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SEQUENTIAL INTERNATIONAL TRADE

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This paper applies the uncertain and sequential trade (UST) model to the analysis of two well known problems in international trade: The purchasing power parity puzzle and the trade in similar goods puzzle. It is shown that the UST model can solve both puzzles without assuming transportation costs, monopoly power and increasing returns to scale.

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1. INTRODUCTION

The following two observations provoked a lot of discussion in the international trade literature.

(a) After converting all prices to dollars, we find large price differences across countries (the purchasing power parity puzzle) and (b) Countries trade in similar goods: They exchange cars for cars, beer for beer and so on.

Here I propose to explain these observations with the uncertain and sequential trade (UST) model. UST models are based on ideas in Prescott (1975) and Butters (1977). Prescott considers an environment in which sellers set prices before they know how many buyers will eventually appear. He assumes that less expensive goods will be sold before more expensive ones and obtains an equilibrium trade-off between the price and the probability of making a sale. A similar trade-off arises in Butters (1977). In both models sellers commit to prices before the realization of demand. In the UST approach taken by Eden (1990), trade is sequential and an equilibrium distribution of prices is obtained even though sellers are allowed to change their prices during trade. The UST approach was recently used by Bental and Eden (1993, 1996), Eden (1994), Lucas and Woodford (1994), Woodford (1996) and Williamson (1996) and Dana (1998).

It was argued in Eden (1994) that since only the distribution of prices must adjust to changes in the money supply, it is possible that sellers will adjust their prices in "jumps". A seller who quoted a price which is at the upper end of the equilibrium price distribution may let inflation erode his real price because he is fully compensated by an increase probability of making a sale. This seller may choose to change his nominal price quotation only when his real price hits the floor of the equilibrium price distribution (because reducing the price below the lower bound of the distribution does not increase the probability of making a sale).

Thus sellers may follow an (S,s) type strategy for adjusting nominal price even when changing nominal prices is costless. In the first part of this paper I apply this idea to explain the purchasing power parity puzzle.

In the second part of the paper I consider a single good world economy in which all countries have access to the same constant returns to scale technology. I argue that differences in the probability distribution of demand across countries will generate trade. But trade may not lead to a Pareto improvement.

2. THE PURCHASING POWER PARITY (PPP) PUZZLE

There is ample evidence that PPP does not hold. Obstfeld and Rogoff (1996) argues that for countries with floating currencies and open capital markets the exchange rates are an order of magnitude more volatile than the ratio of the CPIs which hardly moves at all (page 606, see their Figure 9.2). An immediate corrollary is "that the short-run volatility of real exchange rates is very similar to that of nominal exchange rates".

The distinction between traded and non-traded goods does not provide a full explanation for the PPP puzzle. In a recent article, Engel (1999) looked at five different measures of non-traded goods and real exchange rates. He concludes that relative prices of non-traded goods do not account for movements in the US real exchange rate. Transportation costs also fail to provide a full explanation. Engel (1993) compares US with Canadian consumer price data. In more than 2000 pairwise comparison of prices along the US/Canadian border he finds that the between countries variability is much larger than the within countries variability.

Rogoff (1996) provides a survey of the extensive literature that study PPP. He concludes that "International goods market, though becoming more integrated all the time, remain quite segmented, with large trading frictions across a broad range of goods. These frictions may be due to transportation costs, threatened or actual tariffs, nontariff barriers, information costs, or lack of labor mobility. As a consequence of various adjustment costs, there is a large buffer within which nominal exchange rates can move without producing an immediate proportional response in relative domestic prices." Rogoff (1996, page 665).

Here I attempt to explain the PPP puzzle in a fully integrated world economy in which there are no transportation costs and other barriers to trade. The only so called friction is the UST friction: buyers arrive sequentially and some irreversible trade must be made before the complete resolution of uncertainty about demand.

THE MODEL

Since in a UST equilibrium sellers are indifferent about prices in the equilibrium range they may let exchange rate shocks affect their real price provided that the real price stays in the equilibrium range. Thus the same argument that was used to explain the "bad" behavior of prices across individual stores can be used to explain the "bad" behavior of prices across individual countries.

I use a cash-in-advance economy and assume that all currencies can be used to satisfy the cash-in-advance constraint. Therefore as in Karaken and Wallace (1981) the expected rate of return on all currencies must be the same.

I consider a world economy with J currencies (indexed j) and N infinitely lived households. As in Lucas (1980), each household consists of a seller (worker) and a buyer pair. At the beginning of the period the worker goes to work. The buyer learns about his demand (taste shock) and if he wants to consume he takes the available money and goes shopping. After trade in the goods market is completed the seller takes his revenue in the form of cash (a portfolio of currencies). At the end of the period the two members of the household reunite and consume whatever the shopper has bought.

