EFACR 7/26/02
DIAZ/LUENGO-PRADO 11:15AM

Durable Goods and the Wealth Distribution*

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Abstract

In this paper we study the role of durable consumption goods and collateral lending in shaping the wealth distribution in an otherwise standard heterogeneous agents model economy with idiosyncratic uncertainty. The model accommodates two kind of assets, durable goods and financial assets, and therefore it can be used to analyze wealth composition issues. We find that the model can reproduce several features of the data: (i) The distribution of total wealth is much more concentrated than that of earnings, (ii) the distribution of financial assets is even more concentrated than that of total wealth, (iii) the distributions of earnings and durables are very similar and (iv) durables as a fraction of total wealth decrease with wealth.

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1 Introduction

U.S. data shows that the composition of household wealth (net worth) differs significantly across households. For instance, the 1998 *Survey of Consumer Finances* reports that the poorest 80 percent of households hold, on average, 72.361 percent of their wealth as real state and automobiles, whereas this percent goes down to 23.14 for the top 20 percent in the distribution of total wealth. Moreover, the distribution of financial assets (net worth minus real state and automobiles) is extremely unequal: the top 20 percent of households holds 101.1 percent of these assets. Additionally, 92.16 percent of all available credit to households is for the purchase of houses and automobiles (collateral credit). The average ratio of collateral credit to total debt across households is 78.71 percent! These figures suggest that for a high proportion of the population net worth consists of durable goods and debts, typically collateralized debts. All this implies that the debt and wealth positions of households depend not only on their earnings history and the consumption stream that they choose, but on the financial conditions that determine the amount of collateral needed to obtain credit for the purchase of houses and automobiles. In this paper we want to investigate the effects that the explicit consideration of the existence of durable goods and collateral credit has on the shape of wealth distribution, wealth composition, and its effects on the level of precautionary savings. Our model is an otherwise standard heterogeneous agents model economy with idiosyncratic uncertainty.

First of all, we document some features of the wealth distribution of the U.S. economy. Our source is the 1998 *Survey of Consumer Finances.* We report inequality statistics for three measures of wealth: total wealth, durable goods, and financial assets. *Total wealth* is defined as net worth: total assets minus total liabilities. The category *durables* comprises residential stock and automobiles and *financial assets* are equal to net worth minus the value of the stock of durables. Thus, all debts, including mortgages, home equity loans and car loans, are a negative position of financial assets. We also report inequality statistics for earnings.

The first thing we notice is that the distribution of total wealth is much more concentrated than that of earnings, their respective Gini indices being 0.796 and 0.611. The distribution of financial assets is even more concentrated than that of total wealth: its Gini index is 0.953. Durables and earnings have very similar distributions: their Gini indices are, respectively, 0.626 and 0.611, while the mean to median ratio is 1.52 for durables and 1.57 for earnings. The similarity of these distributions is a feature that appears not only in the 1998 *Survey of Consumer Finances*, but also in the years 1992, and 1989. Thus, we think this is an empirical regularity of the data. Finally, the value of durables as a fraction of total wealth decreases with the level of wealth: the bottom 40 percent of households hold 317.08 percent of their wealth as durables, whereas the top 20 percent only hold 29.19 percent of their wealth as houses and automobiles. Table 4 summarizes these facts.

In order to construct a model that accounts for all these patterns, we need to take into account the specific features that differentiate durables from other assets: First, households derive utility from the services they provide but typically they are not part of the productive capital (it is financial assets that usually materialize into productive capital). Second, there may be down payment requirements when buying durable goods.

Third, durable goods can be used as a collateral for credit. That is, individuals can borrow up to some fraction of the amount of durable goods they own. Fourth, selling or buying durable goods may be subject to adjustment costs.

We build a model economy of ex-ante identical households who face uninsurable idiosyncratic shocks to their labor endowments that accommodates the four features of durables aforementioned. In our model, households save not only for precautionary reasons but to accumulate enough assets to satisfy the down payment required to purchase durable goods. Moreover, in this economy, households can borrow against the value of their holdings of durables. Thus, there are two reasons why households hold them: they provide services and they provide collateral for credits (durable equity loans). We consider the model with and without adjustment costs. For simplicity, we allow no other form of credit. We also assume that there is no rental market for durable services. We calibrate our model economy so its steady state aggregate statistics match some of the aggregate statistics of the U.S. economy and the data on earnings distribution. The wealth distribution and wealth composition statistics delivered by the model resemble quite closely those of the data in all the dimensions previously discussed.

We hypothesize that the similar distributions of durables and earnings may be the result of a high persistence in the earnings process. In order to verify this, we perform an experiment with an earnings process with much higher volatility than the benchmark one and the same stationary distribution. We find that higher volatility leads to a much more egalitarian wealth distribution in all its three dimensions: total wealth, financial assets and durables. In this case, the distribution of durables no longer resembles that of earnings. The reason is that the distribution of durables depends on the distribution of lifetime income. If the earnings process is very persistent, lifetime income is highly correlated with that of earnings and so is the distribution of durables. If, on the contrary, earnings are very volatile, lifetime income correlation with current earnings is low and so is the correlation of durables with earnings.

Next, we consider the aggregate and distributional effects of changes in collateral requirements. As down payments increase, the ratio of financial assets to output increases, the ratio of durables to output decreases and total output increases. As for the distributional effects, total wealth inequality decreases and so does inequality in financial assets. The distribution of durables remains almost unchanged. The explanation is twofold. On one hand, a higher down payment implies that households have to finance a higher fraction of the durable purchased with their savings. This makes inequality in financial assets and, hence, total wealth to decrease. On the other hand, the amount of credit they can obtain as durable equity loans is smaller. Thus, households fear more a bad shock in earnings and consequently the amount of their wealth held as durables decrease.

Finally, we consider the effects of a tax on the stock of durables to highlight the similarity between the existence of adjustment costs and taxes. We could interpret the existence of adjustment costs in durables as a tax that is paid whenever the stock of durables is changed. Adjustment costs imply that changing the durable is costly and, consequently, wealth poor households hold a higher fraction of their wealth as durables to avoid frequent changes. The opposite occurs to wealthier households who find durables less attractive: they hold a higher fraction of their wealth in the form of the liquid asset. When a tax on the durable stock is present, it has to be paid every period. In this case, the effective return to durables is lower and as a result all households want to hold a lower fraction of their wealth as durables. This effect is stronger for wealthier households. Inequality in total wealth and financial assets decreases in both cases. As for the aggregate effects, the ratio of financial assets to output and aggregate output increases in both scenarios.

The previous literature on wealth inequality uses a common framework: economies populated by a large number of ex-ante identical households with standard preferences that are subject to idiosyncratic uninsurable shocks in their labor endowments. In this framework households face a process on labor earnings that is random, household-specific and non-insurable and accumulate wealth in part to smooth their consumption. The first paper to address this issue was Aiyagari (1994). Quadrini and Ríos-Rull (1997) review the literature up to that date. Lately, Castañeda, Díaz-Giménez, and Ríos-Rull (2000) have provided a theory to jointly understand the distribution of earnings and net worth with great success. To our knowledge, the literature on wealth inequality has abstracted from the fact that households differ in the composition of their wealth. In other research areas, the explicit consideration of durable goods has had a great success in accounting for the behavior of aggregate consumption. For instance, it has proved to be key to understand the "excesses" in aggregate consumption (see for instance, Farr and Luengo-Prado 2001) but so far, the wealth distribution implications of the existence of durable goods has not been explored yet. This project attempts to fill this gap.

