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The Time Consistency of Optimal Monetary Policy with Heterogeneous Agents

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Abstract

This paper studies the structure and time consistency of optimal monetary policy from a public finance perspective in an economy where agents differ in transaction patterns and asset holdings. I find that heterogeneity breaks the link between lack of government commitment and high inflation which characterizes representative agent models of optimal fiscal and monetary policy. Even under commitment, it may be optimal to depart from Friedman's rule for setting nominal interest rates. Moreover, optimal monetary and fiscal policy are time consistent. Time consistency does not require outstanding nominal claims on the government to be zero. If the Friedman rule is optimal, time consistency is ensured even if the government cannot issue real and nominal debt of all maturities. I relate these findings to key historical episodes of inflation and deflation.

Keywords: Inflation, Heterogeneity, Distribution, Time Consistency

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1. Introduction

This paper explores the structure and time consistency of optimal monetary policy from a public finance perspective in an economy where agents are heterogeneous in transaction patterns and asset holdings. With a representative agent and no lump sum taxes, two main results hold. First, Friedman's rule for setting nominal interest rates is optimal in a large class of economies, if the government can commit to future policies. Second, as shown by Calvo (1978), if outstanding nominal claims on the government are positive, the incentive to reduce their present value via unanticipated inflation makes optimal fiscal and monetary policy time inconsistent. It follows that, with a representative agent, high inflation is associated with lack of commitment in government policy. The findings in this paper suggest that heterogeneity breaks the link between lack of commitment and high inflation. Even under commitment, it may be optimal to depart from the Friedman rule. In addition, the government's incentive to inflate away nominal liabilities depends crucially on the distribution of currency and other nominal claims on the government. Optimal monetary and fiscal policy can be made time consistent. Time consistency does not require that outstanding nominal claims on the government are zero. In addition, if the Friedman rule is optimal, optimal fiscal and monetary policy can be made time consistent even when the government cannot issue real and nominal debt of all maturities.

I analyze optimal policy and the issue of time consistency following the approach of Lucas and Stokey (1983). I consider an economy where agents are infinitely lived, they value leisure and consumption and hold currency for transaction purposes. Agents differ in labor productivity. Agents with higher labor productivity also have higher labor income. Economies of scale in the transaction technology imply that, in equilibrium, they hold less cash as a fraction of total purchases, consistent with empirical evidence on transaction patterns as a function of income. The government must finance an exogenous stream of government consumption by labor income taxes, seignorage or by issuing real and nominal debt of different maturities. I first solve the Ramsey problem, where policy is chosen once and for all. This amounts to assuming that the initial government has a commitment technology that binds the actions of future governments. In monetary economies with a representative agent, the Friedman rule solves the Ramsey problem when preferences satisfy homotheticity and separability (in consumption and leisure) conditions, as shown by Chari, Christiano and Kehoe (1996). Correia and Teles (1999) establish that the Friedman rule is a robust feature of optimal government policy in monetary economies with a representative agent. Alvarez, Kehoe and Neumeyer (2001) illustrate that the conditions for optimality of the Friedman rule correspond to utility specifications that are consistent with balanced growth. Chari, Christiano and Kehoe (1996) illustrate a connection between optimality of the Friedman rule in monetary economies and the principle of uniform commodity taxation in public finance, studied in Atkinson and Stiglitz (1972). The findings in this paper suggest that this connection also holds in monetary economies where agents are heterogeneous, and inflation and labor income taxation have distributional effects. Under a preference specification for which the Friedman rule is optimal in the corresponding representative agent economy, I find that the Friedman rule is optimal when the labor income tax schedule is sufficiently unconstrained. Otherwise, there is a conflict between distribution and efficiency which can give rise to departures from the Friedman rule. This parallels the results in Atkinson and Stiglitz (1976). They show that with heterogeneous agents and weakly separable utility, uniform commodity taxation is optimal, even with distributional objectives, when the labor income tax schedule is sufficiently unconstrained.

I then analyze time consistency of Ramsey policies. Various papers study this issue in representative agent economies. Lucas and Stokey (1983) find that, in a real economy, there may be an incentive to revise the path of taxes to depreciate the value of government debt. This *real time inconsistency* can be eliminated by appropriately restructuring outstanding real claims on the government. They show that in a monetary economy optimal fiscal and monetary policy cannot be made time consistent in general. This is because time consistency of Ramsey policies requires that both real and nominal government debt held by the private sector is non-zero. But a positive level of outstanding nominal claims on the government give rise to *nominal time inconsistency* - the incentive to inflate away nominal claims on the government. They argue that commitment to a path for nominal prices is needed for time consistency in a monetary economy. This ensures that payments on nominal debt represent a binding real commitment. Alvarez, Kehoe and Neumeyer (2001) establish that optimality of the Friedman rule is a necessary and sufficient condition for time consistency of optimal monetary and fiscal policies. Their result is based on the observation that, under the Friedman rule, a monetary economy is equivalent to a real economy. Then, the restriction that the present value of nominal government liabilities must be zero, which is required for nominal time consistency, is inconsequential. Real debt restructuring is sufficient to make Ramsey policy time consistent, following the logic in Lucas and Stokey (1983). In Alvarez, Kehoe and Neumeyer (2001), as well as in Lucas and Stokey (1983), nominal time inconsistency is particularly severe since agents can adjust their holdings of currency in response to unanticipated inflation, and there are no costs associated with unanticipated inflation.

Nicolini (1998) shows that nominal time inconsistency can be moderated if unanticipated inflation is not a lump sum tax due to the inability of some agents to adjust their holdings of currency in response to changes in the price level. He finds that optimal monetary policy is not time consistent in general but that for certain conditions the optimal deviation involves a fall in the price level. Nicolini does not consider labor income taxation and stops short of analyzing the case with nominal government debt.

The findings in this paper suggest that optimal monetary and fiscal policy can be made time consistent by influencing the distribution of government debt. I first assume that agents cannot adjust their holdings of currency in response to changes in the price level, as in Svensson (1985). If the Friedman rule is optimal, the Ramsey equilibrium is time consistent. This is true if the government can issue real and nominal debt of all maturities. It is also true when the government is restricted to issuing real and nominal debt of one period maturity, and even when it can only issue currency. If the Friedman rule is not optimal, the Ramsey equilibrium is time consistent only if the government can issue both real and nominal debt of all maturities. The intuition for these findings is that removing nominal time inconsistency requires placing constraints on the *distribution* of nominal claims on the government, both currency and bonds. On the other hand, removing real time consistency places constraints on the *level* of both real and nominal government debt held by the private sector. These two sets of restrictions are mutually compatible. The same logic applies if agents can adjust their holdings of currency in response to changes in the price level, as in Lucas and Stokey (1983). If the government can issue both real and nominal debt of all maturities Ramsey policies are time consistent. If the government can issue nominal debt only, Ramsey policies are time consistent if the Friedman rule is optimal.

The possibility of removing the nominal time inconsistency problem, even when the Friedman rule is not optimal, has roots in the distributional consequences of deviations from the continuation of the time 0 Ramsey equilibrium and in the possibility of manipulating the distribution of nominal claims on the government across agents, as well as the maturity structure of claims on the government. The Friedman rule is optimal when agents with low labor productivity and low labor income are weighted more heavily in the government's objective function. Low income households hold more cash as a fraction of their total purchases and are more exposed to inflation. Under Svensson timing, the fact that the cost of unanticipated inflation is relatively high for low income agents makes the Ramsey equilibrium time consistent even when the government cannot issue nominal and real debt of all maturities, when the Friedman rule is optimal.

When the Friedman rule is not optimal,the government has a relative preference for high income households. Then, both distribution and efficiency objectives generate a potential for nominal time inconsistency. However, if the government can issue real and nominal debt of all maturities, than the Ramsey equilibrium can be made time consistent by appropriately choosing the maturity structure and the distribution of nominal claims on the government (other than currency), thereby affecting the distributional costs of inflation[1].

An additional important finding is that time consistency does not require that the present value of nominal liabilities is zero at any point in time. The conditions for time consistency of the Ramsey equilibrium impose restrictions instead on the shadow present value of nominal claims on the government. This is a weighted sum of the present value of nominal claims held by each type of household, where the weights reflect efficiency and distributional considerations. A characterization of distribution of nominal claims on the government that guarantee time consistency points to asymmetric holdings of real and nominal claims against the government as a key feature of time consistent Ramsey equilibria. Total nominal claims on the government are positive and ranging from 30 to 45% of total output. The same holds true for Lucas and Stokey timing.

These findings are in line with an observation put forth by Hamilton (1795). He argued in favor of the Federal assumption of the states' war debt as a way to reduce the risk of monetization. Debt assumption would provide powerful government creditors with a strong incentive to support Federal tax legislation, making the use of inflation to raise revenues less likely. A number of key historical episodes of large inflations and deflation provide clear evidence of the importance of distributional consequences of unanticipated inflation in shaping government incentives, conditional on the political influence of different groups of agents on monetary and fiscal policy decisions.

The plan of the paper is as follows. Section 2 describes the model. Section 3 studies optimal fiscal and monetary policy under commitment. Section 4 characterizes the sufficient conditions for time consistency. Section 5 reviews a number of historical episodes of inflation and disinflation. Section 6 concludes.

[1] The argument that the distributional costs associated with deviations that would be optimal with a representative agent may remove the incentive to deviate from the ex ante optimal policy is also made in Rogers (1986), who studies optimal wage and interest taxation in a two-period, multiple consumer economy. She finds that the incentive to raise interest taxation may be moderated if the resulting create utility distribution is unacceptable.

2. A Cash-Credit Good Economy with Heterogenous Households

In this section, I describe a version of Lucas and Stokey's cash-credit good economy. The economy is populated by households, firms and a government. Households consume, supply labor and trade in assets in each period. Households differ in labor productivity but have identical preferences. They make purchases with currency or with an alternative payment technology. A fixed cost associated with avoiding the use of cash implies that households with lower labor productivity and lower income hold more currency as a fraction of total purchases. This feature of the economy is consistent with cross–sectional evidence on transaction patterns and asset holdings[2]. In each period trade in goods and labor precedes trade in assets, as in Svensson (1985). Firms have access to a linear production technology that requires labor for the production of consumption goods. They are perfectly competitive. The government finances an exogenous stream of spending by issuing real and nominal debt of different maturities, printing money and taxing labor income at a uniform proportional rate. There is no uncertainty.

I now illustrate the problems faced by the agents in our economy in detail.

2.1. Firms

Firms live for one period. They hire labor to produce consumption goods with a linear technology, given by:

$$\sum_{j=1}^{2} y_{jt} \leq n_t.$$

Here y_{1t} is total production of cash goods and y_{2t} total production of credit goods at time t and n_t is aggregate labor. Perfect competition implies:

$$P_{1t} = P_{2t} = P_t = W_t, \tag{2.1}$$

where P_t is the price charged for consumption goods and W_t the nominal wage at time t.

Purchases of consumption goods without currency need to be arranged. The services required to arrange to purchase consumption goods on credit are provided

[2] Erosa and Ventura (2000) report that in the US low income households use cash for a greater fraction of their total purchases relative to high income households. Mulligan and Sala-i-Martin (2000) estimate the probability of adopting financial technologies that hedge against inflation and find that is positively related to the level of household wealth and inversely related to the level of education. Attanasio, Guiso and Jappelli (2001) find that the probability of using an interest bearing bank account increases with educational attainment, income and average consumption, based on cross-sectional household data for Italy.

by competitive financial firms. Their profit per good is given by:

$$\pi_t(j) - W_t\theta(j), \tag{2.2}$$

where $\theta(\cdot)$ is measured in efficiency units of labor and satisfies $\theta' > 0$. π_t is the dollar charge for arranging purchases of consumption good j without currency. Profit maximization implies: $\pi_t(j) = W_t\theta(j)$ for all t and all $j \in [0,1]$.

2.2. Households

There is a continuum of unit measure of households, divided into two types, where $0 < \nu_i < 1$ is the fraction of type i agents, with $i = 1, 2$ and $\sum_i \nu_i = 1$. Households of the same type are identical. Households have preferences defined over consumption of cash goods c_{i1}, consumption of credit goods c_{i2} and over hours worked n_i. Preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t U(c_{it}, n_{it}),$$

$$c_i = \left[(1 - z_i) c_{i1}^{\rho} + z_i c_{i2}^{\rho}\right]^{\frac{1}{\rho}}, \tag{2.3}$$

where $\rho \in (0,1)$ and z_i is the fraction of consumption goods purchased without the use of cash, and c_{i1t}, c_{i2t} is the level of consumption of goods purchased with and without currency, respectively. I assume:

$$U(c_i, n_i) = h(c_i) + v(n_i),$$

where h is strictly increasing and strictly concave, while v is strictly decreasing and concave.

Households choose z_{it}, purchase consumption goods, supply labor, accumulate currency and trade one-period nominal discount bonds in each period. They enter a period with M_{it} units of currency and are subject to a cash in advance constraint, given by:

$$P_t c_{i1t}(1 - z_{it}) - M_{it} \leq 0. \tag{2.4}$$

The asset market session follows trading in the goods and labor market. During the asset market session households receive labor income net of taxes, clear consumption liabilities and trade nominal and real bonds of different maturities issued by other households or by the government. Nominal (real) bonds purchased at time t entitle holders to one unit of currency (consumption) in the asset market section at $t+1$. I assume that the government and private agents are committed to debt repayments. This implies that agents are indifferent between holding privately or government issued bonds. The price in terms of currency of a nominal bond of maturity s at time t is $Q_{t,t+s}$. Analogously, the price in terms of currency of a real bond with maturity s at time t is $P_t q_{t,t+s}$. If the government does not issue debt, the bonds will be in zero net supply. Total holdings of nominal and real bonds by agent i at the end of time t are denoted with $B_{it,t+s}$ and $b_{it,t+s}$ for $i = 1, 2$ and $s > 0$.

Households face the following constraint on the asset market:

$$
\begin{aligned}
& M_{t+1} + \sum_{s>0} \left(Q_{t,t+s} B_{it,t+s} + q_{t,t+s} P_t b_{it,t+s} \right) && (2.5) \\
\leq \; & M_{it} + \sum_{\hat{t}=-1}^{t-1} \left(B_{i\hat{t},t} + P_t b_{i\hat{t},t} \right) - P_t c_{i1t} \left(1 - z_{it}\right) - P_t c_{i2t} z_{it} - \int_{\underline{z}}^{z_{it}} \pi_t(j)\, dj + W_t \xi_i \left(1 - \tau_t^i\right) n_{it},
\end{aligned}
$$

where W_t is the nominal wage, ξ_i denotes labor productivity, τ_t^i is the tax rate on labor income and $\int_{\underline{z}}^{z_{it}} \pi_t(j)\, dj$ the currency cost of arranging purchases of consumption goods with credit. In addition, the no-Ponzi game condition:

$$
\left(Q_{t,t+1}^{-1} M_{it+1} + B_{it+1} \right) \Phi_{t+1} + \sum_{s=1}^{\infty} \Phi_{t+s} W_{t+s} \left(1 - \tau_{t+s}\right) \xi_i \geq 0, \qquad (2.6)
$$

is also required, with $\Phi_t = \prod_{t'=0}^{t-1} Q_{t',t'+1}$, $\Phi_0 = 1$.

2.3. Government

The government finances an exogenous stream of consumption $\bar{g}$ and is subject to the following dynamic budget constraint:

$$
P_t \bar{g}_t + M_t + \sum_{\hat{t}=0}^{t-1} \left(B_{\hat{t},t} + P_t b_{\hat{t},t} \right) = \sum_{s>0} \left(Q_{t,t+s} B_{t,t+s} + q_{t,t+s} P_t b_{t,t+s} \right) + M_{t+1} + W_t T_t, \qquad (2.7)
$$

where M_t, B_t , b_t are the supply of currency, nominal and real bonds, respectively, and:

$$
T_t = \sum_i \nu_i \tau_t^i \xi_i n_{it}.
$$

I will consider the possibility that government policy is constrained to satisfy certain restrictions, captured in the constraint:

$$
\kappa \left(\tau_t^1, \tau_t^2 \right) \leq 0. \qquad (2.8)
$$

I will refer to a particular specification of $\kappa(\cdot)$ as a *fiscal constitution*[3].

2.4. Private Sector Equilibrium

The timing of events in each period is as follows:

1. Households come into the period with holdings of currency and debt given by M_{it} and B_{it}, b_{it}. They choose z_{it}.

