What Measure of Inflation Should a Central Bank Target?

N. Gregory Mankiw
Harvard University

Ricardo Reis
Harvard University

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Abstract

This paper assumes that a central bank commits itself to maintaining an inflation target and then asks what measure of the inflation rate the central bank should use if it wants to maximize economic stability. The paper first formalizes this problem and examines its microeconomic foundations. It then shows how the weight of a sector in the stability price index depends on the sector's characteristics, including size, cyclical sensitivity, sluggishness of price adjustment, and magnitude of sectoral shocks. When a numerical illustration of the problem is calibrated to U.S. data, one tentative conclusion is that the central bank should use a price index that gives substantial weight to the level of nominal wages.

Over the past decade, many central banks around the world have adopted inflation targeting as a guide for the conduct of monetary policy. In such a regime, the price level becomes the economy's nominal anchor, much as a monetary aggregate would be under a monetarist policy rule. Inflation targeting is often viewed as a way to prevent the wild swings in monetary policy that were responsible for, or at least complicit in, many of the macroeconomic mistakes of the past. A central bank committed to inflation targeting would likely have avoided both the big deflation during the Great Depression of the 1930s and the accelerating inflation of the 1970s (and thus the deep disinflationary recession that followed).

This paper takes as its starting point that a central bank has adopted a regime of inflation targeting and asks what measure of the inflation rate it should target. Our question might at first strike some readers as odd. Measures of the overall price level, such as the consumer price index, are widely available and have been amply studied by index-number theorists. Yet a price index designed to measure the cost of living is not necessarily the best one to serve as a target for a monetary authority.

This issue is often implicit in discussions of monetary policy. Many economists pay close attention to "core inflation," defined as inflation excluding certain volatile prices, such as food and energy prices. Others suggest that commodity prices might be particularly good indicators because they are highly responsive to changing economic conditions. Similarly, during the U.S. stock market boom of the 1990s, some economists called for Fed tightening to dampen "asset price inflation," suggesting that the right index for monetary policy might include not only the prices of goods and services but asset prices as well. Various monetary proposals can be viewed as inflation targeting with a nonstandard price index: The gold standard uses only the price of gold, and a fixed exchange rate uses only the price of a foreign currency.

Some formal models also indicate that monetary policymakers should use an index other than the CPI. Phelps (1978) and Mankiw and Reis (2001b) propose that the central bank target the nominal wage and allow the price of goods and services to move with supply shocks, such as changes in productivity. Aoki (2001) examines a model with two sectors – one with sticky prices and one with flexible prices – and concludes that monetary policy should target inflation in the sticky-price sector.

In this paper, we propose and explore an approach to choosing a price index for the central bank to target. We are interested in finding the price index that, if kept on an assigned target, would lead to the greatest stability in economic activity. This concept might be called the *stability price index*.

The key issue in the construction of any price index is the weights assigned to the prices from different sectors of the economy. When constructing a price index to measure the cost of living, the natural weights are the share of each good in the budget of typical consumer. When constructing a price index for the monetary authority to target, additional concerns come into play: the cyclical sensitivity of each sector, the proclivity of each sector to experience idiosyncratic shocks, and the speed with which the prices in each sector respond to changing conditions.

Our goal in this paper is to show how the weights in a stability price index should depend on these sectoral characteristics. Section 1 sets up the problem. Section 2 examines the microeconomic foundations for the problem set forth in Section 1. Section 3 presents and discusses the analytic solution for the special case with only two sectors. Section 4 presents a more realistic numerical illustration, which we calibrate with plausible parameter values for the U.S. economy. One tentative conclusion is that the stability price index should give a substantial weight to the level of nominal wages.

1 The Optimal Price Index: Statement of the Problem

Here we develop a framework to examine the optimal choice of a price index. To keep things simple, the model includes only a single period of time. The central bank is committed to inflation targeting in the following sense: Before the shocks are realized, the central bank must choose a price index and commit itself to keeping that index on target.

The model includes many sectoral prices, which differ according to four characteristics. (1) Sectors differ in their budget share and thus the weight their prices receive in a standard price index. (2) In some sectors equilibrium prices are highly sensitive to the business cycle, while in other sectors equilibrium prices are less cyclical. (3) Some sectors experience large idiosyncratic shocks, while other sectors do not. (4) Some prices are flexible, while others are sluggish in responding to changing economic conditions.

To formalize these sectoral differences, we borrow from the so-called "new Keynesian" literature on price adjustment. We begin with an equation for the equilibrium price in sector k:

$$p_k^* = p + \alpha_k y + \varepsilon_k \tag{1}$$

where, with all variables expressed in logs, p_k^* is the equilibrium price in sector k, p is the price level as conventionally measured (such as the CPI), α_k is the sensitivity of sector k's equilibrium price to the business cycle, y is output (or some other measure of economic activity), and ε_k is an idiosyncratic shock to sector k with variance σ_k^2. This equation says only that the equilibrium relative price in a sector depends on the state of the business cycle and some other shock. Sectors can differ in their sensitivities to the cycle and in the variances of their idiosyncratic shocks.

In Section 2 we examine some possible microeconomic foundations for this model, but some readers may be familiar with the equation for the equilibrium price from the literature on price setting under monopolistic competition[1]. The index p represents the nominal variable that shifts supply and demand, and thus the equilibrium prices, in all the sectors. This variable corresponds to a standard price index such as the CPI. That is, if there are K sectors,

$$p = \sum_{k=1}^{K} \theta_k p_k$$

where θ_k are the weights of different sectors in the typical consumer's budget. One interpretation of the shocks ε_k is they represent changes in the degree of competition in sector k. The formation of an oil cartel, for instance, would be represented by a positive value of ε_k in the oil sector.

Sectors may also have sluggish prices. We model the sluggish adjustment by assuming that some fraction of prices in a sector are predetermined. One rationale for this approach,

[1] For a textbook treatment, see Romer (2001, equation 6.45).

following Fischer (1977), is that some prices are set in advance by nominal contracts. An alternative rationale, following Mankiw and Reis (2001a), is that price setters are slow to update their plans because there are costs to acquiring or processing information. In either case, the key feature for the purpose at hand is that some prices in the economy are set based on old information and do not respond immediately to changing circumstances.

Let λ_k be the fraction of the price setters in sector k that set their prices based on updated information, while $1 - \lambda_k$ set prices based on old plans and outdated information. Thus, the price in period t is determined by

$$p_k = \lambda_k p_k^* + (1 - \lambda_k) E(p_k^*) \tag{2}$$

The parameter λ_k measures how sluggish prices are in sector k. The smaller is λ_k, the less responsive actual prices are to news about equilibrium prices. As λ_k approaches one, the sector approaches the classical benchmark where actual and equilibrium prices are always the same.

The central bank is assumed to be committed to targeting inflation. That is, the central bank will keep a weighted average of sectoral prices at a given level, which we can set equal to zero without loss of generality. We can write this as

$$\sum_{k=1}^{K} \omega_k p_k = 0 \tag{3}$$

for some set of weights such that

$$\sum_{k=1}^{K} \omega_k = 1$$

We will call $\{\omega_k\}$ the target weights and $\{\theta_k\}$ the consumption weights. The target weights are choice variables of the central bank. The sectoral characteristics (θ_k, α_k, λ_k, and σ_k^2) are taken as exogenous.