2 The Model Economy

We consider a production economy populated by a continuum of households of measure one that live forever in a general equilibrium model with labor income uncertainty, durable goods and collateralized liquidity constraints. We focus our analysis on steady states. Sections and 2.1, 2.2 and 2.3 describe the technology, preferences and the market arrangements. In section 2.4 we write down the household problem while section 2.5 presents a formal definition of steady state equilibrium.

2.1 Technology

Aggregate output, Y, is produced according to a Neoclassical production function that takes the aggregate capital stock, K, and aggregate labor, L, as inputs: $Y = F(K, L)$. Given constant returns to scale, we can assume without loss of generality that there is a single representative firm.

The final good, can be either consumed, invested in capital or invested in durable goods on a one-to-one basis. Therefore, we can write the feasibility constraint as:

$$C + I_k + I_d = F(K, L), \tag{1}$$

where C is nondurable consumption, I_k is investment in capital and I_d is investment in durable goods. We assume that durable goods and capital depreciate at the rates δ^d and δ^k respectively.

2.2 Preferences

Households derive utility from the consumption of a nondurable good, c, and from the service flow, $s(d)$, provided by a durable good, d. We write the per period utility as $u\left(c, s(d)\right)$, and lifetime utility as $\sum_{t=0}^{\infty} \beta^t u\left(c_t, s(d_t)\right)$, where β is the time discount factor. For the purpose of this paper, durable goods are private residential assets plus automobiles.

2.3 Market arrangements

Each period, households receive an idiosyncratic labor endowment shock $e \in E = \{e_1, ..., e_{n_e}\}$, that they supply inelastically in the labor market. The shock is Markov with transition matrix, $\pi_{e,e'}$. We assume that there are no state contingent markets for the household specific shock.

Households hold durable goods $d \in [0, \infty)$ and financial assets $k \in [\underline{k}, \infty)$, $\underline{k} \in \mathbb{R}$. Durable goods depreciate at rate δ^d and financial assets pay a net interest rate r. For simplicity, we assume that there are no differences between borrowing and lending rates.

Durable goods provide collateral for loans. In particular:

$$k \geq -(1-\theta)d. \tag{2}$$

This liquidity constraint implies that the maximum debt an individual can incur is a fraction $(1-\theta)$ of his durable stock, which determines the lower bound for k, $\underline{k}$. The constraint summarizes several aspects of collateral lending that we see in reality. First, it means that when purchasing a durable good, a household can only finance a fraction $(1-\theta)$ of it. In other words, it must satisfy a *down payment* requirement θ. The constraint also implies that when a household owns a durable good, it can obtain a loan for up to a fraction $(1-\theta)$ of its value (*durable equity loans*). In summary, at any point in time, an agent is only required to keep an accumulated durable equity of θd. Note that the constraint also implies that the household cannot borrow if it does not own any durable good. In practice, financial institutions require down payments for a number of reasons. They reduce the moral hazard problem in the care that owners take in maintaining the value of the durable and they also mitigate the effects of the adverse selection problem that results from asymmetric information in the credit market.

2.4 The household's problem

The household's state variables are its labor endowment shock, its holdings of capital, and its holdings of durable goods, $\{e, k, d\}$. The problem that a household solves is:

$$v(e,k,d) = \max_{c\geq 0, s\geq 0} u(c, s(d)) \; + \; \beta \sum_{e'} \pi_{e,e'} \; v\left(e', k', d'\right) \tag{3}$$

$$\text{s.t.} \qquad c + k' + d' - \left(1-\delta^d\right) d = w \; e + (1+r) \; k, \tag{4}$$

$$k' + (1-\theta)\, d' \geq 0. \tag{5}$$

Note the timing of the model: consumption of the nondurable good takes place after depreciation of the stocks, whereas services of the durables are obtained before the stock depreciates.

It is well known that under certain conditions, problems of this type have a solution that we denote $k' = g^k(e,k,d)$, $d' = g^d(e,k,d)$, $c = g^c(e,k,d)$ with an upper and a lower bound on capital holdings, $\{\underline{k}, \overline{k}\}$, and an upper bound on the stock of durable good, $\overline{d}$, such that $\overline{k} \geq g^k(e,k,d) \geq \underline{k}$ and $\overline{d} > g^d(e,k,d) > 0$, for all $e \in E$, all $d \in \{d \mid 0 \leq d \leq \overline{d}\}$, and all $k \in \{k \mid \underline{k} \leq k \leq \overline{k}\}$. We may use the compact notation $x = \{e,k,d\}$ and $X = \{E \times [\underline{k}, \overline{k}] \times [0, \overline{d}]\}$. With respect to capital, the required condition is that we have a low enough rate of return, $\beta < \frac{1}{1+r}$. See Aiyagari (1994), Huggett (1993), or Quadrini and Ríos-Rull (1997) for details.

It is possible to construct a Markov process for the individual state variables, from the Markov process on the shocks and from the decision rules of the agents (see Huggett (1993) or Hopenhayn and Prescott (1992) for details). Let $\mathcal{B}$ be the σ-algebra generated in X by, say, the open intervals. A probability measure μ over $\mathcal{B}$ exhaustively describes the economy by stating how many households are of each type. Note that the first moment of μ over e yields the aggregate labor input while the first moment over k yields aggregate capital.

Let $Q(x, B)$ denote the probability that a type $\{x\}$ has of becoming of a type in $B \subset \mathcal{B}$. The function Q naturally describes how the economy moves over time by generating a probability measure for tomorrow μ' given a probability measure, μ, today. The exact way in which this occurs is

$$\mu'(B) = \int_X Q(x, B)\ d\mu. \tag{6}$$

If the process for the earnings shock is normal in the sense that it has a unique stationary distribution, then the economy will also have a unique stationary distribution.[1] Furthermore, this unique stationary distribution is the limit to which the economy converges under any initial distribution.[2]

2.5 Equilibrium

We have almost all the ingredients to define a steady state equilibrium. We only need to add the condition that marginal productivities yield factor prices as functions of μ. Note that to obtain a steady state, we look for a measure of households μ such that given the prices implied by that measure, households actions reproduce the same measure μ in the following period. Formally, a steady state equilibrium for this economy is a set of functions for the household problem $\{v, g^k, g^d, g^c\}$, and a measure of households, μ, such that:

[1] For example if it satisfies the "American–dream American–nightmare" condition stated in Ríos-Rull (1995), then there is a unique stationary distribution of households over earning shocks, assets holdings and stock of habits.

[2] This does not mean that this will happen in equilibrium outside the steady state since the transition Q has been constructed under the assumption that the households think that prices are constant.

1. Factor inputs are obtained aggregating over households: $K = \int_X k \, d\mu$, and $L = \int_X e \, d\mu$.

2. Factor rental prices are factor marginal productivities, $r = F_1(K, L) - \delta^k$, and $w = F_2(K, L)$.

3. Given μ, K, L and D, the functions $\{v, g^k, g^d, g^c\}$ solve the households' decision problem described in Subsection 2.4.

4. The markets for capital, nondurables, and durables clear: $\int_X \left[g^k(x) - \left(1 - \delta^k\right) k\right] d\mu = I_k$, $\int_X g^c(x) \, d\mu = C$, $\int_X \left[g^d(x) - \left(1 - \delta^d\right) d + \tau\left(g^d(x), d\right)\right] d\mu = I_d$.

5. The measure of households is stationary: $x(B) = \int_X Q(x, B) \, d\mu$, for all $B \subset \mathcal{B}$.

3 Calibration

In this section we discuss the calibration of our model economy. We are going to compare the predictions of our model with those delivered by a standard one asset economy that abstract from the composition of wealth. Before doing so, we discuss the relationship between both frameworks as this will help us to understand their different implications on wealth distribution.

