EFEL 7/24/02
BOND/KRISHNAMURTHY
12:30 PM

Regulating Exclusion from Financial Markets*

Philip Bond† Arvind Krishnamurthy‡

Date of last revision: June 24, 2002

Abstract

We study optimal enforcement in credit markets in which the only threat facing a defaulting borrower is restricted access to financial markets. This environment has been studied in the literature initiated by Kehoe and Levine (1993) and Kocherlakota (1996). Rather than presuppose the form of regulation in this environment, we solve for the optimal regulation and link it to observed institutional arrangements. Regulation in this environment must accomplish two objectives. First, it must prevent borrowers from defaulting on one bank and transferring their resources to another bank. Second, and less obviously, it must give banks the incentive to make sizeable loans, and to honor their promises of future credit. We establish that the optimal regulation will resemble laws governing default on debt and bankruptcy. Moreover it can be implemented in a way that does not require the regulator to have information about either the borrower or lender. Finally, the important aspect of observed bankruptcy codes is restricting fraudulent conveyance – which boils down to preventing a borrower in default from transferring their resources to another bank. Restricting the availability of credit to a defaulted borrower is not a threat, in and of itself, that motivates borrowers to repay loans.

Corresponding author: Arvind Krishnamurthy, Finance Department, Northwestern University, 2001 Sheridan Road, Evanston IL 60208-2001.
E-mail:a-krishnamurthy@northwestern.edu.

*We thank Kaushik Basu, Denis Gromb, Mike Fishman, Adriano Rampini, and members of seminar audiences at London Business School, Northwestern University, Stanford University, and the NEUDC conference for helpful comments. Any remaining errors are of course our own.

†Northwestern University. E-mail:p-bond@northwestern.edu.

‡Northwestern University. E-mail:a-krishnamurthy@northwestern.edu.

1 Introduction

There is currently considerable interest in the workings of credit markets in which the only threat facing a defaulting borrower is restricted access to financial markets. In the development context, where collateral is scarce, accounts of lending often give importance to credit denial as a mechanism in ensuring loan repayment — see, e.g., Robinson's (2001) description of the Bank Rakyat Indonesia.[1] At a theoretical level, Kehoe and Levine (1993) and Kocherlakota (1996) (henceforth KL-K) have initiated an important literature that studies risk sharing and asset pricing in economies with endogenous debt-constraints. The punishment to a defaulting borrower is denial of access to savings, insurance, and credit markets. This punishment places an endogenous limit on the amount that an agent can borrow.[2]

In the KL-K setting, agents are expected to deliver on a sequence of time- and state-contingent payments. Default on any of these payments results in agents being restricted to consume their autarchic endowment streams. However, the vast majority of the literature that uses the KL-K framework is silent on how exclusion from financial markets is actually enforced. The minority of papers that do take this issue seriously take one of two views. (A) Financial exclusion is impossible to implement, since financial markets are competitive and financial intermediaries have strong incentives to trade with a borrower who has defaulted (Bulow and Rogoff, 1989b, Chari and Kehoe, 1993a, and Krueger and Uhlig, 2000). (B) In an infinite horizon setting, the enforcementiss ue is moot because agents are able to play trigger strategies that allow them to support any enforcement rule (see Chari and Kehoe, 1993b, and Kletzer and Wright, 2000).

Neither answer is satisfactory in the context of non-sovereign borrowing, because both effectively assume away the enforcement problem. Response (A) assumes that there is no third-party which can regulate trade between borrowers and financial intermediaries. While this seems reasonable in the context of sovereign debt (as in Bulow and Rogoff, 1989b), courts clearly perform such a function in the case of individual borrowers. Response (B) is fragile. It assumes that all financial arrangements are supported only by trigger strategies of an infinitely repeated game. If financial intermediaries are able to honor their savings/insurance contracts, the equilibrium reverts to autarchy (Krueger and Uhlig, 2000).

Our aim in this paper is to study a version of the KL-K environment and derive the optimal regulations required to support lending. A borrower without collateral seeks credit from one of multiple banks. Unlike the main branch of the literature we do not

[1] Another important mechanism is social sanctions. There is a large literature on group lending, spurred by the success of the Grameen Bank. See Morduch (1999) and Ghatak and Guinanne (1999) for surveys.

[2] Alvarez and Jermann (2000, 2001) develop asset pricing implications in these environments.

presuppose the form of the regulation,[3] but instead specify a general class of possible regulations. Regulation is an issue in our economy because unlike response (B) we grant banks at least limited stocks of pledgeable assets, allowing them to effectively commit to make future payments. Moreover, regulation is possible because unlike response (A) we allow for the possibility of some form of third party enforcement.

We first show that a large class of regulations achieve the constrained optimal outcome. The punishment suggested in the KL-K literature that the borrower is restricted from trading with any of the banks if he ever misses the transfers specified in the constrained optimal solution falls within this class. Although this particular regulation is hard to interpret in terms of observed institutions – i.e. in practice, debtors in default are not banished to autarchy – we show that there are alternatives that achieve the constrained optimal outcome that are more readily interpreted.

We concentrate on a rule whereby a creditor bank seizes any cash deposits a delinquent borrower makes to other banks. The institutional counterpart to this rule is that most bankruptcy codes require a borrower to declare all of his assets during bankruptcy. Hiding some cash by depositing it with another bank constitutes a fraudulent conveyance,[4] implying that such hidden deposits can be seized by creditors ReRgulaRt bankruptcy. We show that a debt-default rule that restricts fraudulentco nveyance of a delinquent borrower achieves the constrained optimal outcome.

Somewhat surprisingly, prohibiting a defaulted borrower from ever borrowing again ("credit prohibition") is not necessary in any optimal rule. Moreover, if this is the only punishment embedded in a rule, the lending market will collapse, implying that credit prohibition is also insufficient.

We next refine the class of optimal rules by relaxing lender commitment. This complicates implementation by introducing the additional requirementthat the lender must have incentives to make loans of the efficient size. One feature of laws governing debt contracts is that the lender can only punish a borrower if the borrower is in default. In the absence of lender commitment, we show that this conditional creditor-right provides a lender with precisely the right sort of incentives. Intuitively, by tying the domain of a lender's rights to the size of his loan, lenders have incentives to make sizeable loans. We are further able to show that all optimal rules must possess this "conditionality" feature.

By seeking to understand the enforcement assumptions that underpin exclusion from financial markets, our work provides something of a link between (1) the KL-K literature

[3] In most cases, our optimal regulation can be implemented either by a centralized law, or by centralized enforcement of private contracts.

[4] For instance, Baird (1993, page 143) summarizes fraudulent conveyance law as follows: "Transfers made and obligations incurred with the intent to delay, hinder, or defraud creditors are fraudulent and void as against creditors."

on the economy-wide implications of endogenous debt constraints, and (2) the optimal financial contracting literature that has sought to explain the widespread use of debt.[5] Two points are particularly worth stressing in this regard.

First, while the KL-K literature commonly refers to "debt" constraints, the key property of most decentralizations discussed in that literature is that an agent's net asset position cannot fall too low. That is, agents are said to be "borrowing constrained", but the source of this centrally specified debt limit is rarely discussed. In contrast, we are able to show that that the efficient allocation can be implemented by the rule discussed above in which any funds placed by a defaulted borrower with other banks can be seized by creditors. The resulting contract resembles a standard debt contract, and does not require any limits on a borrower's asset position.

Second, the punishment inflicted on the debtor in this class of environments is that he is prevented from dealing with other financial institutions. That is, the creditor is granted a monopoly position einExEen the defaulted debtor. Focusing on this particular punishment makes it very clear that in the absence of lender commitment an optimal contract must include features that prevent the lender from threatening the punishment outside of default. Our model provides a new rationale for the conditionality of creditor rights that we observe in standard debt contracts.

We defer a more detailed account of our relation to specific papers in the KL-K literature until the main body of the text below (see in particular Sections 3 and 4). From the optimal contracting literature we are closest to the branch which seeks to characterize when some punishment will be imposed on the debtor. Broadly speaking the literature can be broken into papers where the lender has an explicit right to deny access to the borrower's investmentpro ject or place sanctions on the borrower, and those in which the threat is limited to denying the borrower access to savings/insurance markets.[6] Hart and Moore (1994, 1998), Thomas and Worrall (1994), DeMarzo and Fishman (2000), Albuquerque and Hopenhayn (2000) and Bulow and Rogoff (1989a) all present multi-period models in which the lender's right upon nonrepayment is to prevent the borrower from investing or liquidating the borrower.[7] Bolton and Scharfstein (1990) present a model in which the threat is credit denial, but in an environment where there is some collateral. Gromb (1999) has extended their model to a multi-period setting.[8] In the Bolton-Scharfstein model, the borrower has a project with a fixed size which the lender commits to funding if the borrower repays the loan. If the borrower does not repay the loan, he is restricted from using his limited collateral to borrow from another

[5]In addition to those papers that we cite below, prime examples include Townsend (1979), Gale and Hellwig (1985), and Innes (1990).

[6]There is also a literature on reputation building by borrowers. See, e.g., Diamond (1989).

[7]Also related are the models of Hart and Moore (1994, 1998) and Aghion and Bolton (1992)

[8]Like us, Gromb is also concerned with commitment on the lender side. However, while we will be concerned that the lender will be tempted to punish *too often*, the lender in Gromb's paper faces the opposite problem in that it is tempted to punish *too little*.

lender. This effectively means that he is denied access to his project, and this threat enables repayment. In contrast, in our environment, there is no collateral, and the only punishment involves regulating access to financial markets.

The paper proceeds as follows. Section 2 describes the economy to be analyzed, and computes the constrained efficient outcome for the case where banks can completely coordinate their actions to enforce financial exclusion. Section 3 shows that when banks compete, and there is no central enforcement, then there is no possibility of credit. This generalizes the result of Bulow and Rogoff (1989b). Section 4 defines a general class of enforcement rules, and looks at four leading examples in detail. It is shown that multiple different enforcement rules can be used to obtain the constrained efficient outcome characterized in Section 2. Sections 5 and 6 then look at environments in which banks have only limited commitment ability, and where competition in the initial loan market is restricted. By looking at these (realistic) extensions to our basic model, we are able to remove some of the indeterminacy in the choice of an optimal enforcement rule, and establish some basic properties that an optimal enforcement rule must possess. Section 7 concludes.

2 A benchmark: Constrained efficient outcomes

Time periods are indexed $t = 0, 1, \ldots, T$, where $T \geq 2$. There is a borrower B and M banks, $1, \ldots, M$. The borrower has access to a valuable production technology, but has limited funds of his own with which to invest. His resources entering date t are denoted W_t (endogenously determined), while resources at date 0 are exogenously given as W_0. Banks face no resource constrainta nd have outside investmento pportunities that always yield a return of $r > 0$. Wesha ll assume that this is in the form of a storage technology with return of r. Importantly, storage at rate r is only an option for banks. The production opportunity for borrowers is constant returns to scale and deterministically yields R_t between periods t and $t+1$. Let ρ_t denote the ratio of these return rates, $\rho_t \equiv R_t/r$. Finally, both the borrower and bank are risk neutral and take as their objective the maximization of date T wealth.

We will refer to any period $t < T$ in which the borrower's return exceeds the bank's return (i.e. $R_t \geq r$) as an igeRnac Rga period, and any period $t < T$ in which the bank's return exceeds the borrower's return (i.e. $r > R_t$) as a suoc Rga period. Since the only incentive compatible payment in period T is from the bank to the borrower, we will refer to this period as an investment period also. We assume throughout that period 0 is an investment period.

During each period t, first each bank m simultaneously makes a payment to the borrower, $L_t^m \geq 0$. Second, the borrower simultaneously makes a payment to each bank m,

$P_t^m \geq 0$. We will typically refer to the bank payments L_t^m as "loans" and the borrower payments P_t^m as "payments" or "repayments". We require that all payments P_t^m be feasible for the borrower, in the sense of being less than his resources,

$$\sum_m P_t^m \leq W_t + \sum_m L_t^m$$

Note that we require all payments to be weakly positive — no agent can unilaterally seize resources from another agent. Let $\mathbf{P}_t$ and $\mathbf{L}_t$ respectively denote the M-vectors of payments P_t^m and loans L_t^m.

Except for in Sections 5 and 6 we will be concerned with the situation where only the borrower faces a commitment problem. That is, while the borrower can always choose not to make a promised payment P_t^m, each bank m can fully commit to make a future paymentL_t^m. Thus, at date 0 each bank m can commit to make a paymenta t time $t = 0, \ldots, T$ of

$$l_t^m(\mathbf{P}_0, \ldots, \mathbf{P}_{t-1})$$

In this section we will characterize the maximal level of borrower consumption subject to the banks collectively making a return of at least r (in per-period terms). For this exercise, we can assume without loss that if the borrower ever fails to make a promised payment he suffers the maximal punishment available in our environment, namely that he receive no further payment from any bank.[9] Moreover, we can further assume without loss that the borrower deals only with one of the banks. Consequently, we omit the superscript m from our notation whenever possible.

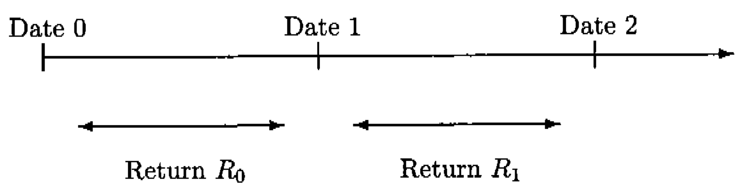

Figure 1: Timeline for $T = 2$

To build intuition, we begin by considering the simplest possible version of our environment — just three periods (i.e. $T = 2$, see Figure 1). Note that since date 2 is the terminal date, the borrower will clearly never hand over any resources at that date. On the other hand, at date 1 the banks can induce the borrower to make a payment by promising to make a payment in return (a new "loan") at date 2. That is, the borrower

[9] i.e. complete exclusion from financial markets.

can be induced to make a date 1 payment by the threat that he will otherwise be denied "credit" at date 2.

Suppose the borrower arrives at date 1 with a wealth level of W_1. We will denote by V_1^M the highest level of profits (in present value terms) that the bank can collectively make from date 1 onwards. There are two distinct cases to consider.

On the one hand, if $R_1 \geq r$ then the banks will have to promise $\$R_1$ for every \$1 they want to the borrower to pay them at date 1. Since the banks' cost of funds is $r \leq R_1$, they will lose money doing this. So the banks cannot make a profit in this case, i.e. $V_1^M = 0$.

On the other hand, if $R_1 < r$ then promising $\$R_1$ for every \$1 the borrower hands over at date 1 is attractive for the banks. Their profits are maximized when they induce the borrower to hand over all his wealth at date 1, i.e. $P_1 = W_1$, in exchange for receiving a "loan" of $L_2 = W_1 R_1$ at date 2. In this case $V_1^M = r^{-1} W_1 (r - R_1) = W_1 (1 - \rho_1)$.

Given the bank profits available at date 1, we can now consider how large a loan the banks will be prepared to make at date 0. In the case when $R_1 \geq r$, they will not make any loan at all, since as discussed above they cannot extract any repaymentfr om the borrower at date 1 without losing still more money. On the other hand, in the case where $R_1 < r$ the banks will receive some repayment at date 1. Since the borrower's date 1 wealth following a loan of L_0 is $W_1 = (W_0 + L_0) R_0$, the largest loan the banks will be prepared to make is given by the solution L_0 to

$$r^{-1} (W_0 + L_0) R_0 (1 - \rho_1) = L_0 \tag{1}$$

Summarizing, the banks will be prepared to make a loan of

$$L_0^* = \frac{\rho_0 \max\{0, 1 - \rho_1\}}{1 - \rho_0 \max\{0, 1 - \rho_1\}} W_0 \tag{2}$$

The borrower's final consumption from such a loan is

$$\frac{R_0 R_1}{1 - \rho_0 (1 - \rho_1)} W_0 \text{if} \quad r > R_1$$

and $R_0 r W_0$ if $r \leq R_1$.

In this simple environment Bulow and Rogoff's (1989b) result that credit denial alone cannot support lending when the borrower has full access to asset markets is very clear. Full access to asset markets would imply that $R_1 \geq r$, and thus the maximal loan size is 0.

Unsurprisingly, the maximal loan size and the borrower's final consumption are increasing in both the period 0 rate of return R_0 and the borrower initial wealth W_0. Less obvious is that they are also increasing as the borrower's date 1 return R_1 decreases. The reason is that when $R_1$is lower, the borrower is prepared to make a larger date 1 repayment in exchange for a "loan" at date 2. Since the date 1 repaymentis higher, it is then possible to increase the size of the original loan. Thus the loan size will be higher in environments where the return on the borrower's project is more seasonal.

Up to now, weha veb een referring to the transfers from the banks to the borrower as loans, and transfers from the borrower to the bank as repayments. While this terminology makes the transfers easiest to interpret, for many of our results it is in fact easier to think of the transfers slightly differently. Recall that the transfers that maximize the borrower's final consumption when $R_1 < r$ are $L_0 = L_0^*, P_1 = W_1$ and $L_2 = R_1 W_1$. Effectively, the banks make a gift of L_0^* to the borrower at date 0. They then recover this gift by acting as deposit-takers between dates 1 and 2, when the borrower's return is low. Their profits from date 1 onwards stem from paying a return of only R_1 on these deposits, while investing funds at a rate of return r. As (1) makes clear, the maximal loan size is determined by setting the profits from acting as a "savings" bank for the borrower equal to the original transfer L_0.[10]

Note that we have been assuming that $\rho_0 (1 - \rho_1) < 1$. If this is not the case, a gift of \$1 to the borrower increases his date 1 wealth by enough that the consequent increase in the banks' date 1 profits more than compensates them for the cost of the gift. If this inequality does not hold there is no sense in which the borrower is credit-constrained, and the problem ceases to be of any economic interest.

All of the above remarks generalize to the T-period version of our environment. Proceeding more formally than before, we compute the maximal level of profits attainable by the coalition of M banks from time t onwards, subject to the borrower obtaining transfers with a present value of v (at the borrower's opportunity cost R_t), the borrower payments being incentive compatible and feasible.

Take any date t, borrower wealth W and present value of transfers to the borrower v (at the borrower's interest rate). Then an upper bound on the present value of the combined profits of the M banks is given by $V_t^M (W, v)$, the solution to

$$\max_{\{P_s \geq 0, L_s \geq 0 : s=t, \ldots T\}} -\frac{1}{r^{T-t}} \sum_{s=t}^{T} r^{T-s} (L_s - P_s) \quad (3)$$

subject to the transfers to the borrower having a present value (at the borrower's discount rate) of v

$$\sum_{s=t}^{T} \left(\prod_{\tilde{s}=s}^{T-1} R_{\tilde{s}} \right) (L_s - P_s) = \prod_{\tilde{s}=t}^{T-1} R_{\tilde{s}} v \quad (4)$$

[10] A similar discussion can be found in Gromb (1999).

and such that at all dates $t' = t, \ldots, T$ the borrower prefers to make the payment $P_{t'}$ over defaulting and receiving no further payments from any bank,

$$\sum_{s=t'}^{T} \left(\prod_{\tilde{s}=s}^{T-1} R_{\tilde{s}} \right) (L_s - P_s) \geq \left(\prod_{\tilde{s}=t'}^{T-1} R_{\tilde{s}} \right) L_{t'} \tag{5}$$

and finally subject to these payments being feasible,

$$P_{t'} - L_{t'} \leq W_{t'} \equiv \prod_{\tilde{s}=t}^{t'-1} R_{\tilde{s}} W + \sum_{s=t}^{t'-1} \left(\prod_{\tilde{s}=s}^{t'-1} R_{\tilde{s}} \right) (L_s - P_s) \tag{6}$$

Since the constraint set is closed and bounded,[11] this problem has a solution provided the constraint set is non-empty.

