

## From Individual to Aggregate Labor Supply\*

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### Abstract

We investigate the mapping from individual to aggregate labor supply using a general equilibrium heterogeneous-agent model with incomplete market. Heterogeneity of the workforce is designed such that the evolution of wages, worker flows between employment and nonemployment, and cross-sectional earnings distribution are consistent with micro data. We find that the aggregate labor-supply elasticity of such an economy is around 1, higher than micro estimates but smaller than those often assumed in aggregate models.

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# 1 Introduction

Despite enormous heterogeneity in the workforce, economists often postulate and analyze an economy populated by identical agents for its simplicity and tractability. A fully specified representative-agent model has become a workhorse in macroeconomics, and it is a common practice to rely on micro evidence to pin down the key parameters of highly aggregated models (e.g., Kydland and Prescott, 1982; Prescott, 1986; King, Plosser, and Rebelo, 1988).

Unfortunately, however, this practice often creates a tension between micro and macro observations.<sup>1</sup> A stylized fact in aggregate fluctuations is that total hours vary greatly over the business cycle without much variation in wages. The explanations range from equilibrium to disequilibrium approaches. According to the intertemporal substitution hypothesis, pioneered by Lucas and Rapping (1969), workers are willing to substitute leisure over the business cycle. To be consistent with the observed movement in hours and wages, this hypothesis requires an elastic labor supply beyond admissible estimates from micro data (e.g., Ghez and Becker, 1975; MaCurdy, 1981; Altonji, 1986; Abowd and Card, 1989).<sup>2</sup> Even in a disequilibrium approach where the role of labor supply is dismissed in the short run, its slope is still important for the welfare cost departing from an equilibrium.<sup>3</sup>

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<sup>1</sup>See Browning, Hansen, and Heckman (1999) that raises warning flags about the current use of micro evidence in calibrating macro models.

<sup>2</sup>For example, Pencavel (1986) reports most estimates between 0 and 0.45 for men. In their parallel survey of research on labor supply of women, Killingsworth and Heckman (1986) present a wide range of estimates, from -0.3 to 14; they do not venture a guess as to which is correct but conclude that the elasticity is probably somewhat higher for women than men. See Blundell and MaCurdy (1999) for a more recent review of the literature. See also Mulligan (1998) on how the current micro estimates may underestimate the workers' willingness to substitute leisure over time.

<sup>3</sup>An alternative equilibrium approach is to introduce shifts in labor supply as well as in demand (e.g., Bencivenga,

The participation margin, the so-called extensive margin, has been recognized as a potential resolution. With heterogeneity among workers, hours fluctuations are accounted for mainly by movement in and out of employment by workers (e.g., Coleman, 1984; Heckman, 1984) with different reservation wages. Then, the slope of aggregate labor supply curve has little to do with intertemporal substitution but rather with the distribution of reservation wages across workers. The well-known lottery economy by Rogerson (1988) and Hansen (1985) is a special case where the reservation-wage distribution is degenerate, yielding a very high elasticity—in fact, infinity.

In this paper, we investigate the mapping from individual to aggregate labor supply using a general equilibrium model economy. Workers face idiosyncratic productivity shocks and the capital market is incomplete. Heterogeneity of the workforce in the model is disciplined by various micro data. The stochastic process of idiosyncratic productivity is estimated by the panel data from the Panel Study of Income Dynamics (PSID) for 1979-1992. The gross worker flows in and out of employment are consistent with those observed in the Current Population Survey (CPS) for 1967:II-2000:IV. The cross-sectional distributions of earnings and wealth are comparable to those from the PSID and Survey of Consumer Finance (SCF).

We find the aggregate labor-supply elasticities of such an economy ranging from 0.8 to 1.7, depending on the degree of heterogeneity and employment rates. These values are higher than typical micro estimates, but smaller than those often assumed in aggregate models. We also compare the cyclical fluctuation of our economy to those of representative-agent economies. When the stochastic productivity shocks are introduced, we find that the hours response of our economy is comparable to those of the representative-agent economy with intertemporal substitution elasticity of leisure between 1 and 2.<sup>4</sup>

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1992; Benhabib, Rogerson, and Wright, 1991; Greenwood and Hercowitz, 1991; Christiano and Eichenbaum; 1992).

<sup>4</sup>An economy with indivisibility at the micro level may be approximated by a representative-agent economy with

Kydland (1984) and Cho (1995), among others, are the closest to our work as they focus on labor-supply elasticity in a stochastic growth model. Kydland constructs an economy with two types of workers, skilled and unskilled, and reproduces some labor-market regularity in relative wages and hours. However, this approach does not reflect the participation margin, a dominant source of variation in total hours in the data. Cho first incorporates the extensive margin into the real-business-cycle model in which workers are *ex ante* identical and *ex post* heterogeneous. This considerably simplifies the computation as consumption is shared among workers. It is, however, clear in the data that persons with greater hours or greater earnings per hour consume more.

Other important works on the labor-market heterogeneity in the context of stochastic general equilibrium include Andolfatto and Gomme (1996), Castañeda, Díaz-Giménez, and Ríos-Rull (1998), Merz (1999), den Haan, Ramey, and Watson (2000), and Gomes, Greenwood, and Rebelo (2001). Andolfatto and Gomme study the Canadian unemployment insurance policy; Castañeda et al. the income distribution and unemployment spells; Merz the cyclical behavior of labor turnover; den Haan et al. the propagation mechanism under labor-market matching and job destruction; Gomes et al. the cyclical behavior of unemployment rates.

The paper is organized as follows. Section 2 presents the model. Section 3 calibrates the model consistent with various micro data. In Section 4, we investigate the aggregate labor supply of the model in both steady state and fluctuations. Comparison with the representative-agent model is also provided. Section 5 is the conclusion.

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divisible labor, as the indivisibility is smoothed by an aggregation over heterogeneous agents. While this point is well illustrated in Mulligan (2001), we have yet to investigate its quantitative implications because the mapping from the micro to the macro function depends crucially on the nature of heterogeneity in the workforce.

## 2 The Model

### 2.1 Environment

The model economy is a version of the stochastic-growth model with a large (measure one) population of infinitely lived workers. Individual workers differ from each other in productivity. Each worker maximizes the expected discounted lifetime utility:

$$U = \max_{\{c_t, h_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \right\},$$

with

$$u(c_t, h_t) = \ln c_t + B \frac{(1 - h_t)^{1-1/\gamma}}{1 - 1/\gamma},$$

where  $E_0[\cdot]$  denotes the expectation operator conditional on information available at time 0,  $\beta$  is the discount factor,  $c_t$  consumption, and  $h_t$  hours worked at time  $t$ . The utility is separable between consumption and leisure and across times. The assumption about the form of utility is popular in both business-cycle analysis (e.g., Hansen, 1985; King, Plosser, and Rebelo, 1988) and the empirical labor-supply literature (e.g., MaCurdy, 1981; Altonji, 1986). The parameter  $\gamma$  denotes the intertemporal substitution elasticity of leisure. Log utility in consumption supports a balanced growth path.

