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Financial Innovation, Market Participation and Asset Prices

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Abstract

This paper proposes that the introduction of non-redundant assets can endogenously modify trader participation in financial markets, which can lead to a lower market premium and a higher interest rate. We demonstrate this mechanism in a tractable exchange economy with endogenous participation. Investors receive heterogeneous random incomes determined by a finite number of macroeconomic factors. They can freely borrow and lend, but must pay a fixed entry cost to invest in risky assets. Security prices and the participation structure are jointly determined in equilibrium. The model reconciles a number of features that have characterized financial markets in the past three decades: substantial financial innovation; a sharp increase in investor participation; improved risk management practices; an increase in interest rates; and a reduction in the risk premium.

Keywords: Endogenous Participation, Epstein-Zin Utility, Financial Innovation, Incomplete Markets, Multiple Risk Factors, Risk Premium, Spanning.

JEL Classification: D52, E44, G12.

1. Introduction

This paper proposes that the introduction of non-redundant assets endogenously modifies trader participation in financial markets. These changes can lead to a simultaneous increase in the interest rate and a reduction in the market premium. Our approach builds on two stylized facts. First, participation in financial markets is costly. Corporate hedging requires the employment of experts able to effectively reduce the firm's risk exposure using existing financial assets. Investors have to sustain learning efforts, and expenses related to the opening and maintenance of accounts with an exchange or a brokerage firm. Statutory and government regulations often create costly barriers to the participation of institutional investors in some markets. Second, when some asset markets are initially missing, financial innovation affects risk-sharing and investment opportunities. For instance, options and futures can provide additional insurance against the price risk of commodities and financial assets.[1] Similarly, asset-backed securities allow lending institutions to reduce their risk exposure to debt contracts such as mortgages, credit card receivables and leasing contracts. For this reason, new assets affect individual incentives to participate in financial markets when trading is limited by transaction or learning costs.

We introduce a two-period economy with incomplete markets and endogenous participation. Agents receive heterogeneous random incomes determined by a finite number of macroeconomic risk factors. They can borrow or lend freely, but have to pay a fixed entry cost to invest in risky assets. Security prices and the participation structure are jointly determined in equilibrium. The model reconciles a number of features that have characterized financial markets in the past three decades: substantial financial innovation, the sharp increase in investor participation, improvements in risk-management practices, the slight increase of real interest rates, and the strong reduction in the risk premium.[2] Our model proposes a precise explanation for these phenomena. New instruments encourage more investors to participate in financial markets for hedging and diversification purposes. This tends to reduce the precautionary demand for savings and thus increase the equilibrium interest rate. Under plausible conditions on the cross-sectional distribution of risk, the new entrants reduce the covariance between

[1] See for instance Ross (1976).

[2] The recent decrease in the risk premium is reported in Blanchard (1993), Cochrane (1997), Fama and French (2000), Vuolteenaho (2000), and Campbell and Shiller (2001). Similarly, Barro and Sala-i-Martin (1990) and Honohan (2000) document a slight increase in real interest rates over the past three decades.

stock returns and the mean consumption of participants, leading to a lower market premium.

Participation can also play an important role in spreading the effects of innovation across markets. When a factor becomes tradable, new agents decide to enter financial markets in order to manage their risk exposure. Under complementarities of learning or increasing returns to trading activities, the new participants trade other assets and can modify the risk premia of securities uncorrelated to the factor. Furthermore, financial innovation differentially affects distinct sectors of the economy and thus has a rich impact on the cross-section of expected returns.

Section 2 introduces a simple asset pricing model with endogenous market participation. Section 3 demonstrates the pricing and participation effects of financial innovation in a one factor model of risk exposure. We consider in Section 4 an economy with multiple risk factors. Calibrated examples show that financial innovation can substantially reduce the equity premium, differentially spread across security markets, and either increase or decrease the interest rate. All proofs are given in the Appendices.

1.1. Review of Previous Literature

This paper builds on two strands of the asset pricing literature that have essentially been developed separately. First, researchers have examined how limited investor participation affects the prices of a fixed set of securities. Second, the price impact of financial innovation has been examined both empirically and theoretically without consideration of participation. The novelty of this paper is to combine these two lines of research in a simple and tractable framework. We show that one of the main consequences of financial innovation could be its effect on participation, which could induce a reduction in the risk premium. This potentially provides useful guidance for future empirical research.

Research on limited participation was pioneered by Mankiw and Zeldes (1991), who reported that only 28% of households owned stocks in 1984, and that only 47% of the households holding other liquid assets in excess of $\$100,000$ held any equity. The fraction of households owning stocks increases with income and education, implying that there could be fixed information costs to participate in financial markets. The consumption of stockholders is also more highly correlated with the stock market than aggregate consumption. The distinction between stockholders and non-stockholders therefore helps explain the equity premium puzzle. Vissing-Jørgensen (1997) gives stronger empirical support for this result, and also documents the increase of stockmarket participation in the US since 1945.

These empirical findings have prompted the development of theoretical models that restrict participation exogenously. In particular, Basak and Cuoco (1998) consider a two-asset exchange economy and succeed in matching the historical risk premium with a low coefficient of relative risk aversion. We differ from their model by considering multiple risk factors and assets, and assuming that agents have heterogeneous risk exposures. We also endogenize participation by considering fixed costs to trading in financial markets. The entry-cost approach has been widely used in finance to analyze issues such as portfolio choice (Campbell, Cocco, Gomes and Maenhout, 2001), volatility (Pagano, 1989; Allen and Gale, 1994*b*; Orosel, 1998), futures risk premia (Hirshleifer, 1988), market size (Allen and Gale, 1990; Pagano, 1993), and the effect of social security reform on capital accumulation (Abel, 2001). We use this setup to analyze how financial innovation affects investor participation and asset prices.

The paper is also related to a line of research that examines the price impact of financial innovation without consideration of participation. Conrad (1989) and Detemple and Jorion (1990) find empirically that the introduction of new batches of options had a substantial price impact between 1973 and 1986. The effect is stronger for underlying stocks, but can also be observed for an industry index that excludes the optioned stock as well as for the market index. Similar empirical evidence is available for other countries and derivative markets (e.g. Jochum and Kodres, 1998). These empirical findings have prompted a rich theoretical literature. In the presence of informational asymmetries, the introduction of an option contract has been shown to affect the volatility of the underlying stock (e.g. Stein, 1987; Grossman, 1989). Another line of research focuses on the risk-sharing component of new derivatives when all investors participate in financial markets (Detemple and Selden, 1991; Huang and Wang, 1997).

Although our model can be applied to a variety of settings, the primary focus is the long-term effect of innovation on market participation and the risk premium. Intuition suggests that the price of a diversified portfolio of assets may be more influenced by risk-sharing than by information asymmetries. It is well-known, however, that risk-sharing models with *exogenous* participation have difficulties explaining the dynamics of the risk premium when new assets are introduced. For instance in standard CAPM economies, financial innovation does not affect the relative price of risky assets relative to bonds (Oh, 1996), but increases the interest rate (Elul, 1997; Calvet, 2001; Angeletos and Calvet, 2001). Innovation in these models thus cannot explain the recent decline of the risk premium documented in the literature. We will show that these difficulties can be resolved when participation is endogenized.

2. A Model of Endogenous Market Participation

We examine an exchange economy with two periods ($t = 0, 1$) and a single perishable good. The economy is stochastic, and all random variables are defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. During his life, each agent h receives an exogenous random endowment $e^h = \left(e_0^h, \tilde{e}^h\right)$, which corresponds for instance to a stochastic labor income. Investors have preferences over consumption streams $(c_0^h, \tilde{c}^h)$, which are represented by a utility function $U^h(c_0^h, \tilde{c}^h)$.

This paper places no restriction on the set of agents H, which can be finite or infinite. To provide a uniform treatment, we endow the space H with a measure μ that satisfies $\mu(H) = 1$. This is equivalent to viewing each element of H as a type, and the measure μ as a probability distribution over all possible types.

At date $t = 0$, agents can exchange two types of real securities. First, they can trade a riskless asset costing $\pi_0 = 1/R$ in date $t = 0$ and delivering one unit of the good with certainty at date $t = 1$. Note that R is the gross interest rate. Second, there also exist J risky assets ($j = 1, ...J$) with price π_j and random payoff $\tilde{a}_j$. We assume for simplicity that all assets are in zero net supply.[3] Investors can freely operate in the bond market but have to pay a fixed entry cost κ in order to invest in one or more risky assets. Note that this assumption is consistent with complementarities of learning in trading activities, and the results of the paper easily generalize to more flexible specifications of the entry cost. Investors are price-takers both in their entry and portfolio decisions, and there are no constraints on short sales. Let π denote the vector of *risky* asset prices, and θ^h the vector of *risky* assets bought (or sold) by investor h. We also consider the dummy variable $1_{\{\theta^h \neq 0\}}$ equal to 1 if $\theta^h \neq 0$, and equal to 0 otherwise. The agent is subject to the budget constraints

$$c_0^h + \theta_0^h/R + \pi \cdot \theta^h + \kappa 1_{\{\theta^h \neq 0\}} = e_0^h,$$
$$\tilde{c}^h = \tilde{e}^h + \theta_0^h + \tilde{a} \cdot \theta^h.$$

These equations are standard, except for the presence of the entry cost in the resource constraint at date 0. We determine the optimal choice $(c_0^h, \tilde{c}^h, \theta_0^h, \theta^h)$ by

[3] A positive supply of assets could be considered by redefining individual endowment as the sum of a labor income and an exogenous portfolio of securities.

calculating the consumption-portfolio decision under entry and non-entry. Comparing the resulting utility levels yields the optimal participation decision.

Let $e_0 = \int_H e_0^h d\mu(h)$ and $\tilde{e} = \int_H \tilde{e}^h d\mu(h)$ denote the average income of the *entire population.*

Definition 1. *A general equilibrium with endogenous participation (GEEP) consists of an interest rate R, a price vector π, and a collection of optimal plans $\left(c_0^h, \tilde{c}^h, \theta_0^h, \theta^h\right)_{h\in H}$ such that*

1. *The good market clears in every state: $\int_H (c_0^h + \kappa 1_{\{\theta^h \neq 0\}}) d\mu(h) = e_0$, and $\int_H \tilde{c}^h(\omega)\, d\mu(h) = \tilde{e}(\omega)$ for all $\omega \in \Omega$.*

2. *The asset markets clear: $\int_H \theta^h d\mu(h) = 0$.*

Under free participation ($\kappa = 0$), the definition coincides with the traditional concept of general equilibrium under incomplete markets (GEI). With positive entry costs, a GEEP equilibrium differs from a GEI through two different channels. First, agents endogenously make their participation decisions, and decide whether to pay the entry cost. Second, trading activities use some of society's resources and thus crowd out private consumption, as seen in the market clearing condition at date $t = 0$. This phenomenon, which we call *the displacement effect*, probably plays a minor role in actual economies. Extensions of our model could transfer a fraction of trading fees to certain consumers (such as exchange owners), or seek to provide a more detailed description of the financial industry.

The existence and constrained efficiency of equilibrium are shown in Appendix A. In order to analyze the effect of financial innovation on participation and prices, we now specialize to a tractable class of CARA-normal economies. Investors have identical utility of the Epstein-Zin type:

$$U(c_0, \widetilde{c}) = -e^{-\chi c_0} - \beta[\mathbb{E}\, e^{-\gamma\widetilde{c}}]^{\chi/\gamma},$$

where γ and χ are positive coefficients. The agent maximizes $-e^{-\chi c_0} - \beta e^{-\chi c_1}$ when she reallocates through time a deterministic income flow. On the other hand, atemporal risky choices only depend on $\mathbb{E}e^{-\gamma\widetilde{c}}$. When future consumption is normally distributed, we can rewrite the utility as

$$-e^{-\chi c_0} - \beta e^{-\chi[\mathbb{E}\widetilde{c} - \gamma Var(\widetilde{c})/2]}.$$

The specification corresponds to the standard expected utility when $\chi = \gamma$.

Individual endowments and the payoffs of risky assets are jointly normal. The securities generate a linear subspace in the set $L^2(\Omega)$ of square-integrable random variables. We assume without loss of generality that the risky assets are centered and mutually independent: $(\tilde{a}_1, .., \tilde{a}_J) \sim \mathcal{N}(0, I)$. Let A denote the span of the *risky* assets, and $A^\perp$ the subspace orthogonal to *all* securities (including the bond). Projections will play an important role in the discussion, and it will be convenient to denote by $\tilde{x}^V$ the projection of a random variable $\tilde{x}$ on a subspace V.

2.1. Individual Entry Decision

We solve the decision problem of an individual trader h by calculating the consumption - portfolio choice under entry and non-entry. Consider the tradable security $\tilde{m}^A \equiv -(R/\gamma)\sum_{j=1}^J \pi_j \tilde{a}_j$, which is determined by risk aversion and market prices. We show in Appendix B:

Theorem 1. *When participating in the risky asset market, the investor buys*

$$\theta_0^{h,p} = \frac{R}{1+R}\left\{e_0^h - \mathbb{E}\,\tilde{e}^h - \kappa - \pi \cdot \theta^{h,p} + \frac{\ln(R\beta)}{\chi} + \frac{\gamma}{2}\left[Var(\tilde{e}^{hA^\perp}) + Var(\tilde{m}^A)\right]\right\}$$

units of the bond, and $\theta_j^{h,p} = -Cov(\tilde{a}_j, \tilde{e}^h) - R\pi_j/\gamma$ *units of risky asset* j*. Consumption is then*

$$\tilde{c}^{h,p} = \mathbb{E}\,\tilde{e}^h + \theta_0^{h,p} + \tilde{m}^A + \tilde{e}^{hA^\perp} \tag{2.1}$$

in the second period.

We can infer from (2.1) that the investor exchanges the *marketable* component $\tilde{e}^{hA}$ of her income risk for the tradable portfolio $\tilde{m}^A$, which allows an optimal allocation of risk and return. Because markets are incomplete, she is also constrained to bear the undiversifiable income risk $\tilde{e}^{hA^\perp}$.

Investment in the riskless asset is the sum of two components, which correspond to intertemporal smoothing and the precautionary motive. First, the agent uses the riskless asset to reallocate her expected income stream between the two periods. Note that she compensates for any discrepancy between her subjective discount factor and the interest rate. Second, she saves more when future prospects are more uncertain. As will be seen in the next section, financial innovation affects this precautionary component by modifying the riskiness of the portfolio $\tilde{m}^A$ and by reducing the undiversifiable income risk $\tilde{e}^{hA^\perp}$.