2. The government sets policy subject to (2.7) and (2.8).

3. Households, firms and the government trade on the goods and labor markets. The households' purchases of cash goods are subject to (2.4). Equilibrium on the goods market requires:

$$\sum_{i=1,2} \nu_i \left(c_{i1t} \left(1 - z_{it}\right) + c_{i2t} z_{it} + \int_{\underline{z}}^{z_{it}} \theta\left(j\right) dj - \xi_i n_{it} \right) + \bar{g}_t = 0. \qquad (2.9)$$

4. Asset markets open. Households purchase bonds and acquire currency to take into the following period subject to the constraint (2.5). Equilibrium in the asset market requires:

$$\begin{aligned} \sum_{i=1,2} \nu_i B_{it,t+s} &= B_{t,t+s}, \text{ for } s > 0, \qquad (2.10) \\ \sum_{i=1,2} \nu_i b_{it,t+s} &= b_{t,t+s}, \text{ for } s > 0, \\ \sum_{i=1,2} \nu_i M_{it+1} &= M_{t+1}. \end{aligned}$$

Definition 2.1. *A private sector equilibrium is given by a government policy* $\{\bar{g}_t, \tau_t^i, M_{t+1}, B_{t,t+s}, b_{t,t+s}\}_{t\geq 0, s>0}$ *, a price system* $\{P_t, W_t, Q_{t,t+s}, q_{t,t+s}, \pi_t\left(j\right)\}_{t\geq 0, s>0, j\in[0,1]}$ *and an allocation* $\{c_{i1t}, c_{i2t}, n_{it}, z_{it}, B_{it,t+s}, b_{it,t+s}\}_{i=1,2, t\geq 0, s>0}$ *such that:*

1. *given the policy and the price system households and firm optimize;*

2. *government policy satisfies (2.7);*

3. *markets clear.*

[3] The constraint (2.8) is introduced to capture the notion that broad features of the fiscal structure, which determine the fiscal instruments available, tend to remain in place for long time periods and acquire an aura of costitutionality, as argued in Buchanan (1967).

The following proposition characterizes the competitive equilibrium.

Proposition 2.2. *An allocation* $\{c_{i1t}, c_{i2t}, n_{it}, z_{it}, B_{it,t+s}, b_{it,t+s}\}_{i=1,2,t\geq 0,s>0}$ *and a price system* $\{P_t, W_t, Q_{t,t+s}, q_{t,t+s}, \pi_t(j)\}_{t\geq 0,s>0,j\in[0,1]}$ *constitute a private sector equilibrium if and only if, for a given government policy* $\{\bar{g}_t, \tau_t^i, M_{t+1}, B_{t,t+s}, b_{t,t+s}\}_{t\geq 0,s>0}$, *(2.9), (2.7) and the following conditions are verified:*

$$0 < Q_{t,t+1} \leq 1,$$

$$W_t = P_t,$$

$$Q_{t,t+1} = \beta \frac{P_t}{P_{t+1}} \frac{u_{i2,t+1}/z_{it+1}}{u_{i2,t}/z_{it}}, \tag{2.11}$$

$$q_{t,t+1} = \beta \frac{u_{i2,t+1}/z_{it+1}}{u_{i2,t}/z_{it}}, \tag{2.12}$$

$$\frac{-u_{i2t}/z_{it}}{u_{int}} = \frac{1}{\xi_i\left(1-\tau_t^i\right)} \text{ for } t \geq 0, \tag{2.13}$$

$$\frac{u_{i1t+1}/\left(1-z_{it}\right)}{u_{i2t+1}/z_{it}} = Q_{t,t+1}^{-1} \equiv R_{t+1}, \tag{2.14}$$

$$\begin{aligned}(R_t - 1)\left(P_{t+1}c_{i1t+1}\left(1-z_{it}\right) - M_{it+1}\right) &= 0,\\ P_{t+1}c_{i1t+1}\left(1-z_{it}\right) &\leq M_{it+1},\end{aligned}$$

$$\left[\left(\frac{1}{\rho}-1\right)\left(1-R_s^{\frac{\rho}{\rho-1}}\right) - \frac{\theta\left(z_{is}\right)}{c_{i2s}}\right]\begin{cases}\leq 0 \text{ for } z_{is} = \underline{z},\\ = 0 \text{ for } z_{is} \in \left(\underline{z}, \bar{z}\right),\\ \geq 0 \text{ for } z_{is} = \bar{z}.\end{cases} \tag{2.15}$$

for $t \geq 0$, *and:*

$$P_0 c_{i10}\left(1-z_{i0}\right) \leq M_{i0}, \tag{2.16}$$

$$\begin{aligned}\hat{u}_{i10}\frac{M_{i0}}{P_0} + \hat{u}_{i20}\frac{B_{i(-1),0}}{P_0} + \hat{u}_{i20}\sum_{t=1}^{\infty}\frac{B_{i(-1),t}}{P_0}\prod_{j=1}^{t}R_j + \sum_{t=0}^{\infty}\beta^t\hat{u}_{i2t}b_{i(-1)t}\\ \leq \sum_{t=0}^{\infty}\beta^t\left[u_{i1t}c_{i1t} + u_{i2t}\hat{c}_{i2t} + u_{int}n_{it}\right].\end{aligned} \tag{2.17}$$

for $i = 1, 2$, *with* $C\left(z_{it}\right) = \int_{\underline{z}}^{z_{it}} \theta\left(j\right) dj$.

Here, $u_{ij} = \partial U\left(c_i, n_i\right)/\partial c_{ij}$, $u_{in} = U_2\left(c_i, n_i\right)$ and $\hat{c}_{i2} = c_{i2} + \frac{C(z_i)}{z_i}$, $\hat{u}_{i1} = u_{i1}/\left(1-z_i\right)$, $\hat{u}_{i2} = u_{i2}/z_i$ for $i, j = 1, 2$. Equation (2.17) is the households' intertemporal budget constraint and it incorporates the transversality condition. The proof of this proposition is in Appendix A.

3. Optimal Policy with Commitment

I define the Ramsey equilibrium as the private sector equilibrium which maximizes the government's objective function, given by the weighted sum of the households' lifetime utility. The Pareto weight on type i agents is η_i, with $\eta_1 + \eta_2 = 1$. I assume that Pareto weights are time-invariant. The case $\eta_i = \nu_i$ corresponds to a utilitarian government.

The Ramsey equilibrium can be characterized by solving a *primal problem*, where the government chooses an allocation at time 0 subject to the constraint that it constitutes a private sector equilibrium. This problem's choice variables are $\{c_{i1t}, c_{i2t}, n_{it}, z_{it}\}_{i=1,2,t\geq 0}$ and P_0. The level of P_0 determines the real value of nominal assets at time 0 and defines the boundary of the agents' intertemporal budget set. High values of P_0 amount to a tax on currency and outstanding nominal claims. The government is constrained to tax all nominal claims at the same rate. The extent to which each household is hit by this tax depends on the distribution of currency and nominal bonds at time 0.

Proposition 3.1. *An allocation $\{c_{i1t}, c_{i2t}, n_{it}, z_{it}\}_{i=1,2,t\geq 0}$ and values of $\{R_t\}_{t\geq 0}$ and P_0 constitute a Ramsey equilibrium if and only if they solve the primal problem:*

$$\max_{P_0,\{c_{i1t},c_{i2t},n_{it},z_{it}\}_{i=1,2,t\geq 0}} \sum_{t=0}^{\infty} \beta^t \sum_{i=1,2} \eta_i U\left(c_{it}, n_{it}\right)$$

subject to:

$$\frac{\hat{u}_{i1t}}{\hat{u}_{i2t}} = R_t, \text{ for } i = 1, 2, \tag{3.1}$$

$$R_t \geq 1, \tag{3.2}$$

$$\kappa\left(\frac{u_{12t}/z_{2t}}{u_{1nt}}\xi_1, \frac{u_{22t}/z_{2t}}{u_{2nt}}\xi_2\right) = 0, \tag{3.3}$$

(2.15) and (2.9) for all t, as well as (2.17) and (2.16).

The proof is in Appendix B.

3.1. Properties of Optimal Policy for $t > 0$

I first illustrate the key properties of Ramsey equilibrium policy for $t > 0$. To this end, I restrict (2.8) to the general affine class:

$$\kappa_1\left(1 - \tau_t^1\right) \geq \left(1 - \tau_t^2\right) + \kappa_0,\ \kappa_0 \geq 0,\ \kappa_1 \geq 1. \tag{3.4}$$

Constraint (3.4) imposes that the average labor income tax rate for high productivity households is higher than for low productivity households. Numerical results are presented in section 3.2 for more general constraints on distribution.

Proposition 3.2. *Assume:*

$$U(c,n) = h(c) + v(n), \tag{3.5}$$

and

$$b_{i(-1),t} = B_{i(-1),t} = 0, \ i = 1, 2, \ t > 0. \tag{3.6}$$

If constraint (3.4) is not binding, $R_t = 1$ for $t \geq 1$ satisfies the necessary conditions for optimality of the primal problem.

The proof is in Appendix B relies on the homotheticity of the consumption aggregator and is analogous to the proof of the optimality of the Friedman rule in the environment with a representative agent and distorting taxes analyzed by Christiano, Chari and Kehoe (1996). Proposition 3.2 also holds for general constraints on distribution as in (2.8). It also encompasses the case where (2.8) is dropped from the Ramsey problem.

Assumption (3.6) is imposed to rule out incentives for setting the nominal interest rate arising from the distribution of outstanding real and nominal debt. These incentives may arise since a tax on interest earnings from real bonds is not available and the present value of nominal debt depends on the path of nominal interest rates. A departure from the Friedman rule is a way to tax interest earnings from real bonds, since interest income accrues at the end of the period. From the standpoint of efficiency, it is optimal to tax interest income from real bonds,since outstanding real bond holdings at time 0 are given. Analogously, it is optimal to minimize the present value of nominal debt at any point in time. If outstanding government debt is positive/negative, this can be achieved by setting the nominal interest rate low/high. These consideration arise with a representative agent. With heterogeneous agents, distributional concerns also play a role. Then, it is the shadow value of aggregate real and nominal government debt that matters i.e. the weighted value of claims on the government, where the weight is the multiplier on the implementability constraint of each type. Assumption (3.6) makes the analysis comparable with Chari, Christiano and Kehoe (1996), who rule out real bonds and allow nominal debt of one period maturity only[4].

[4] In Alvarez, Kehoe and Neumeyer (2001) this problem does not arise since they only allow for consumption taxes, so that debt does not appear in the first order conditions that determine the optimality of the Friedman rule.

Proposition 3.3. *Assume (3.5) and (3.6). If (3.4) is binding, $R_t > 1$ for $t \geq 1$ in the Ramsey equilibrium.*

The proof is in Appendix B. A corollary of this result is that there exists a critical value of η_1, denoted by $\bar{\eta}_1$, such that for $\eta_1 \geq \bar{\eta}_1$, (3.4) will not bind, while it will be binding for $\eta_1 < \bar{\eta}_1$. This can be shown by noticing that the value of the multiplier on (3.4) in the primal problem and $\eta_1$are inversely related. It follows that the Friedman rule is optimal if an only if the weight in government preferences of type 1 -low productivity- households is high enough.

The intuition for this finding lies in the distributional consequences of departures from the Friedman rule. To see this, it is useful to define a type specific consumption price indexes, P_t^i, $\hat{P}_t^i$ for $i = 1, 2$ for $t > 0$:

$$P_t^i = \left[(1 - z_{it}) (R_t)^{\frac{\rho}{\rho - 1}} + z_{it} \right]^{\frac{\rho - 1}{\rho}},$$

$$\hat{P}_t^i = P_t^i + \frac{C(z_{i,t})}{c_{it}}. \tag{3.7}$$

P_i^i measures the cost in efficiency units of labor of one unit of the consumption aggregator c_i for given z_{it}. $\hat{P}_i^i$ measures the cost in efficiency units of labor of one unit of c_i when z_{it} solves (2.15), including the cost of z_{it} [5]. Optimality implies $\hat{P}_t^i \leq R_t$.

For a given level of R_t, (2.15) implies $z_{2t} > z_{1t}$ and $P_t^1 > P_t^2$ and $\hat{P}_t^1 \geq \hat{P}_t^2$. This implies that, for a given tax rate on labor income, the wedge between the marginal utility of leisure and the marginal utility of consumption is higher for low productivity households:

$$(1 - \tau_t) \frac{1}{\hat{P}_t^2} > (1 - \tau_t) \frac{1}{\hat{P}_t^1}. \tag{3.8}$$

[5] This price index is derived from the solution of the following static optimization problem:

$$\max_{c_{i1}, c_{i2}, z_i} \left[(1 - z_i) c_{i1}^{\rho} + z_i c_{i2}^{\rho} \right]^{1/\rho} \text{ subject to}$$

$$w = R c_{i1} (1 - z_i) + c_{i2} z_i + C(z_i),$$

where w is an exogeous endowment of real wealth. Let:

$$c_i = \left[(1 - z_i) c_{i1}^{\rho} + z_i c_{i2}^{\rho} \right]^{1/\rho},$$

and denote the expenditure function with $e(R; \theta)$ and the value function with $v(R; w, \theta)$. Then, the optimal value of c_i solves $c_i = v(R; w, \theta)$ and:

$$\hat{P}^i = \frac{e(R; w, \theta)}{c_i}.$$

Therefore, a departure from the Friedman rule is equivalent to a higher net real wage in efficiency units for high productivity households relative to low productivity households and amounts to redistribution in favor of high productivity households.

As with a representative agent, the intuition for proposition 3.2 and 3.3 lies in the principle of uniform commodity taxation. Atkinson and Stiglitz (1976) show that, if the labor income tax schedule is sufficiently unconstrained, then it is optimal to tax all commodities at the same rate, irrespective of the weighting of different agents in the social welfare function, when utility is weakly separable between consumption and leisure[6]. Their result is based on the following logic. Since the wedge between leisure and consumption is only affected by the sub-utility derived from consumption, as long as resources are scarce and there are no constraints on distribution, a benevolent government seeks to deliver this sub-utility in the cost minimizing way. Constraints on the labor income tax schedule may give rise to a conflict between efficiency and distribution, which induces the government to abandon uniform commodity taxation. For the model economy in this paper, unitary income elasticity of money demand implies that the cost minimizing way of delivering a given sub-utility from consumption is to follow the Friedman rule. However, this policy is sub-optimal when high productivity households have a high Pareto weight and the government faces constraints on redistribution in their favor by labor income taxation.

3.2. Numerical Findings

To evaluate the general impact of distributional incentives, I compute the Ramsey equilibrium as a function of the Pareto weights for a plausibly parametrized version of the economy. I focus on the utility specification:

$$U(c_i, n_i) = \frac{c_i^{1-\sigma} - 1}{1-\sigma} + v(n_i), \text{ for } i = 1, 2, \ \sigma > 0, \tag{3.9}$$

and I present results for:

$$\begin{aligned} v(n_i) &= \psi \log(1 - n_i), \ \psi > 0, & (3.10) \\ &= -\gamma n_i, \ \gamma > 0. & (3.11) \end{aligned}$$

The case corresponding to (3.11) is a useful benchmark. The absence of wealth effects on consumption implies that the distribution of cash holdings across agents

[6] In Atkinson and Stiglitz, imperfect information on the agents' labor productivity imposes incentive compatibility constraints on labor income taxation. Here, I abstract from asymmetric information and focus on exogenous constraints on labor tax rates.

at the end of any period only depends on government policy in the following period. When (3.11) is imposed, I restrict $0 < \sigma < 1$ to ensure that labor supply increases with the real wage. Parameter values are displayed below:

Table 1: Benchmark Parameter Values										
$\underline{z}$	$\overline{z}$	β	γ	ν_1	θ	σ	ψ	ρ	ξ_1	ξ_2
0.10	0.654	0.97	3	0.56	0.021	0.8	3	0.5	1	1.8

I set $\nu_1 = 0.56$, which roughly matches the percentage of US households having no financial assets other than a checking account, according to the 1995 Survey of Consumer Finances. I assume $\rho = 0.5$ and parameterize the transactions technology as follows:

$$\begin{aligned} \theta(j) &= 0 \text{ for } j \leq \underline{z}, \\ &= \theta \text{ for } j \in (\underline{z}, \overline{z}) \\ &= \infty \text{ for } j \geq \overline{z}, \end{aligned}$$

where $0 \leq \underline{z} < \overline{z} \leq 1$. θ is set so that $z_2 = \overline{z}$ for $R \geq 1.10$, and $z_1 = \overline{z}$ for $R \geq 1.50$. I set $\overline{z} = 0.65$ and $\underline{z}= 0.10$. This implies that at $R = 1.06$, money demand velocity in the model is equal to 2.89[7]. When (3.11) is imposed, the value of σ determines the interest elasticity of money demand. I set $\sigma = 0.8$. This implies that for both preference specifications the interest semi-elasticity of aggregate money demand is approximately equal to 4 at $R = 1.06$ [8]. The level of government consumption is set equal to 20% of total employment in equilibrium. Lastly, $\xi_1 = 1$ and $\xi_2 = 1.8$ imply that at a steady state with $R = 1.06$ and $\tau^i = 0.30$, labor income of high productivity households is 2.2 greater than labor income of low productivity households. This percentage is approximately equal to the value of this statistic for the US (see Erosa and Ventura, 2000).