According to our production technology, which will be specified below, labor input enters simply as an effective unit. Thus, a worker who supplies  $h_t$  units of time earns  $w_t x_t h_t$ , where  $w_t$  is the market wage rate for the efficiency unit of labor, and  $x_t$  represents the worker's productivity. We assume that individual productivity  $x_t$  exogenously varies over time according to a stochastic process with a transition probability distribution function  $\pi_x(x'|x) = \Pr(x_{t+1} \leq x' | x_t = x)$ . Since participation is the dominant source of variation in total hours worked (e.g., Coleman, 1984; Heckman, 1984), we abstract from an hours choice and assume that labor supply is indivisible; i.e.,  $h_t$

takes either zero or  $\bar{h} (< 1)$ . A worker can save and borrow by trading a claim for physical capital, which yields the rate of return  $r_t$  and depreciates at rate  $\delta$ . Workers face a borrowing constraint; the level of asset holding,  $a_t$ , cannot be negative at any time. The capital market is incomplete; the physical capital is the only asset available to insure against idiosyncratic risks in  $x$ . A worker's budget constraint is:

$$c_t = w_t x_t h_t + (1 + r_t) a_t - a_{t+1},$$

and

$$a_{t+1} \geq 0.$$

Firms produce output according to constant-returns Cobb-Douglas technology in capital,  $K_t$ , and effective labor,  $L_t$ :

$$Y_t = F(L_t, K_t, \lambda_t) = \lambda_t L_t^\alpha K_t^{1-\alpha},$$

where  $\lambda_t$  is aggregate productivity, following a stochastic process with a transition probability distribution function,  $\pi_\lambda(\lambda'|\lambda) = \Pr(\lambda_{t+1} \leq \lambda' | \lambda_t = \lambda)$ .

It is useful to consider a recursive equilibrium. Suppose  $\mu(a, x)$  denotes the distribution (measure) of workers.<sup>5</sup> Let  $V^E$  and  $V^N$  denote the values of employed and nonemployed. If a worker decides to work, she solves the following Bellman equation by choosing the next period asset holding  $a'$ :

$$V^E(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}} \left\{ u(c, 1 - \bar{h}) + \beta E \left[ \max \{ V^E(a', x'; \lambda', \mu'), V^N(a', x'; \lambda', \mu') \} | x, \lambda \right] \right\} \quad (1)$$

subject to

$$c = w x \bar{h} + (1 + r) a - a',$$

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<sup>5</sup>Let  $\mathcal{A}$  and  $\mathcal{X}$  denote sets of all possible realizations of  $a$  and  $x$ , respectively. The measure  $\mu(a, x)$  is defined over a  $\sigma$ -algebra of  $\mathcal{A} \times \mathcal{X}$ .

$$a' \geq 0,$$

and

$$\mu' = \mathbf{T}(\lambda, \mu).$$

where  $\mathbf{T}$  denotes a transition operator for  $\mu$ .

If the worker decides not to work, her Bellman equation is:

$$V^N(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}} \left\{ u(c, 1) + \beta E \left[ \max \{ V^E(a', x'; \lambda', \mu'), V^N(a', x'; \lambda', \mu') \} \mid x, \lambda \right] \right\} \quad (2)$$

subject to

$$c = (1 + r)a - a',$$

$$a' \geq 0,$$

and

$$\mu' = \mathbf{T}(\lambda, \mu).$$

Having solved (1) and (2), it is straightforward to deal with worker's labor-supply decision:

$$V(a, x; \lambda, \mu) = \max_{h \in \{\bar{h}, 0\}} \{ V^E(a, x; \lambda, \mu), V^N(a, x; \lambda, \mu) \}. \quad (3)$$

## 2.2 Equilibrium

Equilibrium consists of a set of decision rules for consumption, asset holding, and labor supply,

$\{c(a, x; \lambda, \mu), a'(a, x; \lambda, \mu), h(a, x; \lambda, \mu)\}$ , aggregate inputs,  $\{K(\lambda, \mu), L(\lambda, \mu)\}$ , factor prices,  $\{w(\lambda, \mu), r(\lambda, \mu)\}$ ,

and a law of motion for the distribution  $\mu' = \mathbf{T}(\lambda, \mu)$  such that:

### 1. Individual optimization:

Given  $w(\lambda, \mu)$  and  $r(\lambda, \mu)$ , the individual decision rules  $c(a, x; \lambda, \mu)$ ,  $a'(a, x; \lambda, \mu)$ , and  $h(a, x; \lambda, \mu)$

solve (1), (2), and (3).

2. The firm's profit maximization:

$$w(\lambda, \mu) = F_1(L(\lambda, \mu), K(\lambda, \mu), \lambda) \quad (4)$$

$$r(\lambda, \mu) = F_2(L(\lambda, \mu), K(\lambda, \mu), \lambda) - \delta \quad (5)$$

for all  $(\lambda, \mu)$ .

3. The goods market clears:

$$\int \{a'(a, x; \lambda, \mu) + c(a, x; \lambda, \mu)\} d\mu = F(L(\lambda, \mu), K(\lambda, \mu), \lambda) + (1 - \delta)K \quad (6)$$

for all  $(\lambda, \mu)$ .

4. Factor markets clear:

$$L(\lambda, \mu) = \int x h(a, x; \lambda, \mu) d\mu \quad (7)$$

$$K(\lambda, \mu) = \int a d\mu \quad (8)$$

for all  $(\lambda, \mu)$ .

5. Individual and aggregate behaviors are consistent:

$$\mu'(A^0, X^0) = \int_{A^0, X^0} \left\{ \int_{\mathcal{A}, \mathcal{X}} \mathbf{1}_{a'=a'(a, x; \lambda, \mu)} d\pi_x(x'|x) d\mu \right\} da' dx' \quad (9)$$

for all  $A^0 \subset \mathcal{A}$  and  $X^0 \subset \mathcal{X}$ .