The consumption of the non-participating investor is obtained from Theorem 1 by setting $A = \{0\}$ and $\kappa = 0$.

Proposition 1. *When not trading risky assets, the investor saves*

$$\theta_0^{h,n} = \frac{R}{1+R}\left[e_0^h - \mathbb{E}\,\tilde{e}^h + \frac{1}{\chi}\ln(R\beta) + \frac{\gamma}{2}Var\left(\tilde{e}^h\right)\right] \tag{2.2}$$

in the first period, and consumes $\tilde{c}^{h,n} = \tilde{e}^h + \theta_0^{h,n}$ *in the second.*

The non-participating agent bears all the endowment risk in her final consumption. The precautionary demand for the bond therefore depends on the whole variance of future income.

The investor makes her participation choice by comparing utility under entry and non-entry. In the CARA-normal case, this reduces to maximizing the certainty equivalent $\mathbb{E}\tilde{c}^h - \gamma Var(\tilde{c}^h)/2$. As shown in the appendix, the benefit of trading risky assets is $\gamma Var\left(\tilde{e}^{hA} - \tilde{m}^A\right)/2$, while the opportunity cost is κR. This leads to

Theorem 2. *The investor trades risky assets when*

$$\frac{\gamma}{2}Var\left(\tilde{e}^{hA} - \tilde{m}^A\right) > \kappa R, \tag{2.3}$$

and is indifferent between entry and non-entry if the relation holds as an equality.

Relation (2.3) has a simple geometric interpretation in $L^2(\Omega)$, which is illustrated in Figure 1. The agent trades risky assets if the distance between her income risk $\tilde{e}^{hA}$ and her optimal portfolio $\tilde{m}^A$ is larger than $\sqrt{2\kappa R/\gamma}$. The trader thus pays the entry fee only if her initial position is sufficiently different from the optimum, as is standard in decision-theoretic models with adjustment costs.

The theorem has a natural interpretation when all agents have a positive exposure to certain classes of risks. Investors with low exposure to marketable shocks buy the corresponding assets to earn a risk premium; these agents are called *speculators*. On the other hand, agents with a high risk exposure will hedge by selling the corresponding risky assets; these agents are called *hedgers* or *issuers*. The model thus closely matches the type of risk-sharing examined in the futures literature.

2.2. Equilibrium

Let $\mathcal{P} \subseteq H$ denote the set of participants in the risky asset markets. When the class of indifferent agents has measure zero, we can write

$$\mathcal{P} = \left\{ h \in H : \gamma Var(\tilde{e}^{hA} - \widetilde{m}^{A})/2 \geq \kappa R \right\}. \tag{2.4}$$

Market participants can have different income risk characteristics than the entire population. We will show in Sections 3 and 4 that this difference is a driving element of the model.[4] While $\tilde{e}$ denotes the mean income in the population, we define the average endowment *among participants* as

$$\tilde{e}^{p} = \int_{\mathcal{P}} \tilde{e}^{h} d\mu^{p}(h),$$

where μ^{p} is the conditional measure $\mu/\mu(\mathcal{P})$ if $\mu(\mathcal{P}) > 0$, and identically zero otherwise.

In equilibrium, the common consumption risk $\tilde{m}^{A}$ must coincide with the average tradable income risk of participants:

$$\tilde{m}^{A} = \tilde{e}^{pA}. \tag{2.5}$$

We also establish

Theorem 3. *In equilibrium, an asset $\tilde{a}$ is worth*

$$\pi(\tilde{a}) = [\mathbb{E}\,\tilde{a} - \gamma Cov(\tilde{e}^{p}, \tilde{a})]/R. \tag{2.6}$$

The interest rate satisfies

$$\ln R = \ln R_0 + \chi\mu(\mathcal{P}) \left[\kappa + \frac{\gamma}{2} \int_{\mathcal{P}} Var\left(\tilde{e}^{hA} - \tilde{e}^{pA}\right) d\mu^{p}(h) \right], \tag{2.7}$$

where $\ln R_0 = \ln(1/\beta) + \chi(\mathbb{E}\,\tilde{e} - e_0) - (\chi\gamma/2)\int_H Var\left(\tilde{e}^{h}\right) d\mu(h)$.

The participation set and asset prices are thus jointly determined by (2.4) – (2.7).

As in the standard CCAPM, an asset is valuable if it provides a good hedge against the consumption risk $\tilde{e}^{pA}$ of participants. Since participation is endogenous in our setup, financial innovation can change the market endowment $\tilde{e}^{p}$, and

[4] Mankiw and Zeldes (1991) show the empirical importance of this distinction.

therefore the *relative* price $\pi(\tilde{a})/R^{-1} = \mathbb{E}\,\tilde{a} - \gamma Cov(\tilde{e}^p, \tilde{a})$ of a risky asset relative to the bond. The possible effect of financial innovation on the risk premium crucially relies on the endogeneity of participation, and is one of the main properties of the model.[5]

The equilibrium interest rate R is influenced by the two economic effects that correspond to the last two terms of equation (2.7). First, the interest rate tends to be higher when more first period resources $\kappa\mu(\mathcal{P})$ are absorbed in the entry process. The second term of (2.7) corresponds to the precautionary motive. To illustrate this point, recall that the variance of individual consumption is $Var(\tilde{e}^{pA}) + Var(\tilde{e}^{hA^\perp})$ if an agent participates, and $Var(\tilde{e}^{hA}) + Var(\tilde{e}^{hA^\perp})$ otherwise. Entry reduces on average the variance of consumption by

$$\int_{\mathcal{P}} Var\left(\tilde{e}^{hA}\right) d\mu^p(h) - Var\left(\tilde{e}^{pA}\right) = \int_{\mathcal{P}} Var\left(\tilde{e}^{hA} - \tilde{e}^{pA}\right) d\mu^p(h). \tag{2.8}$$

This term is large when many agents participate or many hedging instruments are available. The financial markets then permit agents to greatly reduce their risk exposure, which dampens their precautionary motive, reduces the demand for the riskless asset, and leads to an increase in the equilibrium interest rate.[6]

The entry condition (2.3) suggests that a lower entry fee or improved spanning tends to encourage entry. For instance when the entry cost κ is infinite, no agent trades risky assets and the equilibrium interest rate equals R_0.[7] The equilibrium set of participants, however, may not increase monotonically with the financial structure. This is because the entry condition (2.3) depends on the endogenous variables $\tilde{e}^p$ and R. When new assets are added, a participating agent h may leave the market because the diversification benefit $\gamma Var(\tilde{e}^{hA} - \widetilde{e}^{pA})/2$ has dropped or the opportunity cost has increased. We will provide examples of such behaviors in Sections 3 and 4.

The effect of financial innovation on the interest rate can be predicted when $\tilde{e}^p$ remains constant.

[5] When the set of traders is fixed, an increase in the asset span has no effect on the relative price of a risky asset relative to the bond, as noted in Oh (1996).

[6] This equation is thus consistent with the well-known effect that financial innovation increases the interest rate when the participation structure is exogenous (Weil, 1992; Elul, 1997; Calvet, 2001).

[7] More generally, let $\kappa_{\max}(A)$ denote the essential supremum of $(\gamma/2R_0)Var\left(\tilde{e}^{hA}\right)$ in the population. It is easy to show that when $\kappa \geq \kappa_{\max}(A)$, the economy has a unique equilibrium, in which no agent trades risky assets. On the other hand if $\kappa < \kappa_{\max}(A)$, any equilibrium has a non-negligible set of participants.

Proposition 2. *Financial innovation leads to a higher interest rate when the mean endowment $\tilde{e}^p$ is unchanged.*

The proof has a straightforward intuition. Financial innovation and a decrease in the interest rate would both encourage entry and lead, by (2.7), to a higher interest rate - a contradiction. Thus if the participants' average endowment does not vary, existing asset prices necessarily *decrease* with financial innovation. Changes in $\tilde{e}^p$ thus play a crucial role in determining the impact of financial innovation on asset prices. To better understand this role, we now introduce a factor model of risk exposure.

3. Economies with a Unique Risk Factor

We consider in this section a class of economies with a unique risk factor $\tilde{\varepsilon}$. The participation structure and interest rate are determined by the intersection of two curves, which respectively correspond to the entry condition and the market clearing of the bond. We derive the comparative statics of the economy and develop intuition on the risk premium that will be useful for the multifactor calibrations of Section 4.

We specify the endowment of each investor h as

$$\tilde{e}^h = \mathbb{E}\,\tilde{e}^h + \varphi^h \tilde{\varepsilon}, \tag{3.1}$$

and call φ^h the individual loading of the agent.[8] The factor $\tilde{\varepsilon}$ is a macroeconomic shock that linearly affects all incomes. The model is tractable when the factor and the asset payoffs are jointly normal. Without loss of generality, we assume that $\tilde{\varepsilon}$ has a standard distribution $\mathcal{N}(0,1)$, and that the average loading $\bar{\varphi} = \int_{\mathbb{R}} \varphi d\mu(\varphi)$ in the population is non negative.

When financial markets are incomplete, existing securities span only partially the common shock. The projection of $\tilde{\varepsilon}$ on the asset span, $\tilde{\varepsilon}^A = \sum_{i=1}^{J} Cov(\tilde{\varepsilon}^A, \widetilde{a}_j)\widetilde{a}_j$, is called the tradable component of the factor. The corresponding variance

$$\alpha = Var(\tilde{\varepsilon}^A)$$

is a useful *index of market completeness*, which quantifies the fraction of the risk $\tilde{\varepsilon}$ that is directly insurable. Since $\tilde{\varepsilon}$ has unit variance, the completeness index α is contained between 0 and 1. The values $\alpha = 0$ and $\alpha = 1$ respectively correspond

[8] Purely idiosyncratic shocks are ruled out in this section for expositional simplicity.

to the absence of risky assets ($A = \{0\}$) and the full marketability of the shock ($\tilde{\varepsilon} \in A$). Intermediate values of α arise when agents can only trade the bond and a risky asset imperfectly correlated with the aggregate shock. The portfolio $\tilde{\varepsilon}^A$ and the completeness index α have direct empirical interpretations. We can calculate $\tilde{\varepsilon}^A$ by regressing the factor $\tilde{\varepsilon}$ on the asset payoffs. The corresponding determination coefficient R^2 is then an estimate of the completeness index α.

The one-factor model discussed in this section has a natural interpretation when the factor represents GDP or a market risk that is not directly tradable on organized exchanges (Roll, 1977; Athanasoulis and Shiller, 2000). New assets then help market participants hedge more closely the risk $\tilde{\varepsilon}$, and thus imply an increase in the completeness index α. Similarly, macroeconomic variables such as GDP are observed with measurement errors and lags. Improvements in national accounting can lead to more precise hedging instruments and a corresponding increase in α.

The distribution of the loading φ in the population is specified by a measure μ on the real line. To clarify the intuition, we assume that the measure μ has a continuous density $f(\varphi)$, whose support is the nonnegative interval $[0, \infty)$.[9] The parameters α and κ are also taken to be non-degenerate, in the sense that $\alpha > 0$ and $0 < \kappa < \infty$.[10]

We easily infer from Section 2 the equilibrium conditions. Let φ^p denote the average loading of participants:

$$\varphi^p = \int_{\mathcal{P}} \varphi d\mu^p(\varphi). \tag{3.2}$$

Market entrants have average income $\tilde{e}^p = \mathbb{E}\,\tilde{e}^p + \varphi^p(\tilde{\varepsilon}^A + \tilde{\varepsilon}^{A^\perp})$, and their individual consumption thus satisfies:

$$\widetilde{c}^h = \mathbb{E}\,\widetilde{c}^h + \varphi^p\,\tilde{\varepsilon}^A + \tilde{e}^{hA^\perp}.$$

As seen in Theorem 1, the marketable consumption risk $\varphi^p\,\tilde{\varepsilon}^A$ is identical for all participants. Consider an asset $\widetilde{a}$, $\pi(\widetilde{a}) > 0$, that positively covaries with the factor. The endogenous loading φ^p controls the covariance between the asset and individual consumption, $Cov(\widetilde{c}^h, \widetilde{a}) = \varphi^p Cov(\tilde{\varepsilon}, \widetilde{a})$, and therefore the pricing of risk. Let $\widetilde{R}_a = \widetilde{a}/\pi(\widetilde{a})$ denote the random (gross) return on the asset. By

[9] The theorems of this section are in fact proved for densities $f(\varphi)$ with *arbitrary* unbounded supports.

[10] Degenerate values of α and κ are discussed in Appendix B.

Theorem 3, the *relative risk premium* satisfies

$$\frac{\mathbb{E}\,\widetilde{R}_a - R}{R} = \frac{\gamma\varphi^p Cov(\widetilde{a},\tilde{\varepsilon})}{\mathbb{E}\,\widetilde{a} - \gamma\varphi^p Cov(\widetilde{a},\tilde{\varepsilon})}.$$

We will show that financial innovation can reduce the consumption loading φ^p and thus diminish the risk premium of preexisting securities.

We now turn to the equations that determine the interest rate and the participation set. The equilibrium of the bond market implies

$$\ln R = \ln R_0 + \chi\mu(\mathcal{P})\left[\kappa + \frac{\alpha\gamma}{2} Var_{\mathcal{P}}(\varphi)\right], \tag{3.3}$$

where $Var_{\mathcal{P}}(\varphi) = \int_{\mathcal{P}}(\varphi - \varphi^p)^2 d\mu^p(\varphi)$ denotes the variance of the participants' loadings. By Theorem 2, an agent enters if the diversification benefit $\alpha\gamma(\varphi-\varphi^p)^2/2$ is larger than the opportunity cost κR. As a result, the participation set

$$\mathcal{P} = (-\infty, \varphi^p - \Lambda] \cup [\varphi^p + \Lambda, +\infty) \tag{3.4}$$

is the union of two half-lines that are equidistant from φ^p by length

$$\Lambda = \sqrt{2\kappa R/(\alpha\gamma)}. \tag{3.5}$$

Agents $\varphi \geq \varphi^p + \Lambda$ are hedgers who trade risky assets to reduce their exposure. Conversely, agents with loadings $\varphi \leq \varphi^p - \Lambda$ are speculators who increase their consumption risk in order to earn a higher return. An equilibrium is thus a triplet (R, Λ, φ^p) satisfying equations (3.2) – (3.5).

