# Eliminating Self-Fulfilling Liquidity Crises Through Fundamentals–Revealing Securities

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**Abstract**

We propose a mechanism that could realistically be used to avoid self-fulfilling liquidity crises. It rests on the general idea of "fundamentals revealing" securities. Those securities give a market based assessment of variables such as "future solvency of the country if it receives in the near future a bail-out at reasonable rates". Hence they are likely to be more informative and robust to misspecification that contingent rates based on macro economic variables. We discuss variants and extensions. In all cases self-fulfilling crises are eliminated by the mechanism.

## 1 Position of the problem

We propose a mechanism to avoid self-fulfilling financial crises. For example, there is a self-fulfilling financial crisis when nobody wants to lend to Indonesia because lenders believe that Indonesia will go bankrupt; Indonesia indeed becomes bankrupt. We have a self-fulfilling prophecy.

The traditional solution is a lender of last resort, who will lend at some predetermined rate $r$. The IMF is the embodiment of this idea.

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But the problem is that sometimes the country should not be lent too: the country is indeed running too high deficits, for instance.

So we want a market mechanism that (i) avoids multiple equilibria and (ii) lends at fair market rates, i.e. provides the discipline of high interest rates to countries in fiscal imbalance (e.g., to avoid moral hazard, where the countries would run too high deficits, as it knows it can borrow at rates below the fair rate). We present one below.

## 2 The setup

We will assume rational expectations throughout the paper. For simplicity,

we will take the world interest rate to be $r^* = 0$.

The timing is the following.

$t = 0$. Contracting

$t = 1$. The fundamentals $f$ are determined. If the country's interest rate in the next period is $R$, then the probability of default at $t = 5$ will be $1-F(R, f)$. $f$ can reflect many things like the fiscal stand of the country and the probability of a populist default on the debt. $f$ is observed by market participants. But one cannot directly contract upon it, because there is no security whose value is directly tied to $f$ (our mechanism precisely creates such a security).

$t = 2$. Capital markets decide whether or not to refinance an amount $Q \in \{0, Q^*\}$ to the country. This can be due to multiple equilibria.

$t = 3$. Our mechanism, if it has been put in place.

$t = 4$. Period of normal time, without liquidity needs.

$t = 5$. End. The country can repay a stochastic amount $W(R, f)$. In the example below, we shall take a repayment equal to: (i) nothing (full default), with probability $1-F(R, f)$; and (ii) with probability $F(R, f)$, all its debts if it got interim financing, and nothing otherwise.

We illustrate the idea by a simple example. Say that there are $N$ lenders, each with a capacity[1] 1. The payoffs are:

| | Loan | No Loan |
|---|---|---|
| Lender $i$ | $R_i F(R, f) - 1$ | 0 |
| Country | $U(R)$ | $U(\emptyset)$ |

[1] Equivalently there could be a continuum of atomistic lenders, with a total mass $N$.

$R$ is the average rate at which the country gets its loan. Bank $i$ bids a loan at rate $R_i$. The country needs a loan of size[2] $Q^* \geq 2$. Thus we will have

$$R = \text{mean value of the lowest } Q^* \text{ values of } \{R_1, ..., R_N\}$$

Mathematically $R = \sum_{i=1}^{Q^*} R_{(i)}/Q^*$ where we order $R_{(1)} \leq ... \leq R_{(N)}$.

The utility $U(R)$ of the country it decreasing in $R$. This reflects the direct cash benefits and the potential macroeconomic benefits that come with low interest rates.

There are multiple equilibria.

**Proposition 1** *Call $R^{**}$ the lowest value of $R$ such that $RF(R) - 1 = 0$, assuming it exists. Then there are at least two equilibria: a "loan" equilibrium where all lenders offer a loan at rate $R^{**}$, and an "no loan" equilibrium where no lender offers any loan.*

**Proof.** Each lender has a capacity of 1, and the country needs $Q^* > 1$, so that no lender is pivotal. In the "lending" situation, no country has an incentive to deviate, as it is making 0 profit whether it lends at $R^{**}$ or not. In the no-lending situation, the same is true. ■

### 2.1 Solving the problem via Fundamentals–Revealing Securities

One way to solve the multiple equilibrium problem is to use a Syndicate[3] of banks of size at least $Q^*$. Then, at $t = 2$, the Syndicate at least breaks even if it lends at a rate $R$ such that $V(R, f) \geq 1$ with:

$$V(R, f) := RF(R, f) \tag{1}$$

The competitive first best is:

$$R^o = \min_R \{R \text{ s.t. } V(R, f) \geq 1\}. \tag{2}$$

If the Syndicate has bargaining power, it will ask for a typically higher rate, such as the "hold-up rate"

$$R^H = \arg\max_R \{V(R, f) \text{ s.t. } U(R) \geq U(\emptyset)\}.$$

[2] For simplicity, $Q^*$ is an integer.