At the beginning of the period each household holds many currencies. Let M^h_{jt} denotes the amount of currency j held by househld h at time t and let E_{jt} denotes the value of currency j in terms of dollars. The dollar value of the currencies held by household h is:

(1) $$M^h_t = \Sigma_{j=1}^{J} E_{jt}M^h_{jt}.$$

In addition, all households get a perfectly anticipated lump sum transfer. The transfer payment may be in various currencies and the composition of currencies need not be the same for all households but the dollar value of the transfer payment is the same for all households and is equal to G_t. The average per-household post transfer amount of money is: $M_t = G_t + (1/N)\sum_{h=1}^{N} M_t^h$ dollars. The rate of change in the world money supply is constant over time and is given by: $M_{t+1}/M_t = 1 + \mu$.

It is assumed that exchange rates are determined at the beginning of the period in a random manner but on average they are expected to remain constant:

$$(2) \qquad E(E_{jt+1}|E_{jt}) = E_{jt},$$

where E denotes the expectations operator. Under (2) a risk neutral seller will accept payment in any currency.[2]

The household is risk neutral and its single period utility is: $\theta c_t - v(L_t)$, where c is the quantity consumed, L is the quantity produced and θ is a random variable that takes two possible realizations: 0 if the household does not want to consume and 1 if it does. The cost function v( ) has the standard properties ($v' > 0$ and $v'' > 0$ everywhere) and the household's discount factor is $0 < \beta < 1$. The representative household's utility function is given by:

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[2] To see that, consider a seller who posts the price of 1 dollar. If instead of a dollar the seller gets $1/E_{jt}$ units of currency j he will have next period E_{jt+1}/E_{jt} dollars, which under (2) will equal on average to 1 dollar.

(3) $$E\ \Sigma_{t=0}^{\infty}\ \beta^t[\theta c_t - v(L_t)].$$

There are S possible states of the world. In state s a fraction ϕ_s of the agents experience $\theta = 1$ and want to consume. It is assumed that: $\phi_1 < \phi_2 < \ldots < \phi_S$. The probability of state of the world s is Π_s. The probability that the state of the world is greater than or equal to s is denoted by q_s. For notational convenience I set $\phi_0 = 0$.

The probability that $\theta = 1$ is the same for all agents and is equal to ϕ_s if the aggregate state is s.

Buyers who want to consume spend all the money they have. From the seller's point of view nominal demand in aggregate state s is $\phi_s M_t(1 + \mu)$ dollars or ϕ_s normalized dollars, where normalized magnitudes are nominal magnitudes divided by the post transfer money supply: $M_t(1 + \mu)$.

As in previous UST models buyers arrive sequentially at the market-place. The first ϕ_1 normalized dollars arrive with certainty and buy in the first market at the price p_1 (normalized dollars per unit). After transactions in the first market has been completed there can be two possible events: either trade ends or an additional batch of $\phi_2 - \phi_1$ normalized dollars arrive (with probability q_2). If the second batch arrives it buys in the second market at the price p_2. In general, after the completion of trade in market s there can be two possibilities: either trade ends or an additional batch of $\phi_{s+1} - \phi_s$ opens market s+1 and buys at the price p_{s+1}.

Workers/sellers choose how much to produce and make a contingent plan which specifies the amount that they will sell to each batch if it arrives. This contingent plan is described as an allocation of output across the potential markets: k_s to market s.

The sequence of events within the period is as follows. Dollar prices are announced for each of the S markets and the dollar exchange rate for each of the J currencies. The worker produces and allocates his output across the potential markets.[3] Buyers then receive a transfer payment and learn about their desire to consume. Buyers who want to consume arrive sequentially at the market-place. This is illustrated by Figure 1.

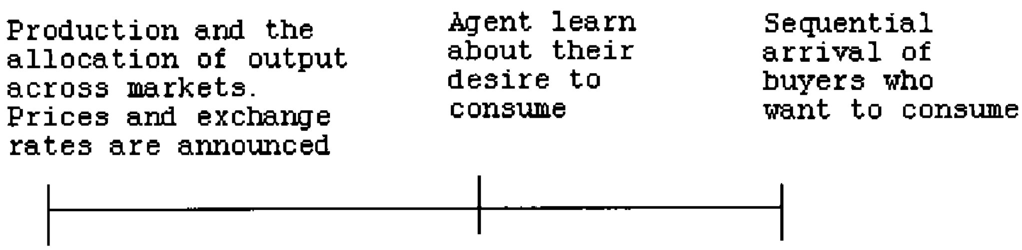

Figure 1

Given that s markets open this period, the expected purchasing power of a portfolio which is worth one normalized dollar is:

$$z_s = \Sigma_{i=1}^{s}(\upsilon_i^s/p_i), \tag{4}$$

where υ_i^s is the probability that the dollar will buy in market i when s markets open: $\upsilon_i^s = (\phi_i - \phi_{i-1})/\phi_s$.