I analyze the Ramsey equilibrium under different constraints on direct distribution.

First, I consider affine restrictions on tax rates. Figure 1 displays the features of Ramsey equilibrium policy as a function of η_1 for $t > 0$, under (3.11) and (3.4)

[7] Dotsey and Ireland (1996) report that average M1 velocity in the US for the post-war period is equal to 5.4.

[8] Computed as:

$$\frac{\partial \log(M/P)}{\partial \log(R)},$$

where M/P are aggregate real money balances. This number is within the range of estimates reported in the literature. For examples, Dotsey and Ireland (1996) report a value of this statistic of 5.9 in post-war US data. Mulligan and Sala-i-Martin (2001) estimate this statistic using cross-sectional household data and report numbers as low as 0.60 at $R = 1.05$.

with equality at $\kappa_1 = 1$ and $\kappa_0 = 0$ i.e. under the constraint that the labor income tax rate is the same across types. Initial real and nominal debt holdings are set at 0 and the distribution of currency is symmetric. The top right panel exemplifies the result in proposition 3.3. The solid line represents the Ramsey equilibrium net nominal interest rate and the dashed line the value of the multiplier on the distribution constraint. There is a value of the Pareto weight, $\bar{\eta}_1$, for which the constraint on tax rates is not binding. For $\eta_1 \geq \bar{\eta}_1$, the Friedman rule is optimal. The tax rate on labor is increasing in η_1, even for $\eta_1 > \bar{\eta}_1$. This is due to the fact that for higher η_1 the multiplier on the implementability constraint on type 2 falls (and the one for type 1 increases). This reduces the shadow cost of raising distortionary taxes from type 2 and induces a rise in the optimal tax rate. The tax rate on labor varies from 0.10 to 0.28, while the net nominal interest rate from 34% to 0.

Figure 2 displays the properties of the Ramsey equilibrium under (3.11) and (3.4) with inequality at $\kappa_1 = 1$ and $\kappa_0 = 0$, i.e.

$$\tau^2 \geq \tau^1. \tag{3.12}$$

The net nominal interest rate is decreasing in η_1 and the Friedman rule obtains only if the multiplier on direct distribution is 0. The tax rate on type 2 households increases with η_1. As long as the constraint on distribution is binding, the tax rate on type 1 agents also increases with η_1. For $\eta_1 \geq \bar{\eta}_1$, it is decreasing with η_1, since in this region of the Pareto space the government wishes to distribute to type 1 agents.

Figure 3 displays the features of Ramsey equilibrium policy as a function of η_1 for $t > 0$, under (3.11) and

$$T_{1t} \leq T_{2t} \text{ for all } t, \tag{3.13}$$

where $T_{it} = \tau_t^i \xi_i n_{it}$ for $i = 1, 2$.

Here, $\bar{\eta}_1$ corresponds to the value of η_1 for which the percentage of labor income tax revenues raised from type 1 (displayed in the bottom right panel) is equal to 50%. For this value of η_1, the multiplier on the distribution constraint is 0. The tax rate on type 2 agents increases with η_1, ranging from 0.05 to 0.52. The tax rate on type 1 decreases for $\eta_1 > \bar{\eta}_1$ and ranges from 0.10 to 0.23. The net nominal interest rate peaks at 26% for $\eta_1 = 0.35$. The behavior of Ramsey policy under (3.13) is very similar to the behavior of Ramsey policy under (3.12).

Figure 4 displays the Ramsey equilibrium for the following restriction on tax rates:

$$\frac{T_{1t}}{W_t \xi_1 n_{1t}} \leq \frac{T_{2t}}{W_t \xi_2 n_{2t}} \text{ for all } t, \tag{3.14}$$

which corresponds to progressive taxation. As before, the Friedman rule is optimal only when the constraint of distribution does not bind. The net nominal interest rate peeks at 26% for the lowest value of η_1. The tax rate on type 2 systematically raises with η_1, from 0.12 to 0.41, while the tax rate on type 1 is increasing in η_1 as long as the constraint on distribution is binding and falls with η_1 otherwise. Under progressive taxation, the maximum share of labor income tax revenues raised from type 1 is 43% and falls below 30% when the constraint on distribution is not binding. Similar results hold under (3.10).

Note that the value of $\bar{\eta}_1$ is different from ν_1, the weight of low productivity households in the population. In particular, $\bar{\eta}_1 < \nu_1$. This is because constraints on distribution as in (3.4) could be binding for efficiency considerations. To see this consider a proportional labor income tax. Since they have higher labor productivity, the distortion induced by the same labor tax rate will be higher for type 2 households. Therefore, even with no distributional objectives it would be optimal to tax labor income generated by high productivity households at a lower rate relative to low productivity households. This can be verified by solving for the value of the multiplier on the implementability constraint (2.17), which measures the shadow cost of raising distortionary taxation from type i agents, for $i = 1, 2$. The value of this variable for type 2 agents is higher than for type 1 agents, as shown in the numerical solution below.

4. Sufficient Conditions for Time Consistency

In this section, I illustrate the potential sources of time inconsistency and derive the sufficient conditions for time consistency of the Ramsey equilibrium. There are three main findings. If the Friedman rule is optimal, the Ramsey equilibrium is time consistent, irrespective of the type of debt that the government can issue. If the Friedman rule is not optimal, the Ramsey equilibrium is time consistent only if the government can issue both real and nominal debt of all maturities. Time consistency of the Ramsey equilibrium does not require the present value of nominal claims on the government to be 0.

The procedure to derive sufficient conditions for time consistency of the Ramsey equilibrium follows Lucas and Stokey (1983). For any $t \geq 0$, define the Ramsey problem at period t analogously to the Ramsey problem for period 0. The Ramsey problem at period t is said to be time consistent for period $t+1$, if the continuation allocation of the solution to the Ramsey problem at period t solves the Ramsey problem at $t+1$. The Ramsey equilibrium is time consistent if the Ramsey problem at time t is time consistent for the Ramsey problem at $t+1$ for $t \geq 0$. In practice, it is sufficient to verify that initial conditions for the time

1 problem exist that would induce the government at time 1 to continue with the allocation that solves the Ramsey problem at time 0.

Time inconsistency may arise in the Ramsey equilibrium given that decisions on asset holdings and on transaction patterns for $t = 1$ are sunk for a government optimizing at time 1. As in Lucas and Stokey (1983), there is a *real* and a *nominal* time inconsistency problem. A real time inconsistency problem may arise from the possibility of manipulating real interest rates and, therefore, the present value of real debt payments by changing the temporal pattern of labor income tax rates. A nominal time inconsistency problem may arise since the elasticity of the inflation tax base at time 1 is lower in the time 1 Ramsey problem relative to the time 0 Ramsey problem. The government's incentive to depart at time 1 from the continuation allocation implied by the solution to the time 0 Ramsey problem depends on the balance of efficiency and distributional concerns. The following propositions hold under Svensson timing.

Proposition 4.1. *If the government is subject to (3.4) and (3.6), the Ramsey equilibrium is time consistent if the Friedman rule is optimal.*

In addition, the following two corollaries hold.

Corollary 4.2. *If the Friedman rule is optimal and*

$$b_{t,j+j} = 0,\ B_{t,t+j} = 0 \text{ for } t = -1, 0, 1, .., \text{ and } j \geq 1, \tag{4.1}$$

is imposed, the Ramsey equilibrium is time consistent.

Corollary 4.3. *If the Friedman rule is optimal and (3.6) is relaxed, the Ramsey equilibrium is time consistent.*

Then, when the Friedman rule is optimal, the Ramsey equilibrium is always time consistent, irrespective of whether the government can issue real and nominal debt of different maturities.

If the Friedman rule is not optimal, time consistency does depend on the type of securities that the government can issue.

Proposition 4.4. *If the Ramsey equilibrium is time consistent and the government is subject to (3.4) and (3.6), the Friedman rule is optimal.*

Proposition 4.5. *If the Friedman rule is not optimal and (3.6) is relaxed, the Ramsey equilibrium is time consistent.*

The proofs of these results are in appendix C. As with a representative agent, the real time consistency problem can be solved by an appropriate choice of the maturity structure of real claims on the government for each type of household. The possibility of removing the nominal time inconsistency problem, even when the Friedman rule is not optimal, has roots in the distributional consequences of deviations from the continuation of the time 0 Ramsey equilibrium and in the possibility of manipulating the distribution of nominal claims on the government across agents.

Under the Svensson timing, an increase in P_1 amounts to a rise in the price of cash goods relative to credit goods. When the Friedman rule is optimal, the cash in advance constraint is non-binding in the continuation of the time 0 Ramsey equilibrium. This implies that the price elasticity of cash good consumption at time 1 is the same in the time 0 and in the time 1 Ramsey equilibrium for small changes in P_1. In addition, when the Friedman rule is optimal, $\eta_1 > \bar{\eta}_1$, by proposition 3.3. Since type 1 households hold more currency as a fraction of their total purchases, distributional considerations weaken the incentive to increase P_1 enough to make the cash in advance constraint binding at $t = 1$ in the time 1 Ramsey equilibrium. The distributional cost of unanticipated inflation for type 1 households makes the Ramsey equilibrium time consistent under much less stringent conditions than with a representative agent. In particular, when the Friedman rule is optimal, the Ramsey equilibrium is time consistent even when the government cannot issue nominal and real debt of all maturities.

When the Friedman rule is not optimal, the cash-in-advance constraint is binding in the continuation of the time 0 Ramsey equilibrium. Therefore, the price elasticity of cash good consumption at time 1 is lower in the time 1 Ramsey equilibrium relative to the time 0 Ramsey equilibrium. In addition, given that the Friedman rule is not optimal for $\eta_1 < \bar{\eta}_1$, and a rise in the relative price of cash goods redistributes to type 2 agents, both distribution and efficiency objectives generate an incentive to increase P_1. Therefore, the Ramsey equilibrium is not time consistent if the government cannot issue real and nominal debt of all maturities. However, if the government can issue real and nominal debt of all maturities, than the Ramsey equilibrium can be made time consistent. It is always possible to find a maturity structure and a distribution of nominal claims on the government (other than currency) such that removes the incentive to increase P_1 in the time 1 Ramsey equilibrium.

It is useful to note that the maturity structure of nominal debt is irrelevant for time consistency or for any other feature of the Ramsey equilibrium when the Friedman rule is optimal. That is because it is only the present value of nominal claims on the government in any period that matters for nominal time inconsistency, given that a change in the price level taxes payments on nominal claims in all future periods at the same rate.

In addition, the following result holds.

Proposition 4.6. *The present value of nominal government liabilities need not be 0 in a time consistent Ramsey equilibrium.*

The proof of this proposition, presented in Appendix C, shows that the sufficient conditions for time consistency of the Ramsey equilibrium pin down the shadow present value of nominal claims on the government, while the actual present value of nominal claims on the government is typically different from 0. The shadow present value of nominal claims on the government is a weighted sum of the present value of nominal claims held by each type of household, where the weight is given by the value of the multiplier on the implementability constraint.

In figure 5, I display an example distribution of currency and nominal and real claims on the government at $t = 1$ that make it optimal for the government to continue with the time 0 Ramsey equilibrium at $t = 1$. Constraint (3.4) is imposed as a weak inequality and the case $\eta_1 \geq \bar{\eta}_1$ is examined, so that the Friedman rule is optimal. Parameters values are in Table 1 and disutility of leisure satisfies (3.11). The initial conditions for the time 0 Ramsey equilibrium are $M_{1,0} = M_{2,0}$ and $b_{i(-1),t} = B_{i(-1),t} = 0$ for both i and all t. Here, I consider the class of currency and asset holdings for which the value of the multipliers of the implementability constraint is the same in the time 0 and the time 1 Ramsey equilibrium. Asset positions are asymmetric and this guarantees that it is optimal for the government to continue with the outcome of the time 0 Ramsey equilibrium. Type 1 households do not hold any nominal claims on the government (except currency), while type 2 households are net nominal debtors to the government. An increase in the price level at time 1 would therefore amount to a transfer to type 2 households, which is suboptimal given $\eta_1 \geq \bar{\eta}_1$. This takes care of nominal time inconsistency. Note that total nominal claims on the government are positive and range between 30% and 45% of total output. Both types of households hold real claims against the government at time 1. Holdings of real debt by type 2 agents are larger which makes an increase in real interest rate between time 1 and time 2, which obtains in equilibrium if the tax rate on labor income is increased at time 1, suboptimal. This takes care of real time inconsistency.

Appendix D works out sufficient conditions for time consistency under the timing convention adopted in Lucas and Stokey (1983). Analogous results hold in this case, despite the fact that unanticipated changes in the price level are lump-sum with this timing.

5. Empirical Relevance

The distribution of nominal wealth and the distribution of political power among classes of agents with different exposure to the effects of inflation played a crucial role in shaping monetary policy decisions in a number of historical episodes of large inflations and deflations.

Johnson (1970) provides a detailed description of the behavior of inflation in England in the aftermath of the Glorious Revolution:

> "When the Bank of England received its charter [1694] ... its directors cultivated all possible contact with parlamentarians, on whom they relied for periodic renewal of the charter."
>
> "Fierce dispute broke out as to what ... should be the remedy. To return to the good old standard would mean ... bankruptcy of many in trade and enrichment of old creditors. Devaluation would ... protect and stabilize domestic trade though initially hit the foreign trader. ... With landed property predominant in government the issue was never in doubt. The recoinage of 1897-1698 returned to Elisabeth's silver standard."
>
> The same political forces played a role in successive episodes of deflation in England, for example in 1815, as Johnson (1970) reports:
>
> "The unitary in monetary interest in the gold standard... included ...the owners of Consols sold to finance the war with Napoleon at a time of skyhigh prices and interest rates. ...In returning to gold, Lord Liverpool thus handed a large bonus to the landed gentry and to a new monied middle class".

Redistributional concerns were also crucial in the large monetization which occurred in France after the Revolution in 1789. White (1896) reports the following:

> "Mirabeu..showed that he was fully aware of the dangers of inflation, but he yielded to the pressure... partly because he thought it important to sell government lands rapidly to the people, and so develop speedily a large class of landholders, pledged to stand by the government who gave them their titles."
>
> "This outgrowth [in money] was the creation of a great debtor class in the nation, directly interested in the depreciation of the currency in which their debts were to be payed. The nucleus of this debtor class was formed by those who had purchased the Church lands from the Government"

Sargent and Velde (1995) document that the downpayments required to purchase church lands were in the range $12 - 30\%$. The rest of the payment was arranged through promissory notes repaid annually over a period ranging between 10 and 12 years at 5% interest.

Hamilton (1788) also highlights the importance of redistributional concerns for the credibility of government debt policy:

> "There are even dissimilar views ... as to the general principle of discharging the public debt. Some of them, either less impressed with the importance of national credit, or because they have little, if any, immediate interest in the question, feel an indifference, if not a repugnance, to the payment of the domestic debt at any rate. ... Others of them, a numerous body of whose citizens are creditors to the public beyond proportion ... in the total amount of the national debt, would be strenuous for some equitable and effective provision."

Based on this view, Hamilton (1795) argued in favor of the Federal assumption of the states' war debt. Debt assumption would provide powerful government creditors with a strong incentive to support the establishment of a Federal tax legislation, thus decreasing the risk of default or monetization.