We assume that the central bank dislikes volatility in economic activity. That is, its goal is to minimize $Var(y)$. We abstract from the problem of monetary control by assuming that the central bank can hit precisely whatever nominal target it chooses. The central question of this paper is the choice of weights $\{\omega_k\}$ that will lead to greatest macroeconomic stability.

Putting everything together, the central bank's problem can now be stated as follows:

$$\min_{\{\omega_k\}} Var(y)$$

subject to

$$\begin{aligned}
\sum_{k=1}^{K} \omega_k p_k &= 0 \\
\sum_{k=1}^{K} \omega_k &= 1 \\
p_k &= \lambda_k p_k^* + (1-\lambda_k)E(p_k^*) \\
p_k^* &= p + \alpha_k y + \varepsilon_k \\
p &= \sum_{k=1}^{K} \theta_k p_k
\end{aligned}$$

The central bank chooses the weights in its targeted price index in order to minimize output volatility, given the constraints the economy imposes on the evolution of prices over time. The solution to this problem will yield the set of weights ω_k in an optimal price index as a function of sector characteristics, which include θ_k, α_k, λ_k, and σ_k^2. We call the resulting measure the *stability price index*, because it is the price index that, if kept on target, would lead to the greatest possible stability in economic activity[2].

At this point, there are two questions that might intrigue readers of this paper. What

[2]The central bank's problem formalized here is more general than might be suggested by the way we have described it. In particular, the variables p_k can be interpreted more broadly than sectoral prices. Almost any nominal variable in the economy whose equilibrium value moves one-for-one with the price level and in response to the business cycle is a candidate. Examples include the monetary aggregates, nominal income, and the exchange rate – three variables that have been suggested as targets for central banks. These variables do not show up in a conventional price index (that is, $\theta_k = 0$), but they can appear in the central bank's targeted index (that is, there is no reason to presume that $\omega_k = 0$). Thus, this framework can be viewed broadly as a way of determining the optimal choice of a nominal anchor as a weighted average of many nominal variables. The choice of an optimal price index for inflation targeting is a restricted application of the more general problem.

This framework could also be useful for a central bank in a monetary union deciding how to weight inflation in different regions. Here, each "sector" is a national economy. For related work, see Benigno (2001).

are the microfoundations behind this problem? What is the solution to this problem? Those interested in the first question should continue on to Section 2. Those interested only in the second question should jump to Section 3.

2 Some Microeconomic Foundations

In this section we build a general equilibrium model that delivers, in reduced form, the problem presented in the previous section. We approach this task aiming for simplicity rather than generality. We suspect that the stability-price-index problem, or some variant of it, arises in settings more general than the one we examine here. Our goal now is to give one example and, at the same time, to relate the stability-price-index problem to the large new Keynesian literature on price adjustment.

2.1 The Economy Without Nominal Rigidities

The economy contains two types of agents–households and firms. The representative household obtains utility from consumption C and disutility from labor L, according to the utility function:

$$U(C, L) = \frac{C^{1-\sigma}}{1-\sigma} - L.$$

There are many types of consumption goods. Following Spence (1976) and Dixit and Stiglitz (1977), we model the household's demand for these goods using a constant elasticity of substitution (CES) aggregate. Final consumption C is a CES aggregate over the goods in the K sectors of the economy:

$$C = \left[\sum_{k=1}^{K} \theta_k^{\frac{1}{\gamma}} C_k^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}. \tag{4}$$

The parameter γ measures the elasticity of substitution across the K sectors. The weights θ_k sum to one and express the relative size of each sector.

Within each sector, there are many firms, represented by a continuum over the unit interval. The sector's output is also a CES aggregate of the firms' outputs:

$$C_k = \left[\int_0^1 C_{ki}^{\frac{\gamma-1}{\gamma}} di\right]^{\frac{\gamma}{\gamma-1}}. \tag{5}$$

Notice that, for simplicity, we have assumed that the elasticity of substitution is the same across sectors and across firms within a sector[3].

The representative household's budget constraint is:

$$\sum_{k=1}^{K}\left(\int P_{ki}C_{ki}di\right) = WL + \Pi.$$

The household obtains income from supplying labor, which earns a nominal wage W, and collecting profits from all the firms, denoted Π. It spends all its income on the consumption goods C_{ki}.

From this household problem, we can derive the demand functions for each sector and each firm. It is useful to begin by first defining these price indices[4]:

$$\begin{aligned} P &= \left[\sum_{k=1}^{K}\theta_k P_k^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \\ P_k &= \left[\int_0^1 P_{ki}^{1-\gamma} di\right]^{\frac{1}{1-\gamma}}. \end{aligned}$$

The demand function can then be expressed as:

$$\begin{aligned} C_k &= \left(\frac{P_k}{P}\right)^{-\gamma}\theta_k C \qquad \text{, and} \\ C_{ki} &= \left(\frac{P_{ki}}{P_k}\right)^{-\gamma} C_k \\ &= \left(\frac{P_{ki}}{P}\right)^{-\gamma}\theta_k C. \end{aligned} \tag{6}$$

[3]As is usual, there are two ways to interpret these CES aggregators. The more common approach is to view them as representing consumers' taste for variety. Alternatively, one can view C as the single final good that consumers buy and the CES aggregators as representing production functions for producing that final good from intermediate goods.

[4]For a derivation of these price indices, see either the original article by Dixit and Stiglitz (1977) or a textbook treatment, such as Obstfeld and Rogoff (1996, p. 664).

The quantity demanded of the good produced by firm i in sector k is a function of its relative price, P_{ki}/P, with an elasticity of demand of γ. It also depends on the sector size θ_k and aggregate consumption C. In addition, we obtain this first-order condition determining the level of aggregate consumption:

$$\frac{W}{P} = C^{\sigma}. \tag{7}$$

This equation, together with the budget constraint, determines the optimal choice of consumption C and labor supply L.

Let's now turn to the supply side of the goods market. We write the desired price of firm i in sector k as:

$$\frac{P^*_{ki}}{P} = m_k MC(Y_{ki}). \tag{8}$$

The relative price of any good is a markup m_k times the real marginal cost of producing the good. The markup m_k can capture many possible market structures from standard monopoly (which here implies $m_k = \gamma/(\gamma - 1)$) to competition ($m_k = 1$). We allow m_k to be both stochastic and a function of the level of economic activity[5]. We express this as

$$m_k = Y^{\phi_k} e^{\varepsilon_k}$$

where ε_k is a random variable with mean μ_k and variance σ_k^2. The parameter ϕ_k governs the cyclical sensitivity of the markups in sector k, and it can be either positive or negative.

The production function takes the simple form $Y_{ki} = L_{ki}$ for all firms, so one unit of labor in firm i in sector k produces one unit of output of that variety. Firms are assumed to hire labor in a competitive market. Thus, the real marginal cost for every firm equals the real wage.