I use lower case letters to denote normalized nominal magnitudes. The household starts with a portfolio which is worth m normalized dollars and receives a transfer worth g normalized

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[3] During trade, there are no incentive to change the allocation of goods across markets or to change prices.

dollars.[4] The Bellman equation which describes the household's choice problem is:

$$(5) \qquad V(m) = \max \Sigma_{s=1}^{S} \Pi_s \phi_s (m + g) z_s - v(\Sigma_{s=1}^{S} k_s)$$

$$+\beta \Sigma_{s=1}^{S} \Pi_s \{\phi_s V[(\Sigma_{i=1}^{s} p_i k_i)\omega] + (1 - \phi_s) V[(m + g + \Sigma_{i=1}^{s} p_i k_i)\omega]\},$$

where $\omega = (1 + \mu)^{-1}$ is used to convert current normalized dollars into next period's normalized dollars and the maximization is with respect to k_s. Here $V(m)$ is the maximum expected utility that the household can achieve if it starts with a pretransfer portfolio which is worth m normalized dollars; $\Sigma_{s=1}^{S} \Pi_s \phi_s (m + g) z_s$ is the expected current consumption; $v(\Sigma_{s=1}^{S} k_s)$ is the utility cost of producing $L = \Sigma_{s=1}^{S} k_s$ units. To understand the future utility terms note that the worker's revenue is $\Sigma_{i=1}^{s} p_i k_i$ if s markets open this period. When $\theta = 1$ the buyer spends everything he has and the household will have at the end of the period a portfolio which is worth $\Sigma_{i=1}^{s} p_i k_i$ normalized dollars which will be worth, in the next period, $(\Sigma_{i=1}^{s} p_i k_i)\omega$ normalized dollars. When $\theta = 0$ the buyer saves everything and the household will start next period with a portfolio which is worth $(m + g + \Sigma_{i=1}^{s} p_i k_i)\omega$ normalized dollars.

To state the first order conditions for an interior solution to (5) I compute the expected utility from a portfolio which is worth one normalized dollar (where expectations are taken before the realization of the taste shock). In aggregate state s, the normalized dollar will buy on average z_s units if the buyer want to consume

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[4] Since we normalize by the post transfer money supply: $g = \mu/(1 + \mu)$.

(with probability ϕ_s) and will be saved and become ω normalized dollars in the next period if the buyer does not want to consume. The expected utility from a normalized dollar is thus:

$$z = \Sigma_{s=1}^{S} \Pi_s [\phi_s z_s + (1-\phi_s)\beta\omega z]. \tag{6}$$

Using $\zeta = \Sigma_{s=1}^{S} \Pi_s (1-\phi_s)$ to denote the unconditional probability of not wanting to consume, we can solve (6) and get:

$$z = (1 - \zeta\beta\omega)^{-1} \Sigma_{s=1}^{S} \Pi_s \phi_s z_s. \tag{7}$$

The expected utility from supplying a unit to market s is $q_s p_s \omega z$: If the market opens (with probability q_s) the seller will get p_s normalized dollars worth which will turn into $p_s\omega$ next period's normalized dollars worth and bring on average $p_s\omega z$ utils. The first order conditions to (5) can therefore be written as:

$$q_s \beta p_s \omega z = \beta p_1 \omega z = v'(\Sigma_{s=1}^{S} k_s), \tag{8}$$

which says that the discounted expected utils from supplying a unit is the same for all markets and is equal to the marginal cost.

Symmetric UST equilibrium requires that the first order conditions (8) are satisfied and markets which are open are cleared:

$$p_s k_s = \phi_s - \phi_{s-1}\ ; \quad \text{for all s}. \tag{9}$$

I now turn to discuss asymmetric equilibria.

Sellers post prices in terms of the local currency:

The above symmetric formulation assumes that the representative household allocates capacity to all S markets. We may think of this allocation as a price setting choice: The seller post many prices and puts the price tag p_s on k_s units.

I now assume that each seller quotes one price only. According to this interpretation a fraction k_s/L chooses to participate in market s and offer to sell at the price p_s. It is important to note that the expected utility of the seller does not depend on his choice of price (market) as long as the price remains in the equilibrium range. To simplify the discussion I assume that the probability distribution of ϕ can be approximated by a continuous distribution and therefore sellers are indifferent about any price in the equilibrium range p_1 to p_S.

I also assume that households are distributed over J groups (countries) and that sellers in group j quote their price in terms of currency j (but nevertheless are willing to accept payment in any other currency). Since sellers are indifferent about prices in the equilibrium range they may not change the price quotation in terms of the local currency in response to changes in the exchange rate. If for example, currency j was devalued the dollar value of the price quoted in currency j decreased. But if it remains in the equilibrium range, the increase in the probability of making a sale fully compensate for the reduction in the dollar value of the price. Thus nominal prices in any individual country may appear to be sticky in spite of the perfect flexibility of the world distribution of prices. I now turn to two examples which illustrate this point.

<u>Example 1:</u> There are two potential markets. In equilibrium, the price in the first market is 1 dollar worth and the price in the second market is 2 dollars worth. The equilibrium supply is one unit to each market.

