

# Can Information Dispersion Explain the Exchange Rate Disconnect Puzzle? <sup>1</sup>

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## **Abstract**

Exchange rates tend to be disconnected from fundamentals over substantial periods of time. The recent “microstructure approach to exchange rates” has shed some important light on this puzzle: most exchange rate volatility at short to medium horizons is related to order flows. This suggests that investor heterogeneity is key to understanding exchange rate dynamics, in contrast to the common representative agent approach in macroeconomic models of exchange rate determination. Here we introduce investor heterogeneity into an otherwise standard monetary model of exchange rate determination. There is both heterogeneous information about fundamentals and non-fundamentals based heterogeneity. The implications of the model are consistent with the evidence on the relationship between exchange rates and fundamentals: (i) the exchange rate is disconnected from fundamentals in the short to medium run, (ii) over longer horizons the exchange rate is primarily driven by fundamentals, (iii) exchange rate changes are a weak predictor of future fundamentals.

# I Introduction

Probably the largest puzzle in international finance is the lack of relationship between exchange rates and macroeconomic fundamentals over short to medium horizons. The puzzle was forcefully demonstrated by Meese and Rogoff [1983] and remains valid today;<sup>1</sup> it has been coined the 'exchange rate disconnect puzzle' by Obstfeld and Rogoff [2001]. The absence of a connection between exchange rates and fundamentals has frustrated both positive and normative analysis in open economy contexts. However, the recent literature on the microstructure approach to exchange rates, in particular Evans and Lyons [2001], has shown that most short-run exchange rate volatility is related to order flows.<sup>2</sup> This suggests that investor heterogeneity plays an important role. Existing models of exchange rate determination give little or no role to heterogeneity. Macroeconomic models commonly assume representative agents, while microstructure models assume that order flow is exogenous, so that the underlying heterogeneity is not modeled either.

The goal of this paper is to shed light on the relationship between exchange rates and fundamentals by explicitly introducing investor heterogeneity in an otherwise standard monetary model of exchange rate determination. We introduce two sources of heterogeneity that have generally been associated with order flow: (i) dispersed information of market participants about future macroeconomic fundamentals, and (ii) non-fundamentals based heterogeneity. The latter category includes liquidity traders, noise traders, or more generally investors who hold different portfolios for non-information based reasons.<sup>3</sup> There is extensive evidence to back up such heterogeneity. Survey evidence shows that investors have dif-

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<sup>1</sup>Cheung, Chinn and Pascual [2002] find that for a much wider range of models than originally considered by Meese and Rogoff, it remains the case that none of them consistently outperforms the random walk in predictive power. For a survey of the evidence, see Jeffrey A. Frankel and Andrew K. Rose [1995].

<sup>2</sup>See also Rime [2001] and Froot and Ramadorai [2002].

<sup>3</sup>Lyons [2001] classifies the types of heterogeneity generating order flow in essentially the same way. Some open economy models introduce the second, but not the first type of heterogeneity. Examples are Jeanne and Rose [2002] and Devereux and Engel [2002], who consider models with noise traders, and Hau and Rey [2001], who introduce an exogenous foreign exchange supply function of banks. It is important to point out that non-fundamentals based trade does not necessarily imply irrational investors. Different investors hold different portfolios for many reasons, including for example legal restrictions imposed on financial institutions.

ferent expectations about future fundamentals and exchange rates.<sup>4</sup> Moreover, investors interpret the same information differently.<sup>5</sup> The two types of heterogeneity are closely interconnected. Without the second type of heterogeneity private information would not play an important role, and its very existence would be questionable. As first argued by Grossman and Stiglitz [1980], in the absence of noise unrelated to fundamentals asset prices fully reveal aggregate information and it does not pay off to collect private information in the first place.

Our model is in the tradition of noisy rational expectations models, where agents extract information about fundamentals from asset prices.<sup>6</sup> Investors can learn indirectly about the information that others possess about fundamentals by observing the relevant asset price. Such models, starting with Grossman and Stiglitz [1980], are usually static, or in some cases have two or three periods. They are therefore not useful to address exchange rate dynamics and have also not been used to address the disconnect between asset prices and fundamentals at short to medium-run horizons. The problem with solving dynamic noisy rational expectations models is what Townsend [1983] called “infinite regress”. Asset prices depend on expectations of fundamentals, expectations of other investors’ expectations, and so on. The dimension of these higher order expectations increases with the horizon. The finance microstructure literature has typically solved this problem by assuming a hierarchical information structure where some investors are fully informed, while others are uninformed.<sup>7</sup> Alternatively, it is assumed that an asset only has a payoff on some terminal date, as in He and Wang [1995].

While these assumptions may be adequate in the context of individual stock prices, they are less appealing in the analysis of foreign exchange markets. We

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<sup>4</sup>For survey evidence on exchange rate expectations, see Ito [1990], Elliott and Ito [1999], and McDonald and Marsh [1996]. McDonald and Marsh [1996] also show that there is a positive relationship between turnover in the foreign exchange market and heterogeneity of exchange rate expectations.

<sup>5</sup>Evans and Lyons [2002] show that macroeconomic news has a substantial impact on order flow and that at least half of the effect of such news on exchange rates is transmitted through order flow.

<sup>6</sup>While there has been some work introducing learning with identical agents (e.g., Gourinchas and Tornell, 2001, and Andersen and Beier, 2000), we are not aware of other rational expectations models with heterogenous expectations in the context of spot exchange rates.

<sup>7</sup>This is done for example Wang [1993, 1994], based on a suggestion from Townsend [1983]. See Brunnermeier [2001, ch.4] for a review of the literature.

will follow the macroeconomics tradition of infinite horizon, where all investors are ex-ante identical, so that information is symmetrically dispersed.<sup>8</sup> We develop a simple numerical solution technique to solving such dynamic noisy rational expectations models that can easily be applied to rather complicated settings. Although in general the exchange rate depends on all information observed by investors between now and the infinite past, we exploit the fact that information from the sufficiently distant past has a negligible impact on the exchange rate today. We are then able to solve a fixed point problem by equating an assumed equilibrium exchange rate equation to the equilibrium exchange rate equation that clears the market when investors solve their portfolio maximization problem under the belief that the assumed exchange rate equation is correct.<sup>9</sup>

Ironically we find that the impact of heterogeneity on the exchange rate is especially large when the heterogeneity itself is limited. When there is not too much private information, and when non-fundamentals trade is limited, investors give a lot of weight to the exchange rate as a source of information about fundamentals. The impact of non-fundamentals trade on the exchange rate can then be significantly amplified as agents rationally misinterpret the resulting exchange rate movements as information about fundamentals. We find that non-fundamentals based trade can become the dominant source of exchange rate volatility in the short to medium run, even when it is so small that it has practically no effect on the exchange rate when all investors have identical information. Information dispersion can also make the impact of non-fundamentals trade on the exchange rate more persistent as it takes time for investors to learn what happened. The impact of non-fundamentals based trade on the exchange rate eventually dies down. As the horizon lengthens the role of fundamentals increases. In the long-run the

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<sup>8</sup>Surprisingly, the macroeconomics literature has devoted little attention to heterogenous information in the last two decades. This contrasts with the 1970s where, following Lucas [1972], there had been active research on rational expectations and heterogenous information (e.g., see King, 1982). Recently, information issues in the context of price rigidity have been brought to the forefront again in contributions by Woodford [2001] and Mankiw and Reis [2002].

<sup>9</sup>The only other paper we are familiar with that has solved a dynamic noisy rational expectations asset pricing model with symmetrically dispersed information is Singleton [1987]. He uses a different solution technique suggested by Townsend [1983] to study the impact of information dispersion on the time series properties of asset prices. He assumes that the unknown fundamental is revealed within 2 periods, which can significantly limit the impact of information dispersion.

exchange rate is primarily driven by fundamentals. This is consistent with the empirical evidence (see Mark, 1995).

The remainder of the paper is organized as follows. Section II describes the model and solution method. Section III discusses the implication of a simplified version of the model without learning in order to develop the intuition for our key results. Section IV presents the results based on the general dynamic model with learning. Section V concludes.

## II A Monetary Model with Information Dispersion

### II.A The Infinite Regress Problem

Our starting point is a simple monetary model of exchange rate determination with three key building blocks: (i) money market equilibrium, (ii) purchasing power parity, and (iii) interest rate parity. As is well known, such models lead to an equation relating the exchange rate to the expectations of future fundamentals. A fourth building block, which is generally implicit, is that all agents have identical information. It is here that we deviate from the standard approach to modeling exchange rate determination. Before describing the precise information structure, we discuss a more general solution to the exchange rate, without imposing common information across agents. In this solution, higher order expectations of other investors' expectations enter the equilibrium exchange rate equation.

Money market equilibrium and purchasing power parity conditions are standard:

$$m_t - p_t = \psi y_t - \alpha i_t \tag{1}$$

$$p_t = p_t^* + s_t \tag{2}$$

where  $m_t$ ,  $p_t$ ,  $s_t$  and  $y_t$  are the logs of money supply, the price level, the exchange rate and output and  $i_t$  is the nominal interest rate. Foreign variables are indicated with a \*.

We slightly generalize the standard interest parity condition by including a portfolio balance effect through a risk-premium term. In order to do so, we must

be precise about preferences and the implied asset demands. We assume that there are overlapping generations of agents that each live for one period and inherit information from the previous generation. This assumption significantly simplifies the presentation and helps in providing intuition, but is not critical for the results. Below we will also discuss the case where agents have infinite horizons. Investors are distributed on the interval  $[0,1]$ . They are identical, but may have different information about future fundamentals. They have constant absolute risk-aversion preferences and can invest in either domestic or foreign bonds. Agent  $i$  maximizes

$$-E^{-\gamma}c_{t+1}^i$$

subject to

$$c_{t+1}^i = (1 + i_t)w_t^i + (s_{t+1} - s_t + i_t^* - i_t)b_t^i$$

where  $w_t^i$  is wealth at the start of period  $t$  and  $b_t^i$  is invested in foreign bonds.  $s_{t+1} - s_t + i_t^* - i_t$  is the log-linearized excess return on investing abroad. The interest rate arbitrage equation is found by combining the first order condition of foreign bond holdings with asset market equilibrium. If  $b_t$  is the supply of foreign bonds, we find:

$$\bar{E}_t(s_{t+1}) - s_t = i_t - i_t^* + \gamma b_t \sigma_t^2 \tag{3}$$

where  $\bar{E}_t$  is the average expectation across all agents and  $\sigma_t^2 = \text{var}_t(s_{t+1})$  is the conditional variance of next period's exchange rate. The latter is the same across all agents for all information structures considered in the paper. The risk-premium term  $\gamma b_t \sigma_t$  depends on the supply of foreign bonds. As standard in the literature on noisy rational expectations models, we interpret  $b_t$  more broadly than the actual supply of bonds. We rather think of it as the net supply generated by agents trading for various non-informational considerations. The structure of the model described so far is similar to that in Jeanne and Rose [2002] when  $b_t$  is interpreted as the net supply of noise traders.<sup>10</sup>

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<sup>10</sup>As an alternative one could keep  $b_t$  constant and introduce other sources of fluctuations in risk-premia that are not associated with fundamentals. One example is fluctuations in the risk-aversion coefficient  $\gamma$ . Another example is to allow for a change in the perceived uncertainty about future fundamentals, which affects  $\sigma_t^2$ , as long as this change is not associated with fundamentals themselves. This is done in Duarte and Stockman [2001], who allow for regime shifts that affect the covariance between monetary and productivity shocks.

Defining the fundamental as  $f_t = (m_t - m_t^*) - \psi(y_t - y_t^*)$ , in the Appendix we use the three building blocks to derive the following equilibrium exchange rate:

$$s_t = \frac{1}{1 + \alpha} \sum_{k=0}^{\infty} \left( \frac{\alpha}{1 + \alpha} \right)^k \bar{E}_t^k (f_{t+k} - \alpha\gamma\sigma_{t+k}^2 b_{t+k}) \quad (4)$$

where  $\bar{E}_t^k(x_{t+k})$  are expectations of order  $k > 1$ , defined as

$$\bar{E}_t^k(x_{t+k}) = \int_0^1 E_t^i(\bar{E}_{t+1}^{k-1}(x_{t+k})) di \quad (5)$$

with  $\bar{E}_t^1(x_{t+1}) = \bar{E}_t(x_{t+1})$  and  $\bar{E}_t^0(x_t) = x_t$ .

Thus, the exchange rate at time  $t$  depends on the fundamental at time  $t$ , the average expectation of the fundamental at time  $t + 1$ , the average expectation of the average expectation of the fundamental at  $t + 2$ , etc. The law of iterated expectations does not apply to average expectations. For example,  $\bar{E}_t \bar{E}_{t+1}(s_{t+2}) \neq \bar{E}_t(s_{t+2})$ .<sup>11</sup> This is a basic feature of asset pricing determination under heterogenous expectations: the expectation of other investors' expectations matters. In a dynamic system, this leads to the infinite regress problem, as analyzed in Townsend (1983): as the horizon goes to infinity the dimensionality of the expectation term goes to infinity.