We can now solve for the economy's equilibrium. Using the first-order condition (7), the pricing equation (8), and the market-clearing condition that $C = Y$, we obtain the following

[5] Rotemberg and Woodford (1999) survey alternative theories of why markups may vary over the business cycle.

equation for the log of the equilibrium price[6]:

$$p_k^* = p + \alpha_k y + \varepsilon_k,$$

where $\alpha_k = \sigma + \phi_k$. Note that this is the equation for the desired price posited in the previous section. In this general equilibrium model, an increase in output influences equilibrium prices both because it raises marginal cost and because it influences the markup. The supply shock ε_k reflects stochastic fluctuations in the markup in sector k.

It will prove convenient to have a log-linearized version of the aggregate price index. Reexpressing the price index in the economy in terms of logs:

$$p = \frac{1}{1-\gamma} \log \left(\sum_{k=1}^{K} \theta_k \exp((1-\gamma)p_k) \right).$$

A first-order approximation to this equation yields:

$$p = \sum_{k=1}^{K} \theta_k p_k.$$

This equation also corresponds to the problem stated in Section 1.

Using this linearized equation for the price level, and the expression for the equilibrium prices in each sector, we can solve for equilibrium output as a function of the parameters and shocks. We find:

$$y^* = -\frac{\sum_{k=1}^{K} \theta_k \varepsilon_k}{\sum_{k=1}^{K} \theta_k \alpha_k}$$

The equilibrium level of output depends on a weighted average of the markups. The larger the markups, the more distorted the economy, and the lower the equilibrium level of output. The expected level of output $E(y^*)$ is $-\sum \theta_k \mu_k / \sum \theta_k \alpha_k$, which depends on $\sum \theta_k \mu_k$ the expected average markup in the economy.

[6]Since all firms in a sector are identical they all have the same desired price. The right hand side of the equation is the same for all i. Therefore we replace p_{ki}^* by p_k^*.

2.2 The Economy With Nominal Rigidities

We now introduce nominal rigidities into the economy. We assume that although all firms in sector k have the same desired price p_k^*, only a fraction λ_k has updated information and is able to set its actual price equal to its desired price. The remaining $1 - \lambda_k$ firms must set their prices without current information and thus set their prices at $E(p_k^*)$. Using a log-linear approximation for the sectoral price level similar to the one used above for the overall price level, we obtain

$$p_k = \lambda_k p_k^* + (1 - \lambda_k)E(p_k^*).$$

The sectoral price is a weighted average of the actual desired price and the expected desired price. As we noted earlier, this kind of price rigidity can be justified on the basis on nominal contracts as in Fischer (1977) or information lags as in Mankiw and Reis (2001a).

The equilibrium in this economy involves $K + 2$ key variables: all the sectoral prices p_k and the two aggregate variables p and y. The above equation for p_k provides K equations (once we substitute in for p_k^*). The equation for the aggregate price index provides another equation:

$$p = \sum_{k=1}^{K} \theta_k p_k.$$

The last equation comes from the policymaker's choice of a nominal anchor:

$$\sum_{k=1}^{K} \omega_k p_k = 0.$$

We do not model how this target is achieved. That is, we do not model the transmission mechanism between the instruments of monetary policy and the level of prices. Instead, our focus is on the choice of a particular policy target, which here is represented by the weights ω_k.

The choice of weights depends on the policymaker's objective function. In this economy, the obvious welfare measure is the utility of the representative household. Recalling that

$Y_{ki} = C_{ki} = L_{ki}$, we can express the household's utility as

$$U = \frac{Y^{1-\sigma}}{1-\sigma} - \sum_{k=1}^{K} \left[\int Y_{ki} di \right]$$

where Y is defined by the CES aggregator in equation (4). We can take a second-order logarithmic approximation to this utility function around expected output:

$$\begin{aligned} E(U) \approx\ & e^{(1-\sigma)E(y)} \left[\frac{1}{1-\sigma} + E(y) + \frac{1-\sigma}{2} E(y - E(y))^2 \right] - \\ & - \sum_{k=1}^{K} \left[\int e^{E(y_{ki})} \left(1 + E(y_{ki}) + \frac{1}{2} E(y_{ki} - E(y_{ki})^2) \right) di \right] \end{aligned} \quad (9)$$

where $y = \log(Y)$ and $y_{ki} = \log(Y_{ki})$. Expected utility depends on expected output, the expected square of output, and the distribution of output across firms.

There are two ways to make the leap from this expression for expected utility to the variance of output, the objective function that we posited in Section 1. One approach is to take the limiting case in which the risk aversion parameter σ is very large. In this case, the $E(y - E(y))^2$ term in equation (9) dominates expected utility. This fact, together with the observation that the natural rate hypothesis holds in the linearized model (so Ey is invariant to policy), implies the policymaker's objective is to minimize the volatility in economic activity. This is exactly what we assumed in Section 1.

A second approach is to take advantage of some features of the CES aggregator to solve out for some of the terms in equation (9). We pursue this approach in Appendix 1. This approach combines the first-order logarithmic approximation of the behavioral model with the second-order logarithmic approximation to the objective function and is thus related to familiar linear-quadratic approximations. The accuracy of this approximation, however, is an open question. Nonetheless, it also leads to the conclusion that the policymaker is interested in minimizing the variance of the output gap.

The bottom line from this analysis is that we can view the stability-price-index problem stated in Section 1 as a reduced form of a model of price adjustment under monopolistic competition. The canonical models in this literature assume symmetry across sectors in order to keep the analysis simple (e.g., Blanchard and Kiyotaki, 1987; Ball and Romer, 1990). Yet sectoral differences are at the heart of our problem. Therefore, we have extended the analysis to allow for a rich set of sectoral characteristics, which are described by the parameters θ_k, α_k, λ_k, and σ_k^2.

3 The Two-Sector Solution

We are now interested in solving the central bank's problem. To recap, it is:

$$\min_{\{\omega_k\}} Var(y)$$

subject to

$$\begin{aligned}
\sum_{k=1}^{K} \omega_k p_k &= 0 \\
\sum_{k=1}^{K} \omega_k &= 1 \\
p_k &= \lambda_k p_k^* + (1-\lambda_k)E(p_k^*) \\
p_k^* &= p + \alpha_k y + \varepsilon_k \\
p &= \sum_{k=1}^{K} \theta_k p_k
\end{aligned}$$

The central bank chooses a target price index to minimize output volatility, given the constraints the imposed by the price-setting process.

To illustrate the nature of the solution, we now make the simplifying assumptions that there are only two sectors ($K = 2$), which we call sector A and sector B, and that the shocks to each sector (ε_A and ε_B) are uncorrelated. We also assume that α_A and α_B are both nonnegative. Appendix 2 derives the solution to this special case. The conclusion is the following equation for the optimal weight on sector A:

$$\omega_A^* = \lambda_B \frac{\alpha_A \sigma_B^2 - \theta_A \lambda_A \left(\alpha_A \sigma_B^2 + \alpha_B \sigma_A^2\right)}{\alpha_B \lambda_A (1-\lambda_B)\sigma_A^2 + \alpha_A \lambda_B (1-\lambda_A)\sigma_B^2}$$

Notice that the optimal target weight depends on all the sectoral characteristics and, in general, need not be between zero and one[7].