There are two countries: US and Germany. At time t, the value of the mark in terms of dollars - the exchange rate - was one. US sellers supplied one unit to market 1 and German sellers supplied one unit to market 2. At time t + 1, the exchange rate changed from 1 dollar per mark to 0.5 dollars per mark and this change was followed by a transfer payment which kept the world money supply constant.

In Table 1, I calculate different alternative adjustments possibilities to the change in the exchange rate. Initially the ratio of dollar prices in the US (P) to dollar prices in Germany (EP*) is 0.5. After the shock sellers in Germany who quote a price of 2 marks will find themselves in market 1. Since after the shock, there are two units in market 1 and no supply to market 2 someone must change his price. One possibility for restoring equilibrium is that German sellers will change their price to 4 marks. In this case the ratio P/EP^* will not change. Another possibility is that US sellers will increase their price. In this case P/EP^* will go up to 2 reflecting the fact that Germany became cheaper. A third possibility is that $^1/_2$ of the US sellers will change their nominal price to 2 dollars and $^1/_2$ of the German sellers will change their nominal price to 4 marks. This will result in a symmetric allocation which also implies that Germany became cheaper after the devaluation. Thus it is possible that after the devaluation, Germany became cheaper and nominal prices look sticky.

Table 1: The adjustment to a change in the exchange rate from one dollar per mark to 0.5 dollars per mark

Quantities supplied	Market 1 ($p_1 = 1$)	Market 2 ($p_2 = 2$)	P/EP^*
Before the shock			0.5
US	1		
Germany		1	
After the shock			
US	1		
Germany	1		
After adjustment (a)			0.5
US	1		
Germany		1	
After adjustment (b)			2
US		1	
Germany	1		
After adjustment (c)			1
US	0.5	0.5	
Germany	0.5	0.5	

With a continuum of countries it is easier to get clean examples in which sellers do not change their price quotation in response to changes in the exchange rate.

Example 2: There is a continuum of countries. The total equilibrium supply of the world economy is 2 units: One allocated to market 1 and one allocated to market 2. Equilibrium prices are also the same as in example 1: the price in market 1 is 1 dollar worth and the price in market 2 is 2 dollars worth.

Suppose that at time t the exchange rate of currency j was 1 and sellers in country j supplied to market 2. Similar to the previous example, currency j depreciated at t + 1 and is now worth 0.5 dollars. If sellers in country j do not change their nominal price quotations (in terms of their currency) they will find themselves supplying to market 1. Since the country is small the movement of supply from market 2 to market 1 does not require any further adjustment. In this case, the dollar price of sellers in the devaluing country will go down by the percentage of the devaluation.

Note that in this example, prices in terms of individual currencies do not move and therefore we will observe the excess volatility of the exchange rates as documented by Obstfeld and Rogoff (1996) and the "border effect" in Engel (1993). Finally, an increase in the volatility of the exchange rates leads to an increase in the volatility of the real exchange rate as in Mussa (1986).

3. THE TRADE IN SIMILAR PRODUCTS PUZZLE

In classical international trade theory (the Heckscher-Ohlin model) countries with a relative large endowment of labor export labor intensive goods and import capital intensive goods. But we observe a lot of international trade in similar products: cars for cars, beer for beer, wine for wine and so on. For example, the USA is the largest car producer and also the largest car importer.

Models with increasing returns to scale and monopolistic competition are often used, to account for these observations. See for example, Chamberlin (1933), Dixit and Stiglitz (1977), and Helpman and Krugman (1985). Here I use the above UST model which does not invoke monopoly power and increasing returns to scale. It is shown that incentive for international trade will emerge in a single good world economy whenever the probability distribution of demand is country specific. However, trade does not always results in a Pareto improvement.

As in the previous section there are S possible states of the world. But here the fraction of households who want to consume varies across countries. I use ϕ_{js} to denote the fraction of the residents in country j who want to consume in state s and assume, for simplicity, that the ordering of the state is the same for all countries: $\phi_{j1} < \phi_{j2} < \ldots < \phi_{jS}$ for all j.

For further simplification, I assume that all countries use dollars and the money supply grow by the same percentage in all countries. Under autarky, dollars may change hands only between two residents of the same country. Under free trade dollars can change hands between any two individuals.

I start with the case of autarky. The expected purchasing power of a normalized dollar held by a country j buyer who wants to consume is:

$$z_{js} = \Sigma_{i=1}^{s}(\upsilon_{ji}^{s}/p_{ji}), \tag{4'}$$

where υ_{ji}^{s} is the probability that the dollar will buy in market i when s markets open: $\upsilon_{ji}^{s} = (\phi_{ji} - \phi_{j,i-1})/\phi_{s}$. The expected utility from a normalized dollar held by a resident of country j is:

$$z_{j} = (1 - \zeta_{j}\beta\omega)^{-1}\Sigma_{s=1}^{S}\Pi_{s}\phi_{js}z_{js}, \tag{7'}$$

where $\zeta_j = \Sigma_{s=1}^{S}\Pi_s(1-\phi_{js})$ is the unconditional probability of not wanting to consume.