The exchange rate also depends on asset supplies and expectations of asset supplies. We will assume that net asset supplies are not observable. Investors therefore cannot know the weighted average of the higher order expectations by observing the exchange rate. We assume that the asset supply follows an AR process:

$$b_t = \rho_b b_{t-1} + \varepsilon_t^b. \quad (6)$$

## II.B The Information Structure

In addition to the currently observable variables, investors are endowed with two pieces of information about future fundamentals. First, investors receive a signal at time  $t$  about the fundamental at  $t + T$ . We will compare the results under public information, where all investors receive the same signal, to that under private

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<sup>11</sup>See Allen, Morris, and Shin [2002] for a discussion.

(heterogeneous) information, where investors receive different signals. We define the fundamental  $T$  periods from now as

$$u_t = f_{t+T}$$

With public information all investors receive the signal

$$v_t = u_t + \varepsilon_t^v \quad \varepsilon_t^v \sim N(0, \sigma_{v,c}^2) \quad (7)$$

where  $\varepsilon_t^v$  is independent of  $u_t$ . Under private information investor  $i$  receives a signal

$$v_t^i = u_t + \varepsilon_t^{vi} \quad \varepsilon_t^{vi} \sim N(0, \sigma_v^2) \quad (8)$$

where  $\varepsilon_t^{vi}$  is independent from  $u_t$  and other agents' signals. Due to the law of large numbers, the average signal received by investors is  $u_t$ , i.e.,  $\int_0^1 v_t^i di = u_t$ . For reasons to be discussed below, the variance  $\sigma_{v,c}^2$  of the public signal ( $c$  stands for common information) may in general be different from the variance  $\sigma_v^2$  of the private signal.

The second piece of information is the process of the fundamental, which is always common knowledge to all agents:

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t^u \quad \varepsilon_t^u \sim N(0, \sigma_u^2) \quad (9)$$

Since investors observe current and lagged values of the fundamental, knowing the process provides information about the fundamental at future dates. The main results of the paper, reported in section IV, are based on the assumption that  $\rho_1 = 1 + \rho$  and  $\rho_2 = -\rho$ , so that the fundamental follows an AR process in first differences:  $u_t - u_{t-1} = \rho(u_{t-1} - u_{t-2}) + \varepsilon_t^u$ . We will take up the much simpler case where  $\rho_1 = \rho_2 = 0$  in the next section.

## II.C Solution Method

The solution of the model is significantly complicated by the infinite regress problem. Our approach will be to first develop intuition by considering a special case of the model where the infinite regress problem does not arise. After that we discuss the results from a numerical solution technique applied to the full dynamic model. The special case, which is taken up in the next section, assumes that  $T = 1$ ,

$\rho_1 = \rho_2 = 0$  and  $\rho_b = 0$ . This case simplifies matters significantly since there is no learning. In other words, information from previous periods is irrelevant in forming expectations about the future. In that case the average expectation at time  $t$  of the average expectation at  $t + 1$  of the fundamental at time  $t + 2$  is zero, and the same for higher order expectations. This is because expectations at time  $t + 1$  and later do not depend on previous information at time  $t$ . We use this simplified case to illustrate the difference between the public and private information models with regards to exchange rate disconnect.

In the more general setup, where  $T > 1$  and  $u_t$  depends on lags of itself, there is learning and we are faced with the infinite regress problem. We adopt a numerical solution method that is described in detail in the Appendix, but briefly outlined here. We can always write  $\bar{E}_t(s_{t+1}) - s_t$  as a linear function of the entire information set of all agents at time  $t$ . This includes current and lagged signals, fundamentals and exchange rates. It then follows from (3), together with (1)-(2), that the exchange rate at time  $t$  can be written as a linear function of the asset supply  $b_t$ , as well as all current and lagged variables in the information set. Only the average signal each period,  $\int_0^1 v_t^i = u_t$  in period  $t$ , enters the exchange rate equation. The equilibrium exchange rate can then be written as

$$s_t = \lambda_s \mathbf{s}_{t-1} + \lambda_f \mathbf{f}_t + \lambda_u \mathbf{u}_t + \lambda_b b_t \quad (10)$$

where  $\mathbf{x}_t = \{x_t, x_{t-1}, \dots\}'$ . Our solution method uses the fact that coefficients tend to zero for variables in the information set with sufficiently long lags. We therefore truncate the exchange rate equation for lags more than  $\bar{T}$  ( $> T$ ). We make sure to choose  $\bar{T}$  such that the coefficients at lag  $\bar{T}$  are very close to zero (less than  $10^{-5}$ ). Further raising  $\bar{T}$  does not affect the results. For  $u$  the longest lag is  $T - 1$  since  $u_{t-T} = f_t$  already enters through  $\mathbf{f}_t$ .

We use the Kalman filter technique to compute the average expectation of excess returns as a function of all variables in the information set. This expectation is based on observing the new equilibrium exchange rate, knowing the exchange rate equation, observing signals of the fundamentals, and information carried over from previous periods. Substituting the average expected excess return into (3) yields a new exchange rate equation. We then solve a fixed point problem by equating the coefficients of the new exchange rate equation to the coefficients of the exchange rate equation on which expectations are based.

## II.D Infinite Horizons

The Appendix also discusses the solution method in the technically more challenging case where investors have infinite horizons. In that case investor  $i$  maximizes

$$-E_t \left( \sum_{s=0}^{\infty} \beta^s e^{-\gamma c_{t+s}^i} \right) \quad (11)$$

subject to

$$w_{t+1}^i = (1 + i_t)w_t^i + (s_{t+1} - s_t + i_t^* - i_t)b_t^i - c_t^i \quad (12)$$

As is well known, such an infinite horizon setup leads to an additional term in the asset demand equation, reflecting a hedge against a change in expected future returns. While this hedging term significantly complicates the solution method and increases computing time, we find that it leaves the results virtually unaltered. The Appendix provides an example to illustrate this.

## III An Example With No Learning

In this section we consider a special case of the model in which there is no learning by setting  $T = 1$ ,  $\rho_1 = \rho_2 = 0$  and  $\rho_b = 0$ . Thus, there is no infinite regress problem, so that we can obtain analytical insight into the solution. In particular, we will show how lack of common knowledge magnifies the impact of non-fundamental shocks, thereby disconnecting the exchange rate from fundamentals. However, this case does not allow the analysis of persistence, since non-fundamentals shocks only have a transitory impact on the exchange rate. This aspect is examined in the following section, where we provide numerical results based on the full dynamic model with learning.

The parameterization adopted here simplifies equation (4) to:

$$s_t = \frac{1}{1 + \alpha} \left[ f_t + \frac{\alpha}{1 + \alpha} \bar{E}_t u_t \right] - \frac{\alpha}{1 + \alpha} \gamma \sigma_i^2 b_t \quad (13)$$

Only the average expectation of  $u_t$  appears, which is next period's fundamental. Higher order expectations have disappeared. This does not mean, however, that investors do not care about the expectations of others. When computing their expectation of  $u_t$ , investors in the private information model attempt to use the equilibrium exchange rate to extract information that other investors have about  $u_t$ .

### III.A Public Information

As a benchmark, we first consider the case of public information. Investors have two pieces of information about  $u_t$ : the public signal  $v_t = u_t + \varepsilon_t^v$  and the fact that  $u_t = \varepsilon_t^u$ . Defining the precision of these signals as  $\beta^{v,c} \equiv 1/\sigma_{v,c}^2$  and  $\beta^u \equiv 1/\sigma_u^2$ , the conditional expectation of  $u_t$  is

$$E_t^i u_t = \bar{E}_t u_t = \frac{\beta^{v,c} v_t}{d} \quad (14)$$

where  $d \equiv 1/\text{var}_t(u_t) = \beta^{v,c} + \beta^u$ . Substitution into (13) yields the equilibrium exchange rate:

$$s_t = \frac{1}{1+\alpha} f_t + \lambda_v v_t + \lambda_{b,t}^c b_t \quad (15)$$

where

$$\lambda_v = \frac{\alpha}{(1+\alpha)^2} \frac{\beta^{v,c}}{d} \quad (16)$$

$$\lambda_{b,t}^c = -\frac{\alpha}{1+\alpha} \gamma \sigma_t^2 \quad (17)$$

In addition to the observable fundamental,  $f_t$ , both the public signal,  $v_t$ , and the non-fundamental factor,  $b_t$ , affect the exchange rate. Notice that in this case the exchange rate is fully revealing, since by observing  $s_t$  investors can perfectly deduce  $b_t$ .

The weight of the public signal naturally depends on its precision  $\beta^{v,c}$ . The weight of the  $b_t$  shock depends on  $\sigma_t^2 = \text{var}_t(s_{t+1})$ , the conditional variance of next period's exchange rate. From (15) at  $t+1$ :

$$\sigma_t^2 = a + b\sigma_{t+1}^4 \quad (18)$$

where  $a = (1 + d\tilde{\alpha}\beta^v/\beta^u)/d(1+\alpha)^2$ ,  $b = \gamma^2\sigma_b^2\tilde{\alpha}$  and  $\tilde{\alpha} = (\alpha/(1+\alpha))^2$ . In the steady state,  $\sigma^2 = \sigma_t^2 = \sigma_{t+1}^2$ . It is easy to see that:

$$\sigma^2 = \frac{1 \pm \sqrt{1-4ab}}{2b} \quad (19)$$

Thus, as long as  $4ab < 1$ , there are two steady states with low and high  $\sigma^2$ . From (18), it can be verified that  $d\sigma_t^2/d\sigma_{t+1}^2 < 1$  around the low  $\sigma^2$  steady state and  $d\sigma_t^2/d\sigma_{t+1}^2 > 1$  for the high  $\sigma^2$  steady state.<sup>12</sup> Since  $\sigma_t^2$  is a forward-looking

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<sup>12</sup>We find  $d\sigma_t^2/d\sigma_{t+1}^2 = 2b\sigma^2 = 1 \pm \sqrt{1-4ab}$ .

variable, only the low  $\sigma^2$  steady state gives a well-defined equilibrium, while the other steady state gives indeterminacy.<sup>13</sup> The high steady state equilibrium is knife-edge, in that it can only be an equilibrium today if one believes that  $\sigma_t$  is exactly the high steady state equilibrium at all future dates. In the model with private information, the results are similar, even though  $\sigma^2$  has to be evaluated numerically. Hence, from now on we focus on the low  $\sigma^2$  steady state.<sup>14</sup>

### III.B Private Information

We now assume that investors get private signals  $v_t^i$  about  $u_t$  as given in (8). Due to the law of large numbers the aggregation of private signals is  $u_t$ , which implies that the exchange rate will be affected directly by  $u_t$ . Therefore, individual investors get information about  $u_t$  by observing the exchange rate  $s_t$ .<sup>15</sup> However, the exchange rate is not fully revealing, as it gives information about a combination of  $u_t$  and  $b_t$ . To determine the information given by  $s_t$ , we need to know the equilibrium exchange rate equation. By analogy to (15), the investor conjectures that:

$$s_t = \frac{1}{1+\alpha} f_t + \lambda_{u,t} u_t + \lambda_{b,t} b_t \quad (20)$$

Since an investor observes  $f_t$ , the signal he gets from the exchange rate can be written

$$u_t = \frac{1}{\lambda_{u,t}} \left( s_t - \frac{f_t}{1+\alpha} \right) - \frac{\lambda_{b,t}}{\lambda_{u,t}} b_t \quad (21)$$

The variance of this signal is  $(\lambda_{b,t}/\lambda_{u,t})^2 \sigma_b^2$ . Consequently, investor  $i$  infers  $E_t^i u_t$  from three sources of information: i) the distribution of  $u_t$ ; ii) the signal  $v_t^i$ ; iii) the exchange rate (i.e., (21)). As usual,  $E_t^i u_t$  is given by a weighted average of the three signals, with the weights determined by the precision of each signal. We have:

$$E_t^i u_t = \frac{\beta^v v_t^i + \beta^s (s_t - \frac{f_t}{1+\alpha}) / \lambda_{u,t}}{D_t} \quad (22)$$

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<sup>13</sup>See, for example, Blanchard and Fischer [1989], ch. 5 for a discussion of these issues.

<sup>14</sup>In the low  $\sigma^2$  equilibrium, it is easy to see that an increase in the precision of  $u_t$  and  $v_t$  shocks decreases  $\sigma^2$  as there is less overall uncertainty. However, an increase in the precision of  $b_t$  has an ambiguous effect.

<sup>15</sup>Under public information,  $s_t$  does not provide any information about  $u_t$ , since all investors know  $v_t$ .

where  $\beta^v = 1/\sigma_v^2$ ,  $\beta_t^s = 1/(\lambda_{b,t}/\lambda_{u,t})^2\sigma_b^2$  and  $D_t = 1/\text{var}_t(u_t) = \beta^v + \beta^u + \beta_t^s$ . For the exchange rate signal, the precision is complex and depends both on  $\sigma_b^2$  and  $\lambda_{b,t}/\lambda_{u,t}$ , the latter being endogenous.

By substituting (22) into (13) and using the fact that  $\int_0^1 v_t^i di = u_t$  in computing  $\overline{E}_t u_t$ , it can be easily seen that (20) indeed holds. The parameters are:

$$\lambda_{u,t} = z_t \frac{\alpha}{(1+\alpha)^2} \frac{\beta^v}{D_t} \quad (23)$$

$$\lambda_{b,t} = z_t \lambda_{b,t}^c \quad (24)$$

The factor  $z_t > 1$ , which is important to the analysis, will be defined below.