### 3 Calibration

A key element in the mapping from individual to aggregate labor supply is the nature of heterogeneity in the workforce. We assume that individual productivity  $x$  follows an AR(1) process in logs:

$$\ln x' = \rho_x \ln x + \varepsilon_x, \quad \varepsilon_x \sim N(0, \sigma_x^2). \quad (10)$$

We view  $x$  reflecting a broad measure of earnings ability and opportunity in the market. Wage data from the PSID for the period of 1979-1992 are used to estimate this process. Appendix A.1 describes the PSID data we use in detail. According to the model, the log wage for individual  $i$  at time  $t$ , denoted by  $\ln w_t^i$ , can be written as  $\ln w_t^i = \ln w_t + \ln x_t^i$ , where  $w_t$  and  $x_t^i$  denote the market wage rate for an effective unit of labor and individual  $i$ 's productivity, respectively. When quasi-differenced, individual wage evolves as:

$$\ln w_t^i = \rho_x \ln w_{t-1}^i + (\ln w_t - \rho_x \ln w_{t-1}) + \varepsilon_{x,t}^i. \quad (11)$$

Equation (11) is estimated with year dummies capturing aggregate effects including  $\ln w_t - \rho_x \ln w_{t-1}$ . The OLS estimate for  $\rho_x$  is .817 with a standard error of .0025 ( $R^2 = .69$ ).<sup>6</sup> The corresponding quarterly persistence is .95(= .817<sup>.25</sup>). Given the persistence we compute the standard deviation of innovation consistent with the cross-sectional wage distribution in the PSID:  $\sigma_x = .194$ . Details of computation is described in Appendix A.2.

In general, the dispersion of productivity of population is larger than that of wage distribution because workers with very low productivity are less likely to participate. To understand the magnitude of this selection bias, consider the following two extreme cases. Suppose employment is randomly determined regardless of productivity. In this case, the standard deviation of wages is a good proxy for  $\sigma_x$ . Consider the other extreme case where employment is completely ordered by the current productivity—that is, a worker with the highest productivity is hired first and so forth.

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<sup>6</sup>As  $x$  reflects a broad measure of individual, both temporary and permanent, we do not include individual characteristics in the regression. When we include age, sex, and years of schooling in the regression to control for individual characteristics, the persistence decreases from .817 to .758. Our estimate is slightly lower than the persistence of idiosyncratic earnings risks in Storesletten, Telmer, and Yaron (1999). The difference is due to the fact that they decompose idiosyncratic shocks into a persistent AR(1) and a purely temporary *i.i.d.* components whereas we assume a single AR(1) process.

In this case, the (observed) wage distribution is a truncated distribution of  $x$ . Under log-normality, when the bottom 40 percent, the average nonemployment rate in the CPS for 1967-200, is truncated, the standard deviation of the underlying distribution is larger than that of truncated distribution by a factor of 1.5 (See Maddala, 1983). For a benchmark case, we use  $\sigma_x = .2425$  by multiplying the standard deviation of wages by 1.25, the mid point of the two extreme cases. In the analysis below, we also consider the lower bound,  $\sigma_x = .194$ , and upper bound,  $\sigma_x = .291 (= .194 \times 1.5)$ .<sup>7</sup>

Other parameters of the model are in accord with business-cycle analysis and empirical labor-supply literature. According to the Michigan Time-Use Survey, a typical household allocates about 33 percent of its discretionary time for paid compensation (e.g., Hill, 1984; Juster and Stafford, 1991):  $\bar{h} = 1/3$ . Most micro estimates of intertemporal substitution elasticity of leisure fall between 0 and .5: we use  $\gamma = .2$ .<sup>8</sup> The labor share,  $\alpha$ , is .64, and the quarterly depreciation rate,  $\delta$ , is 2.5 percent. We search for the weight parameter on leisure,  $B$ , such that the steady-state employment rate is 60 percent, the average from the CPS. The discount factor  $\beta$  is chosen so that the quarterly rate of return to capital is 1 percent.<sup>9</sup> Table 1 summarizes the parameter values.

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<sup>7</sup>While we explore a range of  $\sigma_x$  we argue that a high persistence ( $\rho_x = 0.95$ ) is reasonable for two reasons. First,  $x$  reflects a broad measure, both temporary and permanent, of earnings ability and the individual wages are highly persistent. Second, when idiosyncratic shocks are transient, the model generates unreasonably high worker flows between the market and non-market. With  $\rho_x = .5$ , for example, about 16% of population move between employment and non-employment each quarter, whereas the average flow in the CPS is between 6.82% and 7.01%. With  $\rho_x = .95$ , the worker flows are between 6.33% and 6.87% in our model. This is discussed in detail in the next section.

<sup>8</sup>With discrete choice of hours of work, the value of  $\gamma$  is not so important for the aggregate labor-supply elasticity since it mostly depends on the shape of the reservation-wage distribution.

<sup>9</sup>The discount factor is lower than that in the representative-agent model, because market incompleteness increases savings as noted in Aiyagari (1994).

Finally, when we investigate the response of hours with aggregate fluctuations, we introduce exogenous shifts in labor demand through aggregate technology shocks whose stochastic process is consistent with the post-war total factor productivity:  $\lambda$  can take two values,  $\ln \lambda \in \{-.0224, .0224\}$ , and its transition matrix between the two states is

$$\pi_\lambda = \begin{bmatrix} .95 & .05 \\ .05 & .95 \end{bmatrix}.$$

## 4 Results

### 4.1 Steady State

We first characterize the steady state of the model economy where there is no aggregate uncertainty and the distribution of worker  $\mu(x, a)$  is invariant. A detailed description of computing the equilibrium is provided in Appendix A.4. Even in the absence of aggregate fluctuations there are constant flows of workers in and out of employment due to individual productivity shocks. Table 2 presents employment rate, gross-worker flows, and hazard rates from the model and the CPS. As described in the previous section, employment rate is 60 percent, matching the average in the CPS for 1967:II to 2000:IV. The quarterly gross-worker flows are computed using Robert Shimer's monthly hazard rates as explained in Appendix A.3. On average, 6.82 percent of the population moved from employment to nonemployment each quarter; however, 7.01 percent of the population moved in the opposite direction, from nonemployment to employment.<sup>10</sup> In our benchmark case, this flow is 6.35 percent, slightly lower than those in the data. With different degree of idiosyncratic

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<sup>10</sup>Nonemployment includes both unemployment and non-labor-force. According to Blanchard and Diamond (1990), the flows between employment and non-labor force are as big as those between employment and unemployment.

shocks they are 6.87 ( $\sigma_x=.194$ ) and 6.33 ( $\sigma_x=.291$ ).<sup>11</sup> Although we did not calibrate the model to match these values, the worker flows and hazard rates are fairly close to those in the CPS.