As usual, it is helpful to write the problem recursively. The bank's date t value function $V_t^M(W, v)$ can be written in terms of the date $t+1$ value function as

$$V_t^M(W, v) = \max_{P \geq 0, L \geq 0, v' \geq 0} -(L - P) + \frac{1}{r} V_{t+1}^M(W', v') \tag{7}$$

where

$$W' = (W + L - P) R_t \tag{8}$$

and the maximization is subject to

$$v' = R_t (v - (L - P)) \tag{9}$$

$$v \geq L \tag{10}$$

$$W \geq P - L \tag{11}$$

[11] Constraint (6) can be rewritten

$$P_{t'} - L_{t'} \leq \prod_{\tilde{s}=t}^{t'-1} R_{\tilde{s}} W + \frac{1}{\prod_{\tilde{s}=t'}^{T-1} R_{\tilde{s}}} \left(\sum_{s=t}^{T-1} \left(\prod_{\tilde{s}=s}^{T-1} R_{\tilde{s}} \right) (L_s - P_s) - \sum_{s=t'}^{T} \left(\prod_{\tilde{s}=s}^{T-1} R_{\tilde{s}} \right) (L_s - P_s) \right)$$

which from (4) and (5) implies

$$P_{t'} - L_{t'} \leq \prod_{\tilde{s}=t}^{t'-1} R_{\tilde{s}} W + \frac{1}{\prod_{\tilde{s}=t'}^{T-1} R_{\tilde{s}}} \left(\prod_{\tilde{s}=t}^{T-1} R_{\tilde{s}} v - \left(\prod_{\tilde{s}=t'}^{T-1} R_{\tilde{s}} \right) L_{t'} \right)$$

That is,

$$P_{t'} \leq \prod_{\tilde{s}=t}^{t'-1} R_{\tilde{s}} (W + v)$$

So given any values of W, v, the choice of $\{P_{t'} : t' \geq t\}$ is certainly bounded. Conditions (4) and (5) by themselves imply

$$\left(\prod_{\tilde{s}=t'}^{T-1} R_{\tilde{s}} \right) L_{t'} \leq \left(\prod_{\tilde{s}=t}^{T-1} R_{\tilde{s}} \right) v - \sum_{s=t}^{t'-1} \left(\prod_{\tilde{s}=t}^{t-1} R_{\tilde{s}} \right) (L_s - P_s)$$

It is then straightforward to iteratively establish that the choices $\{L_{t'} : t' \geq t\}$ are also bounded.

(respectively, the participation, incentive and feasibility constraints for the borrower). Thus the maximization problem reduces to[12]

$$V_t^M(W,v) = \max_{P\in[0,W+L],L\in[0,v]} -(L-P) + \frac{1}{r}V_{t+1}^M\left(R_t(W+L-P), R_t(v-(L-P))\right) \tag{12}$$

Exactly as in the $T=2$ case, the banks are able to obtain loan repayments in periods t when the borrower's rate of return is lower than the bank's ($R_t < r$, payment periods) by threatening to deny future loans if the payment is not made. The value of loan repayments that can be obtained is proportional to the wealth of the borrower, and the shortfall of R_t below r. This is easily seen from the "savings market" intuition discussed above — a bigger gap between R_t and r, and a higher level of borrower wealth, allow larger bank profits in the saving market

In the two-period problem, if the bank increases its initial loan by one dollar, then it is able to collect $\rho_0(1-\rho_1)$ present-value dollars more in repayments during a payment period. From (1) this allows us to calculate the maximum initial loan size. In the T-period problem, if the bank increases its initial loan by one dollar, it collects more in repayments during Rıf r payment period in the future. Working out the maximum loan size requires us to sum up this increase over the entire future. We relegate this exercise to the appendix and just state the results here.

Define α_t iteratively by

$$\begin{aligned} \alpha_T &= 0 \\ \alpha_{t-1} &= \begin{cases} \rho_{t-1}\alpha_t & \text{if } t-1 \text{ is an investment period} \\ 1-\rho_{t-1}(1-\alpha_t) & \text{if } t-1 \text{ is a payment period} \end{cases} \quad \text{for } t=1,\ldots,T \end{aligned} \tag{13}$$

The quantity α_t is the present value of future repayments that a lender will collect by increasing his loan at time t by one dollar. So the presentv alue of the banks' profits at date 0 is $-L_0 + (W_0+L_0)\alpha_0$. As before, the largest initial loan can be found by choosing L_0 so that the banks just break even, i.e. $L_0 = W_0\alpha_0/(1-\alpha_0)$.

Proposition 1 ***(An upper bound on borrower welfare)***
mnrFcR $\alpha_0 < 1$M kr R tt R t n cu icu fgnF csai g ua u aR T nF Ha a arR ugn f RfaieRo cui g gg EgR aieRst a n in $\prod_{t=0}^{T-1} R_t W_0/(1-\alpha_0)$M a fug RufriReR o arR ugn ug suoc Rgan$\{L_t^*, P_t^*\}$ R gR un l n

M kr R tt R t tHRieRnu ug $L_0^* = W_0\alpha_0/(1-\alpha_0)$ ua u aR 0M

M g ugo suoc RgasRti t arua l n ug igeRnac RgasRti arR tt R t suon a arR ugn ri n RgatRR u ar iNRM$P_t^* = W_t$M

[12]Note that when the incentive constraint $L \leq v$ holds, then $v' \geq 0$ follows automatically.

M g ugo igeRnac RgasRti $t > 0$ arua l n u suoc RgasRti arR ugn cuR u ug a arR tt R t FFu a arR tt R t n u na suoc Rga fFc FuaR ua arR tE tR t n tuaRl tRaFg R_sMkr ua in $L_t^* = \prod_{s=\tau+1}^{t-1} R_s P_{\tau+1}^*$ rR tRτ in arRigeRnac Rga sRti sti t a tM

M m arRt suoc RganutR0M

Proof: See Appendix.

Both the loan size and the payment amounts grow at the borrower's rate of return R_t. The payments described in Proposition 1 have the same interpretation as those in the two-period problem above.

The banks make an initial loan to the borrower. As before, we can think of this initial loan payment, L_0^*, as a gift the banks maketo the borrower. In every subsequent period in which the borrower's rate of return is below r, the borrower deposits his entire wealth in the banking sector, where he receives an interest rate equal to his private rate of return R_t. So the banks earn profits $r - R_t$ in these periods, which compensates them for making the initial gift (loan).

Clearly if $\alpha_0 > 1$, the bank's optimal strategy is to make an infinite transfer to the borrower, since each dollar transferred increases bank profits by more than a dollar. Under these conditions, the borrower is essentially no longer credit-constrained — he can raise as much financing as he wants. Consequently, to keep the problem of economic interest we will make the following assumption for the remainder of the paper:

Assumption 1 ***(Borrower credit-constrained)*** $\alpha_0 < 1$

Assumption 1 is the exact analogue of condition $\rho_0 (1 - \rho_1) < 1$ discussed in the $T = 2$ case. Note for future use that if $\alpha_0 < 1$ then $\alpha_t < 1$ for all $t > 0$.[13] In general, the requirement that $\alpha_0 < 1$ will be more stringent when T, the total number of time periods, is larger.

Corollary 1 ***(Bulow-Rogoff)***
l $R_t > r$ l t u $t = 0, \ldots, T-1$ arRg arR cu icF c ug niR arR RgRt fug cuR ri R t Ru ig ReRgin Rt M

Proof: Every period is an investmentp eriod. From (13), $\alpha_t = \alpha_{t+1}\rho_t$ for all $t < T$. Since $\alpha_T = 0$, this implies $\alpha_0 = 0$. Thus $\alpha_0 W_0 / (1 - \alpha_0) = 0$. **QED**

[13]For suppose otherwise, i.e. $\alpha_{t^*} \geq 1$ for some t^*. Then from (30) and (31) it would follow that $\alpha_t \geq 1$ for all $t < t^*$, contradicting $\alpha_0 < 1$.

In the two-period problem, we saw that when $R_1 < r$ then the maximum loan size is decreasing in R_1, since the bank's "savings" profits are higher. We also saw that when $R_0 > r$the maximum loan size is increasing in in R_0, since the borrower's wealth is higher, again increasing the bank's saving profits. Both these effects carry over to the T-period problem:

Corollary 2 ***(Seasonality)***
kr RniR l arRtn a ug in natifa o igft Runig ig $|R_t - r|$ l t ugo tM

Proof: We will establish that α_0 is strictly increased by a strict increase in $|R_t - r|$ for any t. Fix t. Clearly an increase in $|R_t - r|$ leaves α_s unaffected for all $s > t$. If period $t$is a paymentp eriod, then $R_t < r$ and an increase in $|R_t - r|$ is equivalent to a decrease in ρ_t. Since $\alpha_t = 1 - (1 - \alpha_{t+1}) \rho_t$ and $\alpha_{t+1} < 1$, we then know that α_t is strictly increased. On the other hand, if period t is an investment period, then $R_t \geq r$ and an increase in $|R_t - r|$ is equivalent to an increase in ρ_t. Since $\alpha_t = \alpha_{t+1}\rho_t$ it again follows that α_t is strictly increased. Finally, note that for any $s < T$ it is immediate from (13) that a strict increase in α_{s+1} implies a strict increase in α_s. Thus α_0 is strictly increased by a strict increase in $|R_t - r|$ for any t.**QED**

3 Borrower welfare in the absence of enforcement

In the previous section we characterized the maximal utility level attainable by the borrower when the M banks can fully coordinate their decisions. This is a (simple) version of the problem that has been studied by numerous previous papers, notably Kehoe and Levine (1993) and Kocherlakota (1996). These papers have had little to say about how this coordination actually occurs. This is the focus of the remainder of the current paper.

As is widely appreciated, competition among banks will undercut their ability to coordinate on excluding a defaulting borrower. Formally, weco nsider competition as occurring as follows. At date 0, each bank m simultaneously announces a lending policy

$$\mathcal{L}^m \equiv \{l_0^m, l_1^m(\mathbf{P}_0), \ldots, l_T^m(\mathbf{P}_0, \mathbf{P}_1, \ldots, \mathbf{P}_{T-1})\}.$$

The borrower then chooses $\{P_t^m\}$ to maximize his final consumption,

$$W_0 \prod_{t=0}^{T-1} R_t + \sum_m \sum_{t=0}^{T} \prod_{s=t}^{T-1} R_s (L_t^m - P_t^m).$$

For any sequence of loans and payments $\{L_t^m, P_t^m\}$, let V_t^m denote the present value of bank m's (at rate r) loans and payments to the borrower from date t onwards,

$$V_t^m = \sum_{s=t}^{T} r^{-(s-t)} \left(P_t^m - L_t^m\right).$$

Our first result is then that competition among banks precludes the possibility of them collectively making positive profits from any date t onwards. The basic argument is straightforward. Suppose to the contrary that banks' combined profits were strictly positive. Then any bank m' whose profits are the lowest could offer a lending policy that replicates the transfers currently made by the M banks together, but which delivers slightly more to the borrower. This raises the profits of m', and so is a profitable deviation. The formal argument (see Appendix) is complicated by the possible interdependence in the banks' lending policies.

Lemma 1 ***(Non-positive profits)***
F ss nR arRtR utR ua Runa a ugn $M \geq 2$M kr Rg ua ugo u aR t arR fci gR gRa lFaFtRst a n l arR ugn cF na Rgg Esni aieR $\sum_m V_t^m \leq 0$ ig ugo RFii tiF cM

Proof: See Appendix.

Since the banks cannot make positive profits from any date t onwards, it follows that they can never obtain a repayment on any loans previously made to the borrower. So competition completely under-cuts the loan market, and implies that the only useful service the banks can offer is to accept deposits and pay an interest rate r on them.

Lemma 2 ***(Bound on borrower's welfare)***
F ss nR arRtR utR ua Runa a ugn $M \geq 2$M kr Rg arR tt R t n g u fgnF csai g fug Rg ct R arug $W_0 \prod_{t=0}^{T-1} \max\{r, R_t\}$Mkr ua in arR tt R t nics o igeRnanig ri n g st Ra rR g $R_t \geq r$ ug nueRh iar arR ugn ua u tuaRr rR g $R_t < r$M

Proof: See Appendix.

Because the consumption level $W_0 \prod_{t=0}^{T-1} \max\{r, R_t\}$ is obtained without the benefit of any loans, it is less than the benchmark level derived in Proposition 1.[14] As discussed above, competition among banks under-cuts their ability to recover any funds lent. This

[14]To establish this formally, note that it is sufficient to proof that $\prod_{s=t}^{T-1} \min\{1, \rho_s\} > 1 - \alpha_t$ at $t = 0$. This follows easily by an inductive argument. The strict inequality is obtaining whenever there is at least one period with $R_t > r$ (i.e. a "strict" investment period) that is followed by another period with $R_t < r$ (payment period).

raises the question of whether the borrower's consumption would actually be higher if the banking sector was monopolistic, i.e. $M = 1$. The answer is no. Although a monopoly bank fF makea loan and then obtain repayment by threatening the borrower with exclusion from saving at rate r, it does even better simply by proposing the lending policy $\mathcal{L}^1$ given by

$$\tilde{l}_t^1 (\mathbf{P}_0, \ldots, \mathbf{P}_{t_1-1}) = R_{t-1} P_{t-1}^1$$

That is, the monopoly bank will simply make monopoly profits in the deposit market, without making any loan. The following Proposition summarizes these observations:

Proposition 2 ***(Borrower's welfare absent enforcement)***
kr R tt R t n g u sRti fgnF csai g in

$$W_0 \prod_{t=0}^{T-1} \max\{r, R_t\} \tag{14}$$

il arRtR utR a t ct R ugn $M \geq 2$ ug

$$W_0 \prod_{t=0}^{T-1} R_t \tag{15}$$

il arRtR in go u nigR ug $M = 1$M

Proof: Consider first the case where $M \geq 2$. We first show there is an equilibrium in which the borrower's final consumption is as in (14). Consider the set of lending policies $\tilde{\mathcal{L}}^m$ consisting simply of

$$\tilde{l}_t^m (\mathbf{P}_0, \ldots, \mathbf{P}_{t_1-1}) = r P_{t-1}^m$$

i.e. each bank will pay a return r on any funds deposited. Clearly the borrower's final period consumption is $W_0 \prod_{t=0}^{T-1} \max\{r, R_t\}$, and all banks make zero profits. Moreover, there is no profitable deviation. For if there were, we would effectively have one deviating bank m' making strictly positive profits while dealing with a borrower with a technology paying $\max\{r, R_t\}$. But in the proof of Proposition 1 (see appendix), wees tablish that the present value of a bank's profits in this situation are no more than $\alpha_0 (W_0 + v) - v$ where v is the present value (at the borrower's rate of return) of transfers to be made to the borrower. Since $\alpha_0 = 0$ when the borrower's return is always weakly above r (see the proof of Corollary 1) and $v \geq 0$, then bank m' must have weakly negative profits.

Next, we note that there cannot exist any equilibrium in which the borrower's final consumption is not as in (14). From Lemma 2, the only possibility is that there is an equilibrium in which it is strictly less. But in this case any bank m could profit be proposing a lending policy $\tilde{\mathcal{L}}^m$ consisting simply of

$$\tilde{l}_t^m(\mathbf{P}_0, \ldots, \mathbf{P}_{t_1-1}) = (r - \varepsilon) P_{t-1}^m$$

for ε sufficiently small.

Finally, we deal with the case $M = 1$. The bank's profits are bounded above by $\alpha_0 (W_0 + v) - v$. Since $\alpha_0 < 1$ (Assumption 1), this upper bound is maximized by $v = 0$. Moreover, inspection of the proof of Proposition 1 reveals that this upper bound is attainable by the monopoly bank $m = 1$ offering a lending policy $\tilde{\mathcal{L}}^1$ defined by

$$\tilde{l}_t^1(\mathbf{P}_0, \ldots, \mathbf{P}_{t_1-1}) = R_{t-1} P_{t-1}^1$$

The resulting final period consumption for the borrower is then simply $W_0 \prod_{t=0}^{T-1} R_t$. **QED**

4 Enforcement

The result of Proposition 2 that competition among banks causes the loan market to fail is most similar to those of Bulow and Rogoff (1989b), Chari and Kehoe (1993a) and Krueger and Uhlig (2000). We generalize their results in that our setting allows a borrower to simultaneously deal with multiple banks. Moreover we allow banks' lending policies to depend on the loans and payments between the borrower and every other bank. In principle, one might think that these interdependencies would give the banks some ability to coordinate without third-party enforcement. It is striking that even given this ability to coordinate, the conclusion of the afore-mentioned papers continues to hold.

4.1 Enforcing exclusion from financial markets

In the main body of the KL-K literature, the result of Proposition 2 is avoided by the introduction of third-party enforcement. For example, Alvarez and Jermann (2001) assume that "default is punished by permanent exclusion from the asset markets." Effectively, banks coordinate by committing to not trade with a defaulting borrower.

By assumption, in KL-K environments (including ours) there is no collateral that can be seized. Enforcement boils down to regulating transfers between borrowers and banks. For example, the exclusion of the borrower from dealing with a bank m can be enforced if the central authority seizes at least some portion of the transfers between the borrower and bank m.

Formally, we assume that a central authority exists and can (1) Observe the net payments made in each period between the borrower and each of the M banks, and (2) Seize some or all these net payments.[15] Notationally, let $\mathbf{NP}_t \equiv \mathbf{P}_t - \mathbf{L}_t$ denote the vector of net payments in period t. An enforcement rule $\mathcal{B}$ is then a specification of how much of each net payment will be seized, where the amount seized can potentially depend on the history of prior net payments. That is, $\mathcal{B}$ is a set of vector-valued functions

$$\mathcal{B} = \left\{\beta_t : \Re^{(t+1)M} \rightarrow [0,1]^M : t = 0, 1, \ldots, T\right\}$$

where $\beta_t^m(\mathbf{NP}_0, \mathbf{NP}_1, \ldots, \mathbf{NP}_t)$ is the fraction of the net payment NP_t^m that is ga nRiR , given the history of net payments $\mathbf{NP}_0, \mathbf{NP}_1, \ldots, \mathbf{NP}_t$.