Equation (24) tells us that the sensitivity of the exchange rate to the non-fundamental supply shock  $b_t$  is the product of two factors. The first is  $\lambda_{b,t}^c$ , which is the sensitivity of the exchange rate to non-fundamental shocks in the public information model. That term, defined in (17), is a result of the risk-premium that the supply shock generates and is therefore proportional to the conditional variance of the exchange rate. We have implicitly assumed that the conditional variance of the exchange rate is the same in the two models. Holding constant the precision of the private signal, one can always change the precision of the public signal to make sure that this is the case. While we will show that this is not critical to the results, it helps focus the analysis on the second factor,  $z_t$ , which magnifies the risk-premium term under heterogeneous information.<sup>16</sup>

### III.C The Magnification of Non-Fundamentals Shocks

The magnification factor can be written as

$$z_t = 1 + x_t^2 \frac{\sigma_v^2}{\sigma_b^2} \quad (25)$$

where  $x_t = \lambda_{u,t}/\lambda_{b,t}$  is the relative weight of  $u_t$  and  $b_t$  in the exchange rate equation. With private information, investors use the value of the exchange rate to make inferences about fundamentals. Since investors do not know whether a change in

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<sup>16</sup>In section IV we show that at the quantitative level the main results are not much different when we instead hold the precision of the public and private signals the same. In that case the conditional variance can be higher in either model. On the one hand, the exchange rate provides an additional piece of information in the private information model, which lowers the conditional variance. On the other hand, the magnification factor  $z_t$  raises the conditional variance in the private information model.

the exchange rate is driven by non-fundamentals shocks or fundamentals information of other investors, they always revise their expectations of fundamentals when the exchange rate changes (equation (22)). This magnifies the impact of non-fundamental shocks on the exchange rate.

It is instructive to consider the impact of some key model parameters on magnification. Since it is hard to derive analytically the relationship between  $x_t$  and model parameters, Figure 1 shows the sensitivity of  $x_t$  and  $z_t$  to  $\sigma_v$  and  $\sigma_b$  based on numerical simulation. A rise in the variance  $\sigma_v^2$  of the private signal at first raises magnification and then lowers it (Panel A). Two opposite forces are at work here. First, as shown in (25), for a given  $x_t$  an increase in the variance of the private signal raises magnification. This is because more weight is given to the exchange rate as a source of information. Second, a rise in the variance of the private signal implies less information and therefore a lower relative weight  $x_t$  of fundamentals in the exchange rate equation (Panel B). This makes the exchange rate less informative about fundamentals and reduces the magnification factor. On the other hand, a rise in the variance  $\sigma_b^2$  of non-fundamentals shocks always reduces magnification since it makes it more difficult to extract information about fundamentals from the exchange rate.<sup>17</sup> Numerical simulation also shows that a rise in the variance  $\sigma_u^2$  reduces the magnification factor. Intuitively, a higher variance of fundamentals shocks raises the risk-premium and therefore increases the relative weight of non-fundamentals shocks. This reduces the information content of the exchange rate.

Numerical simulations show that the *relative* contribution of non-fundamentals shocks to exchange rate volatility is also larger in the private information model. This is mainly driven by the magnification of the non-fundamentals shocks. But we also find that for most parameters the absolute contribution of fundamentals shocks is lower in the private information model. This results primarily from the error  $\varepsilon_t^v$  in the public signal, which leads to exchange rate volatility associated with the macro fundamental  $v_t$  in the public information model. Exchange rate volatility in both models is also driven by the fundamental  $f_t$  or, equivalently,  $u_t$ . While the differences between the two models in this respect are complex, numerically we find that this difference is rather small.<sup>18</sup>

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<sup>17</sup>This can be seen directly from (25). It is only partially offset by the resulting rise in  $x_t$ .

<sup>18</sup>The difference can be seen by comparing  $\lambda_v$  in (16) to  $\lambda_{u,t}$  in (23).

## IV The Full Dynamic Model with Learning

In this section we consider the more general dynamic structure of the model by setting  $T > 1$  and having both fundamentals and non-fundamentals follow an autoregressive process. In this case private information affects both the magnification and persistence of the impact of non-fundamentals shocks on the exchange rate. Persistence is affected by a learning process implied by the dynamic structure of the model. Information is carried over from period to period and it takes time for investors to learn about the presence and magnitude of non-fundamentals trade. This persistent uncertainty leads to a persistent magnification of non-fundamentals trade on the exchange rate.

We first illustrate the key implications of the model with a benchmark parameterization. Then we consider how the main features are affected by the model's parameters, which provides further insight into the operation of the model. We also briefly consider a sticky-price model. Finally, in the last subsection, we show how the model can be calibrated to reproduce the forward discount bias and 'excess' exchange rate volatility.

### IV.A A Benchmark Case

The parameters of the benchmark case are reported in Table 1. They are chosen mainly to illustrate the potential impact of information dispersion; they are not calibrated or chosen to match any data moments. We assume that  $u$ , and therefore the fundamental, follows a random walk ( $\rho = 0$ ). We assume that the extent of private information is small by setting a high standard deviation of the error of the private signal of 0.08, versus a standard deviation in  $u$  of 0.01. Although we have made assumptions about both  $\sigma_b$  and risk-aversion  $\gamma$ , they enter multiplicatively in the model, so only their product matters. We allow for an AR coefficient of 0.8 of the non-fundamentals b-shock. Finally, we assume that  $T = 8$ , so that agents obtain private signals about fundamentals eight periods later.

Figure 2 shows some of the key results from the benchmark parameterization. Panels A and B show the dynamic impact on the exchange rate in response to one-standard deviation shocks in the private and public information models. In both models the non-fundamentals shocks are the b-shocks. In the private information model the fundamentals shocks are the u-shocks. In the public information model

the fundamentals shocks are both u-shocks and  $\varepsilon^v$  shocks, which affect the exchange rate through the publicly observable fundamentals  $f$  and  $v$ .

### **Magnification**

The presence of private information magnifies the impact of b-shocks in a way similar to the simpler model. The magnification factor in the benchmark parameterization turns out to be substantial: 7.2. This is visualized in Figure 2 by comparing the instantaneous response of the exchange rate to the b-shocks in the two models. In order to facilitate comparison, in panel B we have chosen the precision of the public signal such that it leads to the same conditional variance of next period's exchange rate as in the private information model. This is the same approach as taken in section III. The b-shocks then have the same risk-premium effect in the two models. The only reason the impact of a b-shock is so much bigger in the private information model is the magnification factor associated with information dispersion.

### **Persistence**

We can also see from panel A that the non-fundamentals shocks have substantial persistence. The half-life of the impact of the shock on the exchange rate in the private information model is 3 periods. There are two reasons for persistence. The first is a learning factor as it takes time for investors to learn that a change in the exchange rate was driven by a non-fundamentals rather than a fundamentals shock. This explains why the impact of b-shocks on the exchange rate remains much larger than in the public information model for a long period of time. After eight periods the "confusion" about whether a change in the exchange rate is caused by a b-shock or u-shock is entirely resolved. This follows from the exchange rate equation eight periods ago and the fact that  $u_{t-8} = f_t$  is known at time  $t$ . The second factor affecting persistence is the assumed persistence of  $b$  itself. But even if we had set the AR coefficient  $\rho_b$  equal to zero, the impact of b-shocks on the exchange rate would still be persistent because of the learning factor. We will discuss this further in sensitivity analysis below.

### **Exchange rate disconnect in the short and the long run**

Panel C reports the contribution of non-fundamentals trade to the volatility

of  $s_{t+k} - s_t$  at different horizons. In the private information model, 70% of the variance of a 1-period change in the exchange rate is driven by non-fundamentals shocks, while in the public information model it is a negligible 1.3%.<sup>19</sup> Non-fundamentals shocks are dominant in the private information model, while their relative contribution is negligible in the public information model. The difference is a result both of the magnification of the b-shocks in the private information model and the larger impact of fundamentals shocks in the public information model. This is illustrated in Table 2, which shows the contribution of the different shocks to the unconditional variance of a 1-period change in the exchange rate. Fundamentals shocks contribute more to exchange rate volatility in the public information model largely due to the noise in the public signal  $v$ . The noise in signals in the private information model aggregates to zero.

While in the short-run non-fundamentals shocks dominate in the private information model, in the long-run fundamentals shocks dominate. The impact of a change in non-fundamentals trade on the exchange rate gradually dies down, both because investors gradually learn about the size of a b-shock (reducing magnification) and because the shock itself is temporary ( $\rho_b < 1$ ). On the other hand, the impact of  $u$ -shocks on the exchange rate rises over time as we get closer to the time of the rise in the fundamental  $f$  and as agents gradually learn that the  $u$ -shock took place. In panel C we see that the contribution of non-fundamentals shocks to the variance of  $s_{t+k} - s_t$  declines as the horizons  $k$  increases. For a three-period horizon the contribution of the two shocks to exchange rate volatility is about the same, while the contribution of b-shocks declines to less than 20% after ten periods. This is of course mostly illustrative of the qualitative difference between the short, medium and long-run since a “period” in the model can be anything in the data.

In order to determine the relationship between exchange rates and fundamentals, panel D reports the  $R^2$  of a regression of  $s_{t+k} - s_t$  on all current and lagged

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<sup>19</sup>As discussed above, we have set the noise in the public signal such that the conditional variance of the change in the exchange rate is the same in the two models. This implies a standard deviation of 0.033 of the public signal, less than that of the private signal. If instead we assume that the standard deviation of the noise in the signal is the same in the two models, both equal to 0.08, the contribution of b-shocks to exchange rate volatility in the public information model remains small at 2.4%.

*observable* fundamentals. In the private information model this includes all one period changes in the fundamental  $f_t$ , from  $t + k - 1$  to  $t + k$  and back. In the public information model it also includes corresponding one-period changes in the public signal  $v$ . The  $R^2$  is close to 1 for all horizons in the public information model, while it is much lower in the private information model. At the one-period horizon it is only 0.14, and then rises as the horizon increases, to 0.8 for a 20-period horizon. This is consistent with extensive findings of exchange rate disconnect in the short to medium run, starting with Meese and Rogoff [1983], and findings of a much closer relationship between exchange rates and fundamentals over longer horizons.<sup>20</sup> Two factors account for the results in panel D. The first is that the relative contribution of non-fundamentals shocks to exchange rate volatility is large in the short-run and small in the long-run, as illustrated in panel C. The second factor is that through private signals the exchange rate is also affected by future fundamentals that are not yet observable today. This second factor again affects the exchange rate more in the short-run than the long-run.

### Exchange rate and future fundamentals

Recently Engel and West [2002] and Froot and Ramadorai [2002] have reported evidence that exchange rate changes predict future fundamentals, but only weakly so. Our model is consistent with these findings. Panel E of Figure 2 reports the  $R^2$  of a regression  $f_{t+k} - f_t$  on  $s_{t+1} - s_t$  for  $k \geq 2$ . The  $R^2$  is positive, but never reaches above 0.14. The exchange rate is affected by the private signals of future fundamentals, which aggregate to  $u$ . However, most of the short-run volatility of exchange rates is associated with non-fundamentals shocks, which do not predict future fundamentals.

## IV.B Sensitivity to Model Parameters

In order to gain further insight into the operation of the model, we now examine how the main results are affected by parameter values. Figure 3 contains eight panels showing (i) the magnification factor, (ii) the contribution of b-shocks to  $var(s_{t+1} - s_t)$ , and (iii) persistence, measured by the half-life of the impact of

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<sup>20</sup>See MacDonald and Taylor [1993], Mark [1995], Chinn and Meese [1995], Mark and Sul [2001] and Froot and Ramadorai [2002].

b-shocks on the exchange rate. They are each plotted as a function of a specific parameter. Only in Panel H do we consider a parameter that is not in the baseline model by introducing nominal rigidities.

### Private information

Perhaps the most important parameter of the model is the precision of the private signal. Panel A shows the sensitivity to  $\sigma_v$ , the standard deviation of the error in the private signal. In section III we already discussed the impact of  $\sigma_v$  on magnification,  $z_t$ . Even though the current model is much richer, the results in this respect are qualitatively the same as before. In particular, magnification rises up to  $\sigma_v = 0.06$ , and then drops. Consistent with that we also find that the contribution of non-fundamentals shocks to exchange rate volatility first rises, up to  $\sigma_v = 0.08$ , and then drops.

In section III we were unable to discuss persistence since the simplified assumptions implied zero persistence of the non-fundamentals shocks. Here we see that persistence rises as the private signal becomes less precise. In forming expectations about the future fundamental  $u_t$  agents use both new information (from  $f_t$ ,  $v_t^j$  and  $s_t$ ) and information from previous periods, summarized by  $E_{t-1}(u_t)$ . We will argue that as the quality of the signal goes down, more weight is given to the old information, which includes exchange rates from previous periods and therefore lagged values of  $b_t$ . Persistence therefore increases as the quality of the private signal is weaker. The full model does not allow us to show analytically why there is an increased weight on information carried over from the previous period when the quality of the signal is weaker. We therefore illustrate this effect in the context of a much simpler signal extraction problem.