Bordo and Vegh (2002) study fiscal and monetary policy in Argentina for the period 1810-1867 and compare it the policy implemented in the US during the same period. They ascribe the reliance on monetary financing of government expenditures in Argentina to the political influence of constituencies benefitting from inflation.

> "In Argentina, the relaxation of fiscal constraints related to the tax structure and debt financing was made difficult by the presence of dominant political groups who benefitted from inflation- mainly cattle ranchers who profited from the ensuing political depreciation. These groups would oppose raising conventional taxes to retire either debt or notes issued, since they viewed inflation as a more convenient way (from their standpoint) of raising revenue."

Faust (1996) documents that the Federal Reserve Bank's structure is a response to public conflict over inflation's distributional consequences. Demands for debt relief through surprise inflation animated the US political debate from the Revolutionary War, through the *free silver* debate (1870-80's), up until the FED's founding in 1935. The intent behind the Fed's internal power structure is to balance voting power of the financial, agricultural, industrial and commercial interests in the US. The view underlying the distribution of voting rights was, in the words of J. Laurence Laughlin, a monetary economist writing in 1933, that:

> "Politicians find it easy to appeal to the underlying prejudice in favor of inflation in order to ... lift the burden of debt."

The 1935 debates over how to divide FOMC voting power between politically appointed governors and federal reserve president reflect this clearly. Steagall (Congretional Record, 1935, p13706) summarizes it as follows:

> "Under the bill ... the board will stand 5 to 7 giving the people of the country, as contradistinguished by private banking interests, control by a vote of 7 to 5 instead of by a vote of 3 to 2 [as proposed by the Senate]"

Caselli (1997) finds that, for a sample of highly indebted OECD countries in the time period 1970-1990, the interest cost of public debt, which is presumably inversely related to the perceived credibility of the government, depends positively on asymmetries in the distribution of taxes and negatively on the degree of identification of the government with a specific constituency.

6. Concluding Remarks

I describe a monetary economy in which households have different labor productivity, which implies that they are heterogeneous in transaction patterns and asset holdings. Heterogeneity implies that monetary policy has distributional effects and the time consistency of the Ramsey equilibrium depends on the balance between distribution and efficiency. I find that heterogeneity breaks the link between high inflation and time inconsistency of optimal fiscal and monetary policy. First, due to the distributional impact of expected inflation surprisingly high rates of inflation may be optimal even with commitment. Therefore, credibility of government policy does not imply low inflation. On the other hand, due to the distributional costs of unanticipated inflation, the optimal inflation rate under discretion is not necessarily higher than in the Ramsey equilibrium. In particular, optimal fiscal and monetary policy are time consistent in the sense of Lucas and Stokey (1983). If the Friedman rule is optimal, the possibility of making the Ramsey equilibrium time consistent does not depend on the ability of the government to issue real and nominal claims of different maturities. Optimal fiscal and monetary policy are time consistent even if the government only issues currency, when the Friedman rule is optimal. If the Friedman rule is not optimal, then optimal fiscal and monetary policy can be made time consistent only if the government can issue real and nominal debt of all maturities. Time consistency does not require that the present value of nominal claims on the government is 0.

These findings contrast with the results for a representative agent economy. Lucas and Stokey (1983) show that time consistency of optimal fiscal and monetary policy in general are not time consistent. This is because avoiding nominal time inconsistency requires that outstanding nominal claims on the government are 0, but this rules out the possibility of avoiding real time inconsistency. Alvarez, Kehoe and Neumeyer (2001) describe a monetary economy with a representative agent where optimal fiscal and monetary policy can be made time consistent, despite the constraint that nominal claims on the government are 0,if and only if the Friedman rule is optimal. This is because, when the Friedman rule is optimal, a monetary economy is equivalent to a real economy, so that real claims of all maturities are sufficient to solve the real time inconsistency problem and the constraint that nominal claims on the government are 0 does not bind.

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7. Appendix

7.1. A: Characterization of Private Sector Equilibria

Assume that an allocation $\{c_{i1t}, c_{i2t}, n_{it}, z_{it},\ M_{it+1}, b_{it,t+s}, B_{it,t+s}\}_{i=1,2,t\geq 0,s>0}$, with $n_{it} > 0$ for $i = 1,2$ and $t \geq 0$, and a price system $\{P_t,\ W_t,\ Q_{t,t+s},\ q_{t,t+s},\ \pi_t(j)\}_{t\geq 0, j\in[0,1]}$ constitute a private sector equilibrium for a given policy $\{\bar{g}_t, \tau_t, M_{t+1}, B_{t+1}\}_{t\geq 0}$. Then, conditions (2.1) and (2.2) derive from optimality of firm behavior, conditions (2.9) and (2.10) from clearing in the goods and assets markets. The other conditions follow from household optimization.

The Lagrangian for the household problem is given by:

$$\begin{aligned} L &= \sum_{t=0}^{\infty} \beta^t \Big\{ u^i(c_{it}, n_{it}) - \mu_{it}\left(P_t c_{i1t}(1-z_{it}) - M_{it}\right) \\ &\quad -\lambda_{it} \Bigg[M_{it+1} + \sum_{s>0} \left(Q_{t,t+s} B_{it,t+s} + q_{t,t+s} P_t b_{it,t+s}\right) - M_{it} - \sum_{\hat{t}=0}^{t-1} \left(B_{i\hat{t},t} + P_t b_{i\hat{t},t}\right) \\ &\quad - W_t(1-\tau_t)\xi_i n_{it} + P_t c_{i1t}(1-z_{it}) + P_t c_{i2t} z_{it} + \int_0^{z_{it}} \pi_t(j)\, dj \Bigg] \Big\}, \end{aligned}$$

where c_{it} is defined in (2.3) and μ_{it}, λ_{it} are the multipliers on the cash in advance constraint and the wealth evolution equation, respectively. Denote with u_{ijt} and u_{int} the marginal utility of good j and of labor for households $i = 1, 2$.

The necessary conditions for household optimization are given by:

$$u_{i1t} = P_t(\mu_{it} + \lambda_{it})(1 - z_{it}), \tag{7.1}$$

$$\mu_{it}\left(P_t c_{it}(1 - z_{it}) - M_{it}\right) = 0,\ \mu_{it} \geq 0, \tag{7.2}$$

$$u_{i2t} = P_t \lambda_{it} z_{it}, \tag{7.3}$$

$$-u_{int} = W_t(1 - \tau_t)\xi_i \lambda_{it}, \tag{7.4}$$

$$P_t c_{i1t}(\mu_{it} + \lambda_{it}) - P_t c_{i2t}\lambda_{it} - q_t(z_{it})\lambda_{it} \begin{cases} < 0 \text{ for } z_{it} = \underline{z}, \\ = 0 \text{ for } z_{it} \in (\underline{z}, \bar{z}), \\ > 0 \text{ for } z_{it} = \bar{z}, \end{cases} \tag{7.5}$$

$$\lambda_{it} = \beta(\lambda_{it+1} + \mu_{it+1}), \tag{7.6}$$

$$\lambda_{it} Q_{t,t+1} = \beta \lambda_{it+1}, \tag{7.7}$$

$$\lim_{T\to\infty} \beta^T \lambda_{iT} M_{iT} = 0,\ \lim_{T\to\infty} \beta^T \lambda_{iT} B_{it,T} = 0, \tag{7.8}$$

as well as (2.4) and (2.5). To see that (7.8) is a necessary condition for household optimization, suppose it does not hold and

$$\lim_{T\to\infty} \beta^T \lambda_{iT} M_{iT} > 0, \ \lim_{T\to\infty} \beta^T \lambda_{iT} B_{it,T} > 0.$$

(The strictly smaller case is rule out by (2.6).) Then, it is possible to construct a consumption sequence such that the budget constraint is satisfied in each period and utility for each type of household is greater, violating optimality.

Combining (7.1)-(7.3) yields (2.14), while (7.3) and (7.4) determine (7.11). The expression in (2.11) follows from (7.4), (7.7) and (2.1), while (2.16) follows from (7.1)-(7.3) at $t = 0$.

To derive (2.17), multiply (2.5) by λ_{it} and apply (7.2) and (7.6). Use (7.1), (7.3)-(7.5), multiply by β^t and sum over t from 0 to T. Let T go to infinity and apply (7.8). This yields:

$$\sum_{t=0}^{\infty} \beta^t \left(u_{i1t} c_{i1t} + u_{i2t} \left(c_{i2t} + \frac{C(z_{it})}{z_{i,t}} - \frac{B_{i(-1),t}}{P_t z_{it}} - \frac{b_{i(-1),t}}{z_{it}} \right) + u_{int} n_{it} \right) = \frac{u_{i10}}{1 - z_{i0}} \frac{M_{i0}}{P_0}. \tag{7.9}$$

From (7.6)-(7.7):

$$P_t = \beta^t \frac{\hat{u}_{i2t}}{\hat{u}_{i20}} P_0 \prod_{j=1}^{t} R_j \text{ for } t > 1,$$

with $\prod_{j=1}^{1} R_j \equiv R_1$, $\prod_{j=1}^{0} R_j \equiv 1$. Substitute into (7.9), to obtain (2.17).

Now assume that an allocation $\{c_{i1t}, c_{i2t}, n_{it}, z_{it}, M_{it+1}, B_{it+1}\}_{i=1,2,t\geq 0}$, with $n_{it} > 0$ for $i = 1, 2$ and $t \geq 0$, and a price system $\{P_t, W_t, Q_t, q_t(j)\}_{t\geq 0, j\in[0,1]}$ satisfy (2.1)-(2.17) and (2.9) for a given policy $\{\bar{g}_t, \tau_t, M_{t+1}, B_{t+1}\}_{t\geq 0}$ for which (2.7) holds. Then, by (2.1) and (2.12) industrial and credit services firms optimize.

To see that household optimization conditions are satisfied consider an alternative candidate plan $\{c'_{i1t}, c'_{i2t}, n'_{it}, z'_{it}\}_{i=1,2,t\geq 0}$ which satisfies the intertemporal budget constraint for the price system $\{P_t, W_t, Q_t, q_t(j)\}_{t\geq 0, j\in[0,1]}$. This implies that:

$$\Delta \equiv \lim_{T\to\infty} \beta^t \left\{ u_{i1t} \left(c_{i1t} - c'_{i1t} \right) + u_{i2t} \left(c_{i2t} + \frac{C(z_{it})}{z_{it}} - c'_{i2t} - \frac{C(z'_{it})}{z'_{it}} \right) - \gamma \left(n_{it} - n'_{it} \right) \right\} \geq 0,$$

using (2.11) and the fact that $\{c_{i1t}, c_{i2t}, n_{it}, z_{it}\}_{i=1,2,t\geq 0}$ satisfies (2.14)-(2.17) and that the intertemporal budget constraint holds as a weak inequality using (2.6) and (2.5) for the price system $\{P_t, W_t, Q_t, q_t(j)\}_{t\geq 0, j\in[0,1]}$. By concavity of u^i :

$$D \equiv \lim_{T\to\infty} \sum_{t=0}^{T} \beta^t \left(u^i \left(c_{it}, n_{it} \right) - u^i \left(c'_{it}, n'_{it} \right) \right) \geq \Delta,$$

where c'_{it} is defined by (2.3). This establishes the result since (2.10) and (2.9) guarantee market clearing.
Proof. [Proof of 3.1] TBA ■

7.2. B: Solving the Ramsey problem

The Lagrangian for the Ramsey problem can be written as:

$$\begin{aligned}
\Lambda &= \sum_{t=0}^{\infty}\beta^t \sum_i \left\{\eta_i U\left(c_{it}, n_{it}\right) + \lambda_i \left(u_{i1t}c_{i1t} + u_{i2t}\hat{c}_{i2t} + u_{int}n_{it}\right)\right\} \\
&\quad -\sum_{t=1}^{\infty}\beta^t \left[\mu_t\left(1-R_t\right) + \sum_i \mu_{it}\left(\frac{\hat{u}^i_{1t}}{\hat{u}^i_{2t}} - R_t\right)\right] - \sum_{t=0}^{\infty}\beta^t\zeta_t\left(\kappa_1 \frac{u^1_{nt}}{\xi_1 \hat{u}^1_{2t}} + \kappa_0 - \frac{u^2_{nt}}{\xi_2\hat{u}^2_{2t}}\right) \\
&\quad -\sum_i \lambda_i\left(\hat{u}_{i1,0}\frac{M_{i0}}{P_0} + \sum_{t=0}^{\infty}\beta^t \hat{u}_{i2t}b_{i(-1),t} + \hat{u}_{i20}\sum_{t=0}^{\infty}\frac{B_{i(-1),t}}{P_0}\prod_{j=1}^{t}R_j\right) \\
&\quad -\sum_i \mu_{i0}\left(c^i_{10}\left(1-z_{i,0}\right) - \frac{M^i_0}{P_0}\right),
\end{aligned}$$

with choice variables $c_{i1,t}$, $c_{i2,t}$, $z_{i,t}$, n_{it}, R_t, P_0.

The first order conditions for the Ramsey problem at time 0 are as follows.

For $t > 0$ and $i = 1, 2$:

$$0 = \eta_i u_{i1t} + \lambda_i\left(u_{i1t} + u_{i11t}c_{i1t} + u_{i12,t}c_{i2,t}\right) - \frac{\mu_{it}}{\hat{u}_{i2t}}\hat{u}_{i11t} - \nu_i\left(1 - z_{i,t}\right)\omega_t, \quad (7.10)$$

$$\begin{aligned}
0 &= \eta_i u_{i2t} + \lambda_i\left(u_{i12,t}c_{i1,t} + u_{i2,t} + u_{i22t}\hat{c}_{i2t}\right) - \nu_i z_{i,t}\omega_t \qquad (7.11)\\
&\quad + \frac{\mu_{it}}{\hat{u}_{i2t}}\frac{\hat{u}_{i1t}}{\hat{u}_{i2t}}\hat{u}_{i22t} + \frac{\zeta_{it}}{\hat{u}_{i2t}}\frac{u_{int}}{\hat{u}_{i2t}}\hat{u}_{i22t} \\
&\quad -\lambda_i\hat{u}_{i22t}b_{i(-1),t},
\end{aligned}$$

$$\zeta_{1t} = \kappa_1\frac{\zeta_t}{\xi_1},\ \zeta_{2t} = -\frac{\zeta_t}{\xi_2}, \quad (7.12)$$

$$\mu_t\left(1-R_t\right) = 0,\ \mu_t \geq 0,\ r_t \geq 1, \quad (7.13)$$

$$\mu_{it}\left(\frac{\hat{u}_{i1t}}{\hat{u}_{i2t}} - R_t\right) = 0,$$

$$\mu_t + \sum_i\left(\mu_{it} - \frac{\lambda_i\hat{u}_{i2,0}}{R_t}\sum_{s=t+1}^{\infty}\frac{B_{i(-1),s}}{P_0}\prod_{j=1}^{s}R_j\right) = 0. \quad (7.14)$$

For $t \geq 0$:

$$0 = \eta_i u_{int} + \lambda_i \left(u_{int} + u_{innt} n_{it} \right) - \frac{\zeta_{it}}{\hat{u}_{i2t}} u_{innt} + \nu_i \omega_t, \tag{7.15}$$

$$\zeta_t \left(\kappa_1 \frac{u_{nt}^1}{\xi_1 \hat{u}_{2t}^1} + \kappa_0 - \frac{u_{nt}^2}{\xi_2 \hat{u}_{2t}^2} \right) = 0,\ \kappa_1 \frac{u_{nt}^1}{\xi_1 \hat{u}_{2t}^1} + \kappa_0 - \frac{u_{nt}^2}{\xi_2 \hat{u}_{2t}^2} \leq 0,\ \zeta_t \geq 0. \tag{7.16}$$