A rule that specifies exclusion upon a borrower's failure to pay less than some prespecified amount can be represented as follows. Suppose that we want to implement the constrained efficient aggregate payments $\{L_t^*, P_t^*\}$ of Proposition 1 as an equilibrium. Without loss, we can assume that all these payments are made to and from bank 1 — only the borrower faces an incentive problem at this point in the paper. Then in our notation, the enforcement rule suggested by the main KL-K literature is the rule $\mathcal{B}_{KLK}$, defined by

$$\beta_t^m(\mathbf{NP}_0, \mathbf{NP}_1, \ldots, \mathbf{NP}_t) = \begin{cases} 1 & \text{if } NP_s^1 \geq P_s^* \text{ for all } s \leq t \\ 0 & \text{if } NP_s^1 < P_s^* \text{ for some } s \leq t \end{cases}$$

for all all $m \in M$ and $0 \leq t \leq T$. That is, if the net payment from the borrower to bank 1 is strictly less than P_t^* in any period, then this payment and all future transfers between the borrower and the whole banking sector are seized in entirety.

Lemma 3 ***(The KL-K enforcement rule)***
kr RE Rglt fRcRga tF R$\mathcal{B}_{KLK}$ uf riReRnarRfgna tuigR Rfi RgaFa fcR l t s E niai g M

Proof: See Appendix.

Although the KL-K enforcement rule achieves the constrained efficient outcome, it should be clear that the rule is only one in a large class of enforcement rules. Moreover, although the results of Proposition 2 imply that some enforcement is required to achieve the constrained efficiento utcome, there is no a priori reason why enforcement should take the form of $\mathcal{B}_{KLK}$.

The main difficulty with the KL-K rule is that it is hard to identify in observed institutions. Indeed, although the KL-K literature commonly refers to "debt" constraints,

[15] We assume that any funds seized in this way are simply destroyed. Allowing the central authority to distribute funds seized from a transfer between bank m and the borrower to the remaining $M-1$ banks would add nothing of substance, and would serve only to complicate the notation.

the implementation is in fact a complex state- and date-contingent specifications of payments. Exclusion occurs upon failure to make the payment that is specified by a central authority, the computation of which requires intimate knowledge of an agent's production and consumption possibilities and is information intensive. It is difficult to see how this information arises in decentralized settings and becomes available to courts who enforce contracts.

We now turn to some alternative rules that continue to implement the constrained efficient outcome, but which are easier to interpret.

4.2 Enforcing exclusive trading rights

What other enforcement rules might one consider? Referring back to the intuition we provided when characterizing the efficient outcome as a "gift" followed by exclusive trading rights, one possibility is to stipulate that once the borrower has accepted a loan from one bank (i.e. a negative net payment), he is not allowed to deal with any other bank. Effectively we are thus granting each bank the possibility of acquiring a complete monopoly (i.e. R fF nieR trading rights) over the borrower by making a loan. Formally our rule is $\mathcal{B}_{excl}$ defined by

$$\beta_t^m(\mathbf{NP}_0, \mathbf{NP}_1, \ldots, \mathbf{NP}_t) = \begin{cases} 0 & \text{if } NP_s^{m'} < 0 \text{ for some } s \leq t \text{ and } m' \neq m \\ 1 & \text{otherwise} \end{cases}$$

all $m \in M$ and $0 \leq t \leq T$.

Lemma 4 ***(The exclusivity enforcement rule)***
l arRtR utR a t ct R ugn $M \geq 2$ arRg arR R fF nieiao Rglt fRcRga tF R $\mathcal{B}_{excl}$ uf riReRnarR fgna tuigR Rfi Rga Fa fcR l t s ni aig M

Proof: Consider the set of lending policies $\mathcal{L}^m = \left\{L_0^m = L_0^*, L_t^m(\mathbf{P}_0, \ldots, \mathbf{P}_t) = R_{t-1}P_{t-1}^m\right\}$ where L_0^* is as defined in Proposition 1. Given our exclusion rule $\mathcal{B}_{excl}$, the borrower will only ever accept a loan (i.e. negative net payment) from at most one bank — bank 1, say. By construction, accepting the initial loan payment L_0^* from bank 1 and then depositing his entire wealth in each payment period ($R_t < r$) gives the borrower a final consumption as in Proposition 1. All banks makes zero profits under this borrower strategy. The borrower cannot profitably deviate to another strategy. And finally, no bank can unilaterally and profitably deviate to an alternative lending policy. This last point follows since by construction a bank cannot make strictly positive profits while delivering the borrower a final consumption as in Proposition 1. Since $M \geq 2$, this rules out the possibility of a profitable deviation. **QED**

The preceding two lemmas have established that both the KL-K enforcement rule $\mathcal{B}_{KLK}$ and the exclusivity enforcement rule $\mathcal{B}_{excl}$ achievethe constrained efficient outcome. While we find $\mathcal{B}_{KLK}$ hard to interpret in terms of observed institutional arrangements, the exclusivity rule $\mathcal{B}_{excl}$ resembles the sale of the borrower's project to a bank. That is, in exchange for a sale price L_0^*, the buyer of the project obtains property rights over any cash thrown off by the project that is deposited in the financial sector, and has a veto right on any future loans.[16]

4.3 Enforcing debt/default rules

We now consider a third enforcement rule that we call a R a ERluFa enforcement rule $\mathcal{B}_{DD}$. It is defined by

$$\beta_t^m(\mathbf{NP}_0, \ldots, \mathbf{NP}_t) = \begin{cases} \min\left\{\frac{D_t^m}{NP_t^m}, 1\right\} & \text{if } \exists m' \neq m \text{ s.t. } D_t^{m'} - NP_t^{m'} > 0 \\ & \quad \text{and } NP_t^m > 0, D_t^m > 0 \\ 0 & \text{if } \exists m' \neq m \text{ s.t. } D_t^{m'} - NP_t^{m'} > 0 \\ & \quad \text{and } NP_t^m > 0, D_t^m \leq 0 \\ 1 & \text{otherwise} \end{cases}$$

for all $m \in M$ and $0 \leq t < T$. That is, whenever the borrower is indebted to bank m' at the end of period t, he cannot deposit funds with any other bank $m \neq m'$ in excess of the amount he owes to this second bank. Less formally, a borrower in default to one bank is prohibited from saving with any other bank.

We call this a debt-default rule because it resembles laws that regulate a borrower's defaulting on debt and declaring bankruptcy. First, by definition most legal systems dictate that if a borrower is in default, his creditors have the right to seize any of his assets.[17] By assumption, in our model only assets transferred between the borrower and the financial system are seizable. Our enforcement rule $\mathcal{B}_{DD}$ allows for these transfers to be seized until the borrower has repaid his debt. The rule grants a creditor special rights, until the borrower has repaid his debt.

Second, bankruptcy proceedings require the borrower to disclose and surrender all of his assets. In our model the borrower always has the option of instead hiding the portion of his assets represented by his ongoing project. But the stipulation that he should declare them still matters, because it implies that he cannot in the future deposit the proceeds of this project into the financial system — for to do so would reveal to his creditors that

[16] Of course, we continue to respect the fundamental friction of our model that no outsider can unilaterally seize the cash flows produced by the borrower's project.

[17] Different legal systems place different constraints on the ability of a creditor to seize assets or garnish wages without judicial enforcement, and indeed even within the same legal system different constraints are placed on the seizure of different types of asset (see, e.g., White, 1998).

he acted fraudulently in the bankruptcy proceedings and did not disclose all his assets. In legal terms, his hiding of his project would constitute a fraudulent conveyance, and the funds thus hidden are still subject to seizure ReRgulaRt the discharge of his debts in bankruptcy.

Thus, although most legal systems granta borrower the rightto save in the financial system after bankruptcy, this right only applies to assets acquired after bankruptcy. A borrower Rnga have the right to hide assets during bankruptcy and then deposit these in the financial sector post-bankruptcy. This is exactly the punishment that our enforcement rule $\mathcal{B}_{DD}$ imposes.

Lemma 5 ***(The debt-default enforcement rule)***
1 arRtRutRa t ct R ugn $M \geq 2$arRg arRRa ERluFa Rglt fRcRga tFR $\mathcal{B}_{DD}$ ufriReRnarRfgna tuigR Rfi Rga Fa fcR 1 t s ni aig M

Proof: Consider the following set of lending policies. One of the banks — without loss, bank 1 — offers the lending policy $\mathcal{L}^1$defined by

$$L_t^1(\mathbf{P}_0, \dots, \mathbf{P}_t) = \begin{cases} L_t^* & \text{if } P_s^1 = P_s^* \text{ and } P_s^m = 0 \text{ for all } s < t \text{ and } m \neq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\{L_t^*, P_t^*\}$ are as defined in Proposition 1.

Each of the remaining banks $m \neq 1$ offers the "savings" lending policy $\mathcal{L}^m$ defined by $L_t^m(\mathbf{P}_0, \dots, \mathbf{P}_t) = rP_{t-1}^m$. We claim that these polices, along with the borrower paying P_t^* in period t, constitute an equilibrium. Note the borrower's final consumption in this equilibrium is $W_0 \prod_{t=0}^{T-1} \max\{r, R_t\}$, and that all banks make zero profits.

First, we claim that given the lending policies the borrower's behavior is a best-response. Suppose to the contrary that the borrower has a strictly welfare-increasing deviation and that the resulting payments are $\left\{\tilde{L}_t^m, \tilde{P}_t^m\right\}$ between the borrower and the M banks. Given the lending policies of banks $m \neq 1$, $D_{T+1}^m = 0$. So for the deviation to strictly increase the borrower's final consumption, by Proposition 1 we know that $D_{T+1}^1 > 0$. Let τ be the date at which the borrower first deviates. Bank 1 makes no payments to the borrower after this date, so D_t^1 must be weakly decreasing over dates $t \geq \tau + 1$. The debt-default exclusion rule $\mathcal{B}_{DD}$ then implies that at dates $t \geq \tau + 1$ the borrower can never deposit funds with any other bank $m \neq 1$ in the deviation. But then the deviation must be unprofitable, by the definition of $\{L_t^*, P_t^*\}$.

Second, no bank $m \neq 1$ can profitably deviate. Suppose to the contrary that bank 2 deviates to $\tilde{\mathcal{L}}^2$, leading to equilibrium payments $\left\{\tilde{L}_t^m, \tilde{P}_t^m\right\}$ between the borrower and the M banks. For the deviation to strictly increase bank 2's utility, $\tilde{P}_t^2$ must be strictly positive for some date. Let τ_0 be the first such date. Moreover, let τ_1 be the last period in which the net payment between the borrower and bank 2 is non-zero. Note that it must be the case that $NP^2_{\tau_1} < 0$ (i.e. the last payment must be form bank 2 to the borrower). For bank 2 to make strictly positive profits, we need $D^2_{\tau_1} - NP^2_{\tau_1} < 0$. The borrower's final consumption must be equal to at least $W_0 \prod_{t=0}^{T-1} \max\{r, R_t\}$, his payoff from staying with bank 1. Since bank 1 makes no payments to the borrower after the deviation date τ_0, D^1_t must be weakly decreasing over dates $t \geq \tau_0 + 1$. By Proposition 1 we know that $D^1_{T+1} > 0$. So $D^1_t > 0$ for all dates $t \geq \tau_0 + 1$. But $D^2_{t+1} = D^2_t - \beta^2_t NP^2_t$, so $D^2_{t+1} \geq 0$ whenever $D^2_t \geq 0$ for any $t \geq \tau_0$. But since $D^2_{\tau_0} \geq 0$, this contradicts $D^2_{\tau_1} < 0$ completing the claim.

Third, we claim that bank 1 cannot deviate and make strictly positive profits. For suppose to the contrary that there exists a deviation $\tilde{\mathcal{L}}^1$ under which bank does make strictly positive profits. Let $\left\{\tilde{L}^m_t, \tilde{P}^m_t\right\}$ be the resulting payments between the borrower and the M banks. Since bank 1 makes strictly positive profits, $\tilde{D}^1_{T+1} < 0$. Let τ be the last date t at which $\tilde{D}^1_t \geq 0$. So $\tilde{P}^1_\tau - \tilde{L}^1_\tau \geq \tilde{D}^1_\tau > 0$. But this cannot be an equilibrium, since the borrower could instead pay just $\tilde{D}^1_\tau + \tilde{L}^1_\tau$ to bank 1 (thus repaying his debt), and make all future deposits to one of the competitor banks, say bank 2. This must be welfare improving for the borrower since all payments to bank 1 in excess of $\tilde{D}^1_\tau + \tilde{L}^1_\tau$ at date τ, along with all payments to bank 1 after date τ, constitute deposit payments on which bank 1 pays an interest rate of strictly less than r. **QED**

4.4 Enforcing credit denial

Our debt-default enforcement rule $\mathcal{B}_{DD}$ places restrictions on the ability of indebted borrowers to place nueign with other banks. As we discussed above, this restriction on savings is effectively embedded in bankruptcy law. On the other hand, neither the rule $\mathcal{B}_{DD}$, nor most specifications of bankruptcy law, place any direct restriction on access to credit.

Indeed, we next show that altering our rule to restrict a borrower from receiving credit, as opposed to placing savings with the financial system, renders enforcement useless. The equilibrium reverts to the no-enforcement outcome of Proposition 2.

Of course, in practice, lenders may be unwilling to advance credit to a borrower that has declared bankruptcy. However, this is not legally enforced credit denial. (It is presumably a consequence of reputation effects, which are absent in our KL-K environment). Our next result highlights that it is savings denial rather than credit denial that is at the heart of enforcement in the KL-K environment.

Formally, a prohibition on credit to indebted borrowers would be achieved by a ft R aE st r ii aig enforcement rule $\mathcal{B}_{CP}$b y

$$\beta_t^m(\mathbf{NP}_0,\ldots,\mathbf{NP}_t)=\begin{cases}\min\left\{\frac{D_t^m}{NP_t^m},1\right\} & \text{if } \exists m'\neq m \text{ s.t. } D_t^{m'}-NP_t^{m'}>0 \\ & \text{and } NP_t^m<0, D_t^m<0 \\ 0 & \text{if } \exists m'\neq m \text{ s.t. } D_t^{m'}-NP_t^{m'}>0 \\ & \text{and } NP_t^m<0, D_t^m\geq 0 \\ 1 & \text{otherwise}\end{cases}$$

The rule $\mathcal{B}_{CP}$ is exactly analogous to the debt-default rule $\mathcal{B}_{DD}$: Where $\mathcal{B}_{DD}$ preventing an indebted borrower making a payment to another bank that leaves him out of debt, the rule $\mathcal{B}_{CP}$ prevents an indebted borrower receiving a payment from another bank that leaves him indebted.

However, in complete contrast to our debt-default enforcement rule we have the following result:

Lemma 6 ***(The credit-prohibition enforcement rule)***
F ss nR arRfR utR a t ct R ugn $M \geq 2$M kr Rg ig ReRto RFii tiF c FgE R t arR ft R a st r ii aig Rglt fRcRga tF R $\mathcal{B}_{CP}$ arR tt R t n g u fgnF csai g in $W_0\prod_{t=0}^{T-1}\max\{r,R_t\}$.

Proof: See Appendix.

The intuition underlying Lemma 6 is essentially the same as that for Proposition 2 (as is the proof). As in Lemma 1, competition among banks under-cuts the possibility of positive future profits. In any equilibrium the final payments have to be from the banks to the borrower. Consider the first date prior to this (date t_0 say) at which the future profits of the banks are positive. Then over the period from t_0 to the final date, the borrower is effectively saving with the banking sector. Moreover, he must be saving at a rate below r, since the banks are making positive profits. So any one of the banks can undercut the others by offering a higher rate of return on savings. And since this deviation does not involve any new loans, the enforcement rule $\mathcal{B}_{CP}$ never has any effect.

4.5 Discussion

It is widely appreciated that exclusion from financial markets would, in theory, provide borrowers with a powerful incentive to repay their loans. However, in practice the absolute exclusion that the literature has focused on is not observed. Moreover, the dimension of exclusion that is easiest to identify — namely ft R a st r ii aig — is by itself insufficient to provide repayment incentives. Weha vesp ecified a general class of exclusion rules, and exhibited several specific examples that give the constrained efficient outcome. In doing so, we have shown that the extreme of full and indefinite exclusion is (while sufficient) not necessary for efficient lending. The much weaker degree of financial exclusion implied by most bankruptcy regimes is sufficient. The key aspect of bankruptcy for exclusion purposes is that a defaulting borrower must choose between (a) surrendering his assets to his creditors and (b) hiding his assets outside the formal financial sector. Choice (b) is a consequence of the fact that funds hidden from creditors through fraudulent conveyances are still seizable after bankruptcy if discovered.

Under our interpretation of existing enforcement institutions, the debt-default rule $\mathcal{B}_{DD}$ is the most readily identifiable of the efficient enforcement rules we have defined. Given that there are at least several other rules that also lead to efficientlending , a natural question to ask is whether or not the debt-default rule is in any sense superior to other efficient rules.

Our framework allows us to identify two dimensions in which it appears superior. First, unlike the KL-K enforcement rule $\mathcal{B}_{KLK}$ it does not require courts to have any knowledge of the constrained efficient transfers. Courts simply need to observe whether a borrower is in default on his debt, and this only requires knowledge of the uf aFu transfers made between borrowers and lenders. We make this point in more formal terms in the next two sections, in particular in Section 6.

Second, and perhaps more immediate, it gives a lending bank creditor's rights since an indebted borrower is restricted to only trade with the lending bank. Although our formulation of the rule has funds deposited with other banks being seized and thrown away, we can require these funds to be transferred to the lending bank with no substantive changes to our results. For this reason, the rule is also renegotiation proof. In the rule $\mathcal{B}_{KLK}$, if a borrower misses a payment so as to default, he will be excluded from trade from that point on. On the other hand, all agents in the economy would, at this point, prefer to forgivethe borrower's debt in order to realize the lost gains from trade. Under the debt-default rule, this does not hold true since the bank that is defaulted on is granted valuable rights in the event of default.

Finally, we should note that despite the fact that the debt-default rule only restricts the savings options of an indebted borrower, it endogenously restricts his borrowing options. That is to say, a bank only extracts repayments in periods where the borrower has a low R_t, at which points the bank effectively offers the borrower low returns on his savings. However, if a borrower is in debt with one bank, his savings options with another bank are restricted. As a result, the other bank will never receive repayments on any loans made to the borrower. Thus, outcomes under the debt-default rule will look as if a credit-prohibition rule was also in effect. Of course, if the borrower repays the debt of the lending bank, he is free to save/borrow from another bank.