From this equation, we can derive several propositions that shed light on the nature of the solution. We begin with a very special case.

Proposition 1 *If the two sectors are identical (same θ_k, α_k, λ_k, and σ_k^2), then the stability price index gives them equal weight ($\omega_k^* = 1/2$).*

This result is not surprising, as it merely reflects the symmetry of the two sectors.

More interesting results arise when the sectoral characteristics (θ_k, α_k, λ_k, and σ_k^2) vary. Let's start with the two characteristics that describe equilibrium prices.

Proposition 2 *An increase in α_k raises the optimal ω_k. That is, the more responsive a sector is to the business cycle, the more weight that sector's price should receive in the stability price index.*

Proposition 3 *An increase in σ_k^2 reduces the optimal ω_k. That is, the greater the magnitude of idiosyncratic shocks in a sector, the less weight that sector's price should receive in the stability price index.*

Propositions 2 and 3 both coincide with aspects of the conventional wisdom. When economists point to commodity prices as a useful economic indicator for monetary policy, they usually do so on the grounds that these prices are particularly responsive to the business cycle. The index of leading indicators, for instance, includes the change in "sensitive materials prices." Proposition 2 can be used to justify this approach. At the same time, when economists reduce the weight they give to certain sectors, as they do with food and energy sectors in the computation of the core CPI, they do so on the grounds that these sectors are subject to particularly large sector-specific shocks. Proposition 3 can be used to justify this approach.

[7] This equation for the optimal target weight sheds light on the conclusion in Mankiw and Reis (2001b) about the optimality of targeting the nominal wage. The theoretical model in that paper can be viewed as a special case of the model here, including some strong restrictions on the parameter values. If sector A is the labor market and sector B is the goods market, then the earlier model can be written in a form such that $\theta_A = 1$, $\lambda_B = 1$, $\alpha_B = 0$, and $\sigma_A^2 = 0$. In this special case, the equation for the optimal target weight immediately implies that $\omega_A^* = 1$. Below, we conduct an empirical exercise using the prices of both goods and labor, but we do not impose such strong *a priori* restrictions about their behavior.

Let's now consider the effects of price sluggishness on the optimal target weights:

Proposition 4 *If the optimal weight for a sector is not larger than 100 percent ($\omega_k \leq 1$), then an increase in λ_k reduces the optimal ω_k. That is, the more flexible a sector's price, the less weight that sector's price should receive in the stability price index.*

A special case is noteworthy:

Proposition 5 *If the two sectors are identical in all respects except one has full price flexibility prices (same α_k, θ_k, and σ_k^2 but $\lambda_A = 1$, $\lambda_B < 1$), then the monetary authority should target the price level in the sticky-price sector ($\omega_B = 1$).*

This result is parallel to that presented by Aoki (2001). But the very strong conclusion that the central bank should completely ignore the flexible-price sector does not generalize beyond the case of otherwise identical sectors. Even if a sector has fully flexible prices, the optimal target weight for that sector is in general nonzero.

The last sectoral characteristic to consider is θ_k, the weight that the sector receives in the consumer price index.

Proposition 6 *An increase in θ_k reduces the optimal ω_k. That is, the more important a price is in the consumer price index, the less weight that sector's price should receive in the stability price index.*

This proposition is probably the least intuitive one. It illustrates that choosing a price index to aim for economic stability is very different than choosing a price index to measure the cost of living.

What is the intuition behind this surprising result? Under inflation targeting, undesirable fluctuations in the output gap arise when there are shocks ε_k to equilibrium prices, which the central bank has to offset with monetary policy. The effect of a shock in sector k depends on the consumption weight θ_k. The greater is the consumption weight, the more the shock feeds into other prices in the economy, and the more disruptive it is. Thus, to minimize the disruptive effect of a shock, a central bank should accommodate shocks to large sectors. Under inflation targeting, such accommodation is possible by reducing the weight of the sector in the target index. Hence, holding all the other parameters constant, sectors with a larger weight in the consumption index should receive a smaller weight in the target index.

To sum up, the ideal sectoral prices for a central bank to monitor are those that are highly sensitive to the economy (large α_k), experience few sectoral shocks (small σ_k^2), have very sluggish prices (low λ_k), and are relatively small in the aggregate price index (small θ_k).

4 Toward Implementation: An Example

The two-sector example considered in the previous section is useful for guiding intuition, but if a central bank is to compute a stability price index, it will need to go beyond this simple case. In this section, we take a small step toward a more realistic implementation of the stability price index.

4.1 The Approach

We apply the model to annual data for the U.S. economy from 1957 to 2001. We examine four sectoral prices: the price of food, the price of energy, the price of other goods and services, and the level of nominal wages. The first three prices are categories of the consumer price index, while wages refer to compensation per hour in the business sector. All four come from the Bureau of Labor and Statistics. For the level of economic activity y, we use the log of real GDP, obtained from the Bureau of Economic Analysis.

A key question is how to assign parameters to these four sectors. We begin by noting the following equation holds in the model:

$$p_k - Ep_k = \lambda_k(p - Ep) + \alpha_k\lambda_k(y - Ey) + \lambda_k(\varepsilon_k - E\varepsilon_k) \tag{10}$$

That is, the price surprise in sector k is related to the overall price surprise, the output surprise, and the shock. To obtain these surprise variables, we regressed p_k, p, and y on three of its own lags, a constant, and a time trend and took the residual. These surprise variables are the data used in all subsequent calculations.

In principle, one should be able to obtain the parameters by estimating equation (10).

In practice, the identification problem makes formal estimation difficult. Shocks (such as an energy price increase) will likely be correlated with the overall price level and the level of economic activity. Finding appropriate instruments is a task we leave for future work. Here, as a first pass, we adopt a cruder approach that is akin to a back-of-the-envelope calculation.

For the parameter λ_k, which governs the degree of price sluggishness, we rely on bald, but we hope realistic, assumptions. We assume the food and energy prices are completely flexible, so $\lambda_k = 1$. Other prices and wages are assumed to be equally sluggish. We set $\lambda_k = 1/2$, indicating that half of price setters in these sectors base their prices based on expected, rather than actual, economic conditions.

Another key parameter is α_k, the sensitivity of desired prices to the level of economic activity. We estimate this parameter by assuming that the 1982 economic downturn – the so-called Volcker recession – was driven by monetary policy, rather than sectoral supply shocks. Thus, we pick α_k for each sector so that equation (10) without any residual holds exactly for 1982. That is, we are using the price responses during the 1982 recession to measure the cyclical sensitivity of sectoral prices.

With α_k and λ_k, we can compute a time series of $\varepsilon_k - E\varepsilon_k$ and, thus, its variance-covariance matrix. Note that we do not assume that the shocks are uncorrelated across sectors. The previous section made this assumption to obtain easily interpretable theoretical results, but for a more realistic numerical exercise, it is better to use the actual covariances. Thus, if there is some shock that influences desired prices in all sectors (for a given p and y), this shock would show up in the variance-covariance matrix, including the off-diagonal elements.