Assume that  $u$  follows an AR process,  $u_{t+1} = \rho u_t + \varepsilon_t^u$ , and that each period agents receive a signal  $v_t = u_t + \varepsilon_t^v$  (we still assume  $T > 1$ ). In that case

$$E_t(u_t) = \frac{d_{t-1}}{d_t} E_{t-1}(u_t) + \left[1 - \frac{d_{t-1}}{d_t}\right] v_t \quad (26)$$

where where  $d_t = 1/\text{var}_t(u_t)$  and  $d_{t-1} = 1/\text{var}_{t-1}(u_t)$  are the precision associated with the expectation of  $u_t$  at respectively time  $t$  and  $t - 1$ . In a stochastic steady state

$$d_{t-1} = \frac{1}{\rho^2/d_t + \sigma_u^2}$$

It follows from (26) that more weight is given to information from the previous period the larger the relative precision of the information at  $t-1$ , which is

$$\frac{d_{t-1}}{d_t} = \frac{1}{\rho^2 + \sigma_u^2 d_t}$$

It is easily shown that the precision  $d_t$  is smaller when the signal is less informative ( $\sigma_v$  larger). It therefore follows that more weight is given to the information carried over from the previous period. The intuition is that when for all  $t$  the perceived variance of  $u_t$  doubles, the perceived variance of  $u_t$  at time  $t-1$  less than doubles. This is because  $u_t$  also depends on a new innovation  $\varepsilon_t^u$ , which has a constant variance from the perspective of time  $t-1$ .

The overall implication from these results is that the impact of information dispersion on exchange rate dynamics is most powerful for some intermediate range. When  $\sigma_v^2$  is low it is possible to raise the magnification factor, increase the contribution of non-fundamentals shocks to exchange rate volatility, while at the same time making the impact of non-fundamentals shocks more persistent. When information becomes too disperse, however, the magnification factor drops and the contribution of non-fundamentals shocks to exchange rate volatility is lower.

### **Non-fundamental shocks**

Panel B illustrates the role of the standard deviation  $\sigma_b$  of non-fundamentals trade, and panel C the role of persistence  $\rho_b$  of the non-fundamentals shocks. Both a higher standard deviation and more persistence of the non-fundamentals shocks reduce magnification as the exchange rate becomes less informative about fundamentals. In both cases the contribution of non-fundamentals shocks to exchange rate volatility is almost unaffected. The higher contribution of non-fundamentals shocks due to either higher volatility or more persistence of these shocks is offset by the drop in magnification. It is remarkable that one can reduce the variance of non-fundamentals shocks to near zero, while maintaining a large contribution of b-shocks to exchange rate volatility. The magnification factor rises to infinity as  $\sigma_b$  goes to zero.

The persistence of the impact of b-shocks on the exchange rate is not much affected by  $\sigma_b$ , but significantly by  $\rho_b$ . In order to understand this better, Figure 4 shows the half life of  $b_t$ -shocks, which is also the half-life of its exchange rate impact in the public information model, together with the half-life of its impact

on the exchange rate in the private information model. For  $\rho_b < 0.78$  the half-life of the impact of a b-shock on the exchange rate is larger than the half-life of the shock itself, while the opposite is the case for  $\rho_b > 0.78$ . As discussed earlier, two factors drive the persistence of the impact of b-shocks on the exchange rate: a learning factor and the persistence of the b-shock itself. When the shock itself has little persistence, its impact on the exchange rate is more persistent due to the learning factor. But for sufficiently large  $\rho_b$  the impact on the exchange rate can be less persistent than the shock itself since the magnification factor due to information dispersion drops as agents gradually learn about what happened.

One might think that the much stronger relationship between exchange rates and fundamentals in the long-run than the short-run is simply the result of our assumption that non-fundamentals trade is stationary, while the fundamental is non-stationary. This is not the case though. In order to illustrate this, Figure 5 reports the implications of the model for the case where non-fundamentals trade is a random walk. This is the same case as the benchmark parameterization except that  $\rho_b = 1$  and  $\sigma_b = 0.006$ . There is now both a short-run and a long-run impact of non-fundamentals trade on the exchange rate. The long-run impact is the same in the public and private information models and is only associated with the risk-premium channel. In the public information model the short and long-run impact are the same, but in the private information model the short-run impact remains much larger due to the magnification factor (which is 4.1 in this case). It therefore remains the case that the relative contribution of non-fundamentals trade to exchange rate volatility is much larger, and the relationship between exchange rates and fundamentals much weaker, in the short-run than the long-run (panels C and D). Although after 8 periods uncertainty is resolved and the impact on non-fundamentals trade reaches its long-run level, there is substantial persistence of the initial impact. It takes over 6 periods for the impact on the exchange rate to be reduced by 50%.

### **Fundamental shocks**

Panels D and E of Figure 3 show the impact of respectively the standard deviation and persistence of fundamentals shocks. An increase in either  $\sigma_u$  or  $\rho$  reduces the magnification factor. The increased risk raises the risk-premium and therefore the coefficient of  $b_t$  in the exchange rate. This makes it more difficult to extract

information about fundamentals from the exchange rate. The contribution of non-fundamentals shocks to exchange rate volatility is again remarkably insensitive to parameters. Even though more volatile fundamentals now generate more exchange rate volatility, the same is the case for non-fundamentals shocks as a result of the rise in the risk-premium.

### Other parameters

Panel F shows the impact of  $T$ . Magnification rises as agents have private information about fundamentals further into the future ( $T$  bigger). The higher  $T$ , the more information agents have at any point in time, and therefore the lower the uncertainty about next period's exchange rate. This reduces the risk-premium and therefore the coefficient of  $b_t$  in the exchange rate, so that the exchange rate becomes more informative about fundamentals and the magnification factor rises. Persistence also increases as  $T$  goes up. There are two reasons for this. First, it takes  $T$  periods before agents learn the actual size of the b-shock, so a magnification factor remains in place for a longer period of time. Second, since for a large  $T$  agents obtain information about fundamentals well ahead of time, more weight is given to information from previous periods when forming expectations about future fundamentals. This leads to a higher weight on exchange rates from previous periods and therefore higher persistence of non-fundamentals shocks.

### Sticky Prices

The last panel of Figure 3 reports the impact of nominal rigidities. Instead of assuming purchasing power parity, it is assumed that prices adjust with delay to restore purchasing power parity:

$$p_{t+1} - p_t = \phi(s_t + p_t^* - p_t) \quad (27)$$

Such an equation can be derived for example by letting price changes respond to the differences between demand and supply, with demand a function of the relative price  $s_t + p_t^* - p_t$  of foreign to home goods, and supply fixed.  $\phi$  is a parameter between 0 and 1, with a lower number implying more rigid prices.<sup>21</sup> The main impact of nominal rigidities is that they reduce the magnification factor. As

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<sup>21</sup>When  $\phi = 1$ , prices adjust with only one period delay to restore purchasing power parity. This is still slower than in the baseline model, where prices adjust instantaneously.

in the Dornbusch-overshooting model, nominal rigidities lead to higher exchange rate volatility. This raises the risk-premium and therefore the coefficient of  $b$  in the exchange rate. The exchange rate then becomes less informative about fundamentals and the magnification factor drops.

#### IV.C 'Excess' Volatility and the Forward Discount Bias

Although our focus has been primarily on the exchange rate disconnect puzzle, we would like to briefly discuss two other exchange rate puzzles that have received a lot of attention: (i) the puzzle of excess volatility of exchange rates relative to fundamentals and (ii) the forward discount puzzle. The exchange rate disconnect puzzle and the excess volatility puzzle are two distinct puzzles that may not have the same explanation. For example, the Dornbusch-overshooting model can easily generate excess volatility, but exchange rates are driven entirely by fundamentals. In our model the opposite is the case under the benchmark parameterization. The model generates short to medium-run exchange rate disconnect, but the standard deviation of a 1-period change in the exchange rate is only 0.7 times the standard deviation of a 1-period change in the fundamentals. Moreover, the coefficient of a Fama regression of a 1-period change in the exchange rate on the forward discount (interest rate differential) is 0.88. The latter is negative in most empirical estimates (the forward discount puzzle).

This is not the result of some fundamental problem associated with models of information dispersion. There are parameters in the model other than the extent of information dispersion that are critical in accounting for the other puzzles. For example, the same factors that generate excess volatility in representative agent models, such as nominal rigidities, generate excess volatility in our model as well. By choosing appropriate parameters the model can explain all three puzzles at once. In order to illustrate this, Table 3 and Figure 6 report the implications of a very different parameterization, shown in the last column of Table 1. Under this alternative parameterization the standard deviation of 1-period exchange rate changes is more than three times the standard deviation of 1-period fundamentals changes and the Fama coefficient is negative at -0.06. It remains the case that the contribution of non-fundamentals shocks to the variance of  $s_{t+1} - s_t$  is much larger than in the corresponding public information model (76% versus 13%). The

persistence of the impact of non-fundamentals shocks on the exchange rate is about the same as in the benchmark parameterization.

Two factors contribute to the higher exchange rate volatility relative to fundamentals. First, we have borrowed from the Dornbusch overshooting model by introducing nominal rigidities. Second, we have set  $\rho = 0.75$ . This implies that a rise in the fundamental today is expected to lead to a much larger long-run increase in the fundamental, so that the exchange rate changes much more than an unexpected increase in the fundamental.

The increased exchange rate volatility relative to fundamentals also explains the negative Fama coefficient. We know that bond market equilibrium implies that

$$\bar{E}_t(s_{t+1} - s_t) = i_t - i_t^* + \gamma\sigma t^2 b_t$$

In the absence of b-shocks the risk-premium would be constant and the average expected change in the exchange rate is equal to the interest differential. Since investors are rational, they do not make systematic forecast errors. It follows that with a constant  $b_t$  the coefficient of the Fama regression of  $s_{t+1} - s_t$  on  $i_t - i_t^*$  would be equal to 1. Fluctuations in non-fundamentals trade reduce the coefficient below 1 because of a negative correlation between  $b_t$  and  $i_t - i_t^*$ . A drop in  $b_t$  leads to a rise in  $s_t$ , which implies a rise in the interest differential to clear the money market.<sup>22</sup> This effect is strengthened as a result of increased exchange rate volatility, which raises  $\sigma_t$  and therefore the risk-premium for a given level of  $b_t$ . We therefore find, not surprisingly, that any change in model parameters that leads to increased exchange rate volatility (relative to fundamentals) also lowers the Fama coefficient.

There is a tradeoff though between accounting for the disconnect puzzle on the one hand and the other two puzzles on the other hand. Any change in model parameters that leads to more exchange rate volatility (relative to fundamentals), and a lower Fama coefficient, also reduces the magnification of the impact of non-fundamentals trade on the exchange rate. The reason is that a larger risk-premium makes the exchange rate more sensitive to  $b_t$ , which reduces the information content of the exchange rate and therefore the magnification factor associated with information dispersion. Nonetheless the alternative parameterization illustrates

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<sup>22</sup>This happens instantaneously under PPP since the rise in  $s_t$  raises  $p_t - p_t^*$ , which implies a rise in  $i_t - i_t^*$  to clear the money market. It happens with delay in the model with nominal rigidities since the price level reacts gradually to the change in the exchange rate.

that one can account for all puzzles at once, with information dispersion playing a critical role only for the disconnect puzzle.

The point is not that with the right parameters the simple exchange rate model presented here can resolve all outstanding exchange rate puzzles. As the alternative parameterization illustrates, there are many assumptions other than information dispersion that affect exchange rate dynamics. A much richer model would be required if we aim to understand all the stylized facts about exchange rates at once. Our focus here has been on the exchange rate disconnect puzzle, for which information dispersion plays the most critical role.

## V Conclusion

The large volume of trade in the foreign exchange market and the close relationship between order flow and exchange rates suggests that investor heterogeneity is a key element in understanding exchange rate behavior. In this paper we have explored the implications of information dispersion in a simple model of exchange rate determination. We have shown that these implications are rich and that investors' heterogeneity could be an important element in explaining the behavior of exchange rates. In particular, the model can account for some important stylized facts on the relationship between exchange rates and fundamentals: (i) the exchange rate is disconnected from fundamentals in the short to medium run, (ii) over longer horizons the exchange rate is primarily driven by fundamentals, (iii) exchange rate changes are a weak predictor of future fundamentals. Moreover, we show that, by choosing appropriate parameter values, the model also predicts excess volatility of the exchange rate and a forward discount bias.

The paper should be considered as a first step in a promising line of research. The underlying monetary model is certainly too simple to be realistic. Exploring the role of information dispersion in more complete models appears to be of substantial value. For example, it would be interesting to model the microstructure institutions such as foreign exchange dealers in order to better integrate microstructure and macro models of exchange rate determination. The microstructure details of information processing in the foreign exchange market could well have important macroeconomic implications. In another direction, one could introduce investor heterogeneity in the “new open economy macro” models with nominal rigidities.

These models have been used extensively to draw conclusions about optimal monetary and exchange rate policies. These conclusions are likely to be significantly revised in the context of a model where the exchange rate is disconnected from fundamentals over substantial periods of time. It would also be insightful to calibrate a richer version of the model to the data in order to evaluate its performance along a wide range of exchange rate puzzles.

Another direction for extensions relates to the information structure. For example, the information received by agents may differ in its quality or in its timing. There can also be heterogeneity about the knowledge of the underlying model.<sup>23</sup> Finally, as in behavioral finance, one could relax the assumption that all investors are fully rational in their information processing.<sup>24</sup>

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<sup>23</sup>See Honkapohja and Mitra (2002) for a recent analysis in different contexts.

<sup>24</sup>For example, Hong and Stein (1999) explain the overreaction of asset prices in such a context.