Equilibrium in country j requires the first order condition (8) and and the market clearing condition (9) which are rewritten here with an added country index:

$$(10) \qquad q_s\beta p_{js}\omega z_j = \beta p_{j1}\omega z_j = v'(\Sigma_{s=1}^{S} k_{js});$$

$$(11) \qquad p_{js}k_{js} = \phi_{js} - \phi_{js-1}\ ; \quad \text{for all } s.$$

The solution to (10) and (11) depends on the vector $(\phi_{j1}, \phi_{j2}, \ldots, \phi_{jS})$ and will therefore be different across countries. In particular, the price p_{js} are country specific and therefore individuals have an incentive to use money for buying and selling goods in other countries. A seller in country j will have, for example, an incentive to sell in country j' if there is a market s for which: $p_{j's} > p_{js}$.

I now illustrate by a symmetric example, that trade can bring a Pareto improvement even when there is only one homogeneous good and all producers have access to the same constant returns to scale technology.

Example 3: There are two symmetric countries with the same population and the same independent distribution of demand: Either half of the residents want to consume or all of the residents want to consume; with equal probabilities of occurrence.

There are four states of the world which occur with equal probabilities. In state 1 demand is low in both countries, in state 2 and 3 it is low in one country and high in the other and in state 4 it is high in both countries:

$$\phi_{11} = \phi_{21} = {}^1/_2;\ \phi_{12} = 1 \text{ and } \phi_{22} = {}^1/_2; \quad (12)$$

$$\phi_{13} = {}^1/_2 \text{ and } \phi_{23} = 1;\ \phi_{14} = 1 \text{ and } \phi_{24} = 1.$$

Under autarky, there are only two potential markets in each country: Market 1 opens with certainty and face the demand of $^1/_2$ normalized dollar per seller. Market 2 opens with probability $^1/_2$ and if it opens it will face the demand of $^1/_2$ normalized dollar per seller. The expected utility from a normalized dollar (Using equations [4'] and [6'] and dropping the country index) is:

$$z_1 = 1/p_1 \ ; \quad (13)$$

$$z_2 = ({}^1/_2)(1/p_1) + ({}^1/_2)(1/p_2) \ ;$$

$$z = ({}^1/_2)[({}^1/_2)z_1 + ({}^1/_2)\beta\omega z] + ({}^1/_2)z_2.$$

To allow for numerical solutions, I assume $v(L) = L^2$. Under this assumption, the first order condition for the household's problem (5) are:

$$2L = \beta\omega p_1 z = ({}^1/_2)\beta\omega p_2 z; \quad L = k_1 + k_2. \quad (14)$$

And market clearing conditions are:

$$p_1 k_1 = {}^1/_2 \ ; \ p_2 k_2 = {}^1/_2. \quad (15)$$

Equilibrium under autarky (the same for both countries) is a solution (p_1, p_2, z_1, z_2, z, k_1, k_2, L) to (13) - (15). The analytical solution is:

(16) $$p_1 = 2.4/\beta\omega - 0.6 \; ; \; p_2 = 4.8/\beta\omega - 1.2 \; ;$$

$$z_1 = 1.666\beta\omega/(4 - \beta\omega); \; z_2 = 1.25\beta\omega/(4 - \beta\omega); \; z = 4.16\beta\omega/(4 - \beta\omega)^2;$$

$$k_2 = 0.416\beta\omega/(4 - \beta\omega); \; k_1 = 0.833\beta\omega/(4 - \beta\omega); \; L = 1.25\beta\omega/(4 - \beta\omega).$$

To illustrate, Table 2 computes the numerical solution for various levels of $\beta\omega$.

Table 2: Equilibria under autarky (the symmetric case)

	p_1	p_2	z	k_1	k_2	L
$\beta\omega = 1$	1.8	3.6	0.463	0.278	0.139	0.417
$\beta\omega = 0.95$	1.926	3.853	0.426	0.260	0.130	0.39
$\beta\omega = 0.90$	2.066	4.133	0.390	0.242	0.121	0.362
$\beta\omega = 0.5$	4.2	8.4	0.170	0.120	0.060	0.18

Under risk neutrality, welfare is an increasing function of the expected real wage, $w = \beta p_1 \omega z$, and can be measured by:

(17) $$A(w) = \max wL - v(L),$$

where $A(w)$ is the expected utility of the representative consumer as a function of the expected real wage, w. Note that since the first order condition to (17) is $v'(L) = w$, welfare is monotonic in both w and L.