3.1 The model versus a one asset economy

Our goal is to explore the role of durable goods in quantitatively shaping the wealth distribution. In order to do this, we must compare our model economy, which we call a *two-asset economy*, with a *one-asset economy*, essentially a version of Aiyagari (1994). The comparison of both economies is not straightforward. They differ not only in the way preferences are defined over the consumption of services of durable goods but also in the technology. For instance, in the standard model, private residential assets are part of the stock of productive capital whereas in our model they are not, and, therefore, both economies should differ in the share of capital in the aggregate production function. Thus, comparing both economies requires a detailed specification of what we call output and aggregate capital in each model.

Let us assume that there was a perfect rental market for durable goods. The household's budget constraint is:

$$c + r^s s + k' + d' - \left(1 - \delta^d\right) d = we + (1 + r) k + r^s s(d).$$

The rental market for durable services operates in the following way: all households pool their stock of durables together, receive a rent from the flow of services of the stock they own, $r^s s(d)$, and pay the value of the services they consume, $r^s s$. In this type of economy no down payment is required when buying a new unit of the durable good. In equilibrium, the rental price of capital and durable goods should be the same. Therefore, the composition of the household's portfolio is irrelevant when studying the distribution of wealth and we could think of capital and durables as a single asset.

This model economy, however, differs on two accounts from a standard one-asset economy. First, consumption of nondurables and services from durables are not perfect substitutes and, second, the stock of durables is not part of the stock of productive capital. Regarding household preferences, we cannot assume that both types of goods are perfect substitutes, otherwise the demand for each good would be either zero on indeterminate, depending on the relative prices. For this model economy to deliver the same implications on wealth distribution and aggregate savings as a standard one-asset economy, we need one more assumption about the capital share. We know that the equilibrium interest rate depends on the capital share in the production of the final good and on the output-capital ratio. Given that the durable stock is not part of the stock of capital used in the production of the final good, the share of capital should be smaller.

Taking into account all these considerations, the implications of a two-asset economy and a one-asset economy should be the same. Thus, the comparison that we make in section 4 could be thought of as comparing an economy without a rental market for durables and with the collateralized credit constraint with an economy with a perfect rental market and without the constraint. The calibration procedure and the data used are described in detail in Appendix A.

3.2 The earnings process

With respect to the process for earnings, Aiyagari (1994) sets an AR(1) in the logarithm of labor income. The process is fully described by two parameters: its persistence and its volatility. He chooses both values following estimates of Kydland (1984) that used PSID data and of Abowd and Card (1987) and Abowd and Card (1989) that used both PSID and NLS data. Then, he approximates the process by using a seven-state Markov chain following the procedures described in Tauchen (1986). Aiyagari (1994) fails to account for the amount of wealth inequality in the U.S. The main reason for this failure is that the earnings process that he uses is much more egalitarian that what we see in the data. In his benchmark calibration, the Gini Index for earnings is 0.10, where it is around 0.60 for the U.S. economy (see, for instance Budría, Díaz-Gimenez, Quadrini, and Ríos-Rull 2001). We use the earnings process and the transition matrix use by Díaz, Pijoan-Mas, and Ríos-Rull (2000). The process and the transition matrix are constructed to match the Lorenz curves of the U.S distributions of earnings and total wealth.[3] In section 6, we show some results for the same earnings process but with lower persistence.

3.3 Technology, preferences and market arrangements

For preferences over consumption of the nondurable good and services from the durable good we choose $u(c_t, s(d_t)) = \left(\frac{c_t^{1-\sigma}}{1-\sigma} + \gamma\frac{(d_t+\varepsilon)^{1-\sigma}}{1-\sigma}\right)$ as in Farr and Luengo-Prado (2001).

We assume that durable services are proportional to the durable stock and we choose the constant of proportionality to be one. This assumption is not restrictive, since we calibrate γ to match the ratio of nondurable goods to investment in durable goods, $C/I_D =$

[3]See Castañeda, Díaz-Giménez, and Ríos-Rull (2000) and Díaz, Pijoan-Mas, and Ríos-Rull (2000) for a discussion on this calibration choice.

Table 1: THE EARNINGS PROCESS

$$e \in \{e_1, e_2, e_3\} = \{1.00, 5.29, 46.55\}$$

$$\pi_{e,e'} = \begin{bmatrix} 0.96500 & 0.00347 & 0.000333 \\ 0.03937 & 0.95000 & 0.010625 \\ 0.00000 & 0.08300 & 0.917000 \end{bmatrix}$$

Stationary distribution			
$\pi^* =$	0.4983	0.4429	0.05870

Table 2: PARAMETERS FOR THE BENCHMARK CASE.

	Utility			Technology			Market Arrangements
	β	σ	γ	α	δ_k	δ_d	θ
Two Assets	0.769	3	1.4	0.1864	0.081	0.093	0.300
One Asset	0.783	3	-	0.348	0.087	-	-

5.29, that we see in the data. We set the discount factor β so that net interest rate in the steady is 4.63%. We choose $\sigma = 3$.[4]

Feasibility in our model is given by expression (1). Thus, in this model aggregate output corresponds to measured GDP minus housing services. The share of capital is 0.1864. We set depreciation rates so that $\delta^k = \frac{I^k/Y}{K/Y} = \frac{0.1189}{1.4673} = 0.081$ and $\delta^d = \frac{I^d/Y}{D/Y} = \frac{0.1410}{1.5253} = 0.093$.

The parameter θ of the borrowing constraint is set equal to 0.3 to roughly match the average down payment for cars and houses during the period that we consider, 1954-1999. Thus, individuals can borrow up to 70 percent of the value of their holdings of durable good. The procedure for obtaining these numbers is detailed in Appendix A and parameter values are summarized in Table 2.

3.4 The one-asset economy

Preferences over consumption of the unique good are $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$. We choose $\sigma = 3$. The share of capital is 0.3480 for a capital-output ratio target of 2.51. We set the discount factor β so that net interest rate in the steady is 5.23 percent. We calibrate the depreciation rate so that the investment-capital ratio is equal to 0.0871 as in the data. The procedure for obtaining these figures is detailed in Appendix A and parameter values are summarized in Table 2.

[4]Results did not change qualitatively for $\sigma = 1$ and $\sigma = 2$ and nonseparable utility functions.

4 Results

In this section we compare our benchmark two-asset economy and the economy with one-asset in two dimensions: the aggregates and their wealth distribution implications.

4.1 Precautionary savings

In order to analyze how the explicit consideration of durable goods changes the volume of aggregate savings, we compare each economy to its deterministic counterpart (the case with no uncertainty). Aggregate ratios are summarize in Table 3. Both economies are calibrated to produce comparable aggregates.

In the one-asset economy the capital-output ratio more than doubles when idiosyncratic uncertainty is present (increasing from 0.96 to 2.52). The increase of the capital stock due to precautionary savings is 342.3 percent.