[3] Here we consider the case with just one syndicate. A mechanism where several syndicates compete would be of interest.

So we want to avoid two moral hazard problems: (1) Moral hazard (hold-up) by the Syndicate: In a situation of bargaining power the Syndicate would like to ask for $R > R^o$. (2) Moral hazard by the Country: For instance, if the Country was guarantied a credit line at unconditional rate $R$, it would have a high incentive to choose very low fundamentals $f$ at $t = 1$.

To avoid this, we would like to contract on $f$, or on the function $R \mapsto F(R, f)$. It is not directly contractible, but the FR securities allow us to make $F(R, f)$ contractible. An example of mechanism using FR securities is the following. We will present a more realistic mechanism in the next section.

At $t = 2$, the Country announces a desired rate $R$. Two securities are traded:

- "Denominator Security $D$": "pay \$1 if there is a loan at rate $R$, and 0 otherwise". If the probability of the loan at $t = 3$ is $p$, the value of that security should be

$$V_D = p.$$

- "Numerator Security $N$" : "pay the value of the outstanding debt (per notional dollar outstanding), if there is a loan at rate $R$, and 0 otherwise". Its fair value at $t = 3$ is:

$$V_N = p \cdot F(R, f) R.$$

Once securities $D$ and $N$ are traded, if $p > 0$, the trick is that we can infer the value of the fundamental $F(R, f)$ by considering:

$$\widehat{V} := \frac{V_N}{V_D} \tag{3}$$

Indeed, in equilibrium we have:

$$\widehat{V} = V(R, f)$$

where $V(R, f)$, defined in (1), is the fundamental value of the security if there is no crisis.

Thus the mechanism will work if it stipulates "The Syndicate lends at the rate $R$ announced by the country iff $\widehat{V} \geq 1$".

Suppose indeed that somehow, the probability $p$ of a loan at rate $R$ is guaranteed to be positive (we will discuss ways to do this below). Then the ratio $\widehat{V}$ is always well-defined, and the market indicates that rate $R$ does not spoliate the investors, i.e. indicates that $V(R, f) \geq 1$. The market indicates that rate $R$ offers non-negative profits to the Syndicate. A rational Country will choose the lowest possible such $R$, i.e. $R = R^o$. We achieve the first best.

Conceptually, FR are thus very simple. We want to complete the market, to be able to price "value of the fundamentals in case a loan is made at rate $R$", i.e. $V(R, f)$. The mechanism above plays this role. We need to ensure, though, the probability that a loan will be made is non-zero, i.e. $p > 0$, so that we can take the ratio (3). This is the reason for the lottery or the low probability IMF intervention in the mechanism described below.

# 3 The details of the proposed mechanism

We want to elicit the first best interest rate $R^o$, characterized in (2). The problem is that $f$ is non-contractible and the function $R \mapsto F(R, f)$ are not largely unknown[4].

We assume (realistically) that prior to the game the country has an outstanding debt with rate $r$.

At $t = 0$, the Country signs with a Syndicate of banks "market rate based" credit line. It is normally not used (indeed, on the equilibrium path it is never used), except when there is a liquidity crisis (which just happens off the equilibrium path if A3 doesn't apply, and otherwise just happens in period of irrational sudden stop). The Country will pay a fee for the use of this mechanism.

At $t = 3.0$, the Country calls (if it wants) the credit line. Then the following happens:

**Step 1** (at $t = 3.1$). The Country announces a desired interest rate $R$.

- From the existing debt, two new securities are made. The "denominator security," or " contingent interest only" security (IO), gives $r$ for sure if there a loan at rate $R$, and 0 otherwise. To make the IO riskless if there is a loan, the required money from the loan is put in escrow. The "numerator security," or "contingent principal only" gives, if there is a loan at rate $R$, whatever the Country can pay on the principal at $t + 1$ and gives nothing if there is no loan at rate $R$.

[4] A very different way to elicit $F(R, f)$ would be to use macroeconomic variables such as growth and budget deficit and a pre-specified formula that would give a forecast of $F$. This presents several difficulties. This offers high incentives to the government to manipulate the reporting of those macroeconomic variables. Also, the structural relationship between macroeconomic variables and $F$ might be importantly misspecified. In a related way, it will tend to omit important, hard to measure, variables, such as the degree of populism of the country.