The expected real wage under autarky is:

(18) $$w = \beta\omega p_1 z = 9.984\beta\omega/(4 - \beta\omega)^2 - 2.5(\beta\omega)^2/(4 - \beta\omega)^2.$$

Allowing for international trade:

I now allow international trade. The fraction of households who want to consume out of the entire world population is: $^1/_2$ or $^3/_4$ or 1 with probabilities: $^1/_4$, $^1/_2$, $^1/_4$. (The first realization occurs when demand in both countries is low, the second occurs when demand is low in one counrty and high in the other and the third occurs when demand is high in both countries).

There will be three potential markets in this case.

Market 1: Demand (per seller) is $^1/_2$ normalized dollars and it opens with certainty;
Market 2: Demand is $^1/_4$ normalized dollars and it opens with probability $^3/_4$;
market 3: Demand is $^1/_4$ normalized dollars and it opens with probability $^1/_4$.

The expected utility from a normalized dollar can be derived from the following conditions:

(19)
$$z_1 = 1/p_1;$$
$$z_2 = (^2/_3)(1/p_1) + (^1/_3)(1/p_2);$$
$$z_3 = (^1/_2)(1/p_1) + (^1/_4)(1/p_2) + (^1/_4)(1/p_3);$$
$$z = (^1/_4)[(^1/_2)z_1 + (^1/_2)\beta\omega z] + (^1/_2)[(^3/_4)z_2 + (^1/_4)\beta\omega z] + (^1/_4)z_3.$$

The first order conditions for (5) are now given by:

(20) $2L = \beta p_1 \omega z = (^3/_4)\beta p_2 \omega z = (^1/_4)\beta p_3 \omega z; \quad k_1 + k_2 + k_3 = L.$

Market clearing conditions are:

(21) $p_1 k_1 = ^1/_2; \quad p_2 k_2 = ^1/_4; \quad p_3 k_3 = ^1/_4.$

Equilibrium in the integrated economy is a solution $(p_1, p_2, p_3, z_1, z_2, z_3, z, L, k_1, k_2, k_3)$ to (19) - (21). An analytical solution of the main variables is:

(22) $p_1 = (2.286/\beta\omega) - 0.571; \; p_2 = (3.048/21\beta\omega) - 0.286;$

$$p_3 = (9.143/\beta\omega) - 2.286;$$

$$z = 4.594\beta\omega/(4 - \beta\omega)^2; \; L = 1.313\beta\omega/(4 - \beta\omega).$$

To illustrate, I compute in Table 3 the numerical solution for the four alternative values of $\beta\omega$ which were used in Table 2.

Table 3: Equilibria for the integrated world economy (the symmetric case)

	p_1	p_2	p_3	z	L
$\beta\omega = 1$	1.714	2.286	6.857	0.510	0.438
$\beta\omega = 0.95$	1.835	2.446	7.338	0.469	0.409
$\beta\omega = 0.90$	1.968	2.624	7.873	0.430	0.381
$\beta\omega = 0.5$	4.0	5:333	16.0	0.188	0.188

Note that in the integrated economy, the expected real wage is:

(23) $$w = \beta\omega p_1 z = 10.5\beta\omega/(4 - \beta\omega)^2 - 2.623(\beta\omega)^2/(4 - \beta\omega)^2.$$

This is larger than the real wage under autarky (18) and therefore welfare in both countries goes up as a result of trade. Comparing Tables 2 and 3 reveals that the real wage went up by about 5% as a result of trade and therefore the supply in the integrated economy is higher (also went up by about 5%). The reason for the higher real wage is the increase in average capacity utilization.

Thus we have shown that in the symmetric case, trade leads to Pareto improvement. In the next example I show that this is not always the case.

<u>Example 4: Assymetric countries</u>

There are two countries. In country 1 all households always want to consume. Country 2 is identical to the representative country in the previous example: either all the households want to consume or half of the households want to consume. As in the previous example, I assume $v'(L) = 2L$.

Under autarky there is only one market in country 1. Using x to denote the purchasing power of a normalized dollar in country 1 we can define equilibrium as a vector (p, x, L) such that:

$$x = 1/p \tag{24}$$

$$v'(L) = 2L = \beta\omega p x = \beta\omega \tag{25}$$

$$pL = 1. \tag{26}$$

The analytical solution for (24) - (26) is: $p = 2/\beta\omega$, $x = L = \beta\omega/2$. The equilibrium solutions for different values of $\beta\omega$ is given in Table 4.

Table 4: Equilibria for Country 1

	$p = 2/\beta\omega$	$x = \beta\omega/2$	$L = \beta\omega/2$
$\beta\omega = 1$	2	0.5	0.5
$\beta\omega = 0.95$	2.105	0.475	0.475
$\beta\omega = 0.90$	2.222	0.450	0.450
$\beta\omega = 0.5$	4	0.25	0.25

The equilibrium solutions in country 2 for the case of autarky were already computed in Table 2.

Allowing for international trade:

Assume now that these two asymmetric countries open to trade.