5 Limited bank commitment

In the previous section, we established that many different enforcementrules are able to implement the constrained efficient outcome. In this section, we will show that this multiplicity of optimal enforcement rules stems at least in part from the assumption that while the borrower was unable to commit to repaying a lending bank, each bank was able to fully commit to make future payments to the borrower. That is, we now examine a problem in which banks have much weaker commitment powers. In particular, they are no longer able to credibly promise to make the borrower a future loan. Such future loans form part of the constrained efficient allocation. It follows that an optimal enforcement rule must not only give the borrower the incentive to repay the bank, Fa cF naun i eRu ug arRigf RgäeRa cuR u gR ug a arR tt R tM

Formally, we now assume that banks can only commit to future payments at the penultimate date[18] $T-1$. That is, the timing is now as follows. At all dates $0 \le t \le T-2$, the each bank m simultaneously chooses a payment L_t^m to make to the borrower. The borrower then chooses a payment P_t^m to make to each bank. In contrast, at date $T-1$ each bank m simultaneously chooses a payment L_{T-1}^m to make today, and a payment $\mathcal{L}_{T-1}^m(\mathbf{P}_{T-1})$ that will be made at date T, where this payment can be contingent on the vector of date $T-1$ payments from the borrower to the banks. After the M banks have announced $\left(L_t^m, \mathcal{L}_{T-1}^m(\mathbf{P}_{T-1})\right)$ the borrower then chooses the vector of payments $\mathbf{P}_{T-1}$. Finally, at date T each bank honors its commitment $\mathcal{L}_{T-1}^m(\mathbf{P}_{T-1})$.[19]

Once we no longer allow banks to commit at date 0, it is clear that any equilibrium set of payments $\{L_t^m, P_t^m\}$ must possess that property that

$$\sum_{s=t'}^{T} r^{-s}\left(L_s^m - P_s^m\right) \ge 0 \text{ for all dates } 0 \le s \le T-1 \tag{16}$$

i.e. each bank must be making non-negative future profits at all dates s prior to the terminal date T. Proposition 1 of Section 2 characterized the maximum possible level of the borrower's final consumption when the bank's were able to commit (i.e. when constraint (16) is required to hold only at $s=0$). The analogue under constraint (16) is:

Proposition 3 ***(An upper bound on borrower welfare under limited bank commitment)***

[18]For instance, it may be the case that banks only possess pledgeable assets at date $T-1$. The bank then uses these assets to commit to payments at date T

[19]Without loss, we can obviously assume that the borrower will not make any payments in period T.

t ugo $0 \leq t \leq T-1$ R gR

$$\gamma_t = \left(1 - \rho_{T-1}\right) \prod_{s=t}^{T-2} \max\{1, \rho_s\} \tag{17}$$

kr Rg rR g fgna tuiga in tRFit R a r arR tt R t n u aR T fgnF csai g in Fg R u e Ro $\prod_{t=0}^{T-1} R_t W_0 / (1-\gamma_0)$M[20] kr in ReR fug RufriReR o arR ugn ug suoc Rgan$\{L_t^*, P_t^*\}$ R gR un l n

M kr R tt R t tRRRRnu ug $L_0^* = W_0 \gamma_0 / (1-\gamma_0)$ ua u aR 0M

M g ugo suoc RgasRti t arua l n ug igeRnac RgasRti arR tt R t suon ri n RgatRR a iMRMi n u na ug fFc FuaR ua tuaRrM

M g ugo igeRnac RgasRti $t > 0$ arua l n u suoc RgasRti arR ugn cuR u ug a arR tt R t RFu a $W_t \gamma_t / (1-\gamma_t)$

Proof: See Appendix.

The loans and payments described in Proposition 3 have the property that the borrower is paying less 100% of his wealth to the banking sector in payment periods. In terms of the "savings market" intuition of Section 2, this is because the banks are no longer able to credibly promise to repay an arbitrarily large deposit made by the borrower. In other words, limited commitment of banks places an endogenous constraint on the size of "deposits" that the banking sector can accept.

It is clearly inefficient for the borrower not to hand over all is wealth to the banking sector in payment periods — in these periods, the banks havea higher return than the borrower does. Consequently, the maximal consumption level characterized in Proposition 3 is lower than that achievable when banks have the ability to full commit (see Proposition 1):

Corollary 3 ***(Value of full commitment)***
mnFcR arua $\alpha_0 <$ 1M kr Rg arR tt R t n g u fgnF csai g in Rt rR g arR ugn fug go sutaiu o fccia arug rR g arRo fug lF fcciaM kr Rfcs uting in natifa ste iR arRtR in u suoc RgasRti sti t a $T-1$M

Proof: The borrower's maximal final consumption with limited bank commitment is given by $\prod_{t=0}^{T-1} R_t W_0 / (1-\gamma_0)$, while the borrower's maximal final consumption with

[20]Note that we previously assumed that $\alpha_0 < 1$. It is straightforward to verify that this implies that $\gamma_t < 1 \;\forall\; t$.

full bank commitment is given by.$\prod_{s=0}^{T-1} R_s W_0 / (1-\alpha_0)$ (see Proposition 1). Thus we need to establish that $\alpha_0 \geq \gamma_0$. Note that $\gamma_{T-1} = \alpha_{T-1}$. In any investmentp eriod t, $\gamma_t = \rho_t \gamma_{t+1}$ and $\alpha_t = \rho_t \alpha_{t+1}$. However, in any paymentp eriod t, $\gamma_t = \gamma_{t+1}$ while $\alpha_t = 1 - \rho_t (1 - \alpha_{t+1})$. Since $\rho_t < 1$ this later equality implies $1 - \alpha_t < 1 - \alpha_{t+1}$, or $\alpha_t > \alpha_{t+1}$. The result follows. **QED**

To illustrate as starkly as possible the importance of bank commitment, we start by considering the outcome obtained under the exclusivity enforcement rule, $\mathcal{B}_{excl}$. The constrained efficient outcome under limited commitment entails at least one of the banks making payments to the borrower between dates 0 and $T-1$. Intuitively, under the rule $\mathcal{B}_{excl}$ one bank always ends up with complete monopoly power over the borrower, and so (R sna) has no incentive to make these payments. This, in turn, reduces the incentives for the borrower to make repayments, and leads to a smaller initial loan as well as lower final period consumption.

Let $\left\{\hat{L}_t^m, \hat{P}_t^m\right\}$ be a set of equilibrium payments. Let $t_0 \leq T-1$ be the first date at which $\widehat{NP}_{t_0}^m < 0$ for some bank m. That is, date t_0 is the date at which bank m makes a loan to the borrower, after which the borrower is restricted to trade exclusively with bank m.

For all $t \geq t_0$, let $U_t^B (W_t, D_t^m)$ be the final consumption level achieved by the borrower given that his wealth entering period t is W_t, his debt level to bank m is D_t^m, and all actions from date t on are subgame perfect. Define $U_t^m (W_t, D_t^m)$ similarly.[21] Given our exclusivity enforcement rule $\mathcal{B}_{excl}$, clearly $U_{T-1}^B (W_{T-1}, D_{T-1}^m) = W_{T-1} R_{T-1}$ and $U_{T-1}^m (W_{T-1}, D_{T-1}^m) = W_{T-1} (r - R_{T-1}) - r D_{T-1}^m$.

At date $T-2$, the borrower will clearly always pick $\mathbf{P}_{T-2} = 0$, since his value function is strictly increasing in wealth, and making payments reduces wealth.

Likewise, bank m will pick $L_{T-2}^m = 0$. Choosing a positive level of L_{T-2}^m increases D_{T-1}^ma s well as W_{T-1}. However, since $\rho_{T-2} (1 - \rho_{T-1}) \leq \gamma_{T-2}$ and by Assumption 1, $\gamma_{T-2} < 1$, making the loan is not profitable for the bank. We recall that this parameter assumption amounted to assuming that the borrower was always credit constrained.

All other banks $m' \neq m$ will also pick $L_{T-2}^{m'} = 0$, since any payment they make will seized under the exclusivity enforcementrule. Thus $U_{T-2}^B (W_{T-2}, D_{T-2}^m) = W_{T-2} R_{T-2} R_{T-1}$ and $U_{T-2}^m (W_{T-2}, D_{T-2}^m) = W_{T-2} R_{T-2} (r - R_{T-1}) - r^2 D_{T-2}^m$. Straightforward induc-

[21]Note that the value functions are defined in terms of future valued consumption, as opposed to date t present value. This modification simplifies our exposition.

tion implies that for all $t > t_0$,

$$\begin{aligned} U_t^B\left(W_t, D_t^m\right) &= W_t \prod_{s=t}^{T-1} R_s \\ U_t^m\left(W_t, D_t^m\right) &= W_t \prod_{s=t}^{T-2} R_s\left(r - R_{T-1}\right) - r^{T-t} D_t^m \end{aligned}$$

Ata ll of the dates $t \in \ [t_0 + 1, T - 2]$, the borrower makes no payments to any of the banks, and the banks make no loans to the borrower.

Clearly the borrower will never choose $P_t > 0$ for any $t < t_0$, since this reduces his wealth at date t_0, and the value function is increasing in wealth. Thus $D_{t_0}^m = 0$. The final consumption of the borrower and bank m are respectively

$$\begin{aligned} U^B &= \left(W_0 \prod_{s=0}^{t_0-1} R_s - \widehat{NP}_{t_0}^m\right) R_{t_0} \prod_{s=t_0+1}^{T-1} R_s \\ U^m &= \left(W_0 \prod_{s=0}^{t_0-1} R_s - \widehat{NP}_{t_0}^m\right) R_{t_0} \prod_{s=t_0+1}^{T-2} R_s\left(r - R_{T-1}\right) + r^{T-t_0+1} r \widehat{NP}_{t_0}^m \end{aligned}$$

Imposing that $U^m \geq 0$ (i.e. bank m does not make negative profits) allows us to solve for the maximal value of borrower consumption. The constraint $U^m \geq 0$ implies

$$-\widehat{NP}_{t_0}^m \leq W_0 \prod_{s=0}^{t_0-1} R_s \frac{\left(1-\rho_{T-1}\right) \prod_{s=t_0}^{T-2} \rho_s}{1-\left(1-\rho_{T-1}\right) \prod_{s=t_0}^{T-2} \rho_s}$$

giving[22]

$$U^B \leq W_0 \prod_{s=0}^{T-1} R_s \frac{1}{1-\left(1-\rho_{T-1}\right) \prod_{s=t_0}^{T-2} \rho_s}$$

We summarize the results as follows:

Lemma 7 ***(Borrower welfare under the exclusivity enforcement rule)***
F ss nR arR Rglt fRcRga tF R $\mathcal{B}_{excl}$ in ig RR faMkr Rg arR tt R t n g u fgnF csai g in ua cna

$$\max_{t_0} W_0 \prod_{s=0}^{T-1} R_s \frac{1}{1-\left(1-\rho_{T-1}\right) \prod_{s=t_0}^{T-2} \rho_s} \tag{18}$$

[22] Note that it is possible that the lender never makes a loan, and the borrower prefers to simply save at return r at date $T-1$. In this case, the expression for borrower consumption can be arrived at by setting $t_0 = T-1$.

kr in Fg fug RufriReR o gR l arR ugn m nuo cui g u nigR ug a arR tt R t uau u aR $t_0 \leq T-1$MRaR Rg t_0 ug u aR$T-1$ g lFtarRt ugn utRcuR a arR tt R t gt utRtRsuoc RgancuR o arR tt R tM ReRt arRfgnF csai g ReR in ntifa o Rnn arug arR Fg $W_0 \prod_{s=0}^{T-1} R_s/(1-\gamma_0)$ auigR ig t s ni aig rR gReR arRtRin u suoc RgasRti sti t a u aR$T-1$M

Under the exclusivity enforcement rule $\mathcal{B}_{excl}$, a bank is happy to make a loan because after doing so the borrower can deal with no other bank. We have commented before that an alternative way to think about a sequence of loans and repayments is instead as an initial gift followed by the borrower saving with the bank in periods when $R_t < r$. When the borrower is restricted from dealing with all but one of the banks, the remaining bank can effectively make monopoly profits in the "savings" periods. These profits compensate the bank for the initial loan.

Sticking with this intuition, the reason a bank only makes a single loan (before date T) under the exclusivity enforcement rule $\mathcal{B}_{excl}$ is that after the initial loan, he has already gained all the monopoly benefits available. There is no point in him transferring more funds to the borrower, since doing so in no way increases his ability to profit from his position as the only bank the borrower is able to trade with.

In contrast, under the debt-default exclusion rule $\mathcal{B}_{DD}$, a bank would have the incentive to make a second loan if the borrower has repaid the first one. For once the first loan has been repaid, the bank's position as a monopoly bank has vanished. Thus the bank is again prepared to spend funds to (re)acquire this monopoly position. Moreover, the borrower will potentially be prepared to repay the initial loan, since doing so leads to the second loan. And as we have now seen, if the first loan is not repaid the bank will certainly not make the second loan.

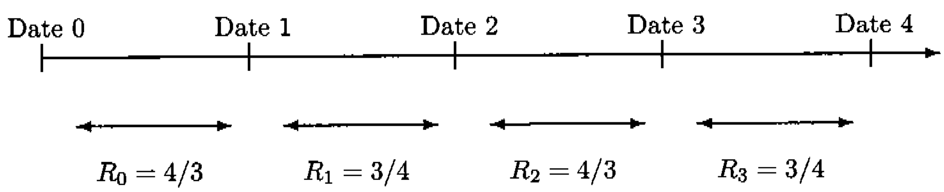

Figure 2: Numerical example

Rather than immediately stating our formal result (see Proposition 4 below), we instead start with an illustrative example. There are 5 periods ($T = 4$) — see Figure 2. The banks' interest rate is $r = 1$. The borrower's rate of return (R_t) alternates between $\frac{4}{3}$ ($t = 0, 2$), and $\frac{3}{4}$ ($t = 1, 3$). The borrower's initial is wealth is $W_0 = 100$.

If the exclusivity enforcement rule is in effect, Lemma 7 implies the borrower's final consumption is no more than 150. One equilibrium to achieve this is bank 1 making an initial loan of 50, followed by the borrower saving all his wealth with bank 1 between dates 3 and 4 at an interest rate of 3/4.

In contrast, when the debt-default enforcement rule is in effect the following is an equilibrium:

- At date 0, bank 1 lends $L_0^1 = 80$.
- At date 1, the borrower enters with wealth $W_1 = (100 + 80) \times \frac{4}{3} = 240$ and debt $D_1^1 = 80$. He repays $P_1^1 = 80$.
- At date 2, the borrower enters with wealth $W_2 = 160 \times \frac{3}{4} = 120$ and debt $D_2^1 = 0$. Bank 1 lends $L_2^1 = 60$.
- At date 3, the borrower enters with wealth $W_3 = (120 + 60) \times \frac{4}{3} = 240$ and debt $D_3^1 = 60$. He repays $P_3^1 = 60$.
- At date 3, the borrower places the rest of his wealth (180) as savings with any bank at the return of one. His final consumption at date 4 is simply 180.

At date 3 the borrower repays 60. If he does not, then he will be restricted to dealing with bank 1, in which case bank 1 will offer to save for the borrower at the return of $\frac{3}{4}$. This leaves the borrower with final wealth of 180. Thus the borrower is indifferent between repaying and paying the 60. Note that if the borrower had wealth W_1 any higher than 240 or debt D_3^1 any lower than 60, the borrower would strictly prefer repayment.

Also, note that if instead of debt-default, the exclusion enforcement rule was in place, the borrower would repay nothing, and would save with bank 1 at the return of $\frac{3}{4}$. The resulting consumption levels for the borrower and bank would be identical to that under debt-default.

Although seemingly irrelevant at date 3, the fact that the borrower has some incentive to repay the loan in full is relevant for the lender incentives at date 2.

As we argued before, if the exclusion enforcement rule is in place, the bank will never make a loan payment to the borrower at date 2. This is because regardless of his making a loan, the borrower would make no repayments at date 3, and the lender's profits would come solely from offering the below-market savings rate of $\frac{3}{4}$ to the borrower at date 3. So the bank is strictly better off if it does not make a new loan.

In contrast, under the debt-default enforcement rule the bank is happy to make a new loan at date 2.

If the lender makes a loan of L_2^1 between 0 and 60, the borrower will strictly prefer to repay the loan at date 3 as opposed to defaulting. In this interval, the marginal loan is fully repaid. In contrast, for loans greater than 60 a larger loan has no marginal impact on the borrower's repayment. This, of course, is the same logic we appealed to when stating that under the exclusion enforcement rule the lender will make no loans beyond an initial one.

All of these considerations only apply if the borrower is out of debt at the start of date 2. If instead he is in debt, bank 1 is already in a monopoly situation, and would be strictly worse off if it made a new loan at date 2.

Finally, note the borrower is persuaded to make the date 1 repayment precisely because there is an equilibrium of the subgame starting at date 2 in which bank 1 makes a new loan if and only the borrower arrives out of debt. The borrower's date 1 repayment incentives are maximized if we choose the subgame equilibrium in which the borrower's consumption is highest, which the above example does in choosing $L_2^1 = 60$.

To summarize, the key property of the debt-default exclusion rule $\mathcal{B}_{DD}$ relative to the exclusivity exclusion rule $\mathcal{B}_{excl}$ is that while both reward a bank for making an initial loan, only the former also gives a lending bank the incentive to make a second loan. Moreover, note that the final consumption level achieved by the borrower in this example is in fact equal to the upper bound derived in Proposition 3.

All important aspects of this example extend to the general T-period environment of the paper. Because of the need to keep track of the value functions of the lending bank and the borrower at all nodes, both in- and out-of-equilibrium, the formal construction of the equilibrium is somewhat lengthy and is relegated to the appendix:

Proposition 4 ***(Limited commitment and the debt-default rule)***
F ss nR arua arR R a ERluFa Rglt fRcRga tF R $\mathcal{B}_{DD}$ in ig RRfaM kr Rg arRfR R inan ug IFii tiF c ig ri fr arR suoc Rgan RaRRg arR tt R t ug gR l arR ugn m^* utR $L_t^{m^*} = L_t^*$ ug $P_t^{m^*} = P_t^*$ rR tR L_t^* ug P_t^* utR un R gR ig t s ni aig M arRt atugnlRtn ffFt RaRRg arR tt R t ug arR arRt ugn $m \neq m^*$M kr ua in FgR t arR R a ERluFa Rglt fRcRga tF R $\mathcal{B}_{DD}$ arR fgna tuigR Rfi Rga Fa fcR R inan un ug IFii tiF cM

Proof: See Appendix.

The equilibrium of Proposition 4 involves the same bank making loans and taking repayments in every period. In fact, it is straightforward to show that there are also efficient equilibria in which a different bank makes the loan in different periods — e.g. bank 1 makes the loan at the initial date and is repaid in the first payment period, bank 2 makes the loan at the next investment date and is then repaid at the following payment date, etc.

6 Properties of all optimal enforcement rules

We now turn to an environment in which we can reach substantive conclusions about the properties of all optimal enforcement rules. As we have discussed, relaxing lender commitment allows us to distinguish between the different rules. Different rules provide differing incentives for lenders to extend loans. An optimal rule must provide the correct incentives to both lenders as well as borrowers.

We fix our environment to include only one set of borrower and lender decisions: $T = 2$; a loan is made at date 0 ($R_0 > r$); a repayment occurs at date 1 ($R_1 < r$); the rule regulates trade at dates 1 and 2. This is a snapshot of the general T-period limited commitment case we have studied, that includes a single investment and payment period.

We assume that only one of the M banks — without loss, bank 1 — has funds to lend to the borrower at date 0. That is, while all banks are present in the lending game at date 0, only one bank has funds available to actually make a date 0 loan. So bank 1 has a limited monopoly over the borrower and must be provided incentives to make a large enough initial loan. Without this assumption, in the $T = 2$ problem competitive pressures would lead to the constrained efficient outcome, obscuring the necessity of providing the bank with incentives. (As we saw in Section 5, when $T > 2$ bank incentives are needed for efficiency even under competition.)

The debt-default rule continues to obtain the constrained efficient outcome of Proposition 1. This can easily be seen from the proof of Lemma 5. There we constructed an equilibrium in which only one of the banks offers to make a loan at date 0, while all other banks offered a savings contract, and showed that the efficient outcome obtained.