Let us now turn to the two-asset economy. When there is uninsurable idiosyncratic risk, the durable-output ratio decreases slightly (compared to the case without uncertainty) while the capital-output ratio increases dramatically. Thus, households increase their holdings of total assets as a response to idiosyncratic uncertainty. Nevertheless, it is not straightforward how to measure the volume of precautionary savings. If we think that the measure is the variation in total wealth, $K + D$, then precautionary savings can be measured by the increase in this total stock, which is 56.6 percent higher than in the case without uncertainty. We could argue that this is the right measure since durables can be sold to smooth nondurable consumption. Nevertheless, durables are a consumption good as well (smoothness is valued) and the right measure of precautionary savings could be the change in what we call *voluntary equity*, which is $K + (1 - \theta)D$, the equity held in excess of the required down payment. Remember that a household can borrow up to the fraction $(1 - \theta)$ of its durable stock. Thus, voluntary equity measures the volume of resources ready to smooth any bad shock, without changing the durable stock. The change in this stock is 57.8 percent. There is another reason why we believe this measure as the right one. The voluntary equity to output ratio in our model economy is 2.51, whereas the capital-output ratio in the economy with one asset is 2.52 (the small difference is due to computation error). Finally, financial assets increase by a 289.2 percent relatively to the deterministic case without uncertainty. Whatever the right measure of precautionary savings, these are lower in our model are than in a model economy that abstracts from durables.

Table 3: AGGREGATE RATIOS

	Model		One Asset		*U.S. Data*	
	Determ.	Benchmark	Determ.	Benchmark	Two Assets	One Asset
Output	1.00	1.329	1.00	1.67	-	-
$K/Y + (1-\theta)D/Y$	1.59	2.51	-	-	*2.53*	-
$K/Y + D/Y$	2.07	2.97	-	-	*2.99*	-
K/Y	0.48	1.45	0.96	2.52	*1.47*	*2.51*
D/Y	1.59	1.52	-	-	*1.52*	-
C/I_D	4.98	5.28	-	-	*5.29*	-
Interest Rate	30.0%	4.63%	27.71%	5.23%	*4.63%*	*5.23%*

4.2 Wealth distribution

Wealth distribution implications are summarized in Table 4. We present quintile shares, Gini coefficients and the mean to median ratio for some key variables–earnings, durables, financial assets and total wealth–in the benchmark model and in the data (*Survey of Consumer Finances*, 1998). For total wealth, we also report distribution measures for the model with one asset. Note that households are ranked according to the variable specified in each row. We also report wealth composition, defined as the fraction of durables in total wealth. The row labelled"adjustment costs" is not relevant for this section and therefore it can be ignored.

Table 4 shows that the distribution of earnings is slightly less concentrated than in the data. Calibrated earnings perform relatively worse for the bottom quintile, but we should keep in mind that in the model, households always have positive earnings by construction, while they can be negative in the data, due to losses reported by the self-employed.

The distribution of durables is very similar to that of earnings, as in the data. This similarity of both distributions is a feature that appears not only in the 1998 *Survey of Consumer Finances*, but also in the years 1992, and 1989. Thus, we believe that it is an empirical regularity. Our model captures well this resemblance. The discussion of the reasons that allow the model to reproduce this feature of the data is postponed to Section 6.

Note that inequality in financial assets is almost as high as in the data and considerably higher than inequality in durables. Thus, durables are less concentrated than financial assets. We should not forget that durables are consumption goods, as well as assets, and therefore their distribution should probably be more equal than that of financial wealth. Moreover, since households can borrow to finance the purchase of durable goods the level of inequality in financial assets will always be higher than in durables in the model (financial assets can be negative, while the durable stock has zero as its lower bound).

Total wealth inequality (or net worth inequality) in our model is slightly lower than in the standard one-asset economy. This is because the dispersion of durable goods is very small, which implies a lower index for net worth. Total wealth is more unequal than earnings and durables are more equal than financial assets in the data and in the model.

In summary, in the data, financial assets are more unequal than total wealth, total wealth is more unequal than earnings, and the distributions of durables and earnings are strikingly similar. Our model can reproduce all of these patterns with more than acceptable order of magnitude.

One advantage of the model presented here over the standard one-asset framework previously used to study wealth distribution issues, is the fact that we can distinguish between two types of assets: durables and financial assets. Let us now turn to the implications of our model for the composition of wealth. Table 4 shows the percentage of a household's net worth held as durables. We can see that this fraction decreases across quintiles, as in the data. The main explanation for this behavior is the concavity of the utility function. Part of the return to durables is the marginal utility from the consumption of its services, which is decreasing. As a result, durables as a fraction of total wealth must diminish, since the return to competing financial assets, the interest rate, is constant. The model fares very well in this dimension for the bottom 40 percent of households. This means that we are able to explain the level of debt of the poorest 40 percent of households. Its predictions for the other quintiles, however, are less satisfactory. Nevertheless, we should keep in mind that our model abstracts from differences in taxation for durables and financial assets, and more importantly, from life cycle effects and adjustment costs in the durables market. The following section explores the implications of adjustment costs for wealth composition.

5 The Adjustment Cost Case

Can adjustment costs help us explain the low durable to total wealth ratio for the rich? In the first part of this paper we have worked under a very strong assumption about the durables market: there were no costs associated with additional changes in the durable stock. This does not reflect reality very well, since one of the well observed characteristics of the durables market is its illiquidity. People do not move every day or change the car they drive very often, and when they do, they encounter several costs (search costs, taxes and transaction costs). In this section, we extend the model to allow for adjustment costs in the durable market. We consider non convex costs of adjustment as proposed by Grossman and Laroque (1990), which generate (S, s) adjustment behavior. In most periods, consumers do not adjust their durable stock, but when they do, they usually make substantial changes. The specification is:

$$\tau(d', d) = I\rho(1 - \delta^d)\, d, \tag{7}$$

where $I = 0$ if $d' - (1 - \delta^d)\, d = 0$ and $I = 1$, when $d' - (1 - \delta^d)\, d \neq 0$. Note that this cost can be seen as a loss in the selling price when changing the durable good. The budget constraint for the household problem becomes:

$$c + k' + d' - \left(1 - \delta^d\right)(1 - I\rho)\, d + \ = we + (1 + r)\, k. \tag{8}$$

We solve the model under this new specification after adjusting some parameter values to guarantee that we are comparing economies with similar aggregate ratios. We choose and adjustment cost parameter $\rho = 0.1$. We need to recalibrate γ, the parameter in the utility function, to maintain the ratio C/I^D constant, and γ turns out to be 2.18. Note that now the durable is more illiquid and households must like it more to keep the same proportion of nondurables to durables in the aggregate. In order to keep the capital-output ratio constant and the interest rate $r = 4.63\%$, we must change β to 0.7639 as well.

Reverting to Table 4, we see that the introduction of adjustment costs improves somewhat the approximation of the model to the data in terms of wealth composition. The poor hold more durables on average to avoid frequent costly adjustment, while the rich find financial assets more attractive. For higher values of the adjustment cost parameter the fit can be even better, but parameters higher than 10% are difficult to justify. This leads us to believe that a better explanation for the wealth composition in the top quintiles requires the consideration of life cycle effects (the old and rich selling their houses), and differential tax treatment between financial and nonfinancial assets.

Since the improvements derived of adjustment costs are minor, we abstract from them in the following sections.