# A Appendix

## A.1 Derivation of equation 4

It follows from (1)-(3) that

$$s_t = \frac{\alpha}{1+\alpha} \bar{E}_t^1(s_{t+1}) + \frac{1}{1+\alpha} f_t - \frac{\alpha}{1+\alpha} \gamma \sigma_t b_t \quad (28)$$

Therefore

$$\bar{E}_t^1(s_{t+1}) = \frac{\alpha}{1+\alpha} \bar{E}_t^2(s_{t+2}) + \frac{1}{1+\alpha} \bar{E}_t^1(f_{t+1}) - \frac{\alpha}{1+\alpha} \gamma \sigma_{t+1} \bar{E}_t^1(b_{t+1}) \quad (29)$$

Substitution into (28) yields

$$s_t = \left( \frac{\alpha}{1+\alpha} \right)^2 \bar{E}_t^2(s_{t+2}) + \frac{1}{1+\alpha} \sum_{k=0}^1 \left( \frac{\alpha}{1+\alpha} \right)^k \bar{E}_t^k (f_{t+k} - \alpha \gamma \sigma_{t+k} b_{t+k}) \quad (30)$$

Continuing to solve for  $s_t$  this way by forward induction yields (4).

## A.2 Derivation of the equilibrium exchange rate equation

We describe the method for solving for the equilibrium exchange rate in the model with investors that have infinite horizons. The case of overlapping generations of one-period-horizon investors is a simplified version of that. We proceed in four steps: i) we conjecture an exchange rate equation; ii) we derive the expectations of  $\mathbf{u}_t$  and  $b_t$  with a Kalman filter, using the conjectured exchange rate equation; iii) we compute the asset demand based on these expectations; iv) we find the equilibrium exchange rate that clears the foreign exchange market; v) we discuss the numerical method used to equate the conjectured exchange rate equation to the one that solves the foreign exchange market equilibrium.

### The exchange rate equation

We conjecture the following equilibrium exchange rate equation

$$s_t = \lambda_s \mathbf{s}_{t-1} + \lambda_f \mathbf{f}_t + \lambda_u \mathbf{u}_t + \lambda_b b_t \quad (31)$$

Our solution method relies on truncating at sufficiently large lags  $\bar{T}$ , so that

$$\begin{aligned} \mathbf{s}_{t-1} &= (s_{t-1}, s_{t-2}, \dots, s_{t-\bar{T}})' & \lambda_s &= (\lambda_s^1, \lambda_s^2, \dots, \lambda_s^{\bar{T}}) \\ \mathbf{f}_t &= (f_t, f_{t-1}, f_{t-2}, \dots, f_{t-\bar{T}})' & \lambda_f &= (\lambda_f^0, \lambda_f^1, \dots, \lambda_f^{\bar{T}}) \\ \mathbf{u}_t &= (u_t, u_{t-1}, u_{t-2}, \dots, u_{t-T+1})' & \lambda_u &= (\lambda_u^0, \lambda_u^1, \dots, \lambda_u^{T-1}) \end{aligned}$$

There are only  $T - 1$  lags for  $u_t$  since  $u_{t-T} = f_t$  after  $T$  periods.

### Kalman Filter

We adopt a notation similar to Hamilton [1994, chapter 13]. Let  $\mathbf{Y}_t^i$  be a vector of observable variables (present and past) for investor  $i$  and  $\xi_t$  be a vector of unobservables. The objective is to determine the estimate of unobservables based on a linear combination of observables, i.e.,  $E_t^i(\xi_t) = \mathbf{A}\mathbf{Y}_t^i$ . In our model the vectors of observables and unknowns are

$$\mathbf{Y}_t^i = (s_t, f_t, v_t^i, s_{t-1}, f_{t-1}, v_{t-1}^i, \dots, s_{t-T}, f_{t-T}, v_{t-T}^i)' \quad \xi_t = (u_t, u_{t-1}, \dots, u_{t-T}, b_t)'$$

We use the following linear state-space representation:

$$\mathbf{Z}\mathbf{Y}_t^i = \mathbf{H}'\xi_t + \mathbf{w}_t^i \quad (32)$$

$$\xi_t = \mathbf{F}\xi_{t-1} + \varepsilon_t \quad (33)$$

where

$$\mathbf{w}_t^i = (0, 0, \varepsilon_t^{v,i})' \quad \varepsilon_t = (\varepsilon_t^u, 0, \dots, 0, \varepsilon_t^b)'$$

$$\mathbf{Z} = \begin{bmatrix} 1 & -\lambda_f^0 & 0 & -\lambda_s^1 & -\lambda_f^1 & 0 & \lambda_s^2 & \lambda_f^2 & 0 & \dots & \dots & \bar{b}_s^T & \bar{b}_f^T & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & \dots & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{H}' = \begin{bmatrix} \lambda_u^0 & \lambda_u^1 & \lambda_u^2 & \dots & \lambda_{T-1}^u & 0 & \lambda^b \\ 0 & 0 & 0 & \dots & 0 & 1 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} 1 + \rho & -\rho & 0 & \dots & 0 & 0 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & \rho_b \end{bmatrix}$$

The first row of (32) represents the exchange rate equation (31), the second row  $u_{t-T} = f_t$ , and the third row the private signal (8).

We also define the variance-covariance matrices:

$$\mathbf{R} \equiv \text{var}(\mathbf{w}_t^i) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_v^2 \end{bmatrix} \quad \mathbf{Q} \equiv \text{var}(\varepsilon_t) = \begin{bmatrix} \sigma_v^2 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & \sigma_b^2 \end{bmatrix}$$

Define the conditional variance-covariance matrices of  $\xi_t$  in the steady state as  $\mathbf{P} = \text{var}(\xi_t|\mathbf{I}_t)$  and  $\tilde{\mathbf{P}} = \text{var}(\xi_t|\mathbf{I}_{t-1})$ , where  $\mathbf{I}_t$  and  $\mathbf{I}_{t-1}$  represent the information available at  $t$  and  $t - 1$ . From Hamilton eq. (13.2.15), we have:

$$E_t^i(\xi_t) = E_{t-1}^i(\xi_t) + \mathbf{M} [\mathbf{Z}\mathbf{Y}_t^i - \mathbf{H}'E_{t-1}^i(\xi_t)] \quad (34)$$

where:

$$\mathbf{M} = \tilde{\mathbf{P}}\mathbf{H} [\mathbf{H}'\tilde{\mathbf{P}}\mathbf{H} + \mathbf{R}]^{-1} \quad (35)$$

$$\mathbf{P} = \tilde{\mathbf{P}} - \mathbf{M}\mathbf{H}'\tilde{\mathbf{P}} \quad (36)$$

$$\tilde{\mathbf{P}} = \mathbf{F}\mathbf{P}\mathbf{F}' + \mathbf{Q} \quad (37)$$

We need to solve for the matrix  $\mathbf{A}$  in  $E_t^i(\xi_t) = \mathbf{A}\mathbf{Y}_t^i$ . Using  $E_{t-1}^i(\xi_t) = \mathbf{F}E_{t-1}^i(\xi_{t-1})$  from (33), (34) gives:

$$\begin{aligned} \mathbf{A}\mathbf{Y}_t^i &= \mathbf{F}\mathbf{A}\mathbf{Y}_{t-1}^i + \mathbf{M}\mathbf{Z}\mathbf{Y}_t^i - \mathbf{M}\mathbf{H}'\mathbf{F}\mathbf{A}\mathbf{Y}_{t-1}^i = \\ &\mathbf{M}\mathbf{Z}\mathbf{Y}_t^i + [\mathbf{F}\mathbf{A} - \mathbf{M}\mathbf{H}'\mathbf{F}\mathbf{A}] \mathbf{Y}_{t-1}^i \end{aligned} \quad (38)$$

We set  $\bar{T}$  sufficiently large such that the last 3 columns of  $\mathbf{A}$  are negligible (less than  $10^{-5}$ ). Thus, we can set the last 3 elements of  $\mathbf{Y}_{t-1}^i$  to zero in the above equation. Define  $\tilde{\mathbf{Y}}_{t-1}^i$  as  $\mathbf{Y}_{t-1}^i$  with the last 3 elements set equal to zero. We then define a transformation matrix  $\mathbf{U}$  such that  $\tilde{\mathbf{Y}}_{t-1}^i = \mathbf{U}\mathbf{Y}_{t-1}^i$ , so that (38) implies:

$$\mathbf{A} = \mathbf{M}\mathbf{Z} + \mathbf{F}\mathbf{A}\mathbf{U} - \mathbf{M}\mathbf{H}'\mathbf{F}\mathbf{A}\mathbf{U} \quad (39)$$

The matrices  $\mathbf{P}$ ,  $\tilde{\mathbf{P}}$  and  $\mathbf{A}$  can be solved from (36), (37) and (39) for a given exchange rate equation.

### Optimal asset demand

The analysis here draws on Wang [1994, Appendix A]. Investors  $i$  maximizes (11) subject to (12). The Bellman equation is

$$U_t^i = \max \left\{ -e^{-\gamma c_t^i} + \beta E_t U_{t+1}^i \right\}$$

where the maximization is done with respect to  $c_t^i$  and  $b_t^i$ . We conjecture that the value function is:

$$U_t^i = -\alpha_1 \exp \left\{ -\alpha_2 w_t^i - \frac{1}{2} \mathbf{Y}_t^i \mathbf{V} \mathbf{Y}_t^{i'} \right\} \quad (40)$$

Define the excess return on foreign bonds as  $q_{t+1} = s_{t+1} - s_t + i_t^* - i_t$ . Then using (12) we have:

$$U_{t+1}^i = -\alpha_1 \exp \left\{ -\alpha_2(1 + i_t)w_t^i + \alpha_2c_t^i - \alpha_2b_t^i q_{t+1} - \frac{1}{2} \mathbf{Y}_{t+1}^i \mathbf{V} \mathbf{Y}_{t+1}^{i'} \right\} \quad (41)$$

Now we want express the last two terms as a function of current observables  $\mathbf{Y}_t^i$ . From (1), (2), and the definition of  $f_t$ , it is easy to see that  $i_t^* - i_t = (f_t - s_t)/\alpha$ . Thus:

$$q_{t+1} = s_{t+1} - \frac{1 + \alpha}{\alpha} s_t + \frac{f_t}{\alpha} \quad (42)$$

We can then use (31) to substitute for  $s_{t+1}$ . We decompose  $q_{t+1}$  such that:  $q_{t+1} = E_t^i(q_{t+1}) + \varepsilon_t^{q,i}$ . Let us define the vector of shocks  $\epsilon_{t+1}^i = (\varepsilon_{t+1}^u, \varepsilon_t^{vi}, \varepsilon_t^b, \tilde{\xi}_t - E_t^i(\tilde{\xi}_t))'$  and  $\tilde{\xi}_t = (u_t, u_{t-1}, \dots, u_{t-T+1})'$ . After a few substitutions we get:

$$q_{t+1} = \Theta_1' \mathbf{Y}_t^i + \Theta_2' E_t^i(\xi_t) + \Theta_3' \epsilon_{t+1}^i \quad (43)$$

Since  $E_t^i(\xi_t) = \mathbf{A} \mathbf{Y}_t^i$ , we get:

$$q_{t+1} = \Theta' \mathbf{Y}_t^i + \Theta_3' \epsilon_{t+1}^i \quad (44)$$

where  $\Theta \equiv \Theta_1 + \mathbf{A}' \Theta_2$ . Thus,  $E_t^i(q_{t+1}) = \Theta' \mathbf{Y}_t^i$  and  $\varepsilon_t^{q,i} = \Theta_3' \epsilon_{t+1}^i$ .

Similarly to  $q_{t+1}$  we can express  $s_{t+1}$ ,  $f_{t+1}$ , and  $v_{t+1}^i$  as functions of  $\mathbf{Y}_t^i$  and  $\epsilon_{t+1}^i$ . This allows us to express  $\mathbf{Y}_{t+1}^i \mathbf{V} \mathbf{Y}_{t+1}^{i'}$  in terms of  $\mathbf{Y}_t^i$  and  $\epsilon_{t+1}^i$ :

$$\mathbf{Y}_{t+1}^i = \mathbf{N}_1 \mathbf{Y}_t^i + \mathbf{N}_2 \epsilon_{t+1}^i$$

We can then rewrite (41) as:

$$U_{t+1}^i = -\alpha_1 \exp \left\{ \begin{array}{l} -\alpha_2(1 + i_t)w_t^i + \alpha_2c_t^i - \alpha_2b_t^i \Theta' \mathbf{Y}_t^i - \frac{1}{2} \mathbf{Y}_t^{i'} \mathbf{N}_1' \mathbf{V} \mathbf{N}_1 \mathbf{Y}_t^i \\ -\alpha_2b_t^i \Theta_3' \epsilon_{t+1}^i - \mathbf{Y}_t^{i'} \mathbf{N}_1' \mathbf{V} \mathbf{N}_2 \epsilon_{t+1}^i - \frac{1}{2} \epsilon_{t+1}^{i'} \mathbf{N}_2' \mathbf{V} \mathbf{N}_2 \epsilon_{t+1}^i \end{array} \right\} \quad (45)$$

Using the normality of the random variables in  $\epsilon_{t+1}^i$ :

$$E_t^i(U_{t+1}^i) = \frac{-\alpha_1}{|\Sigma|^{\frac{1}{2}} |\Omega|^{-\frac{1}{2}}} \exp \left\{ \begin{array}{l} -\alpha_2(1 + i_t)w_t^i + \alpha_2c_t^i - \alpha_2b_t^i \Theta' \mathbf{Y}_t^i - \frac{1}{2} \mathbf{Y}_t^{i'} \mathbf{N}_1' \mathbf{V} \mathbf{N}_1 \mathbf{Y}_t^i \\ + \frac{1}{2} \alpha_2^2 b_t^2 \Theta_3' \Omega \Theta_3 + \mathbf{Y}_t^{i'} \mathbf{N}_1' \mathbf{V} \mathbf{N}_2 \Omega \mathbf{N}_2' \mathbf{V}' \mathbf{N}_1 \mathbf{Y}_t^i \\ + 2\alpha_2 b_t^i \Theta_3' \Omega \mathbf{N}_2' \mathbf{V}' \mathbf{N}_1 \mathbf{Y}_t^i \end{array} \right\} \quad (46)$$

where  $\Sigma = var(\epsilon_{t+1}^i)$  and  $\Omega = [\Sigma^{-1} + \mathbf{N}'_2 \mathbf{V} \mathbf{N}_2]^{-1}$ . Maximizing this expression with respect to  $b_t^i$ , we get

$$b_t^i = \frac{[\Theta' - \Theta'_3 \Omega \mathbf{N}'_2 \mathbf{V}' \mathbf{N}_1] \mathbf{Y}_t^i}{\alpha_2 \Theta'_3 \Omega \Theta_3} \quad (47)$$

The first term in brackets represents the expected return, while the second term represents the hedge against expected return changes.