The equilibrium calculation is simplified by the following observation. Equations (3.2) and (3.4) impose that the loading φ^p is both the center of symmetry and the center of mass of the participation set $\mathcal{P}$. In the Appendix, we show that this restriction implies

Property 1. *For any $\Lambda \geq 0$, there exists a unique $\varphi^p(\Lambda)$ satisfying equations* (3.2) *and* (3.4).

We can now define $\mathcal{P}_\Lambda$ as the participation set $(-\infty; \varphi^p(\Lambda)-\Lambda] \cup [\varphi^p(\Lambda)+\Lambda; +\infty)$ corresponding to a given length Λ. It is easy to show

Property 2. *The participation set $\mathcal{P}_\Lambda$ decreases with the length parameter: $\mathcal{P}_{\Lambda'} \subseteq \mathcal{P}_\Lambda$ for all $\Lambda \leq \Lambda'$.*

Since the sets $\{\mathcal{P}_\Lambda; \Lambda \geq 0\}$ are nested, the length parameter Λ provides a precise ordering of the participation structure. A high value of Λ corresponds to a small set $\mathcal{P}_\Lambda$ and thus a low participation rate $\mu(\mathcal{P}_\Lambda)$.

To develop intuition on the risk premium, consider the simpler model in which the interest rate R is *exogenous.* The formula $\Lambda = \sqrt{2\kappa R/(\alpha\gamma)}$ then expresses the participation parameter as a function of exogenous quantities only. A higher completeness index α reduces Λ and thus increases the participation set $\mathcal{P}_\Lambda$. The implied movement in the loading φ^p then controls changes in pricing of risk.

Property 3. *When the loading density verifies the skewness condition*

$$f(\varphi^p - \Lambda) > f(\varphi^p + \Lambda), \tag{3.6}$$

the center of gravity $\varphi^p(\Lambda)$ locally increases with Λ.

Figure 2 illustrates the mechanism underlying this key result. When Λ decreases, the skewness of the loading density implies that more agents enter to the left (speculators) than to the right (hedgers) of φ^p, which pushes down the average consumption loading φ^p. A majority of the new entrants seeks to buy the factor's marketable component $\tilde{\varepsilon}^A$, bid up its price, and thus drive down the risk premium. The fixed interest rate setup thus illustrates the role of the loading density $f(\varphi)$ on the comparative statics of asset prices.

The equilibrium analysis requires more care in the full-fledged model in which the interest rate is endogenous. Properties 1 and 2 imply that in the (Λ, R) plane, an equilibrium corresponds to the intersection of the two curves:

$$R_1(\Lambda) = \alpha\gamma\Lambda^2/(2\kappa), \tag{3.7}$$

$$R_2(\Lambda) = R_0 \exp\{\chi\mu(\mathcal{P}_\Lambda)\,[\kappa + \alpha\gamma(Var_{\mathcal{P}_\Lambda}\varphi)/2]\}. \tag{3.8}$$

The functions respectively express the entry decision and the equilibrium of the bond market. We observe that $R_1(\Lambda)$ is increasing and quadratic, while $R_2(\Lambda)$ monotonically decreases with Λ (by Property 2). This helps establish

Theorem 4. *There exists a unique equilibrium.*

Figure 3 illustrates the geometric determination of equilibrium, and helps to analyze the effect of financial innovation. An increase in α pushes up both curves in the figure, implying a higher interest rate and an ambiguous change in the participation parameter Λ.

Theorem 5. *The riskless rate R increases with financial innovation. As the completeness index α increases from 0 to 1, the set of participants $\mathcal{P}$ has two possible behaviors. It is either monotonically increasing; or there exists $\alpha^* \in (0,1)$ such that $\mathcal{P}$ increases on $[0, \alpha^*]$ and decreases on $[\alpha^*, 1]$.*

The two behaviors are illustrated in Figure 4. The ambiguous effect of financial innovation on market participation has a simple intuition. On one hand, a higher α increases the benefit $\alpha\gamma(\varphi^h - \varphi^p)^2/2$ of trading risky assets and encourages entry. On the other hand, new assets reduce the precautionary motive and increase the interest rate, thus discouraging participation. The overall movement depends on the sensitivity of the curves R_1 and R_2 to the innovation parameter α.

Let $\eta_{X,\alpha} = d\ln X / d\ln\alpha$ denote the elasticity of an endogenous quantity X. We infer from equation (3.7) that

$$\eta_{\Lambda,\alpha} = (\eta_{R,\alpha} - 1)/2.$$

Financial innovation thus increases the set of participants ($\eta_{\Lambda,\alpha} < 0$) if it only has a weak impact on the interest rate ($\eta_{R,\alpha} < 1$). In addition, we observe that the elasticity of $R_2(\Lambda)$ with respect to α increases with the dispersion of the participants' loadings $Var_{\mathcal{P}_\Lambda}\varphi$. When traders have very heterogeneous incomes, financial innovation allows agents to greatly reduce their average consumption risk, as shown by (2.8). As a result, new assets have a strong impact on the individual precautionary motive and the equilibrium interest rate. In Figure 4, this explains why participation is non-monotonic for the loading density with the highest variance.

The effect of innovation on the risk premium is easily examined. Consistent with Figure 2, we show

Proposition 3. *The relative risk premium locally decreases with financial innovation if $\eta_{\Lambda,\alpha}\left[f(\varphi_p - \Lambda) - f(\varphi_p + \Lambda)\right] < 0$.*

This *local* result is analogous to condition (3.6) derived in the exogenous interest rate case, but now controls for changes in the participation parameter Λ. We can also guarantee a *global* decline in the relative risk premium.

Theorem 6. *As the completeness index α varies from 0 to 1, the relative risk premium monotonically declines if the loading density $f(\varphi)$ decreases on its support and satisfies $\chi\gamma(Var_H\varphi)/2 < 1$.*

The second condition bounds the dispersion of factor loadings, and thus guarantees that the elasticity of $R_2(\Lambda)$ with respect to α is sufficiently small.

The one-factor model thus reconciles a number of changes that have been observed in financial markets in the past thirty years. New financial instruments have encouraged investors to participate in financial markets, which has led to a reduction in the precautionary motive and in the covariance between stockholder consumption and the aggregate shock. These two effects have in turn increased the interest rate and reduced the risk premium.[11] We observe that this argument is consistent with the empirical findings of Mankiw and Zeldes (1991), who show that the consumption of stockholders tends to be more correlated with the market than the consumption of non-stockholders. As financial innovation leads more people to enter the market, the risk premium falls.

In this section, financial innovation consisted of providing a better hedge against a common risk factor. In practice, however, households and firms face multiple sources of income shocks, and innovation often permits to hedge classes of risk that had been previously uninsurable. For this reason, we now examine a multifactor model of risk.

4. Multifactor Economies

We now consider an economy with a finite number of risk factors $(\tilde{\varepsilon}_1, .., \tilde{\varepsilon}_L)$, which correspond to macroeconomic or sectoral shocks affecting individual income. For instance, $\tilde{\varepsilon}_1$ could be an aggregate risk, and $\tilde{\varepsilon}_2, .., \tilde{\varepsilon}_L$ could correspond to industry or firm-specific shocks. We specify the income of each investor h as

$$\tilde{e}^h = \mathbb{E}\,\tilde{e}^h + \sum_{\ell=1}^{L} \varphi_\ell^h \tilde{\varepsilon}_\ell, \tag{4.1}$$

and denote by $\varphi^h = (\varphi_1^h, ..., \varphi_L^h)$ the vector of individual loadings. The model is tractable when the risk factors and the asset payoffs are jointly normal. Without loss of generality, we normalize the factors to have unit variances and no mutual correlation: $(\tilde{\varepsilon}_1, .., \tilde{\varepsilon}_L) \sim \mathcal{N}(0, I)$. The distribution of factor loadings in the population is specified by a continuous density $f(\varphi)$ on $\mathbb{R}^L$.

[11] We show in Appendix B that an increase in the transaction cost implies a higher risk premium when condition (3.6) is satisfied. Like models with exogenously restricted participation (Vissing-Jorgensen, 1997; Basak and Cuoco, 1998), our framework thus helps explain the equity premium puzzle.

The factors may not be fully tradable when financial markets are incomplete. As in the previous section, it is useful to consider their projections $\tilde{\varepsilon}_\ell^A = \sum_{j=1}^J Cov\left(\tilde{\varepsilon}_\ell, \tilde{a}_j\right) \tilde{a}_j$ on the asset span. We interpret $\tilde{\varepsilon}_\ell^A$ as the marketable component of factor ℓ, which can be estimated empirically by regressing $\tilde{\varepsilon}_\ell$ on the asset payoffs. We conveniently stack the projected factors in a vector $\tilde{\varepsilon}^A = (\tilde{\varepsilon}_1^A, .., \tilde{\varepsilon}_L^A)$. The covariance matrix

$$\Sigma^A = Var\left(\tilde{\varepsilon}^A\right)$$

is a generalized index of market completeness, whose diagonal coefficients $\alpha_\ell = Var\left(\tilde{\varepsilon}_\ell^A\right)$ quantify the insurable fraction of each factor.

We assume for simplicity that the projected factors are mutually uncorrelated: $Cov(\tilde{\varepsilon}_\ell^A, \tilde{\varepsilon}_k^A) = 0$ for all distinct ℓ and k. In the comparative statics of the next subsections, this hypothesis will make it more striking that the improved marketability of factor ℓ can affect the risk premium on an uncorrelated component $\tilde{\varepsilon}_k^A$. The covariance matrix is then diagonal:

$$\Sigma^A = \begin{bmatrix} \alpha_1 & & \\ & \ddots & \\ & & \alpha_L \end{bmatrix},$$

with coefficients $\alpha_\ell = Var\left(\tilde{\varepsilon}_\ell^A\right)$ contained between 0 and 1. We note that Σ^A is equal to zero when there are no assets, and to the identity matrix when markets are complete.

The equilibrium calculation follows directly from Section 2. By equation (4.1), the mean endowment of participants satisfies

$$\tilde{e}^p = \mathbb{E}\,\tilde{e}^p + \sum_{\ell=1}^L \varphi_\ell^p \tilde{\varepsilon}_\ell, \tag{4.2}$$

where each φ_ℓ^p represents the traders' average exposure to factor ℓ. The equilibrium of financial markets implies the relations

$$\pi(\tilde{a}) = [\mathbb{E}\,\tilde{a} - \gamma Cov(\tilde{e}^p, \tilde{a})]/R \tag{4.3}$$

and

$$\ln R = \ln R_0 + \chi\mu(\mathcal{P}) \left[\kappa + \frac{\gamma}{2}\sum_{i=1}^2 \alpha_i Var_{\mathcal{P}}(\varphi_i)\right], \tag{4.4}$$

where $\ln R_0 = \ln(1/\beta) + \chi(\mathbb{E}\widetilde{e} - e_0) - (\gamma\chi/2)\sum_{i=1}^{2}\mathbb{E}(\varphi_i^2)$. These equations suggest that when the utility coefficients γ and χ^{-1} are large, financial innovation generates both substantial variations in the pricing of risk and small movements in the interest rate.

The entry condition (2.3) implies the participation set

$$\mathcal{P} = \left\{\varphi : \frac{\gamma}{2}\sum_{\ell=1}^{L}\alpha_\ell(\varphi_\ell - \varphi_\ell^p)^2 \geq \kappa R\right\}. \tag{4.5}$$

When all the coefficients α_ℓ are strictly positive, the participants are located outside an ellipsoid centered at $\varphi^p = (\varphi_1^p, .., \varphi_L^p)$.[12] The lengths $\Lambda_\ell = \sqrt{2\kappa R/(\alpha_\ell\gamma)}$ of the ellipsoid along each axis depend on the completeness index α_ℓ and the endogenous interest rate. As in the one-factor case, we show

Theorem 7. *There exists a unique equilibrium.*

The proof begins by establishing that the lengths $\Lambda = (\Lambda_1, .., \Lambda_L)$ define a unique participation set $\mathcal{P}_\Lambda$. Unlike the one-factor case, however, the set $\mathcal{P}_\Lambda$ need not be decreasing in each component Λ_i because the ellipsoid can move in more than one direction. We show that the market clearing of the bond uniquely determines the interest rate R and thus the lengths $\Lambda_\ell = \sqrt{2\kappa R/(\alpha_\ell\gamma)}$. The proof also provides a simple algorithm for the numerical computation of equilibrium. We now examine the comparative statics of participation and asset prices with respect to financial innovation.

4.1. Calibrated Movements in the Equity Premium

We showed in Section 3 that financial innovation can reduce the risk premium of securities correlated with the common factor. In this subsection, we now investigate the possibility of cross-sectoral effects, i.e. that the improved marketability of a given shock has pricing effects in uncorrelated sectors. Consider for simplicity an economy with two factors $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$. The random variable $\widetilde{\varepsilon}_1$ is an *aggregate shock* to which all investors are positively exposed, while the second risk $\widetilde{\varepsilon}_2$ is *idiosyncratic*. Let $\tilde{a} = \mathbb{E}\,\widetilde{a} + \tilde{\varepsilon}_1^A$, $\pi(\tilde{a}) > 0$, denote an asset or *stock* that is only correlated with the aggregate factor.[13] By equation (4.3), the stock has relative

[12] The participants are located outside a cylinder when some coefficients α_ℓ are equal to zero.

[13] While the assets are assumed to be in zero net supply, we easily reinterpret the model in terms of equity by viewing the endowment as the sum of a labor income and an exogenous

risk premium

$$\frac{\mathbb{E}\,\widetilde{R}_a - R}{R} = \frac{\gamma\varphi_1^p\alpha_1}{\mathbb{E}\,\widetilde{a} - \gamma\varphi_1^p\alpha_1}. \tag{4.6}$$

Consider how this ratio is affected by an increase in the completeness index α_2 of the idiosyncratic shock. If participation were exogenous, the consumption loading φ_1^p would be a constant parameter, and the improved spanning of $\tilde{\varepsilon}_2$ would not affect the relative premium of the stock. In our model, however, financial innovation can affect the consumption loading φ_1^p and the premium (4.6) even though the stock $\tilde{a}$ and the idiosyncratic risk $\tilde{\varepsilon}_2$ are statistically independent.