For $t = 0$:

$$0 = \eta_i u_{i1,0} + \lambda_i \left(u_{i1,0} + u_{i11,0} c_{i1,0} + u_{i12,0} c_{i2,0} \right) - \mu_{i0} \left(1 - z_{i,0} \right) - \lambda_i \hat{u}_{i11,0} M_{i,0} - \nu_i \left(1 - z_{i,0} \right) \omega_0, \tag{7.17}$$

$$\begin{aligned} 0 \ = \ & \eta_i u_{i20} + \lambda_i \left(u_{i12,0} c_{i1,0} + u_{i20} + u_{i220} c_{i20} \right) - \nu_i z_{i,0} \omega_0 \\ & + \frac{\zeta_{i0}}{\hat{u}_{i20}} \frac{u_{in0}}{\hat{u}_{i20}} \hat{u}_{i220} \\ & - \lambda_i \hat{u}_{i22,0} \left(b_{i(-1),0} + \sum_{t=0}^{\infty} \frac{B_{i(-1),t}}{P_0} \prod_{j=1}^{t} R_j \right), \end{aligned} \tag{7.18}$$

$$\sum_{i=1,2} \left(-\lambda_i \hat{u}_{i1,0} M_{i0} - \lambda_i \hat{u}_{i20} \sum_{t=0}^{\infty} \frac{B_{i(-1),t}}{P_0} \prod_{j=1}^{t} R_j + \mu_{i0} M_{i0} \right) \left(\frac{-1}{P_0^2} \right) = 0, \tag{7.19}$$

$$\mu_{i0} \left(c_{i10} \left(1 - z_{i,0} \right) - \frac{M_{i0}}{P_0} \right) = 0,\ \mu_{i0} \geq 0,\ c_{i10} \left(1 - z_{i,0} \right) \leq \frac{M_{i0}}{P_0}.$$

Proof of Proposition 3.2

Combining (7.10) and (7.11) yields:

$$R = \max \left\{ 1, \frac{\eta_i + \lambda_i + \lambda_i \frac{\left(\hat{u}_{i21} c_{i1} + \hat{u}_{i22} \hat{c}_{i2} \right)}{\hat{u}_{i2}} + \zeta_i \frac{u_{in}}{\hat{u}_{i2}} \frac{\hat{u}_{i22}}{\hat{u}_{i2} z_i} + \mu_i R \frac{\hat{u}_{i22}}{\hat{u}_{i2} z_i} - \lambda_i \frac{\hat{u}_{i22}}{\hat{u}_{i2} z_i} b_i}{\eta_i + \lambda_i + \lambda_i \frac{\left(\hat{u}_{i12} \hat{c}_{i2} + \hat{u}_{i11} c_{i1} \right)}{\hat{u}_{i1}} - \mu_i R \frac{\hat{u}_{i11}}{\hat{u}_{i1} (1 - z_i)}} \right\} \tag{7.20}$$

If (3.4) is not binding, $\zeta_t = 0$ for $t \geq 0$. Consider the relaxed Ramsey problem where constraints (3.1) and (3.2) are not imposed. By homotheticity of h^i and the fact that $C\left(\underline{z}\right) = 0$ at $\hat{u}_{i1} = \hat{u}_{i2}$ by (2.15):

$$\frac{u_{i11} c_{i1} + u_{i21} c_{i2}}{u_{i1}} = \frac{u_{i12} c_{i1} + u_{i22} c_{i2}}{u_{i2}},$$

or equivalently:

$$\frac{\hat{u}_{i11} c_{i1} + \hat{u}_{i21} \hat{c}_{i2}}{\hat{u}_{i1}} = \frac{\hat{u}_{i12} c_{i1} + \hat{u}_{i22} \hat{c}_{i2}}{\hat{u}_{i2}}. \tag{7.21}$$

Hence, (3.6) and (7.20) imply $R = 1$. The solution to the relaxed Ramsey problem satisfies the Ramsey problem when constraint (3.4) is not binding. Therefore, $R = 1$ is a necessary condition for the solution to that problem. QED

Note that the Friedman rule satisfies the first order necessary conditions for the primal problem as long as:

$$-\lambda_i \hat{u}_{i22t} b_{i(-1),t} \leq 0 \tag{7.22}$$

holds for all t and $i = 1, 2$. This conditions states that the shadow value of taxing interest income from holdings of real bonds is negative for both types of households.

Proof of Proposition 3.3 Suppose to the contrary that $\zeta > 0$ and $R = 1$. Then, by (7.20) and $\mathrm{C}(z_i) = 0$ for $R_t = 1$:

$$+\zeta_i \frac{u_{in}}{(\hat{u}_{i2})} \frac{\hat{u}_{i22}}{\hat{u}_{i2} z_i} = -\mu_i R \left(\frac{\hat{u}_{i22}}{\hat{u}_{i2} z_i} + \frac{\hat{u}_{i11}}{\hat{u}_{i1} (1 - z_i)} \right),$$

for $i = 1, 2$. This simplifies to:

$$+\zeta_i \frac{u_{in}}{\hat{u}_{i2}} = -\mu_i \left(1 + \frac{\hat{u}_{i11} / (1 - \underline{z})}{\hat{u}_{i22} / \underline{z}} \right).$$

Since at $R = 1$, $\frac{\hat{u}_{i11}/(1-\underline{z})}{\hat{u}_{i22}/\underline{z}}$ is the same for both i, dividing through by $\left(1 + \frac{\hat{u}_{i11}/(1-\underline{z})}{\hat{u}_{i22}/\underline{z}}\right)$, summing the above equality across i and using (7.16), yields:

$$-\zeta \kappa_0 = -\left(1 + \frac{\hat{u}_{i11} / (1 - \underline{z})}{\hat{u}_{i22} / \underline{z}} \right) \sum_i \mu_i.$$

This implies $-\sum_i \mu_i \leq 0$. By (7.14) and (3.6), $-\sum_i \mu_i = \mu_t$, and $\mu_t \geq 0$, this is a contradiction. Hence, $\zeta > 0$ implies $R > 1$.QED

Note that (7.15) and (7.12) imply that η_1 and ζ_t are inversely related. Then, there exists a critical value of η_1, denoted by $\bar{\eta}_1$, such that for $\eta_1 \geq \bar{\eta}_1$, (3.4) will not bind, while it will be binding for $\eta_1 < \bar{\eta}_1$. For example, it can be verified that, for $u_{in} = -\gamma$ and $\kappa_0 = 0$, it is defined by the following equation:

$$\frac{\xi_2}{\xi_1} \kappa_1 = \frac{\bar{\eta}_1}{\nu_1} \left(\frac{\bar{\eta}_2}{\nu_2} \right)^{-1},$$

where $\bar{\eta}_2 = 1 - \bar{\eta}_1$.

7.3. C: Sufficient Conditions for Time Consistency

Proof. [Proof of 4.1] If the Friedman rule is optimal, the distribution constraint is not binding so that $\zeta_t = 0$ for $t \geq 0$, and the Ramsey equilibrium allocation solves the relaxed problem where the constraints corresponding to multipliers μ_t, μ_{it} for $t > 0$ are dropped. Under (3.6), the first order conditions for the continuation allocation of the time 0 Ramsey problem are given by:

$$0 = \eta_i u_{i1t} + \lambda_i \left(u_{i1t} + u_{i11t}c_{i1t} + u_{i21t}\hat{c}_{i2t}\right) - \nu_i\omega_t\left(1 - z_{i,t}\right), \tag{7.23}$$

$$0 = \eta_i u_{i2t} + \lambda_i \left(u_{i12t}c_{i1t} + u_{i2t} + u_{i22t}\hat{c}_{i2t}\right) - \nu_i\omega_t z_{i,t}, \tag{7.24}$$

$$0 = \eta_i u_{int} + \lambda_i \left(u_{int} + u_{innt}n_{it}\right) + \nu_i\xi_i\omega_t, \tag{7.25}$$

plus (2.15) and (7.22) for $t \geq 1$ and $i = 1, 2$. If the Friedman rule is optimal in the time 0 Ramsey equilibrium under (3.6), it will also be optimal in the time 1 Ramsey equilibrium for $t > 1$, since the conditions for optimality of the Friedman rule do not depend on the value of the multipliers in the implementability constraints (see the proof of proposition 3.3). In addition, by $g_t = g$ for all t, the continuation allocation for the time 0 Ramsey problem is stationary. Then, the solution to the time 1 Ramsey problem must be stationary for $t \geq 1$ for time consistency. In addition, $\mu'_{it} = 0$ for $t > 1$, $\zeta'_t = 0$, $z_{i,t} = \underline{z}$ and $\omega'_t = \omega_t$ for $t > 1$, where a prime denotes the multipliers associated with the time 1 Ramsey equilibrium, solve the analogue of (7.23)-(7.25) for $t > 1$ in the continuation of the time 1 Ramsey equilibrium at the continuation allocation for the time 0 Ramsey equilibrium. The first order conditions for $t = 1$ in the time 1 Ramsey problem under (3.6) are given by:

$$0 = \eta_i u_{i1,1} + \lambda'_i\left(u_{i1,1} + u_{i11,1}c_{i1,1} + u_{i2,1}\hat{c}_{i2,1}\right) - \mu'_{i1}\left(1 - z_{i,1}\right) - \lambda'_i\hat{u}_{i11,1}\frac{M_{i,1}}{P_1} - \nu_i\omega'_1\left(1 - z_{i,1}\right), \tag{7.26}$$

$$\begin{aligned} 0 \;=\; & \eta_i u_{i2,1} + \lambda'_i\left(u_{i1,1}c_{i1,1} + u_{i21} + u_{i22,1}\hat{c}_{i2,1}\right) - \nu_i\omega'_1 z_{i,1} \\ & + \frac{\zeta'_{i1}}{\hat{u}_{i2,1}}\frac{u_{in1}}{\hat{u}_{i2,1}}\hat{u}_{i22,1} \\ & - \lambda'_i\hat{u}_{i22,1}\left(b_{i0,1} + \frac{B_{i0,1}}{P_1}\right), \end{aligned} \tag{7.27}$$

$$\sum_{i=1,2}\left(-\lambda'_i\hat{u}_{i1,1}\frac{M_{i,1}}{P_1} - \lambda'_i\hat{u}_{i2,1}\frac{B_{i0,1}}{P_1} + \mu'_{i1}\frac{M_{i,1}}{P_1}\right)\left(\frac{-1}{P_1}\right) = 0, \tag{7.28}$$

$$\mu'_{i1}\left(c_{i1,1}\left(1 - z_{i,1}\right) - \frac{M_{i,1}}{P_1}\right) = 0,\ \mu'_{i1} \geq 0,\ c_{i1,1}\left(1 - z_{i,1}\right) \leq \frac{M_{i1}}{P_1}, \tag{7.29}$$

$$P_1 = P_0 \beta \frac{\hat{u}_{i1,1}}{\hat{u}_{i2,0}}, \tag{7.30}$$

plus (7.15) rewritten for $t \geq 1$, and (7.10), (7.11), (2.15) with primed multipliers appropriately replacing the ones for the time 0 Ramsey problem. $z_{i,1}$ is chosen ahead to government re-optimization and it is taken as given.

I need to show that it is possible to find $\mu'_{i,1}$, ζ'_1, ω'_1 as well as λ'_i, $B_{i0,1}$, $b_{i0,1}$ and M_{i1} so that the continuation allocation for the time 0 Ramsey problem solves the time 1 Ramsey problem. Optimality of the Friedman rule for $t \geq 1$ in the time 0 Ramsey equilibrium implies that we can set $\mu'_{i1} = 0$ for $i = 1, 2$. Using this in (7.28):

$$\sum_{i=1,2} \left(-\lambda'_i \hat{u}_{i2,1} \frac{B_{i0,1}}{P_1} - \lambda'_i \hat{u}_{i1,1} \frac{M_{i,1}}{P_1} \right) = 0. \tag{7.31}$$

(7.31) pins down the distribution of $B_{i0,1}$ as a function of λ'_i and $M_{i,1}$ for $i = 1, 2$. The distribution of currency satisfies:

$$M_{i,1} \geq P_1 c_{i,1} (1 - \underline{z}) \text{ for } i = 1, 2, \tag{7.32}$$

where P_1 is given by (7.30) and $c_{i,1}$ is evaluated at the continuation of the time 0 Ramsey equilibrium. Assuming that (3.4) is non-binding at $t = 1$ in the time 1 Ramsey equilibrium, optimality of the Friedman rule at $t = 1$ implies:

$$-\lambda'_i \frac{\hat{u}_{i22,1}}{\underline{z}} \left(b_{i0,1} + \frac{B_{i0,1}}{P_1} \right) \leq -\lambda'_i \frac{\hat{u}_{i11,1}}{1 - \underline{z}} \frac{M_{i,1}}{P_1} \text{ for } i = 1, 2. \tag{7.33}$$

Evaluating (7.33) at equality yields $b_{i0,1}$ as a function of $B_{i0,1}$ and $M_{i,1}$. Use (7.27) to solve for ω'_1. These values of $b_{i0,1}$, $B_{i0,1}$ and $M_{i,1}$ can be substituted into the time 1 implementability constraint to solve for λ'_i. ■

Notice that I have characterized the initial distribution of nominal claims on the government under the assumption that $\mu'_{i,1} = \zeta_1 = 0$ for $i = 1, 2$. This is only one possible solution. In general, optimality of the Friedman rule at $t = 1$ requires:

$$\zeta'_{i,1} \frac{u_{in,1}}{(\hat{u}_{i2,1})^2} \frac{\hat{u}_{i22,1}}{\underline{z}} - \lambda'_i \frac{\hat{u}_{i22,1}}{\underline{z}} \left(b_{i0,1} + \frac{B_{i0,1}}{P_1} \right) \leq \mu'_{i,1} - \lambda'_i \frac{\hat{u}_{i11,1}}{1 - \underline{z}} \frac{M_{i,1}}{P_1} \text{ for } i = 1, 2. \tag{7.34}$$

In addition, stationarity of the time 1 Ramsey equilibrium imposes:

$$\mu'_{i,1} = \max\{0, (\eta_i + \lambda'_i)(\hat{u}_{i1} + u_{in}) + \lambda'_i (\hat{u}_{i12} c_{i2} + u_{inn} n_i)\}. \tag{7.35}$$

Then, (7.35) can be used in (7.28) to obtain the analogue of (7.31) and (7.33), evaluated at equality, can be used to get un upper bound for $b_{i0,1}$ for $i = 1, 2$, since the assumption is that $\zeta_1 \geq 0$. The upper bound for $b_{10,1}$ will be lower than the one for $b_{20,1}$, conditional on $B_{i0,1}$ and $M_{i,1}$, by (7.12).