Table 4: WEALTH DISTRIBUTION AND WEALTH COMPOSITION

	Share (%)	Quintiles 1st	2nd	3rd	4th	5th	Gini	Mean/Median
EARNINGS	Benchmark	3.53	3.53	11.62	18.7	62.62	0.602	1.069
	US Data	*-0.20*	*4.00*	*13.00*	*22.90*	*60.30*	*0.617*	*1.57*
DURABLES	Benchmark	2.67	3.53	12.12	17.57	64.11	0.606	1.56
	US data	*0.22*	*2.92*	*12.78*	*22.44*	*61.64*	*0.626*	*1.52*
FINANCIAL ASSETS	Benchmark	-2.81	-1.96	-0.28	2.29	102.76	0.913	113.07
	US Data	*-9.52*	*-1.07*	*0.36*	*5.84*	*104.39*	*0.953*	*134.54*
TOTAL WEALTH	Benchmark	0.41	0.85	6.34	9.62	82.78	0.774	3.12
	One Asset	0.00	0.61	5.90	9.48	84.01	0.789	3.28
	US data	*-0.30*	*1.35*	*5.14*	*12.37*	*81.44*	*0.796*	*3.95*
WEALTH COMPOSITION	Benchmark	308.525		102.77	91.56	46.5		
	Adjustment cost*	315.11		100.26	82.30	45.7		
	US Data	*317.08*		*120.28*	*78.12*	*29.2*		

Notes: Data from the *Survey of Consumer Finances*, 1998.
* For the adjustment cost case, $\rho = 10\%$ and $\beta = 0.7639$ and $\gamma = 2.18$ to maintain the same aggregate ratios as in the benchmark model

6 The Distribution of Durables and Earnings

The question that we address in this section is why durables and earnings distributions are so similar. This discussion requires that we refer to lifetime income.

Lifetime income and earnings

Since we abstract from aggregate uncertainty in this model economy, we can write lifetime income at any period t as the sum of current and future earnings plus the value of current financial assets and durable assets:

$$E_t \left\{ w \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} e_s \right\} + (1+r)k_t + \left(1 - \delta^d\right) d_t.$$

We can simplify the previous expression the following way. Note that lifetime earnings depend on the value of the current labor shock e_t and on the transition probability given by the Markov process π:

$$E_t \left\{ w \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} e_s \right\},$$

We could compute a "lifetime earnings shock" the following way:

$$w\widehat{e} = e + \frac{1}{1+r} \sum_{e'} \pi_{e,e'} \widehat{e}'. \tag{9}$$

Lifetime income becomes $w\widehat{e} + (1+r)k + \left(1 - \delta^d\right) d$. Clearly, the higher the persistence in the labor endowment process e, the higher the correlation of current earnings and lifetime income. If the liquidity constraint is not binding, consumption of nondurables and durables is governed by lifetime resources. This should be the key to understanding the similarity of the distributions of earnings and durables: If the earnings process is very persistent, lifetime income and earnings will be very correlated and so will be durables and earnings.

In order to verify this hypothesis, we have constructed a measure of lifetime income using expression (9) from our model economy. The implied "lifetime earnings shocks" are $\widehat{e} \in \{70.96, 134.91, 458.24\}$, which after being normalized by $\widehat{e}_1$, become $\widehat{e} \in \{1.00, 1.90, 6.46\}$. Thus, lifetime earnings are not as unequal as current earnings. The coefficient of correlation between durables and lifetime income is 0.9684, and the correlation between lifetime income and earnings is 0.9686.

Table 5, panel A, shows the implied distribution of household lifetime income with households ordered by lifetime income and by earnings. As we can see, both distributions are very similar. That is, lifetime income distribution depends primarily on earnings. We also report the distribution of household's durables ranked by lifetime income and by durables. Again, the distribution of durables is almost the same. This amounts to say that lifetime income is the main determinant of the distribution of durables: durables follow earnings because lifetime income follows earnings.

Table 5: DURABLES, EARNINGS AND LIFETIME INCOME

	Quintiles Share (%)				
	1st	2nd	3rd	4th	5th
	A. *Persistent* Earnings Process				
EARNINGS	3.53	3.53	11.62	18.70	62.61
INCOME	3.09	3.16	11.32	17.30	65.13
LIFETIME INCOME	9.96	10.02	16.11	20.10	43.82
LIFETIME INCOME BY EARNINGS	10.43	10.44	16.49	21.62	41.03
DURABLES	2.67	3.53	12.12	17.57	64.11
DURABLES BY LIFETIME INCOME	2.67	3.57	12.87	17.32	63.57
	B. *Volatile* Earnings Process				
EARNINGS	3.64	3.64	11.27	19.23	62.23
INCOME	3.16	4.29	12.54	18.36	61.64
LIFETIME INCOME	17.07	17.87	18.73	20.67	25.66
LIFETIME INCOME BY EARNINGS	18.99	18.90	19.36	19.90	22.85
DURABLES	7.88	12.22	17.04	25.55	37.32
DURABLES BY LIFETIME INCOME	8.16	12.41	17.07	25.12	37.24

All Parameters as in Table 2. The persistent earnings process described in Table 1. The *Volatile* earnings process as in Table 6.

Table 6: A VOLATILE EARNINGS PROCESS

Earnings process	
$e \in \{e_1, e_2, e_3\} =$	$\{1.00, 5.29, 46.55\}$
$\pi_{e,e'} =$	$\begin{bmatrix} 0.4983 & 0.4429 & 0.0587 \\ 0.4983 & 0.4429 & 0.0587 \\ 0.4983 & 0.4429 & 0.0587 \end{bmatrix}$
Stationary distribution	
$\pi^* =$	0.4983 0.4429 0.0587

An alternative experiment

We conduct another experiment to further explore the hypothesis that durables are similar to earnings if earnings are similar to lifetime income. We simulate our model economy using a different transition matrix for the earnings process, holding the earnings shocks and the stationary distribution for earnings constant. The way to do this is to assume that earning shocks tomorrow have constant probabilities independent of shocks today and equal to the stationary distribution probabilities. We call this process the *volatile* earnings process, which is presented in Table 6.

We compute lifetime income and lifetime earnings for the model economy. In this case, the implied "lifetime earnings shocks" are $\widehat{e} \in \{125.03, 129.3184, 170.58\}$, which after being normalized by $\widehat{e}_1$, become $\widehat{e} \in \{1.00, 1.03, 1.36\}$. Thus, lifetime income in this case is much less concentrated than earnings. Table 7, shows the distributions of earnings, durables, financial assets and total wealth. We do not report aggregate ratios

Table 7: WEALTH DISTRIBUTION AND COMPOSITION WITH VOLATILE EARNINGS

	Quintiles Share (%)					
	1st	2nd	3rd	4th	5th	Gini
EARNINGS	3.64	3.64	11.27	19.23	62.23	0.601
DURABLES, D	7.88	12.21	17.04	25.55	37.32	0.305
FINANCIAL ASSETS, K	-2.99	-0.15	7.87	29.23	66.05	0.691
TOTAL WEALTH, $K+D$	2.59	6.20	12.64	27.27	51.29	0.501
WEALTH COMPOSITION, $D/(K+D)\%$	184.97	102.57	71.27	48.12	37.56	

Parameters as in Table 2. Aggregate ratios: $D/Y = 1.524$, $C/I_d = 5.289$ and $K/Y = 1.479$.

Table 8: DURABLES AND EARNINGS: MODEL AND DATA

Persistent Earnings	5.10	5.10	14.24	21.79	53.80
Volatile Earnings	17.43	17.25	18.70	20.35	26.27
U.S. Data	*5.69*	*10.14*	*14.57*	*20.93*	*48.64*

Durables ordered by household earnings.

since they do not change significantly with respect to the benchmark economy. The distribution of earnings is not affected by the change in the transition matrix since the stationary distribution remains the same. However, the distribution of wealth is changes dramatically. As we can see, the Gini index for total wealth is much lower now, 0.40 and that for durables is 0.30. Thus, the first lesson that we learn from this exercise is that persistence is needed to match the wealth distribution in its three dimensions.