When only the asset $\tilde{a}$ is initially traded ($\alpha_2 = 0$), non-participants have loadings φ_1 that are close to the market average: $|\varphi_1 - \varphi_1^p| \leq \sqrt{2\kappa R/(\alpha_1\gamma)}$. As α_2 increases, agents with loadings φ_2 sufficiently distant from φ_2^p sustain the entry cost in order to modify their exposure. In addition, the new participants also trade the stock $\tilde{a}$ to achieve an optimal level of diversification. We observe this type of behavior when investors or companies start trading derivative instruments for hedging purposes and then, once acquired a certain level of expertise, act as speculators in unrelated financial markets. The risk premium on $\tilde{a}$ declines if a majority of the new entrants have a small exposure to $\tilde{\varepsilon}_1$ and thus increase the demand for asset $\tilde{a}$.

We assess the magnitude of this effect in calibrated simulations. Assume that exposures to the aggregate and idiosyncratic risks are independent across the population. The cross-sectional density of loadings is then of form $f(\varphi_1, \varphi_2) = f_1(\varphi_1)f_2(\varphi_2)$. This hypothesis will make it perhaps more surprising that the increased marketability of the idiosyncratic risk can modify the equity premium. We specify the loading density $f_2(\varphi_2)$ to be a centered Gaussian $\mathcal{N}(0, \sigma_2^2)$. Aggregate exposure to the idiosyncratic risk is equal to zero, and the symmetry of $f_2(\varphi_2)$ implies that $\varphi_2^p = 0$ in equilibrium.

We next consider a microeconomic structure that relates the loading density $f_1(\varphi_1)$ to aggregate volatility and the distribution of income. The random endowment of an agent is specified as:

$$\widetilde{e}^h = e_0^h(1 + \sigma_1\widetilde{\varepsilon}_1) + \varphi_2^h\widetilde{\varepsilon}_2.$$

The individual loading $\varphi_1^h = \sigma_1 e_0^h > 0$ is thus proportional to the expected income

endowment of stocks. This is a standard convention in asset pricing theory, as discussed for instance in Magill and Quinzii (1996, ch. 3).

e_0^h.[14] The aggregate endowment in period 1 thus satisfies

$$\widetilde{e} = e_0(1 + \sigma_1 \widetilde{\varepsilon}_1).$$

Without loss of generality, we normalize mean income to unity: $e_0 = 1$. The utility coefficients γ and χ^{-1} then coincide with relative risk aversion and the elasticity of intertemporal substitution at the mean endowment point. The quantity κ is also equal to the fraction of mean income used in the entry process.

As is standard in the literature, we assume that a lognormal density $g(e_0^h)$ quantifies the initial allocation of income in population: $\ln e_0^h \sim \mathcal{N}(\mu_z, \sigma_z^2)$. Since mean income is normalized to 1, the parameters μ_z and σ_z^2 satisfy the restriction $\mu_z + \sigma_z^2/2 = 0$. We choose $\mu_z = -0.25$ in a simulation, which corresponds to a reasonable Gini coefficient of 0.4. Since $\varphi_1^h = \sigma_1 e_0^h$, the density of the first loading is given by $f_1(\varphi_1) = \sigma_1^{-1} g(\varphi_1/\sigma_1)$.

We report a representative simulation in Figure 5. The relative risk aversion and the intertemporal elasticity are set equal to $\gamma = 10$ and $\chi^{-1} = 2$. We choose the discount factor $\beta = 0.96$, the entry cost $\kappa = 0.8\%$, and the standard deviation of aggregate income growth $\sigma_1 = 0.04$. The loading density $f_2(\varphi_2)$ is parameterized by $\sigma_2 = 0.10$. We assume that the aggregate shock is only partially tradable. The corresponding completeness index is set to the constant value $\alpha_1 = 0.5$, which is roughly consistent with the correlation between the aggregate labor income shock and the NYSE value-weighted stock return reported in Campbell, Cocco, Gomes and Maenhout (2001). The stock is a traded asset of the form $\widetilde{a} = x + \widetilde{\varepsilon}_1^A$. We select the weighting coefficient x to obtain a risk premium $\mathbb{E}\widetilde{R}_a - R$ equal to 7% before the introduction of new contracts ($\alpha_2 = 0$). This corresponds to the real yield on the equity index over the period 1889-1978. In the absence of a futures market, the economy roughly matches the historical data. The net interest rate R is equal to 1% and the standard deviation of the stock return is $[Var(\widetilde{R}_a)]^{1/2} = 15\%$, implying a Sharpe ratio of about 1/2. We note that these two numbers are consistent with the data reported in the literature (e.g. Mehra and Prescott, 1985; Campbell, Lettau, Malkiel and Xu, 2001).

The simulations show that the risk premium on the stock declines from 7% to 4.5% as α_2 increases from 0 to 1. This is the key result of the calibration. Providing insurance against the idiosyncratic shock substantially decreases the risk premium through changes in participation. The standard deviation of the stock return stays almost constant at 15%, which is consistent with the stationarity of

[14] We assume for simplicity that there is no expected growth between the two periods.

market volatility over the past century (Campbell, Lettau, Malkiel and Xu, 2001). We observe that most of the decline in the risk premium occurs when the hedging coefficient α_2 increases from 0 to 0.5. Furthermore, the value $\alpha_2 = 0.5$ also yields values for participation (60%) and the real net interest rate (2.5%) that are reasonable approximations of the current US economy. This simulation is the main calibration result of the paper and shows strong evidence of the cross-sectoral effects induced by financial innovation.

4.2. Differential and Interest Rate Effects of Financial Innovation

We now explore two additional consequences of introducing new assets in the multifactor model: differential changes in sectoral risk premia, and a possible reduction of the interest rate.

The calibration of subsection 4.1 assumed that the loading density $f_2(\varphi_2)$ is symmetric around zero, implying that an asset correlated only with the second factor yields no risk premium ($\varphi_2^p = 0$). The shock $\widetilde{\varepsilon}_2$ redistributed income across the population, and its marketable component was diversified away by market participants. In this subsection, we consider instead that $\widetilde{\varepsilon}_2$ is a second source of aggregate uncertainty. The loading density $f_2(\varphi_2)$ is skewed and is assumed for simplicity to have a positive support. The risks $\widetilde{\varepsilon}_1$ and $\widetilde{\varepsilon}_2$ are then independent sources of aggregate uncertainty that yield positive and distinct premia. This opens the possibility that financial innovation, such as increase in α_2, can differentially affect asset prices across sectors, and thus have rich effects on the cross-section of expected returns.

To illustrate this possibility, we extend the comparative statics analysis of Figure 2 to the two-factor case. Consider an economy with an *exogenous* interest rate R, and initial indices α_1 and α_2. The ellipse delimiting the participation set is illustrated by a solid line in Figure 6a. It is centered at φ^p and has lengths $\Lambda_\ell = \sqrt{2\kappa R/(\alpha_\ell \gamma)}$ along each axis. We now consider the equilibrium effect of an increase in the second index from α_2 to α_2'. Since the interest is fixed, the limiting boundary in the new equilibrium has the same horizontal length Λ_1 but a shorter vertical length Λ_2'. In dotted lines, we represent the (intermediate) ellipse centered at φ^p with lengths Λ_1 and Λ_2'. Agents in the shaded area have a different center of gravity than φ^p and tend to push the new equilibrium set towards the region of higher mass. Because these agents are more spread out vertically than horizontally, the induced movement in φ^p is expected to be stronger along the vertical axis, i.e. in the direction of innovation. The increased marketability of the shock $\widetilde{\varepsilon}_2$ may thus predominantly influence the average exposure and the risk premium in the second sector.

We demonstrate the validity of this intuition on a (non-calibrated) numerical example with an endogenous interest rate. The marginal densities of the factor loadings are specified as identical log-normals: $f_1 = f_2 = g$. The bivariate density $f(\varphi_1, \varphi_2) = g(\varphi_1)g(\varphi_2)$ has support on the non-negative orthant and is skewed towards the origin. The initial economy has hedging coefficients $\alpha_1 = \alpha_2 = \underline{\alpha}$. As in the previous subsection, we consider two fixed assets $\widetilde{a}_\ell = x_\ell + \widetilde{\varepsilon}_\ell^A$ $(\ell = 1, 2)$, and choose the means x_ℓ to match a risk premium of 7%. The symmetry of the economy imposes that $x_1 = x_2 = x$. As α_2 increases from $\underline{\alpha}$ to 1, the risk premia

$$\mathbb{E}\widetilde{R}_\ell - R = \frac{\varphi_\ell^p R\gamma\underline{\alpha}}{x - \varphi_\ell^p \gamma\underline{\alpha}}, \ \ell = 1, 2,$$

are declining at different rates, as is apparent in Figure 6*b*. As conjectured, the risk premium on the second asset drops more quickly with α_2. The results of the figure are almost unchanged when we exogenously restrict the net interest rate to 2%.

The differential effect of financial innovation is one of the main properties of the multifactor economy. It distinguishes the introduction of sector-specific securities or hedging instruments from changes that jointly affect all security markets, such as a reduction in taxes or transaction costs. In future work, this property may prove useful to explain empirical findings on the price impact of financial innovation.[15]

Multifactor economies also imply novel results for the comparative statics of the interest rate. As discussed in Section 2, the introduction of new assets increases risk-sharing opportunities and weakens the precautionary demand for savings. In models with exogenous participation, this leads to a higher equilibrium interest rate under many specifications, including CARA-normal (Weil, 1992; Elul, 1997; Calvet 2001). The Appendix establishes that when participation is endogenous,

Proposition 4. *The interest rate locally decreases with financial innovation in some multifactor economies.*

This result has a simple geometric intuition. When new assets are introduced, the movement of φ^p pushes the ellipse towards a region of higher mass. In some

[15] See Allen and Gale (1994*a*) for a review of this literature.

economies, this effect is sufficiently strong to reduce the number of participants and the interest rate.

5. Conclusion

This paper develops a tractable asset pricing model with incomplete markets and endogenous participation. Agents receive heterogeneous random incomes determined by a finite number of risk factors. They can borrow or lend freely, but must pay a fixed entry cost to invest in risky assets. Security prices and the participation set are jointly determined in equilibrium. The introduction of non-redundant assets encourages investors to participate in financial markets for hedging and diversification purposes. Under plausible conditions on the cross-sectional distribution of risk, the new entrants reduce the covariance between stock returns and the average consumption of participants, which induces a reduction in the risk premium.

This logic is easily demonstrated in a simple one-factor model. Financial innovation also has strong cross-sectoral effects in economies with multiple sources of risk. When a factor becomes tradable, new agents are drawn to the market in order to manage their risk exposure. Under complementarities of learning or increasing returns to trading activities, the new agents also trade in preexisting markets and can modify the risk premia of securities uncorrelated to the factor. Furthermore, financial innovation differentially affects distinct sectors of the economy and thus has a rich impact on the cross-section of expected returns.

These results suggest several directions for empirical research. Future work will assess the contribution of financial innovation to the decline of the equity premium in the past three decades. Participation changes may also help explain the pricing effects of new derivatives reported in the empirical literature. From a policy perspective, the mechanisms examined in this paper provide useful insights on current debates in public and international economics. When countries face fixed costs to financial integration, the model implies that the creation of new markets can have profound pricing, participation and welfare consequences. An extension of this work could investigate the political economy of the macro markets advocated by Shiller and others. Further research may also evaluate government policies affecting asset creation and participation costs, such as financial regulation, taxes, and social security reform.

6. Appendix A - Existence and Efficiency of Equilibrium

The existence of equilibrium can be established when the state space $\Omega = \{1, .., S\}$ is finite and the economy satisfies standard hypotheses. Assume that the utility function U^h of every agent is continuous, strongly monotone and strictly quasi-concave on $\mathbb{R}^{S+1}_{++}$. At prices where agents are indifferent between entry ($\theta^h \neq 0$) and non-entry ($\theta^h = 0$), individual demand consists of two distinct points, which may lead to discontinuities in aggregate demand. This difficulty can be solved by making the following convexifying hypothesis. There is a finite number of individual types $h = 1, .., H$, and a continuum of agents in each type. We can then show

Theorem A.1. *There exists a GEEP equilibrium.*

Under standard conditions (Aumann, 1966), this result extends to any economy with a continuum of agents.

As in the GEI case, equilibrium allocations are usually Pareto inefficient because the absence of certain markets induces incomplete risk-sharing. With two periods and a single good, however, GEI allocations are known to satisfy a limited or *constrained* form of efficiency. No social planner can improve the utility of all agents when income transfers are constrained to belong to the asset span. This limited form of efficiency easily generalizes to our setting by taking into account the entry fee.

Definition. *An allocation* $(c_0^h, \tilde{c}^h)_{h \in H}$ *is called* feasible *if and only if*

1. *For all* h*, there exists* $(\theta_0^h, \theta^h) \in \mathbb{R} \times \mathbb{R}^J$ *such that* $\tilde{c}^h = \tilde{e}^h + \theta_0^h + \tilde{a} \cdot \theta^h$
2. $\int (c_0^h + \kappa 1_{\{\theta^h \neq 0\}}) d\mu(h) = e_0$, *and* $\int \tilde{c}^h(\omega)\, d\mu(h) = \tilde{e}(\omega)$ *for all* $\omega \in \Omega$.

We can then introduce

Definition. *A feasible allocation* $(c_0^h, \tilde{c}^h)_{h \in H}$ *is called* constrained Pareto-efficient *if no other feasible allocation makes all agents strictly better off.*

We show that any equilibrium allocation is constrained Pareto-efficient.

Theorem A.2. *An equilibrium allocation is constrained Pareto-efficient.*

The theorem implies that the introduction of a new asset cannot make all agents worse off.

6.1. Proof of Theorem A.1

We base our argument on the existence proof provided by Hens (1991) for the standard GEI case.