The last parameter is the consumption weight θ_k. We take this parameter from the "relative importance" of each sector in the consumer price index as determined by the Bureau of Labor Statistics. For nominal wages, θ_k is equal to zero, because nominal wages do not appear in the consumer price index.

With all the parameters in hand, it is now a straightforward numerical exercise to find the set of target weights ω_k that solves the stability-price-index problem as set forth above. Appendix 3 describes the algorithm.

4.2 The Results

Table 1 presents the results from this exercise. The last two columns present the results from the optimization. The last column, denoted ω^c, imposes the constraint that all the sectoral weights in the stability price index be nonnegative. The second to last column, denoted ω^u, allows the possibility of negative weights. The substantive result is similar in the two cases: The price index that the central bank should use to maximize economic stability gives most of its weight to the level of nominal wages.

The intuition behind this result is easy to see. The value of α_k for nominal wages is 0.25, which is larger than the parameter for most other sectors. (This parameter value reflects the well-known fact that real wages are procyclical.) The only other sector that exhibits such a large value of α_k is the energy sector. But the variance of shocks in the energy sector, measured by $Var(\varepsilon_k)$, is very large, making it an undesirable sector for the stability price index. The combination of high α_k and low $Var(\varepsilon_k)$ makes nominal wages a particularly useful addition to the stability price index.[8]

One might suspect that the zero value of θ_k for nominal wages in the consumer price index is largely responsible for the high value of ω_k in the stability price index. That turns out not to be the case. Table 2 performs the same empirical exercise as in Table 1, but it assumes that the economy's true price index gives half its weight to nominal wages (that is, $p = 0.5w + 0.5cpi$). Once again, the most important element of the stability price index is the level of nominal wages[9].

Two other striking results in Table 1 are the large weight on the price of food and the large, negative weight on the price of goods other than food and energy. These results depend crucially on the pattern of correlations among the estimated shocks. If these correlations are set to zero, the target weights for food and other goods are much closer to zero (while the target weight for nominal wages remains close to one). In light of this sensitivity, it would

[8]Indeed, if a better index of wages were available, it would likely be more procyclical, reinforcing our conclusion. See Solon, Barsky, and Parker (1994) on how composition bias masks some of the procyclicality of real wages.

[9]How is the approximate irrelevance of θ_k here consistent with Proposition 6? The proposition examines what happens to ω_k when θ_k changes, holding constant other parameter values. But in this empirical exercise, if we change the weight given to some sector in the price index p, we also change the estimated values of α_k and the variance-covariance matrix of ε_k.

be a mistake to emphasize this aspect of the results. One clear lesson, however, is that the variance-covariance matrix of the shocks is a key input into the optimal choice of a price index.

Finally, it is worth noting that the gain in economic stability from targeting the stability price index rather than the consumer price index is large. It is straightforward to calculate the variance of output under each of the two policy rules. According to this model, moving from a target for the consumer price index to a target for the stability price index reduces the output variance by 57 percent (or by 52 percent with a nonnegativity constraint on the weights). Thus, the central bank's choice of a price index to monitor inflation is an issue of substantial economic significance.

Table 1

Results of Empirical Illustration

Sector	λ	α	$Var(\varepsilon)$	θ	ω^u	ω^c
Energy	1.0	0.31	0.00287	0.07	0.09	0.02
Food	1.0	0.08	0.00024	0.15	0.43	0.23
Other Goods	0.5	0.09	0.00017	0.78	-0.84	0
Wages	0.5	0.25	0.00048	0	1.32	0.75

Correlation Matrix of Epsilon

	Energy	Food	Other Goods	Wages
Energy	1.00	-0.29	0.21	-0.15
Food		1.00	-0.27	-0.03
Other Goods			1.00	0.30
Wages				1.00

Table 2

Results with Alternative Price Index

Sector	λ	α	$Var(\varepsilon)$	θ	ω^u	ω^c
Energy	1.0	0.28	0.00327	0.03	0.06	0.01
Food	1.0	0.05	0.00030	0.08	0.35	0.21
Other Goods	0.5	0.06	0.00023	0.39	-0.61	0
Wages	0.5	0.22	0.00027	0.50	1.20	0.78

Correlation Matrix of Epsilon

	Energy	Food	Other Goods	Wages
Energy	1.00	-0.01	0.43	-0.10
Food		1.00	0.03	-0.32
Other Goods			1.00	0.04
Wages				1.00

5 Conclusion

Economists have long realized that price indices designed to measure the cost of living may not be the right ones for the purposes of conducting monetary policy. This intuitive insight is behind the many attempts to measure "core inflation." Yet, as Wynne (1999) notes in his survey of the topic, the literature on core inflation has usually taken a statistical approach without much basis in monetary theory. As a result, measures of core inflation often seem like answers in search of well-posed questions.

The price index proposed in this paper can be viewed as an approach to measuring core inflation that is grounded in the monetary theory of the business cycle. The stability price index is the weighted average of prices that, if kept on target, leads to the greatest stability in economic activity. The weights used to construct such a price index depend on sectoral characteristics that differ markedly from those relevant for measuring the cost of living.

Calculating a stability price index is not an easy task. Measuring all the relevant sectoral characteristics is an econometric challenge. Moreover, there are surely important dynamics in the price-setting decision that we have omitted in our simple model. Yet, if the calculations performed in this paper are indicative, the topic is well worth pursuing. The potential improvement in macroeconomic stability from targeting the optimal price index, rather than the consumer price index, appears large.

Our results suggest that a central bank should give substantial weight to the growth in nominal wages when monitoring inflation. This conclusion follows from the fact that wages are more cyclically sensitive than most other prices in the economy (which is another way of stating the well-known fact that the real wage is procyclical). Moreover, compared to other cyclically sensitive prices, wages are not subject to large idiosyncratic shocks. Thus, if nominal wages are falling relative to other prices, it indicates a cyclical downturn, which in turn calls for more aggressive monetary expansion. Conversely, when wages are rising faster than other prices, targeting the stability price index requires tighter monetary policy than does conventional inflation targeting.

An example of this phenomenon occurred in the United States during the second half of the 1990s. Here are the U.S. inflation rates as measured by the consumer price index and an index of compensation per hour:

Year	CPI	Wages
1995	2.8	2.1
1996	2.9	3.1
1997	2.3	3.0
1998	1.5	5.4
1999	2.2	4.4
2000	3.3	6.3
2001	2.8	5.8

Consider how a monetary policymaker in 1998 would have reacted to these data. Under conventional inflation targeting, inflation would have seemed very much in control, as the CPI inflation rate of 1.5 percent was the lowest in many years. By contrast, a policymaker trying to target a stability price index would have observed accelerating wage inflation. He would have reacted by slowing money growth and raising interest rates (a policy move that in fact occurred two years later). Would such attention to a stability price index have restrained the exuberance of the 1990s boom and avoided the recession that began the next decade? There is no way to know for sure, but the hypothesis is intriguing.