The wealth and earnings, apart from preference and non-market opportunity which are hard to measure, are probably the most important factors for participation decision. Figure 1 exhibits the Lorenz curves of wealth from the PSID and three models. In the PSID, information on family wealth is available for 1984, 1989, and 1996 survey years. We use 1984 survey because it is around the mid point of our sample. The degree of inequality does not vary significantly across three surveys. Family wealth is obtained by summing net worth of house, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets. Table 3 reports the Gini indices of wealth and earnings. The Gini coefficient of wealth is .76 in the PSID while they are .57 ( $\sigma_x=.194$ ), .60 ( $\sigma_x=.2425$ ) .62. ( $\sigma_x=.291$ ), respectively, in the model. Overall, wealth is more concentrated in the data.

Figure 2 shows the Lorenz curves of earnings. The data is based on family earnings (earnings of head and wife) also from the 1984 PSID. The model and the PSID exhibit similar inequality except that there are more zero earners at the bottom of the distribution—40 percent of population have zero earnings in the model and 20 percent in the model. A family with at least one member worked at some point during the survey year reported a positive earnings in the PSID, whereas the model is calibrated to match the average employment rate of 60 percent in the CPS. This makes Gini index of the model, between .60 and .69, somewhat higher than .54 in the PSID. However, when we use positive earnings only, the Gini indices are .34 ( $\sigma_x = .194$ ), .42 ( $\sigma_x=.2425$ ), and .48 ( $\sigma_x=.291$ ), comparable to .42 in the PSID.

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<sup>11</sup>For each model, we adjust the utility parameter  $B$  and the discount factor  $\beta$  so that the employment rate is 60 percent and quarterly rate of return to capital is 1 percent in steady state.

Table 4 summarizes detailed information on wealth and earnings from the SCF, PSID, and benchmark model.<sup>12</sup> Since the wealth-earnings distributions between the PSID and SCF are similar, we discuss the comparison between the model and PSID only. For each quintile group of the wealth distribution, we calculate the wealth share, the ratio of group average to economy-wide average, and the earnings share of the group. Both in the data and model, the poorest 20 percent own almost nothing. In fact, households in the first quintile of the wealth distribution in the 1984 PSID are in debt. Those in the model own less than one percent, .37 percent, of total wealth. On the contrary, households in the 4th and 5th quintile of the PSID own 18.74 and 76.22 percent of total wealth, respectively. According to the model, they own 23.14 and 62.16 percent, respectively. The average wealth of the 4th and 5th quintile are, respectively, .93 and 3.81 times larger than that of a typical household, while these ratios are 1.15 and 3.11 according to our model. The 4th and 5th quintile group of the wealth distribution earn, respectively, 24.21 and 38.23 percent of total earnings in the PSID. The corresponding groups earn 23.70 and 33.44 percent, respectively, in the model. Overall, the wealth is more concentrated in the data. In particular, the model fails to match the highly concentrated wealth in the right tail of the distribution. About half of total wealth—43 and 53 percent in the PSID and SCF, respectively—is held by the top 5 percent of the population (not shown in the Table). In our model, only 18.4 percent of total wealth is held by them. However, our primary objective is not to explain the behavior of the top 1 or 5 percent of population.<sup>13</sup> We argue that the model economy possess a reasonable heterogeneity to study the average response of

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<sup>12</sup>In terms of Gini indices, the wealth and earnings distributions from the PSID are slightly less concentrated than those in the SCF. According to Díaz-Giménez, Quadrini, and Ríos-Rull, Gini indices are .78 and .63 for wealth and earnings, respectively, in the 1992 SCF.

<sup>13</sup>On the wealth distribution in a dynamic general equilibrium environment, see Huggett (1996), Krusell and Smith (1998), Quadrini (2000), or Meh (2002)

hours, as the stochastic process of wages is estimated from the panel data and the cross-sectional earnings distribution is, by and large, consistent with the data.

For the mapping from individual to aggregate labor supply, the shape of reservation wage distribution is crucial. Figure 3 shows the reservation wages—the productivity that makes workers indifferent between working and not-working—for the benchmark model. At a given asset level, workers with productivity above the line choose to work. The higher the asset holding, the higher is the reservation wage. Consider a person in our economy who has the asset of about 13.3 units, the average asset holding in our model economy. He is indifferent in working at quarterly earnings of about 1.06 ( $=3.18 \times \bar{h}$ ). To illustrate, suppose the unit is one thousand dollars. His net asset is \$13,300 and he is willing to work at quarterly earnings starting \$1,060. Now consider a worker with asset of 133 units, ten times wealthier than the average. According to our model, his reservation wage is 4.1 ( $=12.3 \times \bar{h}$ ), about four times higher than the one with average asset holding.

The reservation-wage schedule and invariant distribution  $\mu(x, a)$  allows us to uncover the aggregate labor-supply curve of the economy. Figure 4 exhibits the aggregate labor supply for three model economies: the benchmark, the lower-bound and upper-bound heterogeneity. We calculate the elasticity at employment rates of 55, 60, and 65 percent in Table 5. For the benchmark economy, the elasticity is 1.02 at the steady-state employment rate of 60 percent. They are 1.32 and .80 at the employment rates of 55 and 65 percent, respectively. With a smaller degree of heterogeneity ( $\sigma_x = .194$ ), the elasticities are 1.68, 1.19, and .98 at employment rates of 55, 60, and 65 percent, respectively. With a larger heterogeneity ( $\sigma_x = .291$ ), the elasticities are smaller as the reservation wage distribution is more dispersed; they are 1.01, .80, and .75 at employment rates of 55, 60, and 65 percent, respectively. While these values are higher than most micro estimates, usually less than .5, they remain at moderate range. In particular, a very high aggregate elasticity—in fact, infinity—generated by the employment lottery, such as Hansen and Rogerson, does not survive a

serious heterogeneity.

## 4.2 Fluctuations

In this section, we introduce exogenous shifts in aggregate productivity and examine the fluctuations of total hours. We do not take a stand on the sources of the business cycle here, but we intentionally exclude other types of aggregate disturbances, especially those that shift the labor-supply curve. Aggregate productivity shocks serve for an instrument, shifting the labor-demand curve, to identify the slope of the aggregate labor-supply curve.

In computing equilibrium fluctuations we adopt the so-called “bounded rationality method” developed by Krusell and Smith, in which agents are assumed to make use of a finite set of moments of the distribution  $\mu$ . The justification of this method is that by using partial information about  $\mu$ , households do almost as well as by using all the information in  $\mu$  when predicting future prices. In fact, Krusell and Smith show that use of the first moment only provides a good approximation in a stochastic-growth model. The details of computation including the accuracy of prediction functions of aggregate prices are provided in Appendix.

We compare our benchmark economy to the representative-agent (with divisible labor) models with various degrees of intertemporal substitution of leisure. The representative-agent economies have the same parameter values as our benchmark economy in Table 1 except for  $\gamma$ .<sup>14</sup> We consider four representative economies with  $\gamma$  equaling to .5, 1, 1.5, and 2. Reference to the real-business-cycle analysis, Prescott (1986) corresponds to  $\gamma = 1$  and Hansen (1985) to  $\gamma = \infty$ .