Individual Excess Demand

Given $p_0 > 0$ and a vector (π_0, π) of asset prices, it is convenient to define $q = (p_0, \pi_0, \pi)$ and the budget set

$$\widehat{B}^h(q) = \left\{ \left(c_0, \theta_0^h, \theta^h\right) : p_0(c_0 + \kappa 1_{\{\theta^h \neq 0\}}) + \pi_0\theta_0^h + \pi \cdot \theta^h \leq p_0 e_0^h \right\}.$$

The no-arbitrage set

$$Q = \left\{ (p_0, \pi_0, \pi) \in \mathbb{R}_{++} \times \mathbb{R}^{J+1} \;\middle|\; \begin{array}{c} \text{there exists } \Lambda \in \mathbb{R}^S_{++} \text{ such that} \\ \pi_j = \Lambda . a_j \text{ for all } j = 0, .., J \end{array} \right\}.$$

is an open convex cone of $\mathbb{R}^{J+2}$, and it is useful to consider its closure

$$\overline{Q} = \left\{ (p_0, \pi_0, \pi) \in \mathbb{R}_{+} \times \mathbb{R}^{J+1} \;\middle|\; \begin{array}{c} \text{there exists } \Lambda \in \mathbb{R}^S_{+} \text{ such that} \\ \pi_j = \Lambda . a_j \text{ for all } j = 0, .., J \end{array} \right\}.$$

Given $q \in Q$, we can calculate the excess demands $Z^{hp}(q) \equiv [c_0^{hp}(q) + \kappa - e_0^h, \theta_0^{hp}(q), \theta^{hp}(q)]$ and $Z^{hn}(q) \equiv [c_0^{hn}(q) - e_0^h, \theta_0^{hn}(q), \theta^{hn}(q)]$ of a participating and non-participating agent of type h. Given a participation decision $d \in \{p, n\}$, the excess demand function $Z^{hd}(q)$ is continuous, homogeneous of degree 0, and satisfies Walras' law. We can then define the excess demand correspondence

$$Z^h(q) = \begin{cases} Z^{hp}(q) & \text{if } \; V^h\left[Z^{hp}(q)\right] > V^h\left[Z^{hn}(q)\right] \\ Z^{hn}(q) & \text{if } \; V^h\left[Z^{hp}(q)\right] < V^h\left[Z^{hn}(q)\right] \\ [Z^{hp}(q), Z^{hn}(q)] & \text{if } \; V^h\left[Z^{hp}(q)\right] = V^h\left[Z^{hn}(q)\right] \end{cases},$$

where $V^h(z)$ denotes the utility $U^h(c_0, \widetilde{e}^h + \theta_0 + \widetilde{a}.\theta)$ associated to an excess demand strategy $z = [c_0 + \kappa 1_{\{\theta \neq 0\}} - e_0^h, \theta_0, \theta]$. We observe that $Z^h(q)$ is homogeneous of degree 0, upper hemi-continuous and satisfies Walras' law.

Consider a vector $\overline{q} \in \overline{Q} \backslash Q$, $\overline{q} \neq 0$, and a sequence $\{q^n\}_{n=1}^{\infty}$ of elements of Q converging to $\overline{q}$. We want to show that $\inf\{\|z\| \,; z \in Z^h(q^n)\} \to +\infty$. Proceed by contradiction and assume that there exists a bounded sequence $\{z^{n_k}\}_{k=0}^{\infty}$, $z^{n_k} \in Z^h(q^{n_k})$ for all k. The sequence $\{z^{n_k}\}_{k=0}^{\infty}$ has then a cluster point $\overline{z}$. Without loss of generality, it is convenient to henceforth neglect subsequence notation and directly assume that $z^n \to \overline{z}$. Given $x \in \widehat{B}^h(\overline{q})$, we know that x is the limit of a sequence $\{x^n\}$, $x^n \in \widehat{B}^h(q^n)$. Since $x^n \in \widehat{B}^h(q^n)$, we know that $V^h(x^n) \leq V^h(z^n)$ for all n. Letting n go to infinity, we infer that $V^h(x) \leq V^h(\overline{z})$ for all $x \in \widehat{B}^h(\overline{q})$, which is absurd. This establishes that $\inf\{\|z\| \,; z \in Z^h(q^n)\} \to \infty$ as $n \to \infty$. We can also consider the matrices $M = [a_0, .., a_J]$ and $N = \begin{bmatrix} 1 & \\ & M \end{bmatrix}$, and show by a similar argument that $\inf\{\|z\| \,; z \in NZ^h(q^n)\} \to \infty$ as $n \to \infty$. Moreover since consumption is non-negative, the set $NZ^h(q^n) \geq -e^h$ is bounded below.

Market Excess Demand

We now define the market excess demand

$$Z(q) \equiv \sum_{h=1}^{H} \mu(h) Z^h(q).$$

The correspondence $Z(q)$ is upper hemi-continuous, convex and compact-valued, homogeneous of degree 0 and satisfies Walras' law: $q.Z(q) \equiv 0$. Moreover consider an arbitrary vector $\widehat{q} \in Q$ and a sequence $\{q^n\}_{n=1}^{\infty}$ of elements of Q converging to a vector $\overline{q} \in \overline{Q} \backslash Q$, $\overline{q} \neq 0$. Since each $NZ^h(q^n)$ is bounded below, we infer that $NZ(q^n)$ is bounded below and $\inf\{\|z\| \,; z \in NZ(q^n)\} \to \infty$. The absence of arbitrage implies that $\widehat{q} = N^{\intercal} \widehat{\Lambda}$ for some $\widehat{\Lambda} \in \mathbb{R}^{S+1}_{++}$. Since $\inf\{\|z\| \,; z \in NZ(q^n)\} \to \infty$, we infer that $\widehat{q}.Z(q^n) = \widehat{\Lambda}.NZ(q^n) > 0$ for n large enough. We then conclude by standard arguments (Debreu, 1956; Grandmont, 1977; Hens, 1991) that there exists an equilibrium price.

6.2. Proof of Theorem A.2

Assume that there exists a feasible allocation $(d_0^h, \widetilde{d}^h)_{h \in H}$ such that $U^h(d_0^h, \widetilde{d}^h) > U^h(c_0^h, \tilde{c}^h)$ for all h. We know that for all h, there exists (η_0^h, η^h) such that $\widetilde{d}^h = \tilde{e}^h + \eta_0^h + \tilde{a} \cdot \eta^h$. Since $(d_0^h, \widetilde{d}^h)$ is strictly preferred to $(c_0^h, \tilde{c}^h)$, it must be that $d_0^h + \pi_0 \eta_0^h + \pi \cdot \eta^h + \kappa 1_{\{\eta^h \neq 0\}} > e_0^h$. We aggregate across consumers: $\int \eta_0^h d\mu = 0$, $\int \eta^h d\mu = 0$ and $\int (d_0^h + \kappa 1_{\{\eta^h \neq 0\}}) d\mu > e_0$, which contradicts feasibility.

7. Appendix B - CARA-Normal Economies (General Case and One Factor)

7.1. Proof of Theorems 1 and 2

The decision problem of a participant consists of maximizing

$$-e^{-\chi c_0} - \beta \left\{ \mathbb{E} e^{-\gamma\left[\tilde{e}^h + \tilde{a}.\theta + R(e_0^h - c_0 - \pi.\theta - \kappa)\right]} \right\}^{\chi/\gamma}$$

with respect to the *unconstrained* variables c_0 and θ. For any choice of these variables, the random consumption $\tilde{c}$ has a normal distribution with mean $\mathbb{E}\widetilde{e}^h + R(e_0^h - c_0 - \pi.\theta - \kappa)$ and variance $Var(\widetilde{e}^{hA^\perp}) + Var(\widetilde{e}^{hA} + \widetilde{a}.\theta)$. With the notation $u(c) = -e^{-\chi c}$, the objective function reduces to

$$u(c_0) + \beta u[\mathbb{E}\tilde{c} - \gamma Var(\tilde{c})/2],$$

or equivalently

$$u(c_0) + \beta u(D - Rc_0) \exp\left[\chi R(\pi.\theta + \kappa) + \chi\gamma Var(\widetilde{e}^{hA} + \widetilde{a}.\theta)/2\right],$$

where $D = \mathbb{E}\widetilde{e}^h + Re_0^h - \gamma Var(\widetilde{e}^{hA^\perp})/2$ is exogenous to the agent.

The utility maximization problem is decomposed in two steps. First, the optimal portfolio θ^h minimizes the quadratic function

$$R(\pi.\theta + \kappa) + \gamma Var(\widetilde{e}^{hA} + \widetilde{a}.\theta)/2.$$

The first order condition implies that $\theta_j^{h,p} = -Cov(\tilde{a}_j, \tilde{e}^h) - R\pi_j/\gamma$. The optimal portfolio has random payoff $\widetilde{a}.\theta^{h,p} = -\tilde{e}^{hA} + \tilde{m}^A$, where $\tilde{m}^A = -(R/\gamma)\sum_{j=1}^J \pi_j \tilde{a}_j$.

Second, the initial consumption c_0 is chosen to maximize

$$u(c_0) + \beta u(D - Rc_0) \exp\left[\chi R(\pi.\theta^{h,p} + \kappa) + \chi\gamma Var(\widetilde{m}^A)/2\right]. \tag{7.1}$$

The first order condition $u'(c_0) = \beta R u'(D - Rc_0) \exp\left[\chi R(\pi.\theta^{h,p} + \kappa) + \chi\gamma Var(\widetilde{m}^A)/2\right]$ can be rewritten $c_0 = -\ln(\beta R)/\chi + D - Rc_0 - [R(\pi.\theta^h + \kappa) + \gamma Var(\widetilde{m}^A)/2]$, which implies

$$c_0^{h,p} = \frac{1}{1+R}\left\{R(e_0^h - \kappa - \pi.\theta^{h,p}) + \mathbb{E}\,\tilde{e}^h - \frac{1}{\chi}\ln(R\beta) - \frac{\gamma}{2} Var(\widetilde{c}^h)\right\}.$$

We then deduce $\theta_0^{h,p}$ from the budget constraint.

Similarly, a non-participant maximizes the function

$$u(c_0) + \beta u(D - Rc_0) \exp\left[\chi\gamma Var(\widetilde{e}^{hA})/2\right]. \tag{7.2}$$

Comparing the functional forms (7.1) and (7.2), we infer that participation is optimal if

$$\gamma Var(\widetilde{e}^{hA})/2 \geq R\left(\pi.\theta^{h,p} + \kappa\right) + \gamma Var(\widetilde{m}^{A})/2.$$

This is equivalent to $\gamma Var(\widetilde{e}^{hA} - \widetilde{m}^{A})/2 \geq \kappa R$.

7.2. Proof of Theorem 3

We obtain the price of risky assets by averaging $\theta_j^{h,p}$ across participating agents. The mean demand $\int_H \theta_0^h d\mu(h)$ for the riskless asset is

$$\frac{R}{1+R}\left[\begin{array}{c} e_0 - \mathbb{E}\tilde{e} + \chi^{-1}\ln(R\beta) + \frac{\gamma}{2}\int_H Var\left(\tilde{e}^h\right) d\mu(h) \\ -\kappa\mu(\mathcal{P}) + \frac{\gamma}{2}\mu(\mathcal{P})Var\left(\tilde{e}^{pA}\right) - \frac{\gamma}{2}\int_{\mathcal{P}} Var\left(\tilde{e}^{hA}\right) d\mu(h) \end{array}\right].$$

In equilibrium, mean demand is zero and the interest rate therefore satisfies (2.7).

7.3. Proof of Proposition 2

Financial innovation increases the assets span to $A' \supset A$, $A' \neq A$. The space A' can be decomposed in two orthogonal subspaces A and $B = A^{\perp} \cap A'$. By definition, R' and $\mathcal{P}'$ solve the system

$$\begin{cases} \mathcal{P}' = \left\{h : \frac{\gamma}{2}\left[Var\left(\tilde{e}^{hA} - \tilde{e}^{A}\right) + Var\left(\tilde{e}^{hB} - \tilde{e}^{B}\right)\right] \geqslant R'\kappa\right\} \\ \ln R' = \ln R_0 + \chi\mu\left(\mathcal{P}'\right)\kappa + \frac{\chi\gamma}{2}\int_{\mathcal{P}'}\left[Var\left(\tilde{e}^{hA} - \tilde{e}^{A}\right) + Var\left(\tilde{e}^{hB} - \tilde{e}^{B}\right)\right] d\mu(h). \end{cases}$$

Assume that $R' < R$. The first equation implies $\mathcal{P} \subseteq \mathcal{P}'$, and we infer from the second equation that $R' \geq R$, a contradiction.

7.4. Degenerate Cases of the One-Factor Economy

We begin by analyzing the special cases $\alpha = 0$ and/or $\kappa = 0$. When assets have no correlation with the risk factor ($\alpha = 0$), the participation set is empty under costly entry, and indeterminate under free entry. In either case, the risk premium is zero and the interest rate is uniquely determined: $R = R_0$. When the completeness index is positive ($\alpha > 0$) and the entry cost is positive and finite, we infer from Proposition 1 and Assumption 3 that the set of participants and non-participants both have a positive measure: $0 < \mu(\mathcal{P}) < 1$,[16] implying $R > R_0$

[16] If everyone participates, $\varphi^p = \overline{\varphi}$ and $\alpha\gamma(\varphi^h - \overline{\varphi})^2/2 \geq \kappa R$ for almost every agent h, which leads to a contradiction since the density f is strictly positive on every neighborhood of $\overline{\varphi}$.

in any equilibrium. Finally, there are no participants ($\mathcal{P} = \emptyset$) when the entry cost is infinite.

7.5. Proof of Properties 1-3

Consider the function

$$G(\varphi_p, \Lambda) = \int_{-\infty}^{\varphi_p - \Lambda} (\varphi - \varphi_p) d\mu + \int_{\varphi_p + \Lambda}^{+\infty} (\varphi - \varphi_p) d\mu$$

with domain $\mathbb{R} \times [0, +\infty)$. For every fixed $\Lambda \geq 0$, the partial function $G_\Lambda(\varphi_p) = G(\varphi_p, \Lambda)$ is continuous, strictly decreasing, and satisfies $\lim_{\varphi_p \to -\infty} G_\Lambda(\varphi_p) = +\infty$, $\lim_{\varphi_p \to +\infty} G_\Lambda(\varphi_p) = -\infty$. The equation $G_\Lambda(\varphi_p) = 0$ has therefore a unique solution, which is denoted by $\varphi_p(\Lambda)$. It is then convenient to define the set $\mathcal{P}_\Lambda = \{\varphi : |\varphi - \varphi^p(\Lambda)| \geq \Lambda\}$.