**Step 2** (at $t = 3.2$). Those securities are traded (this could last for a – fairly dramatic – week). The prices of the securities $V^D$ and $V^N$ are observed at the end of the week. It is worth at this stage calculating the equilibrium value of those securities, as this will motivate step 4 of our mechanism.

Suppose that there will be a loan with probability $p$. Then the fair value of the IO is:

$$V^D = V^{IO} = pr \tag{4}$$

The fair value of the PO is:

$$V^N = V^{PO} = p\left(F\left(R, f\right)R - r\right) \tag{5}$$

as $1 - F\left(R, f\right)$ is the probability of default conditioning on a loan at rate $R$ and the other fundamentals $f$ of the country (e.g. the degree of populism in the country).

The trick, as in section 1, is that we can infer the $F\left(R, f\right)$ by taking the ratio of the contingent $IO$'s and $PO'$s:

$$\widehat{V} := r\left(\frac{V^{PO}}{V^{IO}} + 1\right) \tag{6}$$

Indeed in equilibrium we have

$$\widehat{V} = V\left(R, f\right). \tag{7}$$

$V\left(R, f\right)$ is the fundamental value of the loan contingent on a bailout. $\widehat{V} = V\left(R, f\right)$ means that, by doing the ratio (6), we can infer a measure of $V$ before the bailout itself. This motivates our next step for the mechanism:

**Step 3** (at $t = 3.3$, after the week of trading determining the $\widehat{V}$). If $\widehat{V} \geq 1$, then the Syndicate gives the loan to the country, and the mechanism stops. Otherwise, there is a next step, which has two possible versions:

*IMF-based version.* The IMF may choose to step in, at its discretion, and makes a loan at the rate $R$ (we will discuss later proposals for the "responsible" positions of the IMF[5]). If the IMF doesn't chose to step in, there is no loan.

*Lottery-based version.* We run a lottery with 2 outcomes. With probability $q$ the Syndicate has to make a loan at the rate $R$. Otherwise, the Syndicate is relieved from its obligations.

Step 3 and the mechanism end here.

[5] Basically the only requirement is that the IMF steps in a refinances at rate $R_{i^{**}}$ with a positive probability. This probability can be very low – it just has to be positive, so that $p_{i^{**}} > 0$.

We comment here in the desirable behavior of the IMF. The IMF should make the loan if $V \geq 1$. We need either the IMF or lotteries to make sure all probabilities, or at least the probability of $R$, is positive if it is sustainable. We can insure this via the lottery above, or via the following:

**Definition 2** *We shall say that the IMF behaves in an appropriate way if it makes a loan at rate $R$ with positive probability.*

This requirement is very weak. The IMF could thus declare that it will step in only $q \leq 5\%$ of the cases, and this would be enough for our purposes.

## 3.1 Analysis of the equilibrium

The profits of the Syndicate are, with a loan of size $Q$

$$\Pi = Q\left(V\left(R, f\right) - 1\right)$$

The country's utility function is:

$$U = U\left(R, f, \theta\right) \tag{8}$$

where $R$ is the interest rate at which the Country gets the loan (with $R = \emptyset$ if there is no loan), $f$=fundamentals. $\theta$ is a "taste shock" (e.g., the country turns populist) that motivates the choice of $f$.

Here we take rather special functional forms, for the clarity of the exposition. In section 5, we prove a more general results under more general assumptions.

Both the Lottery-based and IMF based are particular cases of the following General Lottery.

**Definition 3** *We say that a lottery is an admissible lottery if it chooses $R$ with positive probability.*

For instance, the IMF behaving in an appropriate way and the special lottery in Step 3 are both admissible lotteries.

**Proposition 4** *Suppose that the Country calls the mechanism and proposes an admissible $R$, i.e. $R$ s.t. $V\left(R\right) := RF\left(R, f\right) \geq 1$. Then there is a unique equilibrium where the country gets the loan at rate $R$. We also have, in the equilibrium*

$$p = 1 \tag{9}$$

$$V^{IO} = r \tag{10}$$

$$V^{PO} = F\left(R, f\right) R - r \tag{11}$$

*so that with (6), we have*

$$\widehat{V} = V(R, f) \tag{12}$$

*with* $V(R, f) := F(R, f)\, R$.