Proof. [Proposition 4.2] Use (7.31) under (4.1) to characterize $\mu'_{1,1}$ as a function of $\mu'_{2,1}$, $M_{i,1}$ and λ'_i, where the distribution of currency is given by (7.32). Use (7.34) with equality for $i = 1, 2$, imposing (4.1), to solve for the value of ζ_1 and $\mu'_{2,1}$ for given λ'_i. Use (7.27) to solve for ω'_1. Use the implementability constraints to solve for λ'_i. ■

Proof. [Proposition 4.3] I need to show that it is possible to find λ'_i, $B_{i0,t}$ with $t \geq 1$, $b_{i0,t}$ and $M_{i,1}$ so that the continuation allocation for the time 0 Ramsey problem solves the time 1 Ramsey problem. To obtain $b_{i0,t}$ combine the first order conditions for n_i and c_{i2} for $t > 1$ in the time 1 Ramsey problem:

$$(\eta_i + \lambda'_i)(\hat{u}_{i2} + u_{in}) + \lambda'_i(\hat{u}_{i12}c_{i1} + \hat{u}_{i22}\hat{c}_{i2} + u_{inn}n_i) = \lambda'_i\hat{u}_{i22}b'_i,$$

where $\hat{u}_{i11} = u_{i11}/(1 - z_i)$, $\hat{u}_{i12} = u_{i12}/(1 - z_i)$, $\hat{u}_{i22} = u_{i22}/z_i$ and $\hat{u}_{i21} = u_{i21}/z_i$. This implies:

$$b_{i0,t} = b'_i = \left(\frac{\eta_i}{\lambda'_i} + 1\right)\left(\frac{\hat{u}_{i2} + u_{in}}{\hat{u}_{i22}}\right) + \left(\frac{\hat{u}_{i12}}{\hat{u}_{i22}}c_{i1} + \hat{c}_{i2} + \frac{u_{inn}}{\hat{u}_{i22}}n_i\right), \tag{7.36}$$

for $t > 0$. Since the Friedman rule is optimal in the time 0 Ramsey equilibrium, b_i satisfies (7.22). This implies that b'_i also satisfies (7.22). To see this, first note that the right hand side of (7.36) is evaluated at the continuation allocation of the time 0 Ramsey equilibrium for $t > 1$. Therefore, b_i also satisfies this equation with λ_i replacing λ'_i. Then, $\lambda_i, \lambda'_i \geq 0$ implies b'_i, determined from (7.36), satisfies (7.22). To find $B_{i0,t}$, obtain from (7.28) the analogue of (7.31), using (7.35):

$$\sum_{i=1,2}\left(-\lambda'_i\hat{u}_{i2,1}\sum_{t=1}^{\infty}\frac{B_{i0,t}}{P_1} + \max\{-\lambda'_i\hat{u}_{i1,1}, \eta_i\hat{u}_{i1,1} + (\eta_i + \lambda'_i)u_{in} + \lambda'_i(\hat{u}_{i12}\hat{c}_{i2} + u_{inn}n_i)\}\frac{M_{i,1}}{P_1}\right) = 0. \tag{7.37}$$

(7.37) implies that the analogue of (7.14) for the time 1 Ramsey equilibrium:

$$\mu'_t = \sum_i\left(\lambda'_i\hat{u}_{i2,1}\sum_{s=t+1}^{\infty}\frac{B_{i(0),s}}{P_1}\right) \geq 0,$$

is verified for $t > 1$. With these values of b'_i, $\sum_{t=1}^{\infty} B_{i0,t}$ and (7.32), it is possible to procede as for proposition 4.1. ■

Proof. [Proposition 4.4] Assume by contradiction that the Ramsey equilibrium is time consistent, (3.6) is imposed and the Friedman rule is not optimal. Evaluating (7.15) at time $t = 1$ in the time 0 and time 1 Ramsey equilibrium, with $n_{i,t}$ set at the value implied by the continuation of the time 0 Ramsey equilibrium, yields a linear system of four equations in the unknowns $\lambda_i' = \lambda_i$ for $i = 1, 2$, ζ', ω'. This implies that the value of the multipliers on the implementability constraints, on the distribution constraint and on the resource constraint must be the same in the time 1 and time 0 Ramsey equilibrium:

$$\lambda_i' = \lambda_i \text{ for } i = 1, 2, \tag{7.38}$$

$$\zeta_t = \zeta_t', \; \omega_t = \omega_t' \text{ for } t \geq 1. \tag{7.39}$$

But by (7.38)-(7.39) and (7.10) for $t > 1$ in the time 0 and time 1 Ramsey equilibrium, $\mu_{i,t} = \mu_{i,t}'$ for $t > 1$. By the fact that the Friedman rule is not optimal and by (7.20):

$$\zeta_i \frac{u_{in}}{\hat{u}_{i2}} \frac{\hat{u}_{i22}}{\hat{u}_{i2} z_i} + \mu_i R \frac{\hat{u}_{i22}}{\hat{u}_{i2} z_i} > -\mu_i R \frac{\hat{u}_{i11}}{\hat{u}_{i1}(1 - z_i)},$$

for $i = 1, 2$ in the continuation of the time 0 Ramsey equilibrium. Summing this inequality over i and using (7.12) yields:

$$\zeta \left(\frac{\kappa_1}{\xi_1} \frac{u_{1n}}{\hat{u}_{12}} \frac{\hat{u}_{122}}{\hat{u}_{12} z_2} - \frac{u_{2n}}{\hat{u}_{22}} \frac{\hat{u}_{222}}{\hat{u}_{22} z_2} \right) > -R \sum_i \mu_i \left(\frac{\hat{u}_{i11}}{\hat{u}_{i1}(1 - z_1)} + \frac{\hat{u}_{i22}}{\hat{u}_{i2} z_2} \right).$$

But since the distribution constraint is binding: $\frac{\kappa_1}{\xi_1} \frac{u_{1n}}{\hat{u}_{12}} \frac{\hat{u}_{122}}{\hat{u}_{12} z_2} = \frac{u_{2n}}{\hat{u}_{22}} \frac{\hat{u}_{222}}{\hat{u}_{22} z_2}$, which implies $\sum_i \mu_i < 0$. In addition, since under (3.6) the continuation of the time 0 Ramsey equilibrium is stationary, $\mu_{i,1} = \mu_{i,t}$ for $t > 1$. Then, by the analogue of (7.14) under (3.6) in the time 1 Ramsey problem: $\mu_t' = -\sum_i \mu_{i,t}' > 0$, which contradicts $R_t > 1$ by (7.13). Hence, $R_t = 1$. QED ■

Proof. [Proposition 4.5] If the Friedman rule is not optimal, $\mu_t' = \mu_t = 0$ for $t > 1$. Evaluating (7.15) at time 1 in the time 0 and time 1 Ramsey equilibrium, with $n_{i,1}$ set at the value implied by the continuation of the time 0 Ramsey equilibrium, yields four linear equations in four unknowns λ_i' for $i = 1, 2$, ζ', ω'. Then, (7.38) and (7.39) hold. By (7.38)-(7.39) and the first order condition for $c_{i1,t}$ in the time 1 Ramsey equilibrium, time consistency implies:

$$\mu_{it}' = \mu_{it}\ , \; i = 1, 2, \; t > 1. \tag{7.40}$$

The first order condition for $c_{i2,t}$ can be used to derive b_i'. The analogue of (7.14) for $t > 1$ for the time 1 Ramsey equilibrium when the Friedman rule is not optimal is:

$$\sum_i \left(\mu_{i,t}' - \lambda_i \frac{\hat{u}_{i2,1}}{R_t} \sum_{s=t+1}^{\infty} \frac{B_{i0,s}}{P_1} \prod_{j=1}^{s} R_j \right) = 0. \tag{7.41}$$

(7.40) and (7.14) imply:

$$\sum_i \left(\lambda_i \frac{\hat{u}_{i2,0}}{R_t} \sum_{s=t+1}^{\infty} \frac{B_{i(-1),s}}{P_0} \prod_{j=1}^{s} R_j - \lambda_i \frac{\hat{u}_{i2,1}}{R_t} \sum_{s=t+1}^{\infty} \frac{B_{i0,s}}{P_1} \prod_{j=1}^{s} R_j \right) = 0.$$

After manipulation, using (7.30), this is equivalent to:

$$\sum_i \lambda_i \frac{\hat{u}_{i2,0}}{R_t} \sum_{s=t+1}^{\infty} \left(\frac{B_{i(-1),s}}{P_0} - \frac{B_{i0,s}}{\beta P_0} \right) \prod_{j=1}^{s} R_j = 0. \tag{7.42}$$

Using the first order condition for $c_{i1,1}$ in the time 0 and time 1 Ramsey equilibrium yields:

$$-\mu'_{i,1} - \lambda_i \hat{u}_{i,11} \frac{M_{i,1}/P_1}{1 - z_{i,1}} = -\mu_{i,1} R \frac{\hat{u}_{i,11}}{(1 - z_{i,1})\, \hat{u}_{i,1}}, \tag{7.43}$$

which pins down $\mu'_{i,1}$ for given $M_{i,1}$. Since $\mu'_{i,1} \geq 0$ by definition and $\lambda_i > 0$, $u_{i11} < 0$, time consistency requires $\mu_{i,1} > 0$ for $i = 1, 2$. By (7.14), it follows that $\sum_i \left(\lambda_i \frac{\hat{u}_{i2,1}}{R_t} \sum_{s=1}^{\infty} \frac{B_{i0,s}}{P_1} \prod_{j=1}^{s} R_j \right) > 0$ must hold. The last thing to verify is the optimality condition for P_1 in the time 1 Ramsey equilibrium:

$$\sum_{i=1,2} \left(-\lambda_i \hat{u}_{i1,1} \frac{M_{i,1}}{P_1} - \lambda_i \hat{u}_{i2,1} \sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} \prod_{j=2}^{t} R_j + \mu'_{i1} \frac{M_{i,1}}{P_1} \right) = 0.$$

Using (7.43) and (7.42), this condition delivers the value of μ'_{i1}. ■

Proof. [Proposition 4.6] If the Friedman rule is optimal, (7.31) and $\hat{u}_{i2,1} = \hat{u}_{i1,1}$ in the continuation of the time 0 Ramsey equilibrium imply:

$$\sum_{i=1,2} \lambda'_i \hat{u}_{i1,1} \left(\sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} + \frac{M_{i,1}}{P_1} \right) = \sum_{i=1,2} \mu'_{i1} \frac{M_{i,1}}{P_1}.$$

From the first order condition for $c_{i1,1}$ in the time 1 Ramsey equilibrium:

$$\lambda'_i \hat{u}_{i1,1} = -\eta_i \hat{u}_{i1,1} - \lambda'_i \frac{\hat{u}_{i11,1}}{1 - \underline{z}} \left(c_{i1,1} (1 - \underline{z}) - \frac{M_{i,1}}{P_1} \right) - \lambda'_i \left(\frac{u_{i21,1}}{1 - \underline{z}} \hat{c}_{i2,1} \right) + \mu'_{i1} + \nu_i \omega'_1. \tag{7.44}$$

Since the cash in advace constraint is non-binding and (7.32) holds, this reduces to:

$$\sum_{i=1,2} \left(-\eta_i \hat{u}_{i1,1} - \lambda'_i \left(\frac{u_{i21,1}}{1 - \underline{z}} \hat{c}_{i2,1} \right) + \nu_i \omega'_1 \right) \left(\sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} + \frac{M_{i,1}}{P_1} \right) = 0.$$

After rearranging:

$$\omega_1' \sum_{i=1,2} \nu_i \left(\sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} + \frac{M_{i,1}}{P_1} \right) = \sum_{i=1,2} \left(\eta_i \hat{u}_{i1,1} + \lambda_i' \left(\frac{u_{i21,1}}{1-\underline{z}} \hat{c}_{i2,1} \right) \right) \left(\sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} + \frac{M_{i,1}}{P_1} \right).$$

Since ω_1' is the multiplier on the resource constraint and utility is non-satiated, $\omega_1' > 0$, so that:

$$\sum_{t=1}^{\infty} \frac{B_{0,t}}{P_1} + \frac{M_1}{P_1} = \sum_{i=1,2} \frac{1}{\omega_1'} \left(\eta_i \hat{u}_{i1,1} + \lambda_i' \left(\frac{u_{i21,1}}{1-\underline{z}} \hat{c}_{i2,1} \right) \right) \left(\sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} + \frac{M_{i,1}}{P_1} \right) \quad (7.45)$$

If the Friedman rule is not optimal, the analogue of (7.31) is:

$$\sum_{i=1,2} \left(-\lambda_i \hat{u}_{i1,1} \frac{M_{i,1}}{P_1} - \lambda_i \hat{u}_{i2,1} \sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} \prod_{j=2}^{t} R_j + \mu_{i1}' \frac{M_{i,1}}{P_1} \right) = 0.$$

Then:

$$\sum_{i=1,2} \lambda_i \hat{u}_{i2,1} \left(\frac{M_{i,1}}{P_1} + \sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} \prod_{j=2}^{t} R_j \right) = \sum_{i=1,2} \frac{M_{i,1}}{P_1} \left(-\lambda_i \left(\hat{u}_{i1,1} - \hat{u}_{i2,1} \right) + \mu_{i1}' \right).$$

Using (7.44):

$$\begin{aligned} & \sum_{i=1,2} \left(-\eta_i \hat{u}_{i1,1} - \lambda_i' \left(\frac{u_{i21,1}}{1-z_{i,1}} \hat{c}_{i2,1} \right) + \mu_{i1}' + \nu_i \omega_1' \right) \left(\frac{M_{i,1}}{P_1} + \sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} \prod_{j=2}^{t} R_j \right) \\ = \;& R_1 \sum_{i=1,2} \frac{M_{i,1}}{P_1} \left(-\lambda_i \left(\hat{u}_{i1,1} - \hat{u}_{i2,1} \right) + \mu_{i1}' \right), \end{aligned}$$

which implies:

$$\begin{aligned} & \omega_1' \left(\frac{M_1}{P_1} + \sum_{t=1}^{\infty} \frac{B_{0,t}}{P_1} \prod_{j=2}^{t} R_j \right) \\ = \;& (R_1 - 1) \sum_{i=1,2} \frac{M_{i,1}}{P_1} \left(-\lambda_i \hat{u}_{i2,1} + \mu_{i1}' \right) + \sum_{i=1,2} \lambda_i \left(\frac{u_{i21,1}}{1-z_{i,1}} \hat{c}_{i2,1} \right) \frac{M_{i,1}}{P_1} \\ & + \sum_{i=1,2} \left(\eta_i \hat{u}_{i1,1} + \lambda_i \left(\frac{u_{i21,1}}{1-z_{i,1}} \hat{c}_{i2,1} \right) - \mu_{i1}' \right) \sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} \prod_{j=2}^{t} R_j. \end{aligned}$$

■

7.4. D: The Model with Lucas and Stokey Timing

Under Lucas and Stokey timing, the Lagrangian for the household problem is:

$$
\begin{aligned}
L &= \sum_{t=0}^{\infty} \beta^t \Big\{ u^i (c_{it}, n_{it}) - \mu_{it} (P_t c_{i1t} (1 - z_{it}) - M_{it}) \\
&\quad - \lambda_{it} \Bigg[M_{it} + \sum_{s>0} (Q_{t,t+s} B_{it,t+s} + q_{t,t+s} P_t b_{it,t+s}) - M_{it-1} - \sum_{\hat{t}=0}^{t-1} \left(B_{i\hat{t},t-1} + P_{t-1} b_{i\hat{t},t-1} \right) \\
&\quad - W_{t-1} \left(1 - \tau_{t-1}^i\right) \xi_i n_{it-1} + P_{t-1} c_{i1t-1} (1 - z_{it-1}) + P_{t-1} c_{i2t-1} z_{it-1} + \int_0^{z_{it-1}} \pi_{t-1}(j)\, dj \Bigg] \Big\},
\end{aligned}
$$

The solution to this optimization problem, implies the following implementability constraint for $i = 1, 2$:

$$
\sum_{t=0}^{\infty} \beta^t (u_{i1t} c_{i1t} + u_{i2t} \hat{c}_{i2t} + u_{int} n_{it}) = \hat{u}_{i20} \frac{M_{i0}}{P_0} + \sum_{t=0}^{\infty} \beta^t \hat{u}_{i2t} b_{i(-1),t} + \hat{u}_{i20} \sum_{t=0}^{\infty} \frac{B_{i(-1),t}}{P_0} \prod_{j=1}^{t} R_j.
$$

The Lagrangian for the Ramsey problem is:

$$
\begin{aligned}
\Lambda &= \sum_{t=0}^{\infty} \beta^t \sum_i \{ \eta_i U (c_{it}, n_{it}) + \lambda_i (u_{i1t} c_{i1t} + u_{i2t} \hat{c}_{i2t} + u_{int} n_{it}) \} \\
&\quad - \sum_{t=0}^{\infty} \beta^t \left[\mu_t (1 - R_t) + \sum_i \mu_{it} \left(\frac{\hat{u}_{1t}^i}{\hat{u}_{2t}^i} - R_t \right) \right] - \sum_{t=0}^{\infty} \beta^t \zeta_t \left(\kappa_1 \frac{u_{nt}^1}{\xi_1 \hat{u}_{2t}^1} + \kappa_0 - \frac{u_{nt}^2}{\xi_2 \hat{u}_{2t}^2} \right) \\
&\quad - \sum_i \lambda_i \left(\hat{u}_{i2,0} \frac{M_{i0}}{P_0} + \sum_{t=0}^{\infty} \beta^t \hat{u}_{i2t} b_{i(-1),t} + \hat{u}_{i20} \sum_{t=0}^{\infty} \frac{B_{i(-1),t}}{P_0} \prod_{j=1}^{t} R_j \right).
\end{aligned}
$$

The first order conditions for the Ramsey problem at $t > 0$ are the same as for the Svensson timing. They differ as follows for time 0 :

$$
0 = \eta_i u_{i1,0} + \lambda_i (u_{i1,0} + u_{i11,0} c_{i1,0} + u_{i12,0} c_{i2,0}) - \mu_{i0} \frac{\hat{u}_{i11,0}}{\hat{u}_{i2,0}} - \nu_i (1 - z_{i,0}) \omega_0, \quad (7.46)
$$

$$
\begin{aligned}
0 &= \eta_i u_{i20} + \lambda_i (u_{i12,0} c_{i1,0} + u_{i20} + u_{i220} c_{i20}) - \nu_i z_{i,0} \omega_0 \qquad (7.47) \\
&\quad + \frac{\zeta_{i0}}{\hat{u}_{i20}} \frac{u_{in0}}{\hat{u}_{i20}} \hat{u}_{i220} + \mu_{i0} \frac{\hat{u}_{i22,0}}{\hat{u}_{i2,0}} \frac{\hat{u}_{i11,0}}{\hat{u}_{i2,0}} \\
&\quad - \lambda_i \hat{u}_{i22,0} \left(\frac{M_{i,0}}{P_0} + b_{i(-1),0} + \sum_{t=0}^{\infty} \frac{B_{i(-1),t}}{P_0} \prod_{j=1}^{t} R_j \right),
\end{aligned}
$$

$$\sum_{i=1,2}\left(-\lambda_i\hat{u}_{i2,0}\frac{M_{i0}}{P_0}-\lambda_i\hat{u}_{i20}\sum_{t=0}^{\infty}\frac{B_{i(-1),t}}{P_0}\prod_{j=1}^{t}R_j\right)\left(\frac{-1}{P_0^2}\right)=0,$$

$$\mu_{i0}\left(\frac{\hat{u}_{i10}}{\hat{u}_{i20}}-R_0\right)=0, \tag{7.48}$$

$$\mu_0\left(1-R_0\right)=0,\ \mu_0\geq 0,\ R_0\geq 1. \tag{7.49}$$

In addition, (7.14) holds for $t \geq 0$ (and not just for $t > 0$).