We infer from the Implicit Function Theorem that the function $\varphi_p(\Lambda)$ is differentiable. Let $\Delta(\Lambda) = f[\varphi^p(\Lambda) + \Lambda] - f[\varphi^p(\Lambda) - \Lambda]$ and $\nabla(\Lambda) = f[\varphi^p(\Lambda) + \Lambda] + f[\varphi^p(\Lambda) - \Lambda]$. We observe that $\partial G / \partial \varphi_p = -\Lambda \nabla - \mu(\mathcal{P}) < 0$, $\partial G / \partial \Lambda = -\Lambda \Delta$, and therefore

$$\frac{d\varphi_p}{d\Lambda} = -\frac{\Lambda\, \Delta(\Lambda)}{\Lambda\, \nabla(\Lambda) + \mu(\mathcal{P}_\Lambda)}.$$

The sign of $d\varphi_p / d\Lambda$ thus depends on the value of the density f at the endpoints $\varphi^p - \Lambda$ and $\varphi^p + \Lambda$. Since $\left| d\varphi_p / d\Lambda \right| \leq \Lambda \nabla / [\Lambda \nabla + \mu(\mathcal{P}_\Lambda)] \leq 1$, the functions $\varphi^p(\Lambda) - \Lambda$ and $\varphi^p(\Lambda) + \Lambda$ are respectively decreasing and increasing in Λ. We conclude that the set $\mathcal{P}_\Lambda$ is (weakly) decreasing in Λ.

7.6. Proof of Theorem 4

Consider the functions $H_0(\Lambda) = \mu(\mathcal{P}_\Lambda)$ and $H_1(\Lambda) = \mu(\mathcal{P}_\Lambda)(Var_{\mathcal{P}_\Lambda} \varphi)$. The monotonicity of $\mathcal{P}_\Lambda$ implies that $H_0(\Lambda)$ is decreasing in Λ. Similarly, the function

$$H_1(\Lambda) = \int_{-\infty}^{\varphi^p - \Lambda} (\varphi - \varphi_p)^2 f(\varphi) d\varphi + \int_{\varphi^p + \Lambda}^{+\infty} (\varphi - \varphi_p)^2 f(\varphi) d\varphi$$

has derivative $\Lambda^2 \left[f(\varphi_p - \Lambda) \frac{d(\varphi_p - \Lambda)}{d\Lambda} - f(\varphi_p + \Lambda) \frac{d(\varphi_p + \Lambda)}{d\Lambda} \right] + \int_{\mathcal{P}} 2(\varphi - \varphi_p) f(\varphi) d\varphi$, or

$$\frac{dH_1}{d\Lambda} = \Lambda^2 \frac{dH_0}{d\Lambda} < 0.$$

It is thus decreasing in Λ.

In equilibrium, R and Λ are determined by the system (3.7) – (3.8). We observe that R_1 is strictly increasing, R_2 is decreasing, $R_2(0) > R_0 > R_1(0) = 0$, and $R_1(+\infty) = +\infty$. The difference function $R_1(\Lambda) - R_2(\Lambda)$ is therefore strictly increasing and maps $[0, +\infty)$ onto $[-R_2(0), +\infty)$. There thus exists a unique equilibrium.

7.7. Proof of Theorem 5

The equilibrium (R, Λ) is determined by the system

$$\begin{cases} \kappa R - \alpha\gamma\Lambda^2/2 = 0, \\ \ln R - \ln R_0 - \kappa\chi H_0(\Lambda) - \alpha\chi\gamma H_1(\Lambda)/2 = 0. \end{cases}$$

The corresponding Jacobian matrix is

$$J = \begin{pmatrix} \kappa & -\alpha\gamma\Lambda \\ R^{-1} & J_{22} \end{pmatrix} \tag{7.3}$$

where $J_{22} = -\kappa\chi H_0'(\Lambda) - \alpha\chi\gamma H_1'(\Lambda)/2 = -\chi\kappa(1+R)H_0'(\Lambda) > 0$. We infer that $\det J > 0$.

We now infer from Cramer's rule the effect of financial innovation on the interest rate:

$$\frac{dR}{d\alpha} = -\frac{1}{\det J} \begin{vmatrix} -\gamma\Lambda^2/2 & -\alpha\gamma\Lambda \\ -\chi\gamma H_1(\Lambda)/2 & J_{22} \end{vmatrix} > 0. \tag{7.4}$$

Financial innovation therefore increases the interest rate. We similarly infer

$$\begin{aligned} \frac{d\Lambda}{d\alpha} &= -\frac{1}{\det J} \begin{vmatrix} \kappa & -\gamma\Lambda^2/2 \\ R^{-1} & -\chi\gamma H_1(\Lambda)/2 \end{vmatrix} \\ &= -(\kappa/\alpha)\left[1 - \alpha\chi\gamma\mu(\mathcal{P})(Var_{\mathcal{P}}\varphi)/2\right] / \det J, \end{aligned} \tag{7.5}$$

which has an ambiguous sign. The global behavior of Λ is established by a single crossing argument. We know that $\Lambda'(\alpha)$ has the same sign as $\alpha\chi\gamma H_1[\Lambda(\alpha)] - 1 \equiv G(\alpha) - 1$. Since $G(0) = 0$, the function $\Lambda(\alpha)$ is decreasing on a neighborhood of $\alpha = 0$. We observe that

$$G'(\alpha) = \chi\gamma H_1[\Lambda(\alpha)] + \alpha\chi\gamma\Lambda'(\alpha)H_1'[\Lambda(\alpha)].$$

Thus if α satisfies $G(\alpha) = 1$, we know that $\Lambda'(\alpha) = 0$ and $G'(\alpha) = \chi\gamma H_1[\Lambda(\alpha)] > 0$. The equation $G(\alpha) = 1$ has thus at most one solution on $(0, 1]$.

7.8. Proof of Proposition 3

The chain rule implies that

$$\frac{d\varphi^p}{d\alpha} = \frac{d\varphi^p}{d\Lambda}\frac{d\Lambda}{d\alpha}$$

has the same sign as $\eta_{\Lambda,\alpha}[f(\varphi^p + \Lambda) - f(\varphi^p - \Lambda)]$.

7.9. Proof of Theorem 6

We know from (7.5) that $\eta_{\Lambda,\alpha}$ has the same sign as $\alpha\chi\gamma\mu(\mathcal{P})(Var_{\mathcal{P}}\varphi)/2 - 1$. Since

$$\int_{\mathcal{P}} (\varphi - \overline{\varphi})^2 d\mu = \int_{\mathcal{P}} (\varphi - \varphi^p)^2 d\mu + \mu(\mathcal{P})(\varphi^p - \overline{\varphi})^2$$

we infer that $\mu(\mathcal{P})Var_{\mathcal{P}}\varphi \leq \int_{\mathcal{P}}(\varphi - \overline{\varphi})^2 d\mu \leq Var_H(\varphi)$. The condition $\chi\gamma Var_H(\varphi)/2 < 1$ therefore guarantees that $\eta_{\Lambda,\alpha} < 0$.

7.10. Effect of the Entry Fee

We can similarly analyze the effect of the transaction cost κ. We note that

$$\frac{dR}{d\kappa} = -\frac{1}{\det J}\begin{vmatrix} R & -\alpha\gamma\Lambda \\ -\chi H_0(\Lambda) & J_{22} \end{vmatrix}$$

has the sign of $\alpha\chi\gamma\Lambda\mu(\mathcal{P}_\Lambda) - RJ_{22}$, while

$$\frac{d\Lambda}{d\kappa} = -\frac{1}{\det J}\begin{vmatrix} \kappa & R \\ R^{-1} & -\chi H_0(\Lambda) \end{vmatrix} > 0.$$

This implies that the mass of participants decreases with the transaction cost κ. Finally, $d\varphi^p/d\kappa$ has the sign of $f(\varphi^p + \Lambda) - f(\varphi^p - \Lambda)$.

8. Appendix C - Multifactor Economies

8.1. Proof of Theorem 7

For every $\varphi^p \in \mathbb{R}^L$ and $\Lambda = (\Lambda_1, .., \Lambda_L) \in \mathbb{R}^L_{++}$, consider the set

$$\mathcal{P}(\varphi^p, \Lambda) = \left\{\varphi : \sum_{i=1}^{L} \left(\frac{\varphi_i - \varphi_i^p}{\Lambda_i}\right)^2 \geq 1\right\}. \tag{8.1}$$

The boundary of this set is an ellipsoid. An equilibrium consists of φ^p, Λ, and R satisfying

$$\int_{\mathcal{P}(\varphi^p,\Lambda)} (\varphi - \varphi^p) d\mu(\varphi) = 0, \tag{8.2}$$

$$\Lambda_\ell = \sqrt{2\kappa R/\alpha_\ell \gamma}, (1 \leq \ell \leq L),$$

and the market clearing condition

$$\ln R = \ln R_0 + \chi \int_{\mathcal{P}(\varphi^p,\Lambda)} \left[\kappa + \frac{\gamma}{2} \sum_{i=1}^{L} \alpha_i (\varphi_i - \varphi_i^p)^2 \right] d\mu(\varphi). \tag{8.3}$$

The analysis is simplified by

Fact C.1. *For any $\Lambda \in \mathbb{R}^L_{++}$, the equation $\int_{\mathcal{P}(\varphi^p;\Lambda)} (\varphi - \varphi^p) d\mu(\varphi) = 0$ has a unique solution $\varphi^p \in \mathbb{R}^L$.*

Proof. The equation can be conveniently rewritten as a convex optimization problem. More specifically, consider

$$k(\varphi; \varphi^p, \Lambda) = \left[\sum_{i=1}^{L} \left(\frac{\varphi_i - \varphi_i^p}{\Lambda_i} \right)^2 - 1 \right] 1_{\mathcal{P}(\varphi^p,\Lambda)}(\varphi),$$

where $1_{\mathcal{P}(\varphi^p,\Lambda)}$ denotes the indicator function of $\mathcal{P}(\varphi^p, \Lambda)$. Since $k(\varphi; \varphi^p, \Lambda)$ is convex in φ^p and the measure μ has an unbounded support, the function

$$K(\varphi^p, \Lambda) = \frac{1}{2} \int_{\mathbb{R}^L} k(\varphi; \varphi^p, \Lambda) d\mu(\varphi) = \frac{1}{2} \int_{\mathcal{P}(\varphi^p,\Lambda)} \left[\sum_{i=1}^{L} \left(\frac{\varphi_i - \varphi_i^p}{\Lambda_i} \right)^2 - 1 \right] d\mu(\varphi).$$

is *strictly convex* in φ^p. A vector φ^p thus minimizes $K(\varphi^p, \Lambda)$ on $\mathbb{R}^L$ if and only if $\partial K / \partial \varphi^p (\varphi^p, \Lambda) = 0$, which coincides with (8.2). It is therefore equivalent for a vector φ^p to minimize $K(\varphi^p, \Lambda)$ or to be the center of mass of $\mathcal{P}(\varphi^p, \Lambda)$. This observation is very useful for the numerical calculation of equilibrium. From a theoretical standpoint, note that the function $K(\varphi^p, \Lambda)$ is strictly convex on $\mathbb{R}^L$ and diverges to $+\infty$ as $\|\varphi^p\| \to +\infty$. This implies that the function $K(\varphi^p, \Lambda)$ reaches a minimum at a *unique* point φ^p. ■

Let φ^p_Λ denote the unique solution to (8.2), and $\mathcal{P}_\Lambda$ the corresponding participation set. Fact C.1 allows us to rewrite the equilibrium system as an equation of a unique variable, the interest rate R. For every $R > 0$, consider the lengths $\Lambda_\ell(R) = \sqrt{2\kappa R/(\alpha_\ell \gamma)}$, $(1 \leq \ell \leq L)$, and the vector $\Lambda(R) = [\Lambda_1(R), ..., \Lambda_L(R)]$. It is then natural to define the continuous functions $\varphi^p_{\Lambda(R)}$ and $\mathcal{P}_{\Lambda(R)}$, which will be henceforth denoted $\varphi^p(R)$ and $\mathcal{P}(R)$ for simplicity. We also consider the function

$$z(R) = \ln R_0 + \chi \int_{\mathcal{P}(R)} \left\{ \kappa + \frac{\gamma}{2} \sum_{i=1}^{L} \alpha_i \left[\varphi_i - \varphi_i^p(R)\right]^2 \right\} d\mu(\varphi). \tag{8.4}$$

The market clearing of the bond imposes that

$$z(R) = \ln R.$$

An equilibrium exists and is unique when the function $z(R)$ is (weakly) decreasing. We can indeed establish

Fact C.2. *The function $z(R)$ is decreasing in R.*

Proof. We show this property by differentiating $z(R)$ with respect to the interest rate. Note that on the boundary of $\mathcal{P}(R)$, the integrand $\kappa + \frac{\gamma}{2} \sum_{i=1}^{L} \alpha_i \left[\varphi_i - \varphi_i^p(R)\right]^2$ takes the constant value $\kappa(1 + R)$. The chain rule therefore implies

$$z'(R) = \chi\kappa(1+R)\frac{d\mu[\mathcal{P}(R)]}{dR} + \chi\gamma \int_{\mathcal{P}(R)} \left\{ \sum_{i=1}^{L} \alpha_i \left[\varphi_i - \varphi_i^p(R)\right] \frac{d\varphi_i^p(R)}{dR} \right\} d\mu(\varphi). \tag{8.5}$$

The second term is zero because φ^p is the center of mass. Thus,

$$z'(R) = \chi\kappa(1+R)\frac{d\mu[\mathcal{P}(R)]}{dR}. \tag{8.6}$$

This expression is non-positive by Fact C.3 below. ■

Fact C.3. *The mass of participants $\mu[\mathcal{P}(R)]$ is a decreasing function of R.*

Proof. The discussion proceeds in two steps. We first show that the property holds when indifferent agents are located on a sphere. We then extend the result to arbitrary ellipsoids.