However, not all rules will still deliver the efficient outcome. For consider the exclusivity rule ($\mathcal{B}_{excl}$). If the borrower only traded with banks $m \neq 1$, the maximum consumption that the borrower obtains is $W_0 R_0 r$, since the borrower receives no loan in period 0. This is strictly less than the constrained efficient outcome, since $r < R_1/(1 - \alpha_0)$. The borrower can only obtain a higher consumption level if he received funds from bank 1 at date 0. However, since bank 1 receives an exclusive right to trade with the borrower tRı t Rnn of the size of the date 0 loan, the bank can reduce the initial loan size and the borrower would continue to accept the loan. This implies that the equilibrium requires the borrower to have consumption that is no more than $W_0 R_0 r$.

Another alternative is the KL-K enforcement rule. This rule requires a central authority to compute and dictate specific payments to be made by agents. An obvious application of this rule to the current setting is that the central authority computes the efficient loan size, L_0^*, and dictates that the monopoly bank must make this initial loan or be restricted from all future trade. Although this delivers the efficient outcome, as we observed earlier, we find it unappealing because it is information intensive and is not robust to small errors in the information possessed by the central authority. That is, suppose the central authority mistakenly believes that the true value of $L_0^*$is $\tilde{L}_0^* > L_0^*$. In this case it will always be unprofitable for bank 1 to make ugo loan at date 0.

To formalize what it means for a rule to be robust to errors in information, we proceed as follows. Write $\mathcal{X}$ for the set of all possible parameter configurations,

$$\mathcal{X} \equiv \{(W_0, R_0, R_1) : W_0 > 0, R_0 > r, 0 \leq R_1 < r, R_0 (r - R_1) < r^2\}$$

For use below, for any parameter choice $x \in \mathcal{X}$ let $L_0^*(x)$ denote the efficient loan size, i.e.

$$L_0^*(x) = \frac{R_0 (r - R_1)}{r^2 - R_0 (r - R_1)} W_0$$

We regard an exclusivity rule as robust if it does not depend on the borrower-specific parameters x:

Definition 1 ***(Robust enforcement rule)***
t sutucRaRtn $x \in$ $\mathcal{X}$ gRa suoc Rgan $(\mathbf{NP}_0, \mathbf{NP}_1)$ ug ug tRaFg r u RgRtu Rglt fRcRga tF R$\mathcal{B}$ in ***robust*** il

$$\beta_1^m(\mathbf{NP}_0, \mathbf{NP}_1, r, x) = \beta_1^m(\mathbf{NP}_0, \mathbf{NP}_1, r, x') \ \forall \ x, x' \in \mathcal{X}.$$

In other words, courts only need to observe the net payments at date 0 and date 1 as well as the interest rate r in order to implement a robust enforcement rule. While it may be that x indirectly affects repayments – since $\mathbf{NP}_0$ and $\mathbf{NP}_1$ are endogenous – this effect is summarized in the values of the net payments.

Clearly our debt-default enforcement rule is robust in this sense, and we have already shown that it achieves the constrained efficient level of lending for all $x \in \mathcal{X}$. Likewise, the KL-K rule is not robust because it is dependent on knowledge of x. In general, what are the properties of an enforcement rule that delivers efficiento utcomes and is robust?

Lemma 8 ***(Borrower punishment)***
Ra$\mathcal{B}$ Ru t FnaRglt fRcRga tF RnFFr arua l t u $x \in \mathcal{X}$ arR fgna tuigR Rfi Rga Fa fcR R inan un ug FFii tiF cM F ss nR ua u aR1 arR tt R t run luiR a tRsuo u ug a ug 1 iNRMNP$_1^1 < rNP_0^1$M kr Rg arRtF R$\mathcal{B}$ cF na RnFFr arua arR tt R t in

sFg inrR iar u tRnaifaig g ri n u iiao a R sni a lFg n iar arR arRt $M-1$ ugnM tR st Ri nRb arRtR R inan u $\lambda < 1$ nFfr arua l t ugo fr if R l gRa suoc Rgan a arR tRcui gig $M-1$ ugn $\{NP_0^m, NP_1^m : m \neq 1\}$ nFfr arua $NP_0^m = 0$ ug $\sum_{m\neq 1} NP_1^m >$ $rNP_0^1 - NP_1^1$ arRg

$$\sum_{m\neq 1} \beta_1^m \left(\mathbf{NP}_0, \mathbf{NP}_1\right) NP_1^m \leq \lambda \sum_{m\neq 1} NP_1^m \tag{19}$$

In less formal terms, if the borrower is in default with bank 1, his ability to deposit funds with another bank is restricted. In particular, at least a fraction $1-\lambda$ of these funds are seized.

Proof: Suppose otherwise. That is, there exists values of $\widehat{NP}_0^1$ and $\widehat{NP}_1^1$ such that $\widehat{NP}_1^1 < r\widehat{NP}_0^1$ but with the property that for any $\lambda < 1$, there exists a set of payments $\left\{\widehat{NP}_0^m, \widehat{NP}_1^m : m \neq 1\right\}$ such that $\widehat{NP}_0^m = 0$ and $\sum_{m\neq 1} \widehat{NP}_1^m > r\widehat{NP}_0^1 - \widehat{NP}_1^1$ but inequality (19) fails to hold. The proof consists of showing that under these assumptions, there always exists at least some parameter value $x \in \mathcal{X}$ for which the borrower can do strictly better by not repaying bank 1's loan, so that the constrained efficient outcome does not exist as an equilibrium.

Let $\hat{X}$ denote the subset of the parameter space $\mathcal{X}$ for which $L_0^*(x) = \widehat{NP}_0^1$. Suppose $\{NP_0^m, NP_1^m\}$ is an equilibrium that does achieve the constrained efficient outcome. This means that $NP_0^1 = L_0^*\left(\hat{X}\right)$, $NP_0^m = 0$ for $m \neq 1$, and the borrower's final consumption is

$$\hat{U}(x) = \left(W_0 + L_0^*\left(\hat{X}\right)\right) R_0 r - L_0^*\left(\hat{X}\right) r^2$$

and all banks make zero profits.

Suppose for now that the payments $\left\{\widehat{NP}_0^m, \widehat{NP}_1^m : m \neq 1\right\}$ have the property that they completely exhaust the borrower's date 1 wealth, i.e.

$$\sum_{m\neq 1} \widehat{NP}_1^m = \left(W_0 + \widehat{NP}_0^1\right) R_0 - \widehat{NP}_1^1 \tag{20}$$

Choose $\varepsilon > 0$ and $\lambda \in [0,1]$ to be such that the inequality

$$\lambda\left(\left(W_0 + \widehat{NP}_0^1\right) R_0 r - \widehat{NP}_1^1 r\right) - \varepsilon > \left(W_0 + \widehat{NP}_0^1\right) R_0 r - \widehat{NP}_0^1 r^2 = U\left(\hat{X}\right) \tag{21}$$

holds. That is, inequality (21) says that if the borrower can transfer a proportion λ of his date 1 wealth to banks $m \neq 1$ and earn an interest rate r, then he will be strictly better off than repaying bank 1 in full. Note that such choice is always possible, since the left-hand side of (21) is equal to

$$\lambda\left(W_0 + \widehat{NP}_0^1\right) R_0 r - \widehat{NP}_0^1 r^2 + r\left(r\widehat{NP}_0^1 - \lambda\widehat{NP}_1^1\right) - \varepsilon$$

and by supposition $\widehat{NP}_0^1 - \widehat{NP}_1^1 > 0$.

Given inequality (21), it follows that the banks $m \neq 1$ can strictly increase their collective profits to ε by offering to accept saving between dates 1 and 2 at a rate just less than r. Inequality (21) guarantees that the borrower will accept this offer, since doing so yields a utility level strictly greater than $U\left(\hat{X}\right)$. Thus we have established that there cannot be an equilibrium of the type described if (20) holds.

To complete the proof of the lemma it remains only to show that (20) holds for at least some parameter value $x \in \hat{X}$. Consider the line in $\hat{X}$ given by

$$x(\delta) = (W_0, R_0, R_1) = \left(\frac{\delta^2}{r^2 - \delta^2} L_0^*\left(\hat{X}\right), r + \delta, \delta\right) \text{ where } \delta \in (0, r)$$

Since $\widehat{NP}_0^1 = L_0^*\left(\hat{X}\right)$, the borrower's date 1w ealth under the parameter $x(\delta)$ is $\frac{r^2}{r-\delta} L_0^*\left(\hat{X}\right)$. By assumption, $\sum_m \widehat{NP}_1^m > r L_0^*(X)$. So we can always find a value of $\delta \in (0, r)$ such that $\sum_m \widehat{NP}_1^m = \frac{r^2}{r-\delta} L_0^*(X)$, and so (20) holds. **QED**

We have just established that based on considerations of the borrower's repayment incentives, an optimal enforcement rule must "punish" a defaulting borrower. Let us now consider the lender's incentives to forward the initial loan and see how this affects the enforcement rule.

Lemma 9 ***(Borrower rights)***
RaB Ru t FnaRglt fRcRga tF RnFFr arua l t u $x \in \mathcal{X}$ arR fgna tuigR Rfi Rga Fa fcR R inan un ug FFii tiF cM kr Rgl t u u aR0 ug euF Rn$L_0 > 0$ ug ugo euF R l $\lambda < 1$ arR tt R t cF nasnnRnn u fr if Ru suoc Rgan$\mathbf{P}_0$ ug $\mathbf{P}_1$ nFFr arua

$$\sum_{m \neq 1} P_1^m > r\left(L_0 - P_0^1\right) - P_1^1$$

iMRMarR tt R t uaRc san a nueRct Rarug arR nr talu ig ri n ug tRsuoc Rga ug

$$\sum_{m \neq 1} \beta_1^m (\mathbf{NP}_0, \mathbf{NP}_1) P_1^m \geq \lambda \sum_{m \neq 1} P_1^m \tag{22}$$

iMRMua cna u ltuf ai g $1 - \lambda$ l arRnRsuoc RganutR nRiR rR tR $\mathbf{L}_0 = (L_0, 0, \ldots, 0)$ ug $\mathbf{L}_1 = (0, 0, \ldots, 0)$M tRe Rt rR gReR λ in nFF iRgao ri r arR suoc Rgan$\mathbf{P}_0$ ug $\mathbf{P}_1$ cF na nuainlo RiarRt $P_1^1 \geq r\left(L_0 - P_0^1\right)$ t $P_0^m \neq 0$ ncR ug $m \neq 1$M

In less formal terms, Lemma 9 states that the borrower must always have a way of "paying off" his loan and regaining the right to trade freely with all M banks. If the borrower does not possess this right, bank 1 will not have the incentive to make the socially efficient loan at the initial date. Moreover, the payments that correspond to the borrower "paying off" his loan must be such that (with a caveat[23]) the loan is actually repaid — i.e. bank 1 receives a payment that is r times the original loan.

Proof of Lemma 9: Suppose otherwise. That is, there exists an $\hat{L}_0 > 0$ and a $\hat{\lambda} < 1$ such that for all $(\mathbf{P}_0, \mathbf{P}_1)$ with

$$\sum_{m \neq 1} P_1^m > r\left(\hat{L}_0 - P_0^1\right) - P_1^1$$

the inequality (22) does not hold. Fix $W_0 = \hat{W}_0$a rbitrarily, and define the set $\hat{X} \subset \mathcal{X}$ to be set of all parameter values x with wealth level $\hat{}W_0$ and such that $L_0^*(x) = \hat{L}_0$. Observe that $(r - R_1) R_0$is constant over the subset $\hat{X}$.

Note that for the constrained efficient outcome to be an equilibrium at x, bank 1 must use a lending policy with $L_0^1 = \hat{L}_0 = L_0^*\left(\hat{X}\right)$. For any $\varepsilon, \delta > 0$, consider the deviation by bank 1 to a lending policy $\tilde{\mathcal{L}}^1$ with $\tilde{L}_0^1 = \hat{L}_0 - \varepsilon$ and

$$\tilde{l}_2^1(\mathbf{P}_1) = \begin{cases} \left(W_0 + \tilde{L}_0^1\right)(R_1 + \delta) R_0 & \text{if } P_1^1 = \left(W_0 + \tilde{L}_0^1\right) R_0 \\ 0 & \text{otherwise} \end{cases}$$

i.e. at date 1, bank 1 offers to pay a return of $R_1 + \delta$ if the borrower deposit all his wealth. The proof will consist of showing that the lending policy $\tilde{\mathcal{L}}^1$ is a profitable deviation for bank 1.

First, assume that given the policy $\tilde{\mathcal{L}}^1$ that the borrower's best response is $\mathbf{P}_0 = \mathbf{0}$ and $\mathbf{P}_1 = \left(\left(W_0 + \tilde{L}_0^1\right) R_0, 0, \ldots, 0\right)$, i.e. the borrower deposits all his date 1 wealth with bank 1. Then at any $x \in \hat{X}$ bank 1 gets

$$-\tilde{L}_0^1 r^2 + (r - R_1 - \delta)\left(W_0 + \tilde{L}_0^1\right) R_0 = \varepsilon\left(r^2 - (r - R_1) R_0\right) - \delta R_0\left(W_0 + \tilde{L}_0^1\right) \quad (23)$$

where we are using the fact that at any $x \in \hat{X}$ we know $-\hat{L}_0 r^2 + (r - R_1)\left(W_0 + \hat{L}_0\right) R_0 = 0$.

It is sufficient to show that the deviation to $\tilde{\mathcal{L}}^1$ is profitable for some $x \in \hat{X}$ (since we require the rule $\mathcal{B}$ to be robust) and some values $\varepsilon, \delta > 0$.W e select values of x, ε, δ as

[23] The caveat reflects the fact that although we have ruled out message games *per se*, there remains the possibility of the borrower using payments to banks $m \neq 1$ as messages. We have not been able to rule out the case where the borrower has the possibility of making a non-zero payment to a bank $m \neq 1$ to signal that bank 1 made an inefficient loan.

follows. First, choose $\lambda \geq \hat{\lambda}$ such that $(1-\lambda) r^2 < (r - R_1) R_0$for all $x \in \hat{X}$. Choose $\varepsilon \in \left[0, \hat{L}_0\right]$ such that for all $x \in X$

$$(1-\lambda) r^2 < \frac{\varepsilon \left(r^2 - (r - R_1) R_0\right)}{W_0 + \hat{L}_0 - \varepsilon} \tag{24}$$

Such a choice is always possible since as $\varepsilon \rightarrow L_0$ the RHS tends to $\frac{\hat{L}_0}{W_0}\left(r^2 - (r - R_1) R_0\right) = (r - R_1) R_0$. Let $\hat{x} \in \hat{X}$ be such that

$$\lambda r < R_1 \tag{25}$$

(clearly such a choice is always possible by setting R_1hig h enough). Finally, choose δ so that

$$(1-\lambda) r^2 \quad < \quad \delta R_0 \tag{26}$$

$$\delta R_0 \quad < \quad \varepsilon \frac{\left(r^2 - (r - R_1) R_0\right)}{W_0 + L_0 - \varepsilon} \tag{27}$$

where such a choice is possible by inequality (24).

The right-hand side of (23) is strictly positive given inequality (27), and so bank 1's deviation from $\mathcal{L}^1$ to $\tilde{\mathcal{L}}^1$ is strictly profitable st e iR arua arR tt R t tRngn o R sni aE ig u ri n u aR 1 R u ar iar ug 1, i.e., $\mathbf{P}_0 = \mathbf{0}$ and $\mathbf{P}_1 = \left(\left(W_0 + \tilde{L}_0^1\right) R_0, 0, \ldots, 0\right)$. Note that the borrower's final consumption under this choice of $\mathbf{P}_0, \mathbf{P}_1$g iven $\tilde{\mathcal{L}}^1$ is

$$\left(W_0 + \tilde{L}_0^1\right) R_0 \left(R_1 + \delta\right) = \left(W_0 + \tilde{L}_0^1\right) R_0 R_1 + \left(W_0 + \tilde{L}_0^1\right) R_0 \delta$$

Next, consider any other choice of $\mathbf{P}_0, \mathbf{P}_1$. Necessarily it must feature either $P_0^m > 0$ for some m, and/or $P_1^1 < \left(W_0 + \tilde{L}_0^1\right) R_0$. However, if $P_0^m > 0$ for some m then the payment $P_1^1 = \left(W_0 + \tilde{L}_0^1\right) R_0$ is not feasible,[24] and so either way we have $P_1^1 < \left(W_0 + \tilde{L}_0^1\right) R_0$. The borrower's final consumption is then at most

$$\left(\left(W_0 + \tilde{L}_0^1 - \sum_m P_0^m\right) R_0 - P_1^1\right) R_1 + \sum_{m \neq 1} \left(\beta_1^m r - R_1\right) P_1^m$$

For the case

$$\sum_{m \neq 1} P_1^m > r \left(\tilde{L}_0^1 - P_0^1\right) - P_1^1 \tag{28}$$

[24]This is true provided that no bank $m \neq 1$ offers a lending policy $\mathcal{L}^m$ in which deposits earn a rate of return strictly higher than r. Such a lending policy would generate strictly negative profits. Ruling out bank $m \neq 1$ strategies that yield negative out-of-equilibrium profits is consistent with our assumption that only bank 1 has surplus funds available.

then by supposition the borrower's consumption is less than

$$\left(\left(W_0 + \tilde{L}_0^1 - \sum_m P_0^m\right) R_0 - P_1^1\right) R_1 + (\lambda r - R_1) \sum_{m \neq 1} P_1^m$$

By inequality (25) this expression must be strictly less than $\left(W_0 + \tilde{L}_0^1\right) R_0 R_1$. On the other hand, if (28) does not hold then the borrower's consumption is certainly less than

$$r\left(r\left(\tilde{L}_0^1 - P_0^1\right) - P_1^1\right) + R_1\left(W_1 - P_1^1 - \left(r\left(\tilde{L}_0^1 - P_0^1\right) - P_1^1\right)\right)$$

where $W_1 = \left(W_0 + \tilde{L}_0^1 - \sum_m P_0^m\right) R_0$ is the borrower's date 1 wealth. Since

$$\left(W_0 + \tilde{L}_0^1\right) R_0 \geq W_1 \geq r\left(\tilde{L}_0^1 - P_0^1\right) - P_1^1$$

the borrower's consumption under the deviation is less than his consumption from sticking to $\mathbf{P}_0, \mathbf{P}_1$b y at least

$$(R_1 - r + \delta)\left(r\left(\tilde{L}_0^1 - P_0^1\right) - P_1^1\right)$$

Conditions (25) and (26) imply that $R_1 - r + \delta > 0$, again establishing that the borrower will indeed stick to repayments $\mathbf{P}_0, \mathbf{P}_1$. Thus we can conclude that $\mathbf{P}_0 = \mathbf{0}$ and $\mathbf{P}_1 = \left(\left(W_0 + \tilde{L}_0^1\right) R_0, 0, \ldots, 0\right)$ is indeed a strict best response for the borrower, completing the proof. **QED**

To summarize, Lemmas 8 and 9 together establish that all optimal robust enforcement rules must share the following two features: (1) A borrower in default is punished by restricting his access to saving within the financial system; and (2) A borrower must always have the ability to repay his debt so as to have unlimited access to savings.