Proposition 7.1. *If the Friedman rule is optimal, the Ramsey equilibrium is time consistent.*

Proof. I need to show that it is possible to find λ_i', μ_{it}', μ_t', ζ_t', ω_t', $B_{i0,t}$ with $t \geq 1$, $b_{i0,1}$, $b_{i0,t} = b_i'$ for $t > 1$and $M_{i,1}$ so that the continuation allocation for the time 0 Ramsey problem solves the time 1 Ramsey problem. Here, a prime denotes the multipliers on the corresponding constraints in the Lagrangian for the time 1 Ramsey problem. If the Friedman rule is optimal, the constraint on the labor tax rate is not binding in the time 0 Ramsey equilibrium, so that $\zeta_t = 0$ for $t \geq 0$. In addition, when the Friedman rule is optimal, the Ramsey equilibrium allocation solves the relaxed problem where the constraints corresponding to multipliers μ_{it} for $t > 0$ are dropped. Therefore, the continuation allocation of the time 0 Ramsey equilibrium satisfies:

$$0=\eta_i u_{i1t}+\lambda_i\left(u_{i1t}+u_{i11t}c_{i1t}+u_{i21t}\hat{c}_{i2t}\right)-\nu_i\omega_t\left(1-z_{i,t}\right),$$

$$\begin{aligned}0 &= \eta_i u_{i2t}+\lambda_i\left(u_{i12t}c_{i1t}+\mathrm{u}_{i2t}+u_{i22t}\hat{c}_{i2t}\right)-\nu_i\omega_t z_{i,t}\\ &\quad -\lambda_i\beta^t\hat{u}_{i22t}b_{i(-1),t},\end{aligned}$$

$$0=\eta_i u_{int}+\lambda_i\left(u_{int}+u_{innt}n_{it}\right)+\nu_i\xi_i\omega_t,$$

plus (7.15) and the first order condition for z_{it}. The continuation allocation of the time 0 Ramsey equilibrium will also satisfy the first order conditions for $t > 1$ of the time 1 Ramsey equilibrium at the Friedman rule, if $b_{i0,t} = b_i'$ where:

$$b_i'=\left(\frac{\eta_i}{\lambda_i'}+1\right)\left(\frac{\hat{u}_{i2}+u_{in}}{\hat{u}_{i22}}\right)+\left(\frac{\hat{u}_{i12}}{\hat{u}_{i22}}c_{i1}+\hat{c}_{i2}+\frac{u_{inn}}{\hat{u}_{i22}}n_i\right). \tag{7.50}$$

This value of b_i' obtains by combining the first order conditions for n_i and c_{i2} for $t > 1$ in the time 1 Ramsey problem. As for Svensson timing, it can be shown that if b_i satisfies (7.22) and the Friedman rule holds in the time 1 Ramsey equilibrium, b_i' will also satisfy (7.22), as long as λ_i and λ_i' have the same sign for

$i = 1, 2$. Since other conditions for optimality of the Friedman rule do not depend on the value of the multipliers in the implementability constraints, as shown in the proof of proposition 3.3, $\mu'_t = 0$, $\mu'_{it} = 0$ for $t > 1$, $\zeta'_t = 0$ and $z_{i,t} = \underline{z}$ for $t > 1$ are necessary conditions for the continuation allocation of the time 0 RE to solve the first order conditions for the Ramsey problem at $t > 1$. In addition, imposing that n_{it} in the continuation allocation of the time 0 Ramsey equilibrium solves the analogue of (7.15) in the time 1 RE for $t > 1$ determines ω'_t for given λ'_i for $t \geq 1$. The first order conditions for $t = 1$ in the time 1 Ramsey problem are given by:

$$0 = \eta_i u_{i1,1} + \lambda'_i \left(u_{i1,1} + u_{i11,1} c_{i1,1} + u_{i2,1} \hat{c}_{i2,1} \right) - \mu'_{i1} \frac{\hat{u}_{i11,1}}{\hat{u}_{i2,1}} - \nu_i \omega'_1 \left(1 - z_{i,1} \right),$$

$$\begin{aligned} 0 \;=\; & \eta_i u_{i2,1} + \lambda'_i \left(u_{i1,1} c_{i1,1} + u_{i21} + u_{i22,1} \hat{c}_{i2,1} \right) - \nu_i \omega'_1 z_{i,1} \\ & + \frac{\zeta'_{i1}}{\hat{u}_{i2,1}} \frac{u_{in1}}{\hat{u}_{i2,1}} \hat{u}_{i22,1} + \mu'_{i1} \frac{\hat{u}_{i1,1}}{\hat{u}_{i2,1}} \frac{\hat{u}_{i22,1}}{\hat{u}_{i2,1}} \\ & - \lambda'_i \hat{u}_{i22,1} \left(\frac{M_{i,1}}{P_1} + b_{i0,1} + \sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} \prod_{j=1}^{t} R_j \right), \end{aligned} \tag{7.51}$$

$$\sum_{i=1,2} \lambda'_i \hat{u}_{i2,1} \left(\frac{M_{i,1}}{P_1} + \sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} \prod_{j=1}^{t} R_j \right) \left(\frac{-1}{P_1} \right) = 0, \tag{7.52}$$

$$P_1 = P_0 \beta \frac{\hat{u}_{i1,1}}{\hat{u}_{i2,0}},$$

plus (7.15) at $t = 1$. Set $\mu'_{i1} = \zeta'_1 = 0$ and $\omega'_1 = \omega_1$. Then, (7.51) pins down $\frac{M_{i,1}}{P_1} + b_{i0,1} + \sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} \prod_{j=1}^{t} R_j$ as a function of λ'_i for each i, (7.52) pins down the distribution of nominal claims on the government across households as a function of λ'_1, λ'_2. Since (7.52) does not depend on the level of nominal assets, but only on the distribution, while (7.53) depends on the level, they can both be satisfied at the same time. These values of b'_i, $\sum_{t=1}^{\infty} B_{i1,t}$ and M_{i1} can be substituted into the time 1 implementability constraint to solve for λ'_i. ■

Equations (7.51) and (7.52) pin down a family of asset distribution across households of different types that supports the continuation of the time 0 Ramsey equilibrium as an outcome of the time 1 Ramsey equilibrium. Optimality of the Friedman rule implies:

$$-\lambda'_i \hat{u}_{i22,1} \left(\frac{M_{i,1}}{P_1} + b_{i0,1} + \sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} \prod_{j=1}^{t} R_j \right) \leq 0 \text{ for } i = 1, 2, \tag{7.53}$$

which combined with (7.52) implies:

$$\sum_{i=1,2} \lambda_i' \hat{u}_{i2,1} b_{i0,1} \leq 0.$$

Proposition 7.2. *If the Friedman rule is not optimal, the Ramsey equilibrium is time consistent.*

Proof. Evaluate the first order conditions for the time 1 Ramsey equilibrium at the continuation allocation for the time 0 Ramsey equilibrium and assume the Friedman rule is not optimal. Then, $\mu_t' = 0$ for $t \geq 1$ and (7.15) for $i = 1, 2$ for the time 1 Ramsey equilibrium define an equation for $\zeta_t' = \zeta'$ and $\omega_t' = \omega'$ as a function of λ_1', λ_2'. The first order condition for $c_{i1,t}$ in the 1 Ramsey equilibrium determines μ_{it}' for $i = 1, 2$ and $t \geq 1$, and the first order condition for $c_{i2,t}$ can be used to derive $b_{i0,t} = b_i'$ for $t > 1$ as a function of λ_i'. The analogue of (7.14) for $t > 1$ for the time 1 Ramsey equilibrium when the Friedman rule is not optimal is:

$$\sum_i \left(\mu_{i,t}' - \lambda_i' \frac{\hat{u}_{i2,1}}{R_t} \sum_{s=t+1}^{\infty} \frac{B_{i0,s}}{P_1} \prod_{j=1}^{s} R_j \right) = 0,$$

which determines the maturity structure of nominal debt. If condition (7.41) did not hold, it would be possible to increase the value of the planner's objective in the time 1 Ramsey equilibrium by decreasing R_t (or equivalently by changing P_1). This would reduce the shadow present value of nominal government liabilities, thus relaxing both implementability constraints, while satisfying the resource constraint and other optimality conditions. $b_{i0,1} + \frac{M_{i1}}{P_1} + \sum_{s=1}^{\infty} \frac{B_{i0,s}}{P_1} \prod_{j=1}^{s} R_j$ can be obtained from the first order condition for $c_{i2,1}$ and (7.52) pins down the distribution of nominal claims on the government across agents of different types. The implementability constrains can then be used to determine λ_i' for $i = 1, 2$. ∎

Lemma 7.3. *The present value of nominal government liabilities need not be* 0 *in a time consistent Ramsey equilibrium.*

Proof. (7.52) implies:

$$\sum_{i=1,2} \lambda_i' \frac{\hat{u}_{i1,1}}{R_1} \left(\sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} + \frac{M_{i,1}}{P_1} \right) = 0.$$

Using:

$$\lambda_i' \hat{u}_{i1,1} = -\eta_i \hat{u}_{i1,1} - \lambda_i' \left(\frac{\hat{u}_{i11,1}}{1-\underline{z}} c_{i1,1} + \frac{u_{i21,1}}{1-\underline{z}} c_{i2,1} \right) + \mu_{i1}' \frac{\hat{u}_{i11,1}}{\hat{u}_{i2,1}} + \nu_i \omega_1',$$

which obtains from the first order condition for $c_{i1,1}$ in the time 1 Ramsey equilibrium, this equation reduces to:

$$\sum_{i=1,2} \frac{1}{R_1} \left(-\eta_i \hat{u}_{i1,1} - \lambda_i' \left(\frac{\hat{u}_{i11,1}}{1-\underline{z}} c_{i1,1} + \frac{u_{i21,1}}{1-\underline{z}} c_{i2,1} \right) + \mu_{i1}' \frac{\hat{u}_{i11,1}}{\hat{u}_{i2,1}} + \nu_i \omega_1' \right) \left(\sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} + \frac{M_{i,1}}{P_1} \right) = 0.$$

Rearranging:

$$\sum_{t=1}^{\infty} \frac{B_{0,t}}{P_1} + \frac{M_1}{P_1} = \sum_{i=1,2} \frac{R_1}{\omega_1'} \left(\eta_i \hat{u}_{i1,1} + \lambda_i' \left(\frac{\hat{u}_{i11,1}}{1-\underline{z}} c_{i1,1} + \frac{u_{i21,1}}{1-\underline{z}} \hat{c}_{i2,1} \right) - \mu_{i1}' \frac{\hat{u}_{i11,1}}{\hat{u}_{i2,1}} \right) \left(\sum_{t=1}^{\infty} \frac{B_{i0,t}}{P_1} + \frac{M_{i,1}}{P_1} \right), \tag{7.54}$$

since ω_1' is the multiplier on the resource constraint and utility is non-satiated, $\omega_1' > 0$, so that $\sum_{t=1}^{\infty} \frac{B_{0,t}}{P_1} + \frac{M_1}{P_1} \neq 0$. ∎

Figure 1

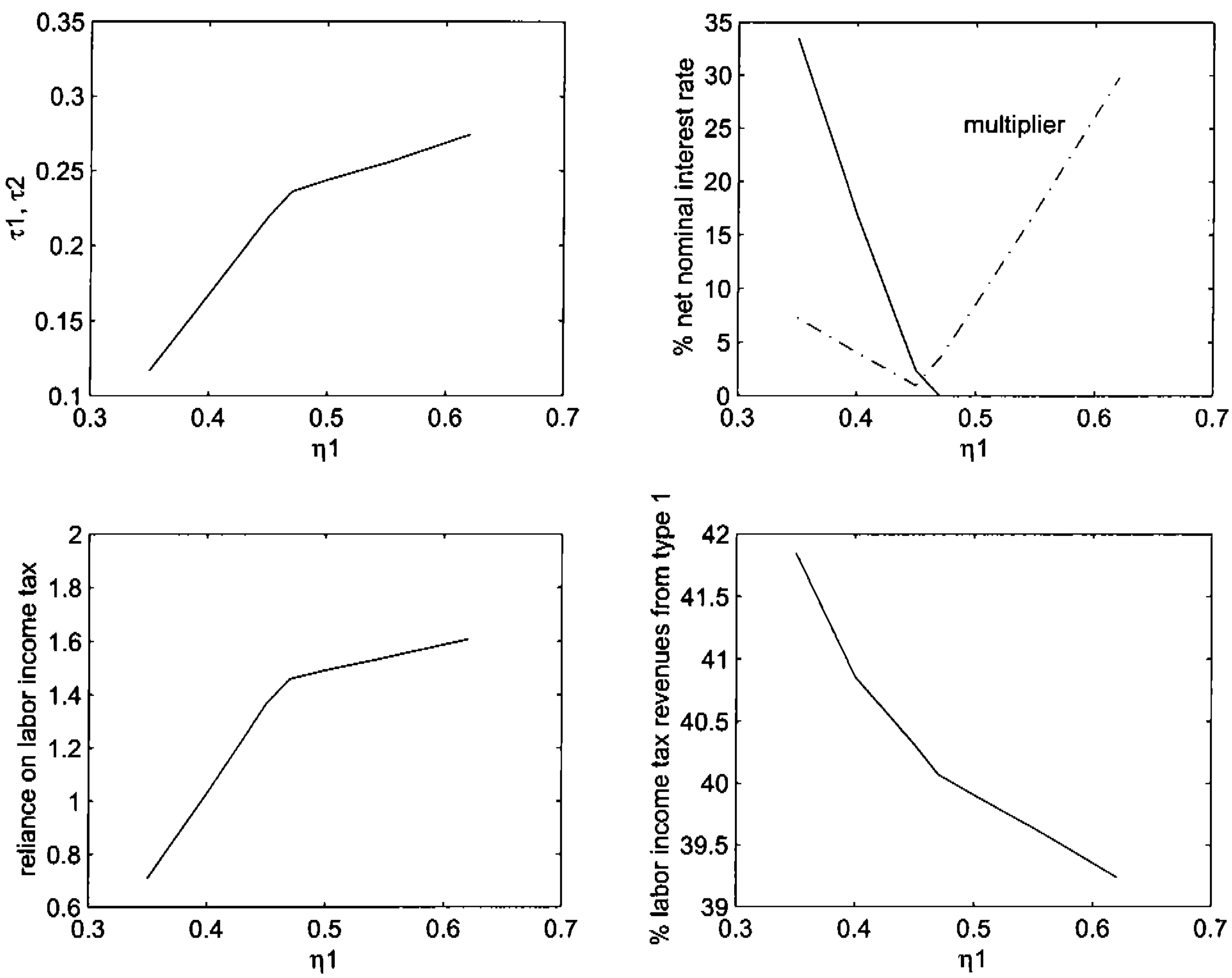

τ1, τ2
η1
% net nominal interest rate
multiplier
η1
reliance on labor income tax
η1
% labor income tax revenues from type 1
η1