Table 5 shows the implied distribution of household lifetime income ordered by lifetime income and earnings. As we can see, both distributions are very similar. We also report the distribution of household durables ranked by lifetime income and by durables. Again, the distribution of durables is the same but now much more egalitarian than in the benchmark model. The reason is that lifetime income distribution is now much more egalitarian.

The coefficient of correlation between durables and lifetime income now is 0.9198, and the correlation between lifetime income and earnings is only 0.7333. The fall in the latter correlation is what explains why the distribution of durables and earnings are no longer similar.

What happens in the data?

We have explained what makes earnings and durables distributions so similar in our model. Is this a plausible explanation for this feature of the data? Table 8 presents the distribution of durables ordered by earnings in both model economies, the benchmark economy with persistent earnings and the economy with volatile earnings shocks. The table also reports the distribution of durables in the data, for households whose head is of age 30-65, to abstract from life-cycle considerations.

As we can see in the table, the model with persistent earnings approximates the data better. This may be seen as indirect evidence of persistent earnings process in reality. Since we are dealing with an infinite-horizon model, this can also be seen as a sign of low social mobility.

7 Changes in the Down Payment Requirement

Over the last decades we have seen a fall in the down payment required by financial institutions for collateral lending as well as the proliferation of home equity loans. In this section, we study the effects of such measures on aggregate ratios and on the distribution of assets. In our model, a decrease in the parameter θ represents these financial improvements (although it cannot disentangle one from the other).

We simulate our model economy for different values of the down payment. Table 9 shows the main aggregate statistics and the Gini indices for different values of θ. Note that the capital-output ratio is lower the lower the down payment, whereas the durable-output ratio increases; thus, aggregate output falls. The overall inequality increases as the down payment increases. The reason is the following: Households want to buy durable goods, if the down payment is lower, they can afford higher consumption of durable good at low levels of income but they need to borrow more. Durable good dispersion remains fairly constant but financial wealth varies greatly, accounting for the increase in total wealth inequality. The changes in the distribution of financial assets are shown in Table 10. Composition of wealth for the bottom 40 percent of households varies significantly with changes in the down payment. The reason is, of course, that households for which the liquidity constraint is binding are concentrated in those two quintiles. Thus, a decrease in the down payment implies higher inequality and lower aggregate output. Nevertheless, households may be better off with lower down payments due to the ease of liquidity constraints.

This experiment illustrates that it is possible for total wealth to decrease and, at the same time, the amount of precautionary savings to increase. Table 9 shows the variation in precautionary savings, measured as voluntary equity ($K + (1-\theta)D$), with respect to the benchmark case $\theta = 0.3$. Note that as the down payment decreases the capital-output ratio decreases but households can borrow higher amounts against the value of their holdings of durables.

8 Taxes on Durables

The tax treatment for housing varies very much across countries. ¿From the point of view of optimal taxation, durables should be taxed, whereas financial assets should not, since durables are consumption goods and they should bear the burden of taxation (see Chamley 1986). In this section we report the following experiment: we assume that there is a tax of 5 percent on the household stock of durables. This tax finances the production of a public good which affects neither marginal returns nor marginal utilities. Table 11 presents the effects of the tax on the main aggregate ratios as well as its distributional impact.

¿From the household's point of view, a tax on the durable stock has the same effect as a larger depreciation rate. Thus, the return to durables falls for a given level of consumption of the durable good. Since the return to financial assets is constant this

Table 9: AGGREGATE RATIOS AND INEQUALITY INDEXES FOR DIFFERENT DOWN PAYMENTS

Down Payment θ	1	0.5	0.3	0.2
Y	1.01	1.00	1.00	0.99
K/Y	1.53	1.47	1.45	1.42
D/Y	1.48	1.51	1.52	1.52
$(K+D)/Y$	3.01	2.98	2.97	2.94
$\Delta(K+D)$ (%)	2.60	0.50	-	-1.49
$(K+(1-\theta)D)/Y$	1.53	2.22	2.51	2.64
$\Delta Prec.Savings$ (%)	-38.36	-11.44	-	4.73
C/I_D	5.37	5.30	5.28	5.27
r	3.95%	4.48%	4.63%	4.85%
Gini Index, K	0.877	0.910	0.913	0.915
Gini Index D	0.608	0.603	0.606	0.604
Gini Index, $K+D$	0.738	0.766	0.774	0.777

Table 10: FINANCIAL ASSETS AND WEALTH COMPOSITION FOR DIFFERENT DOWN PAYMENTS

	Quintiles				
θ	1st	2nd	3rd	4th	5th
FINANCIAL ASSETS (share %)					
1	0	0.01	1.295	3.05	95.645
0.5	-2.45	-1.38	-0.16	2.23	101.76
0.3	-2.81	-1.96	-0.28	2.29	102.76
0.2	-3.19	-2.27	-1.34	2.11	104.71
WEALTH COMPOSITION ($D/(D+K)$)					
1	100.00	98.53	86.99	84.21	47.29
0.5	200.00	185.71	103.29	92.59	46.58
0.3	333.33	283.72	102.77	91.56	46.50
0.2	500.00	390.74	104.97	97.36	45.21

Table 11: Wealth Distribution and Composition with a tax on Durables

	Quintiles Share (%)					
	1st	2nd	3rd	4th	5th	Gini
Durables, D	3.02	3.63	12.09	17.94	63.32	0.598
Financial Assets, K	−2.26	−1.82	0.50	3.03	100.55	0.907
Total Wealth, $K+D$	0.42	0.78	6.14	9.50	83.15	0.778
Wealth Composition, $D/(K+D)\%$	309.61		95.90	86.88	42.22	

Tax rate of 5%. Benchmark parameters as in Table 2. The interest rate r is allowed to vary. In equilibrium, $r = 4.05\%$. Aggregate figures: $D/Y = 1.325$, $C/I_d = 5.65$ and $K/Y = 1.534$.

implies that all households will diminish their holdings of durables. As a result, durables as a fraction of total wealth decrease. Wealthy households decrease their holdings of durables proportionally more and then total wealth inequality increases slightly. Aggregate output increases by 1 percent. As for precautionary savings, voluntary equity falls by -0.6 percent.

It may be argued that this experiment makes no sense since households do not value leisure. We think we do not lose much generality since taxes on durables are taxes on a stock and do not affect leisure at the margin. Thus, all effects on labor supply would be income effects, that is, hours worked would increase and all results we describe would be reinforced.

This exercise points out the importance of differences in tax treatment between financial assets and durable goods in understanding the composition of wealth and the distributional effects that taxes have on real estate. As we can see in Table 11 all quintiles but the bottom 40 percent reduce their holdings of durables as a fraction of total wealth, being the reduction the highest for the third and the fifth quintile.

The effect of a tax on the stock is different from the effect of an adjustment cost. We could think of the adjustment cost as a tax that is paid only whenever the durable good is traded, whereas this estate tax is paid every year. From this point of view the intertemporal burden of a 5 percent tax on the durable stock is higher that the burden imposed by a 10 percent adjustment cost. Moreover, the tax affects all households in the same manner whereas the adjustment costs affects some households more than others depending on the relative volatility of their earnings.

9 Final comments

In this paper, we have shown that a heterogeneous agent model with uninsurable idiosyncratic risk that explicitly considers the role of durable goods and collateral lending can account fairly well for the wealth distribution and wealth composition of US households.

In this study we have abstracted from some important issues that we plan to explore in future research. We may try to incorporate rental markets into the framework to determine if the results presented here in terms of wealth composition are robust to the existence of these markets. We also intend to consider life-cycle effects. It would be interesting to explore the portfolio composition of households of different ages and analyze if the model can account for life-cycle patterns of wealth holding and wealth composition.