Appendix 1 - Approximation of the utility function

In this appendix, following Woodford (2002), we derive the objective function of the policymaker as a Taylor 2^{nd} order log-linear approximation of the utility function.

The first issue to address is the choice of the point around which to linearize. A natural choice is the expected equilibrium in the economy. Yet, as Woodford discusses, it is important for the accuracy of the log-linearization that this is also close enough to the efficient equilibrium of the economy. To ensure this is the case, we assume the average markup is zero for all sectors: $\mu_k = 0$. One way to make this consistent with the monopolistic competition model is to introduce a production subsidy to firms funded by lump-sum taxes on consumers[10].

We start by deriving the expected equilibrium. Taking expectations of equation (2) combined with equation (1), the expected price in each sector is:

$$Ep_k = Ep + \alpha_k Ey. \tag{A.1}$$

Multiplying both sides by the weight of sector k in the overall price index θ_k, and adding up across sectors, we obtain:

$$Ep = Ep + Ey \sum_k \theta_k \alpha_k,$$

where we used the fact that the expectation is a linear operator and so $\sum \theta_k Ep_k = E \sum \theta_k p_k = Ep$. The first result then follows that the expected value of output is equal to zero:

$$E(y) = 0.$$

Using the policy rule in equation (3) we find the expected price level in the economy $E(p) = 0$, and using equation (A.1) the expected price in each sector is $Ep_k = 0$. Finally using the demand functions in equation (6), expected output in each firm and sector is:

[10] Alternatively we could allow the markups to be of first or higher stochastic order.

$$E(y_{ki}) = E(y_k) = \log(\theta_k)$$

It is important to note that the economy respects the natural rate property – all the expected output variables ($E(y)$, $E(y_{ki})$, $E(y_k)$) are independent of monetary policy (ω_k), and so are beyond the control of the policymaker

We can now turn to the linearization of the utility function:

$$U(.) = \frac{Y^{1-\sigma}}{1-\sigma} - \sum_{k=1}^{K} \left(\int_0^1 Y_{ki} di \right).$$

We approximate each of its two additive components in term.

On the first term, a 2^{nd} order linear approximation of y around its expected value $E(y) = 0$ yields the expression:

$$\begin{aligned} \frac{Y^{1-\sigma}}{1-\sigma} &= \frac{e^{(1-\sigma)y}}{1-\sigma} = \\ &\approx \frac{1}{1-\sigma}(e^{(1-\sigma)Ey} + (1-\sigma)e^{(1-\sigma)Ey}y + \frac{1}{2}(1-\sigma)^2 e^{(1-\sigma)Ey}y) = \\ &= \frac{1}{1-\sigma} + y + \frac{1}{2}(1-\sigma)y^2 = \\ &\approx y + \frac{1}{2}(1-\sigma)y^2. \end{aligned} \tag{A.2}$$

The last approximation involves dropping a term that enters the expression additively and which the policymaker can not affect, and which therefore does not influence the results from the optimization.

As for the second term, we also approximate it using a Taylor 2^{nd} order approximation of the log of its components around their expected value. We do this in three steps, (1) approximating each firm's output around its expected value, (2) adding up within sectors, and then (3) adding up across sectors. The log linear approximation of Y_{ki} around the expected value $E(y_{ki}) = \log(\theta_k)$ is given by:

$$\begin{aligned} Y_{ki} &= e^{y_{ki}} = \\ &\approx e^{Ey_{ki}} + e^{Ey_{ki}}(y_{ki} - E(y_{ki})) + \frac{1}{2}e^{Ey_{ki}}(y_{ki} - E(y_{ki}))^2 = \\ &\approx \theta_k \left\{ \hat{y}_{ki} + \frac{1}{2}\hat{y}_{ki}^2 \right\} \end{aligned}$$

where the last line involves again dropping terms beyond the control of the policymaker, and a hat over a variable denotes the deviation from its expected value: $\hat{y}_{ki} = y_{ki} - \log(\theta_k)$.

The next step is to add this up across firms:

$$\int_0^1 Y_{ki} di = \theta_k \left\{ \int_0^1 \hat{y}_{ki} di + \frac{1}{2} \int_0^1 \hat{y}_{ki}^2 di \right\}$$

Introducing the notation $E_i(\hat{y}_{ki})$ to stand for the cross sectional average of output across firms in sector k, $E_i(\hat{y}_{ki}) = \int_0^1 \hat{y}_{ki} di$, we can re-write the expression above as:

$$\int_0^1 Y_{ki} di = \theta_k \left\{ E_i(\hat{y}_{ki}) + \frac{1}{2} E_i(\hat{y}_{ki}^2) \right\}. \tag{A.3}$$

To find what these cross-sectional averages are, we use the market clearing condition $C_k = Y_k$, to re-write the CES aggregate defining the sectoral output in equation (5) as:

$$e^{\frac{\gamma-1}{\gamma} y_k} = \int_0^1 e^{\frac{\gamma-1}{\gamma} y_{ki}} di,$$

and approximate both sides in turn. Starting with the left-hand side we approximate it with respect to y_k around the point $E(y_k) = \log(\theta_k)$:

$$\begin{aligned} e^{\frac{\gamma-1}{\gamma} y_k} &\approx e^{\frac{\gamma-1}{\gamma} Ey_k} + \frac{\gamma-1}{\gamma} e^{\frac{\gamma-1}{\gamma} Ey_k}(y_k - Ey_k) + \frac{1}{2}\left(\frac{\gamma-1}{\gamma}\right)^2 e^{\frac{\gamma-1}{\gamma} Ey_k}(y_k - Ey_k)^2 = \\ &= \theta_k^{\frac{\gamma-1}{\gamma}} \left\{ 1 + \frac{\gamma-1}{\gamma}\hat{y}_k + \frac{1}{2}\left(\frac{\gamma-1}{\gamma}\right)^2 \hat{y}_k^2 \right\} \end{aligned}$$

To approximate the right hand side, we start by approximating each of the terms in the integral with respect to y_{ki} around the expected point Ey_{ki}, and then add up:

$$\begin{aligned}
\int_0^1 e^{\frac{\gamma-1}{\gamma}y_{ki}}di &\approx \int_0^1 \theta_k^{\frac{\gamma-1}{\gamma}} \left\{1+\frac{\gamma-1}{\gamma}\hat{y}_{ki}+\frac{1}{2}\left(\frac{\gamma-1}{\gamma}\right)^2 \hat{y}_{ki}^2\right\} di = \\
&= \theta_k^{\frac{\gamma-1}{\gamma}} \left\{1+\frac{\gamma-1}{\gamma}E_i\hat{y}_{ki}+\frac{1}{2}\left(\frac{\gamma-1}{\gamma}\right)^2 E_i\hat{y}_{ki}^2\right\}
\end{aligned}$$

Equating right and left hand sides and rearranging we obtain an approximation for $E_i\hat{y}_{ki}^2$:

$$\frac{1}{2}E_i\hat{y}_{ki}^2 = \frac{\gamma}{\gamma-1}\hat{y}_k + \frac{1}{2}\hat{y}_k^2 - \frac{\gamma}{\gamma-1}E_i\hat{y}_{ki}.$$