Figure 2

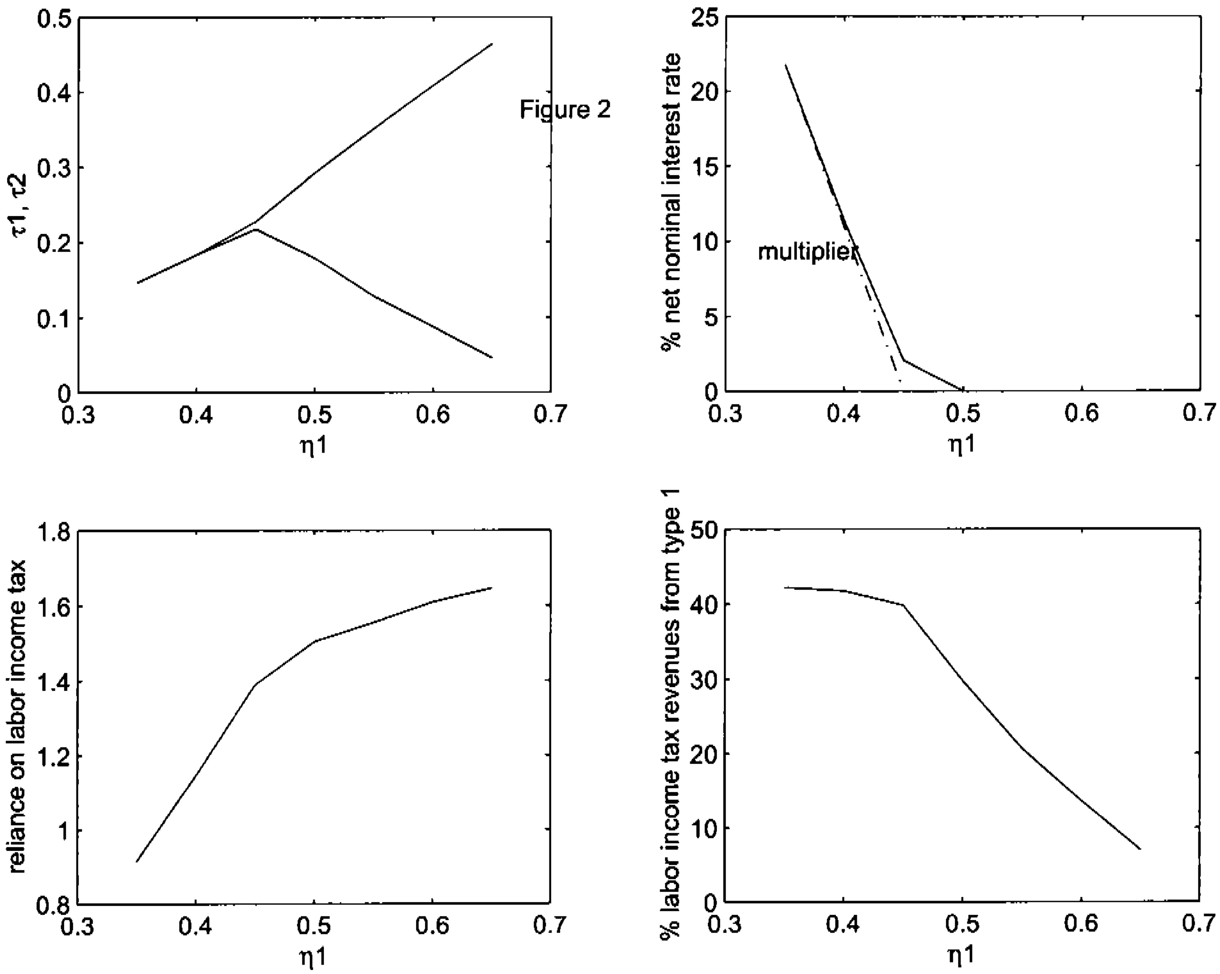
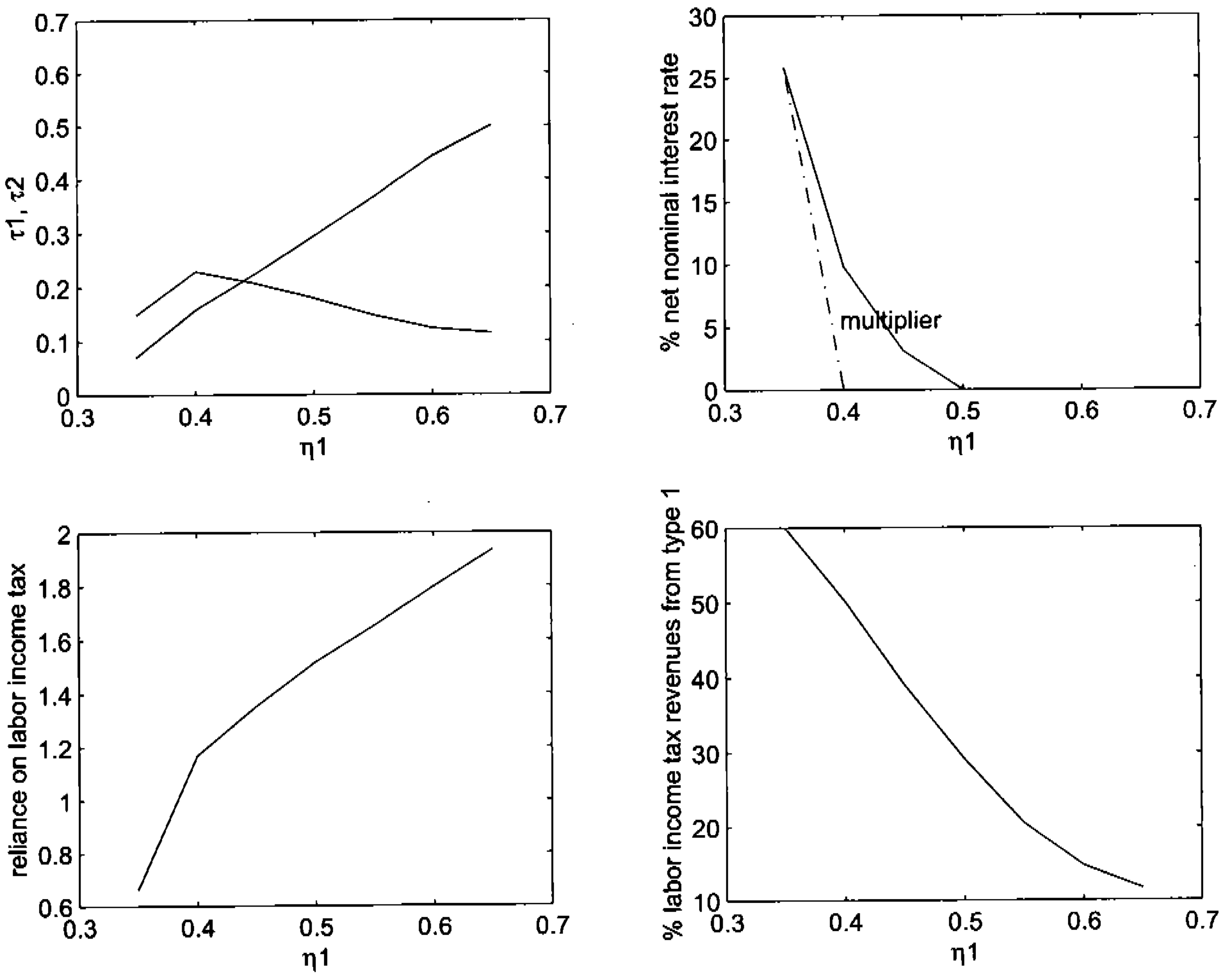
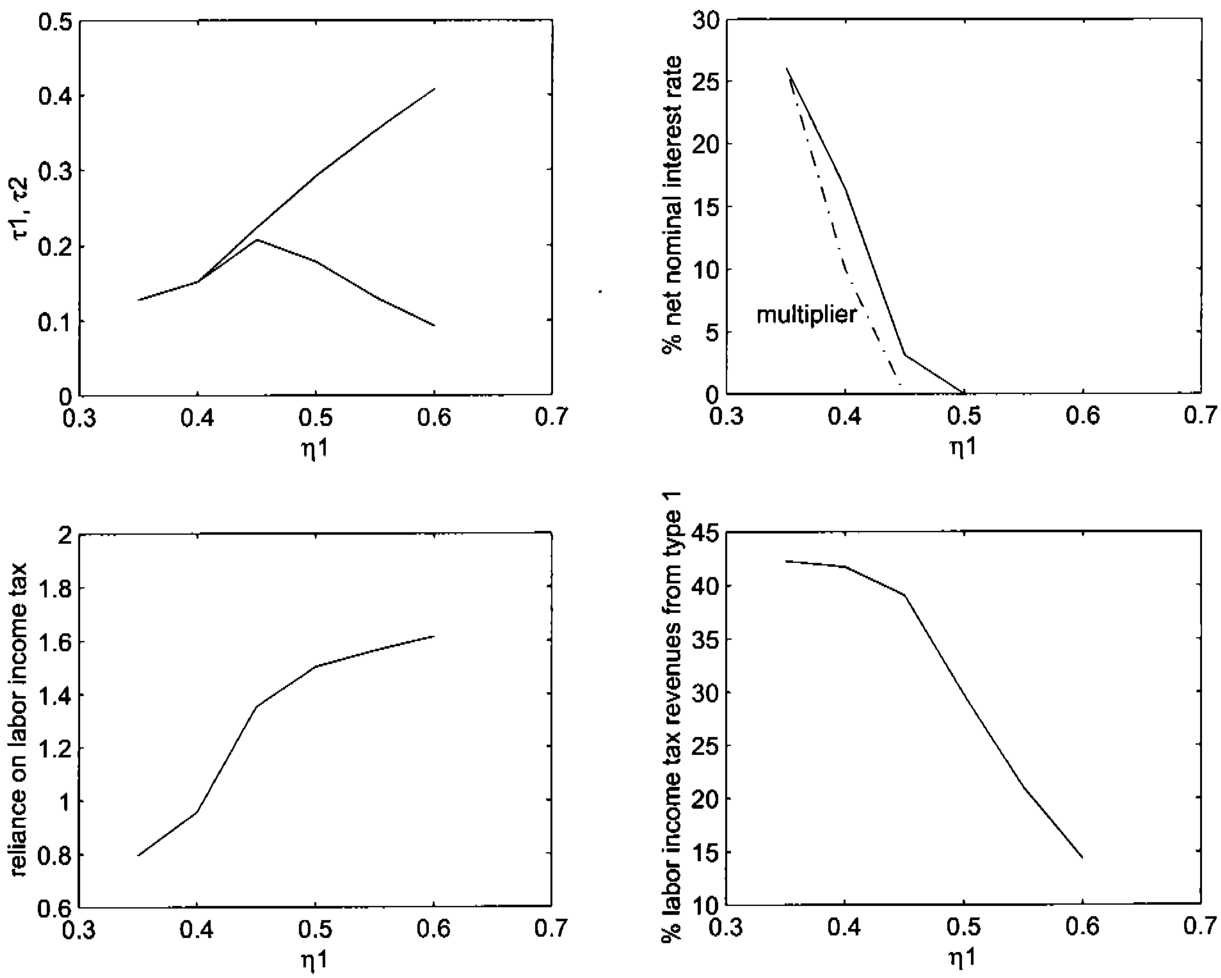

0.5
0.4
0.3
0.2
0.1
0
τ1, τ2
0.3
0.4
0.5
0.6
0.7
η1
25
20
15
10
5
0
% net nominal interest rate
multiplier
0.3
0.4
0.5
0.6
0.7
η1
1.8
1.6
1.4
1.2
1
0.8
reliance on labor income tax
0.3
0.4
0.5
0.6
0.7
η1
50
40
30
20
10
0
% labor income tax revenues from type 1
0.3
0.4
0.5
0.6
0.7
η1

Figure 3



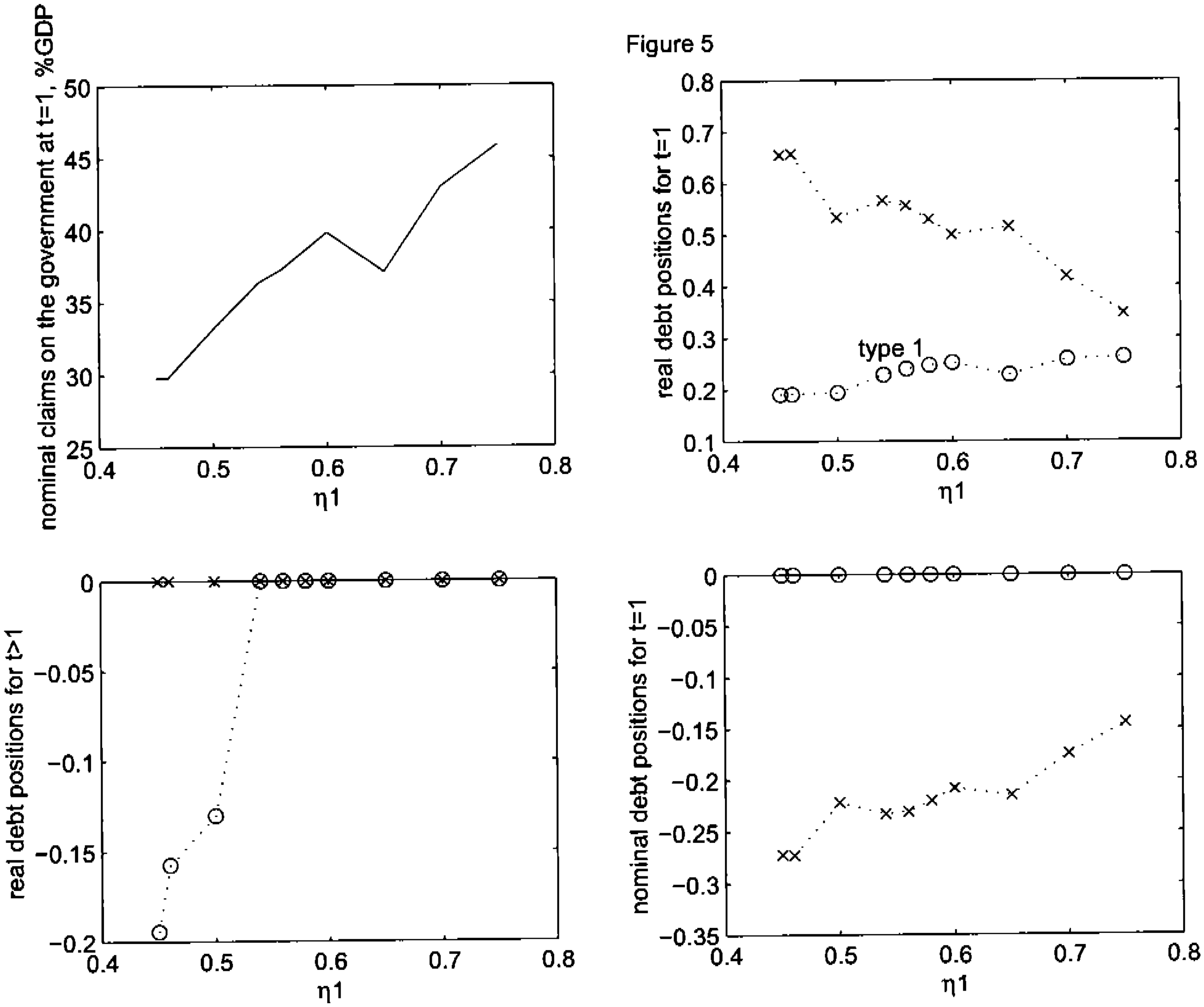
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
τ1, τ2
0.3 0.4 0.5 0.6 0.7
η1
30
25
20
15
10
5
0
% net nominal interest rate
multiplier
0.3 0.4 0.5 0.6 0.7
η1
2
1.8
1.6
1.4
1.2
1
0.8
0.6
reliance on labor income tax
0.3 0.4 0.5 0.6 0.7
η1
60
50
40
30
20
10
% labor income tax revenues from type 1
0.3 0.4 0.5 0.6 0.7
η1

Figure 4



τ1, τ2
η1
% net nominal interest rate
multiplier
η1
reliance on labor income tax
η1
% labor income tax revenues from type 1
η1

Figure 5


nominal claims on the government at t=1, %GDP
η1
real debt positions for t=1
type 1
real debt positions for t>1
nominal debt positions for t=1