**Proof.** Call the event $\Omega = \{\text{there is a loan at rate } R\}$. Because of the lottery, $p = \Pr(\Omega)$ is strictly positive. The fair values of the IO and PO are (4) and (5). So we have $\widehat{V} = V$ with (6) and (**??**). As by assumption of sustainability, we have $V \geq 1$, we have $\widehat{V} \geq 1$. So at equilibrium, the loan is realized, and $p = 1$. ■

We have immediately:

**Corollary 5** *Even if the market didn't refinance the Country, the Country can still achieve the first best interest* $R^o$ *by using the mechanism.*

**Proof.** Indeed, given (2), we have $R^o F(R^o, f) \geq 1$, which is realized by the previous Proposition. ■

Also:

**Proposition 6** *Suppose that the Country offers to the market a rate such that* $RF(R, f) > 1$. *Then there is a unique equilibrium, where the market accepts this rate, and the mechanism is never used. By offering rates close to* $R^o$*, the Country can thus get arbitrarily close to the First Best, in a unique equilibrium. The mechanism is never used this equilibrium.*

**Proof.** Follows immediately from the Propositions and Corollary above. ■

## 4 Comments

We think that the proposed mechanism could realistically be implemented. Having this in mind, we here provide some comments on its design and its robustness.

**Market manipulation** A key issue is market manipulability. There is an incentive for Country and the Syndicate to manipulate the prices of the FR securities. This is why we propose creating them from a large, existing market (the existing Country debt). New securities might be traded thinly, and hence much easier to manipulate. A trick here is that in equilibrium, we will have $p = 1$, so that the market for the important FR security is very large – the size of the Country bond market.

Indeed, in equilibrium, $V^{IO} = r$. Given that the IO is a sure asset, it is very easy to arbitrage and its price will be at the fundamentals. The value of PO is $V^{PO} = F(R, f)(R - r)$, or essentially the value of the debt outstanding (minus the interest part). This is a very large market, given the size of the Country. For this reason, it is also very difficult to manipulate [6].

Hence our mechanism acquires some sort of automatic robustness this way. This is why we based the FRS on the existing debt market, rather than new securities as in the simple example in Section 1. It is likely that an entirely new market would be traded too thinly and would be too easy to manipulate.

**IO and POs** We use IOs and POs for the reason above, and to save a bit on transaction costs. It is clear that the FRS idea is very general, and that many similar mechanisms could be used. In general there will be the need for a denominator and one or more numerator securities.

**IMF** The IMF would step in very rarely here. Indeed, in equilibrium, multiple equilibria are eliminated and the IMF never intervenes. But even if there is a Sudden Stop, hence the mechanism is put in motion, the IMF just has to promise to intervene with some positive probability (e.g. 5%). Hence the IMF intervenes drastically less often that it would without our mechanism[7].

**Robustness** The mechanism doesn't use assumptions about specific functional forms of the utility function etc, as is common in the implementation literature. In that sense it is very robust to misspecification of "tastes."

In the mechanism above, the Country proposes just one interest rate $R$. Given that the state or beliefs of the market can be hard to assess, it would

[6] A model with limited arbitrage would deliver those results. It is outside the scope of this paper, though.

[7] A word on lotteries. Lottery are distasteful. This is why we propose rather the IMF-based version of the mechanisms. But some lottery (the IMF is just a disguised way to have a lottery) is useful and maybe necessary here [or impossibility theorem?], because they force the important states of the world to have positive probability. The mechanism might still be emotionally acceptable with them, it scarcely involves any drama. As $q$ is small, the outcome of the lottery is fairly known in advance. Another defense is that, without sudden stops, in equilibrium our mechanism (in particular, its lotteries) is never used. Also, one might be without lotteries if one think that there is enough diversity of opinion to ensure that every state of the world is priced.

be safer to offer a menu of interest rates. This is what the mechanism in Appendix A does. It is a bit more complex, but the conclusions are not changed.

**Loose ends to take care of in next iteration** Making sure that the country doesn't want to play the lottery anyway (easy, with a penalty of using the lottery, and a small probability that the lottery chooses the "Lending" outcome.

# 5 Extension to multiperiod setting

There are $T+1$ periods. The country chooses fundamentals $f_1, ..., f_T$. The world ends at $T+1$.

We assume that there is a "good" equilibrium with always "refinancing". We want to select that equilibrium.

Mechanically, we know that a mechanism via Fundamentals Revealing Securities will work: the event to select is just (loosely speaking) $\Omega$="low rates at $t = 1,...,T$", and we would make Numerator and Denominator securities to reveal the fundamentals in that state of the world. In practice, that is complicated, as it requires the Syndicate to make $T-$period loans — something that most institutions would be reluctant to do for a variety of reasons.