We can then use this to replace for the $E_i\hat{y}_{ki}^2$ term in our original linearization for L_k in equation (A.3) to obtain:

$$\int_0^1 Y_{ki}di = \theta_k \left\{\frac{\gamma}{\gamma-1}\hat{y}_k + \frac{1}{2}\hat{y}_k^2 + \frac{1}{1-\gamma}E_i(\hat{y}_{ki})\right\}.$$

The third and final step in approximating L is to add the expression above across sectors:

$$\begin{aligned}
L &= \sum_{k=1}^K \int_0^1 Y_{ki}di = \\
&= \sum_{k=1}^K \theta_k \left\{\frac{\gamma}{\gamma-1}\hat{y}_k + \frac{1}{2}\hat{y}_k^2 + \frac{1}{1-\gamma}E_i(\hat{y}_{ki})\right\} = \\
&= \frac{\gamma}{\gamma-1}E_k(\hat{y}_k) + \frac{1}{2}E_k(\hat{y}_k^2) + \frac{1}{1-\gamma}E_k(E_i(\hat{y}_{ki})) \qquad \text{(A.4)}
\end{aligned}$$

In the last line, we introduce $E_k(\hat{y}_k)$ to stand for the cross sectional average of output across sectors, now weighted by the sectoral weights θ_k: $E_k(\hat{y}_k) = \sum_{k=1}^K \theta_k\hat{y}_k$. Using the CES aggregator for C, together with market clearing:

$$e^{\frac{\gamma-1}{\gamma}y} = \sum_{k=1}^K \theta_k^{\frac{1}{\gamma}} e^{\frac{\gamma-1}{\gamma}y_k}$$

we can approximate the left hand side around $E(y)$:

$$e^{\frac{\gamma-1}{\gamma}y} \approx 1 + \frac{\gamma-1}{\gamma}y + \frac{1}{2}\left(\frac{\gamma-1}{\gamma}\right)^2 y^2,$$

and the right hand side around $E(y_k)$:

$$\begin{aligned}\sum_{k=1}^{K}\theta_k^{\frac{1}{\gamma}}e^{\frac{\gamma-1}{\gamma}y_k} &\approx \sum_{k=1}^{K}\theta_k^{\frac{1}{\gamma}}\theta_k^{\frac{\gamma-1}{\gamma}}\left\{1 + \frac{\gamma-1}{\gamma}\hat{y}_k + \frac{1}{2}\left(\frac{\gamma-1}{\gamma}\right)^2 \hat{y}_k^2\right\} = \\ &= 1 + \frac{\gamma-1}{\gamma}E_k(\hat{y}_k) + \frac{1}{2}\left(\frac{\gamma-1}{\gamma}\right)^2 E_k(\hat{y}_k^2).\end{aligned}$$

Combining right and left hand side we obtain an expression for $E_k(\hat{y}_k^2)$:

$$\frac{1}{2}E_k(\hat{y}_k^2) = \frac{\gamma}{\gamma-1}y + \frac{1}{2}y^2 - \frac{\gamma}{\gamma-1}E_k(\hat{y}_k)$$

Replacing this in our previous expression for L in eq. (11), we obtain the desired approximation:

$$L = \frac{\gamma}{\gamma-1}y + \frac{1}{2}y^2 + \frac{1}{1-\gamma}E_kE_i(\hat{y}_{ki}) \qquad \text{(A.5)}$$

Returning to the original problem of approximating the utility function, we combine the two terms of the utility function approximated in equations (11) and (A.5) to obtain our log-linear 2^{nd} order approximation of the utility function:

$$\begin{aligned}U(.) &= y + \frac{1}{2}(1-\sigma)y^2 - \frac{\gamma}{\gamma-1}y - \frac{1}{2}y^2 - \frac{1}{1-\gamma}E_kE_i(\hat{y}_{ki}) = \\ &= -\frac{\sigma}{2}y^2 + \frac{1}{1-\gamma}y - \frac{1}{1-\gamma}E_kE_i(\hat{y}_{ki})\end{aligned}$$

Taking unconditional expectations, and since the economy satisfies the natural rate property so $E(y)$ is beyond the control of the policymaker:

$$\begin{aligned}E(U(.)) &\approx -\frac{\sigma}{2}E(y^2) = \\ &= -\frac{\sigma}{2}VAR(y)\end{aligned}$$

The policymaker wishes to minimize the unconditional variance of aggregate output.

Appendix 2 - Results for the two-sector case

In this appendix, we prove the results and propositions presented in section 4 of the text.

The optimal weights in the Stability Price Index

First, express all variables as deviations from their expected value. Letting a hat over a variable denote its deviations from its expected value ($\hat{x} = x - E(x)$), the model can be written as:

$$\begin{aligned}
\hat{p}_k^* &= \hat{p} + \alpha_k \hat{y} + \hat{\varepsilon}_k \\
\hat{p}_k &= \lambda_k \hat{p}_k^* + (1 - \lambda_k) E(\hat{p}_k^*) \\
\hat{p} &= \theta_A \hat{p}_A + \theta_B \hat{p}_B \\
0 &= \omega_A \hat{p}_A + \omega_B \hat{p}_B.
\end{aligned}$$

Next, we use the facts that (i) there are only 2 sectors in this application ($k = A, B$), (ii) the expected value of any variable with a hat is zero, and (iii) the weights must sum to one, to re-express the system as:

$$\begin{aligned}
\hat{p}_A &= \lambda_A \left(\hat{p} + \alpha_A \hat{y} + \hat{\varepsilon}_A\right) \\
\hat{p}_B &= \lambda_B \left(\hat{p} + \alpha_B \hat{y} + \hat{\varepsilon}_B\right) \\
\hat{p} &= \theta_A \hat{p}_A + (1 - \theta_A) \hat{p}_B \\
0 &= \omega_A \hat{p}_A + (1 - \omega_A) \hat{p}_B.
\end{aligned}$$

This is a system of 4 equations in 4 variables ($\hat{p}_A, \hat{p}_B, \hat{p}, \hat{y}$). Solving for the variable of interest $\hat{y}$, we obtain:

$$\hat{y} = -\frac{\left[\omega_A + \lambda_B \left(\theta_A - \omega_A\right)\right] \lambda_A \varepsilon_A + \left[(1 - \omega_A) - \lambda_A(\theta_A - \omega_A)\right] \lambda_B \varepsilon_B}{\alpha_B \lambda_A + \alpha_A \omega_A \left(\lambda_A - \lambda_B\right) + \lambda_A \lambda_B \left(\omega_A - \theta_A\right)\left(\alpha_B - \alpha_A\right)}$$

The policymaker will then choose the weight ω_A in order to minimize the variance of the expression above. Using the first-order condition and rearranging we find the optimal ω_A^*

given by:

$$\omega_A^* = \lambda_B \frac{\alpha_A \sigma_B^2 - \theta_A \lambda_A \left(\alpha_A \sigma_B^2 + \alpha_B \sigma_A^2\right)}{\alpha_B \lambda_A (1-\lambda_B)\sigma_A^2 + \alpha_A \lambda_B (1-\lambda_A)\sigma_B^2} \tag{A.6}$$

The optimal ω_B^* is just given by $\omega_B^* = 1 - \omega_A^*$.