The fraction of households who want to consume out of the entire world population is: $^3/_4$ or 1 with equal probabilities of occurrence. There will therefore be two markets. Market 1 will open with certainty and will face the demand of $^3/_4$ normalized dollars per seller. Market 2 will open with probability $^1/_2$ and will face the demand of $^1/_4$ normalized dollars per seller.

The expected purchasing power of a normalized dollar if exactly s markets open is now given by:

(27) $$z_1 = 1/p_1$$

(28) $$z_2 = (^3/_4)(1/p_1) + (^1/_4)(1/p_2).$$

The expected utility from a normalized dollar is not the same for the residence of both countries because of the difference in the desire to consume. I use x (y) to denote the expected utility from a normalized dollar held by a resident in country 1 (country 2). These are defined by:

(29) $$x = (^1/_2)z_1 + (^1/_2)z_2$$

(30) $$y = (^1/_2)[(^1/_2)z_1 + (^1/_2)\beta\omega y] + (^1/_2)z_2.$$

Since the expected utility from a normalized dollar varies across countries the real wage and labor supply varies across countries. Using L_j to denote the labor supply in country j and using

$v'(L) = 2L$, leads to the following first order conditions (assuming an interior solution):

(31) $$2L_1 = \beta\omega p_1 x,$$

(32) $$2L_2 = \beta\omega p_1 y,$$

(33) $$p_1 = (^1/_2) p_2.$$

In equilibrium, markets which open must clear:

(34) $$p_1 k_1 = {}^3/_4,$$

(35) $$p_2 k_2 = {}^1/_4,$$

(36) $$k_1 + k_2 = (^1/_2)(L_1 + L_2),$$

where the last equation requires that the total supply is equal to the average per seller labor supply.

Equilibrium for this world is a vector $(p_1, p_2, z_1, z_2, x, y, L_1, L_2, k_1, k_2)$ that satisfy (27) - (36).

The full analytical solution for all the variables is rather complicated. But the analytical solution for L_1 is simple. It is: $L_1 = 0.469\beta\omega$. This is less than the supply in country 1 under autarky $(0.5\beta\omega)$. Since labor supply is increasing in the expected real wage and welfare, we conclude that welfare in country 1 went down as a result of trade.

The analytical solution for the labor supply in country 2 is: $1.375\beta\omega/(4 - \beta\omega)$. This is larger than the labor supply in country 2 under autarky ($1.25\beta\omega/(4 - \beta\omega)$), indicating an improvement in welfare as a result of trade. For the sake of comparison, the solutions for alternative values of $\beta\omega$ are in Table 5.

Table 5: Equilibria for the integrated world economy (the asymmetric case)

	p_1	p_2	x	y	L_1	L_2	k_1	k_2
$\beta\omega = 1$	1.888	3.775	0.499	0.486	0.469	0.458	0.397	0.066
$\beta\omega = 0.95$	2.003	4.006	0.468	0.450	0.445	0.428	0.374	0.062
$\beta\omega = 0.90$	2.131	4.263	0.440	0.416	0.422	0.399	0.352	0.059
$\beta\omega = 0.5$	4.062	8.124	0.231	0.193	0.234	0.196	0.185	0.031

The result that trade is not always Pareto improving is surprising because here each country is represented by a single agent. This occurs because in the UST model trade may lead to a decline in average capacity utilization. As will be seen in the next section, this is special to the UST model: In the standard model, capacity is always fully utilized and therefore trade leads to a Pareto improvement.

Comparison with the standard Walrasian model:

In the standard Walrasian model, information about the state of the world becomes public knowledge before the beginning of trade. Let p_1 denote the Walrasian price when world demand is high and p_2 denote the Walrasian price when world demand is low. The purchasing power of a normalized dollar for the residents in country 1 and 2 are given by:

(37) $$x = (^1/_2)(1/p_1) + (^1/_2)(1/p_2)$$

(38) $$y = (^1/_2)[(^1/_2)(1/p_1) + (^1/_2)\beta\omega y] + (^1/_2)(1/p_2).$$

Supply decisions are made prior to the realization of demand and depend on the average price:

(39) $$p = (^1/_2)p_1 + (^1/_2)p_2.$$

Labor supply is determined by:

(40) $$2L_1 = \beta\omega px$$

(41) $$2L_2 = \beta\omega py$$

(42) $$L = (^1/_2)(L_1 + L_2),$$

where L_i is the supply of labor in country i and L is the average per household labor supply.

The demand is 3/4 normalized dollars per seller in state 1 and one normalized dollar per seller in state 2. Market clearing conditions are therefore given by:

(43) $$p_1L = 3/4,$$

(44) $$p_2L = 1.$$

A standard Walrasian equilibrium in the integrated economy is a solution (p_1, p_2, p, x, y, L_1, L_2, L) to (37) - (44). The solution to the labor supply in country 1 is 0.510($\beta\omega$). This is more than under autarky (0.5$\beta\omega$) and indicates an improvement in welfare. The solution for the four levels of $\beta\omega$ is given in Table 6.