It turns out, however, that a much simpler mechanism will ensure the selection of the good equilibrium. All we need is a juxtaposition of one-shot mechanisms. It works the following way. For each period $t$.

- At $t$, the country chooses $f_t$. $f_t$ is observable by all participants.

- Then the market decides to refinance, as some rate $R_t^1$.

- If the country chooses so, it can call the 1-period mechanism, with a rate $R_t$ it chooses. The 1-period mechanism is then run.

- The outcome is then either refinancing or no refinancing. If there is no refinancing, the game ends. If there is refinancing, we move to period $t+1$.

The nice property of this is that *the multiple equilibria are eliminated*, and only the "good" equilibrium is selected. The proof is simple and works by backward induction. Again, in equilibrium the mechanism is never called.

We will actually state the proof under a fairly general structure. This way we will provide more general results than the Propositions above. We need some notations. The first best rate realizes minimum interest rates subject to:

$$E_t \left[ M_{t+1} W_{t+1} \left( R_t, \mu_{R_{>t}} \right) \right] = 1. \tag{13}$$

where $\mu_{R>t}$ represents the distribution of interest rates at date later than $t$, and $W_{t+1}\left(R_t, \mu_{R>t}\right)$ is a stochastic variable representing the amount repayment to the creditors. Call $D_{t+1}$ =the event of a default at $t+1$, an example of $W_{t+1}$ is:

$$W_{t+1}\left(R_t, \mu_{R>t}\right) = 1_{D_{t+1}} R_{\text{default}} + \left(1 - 1_{D_{t+1}}\right) R_t.$$

But we won't assume any functional form here.

**Proposition 7** *In the mechanism above, in equilibrium, there is always refinancing. There are no multiple equilibria.*

**Proof.** We proceed by backward induction, proving

*Lemma$_t$*: If the country will get refinanced until $t-1$, then the country is refinanced at time $t$.

To prove this lemma, observe first that the Lemma$_t$ is true for $t = T$. Indeed, we apply the analysis of the 1-step mechanism, proving that there is just one equilibrium, where the country is refinanced.

Now suppose that the Lemma is true for $t+1$. We show it is true for $t$. The event is

$$\Omega_t = \left\{\text{there is a loan at rate } R_t^a\right\}.$$

The numerator and denominator securities give 0 if $\Omega_t$ isn't realized, and otherwise give respectively the $W_{t+1}\left(R_t, \mu_{R>t}\right)$, and 1. So the fair values are:

$$\begin{aligned} V_{N,t} &= E\left[1_{\Omega_t} M_{t+1} W_{t+1}\left(R_t, \mu_{R>t}\right)\right] \\ V_{D,t} &= E\left[1_{\Omega_t} M_{t+1}\right] \end{aligned}$$

We define:

$$\widehat{V}_t = \frac{V_{N,t}}{V_{D,t}}$$

First observe that if the loan is realized, then $1_{\Omega_t} = 1$ a.s., and we have:

$$\widehat{V}_t = E\left[M_{t+1} W_{t+1}\left(R_t, \mu_{R>t}\right) \mid \Omega_t\right] = V_t$$

As by assumption of sustainability, we have $V_t \geq 1$, we have $\widehat{V}_t \geq 1$.

Suppose that at the equilibrium $\widehat{V}_t < 1$. The loan is not automatically realized. The lottery selects it with a probability $\varepsilon > 0$. So we have:

$$V_{N,t} = E\left[1_{\Omega_t} M_{t+1} W_{t+1}\left(R_t, \mu_{R>t}\right)\right]$$

$$V_{D,t} = p\left(\Omega_t\right) = \varepsilon > 0$$

and $\widehat{V}_t = V_t \geq 1$. So we cannot have $\widehat{V}_t < 1$ in the equilibrium.

So we have $\widehat{V}_t \geq 1$, and the loan is made for sure. Lemma$_t$ is proven. By backward induction, Lemma$_1$ is true, and the Proposition is proven. ■

So: *Putting 1-step ahead mechanisms in place is enough to ensure that no self-fulfilling financial crisis will happen.*

[A worry: This relies a lot on backward induction and common knowledge of rationality. This may be a problem ($p-$beauty contests, centipede game)].