Proof of the Propositions

Proposition 1: Using the values $\alpha_A = \alpha_B$, $\sigma_A^2 = \sigma_B^2$, $\lambda_A = \lambda_B$, $\theta_A = \theta_B = 1/2$ in the formula for ω_A^* above we find that $\omega_A^* = 1/2$.

Proposition 2: Taking derivatives of (A.6) with respect to α_A, we find that

$$\frac{\partial \omega_A^*}{\partial \alpha_A} = \alpha_B \frac{\lambda_A \lambda_B \sigma_A^2 \sigma_B^2 (1 - \theta_A \lambda_A - (1-\theta_A)\lambda_B)}{\left(\alpha_B \lambda_A (1-\lambda_B)\sigma_A^2 + \alpha_A (1-\lambda_A)\lambda_B \sigma_B^2\right)^2}$$

The denominator is clearly non-negative, and so is the numerator since $\lambda_k \leq 1$ and $\theta_k \leq 1$, so we can sign $\partial \omega_A^* / \partial \alpha_A \geq 0$. By very similar steps, it is easy to show:

$$\begin{aligned} \partial \omega_B^* / \partial \alpha_B &= -\partial \omega_A^* / \partial \alpha_B = \\ &= \frac{\alpha_A}{\alpha_B} \frac{\partial \omega_A^*}{\partial \alpha_A} \end{aligned}$$

from where $\partial \omega_B^* / \partial \alpha_B \geq 0$.

Proposition 3: Taking derivatives of the solution (A.6):

$$\frac{\partial \omega_A^*}{\partial \sigma_A^2} = -\frac{\alpha_A \alpha_B \lambda_A \lambda_B \sigma_B^2 (1 - \theta_A \lambda_A - (1-\theta_A)\lambda_B)}{\left(\alpha_B \lambda_A (1-\lambda_B)\sigma_A^2 + \alpha_A (1-\lambda_A)\lambda_B \sigma_B^2\right)^2}$$

which by the same argument as in the previous proposition, implies $\partial \omega_A^* / \partial \sigma_A^2 \leq 0$. Similarly:

$$\begin{aligned} \partial \omega_B^* / \partial \sigma_B^2 &= -\partial \omega_A^* / \partial \sigma_B^2 = \\ &= \frac{\sigma_A^2}{\sigma_B^2} \frac{\partial \omega_A^*}{\partial \sigma_A^2} \end{aligned}$$

and so $\partial \omega_B^* / \partial \sigma_B^2 \leq 0$.

Proposition 4: Taking derivatives of ω_A^* with respect to λ_A:

$$\frac{\partial \omega_A^*}{\partial \lambda_A} = -\frac{\alpha_A \lambda_B \sigma_B^2 \left[\alpha_B \sigma_A^2 - (1-\theta_A)\lambda_B(\alpha_B \sigma_A^2 + \alpha_A \sigma_B^2)\right]}{\left(\alpha_B \lambda_A (1-\lambda_B)\sigma_A^2 + \alpha_A (1-\lambda_A)\lambda_B \sigma_B^2\right)^2}.$$

From the solution for ω_A^*:

$$\begin{aligned} \omega_A^* &\leq 1 \Leftrightarrow \\ \alpha_B \sigma_A^2 &\geq (1-\theta_A)\lambda_B(\alpha_B \sigma_A^2 + \alpha_A \sigma_B^2). \end{aligned}$$

Therefore, as long as $\omega_A^* \leq 1$, then $\partial \omega_A^* / \partial \lambda_A \leq 0$. Taking derivatives with respect to λ_B:

$$\frac{\partial \omega_B^*}{\partial \lambda_B} = -\frac{\alpha_B \lambda_A \sigma_A^2 \left[\alpha_A \sigma_B^2 - \theta_A \lambda_A(\alpha_B \sigma_A^2 + \alpha_A \sigma_B^2)\right]}{\left(\alpha_B \lambda_A (1-\lambda_B)\sigma_A^2 + \alpha_A (1-\lambda_A)\lambda_B \sigma_B^2\right)^2}.$$

Realizing that:

$$\begin{aligned} \omega_B^* &\leq 1 \Leftrightarrow \\ \alpha_A \sigma_B^2 &\geq \theta_A \lambda_A(\alpha_B \sigma_A^2 + \alpha_A \sigma_B^2) \end{aligned}$$

then if $\omega_B^* \leq 1$, it follows that $\partial \omega_B^* / \partial \lambda_B \leq 0$.

Proposition 5: Follows from evaluating the optimal solution ω_A^* at the point: $\alpha_A = \alpha_B$, $\sigma_A^2 = \sigma_B^2$, $\theta_A = \theta_B = 0.5$, $\lambda_A = 1$, $\lambda_B < 1$, to obtain $\omega_A^* = 0$.

Proposition 6: Taking derivatives of ω_A^* with respect to θ_A, we obtain:

$$\frac{\partial \omega_A^*}{\partial \theta_A} = -\frac{\lambda_A \lambda_B(\alpha_B \sigma_A^2 + \alpha_A \sigma_B^2)}{\alpha_B \lambda_A \sigma_A^2 (1-\lambda_B) + \alpha_A \lambda_B \sigma_B^2 (1-\lambda_A)},$$

which is clearly negative. Then clearly $\partial \omega_B^* / \partial \theta_B$ is also negative.

Appendix 3 - Multi-sector problems

In this appendix, we describe how to find the optimal price index in a K sector problem as in section 4 of the text. The algorithm has 3 steps. First, we solve for the equilibrium output in the economy, by solving the set of $K+2$ equations:

$$\begin{aligned}\hat{p}_k &= \lambda_k(\hat{p} + \alpha_k \hat{y} + \hat{\varepsilon}_k), \quad k = 1, ..., K \\ \hat{p} &= \sum_{k=1}^{K} \theta_k \hat{p}_k \\ 0 &= \sum_{k=1}^{K} \omega_k \hat{p}_k.\end{aligned}$$

in $K+2$ variables ($\hat{y}$, $\hat{p}$, and the $\hat{p}_k$), for the variable $\hat{y}$, in terms of the parameters and the innovations $\hat{\varepsilon}_k$. Second, we take the unconditional expectation of the square of $\hat{y}$, to obtain the variance of output as a function of α_k, θ_k, λ_k, ω_k and the variances $\sigma_k^2 = E(\hat{\varepsilon}_k^2)$ and covariances $\sigma_{kj} = E(\hat{\varepsilon}_k \hat{\varepsilon}_j)$:

$$Var(y) = f(\alpha_k, \theta_k, \lambda_k, \omega_k, \sigma_k^2, \sigma_{kj}).$$

Given values for (α_k, θ_k, λ_k,σ_k^2,σ_{kj}) the third step is to numerically minimize $f(.)$ with respect to the ω_k, subject to the constraint that $\sum_k \omega_k = 1$, and possibly additional non-negativity constraints: $\omega_k \geq 0$.

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