Appendices

A Calibration

In this Appendix we describe in detail the calibration of the share of capital in each model economy.

A.1 National Accounts

In our calibration, we use data from the *National Income and Product Accounts* (NIPA) and *Fixed Reproducible Wealth* published by the Bureau of Economic Analysis for the years 1954-1999. This section discusses the relationship between the one-asset and the two-asset economies and the corresponding NIPA categories.

Let us write measured GDP as follows:

$$(c + sh + i_{cd}) + (i_{prk} + i_{pnrk}) + g + nx + \Delta inv = GDP, \tag{10}$$

Expression (10) presents final expenditure components of GDP, where we have grouped those categories that belong to the same type in parentheses. $(c + sh + i_{cd})$ are expenditures on private consumption. They are comprised of expenditures on nondurable and services excluding housing (c), housing services (sh), and expenditures on consumer durable goods (i_{cd}). The category $(i_{prk} + i_{pnrk})$ is total private investment, which is the sum of residential and nonresidential private investment. The other categories are public expenditures, g, net exports, nx, and the change in inventories, Δinv.

We can also write output as value added:

$$GDP = we + r_{prk}prk + r_{pnrk}pnrk \tag{11}$$

GDP is equal to the sum of wages plus rents of residential and nonresidential private stocks of capital. Of course, it should be the case that $sh = r_{prk}prk$.

A.1.1 The economy with only one asset

In order to properly calibrate a model of heterogeneous agents with only one asset and without taxes and government expenditure, we have to make some appropriate imputations. First, we should treat expenditures on consumer durables as part of total investment. Second, we must add public expenditure to consumption. Third, we should augment investment with net exports since we are assuming a closed economy. Therefore:

$$(c + sh + g) + (i_{prk} + i_{pnrk} + i_{cd} + nx + \Delta inv) = GDP.$$

Since services of consumer durables, $r_{cd}cd$, do not appear in measured GDP, we impute them to GDP and our measure of output is (from the perspective of expenditure and value added) becomes:

$$(c + sh + g + r_{cd}cd) + (i_{prk} + i_{pnrk} + i_{cd} + nx + \triangle inv) = we + r_{prk}prk + r_{pnrk}pnrk + r_{cd}cd.$$

The share of capital is:

$$\text{share of capital } = \frac{r_{prk}prk + r_{pnrk}pnrk + r_{cd}cd}{we + r_{prk}prk + r_{pnrk}pnrk + r_{cd}cd}.$$

Note also that the implied interest rate is a weighted average of the returns of residential assets, non residential assets and consumer durable goods.

A.1.2 The economy with two assets

Now, we want to explicitly consider the existence of durable goods that comprise consumer durable goods and residential assets. Thus, we define a new category, durables, $d = prk + cd$. Consequently, investment in durables is $i_{pnrk} + i_{cd}$. Rearranging:

$$(c + g) + (sh + r_{cd}cd) + (i_{pnrk} + nx + \triangle inv) + (i_{prk} + i_{cd}) = we + r_{pnrk}pnrk + (r_{prk}prk + r_{cd}cd).$$

Investment in capital is nonresidential investment, and investment in durables is the sum of residential investment and expenditure on consumer durable goods.

Let us now define our measure of output taking into account several considerations. First, in the aggregate, $sh + r_{cd}cd = r_{prk}prk + r_{cd}cd$. Second, services of durable goods are considered final consumption. From this point of view, we should include it as another category in consumption and keep the rental income of durables, $(r_{prk}prk + r_{cd}cd)$, as part of value added. However, since there is not a rental market for durable goods in our model economy, there is not a price for durable services and we can only define its opportunity cost. The absence of this rental market dictates our definition of output in the model:

$$(c + g) + (i_{pnrk} + nx + \triangle inv) + (i_{prk} + i_{cd}) = we + r_{pnrk}pnrk. \tag{12}$$

In order to represent value added as a Cobb-Douglas function of labor and capital (non residential stock of capital), we need to calibrate the share of capital as:

$$\text{share of capital } = \frac{r_{pnrk}pnrk}{we + r_{pnrk}pnrk}.$$

Thus, the definition of output and the share of capital differ depending on which model economy we calibrate. Note that the economy with just one asset could be thought of as an economy with a perfect rental market for durable goods that eliminates the friction of the down payment. In such an economy, the distribution of wealth would be invariant with respect to the composition of a household's portfolio.

A.2 The One-Asset Economy

A question we have not addressed above is whether we should include government owned capital as part of aggregate capital and, consequently, augment measured GDP with the imputed flow of services of this capital. We think we should not, since the focus of this paper is on the distribution of privately owned wealth. We construct broad measures of output, Y, investment, I, and aggregate capital, K, according to the organization discipline described before:

$$\begin{aligned} Y &= we + r_{prk}prk + r_{pnrk}pnrk + r_{cd}cd, \\ K &= PRK + PNRK + CD + INV, \\ I &= i_{prk} + i_{pnrk} + i_{cd} + nx + \triangle inv. \end{aligned} \tag{13}$$

To construct an appropriate measure of output, we need to impute the flow of services of durable goods as in Cooley and Prescott (1995). Income from capital is related to the stock of capital by the following expression:

$$Y_{K_p} = \left(i + \delta_{K_p}\right) K_p, \tag{14}$$

where Y_{K_p} is income of private fixed capital, $r_{prk}prk + r_{pnrk}pnrk$, and δ_{K_p} is the depreciation rate of that capital stock. Given measured values of the capital stock, for the capital income and a measured value for depreciation, we can obtain an estimate of i, the return to capital. For us, measured K_p includes the net stock of private capital (not including the stock of consumer durable goods), and the stock of inventories, $PRK + PNRK + INV$. Both measures are taken from the *from the Fixed Reproducible Wealth* study.

There are categories of income in NIPA that are unambiguously capital income–rental income, corporate profits and net interest– whereas some other categories, such as proprietors's income cannot clearly be imputed to capital in full. Define *unambiguous capital income* (UCI) and *ambiguous capital income* (ACI) as follows:

$$\begin{aligned} UCI &= \text{Rental Income} + \text{Corporate Profits} + \text{Net Interest}, \\ ACI &= \text{Proprietors Income} + \text{Net National Product - National Income}. \end{aligned}$$

Write Y_{K_p} as:

$$Y_{K_p} = UCI + \theta ACI + \text{Depreciation } = \theta GNP,$$

where Depreciation is consumption of Fixed Capital, $\delta_{K_p} K_p$. This equation can be solved for θ,

$$\theta = \frac{UCI + \text{ Depreciation}}{GNP - ACI}.$$

The value of Y_{K_p} is obtained as θGNP and $i = Y_{K_p}/K_p - \delta_{K_p}$. The average estimated value of the return to capital for the period 1954-99 is 8.42%. We then use this estimated return to calculate services from the stock of consumer durable goods, $r_{cd}cd$, given the measured value of the stock of consumer durables and the measured value of depreciation.

We have defined output as measured GDP plus the estimated value of the flow of services of consumer durable goods. The stock of capital is the sum of private fixed assets, the stock of inventories and the stock of consumer durables. Therefore, the share of capital is then computed as the ratio:

$$\alpha = \frac{Y_{K_p} + r_{cd}cd}{GDP + r_{cd}cd}.$$

The estimated value for the period 1954-1999 is 34.80 percent.