In practice, these are characteristics of virtually all laws that govern debt, default and bankruptcy. Of the rules formally defined in this paper — the KL-K rule $\mathcal{B}_{KLK}$, the exclusivity rule $\mathcal{B}_{excl}$, the debt-default rule $\mathcal{B}_{DD}$, and the credit prohibition rule $\mathcal{B}_{CP}$ — only the debt-default rule $\mathcal{B}_{DD}$ possesses both these properties.

Finally, we should note that if we were to make the additional restriction that the fraction of the transfer between the borrower and a bank $m \neq 1$ (i.e. NP_1^m) that is seized by the exclusion rule must be constant with respect to the transfer NP_1^m (though not, of course, with respect to the transfers to bank 1, NP_0^1 and NP_1^m), then we could establish the even stronger result that the debt-default rule is the go robust rule to achieve the efficient outcome for all $x \in \mathcal{X}$.

7 Conclusion

We have studied optimal enforcement in credit markets where enforcement is limited to regulating transfers between borrowers and banks. Our results have bearing for both the burgeoning theoretical literature on risk sharing with limited commitment, as well as for understanding credit in environments with no collateral.

In the latter regard, our results re-enforce and extend some of the results in the literature (e.g. Bulow and Rogoff, 1989b). Credit denial in and of itself does not serve as a sufficient threat to motivate borrowers to repay loans. Some form of enforcement is also necessary to sustain lending. This result is relevant for understanding lending and regulation in developing countries, where collateral is likely to be scarce.

The literature on risk sharing with limited commitment has, for the most part, neglected studying optimal enforcement and tying this to observed institutional arrangements. This is our main contribution. We have shown that optimal enforcement will resemble laws governing default on debt and bankruptcy. We have also shown that the important aspect of enforcement is restricting fraudulent conveyance – which in our model is preventing a borrower in default from saving with the financial sector. This result re-enforces our statement that restricting the available credit to a defaulted borrower is not the central threat of enforcement in these environments.

Our results on lending where banks have limited commitment is also of independent interest for theories of banking. The multi-period model of Section 5 showed that if a bank is able to commit to even a single period of savings, via some pledgeable assets for example, it can increase lending activities. Moreover, in a repeated lending situation, longer relationships lead to larger loans. In a sense, the bank is able to leverage a limited amount of commitment to sustain lending. For developing economies this limitation is an important constraint and may be what limits the size of the banking sector. This leads us to speculate that the development of the banking sector has much to do with limited pledgeable assets. We plan on investigating this issue in future research.

References

Philippe Aghion and Patrick Bolton. An incomplete contracts approach to financial contracting. ReiR l f gcif a FiRn, 59(3):473–94, July 1992.

Rui Albuquerque and Hugo Hopenhayn. Optimal lending contracts and firm dynamics. ti g u sRt, 2000.

Fernando Alvarez and Urban J Jermann. Efficiency, equilibrium, and asset pricing with risk of default. f gcRatif u, 68(4), July 2000.

Fernando Alvarez and Urban J Jermann. Quantitative asset pricing implications of endogenous solvency constraints. ReiR l igugfiu a FiRn, 14(4):1117–1151, Winter 2001.

Douglas G Baird. kr RR c Rgal u gtFsafo . Foundation Press, 1993.

Patrick Bolton and David S Scharfstein. A theory of predation based on agency problems in financial contracting. mcRtif ug f gcif ReiR , 80(1):93–106, March 1990.

Jeremy Bulow and Kenneth Rogoff. A constant recontracting model of sovereign debt. Ftgu l iaif u f gco , 97(1):155–78, February 1989a.

Jeremy Bulow and Kenneth Rogoff. Sovereign debt: Is to forgive to forget? mcRtif ug f gcif ReiR , 79(1):43–50, March 1989b.

V V Chari and Patrick J Kehoe.Susta inable plans and debt. Ftgu l f gcif kr Rto , 61(2):230–61, December 1993a.

V V Chari and Patrick J Kehoe. Sustainable plans and mutual default. ReiR l f gcif a FiRn, 60(1):175–195, January 1993b.

Peter M DeMarzo and Michael J Fishman. Optimal long-term financial contracting with privately observed cash flows. ti g u sRt, 2000.

Douglas W Diamond. Reputation acquisition in debt markets. Ftgu l iaif u f gco , 97(4):828–862, August 1989.

Douglas Gale and Martin Hellwig. Incentive-compatible debt contracts: The one-period problem. ReiR l f gcif a FiRn, LII:647–663, 1985.

Maitreesh Ghatak and Timothy Guinnane. The economics of leinding with joint liability: A review of theory and practice. Ftgu l ReRs cRg a f gcif n, 60(1):195–228, 1999.

Denis Gromb. Renegotiation in debt contracts. ti g u sRt, 1999.

Oliver Hart and John Moore. A theory of debt based on the inalienability of human capital. F utaRto Ftgu l f gcif n, 109(4):841–79, November 1994.

Oliver Hart and John Moore. Default and renegotiation: A dynamic model of debt. F utaRto Ftgu l f gcif n, 113(1):1–41, February 1998.

Robert D Innes. Limited liability and incentive contracting with ex-ante action choices. Ftgu l f gcif kr Rto , 52(1):45–67, October 1990.

Timothy J Kehoe and David K Levine. Debt-constrained asset markets. ReiR l f gcif a FiRn, 60:865–888, 1993.

Kenneth M Kletzer and Brian D Wright. Sovereign debt as intertemporal barter. mcRtE ifug f gcif ReiR , 90(3):621–639, June 2000.

Narayana Kocherlakota. Implications of efficient risk sharing without commitment. ReiR l f gcif a FiRn, 63:595–609, 1996.

Dirk Krueger and Harald Uhlig. Competitive risk sharing contracts with one-sided commitment. ti g u sRt, 2000.

Jonathan Morduch. The microfinance promise. Ftgu l f gcif iaRt uaRtR, 37(4), December 1999.

Marguerite S Robinson. kr Ri ft g ugfR Re F ai g F naiguR g ugf Rl t arRs t . World Bank, 2001.

Jonathan Thomas and Tim Worrall. Foreign direct investment and the risk of expropriation. ReiR l f gcif a FiRn, 61:61–108, 1994.

Robert M Townsend. Optimal contracts and competitive markets with costly state verification. Ftgu l f gcif kr Rto , 21:265 – 293, 1979.

Michelle White. Why it pays to file for bankruptcy: A critical look at incentives under U.S. bankruptcy laws and a proposal for change. g ieRtnia l r if u u ReiR , 65(3):685–732, Summer 1998.

A Mathematical proofs

A.1 Proof of Proposition 1

To establish the result, we will first compute the solution to the problem of planning problem of maximizing the objective (3) subject to the constraints (4), (5) and (6). In the main text we showed this problem can be written recursively as

$$V_t^M(W,v) = \max_{P\in[0,W+L],L\in[0,v]} -(L-P) + \frac{1}{r}V_{t+1}^M(R_t(W+L-P), R_t(v-(L-P))) \tag{12}$$

Recall that $V_t^M(W,v)$ is the maximal present value of profits from date t onwards attainable by the coalition of all M banks.

Trivially in the final period T the bank's value function is just $V_T^M(W,v) = -v$. From the linearity of the problem and the form taken by $V_T^M(W,v)$, we guess (and verify below) that for all $t < T$ the value function $V_t^M(W,v)$ is linear in W and v with coefficients that sum to 1,

$$V_t^M(W,v) = \alpha_t W - (1-\alpha_t)v \tag{29}$$

Thus $\alpha_T = 0$, and

$$V_t^M(W,v) = \max_{P\in[0,W+L],L\in[0,v]} (L-P)(\rho_t - 1) + (W\alpha_{t+1} - v(1-\alpha_{t+1}))\rho_t$$

In any paymentp eriod t, the bank's rate of return is higher than the borrower's. In these periods the borrower is best off transferring all his resources to the bank, so that the bank can invest them at the higher rate $r > R_t$. Formally, since $\rho_t < 1$ we must have $P - L = W$. Without loss, we can set $L = 0$ and $P = W$, i.e. no new loan, and borrower transfers all his wealth to the bank. In this case,

$$V_t^M(W,v) = W(1-(1-\alpha_{t+1})\rho_t) - v(1-\alpha_{t+1})\rho_t$$

so that

$$\alpha_t = 1-(1-\alpha_{t+1})\rho_t \tag{30}$$

Note also that $W' = 0$ and $v' = R_t(v+W)$ — the borrower now has no wealth, but the amount that the bank must transfer to the borrower has increased from v to $R_t(v+W)$.

In any investmentp eriod t, the bank's rate of return is lower than the borrower's, so there is potentially scope for lending. Since $R_t \geq r$, then $\rho_t \geq 1$ and so $P = 0$ and $L = v$ is optimal. In this case,

$$V_t^M(W,v) = W\alpha_{t+1}\rho_t - v(1-\alpha_{t+1}\rho_t)$$

so that

$$\alpha_t = \alpha_{t+1}\rho_t \tag{31}$$

Note also that $W' = R_t(v + W)$ and $v' = 0$ — the bank no longer "owes" the borrower anything, and will make no further transfers until after the borrower has made some payments to the bank

We now turn to a description of the actual payments. First note that if t is an investment period that is followed by another investment period, since $v_{t+1} = 0$ we know that no payments are made in period $t+1$. Likewise, if t is a payment period that is followed by a payment period, since $W_{t+1} = 0$ we again know that no payments are made in period $t+1$. Thus the funds are transferred between the bank and borrower only in some subset of periods $t_0 = 0, t_1, t_2, \ldots, t_\tau$ where t_{i+1} is an investment (respectively, payment) period if and only if t_i and $t_{i+1}-1$ are payment (respectively, investment) periods. Note that since T is an investment period it follows that t_τ must be an investment period.

The resulting payments are as follows. At $t_0 = 0$ the bank makes an initial loan of $L_0 = v_0$. In the first payment period t_1, the borrower repays the bank all his wealth, $P_{t_1} = W_{t_1} = (W_0 + L_0)\prod_{s=t_0}^{t_1-1} R_s$. In the next investment period, t_2, the bank makes a new loan of $L_{t_2} = v_{t_2} = P_{t_1}\prod_{s=t_1}^{t_2-1} R_s$. The cycle then continues until at t_τ the bank makes the last "loan", $L_{t_\tau} = P_{t_{\tau-1}}\prod_{s=t_{\tau-1}}^{t_\tau-1} R_s$.

Finally, the maximal loan size and final borrower consumption can easily be determined as follows. For the banks to be collectively break-even we must certainly have

$$V_0(W_0, v_0) = \alpha_0 W_0 - (1 - \alpha_0)v_0 \geq 0$$

and so

$$v_0 = L_0 \leq \frac{W_0\alpha_0}{1 - \alpha_0}$$

The borrower's maximal final consumption is then

$$\left(W_0 + \frac{W_0\alpha_0}{1 - \alpha_0}\right)\prod_{t=0}^{T-1} R_t = \frac{W_0}{1 - \alpha_0}\prod_{t=0}^{T-1} R_t$$

A.2 Proof of Lemma 1

Suppose to the contrary that $\sum_m V_t^m > 0$ for some t. Denote the equilibrium loans and payments by $\{L_t^{*m}\}$ and $\{P_t^{*m}\}$. Let t_0 be the last date at which this is true. Also, let t_1 be the first date after t_0 such that the net payment to the borrower is weakly positive, i.e. $L_t \geq P_t$. Note that t_1 is well-defined, since at date T the borrower will not make any payments to the banks ($P_T = 0$), so that certainly $\sum_m V_T^m \leq 0$. Thus $t_0 < t_1 \leq T$.

Let m' be a bank whose future profits at date t_0 are minimal, i.e. $V_{t_0}^{m'} \leq V_{t_0}^{m}$ for all $m \in M$. Then consider the following deviation for bank m'. Define a lending policy $\tilde{\mathcal{L}}^{m'}$ as follows. For dates $t < t_0$, let $\tilde{\mathcal{L}}^{m'}$ be as before. For dates $t \geq t_0$o ther than t_1, let

$$\tilde{l}_t^{m'}(\mathbf{P}_0, \ldots, \mathbf{P}_{t-1}) = \max\left\{0, \sum_m (L_t^{*m} - P_t^{*m})\right\}$$

provided $P_s^{m\prime} = P_s^{*m'}$ for $0 \leq s < t_0$ and $P_s^{m'} = \max\{0, \sum_m (P_s^{*m} - L_s^{*m})\}$ for $t_0 \leq s < t$. Otherwise let $\tilde{l}_t^{m'}(\mathbf{P}_0, \ldots, \mathbf{P}_{t-1}) = 0$. Finally, let

$$\tilde{l}_{t_1}^{m'}(\mathbf{P}_0, \ldots, \mathbf{P}_{t_1-1}) = \max\left\{0, \sum_m \left(L_{t_1}^{*m} - P_{t_1}^{*m}\right)\right\} + \varepsilon$$

provided $P_s^{m'} = P_s^{*m'}$ for $0 \leq s < t_0$ and $P_s^{m'} = \max\{0, \sum_m (P_s^{*m} - L_s^{*m})\}$ for $t_0 \leq s < t_1$. Otherwise let $\tilde{l}_{t_1}^{m'}(\mathbf{P}_0, \ldots, \mathbf{P}_{t-1}) = 0$.

Given this deviation by bank m', the borrower can now obtain an improvement of ε in his utility if he pays $\max\{0, \sum_m (P_s^{*m} - L_s^{*m})\}$ to bank m' at all dates from t_0 onwards. So he will certainly deviate to some alternative sequence of payments $\left\{\tilde{P}_t^m\right\}$. Moreover, whatever his deviation is it must entail $\tilde{P}_t^m = P_t^m$ for all $t < t_0$ and

$$\tilde{P}_t^{m\prime} = \max\left\{0, \sum_m (P_s^{*m} - L_s^{*m})\right\}$$

for $t_0 \leq t < t_1$.

Since

$$\sum_{s=t_1+1}^{T} \sum_m r^{-s}(P_s^{*m} - L_s^{*m}) \leq 0$$

The date 0 profits of bank m' under the deviation are of the form

$$-\varepsilon r^{-t_1} + \sum_{s=0}^{t_0-1} r^{-s}\left(P_s^{*m'} - L_s^{*m\prime}\right) + \sum_{s=t_0}^{\tau} \sum_m r^{-s}(P_s^{*m} - L_s^{*m})$$

for some $\tau \geq t_1$ (i.e. the borrower does not deviate before date t_1). This expression rewrites to

$$\begin{aligned} & -\varepsilon r^{-t_1} + \sum_{s=0}^{t_0-1} r^{-s}\left(P_s^{*m'} - L_s^{*m\prime}\right) + r^{-t_0}\sum_m V_{t_0}^m - r^{-(\tau+1)}\sum_m V_{\tau+1}^m \\ \geq \; & -\varepsilon r^{-t_1} + \sum_{s=0}^{t_0-1} r^{-s}\left(P_s^{*m'} - L_s^{*m\prime}\right) + r^{-t_0}\sum_m V_{t_0}^m . \end{aligned}$$

where the inequality follows from $\sum_m V_{\tau+1}^m \leq 0$ since $\tau + 1 > t_0$. Since the bank m' was chosen such that its date t_0 future profits $V_{t_0}^{m'}$ were minimal, it follows that the lending policy $\tilde{\mathcal{L}}^{m'}$ results in strict increase in date 0 profits for ε sufficiently small.

A.3 Proof of Lemma 2

Consider any date t, with the borrower's wealth level given by W and such that the present value (at the borrower's interest rate) of future payments to the borrower is v. When $M \geq 2$, Lemma 1 implies that the present value of future bank profits (at rate r) must be non-positive at all future dates $t+1, \ldots, T$. So an upper bound on the combined present value is given by the solution $V_t^M(W, v)$ to the maximization problem described in Section 2 consisting of maximizing (3) subject to the constraints (4), (5) and (6), and to the additional constraint that at all $t' > t$

$$-\frac{1}{r^{T-t'}} \sum_{s=t'}^{T} r^{T-s} (L_s - P_s) \leq 0$$

As before, we can write this problem recursively as

$$V_t^M(W, v) = \max_{P \in [0, W+L], L \in [0, v]} -(L - P) + \frac{1}{r} V_{t+1}^M (R_t (W + L - P), R_t (v - (L - P)))$$

subject to the additional constraint that

$$V_{t+1}^M (R_t (W + L - P), R_t (v - (L - P))) \leq 0$$

As before, $V_T^M(W, v) = -v$. We will again guess and verify that $V_t^M(W, v)$is linear in W and v, and of the form $V_t^M(W, v) = \gamma_t W - (1 - \gamma_t) v$ with $\gamma_t < 1$. Note that $\gamma_T = 0$. Substituting this guess into the problem, we have

$$V_t^M(W, v) = \max_{P \in [0, W], L \in [0, v]} (\rho_t - 1)(L - P) + \rho_t (\gamma_{t+1} W - (1 - \gamma_{t+1}) v) \tag{32}$$

subject to

$$L - P \leq -\gamma_{t+1} W + (1 - \gamma_{t+1}) v \tag{33}$$

In payment periods, $\rho_t < 1$ and we want to set $L - P$ as low as possible. So set $L = 0$, $P = W$, and the additional constraint (33) does not bind since $\gamma_{t+1} < 1$. Thus for t a payment period,

$$\begin{aligned} V_t^M(W, v) &= -(\rho_t - 1) W + \rho_t (\gamma_{t+1} W - (1 - \gamma_{t+1}) v) \\ &= W (1 - (1 - \gamma_{t+1}) \rho_t) - v (1 - \gamma_{t+1}) \rho_t \end{aligned}$$

and so $\gamma_t = (1 - (1 - \gamma_{t+1}) \rho_t) < 1$.

In investment periods, $\rho_t \geq 1$ and so it is optimal to set $L - P$ as high as possible. This time the constraint (33) will bind, so we set $L - P = -\gamma_{t+1}W + (1 - \gamma_{t+1})\, v$. Substituting into the objective (32) then gives

$$V_t^M(W, v) = \gamma_{t+1}W - (1 - \gamma_{t+1})\, v$$

so that for any investment period we have $\gamma_t = \gamma_{t+1} < 1$.

Our guess as to the form of $V_t^M(W, v)$ is thus verified, and we have

$$V_0^M(W, v) = \gamma_0 W_0 - (1 - \gamma_0)\, v$$

Since certainly $V_0^M(W, v) \geq 0$ if the banks are to collectively break even, we must have $v \leq W_0\gamma_0 / (1 - \gamma_0)$. Thus the borrower's final period consumption can be no more than

$$\prod_{t=0}^{T-1} R_t \left(W_0 + \frac{W_0\gamma_0}{(1 - \gamma_0)} \right) = \frac{W_0}{1 - \gamma_0} \prod_{t=0}^{T-1} R_t$$

To complete the proof it is sufficient to establish that $\frac{1}{1-\gamma_t} \prod_{s=t}^{T-1} R_s = \prod_{s=t}^{T-1} \max\{r, R_s\}$ for all t. We proceed inductively. Since $\gamma_T = 0$ the relation is clearly satisfied for $t = T$. Let $t < T$ be a payment period, i.e. $\rho_t < 1$. Then $\frac{1}{1-\gamma_t} = \frac{1}{1-\gamma_{t+1}} \frac{1}{\rho_t}$ so that applying the inductive step we have

$$\frac{1}{1 - \gamma_t} \prod_{s=t}^{T-1} R_s = \frac{R_t}{\rho_t} \frac{1}{1 - \gamma_{t+1}} \prod_{s=t+1}^{T-1} R_s = r \prod_{s=t+1}^{T-1} \max\{r, R_s\} = \prod_{s=t}^{T-1} \max\{r, R_s\}.$$

Finally, let $t < T$ be an investment period, i.e. $\rho_t \geq 1$. Then $\gamma_t = \gamma_{t+1}$ and so again applying the inductive step,

$$\frac{1}{1 - \gamma_t} \prod_{s=t}^{T-1} R_s = \frac{R_t}{1 - \gamma_{t+1}} \prod_{s=t+1}^{T-1} R_s = R_t \prod_{s=t+1}^{T-1} \max\{r, R_s\} = \prod_{s=t}^{T-1} \max\{r, R_s\},$$

completing the proof.