After maximizing the Bellman equation with respect to  $c_t^i$  and substituting the result back into the Bellman equation, it is easily verified that our conjecture of the value function (41) is correct if  $\alpha_2 = \gamma i$  and

$$\mathbf{V} = \frac{1}{1+i} \left( \frac{(\Theta - \mathbf{N}'_1 \mathbf{V} \mathbf{N}_2 \Omega' \Theta_3)(\Theta' - \Theta'_3 \Omega \mathbf{N}'_2 \mathbf{V} \mathbf{N}_1)}{\Theta'_3 \Omega \Theta_3} + \mathbf{N}'_1 \mathbf{V} \mathbf{N}_1 - \mathbf{N}'_1 \mathbf{V} \mathbf{N}_2 \Omega \mathbf{N}'_2 \mathbf{V}' \mathbf{N}_1 \right) \quad (48)$$

This contains an implicit solution for  $\mathbf{V}$ .

### Foreign exchange market equilibrium

Foreign exchange market equilibrium implies  $\int_0^1 b_t^i = b_t$ . Aggregating (47), this implies

$$\alpha_2 \Theta'_3 \Omega \Theta_3 b_t = \omega' \mathbf{Y}_t \quad (49)$$

where  $\mathbf{Y}_t$  is the average  $\mathbf{Y}_t^i$  (i.e.,  $v_t^i, v_{t-1}^i, \dots$  are replaced by  $u_t, u_{t-1}, \dots$ ) and the vector  $\omega$  is defined as:

$$\omega = \Theta - \mathbf{N}'_1 \mathbf{V} \mathbf{N}_2 \Omega \Theta_3.$$

Equation (49) can be solved for the exchange rate:

$$s_t = \lambda'_s s_{t-1} + \lambda'_f \mathbf{f}_t + \lambda'_u \mathbf{u}_t + \lambda'_b b_t \quad (50)$$

with the parameters a function of  $\alpha_2 \Theta'_3 \Omega \Theta_3$  and  $\omega$ .

### Numerical Solution

In order to solve for the exchange rate equation, we need to simultaneously solve for the matrices  $\mathbf{A}$ ,  $\mathbf{P}$ ,  $\tilde{\mathbf{P}}$ ,  $\mathbf{V}$ , the vectors  $\lambda_f$ ,  $\lambda_u$ ,  $\lambda_s$ , and  $\lambda_b$ . This is done as follows. For a given  $\mathbf{V}$  we solve for all the other parameters with the Gauss non-linear least squares routine. The equations solved are (36), (37), (39),  $\lambda_f = \lambda'_f$ ,  $\lambda_s = \lambda'_s$ ,  $\lambda_u = \lambda'_u$  and  $\lambda_b = \lambda'_b$ . We start with  $\mathbf{V} = 0$  and solve for the other parameters with non-linear least squares. Given the other parameters we then

solve for  $\mathbf{V}$  using (48). The solution to (48) is found as follows. We first set  $\mathbf{V} = \mathbf{0}$  on the right hand side. The expression then gives us a new  $\mathbf{V}$  on the left hand side. We then substitute this on the right hand side, and so on, until it converges to the solution. Once we obtain the solution of  $\mathbf{V}$  to (48), we use the Gauss non-linear least squares routine to solve again for the other parameters. After that we compute a new  $\mathbf{V}$  from (48), and so on, until the process converges (with changes in parameters becoming negligibly small from one iteration to the next).

The solution method is easier for the case of overlapping generations of investors with one-period horizons. In that case the solution is exactly the same with  $\mathbf{V} = 0$  and  $\alpha_2 = \gamma$ . We solve all parameters as discussed above with the Gauss non-linear least squares routine. Notice that the  $\gamma$  in the infinite horizon model is not exactly the same as the  $\gamma$  in the overlapping generations model. In the infinite horizon model  $\alpha_2$  is the rate of absolute risk-aversion with respect to wealth, which is comparable to the absolute risk-aversion parameter  $\gamma$  in the overlapping generations model. When comparing the two models below we therefore set  $\alpha_2 = \gamma$  equal to  $\gamma$  in the overlapping generations model.

### A.3 Infinite Horizon Results

The results for the infinite horizon model are practically identical to those for the overlapping generations model. In order to illustrate this, we report some results here based on the benchmark parameterization. The equilibrium exchange rate equations are very similar. For respectively the overlapping generations model and the infinite horizons model, they are (only reporting one lag for space limitations):

$$s_t = -0.0189s_{t-1} + 0.2922f_t + 0.0239f_{t-1} + 0.1675u_t + 0.1271u_{t-1} + \dots - 0.5443b_t$$

$$s_t = -0.0228s_{t-1} + 0.2868f_t + 0.0253f_{t-1} + 0.1736u_t + 0.1305u_{t-1} + \dots - 0.5337b_t$$

The magnification factor of the impact of b-shocks on the exchange rate is 7.2326 for the overlapping generations model and 7.3225 for the infinite horizons model. The contribution of b-shocks to  $var(s_{t+1} - s_t)$  is 69.37% for the overlapping generations model and 68.67% for the infinite horizon model. Finally, the half-life of the impact of b-shocks on the exchange rate is 2.9523 for the overlapping generations model and 2.9275 for the infinite horizon model.

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	Benchmark parameterization	Alternative parameterization
$\sigma_b$	0.01	0.008
$\sigma_v$	0.08	0.08
$\sigma_u$	0.01	0.003
$\rho$	0	0.75
$\rho_b$	0.8	0.92
$\alpha$	10	9
$\gamma$	500	500
$T$	8	8
prices	flexible	rigid
$\phi$	-	0.15

Table 1: Parameterization

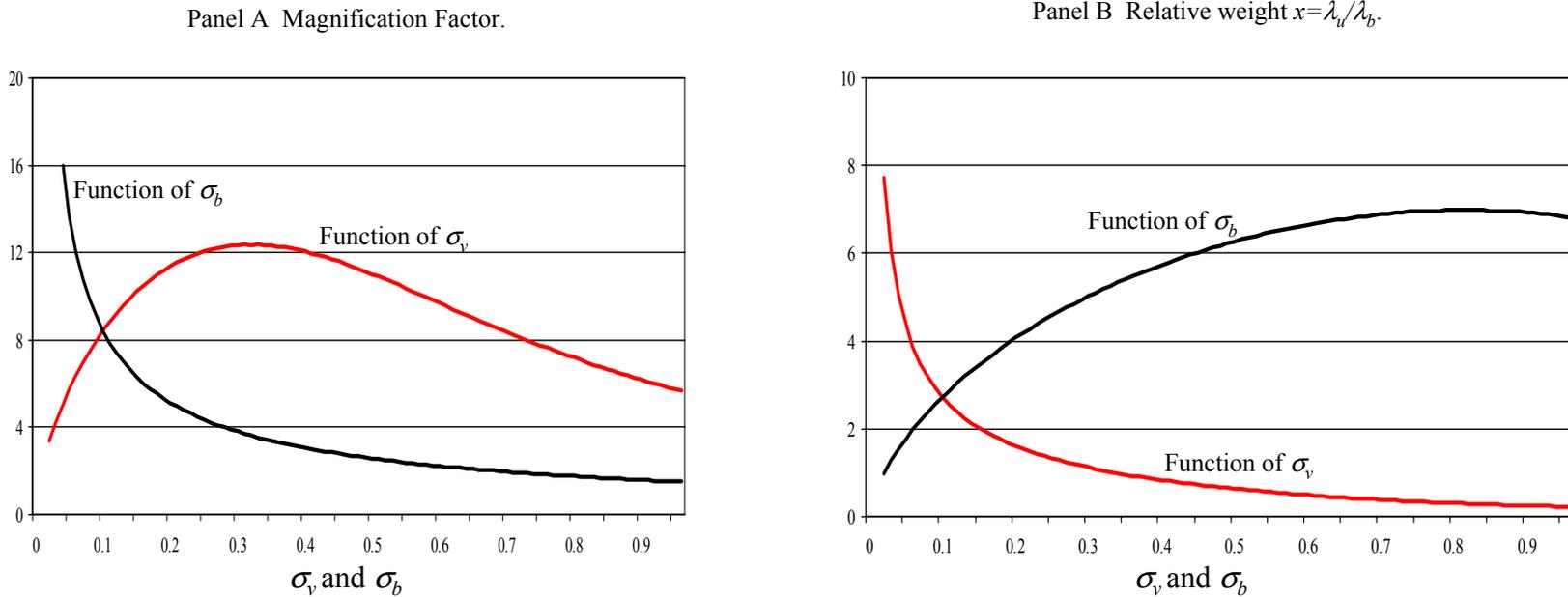
	private information model	public information model
b-shocks	3.36E-05	0.063E-05
u-shocks	1.48E-05	1.86E-05
$\epsilon^v$ -shocks	-	2.87E-05
total	4.84E-05	4.973E-05

Table 2: Contribution to  $var(s_{t+1} - s_t)$

	Benchmark parameterization	Alternative parameterization
magnification	7.23	2.40
contribution b-shocks to $var(s_{t+1} - s_t)$ in private information model	69.4%	76%
contribution b-shocks to $var(s_{t+1} - s_t)$ in public information model	1.3%	13%
$R^2$ of regression of $s_{t+1} - s_t$ on current and lagged fundamentals in private information model	14.8%	20.5%
$R^2$ of regression of $s_{t+1} - s_t$ on current and lagged fundamentals in public information model	98.7%	87.1%
half-life b-shocks	2.95	2.82
$\sigma_{(s_{t+1}-s_t)}/\sigma_{(f_{t+1}-f_t)}$	0.70	3.03
Fama coefficient	0.89	-0.06

Table 3: Comparison Benchmark and Alternative Parameterization

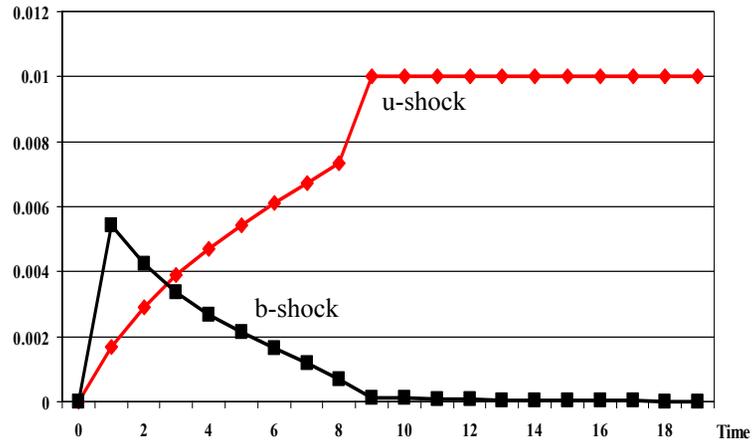
Figure 1 Magnification and relative weight  $x$  in model without learning\*



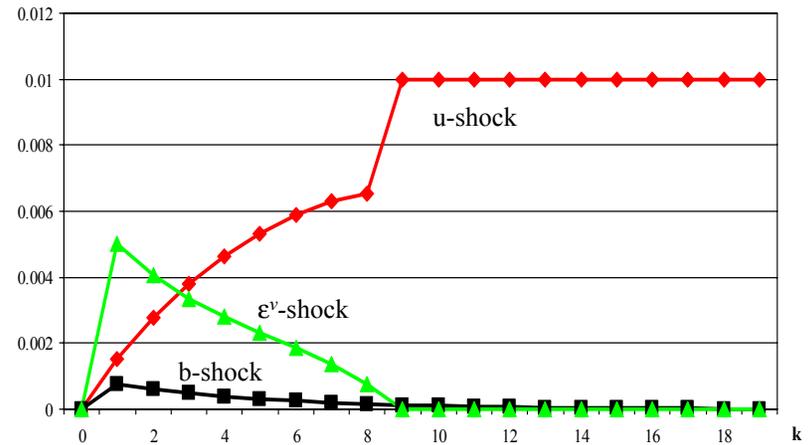
\*These figures are based on the simulation of the model without learning, where  $T=1$ ,  $\rho_1 = \rho_2 = \rho_b = 0$ . The qualitative results do not depend on other model parameters. We set  $\alpha=10$ ,  $\gamma=50$ , and all standard deviations of the shocks equal to 0.1, unless varied within the Figure.