With the definitions shown in (13), we can compute an average capital-output ratio for the period mentioned equal to 2.5061 and an investment-capital ratio equal of 0.0871. Note that with a Cobb-Douglas technology, in a steady state, the interest rate implied by these numbers is

$$r = \alpha\frac{Y}{K} - \frac{I}{K} = 0.0523.$$

A.3 The Economy with Two Assets

The explicit consideration of durable goods requires to redefine all aggregate variables:

$$\begin{aligned} Y &= we + r_{prk}prk, \\ K &= PNRK + INV, \\ D &= PRK + CD, \\ I_k &= i_{pnrk} + nx + \triangle inv, \\ I_d &= i_{prk}i_{cd}. \end{aligned} \tag{15}$$

As before, we follow Cooley and Prescott (1995), but now we subtract from capital income the item of housing services, sh, and Y_{K_p} is just equal to $r_{prk}prk$. Thus, we have:

$$Y_{K_p} = UCI - sh + \theta ACI + \text{Depreciation} \;= \theta\left(GNP - sh\right).$$

Proceeding as above:

$$\alpha = \frac{Y_{K_p}}{GDP - sh}.$$

Note now that the share of capital is equal to θ, which depends on the difference between GDP and GNP. The estimated share of capital is 0.18639.

With the definitions shown in (15), we compute an average capital-output ratio $K/Y = 1.4673$, an investment-capital ratio $I_K/K = 0.0814$, a durable stock to output ratio, $D/Y = 1.5253$, an investment-durable stock ratio, $I_D/D = 0.0927$ and a nondurable to durable investment ratio, $C/I_D = 5.2889$. Note that with a Cobb-Douglas technology, in a steady state, the interest rate implied by these numbers is:

$$r = \alpha \frac{Y}{K} - \frac{I_K}{K} = 0.0463.$$

B Data on Distribution

We use data from the 1998 *Survey of Consumer Finances*, SCF98 hereafter. Our variable Total Wealth corresponds to what the SCF98 calls net worth. The data on durable goods corresponds to the reported value of three items: value of vehicles, (variable VEHIC), value of residential assets (variable HOUSES) and other residential assets, (ORESE). Thus, consistent with our definition of durables in our model economy, the value of durables is always non-negative. There is however an inconsistency. In aggregate data we have information on the total stock of durables (housing, vehicles and other durable goods), whereas at the disaggregate level we only have information on real estate and vehicles. Nevertheless, we think that the discrepancy is not important in quantitative terms.

C Computational Procedures

C.1 Reformulation of the model

In order to compute the equilibrium of the model, it is convenient to reformulate the household problem. Define voluntary equity, $q = k + (1-\theta)d$, as the wealth held in excess of the required down payment. A household's state variables are its shock, its holdings of voluntary equity, and its holdings of durable good, $\{e, q, d\}$. In this way, we have two assets whose values are restricted to be non-negative. This greatly simplifies the problem imposed by the endogenous liquidity constraint in the solution of the household problem. Define Q as aggregate voluntary equity. We write feasibility as a function of this new variable,

$$C + Q' - (1-\delta_k)Q + \theta D' + \left[(1-\delta_k)(1-\theta) - (1-\delta_d)\right] D = F\left(Q - (1-\theta)D, L\right).$$

Factor prices can also be written as functions of Q and D.

Note that our individual state space contains two endogenous individual variables (voluntary equity and durable assets) as well as the exogenous idiosyncratic shock. This implies the need to create a two dimensional grid for the endogenous state and interpolate for solutions of both types of assets choices different from the grid points. We use *two-dimensional interpolation.*

C.2 Solving the household problem

The household problem can be rewritten as follows:

$$v(e,q,d) = \max_{q' \geq 0, d'} \left\{ u(c,d) \ + \ \beta \sum_{e'} \pi_{e,e'} \ v\left(e', q', d'\right) \right\}, \tag{16}$$

where

$$c = we + (1+r)\ q + \left[\left(1-\delta^d\right) - (1-\theta)\,(1+r)\right]\ d - q' - \theta d' \tag{17}$$

Note that by reformulating the model this way, we deal with an exogenous borrowing limit, zero, in voluntary equity, q, instead of the endogenous liquidity constraint in capital, k, shown in (2). This transformation allows us to work with a grid in q that starts at zero, instead of starting at some (endogenous) negative number, which would be required if we worked with the original state variable k.

We solve this problem by value function iteration. We guess an initial value function v^0, and make the individuals choose next period value of q' and d' in the grid. The grid for the durable good starts at a positive value instead of zero. Alternatively, we could have assumed that marginal utility of durable goods at zero was positive. In principle, choosing values of the policy function in the grid may be very costly computationally in a heterogeneous agents model. We avoid this problem by using a policy function accelerator described in Judd (1998):

1. Given the initial guess of the value function v^0 obtain the associated policy functions choosing their values within the grid, $g_0^q(e_i, q_j, d_l)$ and $g_0^d(e_i, q_j, d_l)$.

2. Iterate a sufficiently large number of times on the value function using g_0^q and g_0^d. This is the accelerator. Call the resulting function v^1. If v^1 and v^0 are close enough we reached the fixed point. The associated policy functions are those that solve the household's problem. If they are not, rename $v^0 = v^1$ and go back to step 1.

By the N-Stage Contraction Theorem there is a unique fixed point v that solves the household problem. Moreover, the speed of convergence is much greater than in the regular case, which allows us to use a grid for both assets of 100x100 points. This implies solving the household's problem for 30,000 points at each iteration

C.3 Solving for the Steady State

We outline the algorithm below.

Step 1. For an initial guess of the aggregate stock of durables, D_0, we choose an initial guess Q_0 (so that the interest rate is less than the discount factor and durables depreciation factor) and compute the implied factor prices $\{r_0, w_0\}$.

Step 2. Given those factor prices, we solve the household problem to get the associated policy functions g_0^q and g_0^d.

Step 3. We find the aggregate voluntary equity, Q_1, and the aggregate stock of durables D_1 implied by individual optimal saving behavior at the given factor prices $\{r_0, w_0\}$. To do this, we guess an initial sample of individuals of size 9,000 and apply to them g_0^q, and g_0^d and the earnings Markov process 15,000 times, which ensures in all experiments that the main statistics of the sample are almost constant. To ensure no sampling error is spoiling the convergence in Q we use the same seed to initialize the random number generator in each iteration. If Q_0 and Q_1 are sufficiently close, stop. Otherwise, update $Q_0 = (1-\varepsilon)Q_0 + \varepsilon Q_1$ and go back to step 1 *without changing the initial guess* D_0. Once we have found a fixed point for Q, we have an aggregate stock of durables $D_1(D_0)$.

Reformulating the model this way is especially advantageous at this step. The reason is the following: Aggregate voluntary equity is a non-negative function of the interest rate, whereas aggregate capital, K, is not. Aggregate capital becomes negative for sufficiently small levels of the interest rate and is very elastic. Hence, convergence in K may be slow since aggregate capital may overshoot with small changes in interest rate. This is not the case for aggregate voluntary equity.

Step 4. Once Q_1 has converged, update $D_0 = D_1(D_0)$ and repeat step 3 until a fixed point for D is found.[5]

In this setting the implementation of the adjustment cost is straightforward. Set $I = 0$ if $d' - (1-\delta^d)\ d \leq \kappa$ and $I = 1$, when $d' - (1-\delta^d)\ d > \kappa$ in equation 7, where κ is chosen appropriately. Note, that the precision of our solution increases as κ falls but his "work around" solution may have some economic significance. It may be possible for the agent to make small changes to his durable stock, such as repairs, which do not require significant adjustment costs. If this is the case, the numerical implementation described here would be most appropriate.

[5]In practice, three iterations for step 4 were sufficient to find convergence of D.

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