Table 6: Equilibria for the integrated economy in the standard model (the assymetric case)

	p_1	p_2	p	x	y	L_1	L_2	L
$\beta\omega = 1$	1.435	1.914	1.674	0.610	0.639	0.510	0.535	0.523
$\beta\omega = 0.95$	1.524	2.031	1.777	0.574	0.592	0.484	0.500	0.492
$\beta\omega = 0.90$	1.621	2.162	1.892	0.540	0.547	0.459	0.466	0.463
$\beta\omega = 0.5$	3.097	4.129	3.613	0.283	0.254	0.255	0.229	0.242

Note that uncertainty about demand leads to trade also in the standard Walrasian model. For example, if the money held by the buyers of country 1 is larger than the value of the supply in country 1, then country 1 buyers will buy in country 2. But it is always the case that only one country imports and only one country exports. In the UST model, sellers from all countries and buyers from all countries participate in all markets and therefore in general we will get that all countries export and import in the same period. This is consistent with the observation that in each period we see the same country both import and export the same good.

CONCLUSIONS

In this paper we used the sequential trading model to address two problems in international trade: The purchasing power parity puzzle and the trade in similar goods puzzle. These problems were addressed in an ideal environment in which all countries have access to the same constant returns to scale technology and there are no transportation costs.

The sequential trade model abandon the law of one price in favor of an equilibrium price distribution. Therefore there is no difficulty in getting deviations from PPP and in getting a high correlation between the real and the nominal exchange rates. What may be more surprising is that uncertainty about demand may lead countries to simultaneously export and import the same good. This follows from the observation that under autarky the distribution of equilibrium prices will in general be different across countries, creating an incentive for international trade in the same good.

It is also shown that international trade does not always lead to Pareto improvement. This may occur in the UST model because trade may increase demand uncertainty and reduce average capacity utilization.

REFERENCES

Butters, G. "Equilibrium Distribution of Sales and Advertising Prices" Review of Economic Studies 44:467-491 (1977).

Bental, Benjamin and Benjamin Eden "Inventories in a Competitive Environment" The Journal of Political Economy, October 1993, Vol.101, No.5, 863-886.

_____ "Money and Inventories in an Economy with Uncertain and Sequential Trade", Journal of Monetary Economics, 37 (1996) 445-459.

Chamberlin, Edward H. The Theory of Monopolistic Competition. Cambridge, Mass.: Harvard University Press, 1933.

Dana James D. Jr. "Advance-Purchase Discounts and Price Discrimination in Competitive Markets" Journal of Political Economy, Vol.106, Number 2, April 1998, 395-422.

Dixit, Avinash, and Stiglitz, Joseph E. "Monopolistic Competition and Optimum Product Diversity." American Economic Review 67 (1977): 297-308.

Eden, Benjamin. "Marginal Cost Pricing When Spot Markets are Complete" Journal of Political Economy, Dec. 1990. Vol. 98, No.6,1293-1306.

_______ "The Adjustment of Prices to Monetary Shocks When Trade is Uncertain and Sequential" Journal of Political Economy, Vol. 102, No.3, 493-509, June 1994.

Engel, Charles M. "Real exchange rates and relative prices: An empirical investigation" Journal of Monetary Economics, August 1993, 32, 35-50.

_______ "Accounting for U.S. Real Exchange Rate Changes" Journal of Political Economy, June 1999, 107, 507-38.

Helpman, Elhanan and Paul R. Krugman. Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition and the International Economy M.I.T press, 1985.

Karaken John and Neil Wallace. "The indeterminacy of equilibrium exchange rates". Quarterly Journal of Economics, May 1981, 96, 207-22.

Lucas, Robert. E., Jr. "Equilibrium in a Pure Currency Economy" in Models of Monetary Economies, John H. Karaken and Neil

Wallace, editors, Published by the Federal Reserve Bank of Minneapolis, 1980.

Lucas, Robert. E., Jr. and Michael Woodford "Real Effects of Monetary Shocks In an Economy With Sequential Purchases" Preliminary draft, The University of Chicago, April 1994.

Mussa, Michael "Nominal exchange rate regimes and the behavior of the real exchange rates: Evidence and implications" Carnegie-Rochester Series on Public Policy ,1986 (Autumn), 25, 117-214.

Obstfeld Maurice and Kenneth Rogoff. Foundations of International Macroeconomics MIT Press 1996.

Prescott, Edward. C., "Efficiency of the Natural Rate" Journal of Political Economy, 83 (Dec. 1975): 1229-1236.

Kenneth Rogoff "The Purchasing Power Parity Puzzle" Journal of Economic Literature" Vol. XXXIV (June 1996), pp. 647-668.

Williamson, Stephen D. "Sequential Markets and the Suboptimality of the Friedman rule" Journal of Monetary Economics; 37(3), June 1996.

Woodford, Michael "Loan Commitments and Optimal Monetary Policy" Journal of Monetary Economics; 37(3), June 1996, 573-605.