# Figure 2 Results for the Benchmark Parameterization\*

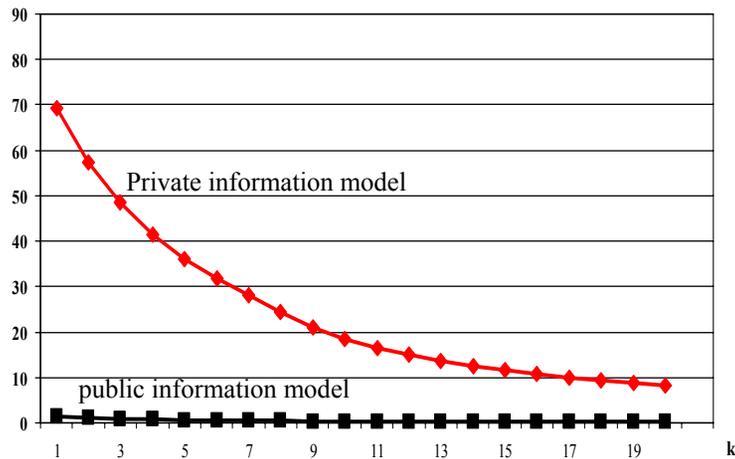
Panel A Impulse Response Functions in Private Information Model



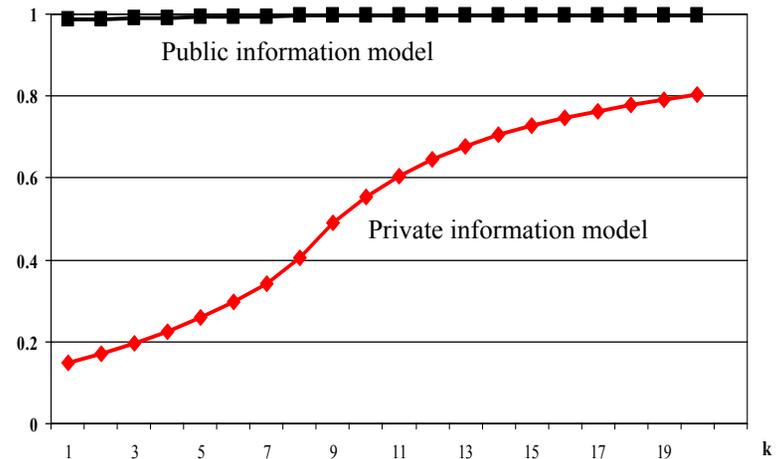
Panel B Impulse Response Functions in Public Information Model



Panel C Percent contribution b-shocks to  $\text{var}(s_{t+k}-s_t)$



Panel D  $R^2$  of regression of  $s_{t+k}-s_t$  on current and lagged fundamentals



\* See Table 1.

Figure 2 Results for the Benchmark Parameterization-continued.

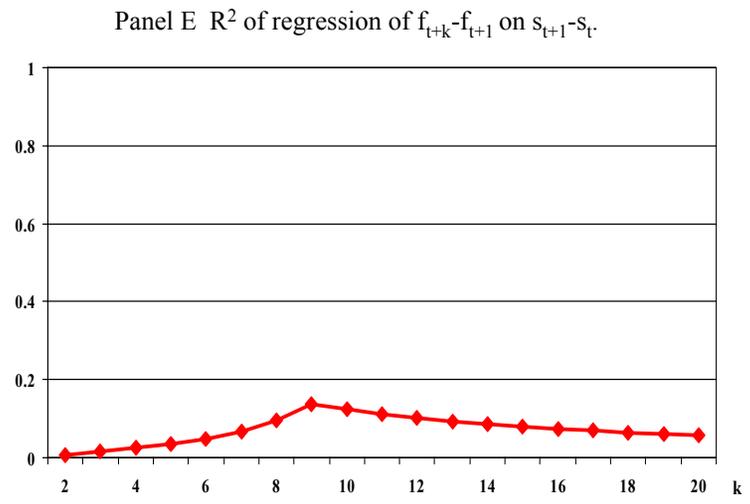
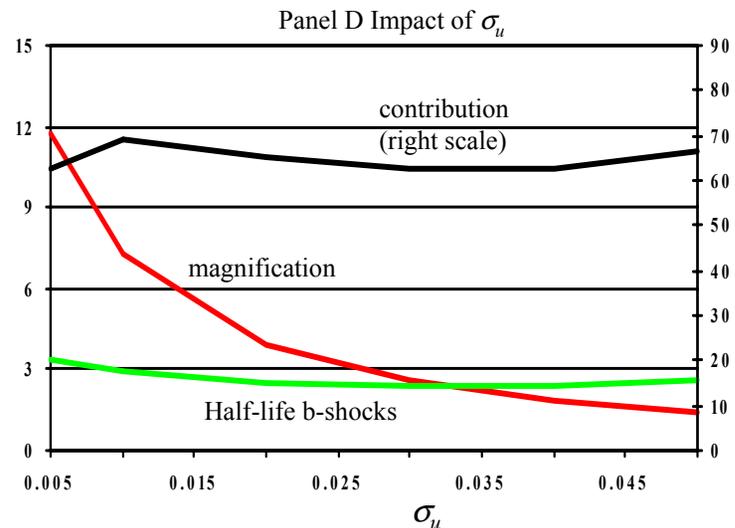
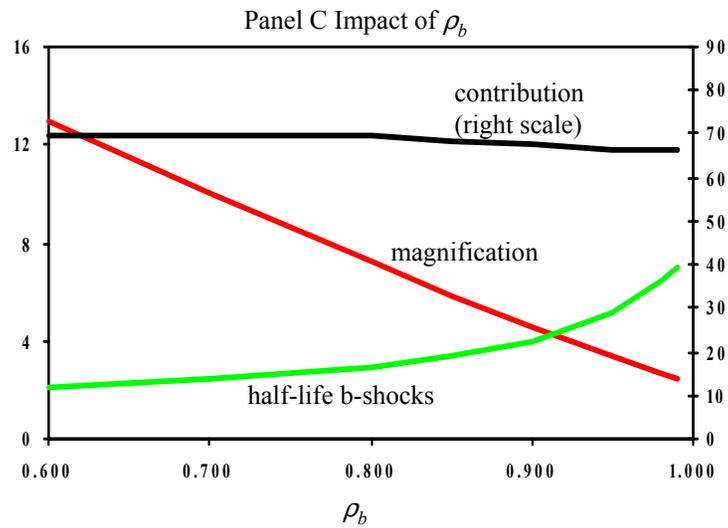
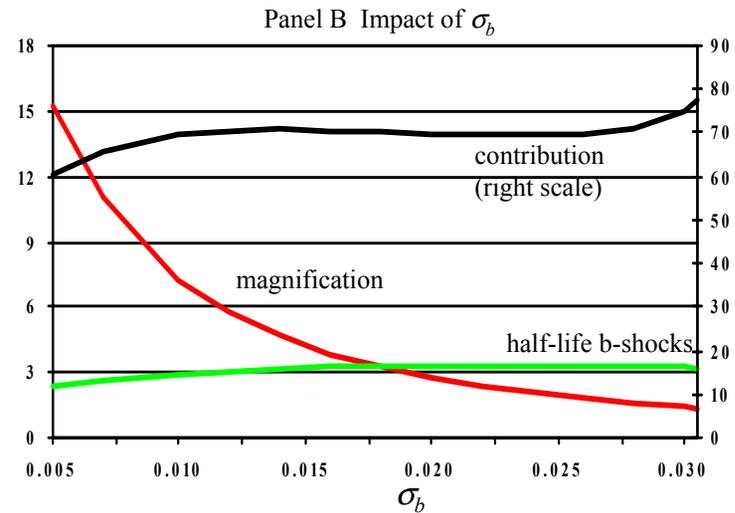
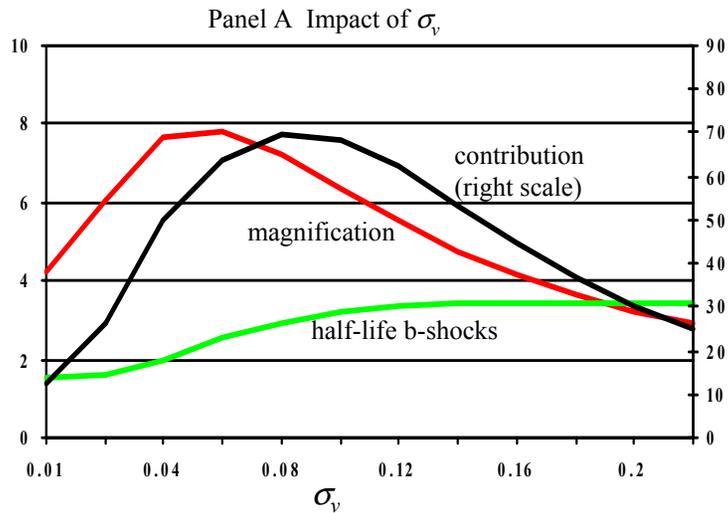
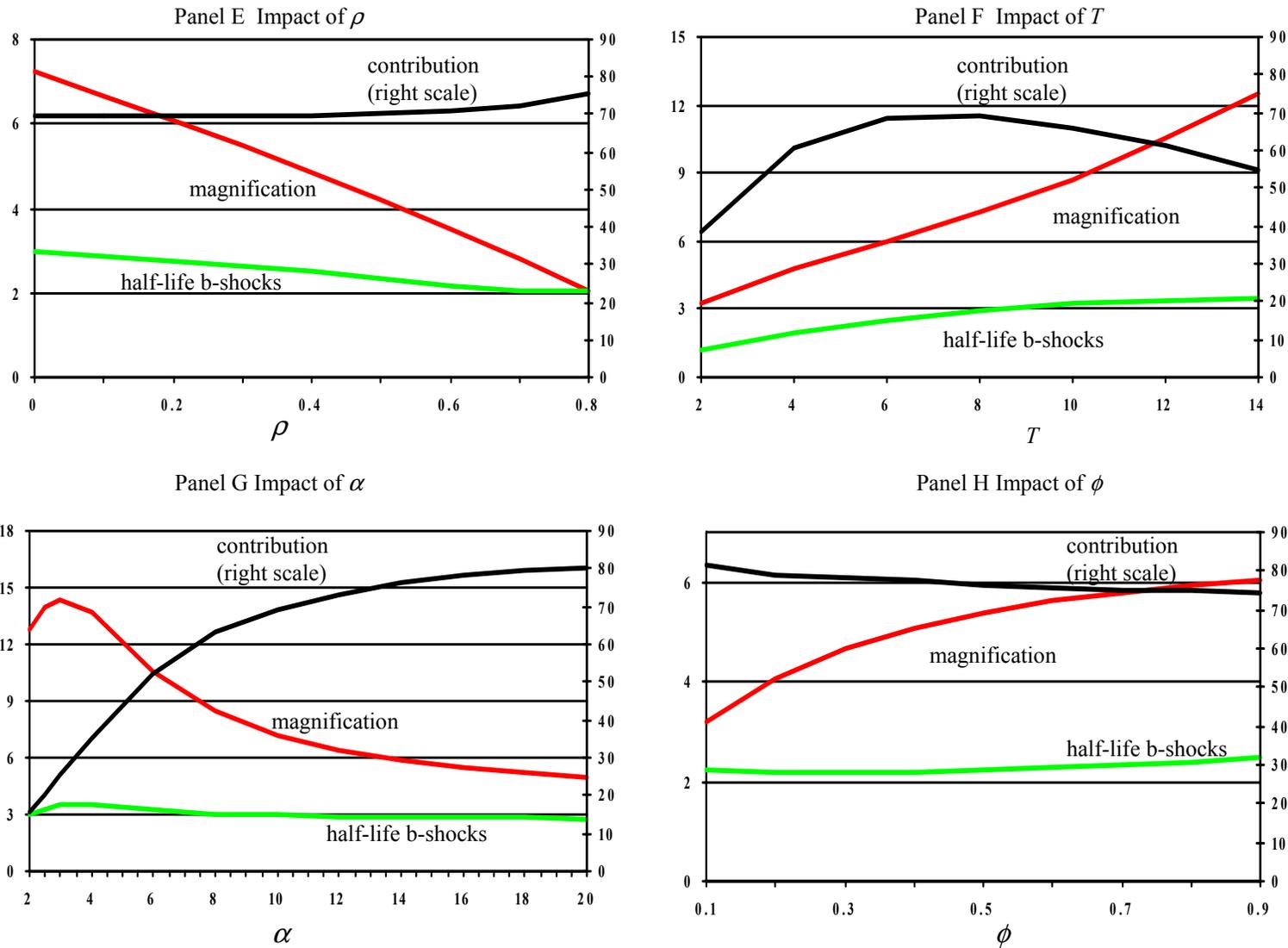