Consider economies such that $\alpha_\ell = 1$ for all ℓ. The boundary of a participation set $\mathcal{P}(R)$ is a sphere, which is denoted $S(R)$. Given two positive numbers R and δ, $\delta < R$, we seek to show that

$$\mu[\mathcal{P}(R)] \leq \mu[\mathcal{P}(R - \delta)]. \tag{8.7}$$

The inequality is trivially satisfied when $\mathcal{P}(R) \subseteq \mathcal{P}(R-\delta)$. We now focus on the case $\mathcal{P}(R) \subsetneq \mathcal{P}(R-\delta)$. Since the indifference sets $S(R)$ and $S(R-\delta)$ are spheres, their intersection is contained in a hyperplane H:

$$S(R) \cap S(R-\delta) \subset H.$$

Without loss of generality, we choose the axes so that the hyperplane H is described by the equation $\varphi_1 = 0$, and the center of gravity $\varphi^p(R) = (x, 0 \ldots 0)$ has a positive first coordinate x. It is straightforward to show that $\varphi^p(R-\delta)$ has coordinates $(y, 0 \ldots 0)$, where $y < x$.[17] We denote by $\mathcal{P}_- = \mathcal{P}(R) \backslash \mathcal{P}(R-\delta)$ the set of participants lost in moving from R to $R-\delta$, by $\mathcal{P}_+ = \mathcal{P}(R-\delta) \backslash \mathcal{P}(R)$ the set of gained participants, and by $\mathcal{P}_C = \mathcal{P}(R) \cap \mathcal{P}(R-\delta)$ the common intersection. Figure C1 illustrates these definitions. The subset $\mathcal{P}_-$ is contained in the half-space $\varphi_1 < 0$, and the subset $\mathcal{P}_+$ in the half-space $\varphi_1 > 0$. Since $\mathcal{P}(R) = \mathcal{P}_- \cup \mathcal{P}_C$ and $\mathcal{P}(R-\delta) = \mathcal{P}_+ \cup \mathcal{P}_C$, we infer that

$$\begin{aligned}
\int_{\mathcal{P}_-} \varphi_1 d\mu(\varphi) + \int_{\mathcal{P}_C} \varphi_1 d\mu(\varphi) &= x\, \mu(\mathcal{P}_- \cup \mathcal{P}_C), \\
\int_{\mathcal{P}_+} \varphi_1 d\mu(\varphi) + \int_{\mathcal{P}_C} \varphi_1 d\mu(\varphi) &= y\, \mu(\mathcal{P}_+ \cup \mathcal{P}_C).
\end{aligned}$$

Substracting these equalities implies

$$\int_{\mathcal{P}_+} \varphi_1 d\mu(\varphi) - \int_{\mathcal{P}_-} \varphi_1 d\mu(\varphi) = y\, \mu(\mathcal{P}_+ \cup \mathcal{P}_C) - x\, \mu(\mathcal{P}_- \cup \mathcal{P}_C).$$

The left-hand side of the equation is positive because $\mathcal{P}_+$ is contained in the half-space $\varphi_1 > 0$ and $\mathcal{P}_-$ is contained in the half-space $\varphi_1 < 0$. This implies the inequality: $y\, \mu(\mathcal{P}_+ \cup \mathcal{P}_C) \geq x\, \mu(\mathcal{P}_- \cup \mathcal{P}_C)$. Since $x > y$, we infer that

$$x\, \mu(\mathcal{P}_+ \cup \mathcal{P}_C) \geq y\, \mu(\mathcal{P}_+ \cup \mathcal{P}_C) \geq \ \mu(\mathcal{P}_- \cup \mathcal{P}_C),$$

and conclude that inequality (8.7) holds in the spherical case.

When the coefficients α_ℓ are arbitrary, a linear change of variables allows us to return to the spherical case we just examined. Thus, consider the linear rescaling $\varphi_\ell^* = \Phi_\ell(\varphi) = \varphi_\ell \sqrt{\alpha_\ell}$, and the corresponding measure $\mu^* = \mu \circ \Phi^{-1}$. Note that this transformation does not involve a particular choice of R. For every $R > 0$,

[17] The condition $\mathcal{P}(R) \subsetneq \mathcal{P}(R-\varepsilon)$ implies that $y - \Lambda_1(R-\varepsilon) < x - \Lambda_1(R)$ and thus $y < x + \Lambda_1(R-\varepsilon) - \Lambda_1(R) < x$.

the rescaled set $\mathcal{P}^*(R) = \Phi[\mathcal{P}(R)]$ has a spherical boundary centered around $\varphi^{*p}(R) = \Phi[\varphi^p(R)]$:

$$\mathcal{P}^*(R) = \left\{ \varphi : \frac{\gamma}{2} \sum_{\ell=1}^{L} [\varphi_\ell^* - \varphi_\ell^{*p}(R)]^2 \geq \kappa R \right\}.$$

Furthermore, the condition $\int_{\mathcal{P}(R)} [\varphi - \varphi^p(R)] d\mu(\varphi) = 0$ implies that $\varphi^{*p}(R)$ is the center of gravity of $\mathcal{P}^*(R)$. We then conclude from the previous paragraph that the function $\mu[\mathcal{P}(R)] = \mu^*[\mathcal{P}^*(R)]$ is decreasing in R. ■

8.2. Proof of Proposition 4

We provide an example in an economy with two uncorrelated factors $(\varepsilon_1, \varepsilon_2)$ and a finite number of types. Letting $\delta = 0.01$, we consider $\varphi^A = (-2, 0)$, $\varphi^B = (1+\delta, 0)$, $\varphi^C = (2, 0)$, $\varphi^- = (0, -1+\delta)$ and $\varphi^+ = (0, 1-\delta)$, with respective weights $m^A = m^B = 1/5$, $m^C = 1/10$, $m^+ = m^- = 1/4$. The other parameters of the economy are $\gamma = \chi = 0.7$, $\kappa = 0.3$, $e_0 = \mathbb{E}\widetilde{e} = 1$, $\alpha_2 = 0.9$.

A straightforward extension of Theorem 7 implies that a unique equilibrium exists for any given value of α_1. When $\alpha_1 = 0.55$, we check that the participation set contains all the agents of type $A, C, +$ and $-$. The fraction of participants is $4/5$ and the net rate is approximately 7.9%.

On the other hand when $\alpha_1 = 0.9$, the participation set contains all the agents of type A, B, C. The participation rate has now fallen to $1/2$ and the net interest rate is now approximately 5.7%.

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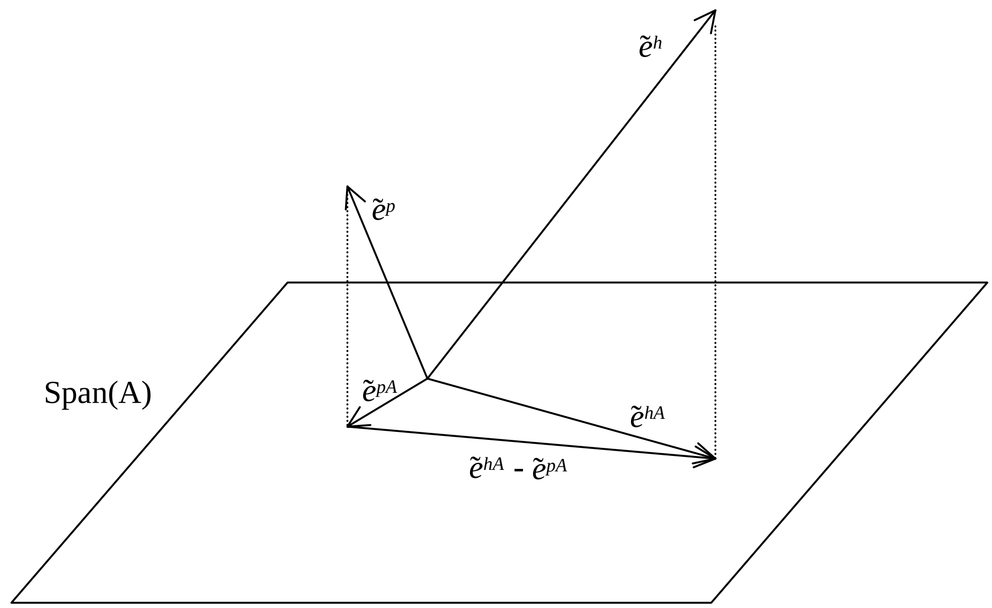