A.4 Proof of Lemma 3

We will establish that the following lending policies and borrower payments constitute an equilibrium, given the enforcement rule $\mathcal{B}_{KLK}$: (a) Bank 1's lending policy $\mathcal{L}^1$ is $l_t^1 \equiv L_t^*$, (b) Every other bank $m \neq 1$ offers the lending policy $\mathcal{L}^m$, $l_t^m(\mathbf{P}_0, \ldots, \mathbf{P}_{t-1}) \equiv rP_{t-1}^m$ (i.e. take deposits at rate r), (c) The borrower repays $P_t^1 = P_t^*$ and $P_t^m = 0$. By construction, in this equilibrium all banks make zero profits and the borrower's final consumption is $W_0 \prod_{t=0}^{T-1} \max\{r, R_t\}$.

To show that we have actually described an equilibrium, start by noting that since the payments P_t^* were defined to be feasible and incentive compatible given loans L_t^* and the threat of full exclusion, the borrower's payment strategy is certainly a best response given lending policies $\mathcal{L}^1$a nd $\mathcal{L}^m$ and the enforcement rule $\mathcal{B}_{KLK}$.

Next, we claim that $\mathcal{L}^1$ is a best response to $\mathcal{L}^m$. For suppose to the contrary that there exists $\hat{\mathcal{L}}^1$ delivering strictly positive profits to bank 1. Let $\hat{L}_t^1$ and $\hat{P}_t^1$ be the equilibrium loan payments under this deviation. The loan payments $\hat{L}_t^1$ must differ from the original loan payments L_t^* at at least one date. Let τ be the first such date. If $\hat{L}_\tau^1 < L_\tau^*$ then at the first payment period to follow τ, the borrower will have insufficient resources to repay the bank P_t^*. But then date τ is the last period in which any transfer occurs between the borrower and the bank. However, this then implies that the borrower would not have made the payment $P_{\tau'+1}^*$ where τ' is the last investment period prior to τ, since be definition $L_t^* = \prod_{s=\tau'+1}^{\tau-1} R_s P_{\tau'+1}^*$. Since any payment less than $P_{\tau'+1}^*$ leads to full exclusion, the borrower will simply pay 0 at this date. On the other hand, if $\hat{L}_\tau^1 > L_\tau^*$ then the borrower's incentive constraint is now violated at the first payment period to follow τ. In either case, bank 1 is left with negative profits.

Finally, we claim that $\mathcal{L}^m$ is a best response to $\mathcal{L}^1$ for every $m \neq 1$. For under any strictly profitable deviation, bank m's deviation must involve him making a positive payment to the borrower before receiving any payments from the borrower, since the borrower must pay all his wealth to bank 1 in each payment period.[25] By the same argument as above, the borrower now prefers to default on everyone over repaying both banks 1 and m. Moreover, if he just defaults on bank 1, his payment to bank m is seized. So regardless of whether he defaults on both banks, just on bank m, or just on bank 1, bank m will receive no payment after the initial loan in period τ. Thus his deviation cannot have been profitable.

A.5 Proof of Lemma 6

The proof is exactly analogous to the proof of Proposition 2. The only non-trivial issue to show that the analogue to Lemma 1 continues to hold. First note that a straightforward inductive argument implies that under rule $\mathcal{B}_{CP}$ a borrower is indebted to at most a single bank at any date t. Lemma 1 established that the banks cannot be collectively making strictly positive future profits at any date t. The key step in the proof was to consider a deviation in which one of the banks (bank m' say) deviated to a lending policy in which u the payments between the borrower and the M banks from a date t_0 onwards are made through bank m'. There are two cases to consider.

[25]Technically there is also the possibility that the borrower could make a payment to the deviating bank m in an investmentp eriod. But then bank m would have to pay an interest rate of $R_t > r$ on this deposit to avoid the borrower defaulting at the next payment period.

First, suppose that the borrower is not indebted to a bank with minimal future profits at date t_0. In this case, under the deviation

$$\tilde{D}_t^{m'} = D_{t_0}^{m'} r^{t-t_0} - \left(\sum_{m \in M} V_{t_0}^m \right) r^{t-t_0} + \sum_{m \in M} V_t^m$$

where $\tilde{D}_t^{m'}$ is the debt level under the deviation. Since by the deviation date t_0 was chosen to be the last date at which aggregate future bank profits are positive, and $D_{t_0}^{m'}$, it follows that $\tilde{D}_t^{m'} < 0$. The enforcement rule $\mathcal{B}_{CP}$ can then have no impact on this deviation, since the borrower is never indebted to the deviating bank m'.

Second, suppose that there is a unique bank with minimal future profits at date t_0, and the borrower is indebted to this bank. Denote this bank m'. Consequently the borrower cannot be indebted to any bank $m \neq m'$. We can then select bank m' as the deviating bank. The enforcement rule $\mathcal{B}_{CP}$ has not impact on the payments between the borrower and bank m' under the deviation The rest of the proof is as before.

A.6 Proof of Proposition 3

Let $V_t^M(W, v)$ be the maximum present value of payments achievable by the coalition of M banks facing a borrower with wealth W, subject to the constraint that the present value of transfers to the borrower v (at the borrower's interest rate). Thus $V_t^M(W, v)$ is the solution to the maximization problem described in Section 2 consisting of maximizing (3) subject to the constraints (4), (5) and (6), along with the additional constraint (16) that the banks' future profits are non-negative at all dates

As before, we can write this problem recursively as

$$V_t^M(W, v) = \max_{P \in [0, W+L], L \in [0, v]} -(L - P) + \frac{1}{r} V_{t+1}^M (R_t (W + L - P), R_t (v - (L - P)))$$

subject to the additional constraint

$$V_{t+1}^M (R_t (W + L - P), R_t (v - (L - P))) \geq 0$$

for all $t \leq T - 2$. As before, we will guess and verify that $V_t^M(W, v)$ is linear in W and v, and of the form $V_t^M(W, v) = \gamma_t W - (1 - \gamma_t) v$ with $\gamma_t < 1$.F rom Proposition 1, we know

$$V_{T-1}^M(W, v) = \gamma_{T-1} W - \left(1 - \gamma_{T-1}\right) v$$

where $\gamma_{T-1} = \rho_{T-1}$.

Substituting the guess $V_t^M(W, v) = \gamma_t W - (1 - \gamma_t) v$ into the problem, we have

$$V_t^M(W, v) = \max_{P \in [0, W+L], L \in [0, v]} (\rho_t - 1)(L - P) + \rho_t \left(\gamma_{t+1} W - \left(1 - \gamma_{t+1}\right) v\right) \qquad (34)$$

subject to

$$L - P \geq -\gamma_{t+1} W + (1 - \gamma_{t+1}) v \tag{35}$$

In investment periods, $\rho_t \geq 1$ and so it is optimal to set $L - P$ as high as possible. In this case the constraint (35) is not binding. Thus we set $L = v$, $P = 0$. Substituting into the objective (34) then gives

$$V_t^M(W, v) = \rho_t \gamma_{t+1} W - (1 - \rho_t \gamma_{t+1}) v.$$

So in any investment period t we have $\gamma_t = \rho_t \gamma_{t+1}$

In payment periods, $\rho_t < 1$ and we want to set $L - P$ as low as possible. This time the constraint (35) is binding. Substituting (35) at equality into the objective (34) then gives

$$V_t^M(W, v) = \gamma_{t+1} W - (1 - \gamma_{t+1}) v.$$

So in any payment period $\gamma_t = \gamma_{t+1}$. Without loss, we can assume this is achieved by $L = 0$ and $P = \gamma_{t+1} W - (1 - \gamma_{t+1}) v$.

Our guess as to the form of $V_t^M(W, v)$ is thus verified, and we have

$$V_0^M(W, v) = \gamma_0 W_0 - (1 - \gamma_0) v$$

Since certainly $V_0^M(W_0, v) \geq 0$ if the banks are to collectively break even, we must have $v \leq W_0 \gamma_0 / (1 - \gamma_0)$. Thus the borrower's final period consumption can be no more than

$$\prod_{t=0}^{T-1} R_t \left(W_0 + \frac{W_0 \gamma_0}{1 - \gamma_0} \right) = \frac{W_0}{1 - \gamma_0} \prod_{t=0}^{T-1} R_t$$

where $\gamma_0 = (1 - \rho_{T-1}) \prod_{t=0}^{T-2} \max\{1, \rho_t\}$.

Finally, we characterize a set of payments that lead to this upper bound. Let date t be an investment period and date τ the following payment period. Assume that at date t, $L_t = W_t \gamma_t / (1 - \gamma_t)$. At all dates $s = t + 1, \ldots, \tau - 1$ we have $L_t = P_t = v = 0$. So at date τ we have $v = 0$, and so $P_\tau = \gamma_\tau W_\tau$. Note that $W_\tau = \prod_{s=t}^{\tau-1} R_s W_t / (1 - \gamma_t)$ and $\gamma_t = \gamma_\tau \prod_{s=t}^{\tau-1} \rho_s$. Then

$$P_\tau = \gamma_\tau \prod_{s=t}^{\tau-1} R_s W_t / (1 - \gamma_t) = \frac{\gamma_t}{\prod_{s=t}^{\tau-1} \rho_s} \prod_{s=t}^{\tau-1} R_s W_t / (1 - \gamma_t) = r^{\tau - t} L_t$$

i.e. the borrower repays the loan in full.

Next, let $\hat{\tau}$ be the first investment period following the payment period τ. At all dates $s = t + 1, \ldots, \tau - 1$ we have $L_t = P_t$. At date $\tau + 1$ we have $v = R_\tau \gamma_\tau W_\tau$, so at date $\hat{\tau}$ we have $v = \prod_{s=\tau}^{\hat{\tau}-1} R_s \gamma_\tau W_\tau$. The borrower's wealth at date $\hat{\tau}$ is $W_{\hat{\tau}} = (W_\tau - \gamma_\tau W_\tau) \prod_{s=\tau}^{\hat{\tau}-1} R_s$. So the loan size at date $\hat{\tau}$ is

$$v = \frac{\gamma_\tau}{1 - \gamma_\tau} W_{\hat{\tau}} = \frac{\gamma_{\hat{\tau}}}{1 - \gamma_{\hat{\tau}}} W_{\hat{\tau}}$$

A.7 Proof of Proposition 4

A.7.1 Proof outline

We proceed as follows:

1. We characterize the payoffs of an equilibrium of the subgame starting at date $T-1$ in which the borrower is indebted to only one of the banks (without loss, bank 1).
2. Proceeding inductively, we then characterize the payoffs and payments of an equilibrium of the subgame starting at any prior date $t < T-1$, under the assumption that go ug 1 in st RnRga
3. Finally, we show that the equilibrium constructed is still an equilibrium when remaining $M-1$ banks are present at dates $t < T-1$.

A.7.2 Step 1: The subgame at date $t = T-1$

Lemma 10 F ss nR arua ua u aR$T-1$ arR tt R t run R u ar W u R a ReR iar ug 1 l D ri R $D^{m}_{T-1} = 0$ l t u arRt ugn $m \neq 1$M Ra$\gamma_{T-1} = 1-\rho_{T-1}$ un R gR ig M kr Rg arRtRin u nF u c RsRtlHa FFii tiF c ig ri fr arRg u fgnF csai g l arR tt R t in i eRg o

$$U^{B}_{T-1}(W,D) = \begin{cases} Wr & \text{il } D<0 \\ (W-D)\,r & \text{il } D \in [0, W\gamma_{T-1}] \\ WR_{T-1} & \text{il } D > W\gamma_{T-1} \end{cases}$$

ri R arRg u st a n l ug 1 ig u aRT aRtcn utRi eRg o

$$U^{1}_{T-1}(W,D) = \begin{cases} -Dr & \text{il } D<0 \\ 0 & \text{il } D \in [0, W\gamma_{T-1}] \\ (-D+W\gamma_{T-1})\,r & \text{il } D > W\gamma_{T-1} \end{cases}$$

m l arRtRcui gig ugn $m \neq 1$ rueRu g u st a l RtM

Proof: If $D<0$ then it is straightforward to show that it is an equilibrium for all banks $m \in M$ to set $L^{m}_{T-1} = 0$ and $l^{m}_{T}(\mathbf{P}_{T-1}) = rP^{m}_{T-1}$ (i.e. offer to accept savings at a rate r). The payoffs are then immediate.

Suppose on the other hand that $D \geq 0$. Then it is an equilibrium for banks $m \neq 1$ to set $L_{T-1}^m = 0$ and $l_T^m(\mathbf{P}_{T-1}) = rP_{T-1}^m$, while bank 1 sets $L_{T-1}^1 = 0$ and

$$l_T^1(\mathbf{P}_{T-1}) = \begin{cases} (W-D)\,r & \text{if } P_{T-1}^1 = W \text{ and } D \in [0, W\gamma_{T-1}] \\ WR_{T-1} & \text{if } P_{T-1}^1 = W \text{ and } D > W\gamma_{T-1} \\ 0 & \text{otherwise} \end{cases}$$

and for the borrower to respond by paying all his wealth to bank 1, i.e. $\mathbf{P}_{T-1} = (W, 0, \ldots, 0)$.

There are two cases to deal with in verifying that $\mathbf{P}_{T-1} = (W, 0, \ldots, 0)$ is indeed a best response for the borrower:

Case (A): $D \in [0, W\gamma_{T-1}]$. If he sets $P_{T-1}^1 = W$ he gets $(W-D)\,r$. If he sets $P_{T-1}^1 \in [D, W)$ he can get at most $(W - P_{T-1}^1)\,r$, which is at least weakly worse. If he sets $P_{T-1}^1 \in [0, D)$ the debt-default rule $\mathcal{B}_{DD}$ prevents him from transferring any funds to any bank $m \neq 1$ and so he gets $(W - P_{T-1}^1)\,R_{T-1} \leq WR_{T-1} \leq (W-D)\,r$, where the second inequality follows from the fact that $D \leq W\gamma_{T-1}$.

Case (B): $D > W\gamma_{T-1}$. If he sets $P_{T-1}^1 = W$ he gets WR_{T-1}. If he sets $P_{T-1}^1 \in [D, W)$ he can get at most $(W - P_{T-1}^1)\,r \leq (W-D)\,r \leq WR_{T-1}$where the second inequality follows from the fact that $D \leq W\gamma_{T-1}$. If he sets $P_{T-1}^1 \in [0, D)$ the debt-default rule $\mathcal{B}_{DD}$ prevents him from transferring any funds to any bank $m \neq 1$ and so he gets $(W - P_{T-1}^1)\,R_{T-1} \leq WR_{T-1}$.

Given the lending policies of the banks $m \neq 1$, bank 1's lending policy is clearly a best response. Likewise, the lending policies of banks $m \neq 1$ are a best response to bank 1's lending policy. The payoffs stated in the Lemma are then immediate. **QED**

A.7.3 Step 2: Periods $t < T-1$

We now proceed to construct a subgame perfect equilibrium of the game in which prior to period $T-1$ only bank 1 is present, and from period $T-1$ onwards the borrower and the banks play the equilibrium described in Lemma 10.

Let the equilibrium payoffs at date t be denoted by $U_t^B(W, D)$ and $U_t^1(W, D)$ for the borrower and bank 1 respectively, where D is the borrower's level of indebtedness at that date to bank 1 and W is his wealth level. From Lemma 10 we guess (and will verify) that $U_t^B(W,D)$ and $U_t^1(W,D)$ take the forms

$$U_t^B(W,D) = \begin{cases} Wa_{1,t} & \text{if } D<0 \\ (W-D)\,a_{2,t} & \text{if } D\in[0,W\delta_t] \\ Wa_{3,t} & \text{if } D>W\delta_t \end{cases}$$

$$U_t^1(W,D) = \begin{cases} -Da_{4,t} & \text{if } D<0 \\ 0 & \text{if } D\in[0,W\delta_t] \\ (-D+Wa_{5,t})\,r^{T-t} & \text{if } D>W\delta_t \end{cases} \tag{36}$$

Investment periods

Lemma 11 F ss nR R utRig ug igeRnac RgasRti $t<T-1$ iMRM $R_t>r$ ug arua arR suo lFgf ai gn U_{t+1}^B ug U_{t+1}^1 utRl arRl tc i eRg ig iar f Rfi Rgannuainldg

$$a_{1,t+1} \leq a_{2,t+1} \tag{37}$$

$$(1-\delta_{t+1})\,a_{2,t+1} \leq a_{3,t+1} \tag{38}$$

$$a_{5,t+1} \leq \delta_{t+1} \tag{39}$$

$$\delta_{t+1}\rho_t < 1 \tag{40}$$

kr Rg U_t^B ug U_{t+1}^1 utRu n l arRl tc iar f Rfi Rgan

$$\begin{aligned} a_{1,t} &= R_t a_{1,t+1} \\ a_{2,t} &= R_t \frac{1-\delta_{t+1}}{1-\rho_t\delta_{t+1}} a_{2,t+1} \\ a_{3,t} &= R_t a_{3,t+1} \\ a_{4,t} &= r a_{4,t+1} \\ a_{5,t} &= \rho_t a_{5,t} \\ \delta_t &= \rho_t \delta_{t+1} \end{aligned}$$

u g l cuRn ug ug l $\frac{W\rho_t\delta_{t+1}-D}{1-\rho_t\delta_{t+1}}$ st e iR arua arR R a ReR D lu n RaRRg 0 ug $W\delta_t$ ug arRti nR cuRn g ugM kr R tt R t Rn ga tRsuo ugoarig ig arin sRti M

Proof: Given W,D,L,P,

$$\begin{aligned} W_{t+1} &= (W+L-P)\,R_t \\ D_{t+1} &= (D+L-P)\,r \end{aligned}$$

and so

$$W_{t+1} - D_{t+1} = WR_t - Dr + (L - P)(R_t - r)$$
$$D_{t+1} \leq W_{t+1}\delta_{t+1} \Longleftrightarrow L - P \leq \frac{W\rho_t\delta_{t+1} - D}{1 - \rho_t\delta_{t+1}}$$
$$D_{t+1} \geq 0 \Longleftrightarrow L - P \geq -D$$

where the second equivalence follows from condition (40). Substituting into the expressions for (36)

$$U^B_{t+1}(W, D) = \begin{cases} (W + L - P) R_t a_{1,t+1} & \text{if } L - P < -D \\ (WR_t - Dr + (L - P)(R_t - r)) a_{2,t+1} & \text{if } -D \leq L - P \leq \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}} \\ (W + L - P) R_t a_{3,t+1} & \text{if } L - P > \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}} \end{cases}$$
$$U^1_{t+1}(W, D) = \begin{cases} -(D + L - P) r a_{4,t+1} & \text{if } L - P \leq -D \\ 0 & \text{if } -D < L - P \leq \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}} \\ (-(D + L - P) r \\ \quad + (W + L - P) R_t a_{5,t+1}) r^{T-(t+1)} & \text{if } L - P > \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}} \end{cases} \quad (41)$$

Note that when $L - P = -D$,

$$WR_t - Dr + (L - P)(R_t - r) = (W - D) R_t = (W + L - P) R_t$$

while when $L - P = \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}}$

$$\begin{aligned} WR_t - Dr + (L - P)(R_t - r) &= (W - D)\frac{R_t(1 - \delta_{t+1})}{1 - \rho_t\delta_{t+1}} \\ &= (1 - \delta_{t+1})(W + L - P) R_t \end{aligned}$$

It follows from $R_t > r$ and conditions (37) and (38) that the borrower's utility is a strictly decreasing function of the payment P. Thus $P = 0$.