Table 6 displays the statistics of five model economies (our benchmark economy and four representative-agent economies with divisible labor) and the U.S economy. The upshot is that the

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<sup>14</sup>For each representative economy, the parameter  $B$  is also adjusted to yield  $\bar{h} = 1/3$  in the steady state.

response is similar to those from the representative-agent economies with  $\gamma$  between 1 and 2. The volatility of output of our benchmark economy is 1.81, slightly smaller than that of the economy with  $\gamma=1$  (1.93). The volatility of consumption (.49) is the same as that with  $\gamma=1.5$ . The volatility of investment (5.92) is somewhat smaller than that with  $\gamma = 1$  (6.51) and greater than that with  $\gamma = .5$  (5.69). The volatility of hours in our model is 1.24 which is close to that with  $\gamma = 1.5$  (1.28). The volatility of hours relative to labor productivity is 1.51, almost similar to that with  $\gamma = 1.5$  (1.52). The volatility of hours relative to output is .69, slightly higher than that with  $\gamma = 2$  (.66).

For a representative agent with the same type of utility function as ours, the Frisch labor-supply function linearized around the steady state is

$$\widehat{h}_t = \psi(\widehat{w}_t - \widehat{c}_t), \quad \psi = \frac{1 - \bar{h}}{\bar{h}}\gamma, \quad (12)$$

where the circumflexes denote the variable's percentage deviation from its steady-state value. The Frisch labor-supply elasticity,  $\psi$ , also known as compensated labor-supply elasticity, represents the elasticity of hours with respect to wages holding consumption (or wealth) constant. Due to incomplete market, the aggregation theorem does not hold in our model economy. Yet it is still of interest to estimate Equation (12) pretending that the data were generated by a representative agent. Since there is only one aggregate disturbance which exogenously shifts the labor demand curve, a simple OLS (without any instrumental variable) is sufficient to reveal the slope of labor supply. When we estimate (12) using the aggregate time series from our benchmark economy, the estimate for  $\psi$  is 1.08 consistent with the value obtained from the steady-state reservation-wage distribution.

The response of aggregate hours to shifts in demand is moderate as the reservation-wage distribution is scattered. For example, the dispersion of individual productivity, measured by the cross-sectional standard deviation of log wages in the PSID (.606), is larger than that of aggregate

productivity, measured by the time-series standard deviation of aggregate TFP (.0224) by a factor of nearly 27.<sup>15</sup>

## 5 Conclusion

The labor supply is at the heart of macroeconomic research. It is a cornerstone of the equilibrium approach that relies on intertemporal substitution of leisure. In a disequilibrium approach, in which the role of labor supply is dismissed in the short run, the labor supply is still crucial for the welfare loss of the economy departing from the equilibrium. Aggregate models often assume elastic labor supply, despite the low estimates from empirical studies based on individual data. The fact that fluctuations of hours are mainly accounted for by participation suggests that the labor supply in aggregate has little to do with intertemporal substitution rather with the distribution of reservation wages.

We construct a model economy where heterogeneous agents decide on labor-market participation and the capital market is incomplete. Heterogeneity of workforce is designed such that evolution of wages, gross worker flows, and cross-sectional income distribution are consistent with the micro data. We find the aggregate labor supply elasticity of such an economy around 1, higher than typical micro estimates but smaller than those often assumed in aggregate models.

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<sup>15</sup>We abstract from the variation of hours per worker to isolate the effect of participation margin only. Allowing for an intensive margin may generate a bigger response of labor supply. However, under the small intertemporal substitution elasticity of leisure, the effect on aggregate labor supply would be small.

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## A Appendix

### A.1 The PSID Data

The PSID sample period used is 1979-1992. The sample consists of heads of households and wives. We start in 1979 because the wage data for wives are available only since 1979. Wages are annual hourly earnings (annual labor incomes divided by annual hours). Nominal wages are deflated by the Consumer Price Index. The base year is 1983. Workers who worked less than 100 hours per year or whose hourly wage rate was below \$1 (in 1983 dollars) are viewed as nonemployed even though their employment status is reported as employed in the survey. We use workers who were employed in non-agricultural sectors and not self-employed. We also restrict the sample to hourly earnings less than or equal to \$500. In the PSID, the wealth data are available for 1984, 1989 and 1996 only. We use 1984 data as it is around the mid point of our sample. The distributions are similar across the three surveys. The wealth is defined as the sum of net worth of all family members resulting from the aggregation of the following components: house (main home), other real estate, vehicles, farms and businesses, stocks, bonds, cash accounts, and other assets. Family earnings is the sum of earnings of head and wife. The descriptive statistics for our PSID data are summarized in Table A.1.

### A.2 Conversion between Annual and Quarterly Variances

After controlling for aggregate price of an effective unit of labor, the wage evolves according to  $x$ . Since the wages in the PSID are annual averages:<sup>16</sup>

$$\ln \tilde{x}_\tau = \frac{1}{4} \left\{ \ln x_{(\tau,1)} + \ln x_{(\tau,2)} + \ln x_{(\tau,3)} + \ln x_{(\tau,4)} \right\},$$

---

<sup>16</sup>Note that  $\frac{1}{4} \sum_{q=1}^4 \ln x_{(\tau,q)}$  can be interpreted as a log-linear approximation of the arithmetic average  $\ln \tilde{x}_\tau = \ln \left[ \frac{1}{4} \sum_{q=1}^4 x_{(\tau,q)} \right]$ .

Table A.1: Summary Statistics for the PSID

Variable	Mean	S.D.	Obs.
Real Wage Rate (in 1983 \$)	10.95	9.66	45207
Log Real Wage (net of aggregate effect)	0	.606	45207
Annual Hours of work	2079.1	9.65	45207
Age	38.3	10.7	45207
Years of schooling	13.32	2.35	45207
Sex (male =1)	.584	.49	45207
Wealth	60,524.4	231103.1	5515
Earnings	17,933.1	19681.9	6918

Note: Log real wages (net of aggregate effect) are the residuals from the regression of log wages on year dummies. Family wealth and earnings are based on 1984 survey.

where  $\tilde{x}_\tau$  is annual average and  $x_{(\tau,q)}$  denotes the wage of the  $q$ th quarter in year  $\tau$ . According to a AR(1) process for  $x$ , the quarterly persistence  $\rho_x$  is simply  $.95(= .817^{.25})$  where 0.817 is the annual estimate from the PSID. Furthermore,

$$\ln \tilde{x}_\tau = \frac{1}{4} \left\{ (1 + \rho_x + \rho_x^2 + \rho_x^3) \ln \tilde{x}_{\tau-1} + (1 + \rho_x + \rho_x^2) \varepsilon_{x,(\tau,2)} + (1 + \rho_x) \varepsilon_{x,(\tau,3)} + \varepsilon_{x,(\tau,4)} \right\}.$$

The standard deviation of annual average is:

$$\sigma(\ln \tilde{x}) = \frac{1}{4} \sigma_x \left\{ \frac{(1 + \rho_x + \rho_x^2 + \rho_x^3)^2}{1 - \rho_x^2} + (1 + \rho_x + \rho_x^2)^2 + (1 + \rho_x)^2 + 1 \right\}^{1/2}.$$

When we replace  $\sigma(\ln \tilde{x})$  with .606, the standard deviation of log annual hourly earnings (net of aggregate effects) from the PSID, we obtain  $\hat{\sigma}_x = .194$ .