Figure 1: **Geometry of the Entry Condition**

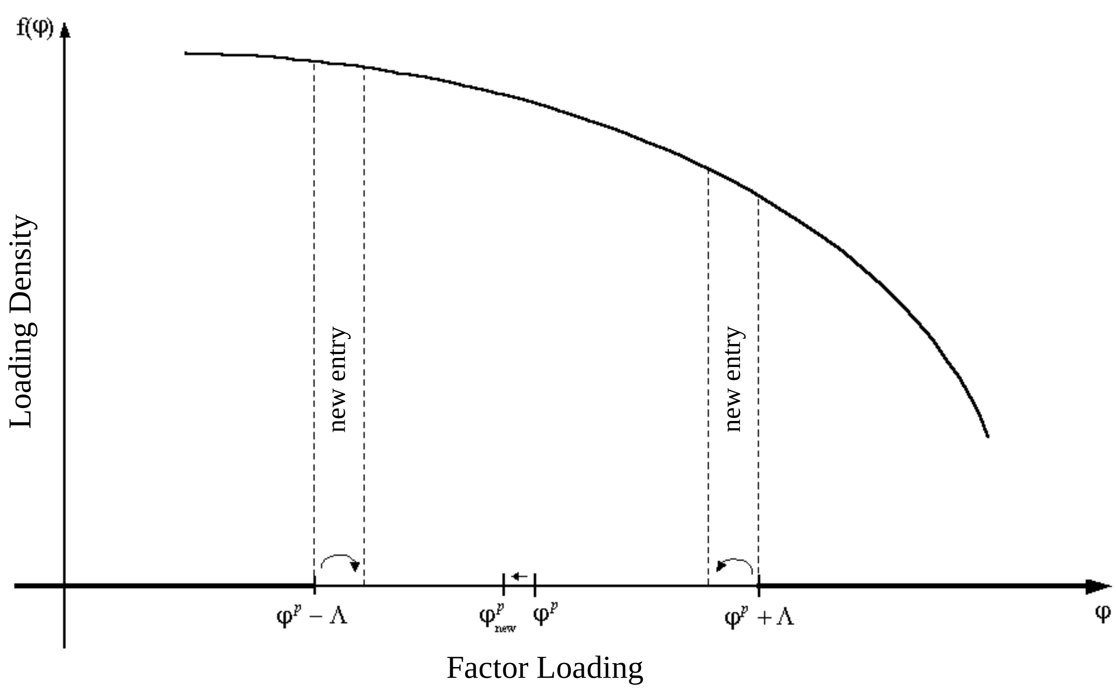

Figure 2: **Effect on Participation of a Decrease in Λ**

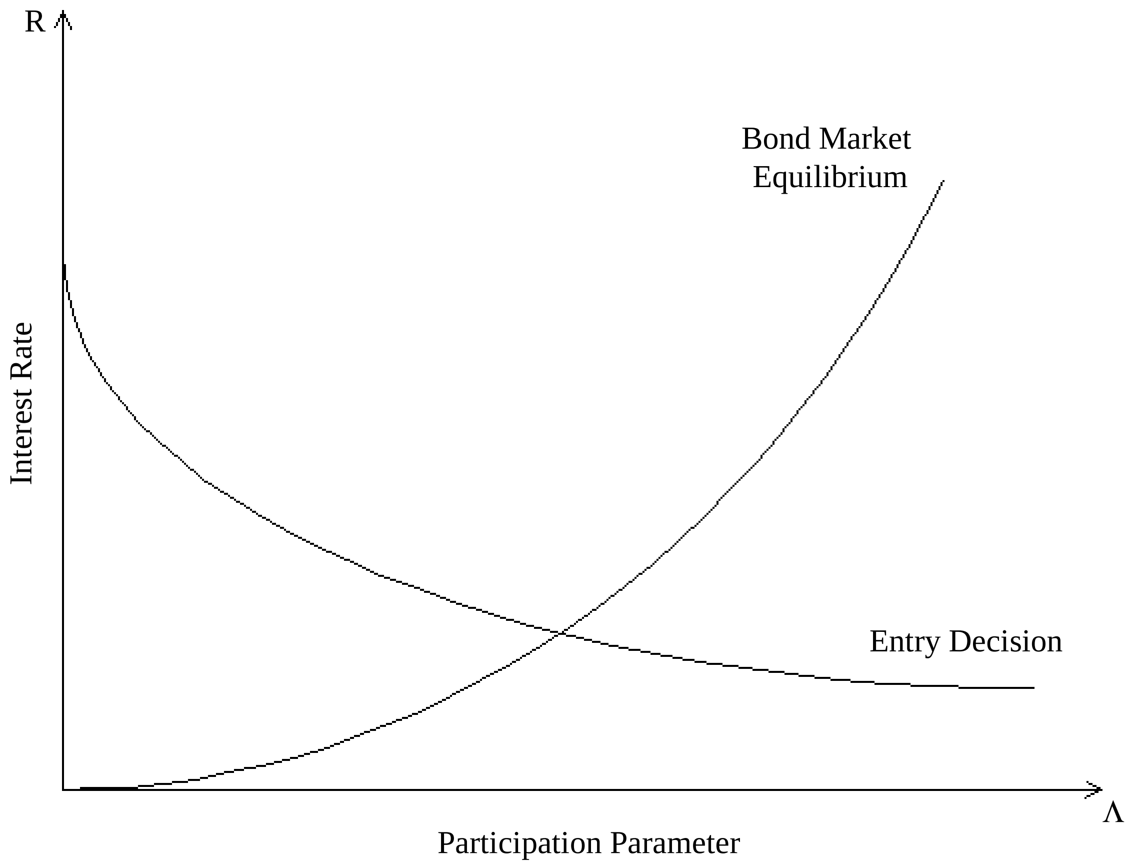

Figure 3: **Equilibrium of the One-Factor Economy**

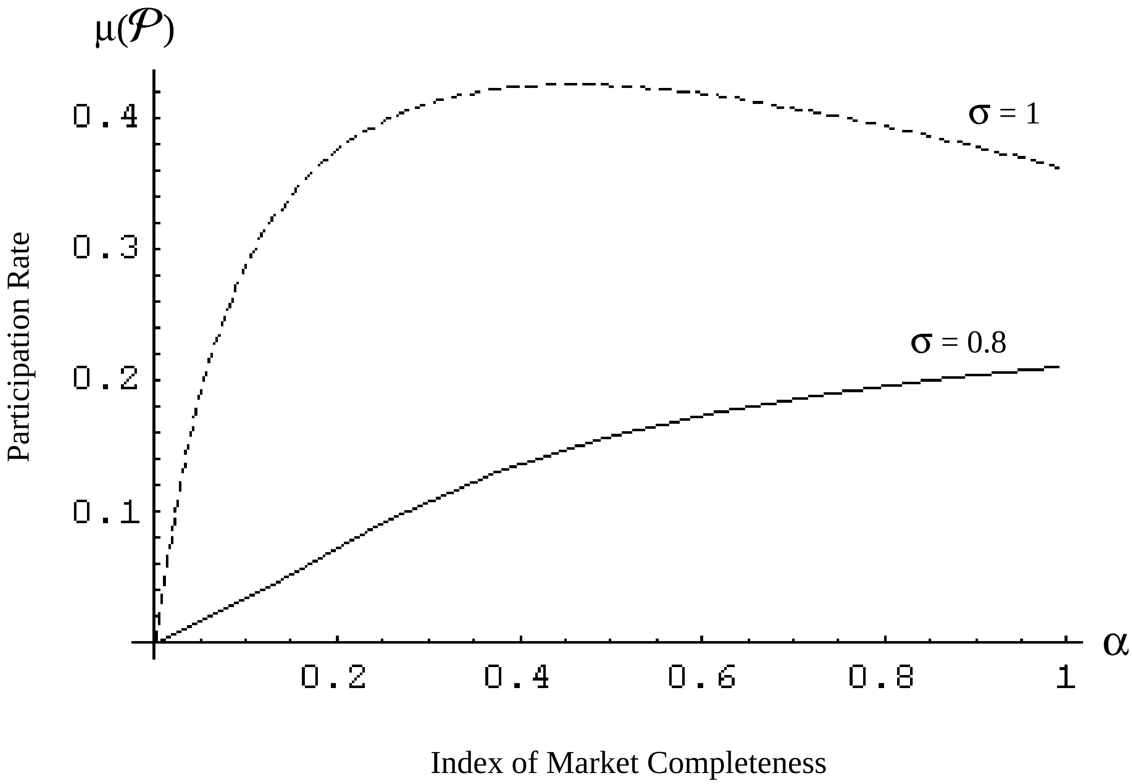

Figure 4: **Effect of Financial Innovation on Market Participation.** The cross-sectional loading distribution is log-normal: $\ln(\varphi) \sim \mathcal{N}(0, \sigma^2)$. The solid curve corresponds to $\sigma = 0.8$, and the dashed curve to $\sigma = 1$. The other parameters of the economy are: $\gamma = \chi = 1$, $\kappa = 1$ and $\beta = 1$.

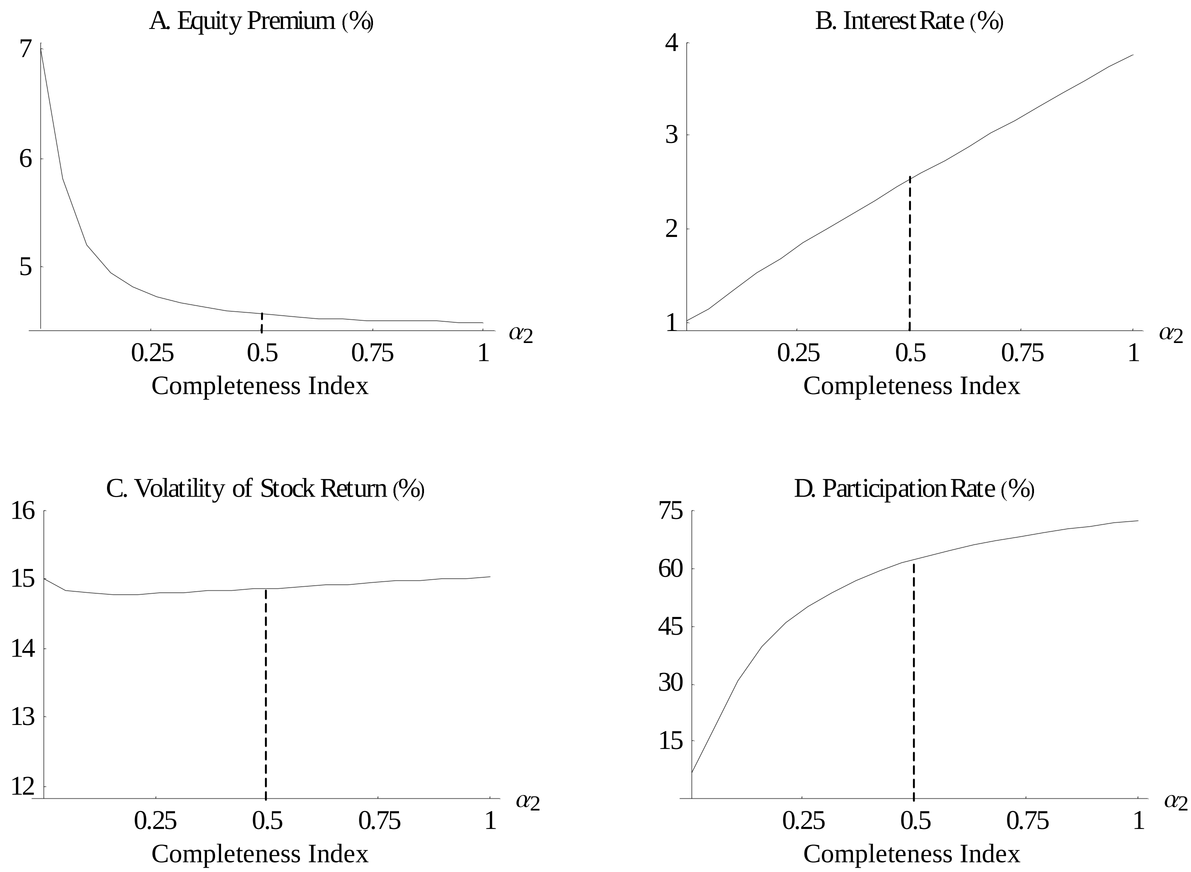

Figure 5: **Calibration of a Two-Factor Economy.** Individual labor income is exposed to an aggregate shock ε_1 and an idiosyncratic risk ε_2. The aggregate shock is partially insurable ($\alpha_1 = 0.5$). The idiosyncratic risk is uncorrelated to the existing asset when $\alpha_2 = 0$ and is fully insurable when $\alpha_2 = 1$. The other calibration parameters are $\beta = 0.96$, $\gamma = 10$, $\chi = 0.5$, $\kappa = 0.8\%$, $\sigma_1 = 4\%$ and $\sigma_2 = 10\%$.

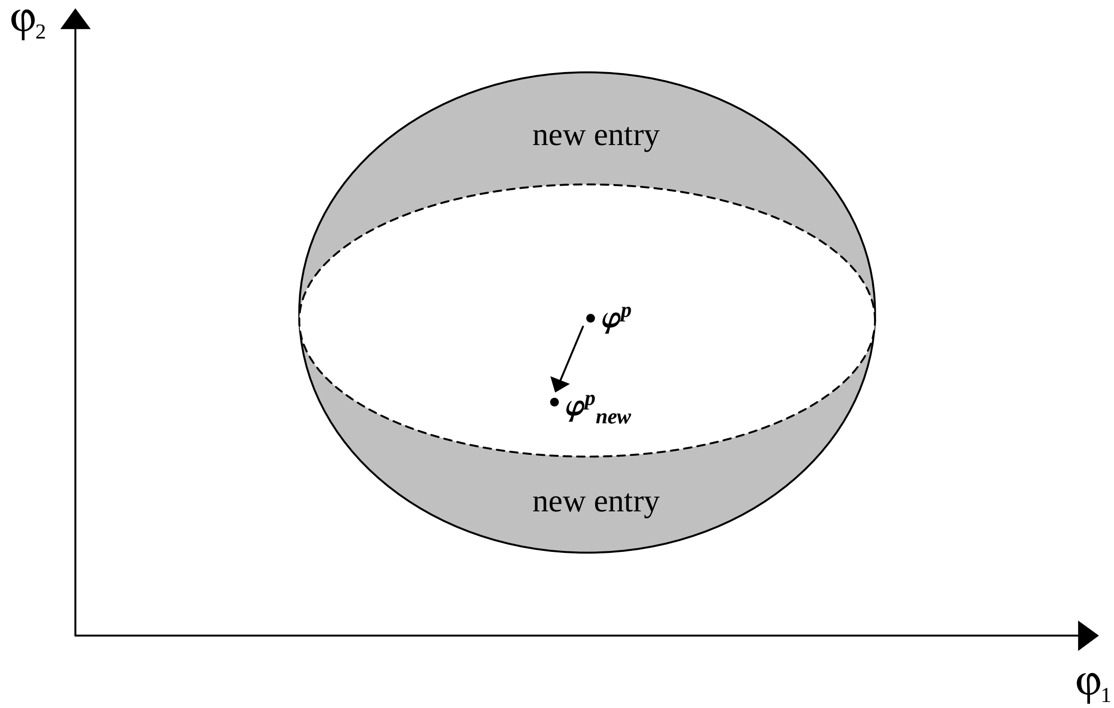

Figure 6a: **Effect of an Increase in α_2 on the Set of Participants.**

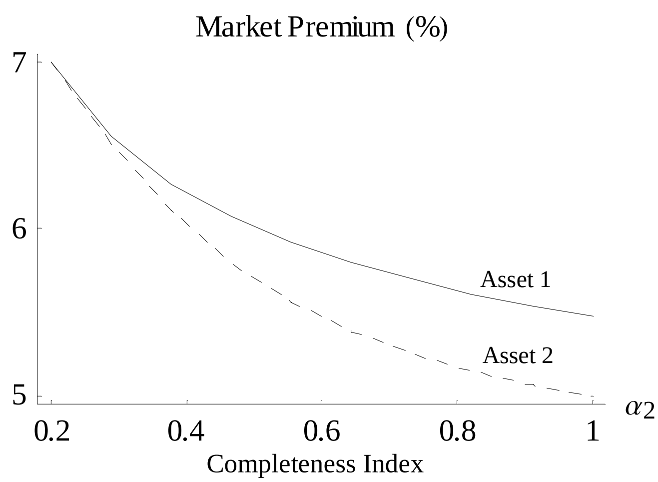

Figure 6b: **Differential Effects of Financial Innovation.** The cross-sectional loading density is the product: $f(\varphi_1, \varphi_2) = g(\varphi_1)\, g(\varphi_2)$, where the function g is the density of a log-normal variable Z: $\ln Z \sim \mathcal{N}(-3.5, 1)$. The other parameters of the economy are: $\alpha_1 = 0.2$, $\gamma = 10$, $\chi = 0.5$, $\beta = 0.96$ and $\kappa = 0.8\%$.

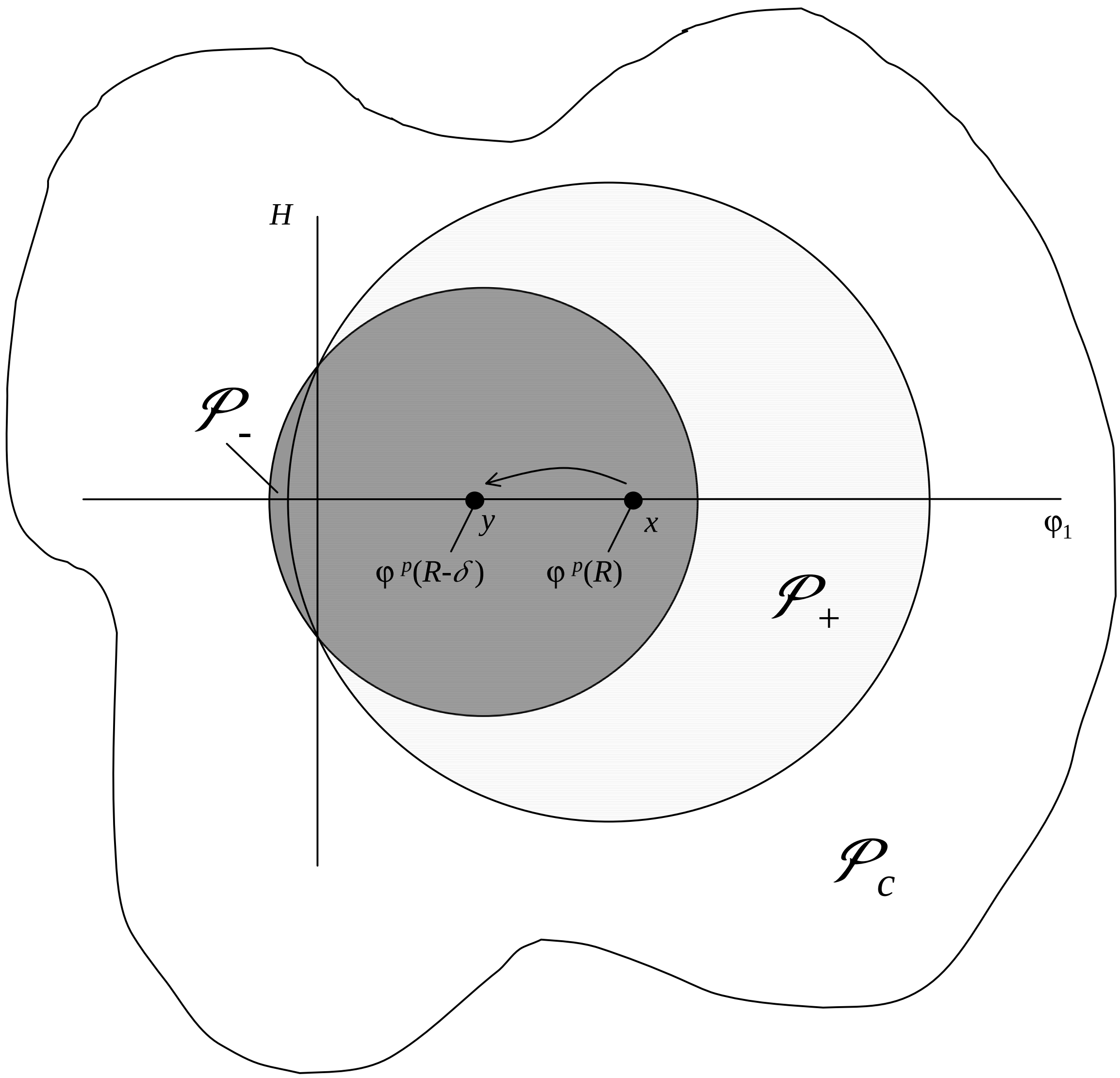

Figure C1: **Geometry of the Participation Sets**