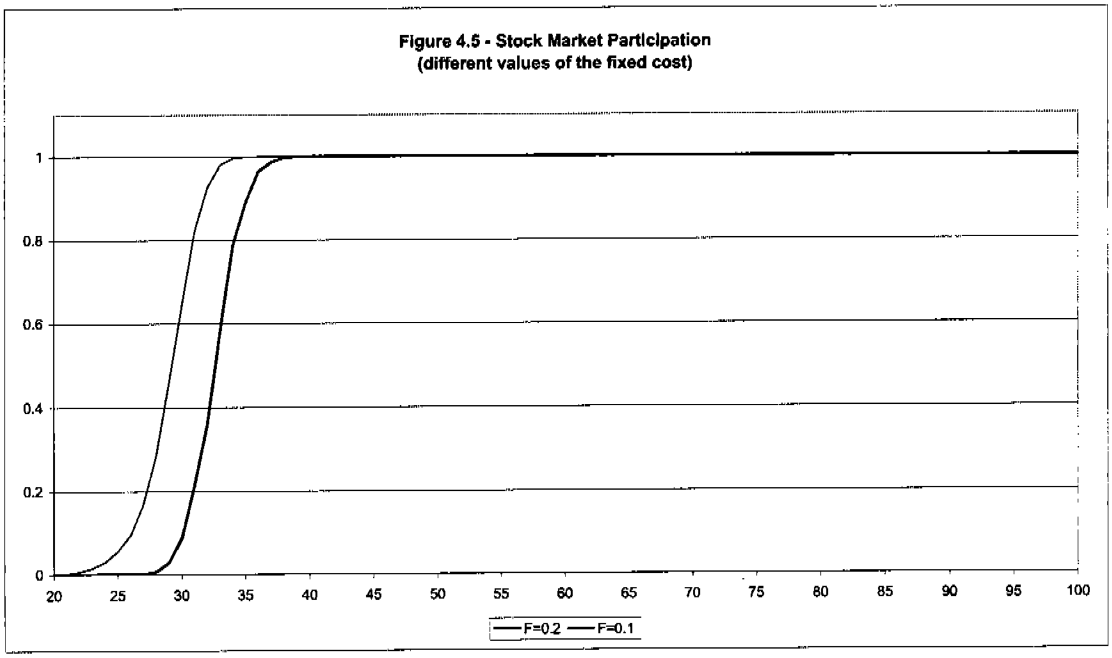


Figure 4.5 - Stock Market Participation
(different values of the fixed cost)
0
0.2
0.4
0.6
0.8
1
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
F=0.2
F=0.1

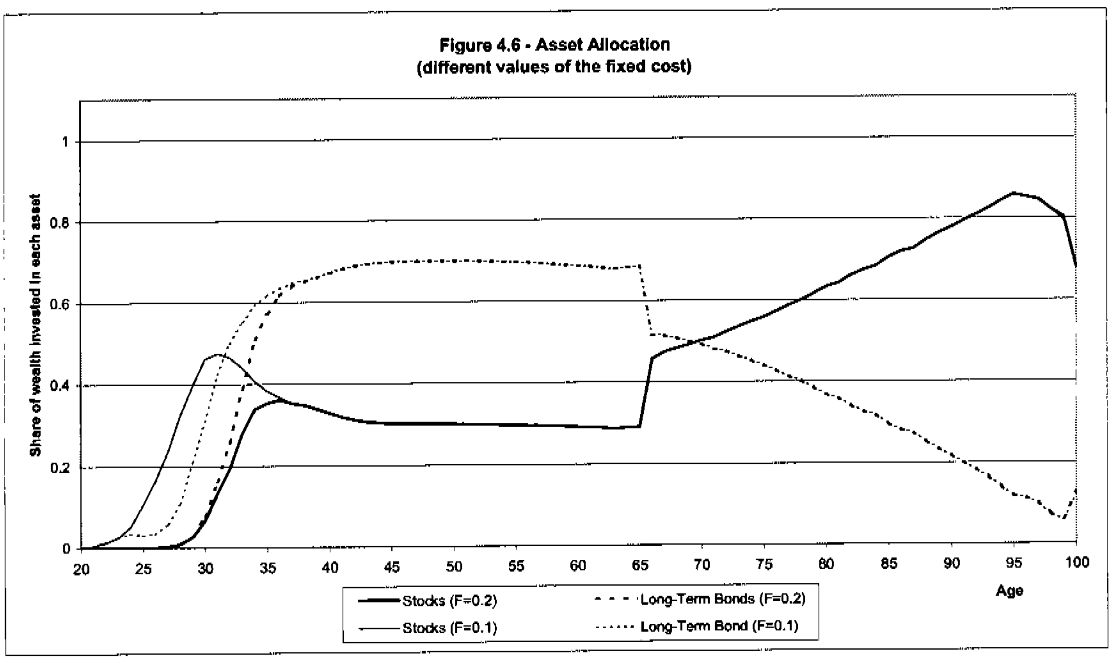


Figure 4.6 - Asset Allocation
(different values of the fixed cost)
Share of wealth invested in each asset
0
0.2
0.4
0.6
0.8
1
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
Age
Stocks (F=0.2)
Stocks (F=0.1)
Long-Term Bonds (F=0.2)
Long-Term Bond (F=0.1)