Next, we turn to the lender's payment L. If $D < 0$, then the lender cannot do better than set $L = 0$. On the other hand, if $D \geq 0$ then the only way for the lender to achieve non-negative utility is to choose L such that $0 \leq L \leq \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}}$. Since the borrower's utility is an increasing function of L (by the same argument that it is a decreasing function of P) we have $L = \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}}$ provided this is non-negative, i.e. provided $D \leq W\rho_t\delta_{t+1}$, and $L = 0$ otherwise. Substituting into our expressions for U^B_{t+1} and U^1_{t+1} we obtain

$$U^B_t(W, D) = \begin{cases} WR_t a_{1,t+1} & \text{if } D < 0 \\ (W - D)\frac{R_t(1-\delta_{t+1})}{1-\rho_t\delta_{t+1}} a_{2,t+1} & \text{if } D \in [0, W\rho_t\delta_{t+1}] \\ WR_t a_{3,t+1} & \text{if } D > W\rho_t\delta_{t+1} \end{cases}$$
$$U^1_t(W, D) = \begin{cases} -Dra_{4,t+1} & \text{if } D < 0 \\ 0 & \text{if } D \in [0, W\rho_t\delta_{t+1}] \\ (-D + W\rho_t a_{5,t+1}) r^{T-t} & \text{if } D > W\rho_t\delta_{t+1} \end{cases}$$

which completes the proof. **QED**

Payment periods

Lemma 12 F ss nR R utRig u suoc RgasRti $t < T$ iMRMR$_t \leq r$ ug arua arRsuo lFgfai gn U^B_{t+1} ug U^1_{t+1} utRl arRl tc i eRg ig iar f Rfi Rgannuainldg

$$\frac{a_{2,t+1}}{a_{3,t+1}} \geq \frac{1}{1-\rho_t\delta_{t+1}} \tag{42}$$

$$a_{5,t+1} \leq \delta_{t+1} \tag{43}$$

$$\delta_{t+1}\rho_t < 1 \tag{44}$$

kr Rg U^B_t ug U^1_t utRu n l arRl tc iar f Rfi Rgan

$$\begin{aligned}
a_{1,t} &= R_t a_{1,t+1} \\
a_{2,t} &= R_t a_{2,t+1} \\
a_{3,t} &= R_t a_{3,t+1} \\
a_{4,t} &= r a_{4,t+1} \\
a_{5,t} &= \rho_t a_{5,t+1} \\
\delta_t &= 1 - \frac{a_{3,t+1}}{a_{2,t+1}}
\end{aligned}$$

u g 1 Rnga cuR ugo ug ig arin sRti M kr R tt R t tRsuon ri n R a D ig lF st e iR ia RaRRg 0 ug $W\delta_t$ ug Rnga tRsuo ugoarig arRti nRM

Proof: Given W, D, P, L we obtain exactly the same characterization of U^B_{t+1} and U^1_{t+1} as in for investment periods (see (41) in the proof of Lemma 11). We will first analyze the choice of the borrower's payment ignoring the wealth constraint $P \leq W + L$, and then show it does not bind.

Clearly the borrower will never choose to make a strictly positive payment P such that $L - P < -D$ or $L - P > \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}}$,since in this range a lower payment would be strictly better. On the other hand, since $r \geq R_t$ the borrower weakly prefers $P = D + L$ to all other payments P for which $-D \leq L - P \leq \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}}$. So (ignoring the wealth constraint) the borrower's choice reduces to one between $P = 0$ and $P = D + L$. Choosing $P = L$ leads to

$$U^B_{t+1}(W, D) = \begin{cases} (W + L) R_t a_{1,t+1} & \text{if } L < -D \\ (WR_t - Dr + L(R_t - r)) a_{2,t+1} & \text{if } -D \leq L \leq \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}} \\ (W + L) R_t a_{3,t+1} & \text{if } L > \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}} \end{cases}$$

while $P = L + D$ gives

$$U^B_{t+1}(W, D) = (W - D) R_t a_{2,t+1}$$

Now, $P = L + D$ is only a feasible choice if it is non-negative, so if $L < -D$ the borrower chooses $P = 0$. For $-D < L \leq \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}}$ then $P = L + D$ is trivially the better choice. Finally, for $L > \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}}$ then $P = L + D$ is better if

$$(W - D) R_t a_{2,t+1} \geq (W + L) R_t a_{3,t+1}$$

or equivalently

$$L \leq (W - D) \frac{a_{2,t+1}}{a_{3,t+1}} - W \tag{45}$$

When $W - D \geq 0$, then given condition (42) the inequality

$$L \leq \frac{W\rho_t\delta_{t+1} - D}{1 - \rho_t\delta_{t+1}} = \frac{(W - D)}{1 - \rho_t\delta_{t+1}} - W$$

holds whenever (45) does. On the other hand, if $W - D < 0$ then there is no $L \geq 0$ satisfying $-D < L \leq \frac{W\rho_t\delta_{t+1} - D}{1-\rho_t\delta_{t+1}}$ or (45). We have now almost established that $P = L + D$ whenever $L \geq 0$ and

$$-D \leq L \leq (W - D) \frac{a_{2,t+1}}{a_{3,t+1}} - W \tag{46}$$

and $P = 0$ otherwise. It remains only to check that the wealth constraint is satisfied. If $P = 0$ there is nothing to check, while we have just argued that $P = L + D$ is only chosen if $W \geq D$, in which case the wealth constraint $P \leq W + L$ is satisfied.

We now turn to the bank's choice of L. Given L, his interim utility $u^1_t(W, D, L)$ is

$$u^1_t(W, D) = \begin{cases} -(D + L) r a_{4,t+1} & \text{if } L < -D \\ 0 & \text{if } -D \leq L \leq (W - D) \frac{a_{2,t+1}}{a_{3,t+1}} - W \\ \begin{array}{l}(-(D + L) r \\ + (W + L) R_t a_{5,t+1}) r^{T-(t+1)}\end{array} & \text{if } L > (W - D) \frac{a_{2,t+1}}{a_{3,t+1}} - W \end{cases}$$

Now, from conditions (44) and (43) $-(D + L) r + (W + L) R_t a_{5,t+1}$ is negative if and only if

$$L > \frac{W - D}{1 - \rho_t a_{5,t+1}} - W$$

which by conditions (43) and (42) holds whenever $L > (W - D) \frac{a_{2,t+1}}{a_{3,t+1}} - W$. So if $D < 0$, the lender's best choice is $L = 0$, while if $D \geq 0$ the lender cannot do better than set $L = 0$ if $(W - D) \frac{a_{2,t+1}}{a_{3,t+1}} - W \geq 0$ (note that the borrower does not care about the choice of L between 0 and $(W - D) \frac{a_{2,t+1}}{a_{3,t+1}} - W$, since he will just pay $P = D + L$).

Finally, if $(W-D)\frac{a_{2,t+1}}{a_{3,t+1}} - W < 0$ then $L = 0$ is the lender's best choice by conditions (44) and (43) again. Substituting into our expressions for U^B_{t+1} and U^1_{t+1} we obtain

$$U_t^B(W,D) = \begin{cases} WR_t a_{1,t+1} & \text{if } D < 0 \\ (W-D)\,R_t a_{2,t+1} & \text{if } D \in \left[0, W\left(1 - \frac{a_{3,t+1}}{a_{2,t+1}}\right)\right] \\ WR_t a_{3,t+1} & \text{if } D > W\left(1 - \frac{a_{3,t+1}}{a_{2,t+1}}\right) \end{cases}$$

$$U_t^1(W,D) = \begin{cases} -Dra_{4,t+1} & \text{if } D < 0 \\ 0 & \text{if } D \in \left[0, W\left(1 - \frac{a_{3,t+1}}{a_{2,t+1}}\right)\right] \\ (-D + W\rho_t a_{5,t+1})\, r^{T-t} & \text{if } D > W\left(1 - \frac{a_{3,t+1}}{a_{2,t+1}}\right) \end{cases}$$

which completes the proof. **QED**

Verifying the form of the payoff functions $U_t^B(W,D)$ and $U_t^1(W,D)$ Next, we confirm that the conditions needed to apply Lemmas 11 and 12 are in fact satisfied:

Lemma 13 F ss nR arua l t $t = 0, 1, \ldots, T-1$ arRf Rfi Rgan $a_{1,t}, a_{2,t}, a_{3,t}, a_{4,t}, a_{5,t}, \delta_t$ utRR gR iaRtuaieRb o

$$(a_{1,T-1}, a_{2,T-1}, a_{3,T-1}, a_{4,T-1}, a_{5,T-1}, \delta_{T-1}) = (r, r, R_{T-1}, r, 1-\rho_{T-1}, 1-\rho_{T-1}) \quad (47)$$

ug

$$a_{1,t} = R_t a_{1,t+1} \quad (48)$$

$$a_{2,t} = \begin{cases} R_t \frac{1-\delta_{t+1}}{1-\rho_t\delta_{t+1}} a_{2,t+1} & \text{il } R_t > r \\ R_t a_{2,t+1} & \text{il } R_t < r \end{cases} \quad (49)$$

$$a_{3,t} = R_t a_{3,t+1} \quad (50)$$

$$a_{4,t} = r a_{4,t+1} \quad (51)$$

$$a_{5,t} = \rho_t a_{5,t+1} \quad (52)$$

$$\delta_t = \begin{cases} \rho_t \delta_{t+1} & \text{il } R_t > r \\ 1 - \frac{a_{3,t+1}}{a_{2,t+1}} & \text{il } R_t < r \end{cases} \quad (53)$$

kr Rg l t ugo $t = 1, \ldots, T-1$

$$\delta_t = \gamma_t = (1-\rho_{T-1}) \prod_{s=t}^{T-2} \max\{1, \rho_s\} \quad (54)$$

$$\delta_t \rho_{t-1} < 1 \quad (55)$$

$$a_{1,t} \leq a_{2,t} \quad (56)$$

$$a_{5,t} \leq \delta_t \quad (57)$$

$$\frac{a_{2,t}}{a_{3,t}} = \frac{1}{1-\delta_t} \quad (58)$$

tRe Rt ig ugo suoc RgasRti $t < T - 1$ iMRM$R_t < r$

$$\frac{a_{2,t+1}}{a_{3,t+1}} \geq \frac{1}{1 - \rho_t \delta_{t+1}} \tag{59}$$

Proof. We will proceed by induction. Fix t, and suppose that the result holds for all $s > t$.

First, consider the case where t is a payment period ($R_t < r$). If $R_{t+1} > r$ then

$$\delta_t = 1 - \frac{a_{3,t+1}}{a_{2,t+1}} = 1 - \frac{a_{3,t+2}}{a_{2,t+2}} \frac{1 - \rho_{t+1}\delta_{t+2}}{1 - \delta_{t+2}} = 1 - \left(1 - \rho_{t+1}\delta_{t+2}\right) = \rho_{t+1}\delta_{t+2} = \delta_{t+1}$$

while if $R_{t+1} < r$ then

$$\delta_t = 1 - \frac{a_{3,t+1}}{a_{2,t+1}} = 1 - \frac{a_{3,t+2}}{a_{2,t+2}} = \delta_{t+1}$$

Condition (54) follows since $\rho_t < 1$ and so $\delta_t = \max\{1, \rho_t\}\, \delta_{t+1}$, and condition (55) is then immediate from Assumption 1. Condition (56) follows trivially by induction since when $R_t < r$ we have $a_{1,t} = R_t a_{1,t+1}$ and $a_{2,t} = R_t a_{2,t+1}$. If $R_{t+1} > r$ then condition (57) follows immediately by induction. On the other hand, if $R_{t+1} < r$ then substituting in for $a_{5,t}$ and δ_t, condition (57) is equivalent to

$$\rho_t a_{5,t+1} \leq 1 - \frac{a_{3,t+1}}{a_{2,t+1}}$$

which holds since by induction $\rho_t a_{5,t+1} \leq \rho_t \delta_{t+1}$ and $1 - \frac{a_{3,t+1}}{a_{2,t+1}} \geq \rho_t \delta_{t+1}$. Finally, condition (58) holds since

$$\frac{a_{2,t}}{a_{3,t}} = \frac{a_{2,t+1}}{a_{3,t+1}} = \frac{1}{1 - \delta_t}$$

Next, turn to the case in which t is an investment period ($R_t > r$). The characterization (54) of δ_t is immediate and condition (55) again follows from Assumption 1. Condition (56) holds since $\rho_t > 1$ and thus $\frac{1-\delta_{t+1}}{1-\rho_t\delta_{t+1}} > 1$, and so

$$a_{1,t} = R_t a_{1,t+1} \leq R_t \frac{1 - \delta_{t+1}}{1 - \rho_t \delta_{t+1}} a_{2,t+1} = a_{2,t}$$

given that $a_{1,t+1} \leq a_{2,t+1}$. Condition (57) follows trivially by induction. Finally, condition (58) holds since

$$\frac{a_{2,t}}{a_{3,t}} = \frac{1 - \delta_{t+1}}{1 - \rho_t \delta_{t+1}} \frac{a_{2,t+1}}{a_{3,t+1}} = \frac{1 - \delta_{t+1}}{1 - \rho_t \delta_{t+1}} \frac{1}{1 - \delta_{t+1}} = \frac{1}{1 - \rho_t \delta_{t+1}} = \frac{1}{1 - \delta_t}$$

It remains only to check that when period t is payment period ($R_t < r$) condition (59) holds. But this follows easily since

$$\frac{a_{2,t+1}}{a_{3,t+1}} = \frac{1}{1 - \delta_{t+1}} \geq \frac{1}{1 - \rho_t \delta_{t+1}}$$

as $\rho_t < 1$. **QED**

The equilibrium payments Suppose the borrower starts with no debt in period 0 ($D_0 = 0$) and a wealth level W_0. Then from Lemmas 10 -1 3 the equilibrium we have constructed is as follows:

In each **payment period** t ($R_t \leq r$) the borrower repays any debt he has (D_t). Note that this implies that if several savings periods follow each other, the only repayment occurs in the first of these.

In each **investment period** t ($R_t > r$) the lender extends a loan of

$$L_t = \frac{W_t\delta_t - D_t}{1-\delta_t}$$

where W_t is the borrower's wealth level, D_t is the borrower's debt level and δ_t is as given by (54). Note that whenever an investment period follows another investment period, then since $D_t = r\left(D_{t-1} + L_{t-1}\right)$, $W_t = R_{t-1}\left(W_{t-1} + L_{t-1}\right)$ and $R_{t-1}\delta_t = r\delta_{t-1}$ we have

$$\begin{aligned} W_t\delta_t - D_t &= r\delta_{t-1}\left(W_{t-1} + L_{t-1}\right) - r\left(D_{t-1} + L_{t-1}\right) \\ &= r\left(W_{t-1}\delta_{t-1} - D_{t-1}\right) - r\left(1-\delta_{t-1}\right)L_{t-1} = 0 \end{aligned}$$

Thus if several investment periods occur consecutively, a loan of

$$\frac{W_t\delta_t}{1-\delta_t} = \frac{W_t\gamma_t}{1-\gamma_t} = L_t^*$$

is granted in the first of these, and no loan is granted in the ones that follow.

In the **penultimate period** $T-1$ the borrower pays of all his debt if he has not already done so.

Since δ_t is larger for smaller t,the **ratio of loan size to wealth** falls as the relationship nears its end.

Bank 1's final profit is 0 (i.e. it exactly breaks even). To find the borrower's utility, note that from Lemma 13

$$a_{2,0} = \frac{a_{3,0}}{1-\delta_0} = \frac{\prod_{s=0}^{T-1} R_s}{1-\delta_0} = \frac{\prod_{s=0}^{T-1} R_s}{1-\gamma_0}$$

Since the borrower's initial debt is $D_0 = 0$, we then have

$$U_0^B\left(W_0, D_0\right) = \frac{W_0}{1-\gamma_0}\prod_{s=0}^{T-1} R_s$$

A.7.4 Step 3: Reintroducing the other banks

Finally, we need to show that the equilibrium we have constructed is still an equilibrium when the remaining $M-1$ banks are present prior to date $T-1$.

To see this, suppose to the contrary that there exists a sequence of loans and payments $\left\{\tilde{L}_t^m, \tilde{P}_t^m\right\}$ such that the borrower's final consumption is strictly increased, with $\mathbf{L}_t^1 = (L_t^*, 0, \ldots, 0)$ and $\mathbf{P}_s^1 = (P_s^*, 0, \ldots, 0)$ for all $t \leq \tau$ and $s < \tau$ (i.e. τ is the deviation date), and $\left\{\tilde{L}_t^m, \tilde{P}_t^m\right\}$ subgame perfect for dates $t \geq \tau + 1$. Denote the debt levels under the deviation by $\tilde{D}_t^m$.

Since the equilibrium we have constructed achieves the upper bound on the borrower's utility characterized in Proposition 3, bank 1's final profits under these alternative payments must be strictly negative. Thus $\tilde{D}_T^1 < 0$, since the only transfers in period T are from the bank to the borrower.

If $\tilde{D}_{T-1}^m \geq 0$, the debt-default rule $\mathcal{B}_{DD}$ implies that $\tilde{D}_T^m \geq 0$ since the borrower is restricted from depositing funds with any bank $m \neq 1$. But then $\tilde{L}_T^m = 0$, since otherwise bank m would be making negative profits. But then $\tilde{P}_{T-1}^m = 0$ and so $\tilde{L}_{T-1}^m = 0$. Iterating establishes that $\tilde{P}_t^m = \tilde{L}_t^m = 0$ for all $m \neq 1$ and all t.

Similarly, if $\tilde{D}_{T-1}^m < 0$ the debt-default rule $\mathcal{B}_{DD}$ implies that $\tilde{P}_{T-1}^m = 0$ since any positive payment would be seized in entirety. So again we can conclude $\tilde{P}_t^m = \tilde{L}_t^m = 0$.

Thus the deviation must be such that no bank $m \neq 1$ is involved. But in constructing the equilibrium we have already shown that no deviation involving payment just to bank 1 is profitable.