# 6 Conclusion

Recent proposals advocate changing the structure of the underlying economies (e.g. by adjusting the balance sheet or the amount of pledgeable capital) to cure self-fulfilling prophecies. Here we point in a different: if the problems are of multiple equilibria, information-revealing mechanisms can be enough to force coordination on the "right" equilibrium. The idea behind FRS is simple. We complete the market to reveal information of the type "in state of the world $s$, then the value of variable $X$ will be $x$". Some features emerging from the present analysis are likely to be robust: need for lotteries (maybe disguised as the IMF), to ensure that the important states of the world have positive probability, hence are priced. This paper illustrates the idea, and proposes a mechanism that could realistically be implemented.

Perhaps surprisingly, a series of one-time commitments are enough to ensure that in a multi-period setting no self-fulfilling financial crisis happens. This makes the mechanism easier to use and accept, as it requires only short-term loans by the Syndicate of banks. Moreover, in equilibrium, this mechanism is never used.

Conceptually, this proposal is quite straightforward. Its workings are also fairly simple. We think that the current proposal could be realistically implemented, and avoid violent and gratuitous economic pains caused by self-fulfilling liquidity crises.

# 7 Appendix A: A richer mechanism, where the Country proposes a menu of interest rates

## 7.1 Mechanism (to revise a bit)

**Step 1** (at $t = 3.1$). The contract specified a limited number of interest rates $\overline{m}$, and a lower bound on the interest rates $\underline{R}$ (e.g. $\underline{R} = 1$).

- The Country decides on a vector $C = (R_1, ..., R_m)$ of candidate interest rates[8] at which it will be lent by the Syndicate, with $m \leq \overline{m}$ and $R_i \geq \underline{R}$. We also add a symbolic rate $R_0 = \emptyset$, which means that their is no bailout by the Syndicate.

- The existing debt, assumed to mature at $t = 4$, is stripped into Principal Only (PO), and Interest Only (IO) parts[9]. The PO pays the principal, the IO the interest rate. If the bailout works, the money for the interest rates will be put in escrow in a foreign bank.

- From the IO and PO, $m$ securities are made. If the original security $S$ pays $x_S$ at $t = 4$, then we make $m + 1$ new securities $S_0, ..., S_m$. Security $i$ pays $x_S$ in state $i$ and nothing otherwise. As the events $i = 0...m$ span the states of the world, the sum of the payoffs of the $S_i$ is equal to the payoff of $S$. As both the IO and PO are stripped into $m+1$ securities, there are now $2m + 2$ new securities trading: $IO_i, PO_i$ for $i = 0...m$. The sum of those securities is the original bond.

**Step 2** (at $t = 3.2$). Those securities are traded (this could last for a – fairly dramatic – week). The prices of the securities $V_i^{IO}$ and $V_i^{PO}$ are observed at the end of the week. At this stage it is worth calculating the equilibrium value of those securities, as this will motivate step 4 of our mechanism.

Suppose that event $i$ has probability $p_i$. Then the fair value of the IO is:

$$V_i^{IO} = p_i r \tag{14}$$

Indeed the IO is riskless, conditional on $i$ being realized, as the money has been put in escrow (that's why we do not have $V_i^{IO} = p_i r F(R, f)$). The fair value of the PO is:

$$V_i^{PO} = p_i F(f, R_i)(R_i - r) \tag{15}$$

[8] Without uncertainty $\overline{m} = 1$ is enough. In practice, to increase the robustness of the mechanism and allow for uncertainty, we could take $\overline{m} = 2$ or 3.

[9] This is inspired by the mortgage-backed securities market, which has a very large (precise...) IO and PO market.

as $F(f, R_i)$ is the probability of default conditioning on a loan at rate $R_i$ and the other fundamentals $f$ of the country (e.g. the degree of populism in the country).

The trick, as in section 1, is that we can infer the $F(f, R_i)$ by taking the ratio of the contingent $IO$'s and $PO'$s:

$$\widehat{V}_i := r\left(\frac{V_i^{PO}}{V_i^{IO}} + 1\right) \tag{16}$$

Indeed in equilibrium we have

$$\widehat{V}_i = F(f, R_i) R_i. \tag{17}$$

This motivates our next step for the mechanism:

**Step 3** (at $t = 3.3$, after the week of trading determining the $\widehat{V}_i$). Define

$$R_{i^*} = \min A^* \text{ if the set } A^* = \left\{R_i \text{ s.t. } \widehat{V}_i \geq 1\right\} \text{ is not empty} \tag{18}$$

$$i^* = 0 \text{ otherwise.} \tag{19}$$

The Country can either choose $R_{i^*}$, or choose to call the next step, which has two possible versions:

*IMF-based version.* The IMF may choose to step in, at its discretion, and makes a loan at one of the rates $R_i$ (we will discuss later proposals for the "responsible" positions of the IMF[10]). If the IMF doesn't chose to step in, the Syndicate makes the loan at rate $R_{i^*}$ if $i^* > 0$, and no loan if $i^* = 0$.