Figure 3 Sensitivity Analysis\*



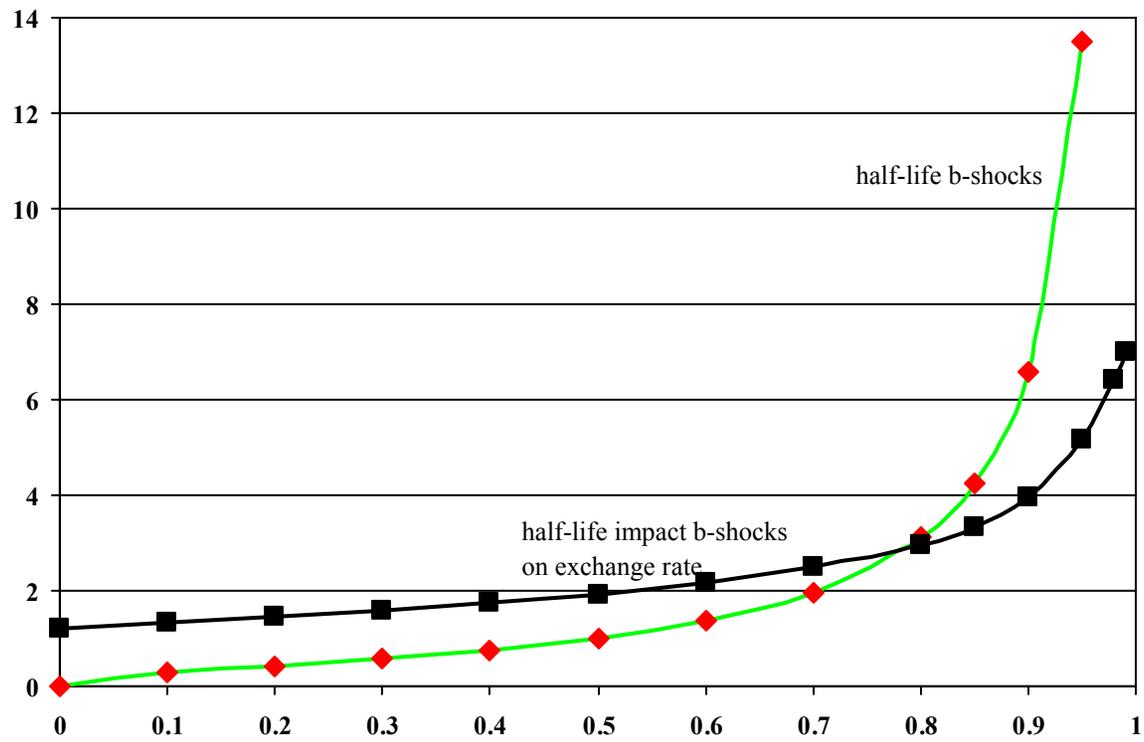
\* Contribution=the percentage of  $\text{var}(s_{t+1}-s_t)$  contributed by non-fundamental shocks (b-shocks). It is measured in percentage terms on the right scale. Magnification= factor by which information dispersion amplifies the exchange rate response to b-shocks (measured on the left scale). Half life=number of periods after which the impact of a B-shock on the exchange rate is reduced by half relative to the initial impact (measured on the left scale).

Figure 3 Sensitivity Analysis-continued\*



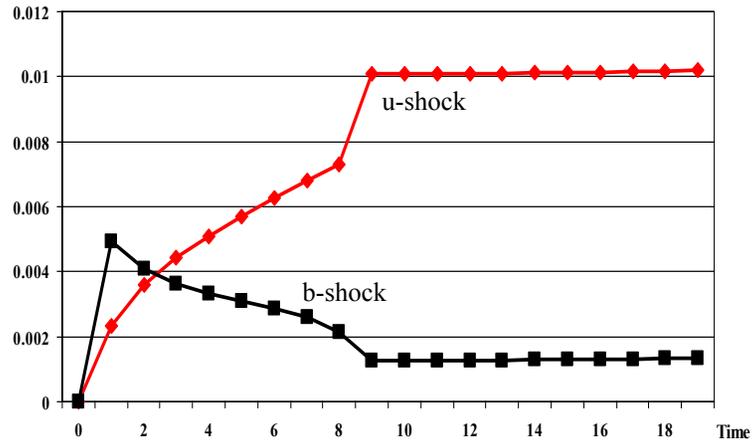
\* Contribution=the percentage of  $\text{var}(s_{t+1}-s_t)$  contributed by non-fundamental shocks (b-shocks). It is measured in percentage terms on the right scale. Magnification= factor by which information dispersion amplifies the exchange rate response to b-shocks (measured on the left scale). Half life=number of periods after which the impact of a B-shock on the exchange rate is reduced by half relative to the initial impact (measured on the left scale).

Figure 4 Half-Life Non-Fundamentals Shocks

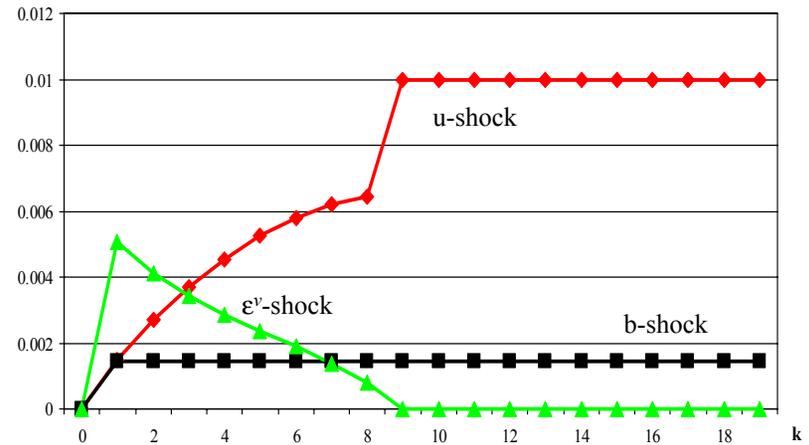


# Figure 5 Results with persistent non-fundamentals shocks\*

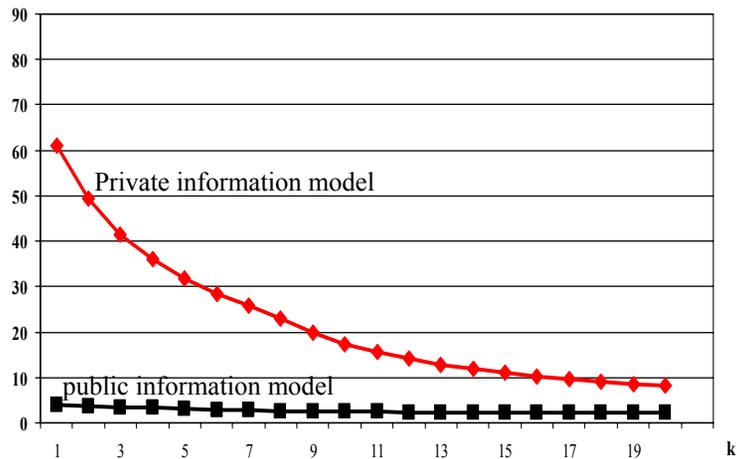
Panel A Impulse Response Functions in Private Information Model



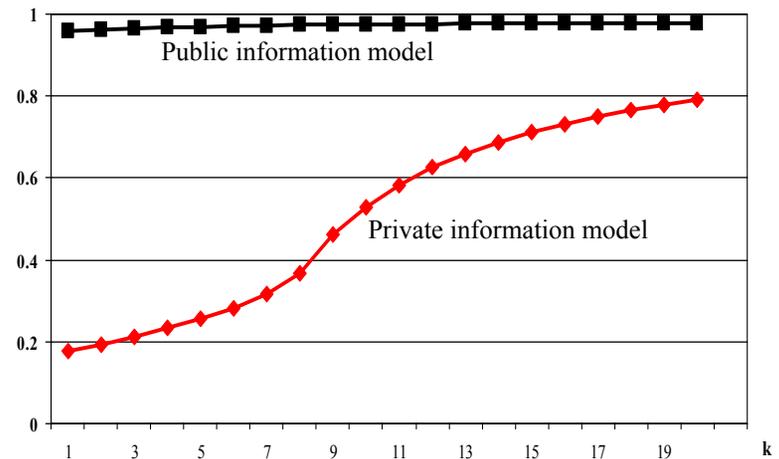
Panel B Impulse Response Functions in Public Information Model



Panel C Percent contribution b-shocks to  $\text{var}(s_{t+k}-s_t)$



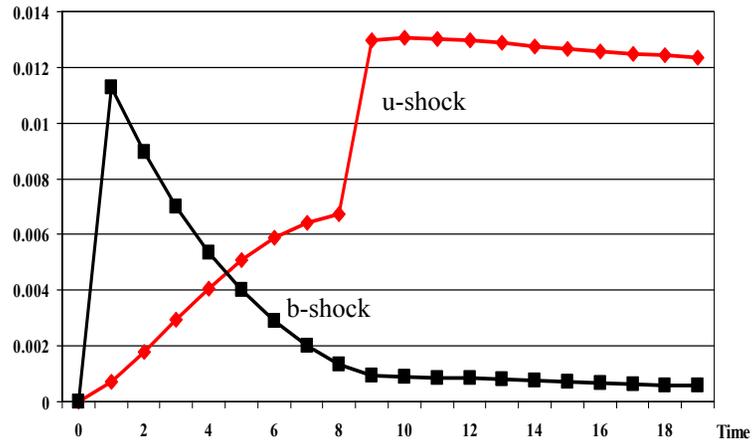
Panel D  $R^2$  of regression of  $s_{t+k}-s_t$  on current and lagged fundamentals



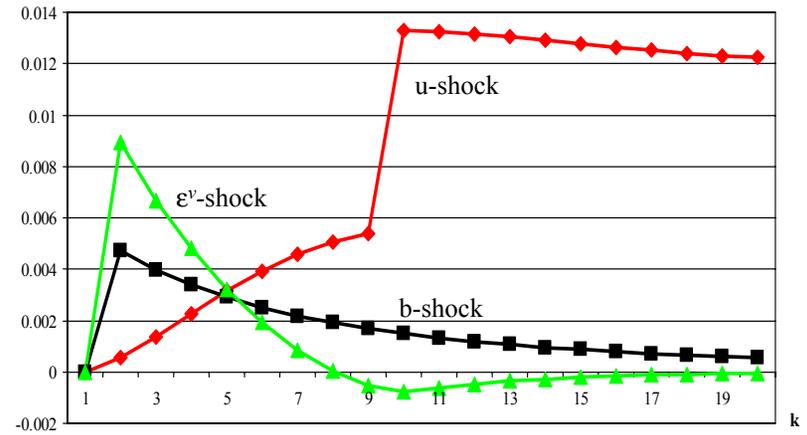
\* The parameters are the same as in the benchmark parameterization, except that now  $\rho_b=1$  and  $\sigma_b=0.006$ .

# Figure 6 Results for Alternative Parameterization\*

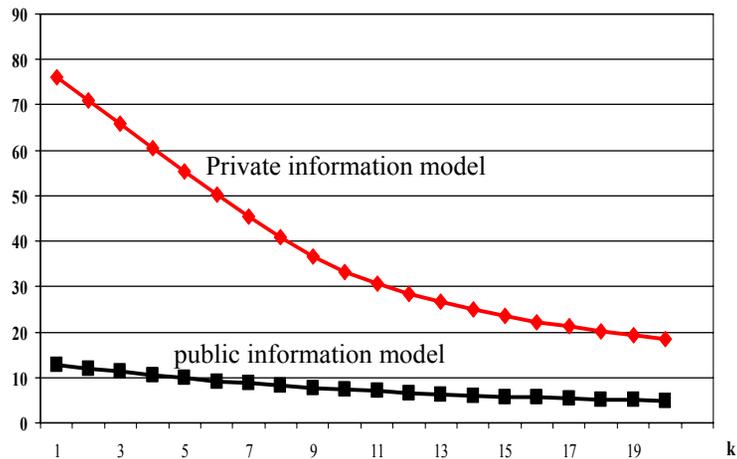
Panel A Impulse Response Functions in Private Information Model



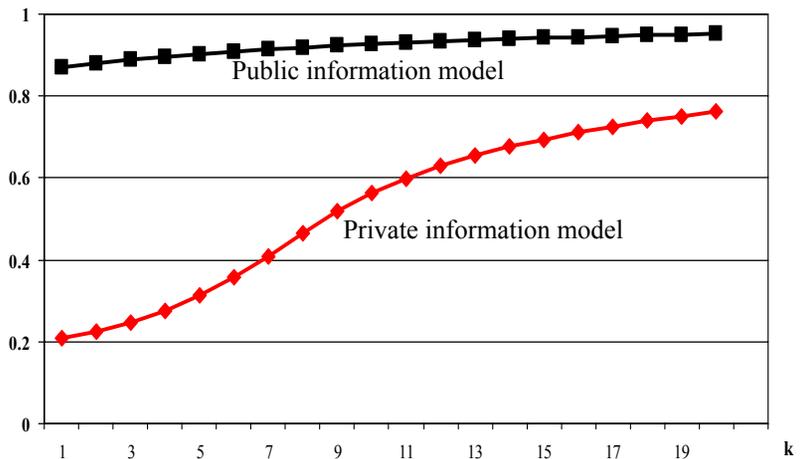
Panel B Impulse Response Functions in Public Information Model



Panel C Percent contribution b-shocks to  $\text{var}(s_{t+k}-s_t)$



Panel D  $R^2$  of regression of  $s_{t+k}-s_t$  on current and lagged fundamentals



\* See Table 1.