### A.3 The Worker-Flow Data

We compute quarterly worker flows from the seasonally adjusted monthly hazard rates in the CPS for 1967-2000, obtained from Robert Shimer as follows. There are three possible labor-market status: employment, unemployed, and non-labor-force, denoted by  $e$ ,  $u$ , and  $n$ , respectively. The

flow of workers from labor-market status  $i$  to  $j$  during the quarter, denoted by  $f_{ij}$ :

$$f_{ij} = \bar{i} \times \left\{ \sum_{k,l \in \{e,u,n\}} h_{ik}^1 h_{kl}^2 h_{lj}^3 \right\} \quad i, j \in \{e, u, n\},$$

where  $\bar{i}$  denotes the number of workers in status  $i$  in the beginning of the quarter, and  $h_{kl}^m$  the monthly hazard rate from status  $k$  to  $l$  in the  $m$ -th month of the quarter. This takes into account all possible paths, direct and indirect, from  $i$  to  $j$  during a quarter, and avoids a potential double counting in a simple sum of monthly flows. Because of survey redesigns and privacy restrictions, hazard rates are not available in January 1976, January 1978, July 1985, October 1985, January 1994, and June to October 1995. For these months we interpolate with the values from nearby periods.

## A.4 Computational Procedures

### A.4.1 Steady-State Equilibrium

The distribution of workers  $\mu(x, a)$  is invariant in the steady state; so are the factor prices. In finding the invariant  $\mu$ , we use the algorithm suggested by Ríos-Rull (1999). We search for the discount factor  $\beta$  that clears the capital market given the quarterly rate of return of 1 percent. Computing the steady-state equilibrium amounts to finding the value functions, the associated decision rules, and time-invariant measure of workers. Details are as follows:

1. First, we choose the grid points for  $a$  and  $x$ . The number of grids are denoted by  $N_a$  and  $N_x$ , respectively:  $N_a = 936$ ,  $N_x = 17$ . The asset holding  $a_i$  is in the range of  $[0, 260]$ , where the average asset holding is 13.3. The grid points of assets are not equally spaced. We assign more points on the lower asset range to better approximate savings decisions of workers with lower assets. For productivity,  $x_j$ , we construct grid vectors of  $N_x$  equally spaced points in which  $\ln x_j$ 's lie on the range of  $\pm 3\sigma_x / \sqrt{1 - \rho_x^2}$ .

2. Given  $\beta$ , we solve the individual optimization problem in (1), (2), and (3) at each grid point of the individual states. In this step, we also obtain the optimal decision rules for asset holding  $a'(a_i, x_j)$  and labor supply  $h(a_i, x_j)$ . This step involves the following procedure:

(a) Initialize value functions  $V_0^E(a_i, x_j)$ ,  $V_0^N(a_i, x_j)$ , and  $V_0(a_i, x_j)$ .

(b) Update value functions by evaluating the discretized versions of (1), (2), and (3):

$$V_1^E(a_i, x_j) = \max \left\{ u(w\bar{h}x_j + (1+r)a_i - a', 1 - \bar{h}) \right. \\ \left. + \beta \sum_{j'=1}^{N_x} \max [V_0^E(a', x_{j'}), V_0^N(a', x_{j'})] \pi_x(x_{j'}|x_j) \right\}, \quad (\text{A.4.1})$$

$$V_1^N(a_i, x_j) = \max \left\{ u((1+r)a_i - a', 1) \right. \\ \left. + \beta \sum_{j'=1}^{N_x} \max [V_0^E(a', x_{j'}), V_0^N(a', x_{j'})] \pi_x(x_{j'}|x_j) \right\}, \quad (\text{A.4.2})$$

and

$$V_1(a_i, x_j) = \max \{ V_1^E(a_i, x_j), V_1^N(a_i, x_j) \}, \quad (\text{A.4.3})$$

where  $\pi_x(x_{j'}|x_j)$  is the transition probabilities of  $x$ , which is approximated using Tauchen's (1986) algorithm.

(c) If  $V_1$  and  $V_0$  are close enough for all grid points, then we found the value functions.

Otherwise, set  $V_0^E = V_1^E$  and  $V_0^N = V_1^N$ , and go back to step 2-(b).

3. Using  $a'(a_i, x_j)$ ,  $\pi_x(x_{j'}|x_j)$  obtained from step 2, we obtain time-invariant measures  $\mu^*(a_i, x_j)$  as follows:

(a) Initialize the measure  $\mu_0(a_i, x_j)$ .

(b) Update the measure by evaluating the discretized version of (9):

$$\mu_1(a_{i'}, x_{j'}) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \mathbf{1}_{a_{i'}=a'(a_i, x_j)} \mu_0(a_i, x_j) \pi_x(x_{j'}|x_j) \quad (\text{A.4.4})$$

(c) If  $\mu_1$  and  $\mu_0$  are close enough for all grid points, then we found the time-invariant measure. Otherwise, replace  $\mu_0$  with  $\mu_1$ , and go back to step 3(b).

4. We calculate the real interest rate as a function of  $\beta$ , i.e.,  $r(\beta) = \alpha(K(\beta)/L(\beta))^{1-\alpha} - \delta$ , where  $K(\beta) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} a_i \mu^*(a_i, x_j)$  and  $L(\beta) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} h(a_i, x_j) \mu^*(a_i, x_j)$ . Other aggregate variables of interest are calculated using  $\mu^*$  and decision rules. If  $r(\beta)$  is close enough to the assumed value of the real interest rate, we found the steady state. Otherwise, choose a new  $\beta$  and go back to step 2.