*Lottery-based version.* We run a lottery with $m + 1$ outcomes. With probability $q/m$ one of the rates $i = 1, ..., m$ are drawn, and the Syndicate makes a loan at the rate $R_i$. With probability $1-q$ none of the rate is drawn, and we select the interest rate $R_{i^*}$. If the set of interest rates satisfying (18) is not empty, the Syndicate lends (up to a specified amount of money, for robustness[11]) to the country, at rate $R_{i^*}$. Otherwise, the Syndicate is relieved from its obligations.

Step 3, and the mechanism, end here.

We comment here in the desirable behavior of the IMF. The best realizable rate, given $R_1, ..., R_m$, is

$$R_{i^{**}} = \min A^{**} \text{ if the set } A^{**} = \{R_i \text{ s.t. } R_i F(R_i, f) \geq 1\} \text{ is not empty}$$

$$R_{i^{**}} = \emptyset \text{ otherwise}$$

[10]Basically the only requirement is that the IMF steps in a refinances at rate $R_{i^{**}}$ with a positive probability. This probability can be very low – it just has to be positive, so that $p_{i^{**}} > 0$.

[11]Alternatively, the country could propose menus of $(R_i, Q_i)$, where $Q_i$ is the amount it is lend to if rate $i$ is chosen. That would be theoretically more desirable. Practically, this has the drawback of making things more complicated.

We need either the IMF or lotteries, so as to make sure that all probabilities, or at least the probability of the best rate $R_{i^{**}}$, is positive. We can insure this via the lottery above, or via the following:

**Definition 8** *We shall say that the IMF behaves in an appropriate way if it makes a loan at rate $R_{i^{**}}$ with positive probability. But it steps in with probability $\leq q$, for some specified $q$.*

This requirement is very weak. The IMF could thus declare that it will step in only $q \leq 5\%$ of the cases, and this would be enough for our purposes.

### 7.2 Analysis of the equilibrium

With a loan of size $Q$ the profits of the Syndicate are:

$$\Pi = Q\left(R_i F\left(R_i, f\right) - 1\right)$$

The country's utility function is:

$$U = U\left(R, f, \varepsilon\right) \tag{20}$$

where $R$ is the interest rate at which the country gets the loan (with $R = \emptyset$ if there is no loan), $f$=fundamentals. $\varepsilon$ is a "taste shock" (e.g., the country turns populist) that motivates the choice of $f$.

Both the Lottery-based and IMF based are particular cases of the following General Lottery.

**Definition 9** *We say that a lottery is an admissible lottery if it is activated with a non-zero probability, choose event $i^{**}$ with a positive probability, and the probability that the lottery it chooses one of the events $i \neq i^{**}$is $\leq q$.*

For instance, the IMF behaving in an appropriate way, and the special lottery in Step 3 are both admissible lotteries.

**Proposition 10** *Given the set of interest rates $S$ chosen by the country, there is just one equilibrium set of prices. We have $i^* = i^{**}$, and the probability of event $i^{**}$ satisfies*

$$p_{i^{**}} \geq 1 - q$$

*We also have:*

$$V_i^{IO} = p_i r \tag{21}$$

$$V_i^{PO} = p_i F\left(f, R_i\right) R_i \tag{22}$$

*for all i's, so that, for those i such that $p_i > 0$, defining*

$$\widehat{V}_i := \frac{rV_i^{PO}}{V_i^{IO}}$$

*we have*

$$\widehat{V}_i = R_i F(f, R_i)$$

**Proof.** As the lottery is admissible, we have $p_{i^{**}} > 0$. In the equilibrium, we have thus automatically $i^* = i^{**}$. $i^{**}$ is chosen for sure when the lottery is not activated, which happens with probability $\geq 1 - q$. Thus we have $p_{i^{**}} \geq 1 - q$. ■

**Proposition 11** *As $q \to 0$, we approach full efficiency.*

**Proof.** Trivial given the Proposition above. Write down the utility functions (do it) ■

Finally, we observe the following:

**Proposition 12** *Assume that the function $R \mapsto RF(R, f)$ is locally increasing at $R = R^o$. If at time $t = 2$ the Country issues debt at rate $R^o + \varepsilon$, for some $\varepsilon > 0$, it will be supplied the liquidity for sure. The mechanism is never used in equilibrium.*

**Proof.** Indeed, because of the mechanism, it is sure that the country will get refinancing at $t = 3$. So an investor who lends at rate $R = R^o + \varepsilon$ will make a positive profit, as $RF(R, f) > R^o F(R^o, f) = 1$. It will make zero profit if he delays, as the Syndicate makes zero profit and investors, who are not members of the Syndicate, don't lend at $t = 3$. So the investor prefers to lend at $t = 2$. As all investors do this, there is no liquidity crisis. ■

# 8 Appendix B: A simple mechanism for the 1-period settings

The mechanisms in this Appendix are less general than the ones above, but they do the job in the case of 1-period liquidity crises. There is no need for a Syndicate. They have some information drawbacks mentioned at the end. The concept of Fundamentals Revealing Securities is more general [ study that more in details].

**Definition 13** *(Contingent lending fixed rate contract). At* $t = 3$*. the country announces a rate* $R$ *and issues an instrument whereby a potential lenders says: "if at least* $Q^*$ *lenders subscribe to that instrument, I will lend to the Country a quantity 1, at a rate* $R$*." The country pays a small fee* $\varepsilon/N > 0$ *to those willing to buy the contract.*

**Proposition 14** *With the "contingent lending contract above," there is just one equilibrium outcome if* $RF(R, f) \geq 1$*. Lenders lend to the Country a quantity 1.*

**Proof.** Suppose that the outcome is "no lending". Then a potential lender should accept the contract, because then he receives $\varepsilon > 0$ without any obligation. But then "no lending" is not the outcome, as $N \geq Q^*$ lenders are subscribing to the instrument. In the "lending" situation, any investor is better off accepting the contract, as he gets a profit $\varepsilon/N + q(RF(R, f) - 1) > 0$. So it is a strictly dominating strategy for the investors to accept the instrument. ■

The best action of a rational Country is obviously:

**Proposition 15** *The best action of the Country is to propose the rate* $R^o$*. Hence the First Best is achieved, within* $\varepsilon$*.*

**Proof.** The Country can realize any rate $R$ s.t. $RF(R, f) \geq 1$. So it should choose $R^o$ defined in (2). ■

This scheme is very simple, and general in those cases of Multiple equilibria.

Problems: Needs participants to react quickly (Syndicates "mobilizes" participants) [pretty weak argument].

Better scheme for information aggregation.

**Definition 16** *(Contingent lending variable rate contract). At* $t = 3.0$*, the country issues an instrument, whereby a potential lender* $j$ *says "I announce the rate* $R_j$*. The Country will order the rates* $R_{(1)} \leq ... \leq R_{(N)}$*, and accept a loan at rate* $R_{(Q^*)}$ *from those who have agreed to a rate* $R_{(j)} \leq R_{(Q^*)}$*. The subscriber to* $R_{(j)}$ *will receive, in addition* $\varepsilon(N + 1 - j)/N^2$*, for some* $\varepsilon > 0$ *(with averaging in case of ties).*

**Proposition 17** *With the "contingent lending contract above," there is just one equilibrium outcome at* $R^o$*. The Country can refinance its debt. The First Best is achieved, within* $\varepsilon$*.*

**Proof.** Because of the $\varepsilon$ premium, lenders offer all a rate as low as possible. The lowest rate compatible with equilibrium is $R^o$. So all investors lend to the Country at rate $R^o$. The Country just pays a total premium $\leq \varepsilon$. ■

This contract has better "information aggregation" properties (presumably). Still, FRS do a better job. Suppose an extreme separation between brains and money; the people with information (or insight) have very little money, the large lender has no information. In the FRS scheme, the informed agents are going to price the FRS, so that prices reflect their information. The lender will just lend at that revealed rate. In the Contingent Lending Variable R Contract, we need the lenders have to have the information.

Comments.

In cases of Multiple Equilibria, the Coase theorem fails: People don't reach the efficient outcomes. What imperfection causes that?

When liquidity can dry up: Contingent lending contract does not work. One needs a "stable" group of investors who can lend. But then, they have large bargaining power. The mechanism with fundamentals–revealing securities still works then.

# References


[1] Angeletos, George-Marios (2002) "Fiscal Policy with Noncontingent Debt and the Optimal Maturity Structure", *Quarterly Journal of Economics*, 117(3), p.1105-1131