#### A.4.2 Equilibrium with Aggregate Fluctuations

Approximating the equilibrium in the presence of aggregate fluctuations requires us (i) to include the measure of workers and the aggregate productivity shock in the list of state variables, and (ii) to keep track of the evolution of the measure  $\mu$  over time. Since  $\mu$  is an infinite dimensional object, it is almost impossible to implement these tasks as they are. We follow the procedure suggested by Krusell and Smith (1998). We assume that agents make use of its first moment only. The law of motion for aggregate capital stock is restricted to the log-linear function. Therefore, computing the equilibrium with aggregate fluctuations amounts to finding the value functions, decision rules, and law of motion for the aggregate capital within the class of a parametric form of the log linear in  $K$  and  $\lambda$ . Details are as follows:

1. In addition to the grids for individual state variables specified above, we choose 7 grid points for the aggregate capital  $K$  in the range of  $[\.93K^*, 1.07K^*]$ , where  $K^*$  denotes the steady-state aggregate capital. In our numerous simulations, the capital stock has never reached the upper or lower bound. The stochastic process for the aggregate productivity shock  $\pi_\lambda(\lambda'|\lambda)$  is described in the text.

2. Let the parametric law of motion for the aggregate capital take a log linear in  $K$  and  $\lambda$ :

$$\ln K_{t+1} = \kappa_0^0 + \kappa_1^0 \ln K_t + \kappa_2^0 \ln \lambda_t. \quad (\text{A.4.5})$$

In order for individuals to make their decisions on savings and labor supply they have to know (or predict) the interest rate and wage rate for an effective unit of labor. While the factor prices depend on aggregate capital and labor aggregate labor input is not known to individuals at the moment when they make decisions. Thus, individuals need to predict the factor prices. These predictions on factor prices, in turn, must be consistent with the outcome of individual actions—the factor market clearing in (7) and (8). We also assume that individuals predict the market wage and the interest rate using a log-linear function of  $K$  and  $\lambda$ :

$$\ln w_t = b_0^0 + b_1^0 \ln K_t + b_2^0 \ln \lambda_t. \quad (\text{A.4.6})$$

and

$$\ln r_t = d_0^0 + d_1^0 \ln K_t + d_2^0 \ln \lambda_t \quad (\text{A.4.7})$$

3. We choose the initial values for the coefficients  $\kappa^0$ 's,  $b^0$ 's and  $d^0$ 's. Good initial values may come from a representative-agent model.

4. Given the law of motion for the aggregate capital and the prediction functions for factor prices, we solve the individual optimization problem in (1), (2), and (3). This step is analogous to step 2 in the steady-state computation:

(a) We have to solve for the value functions and the decision rules over a bigger state space.

Now the state variables are  $(a, x, K, \lambda)$ .

(b) Computation of the conditional expectation involves the evaluation of the value functions not on the grid points along the  $K$  dimension since  $K'$  predicted by (A.4.5) need not be

a grid point. We polynomial-interpolate the value functions along  $K$  dimension when necessary.

5. Using  $a'(a_i, x_j, K_t, \lambda_m)$ ,  $h(a_i, x_j, K_t, \lambda_m)$ ,  $\pi_x(x_{j'}|x_j)$ ,  $\pi_\lambda(\lambda_{m'}|\lambda_m)$ , and the assumed law of motion for the aggregate capital, we generate a set of artificial time series data  $\{K_t, w_t, r_t\}$  of the length of 5,000 periods. Each period,  $\{K_t, w_t, r_t\}$  is calculated by aggregating labor supply and assets of 50,000 individuals.
6. We obtain new values for coefficients  $\kappa^1$ 's,  $b^1$ 's and  $d^1$ 's by the OLS from the simulated data. If  $\kappa^1$ 's,  $b^1$ 's and  $d^1$ 's are close enough to  $\kappa^0$ 's,  $b^0$ 's, and  $d^0$ 's, respectively, we found the law of motion. Otherwise, update coefficients by setting  $\kappa^0 = \kappa^1$ ,  $b^0 = b^1$ 's and  $d^0 = d^1$ 's, and go back to step 4.

The estimated law of motion for capital and prediction functions and their accuracy, measured by  $R^2$  for the prediction equations are as follows.

- the law of motion for aggregate capital in equation (A.4.5):

$$\ln K_{t+1} = .1297 + .9500 \ln K_t + .1099 \ln \lambda_t, \quad R^2 = .9998$$

- the market wage rate in equation (A.4.6):

$$\ln w_t = -.3110 + .4539 \ln K_t + .7988 \ln \lambda_t, \quad R^2 = .9894$$

- the interest rate in equation (A.4.7):

$$\ln r_t = .182 - .0282 \ln K_t + .0476 \ln \lambda_t, \quad R^2 = .9687$$

The law of motion for aggregate capital provides the highest accuracy. The wage function is more accurate than the interest rate function. Overall, predictions functions are fairly precise as  $R^2$ 's are close to 1.

Table 1: Parameters of the Benchmark Economy

Parameter	Description
$\alpha = .64$	Labor share in production function
$\beta = .97724$	Discount factor
$\gamma = .2$	Intertemporal substitution elasticity of leisure
$B = 1.019$	Utility parameter
$\bar{h} = 1/3$	Steady-state hour
$\rho_x = .95$	Persistence of idiosyncratic productivity shock
$\sigma_x = .2425$	Standard deviation of innovation to idiosyncratic productivity

Table 2: Labor Market Steady States

Variable	CPS	Model		
		$\sigma_x = .194$	$\sigma_x = .2425$	$\sigma_x = .291$
Employment rate	60.02	60.05	60.02	60.33
Flow out of employment	6.82	6.87	6.35	6.33
Flow into employment	7.01	6.87	6.35	6.33
Hazard rate out of nonemployment	17.52	17.18	15.87	15.83
Hazard rate out of employment	11.36	11.45	10.58	10.55

Note: All variables are in percentage. Data are quarterly averages for 1967:II-2000:IV as described in Appendix A.3.

Table 3: Gini Indices for Wealth and Earnings

Variable	PSID	Model		
		$\sigma_x = .194$	$\sigma_x = .2425$	$\sigma_x = .291$
Wealth	.76	.57	.60	.62
Earnings	.53	.60	.65	.69
Earnings (non-zeros)	.42	.34	.42	.48

Note: Data statistics are family wealth and labor income from the PSID 1984.

Table 4: Characteristics of Wealth Distribution

	Quintile					Total
	1st	2nd	3rd	4th	5th	
<u>SCF</u>						
Share of wealth	-.39	1.74	5.72	13.43	79.49	100
Group average/population average	-.02	.09	.29	.67	3.97	1
Share of earnings	7.05	14.50	16.48	20.76	41.21	100
<u>PSID</u>						
Share of wealth	-.52	.50	5.06	18.74	76.22	100
Group average/population average	-.02	.03	.25	.93	3.81	1
Share of earnings	7.51	11.31	18.72	24.21	38.23	100
<u>Benchmark Model</u>						
Share of wealth	.37	3.80	10.54	23.14	62.16	100
Group average/population average	.02	.19	.53	1.15	3.11	1
Share of earnings	9.43	14.72	18.70	23.70	33.44	100

Note: The statistics for the SCF are from Diaz-Gimenez, Quadrini, and Rios-Rull (1997).

Table 5: Aggregate Labor Supply Elasticities

Model	Elasticity at Employment Rate		
	$E = 55\%$	$E = 60\%$	$E = 65\%$
$\sigma_x = .194$	1.68	1.19	.98
$\sigma_x = .2425$	1.32	1.02	.80
$\sigma_x = .291$	1.01	.80	.75

Note: Elasticities are evaluated at employment rates of 55%, 60% and 65%, respectively.

Table 6: Comparison with Divisible Labor Economies

	Benchmark	Divisible Labor				U.S. Data 1955:I-1998:4
		$\gamma = .5$	$\gamma = 1$	$\gamma = 1.5$	$\gamma = 2$	
$\sigma(Y)$	1.81	1.70	1.93	2.08	2.17	2.10
$\sigma(C)$	.49	.42	.47	.49	.51	.84
$\sigma(I)$	5.92	5.69	6.51	7.04	7.40	4.77
$\sigma(N)$	1.24	.69	1.05	1.28	1.44	1.58
$\sigma(N)/\sigma(Y)$	.69	.41	.54	.62	.66	.75
$\sigma(N)/\sigma(Y/N)$	1.51	.67	1.15	1.52	1.78	1.35

Note: Data are from the DRI data base.  $Y$  = GDP-government spending;  $C$  = consumption of non durables and services;  $I$  = non-residential fixed investment;  $N$  = total employed hours in non-agricultural private sector from the establishment survey. All variables are divided by civilian noninstitutional population over 16. All variables are detrended by the H-P filter.

Figure 1: Lorenz Curves for Wealth

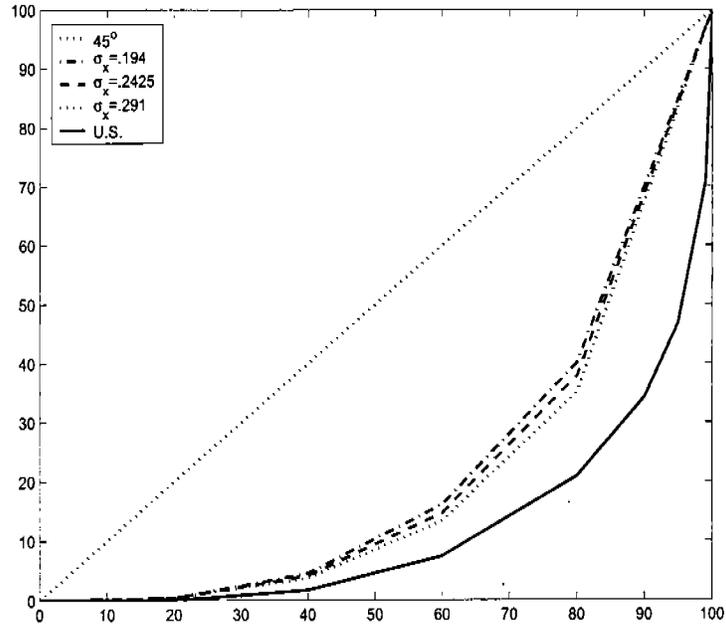


Figure 2: Lorenz Curves for Earnings

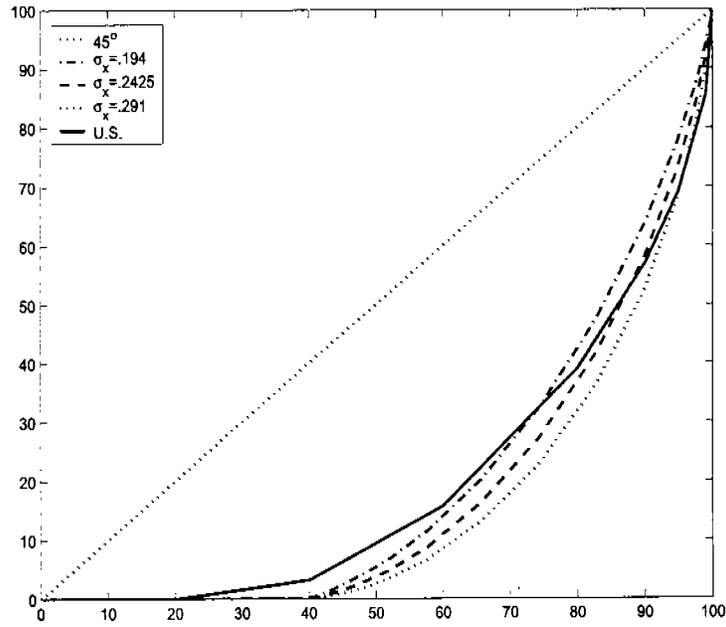


Figure 3: Working Decisions

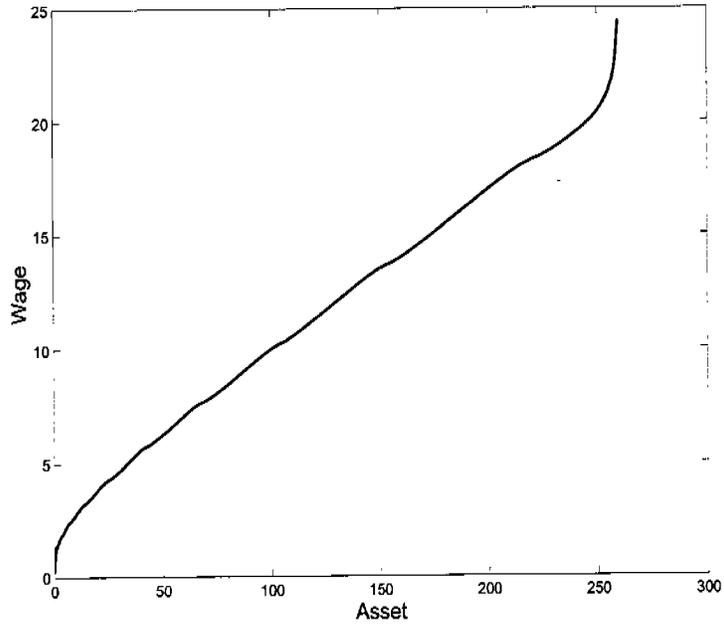


Figure 4: Aggregate Labor Supply Curves

