National Income in Dynamic Economies.

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Abstract

Which index number best measures welfare changes in a dynamic economy? A significant number of economists now disagree with the traditional answer, national income. Here we develop the theoretical case for a new welfare index with two attractive properties: it meets all criticisms of standard measures, those from the environmental and the growth communities, and has an intellectual rationale that is solidly founded in welfare economics. Called National Wealth (NW), this measure has all the properties possessed by an ideal welfare or income measure in a static context: an increase in national wealth is a necessary and sufficient condition for a potential Pareto improvement, a property not shared by any other measure proposed in the literature. National wealth is the value of a consumption plan at supporting prices, a generalization of the static measure, although in the dynamic context it appears unfamiliar. Changes in NW can be inferred from observable contemporaneous data although the full measure depends on future variables that are not currently observable.

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1 Welfare in a growing economy

Which index number best measures welfare changes in a dynamic economy? And is national income interest on a measure of national wealth? We show below that these apparently distinct issues are in fact two aspects of the same question. A significant number of economists now disagree with the traditional answer to the first question, and believe that national income is not the best measure of welfare changes. In their 1973 paper "Is growth obsolete?" William Nordhaus and James Tobin [23] established the dissenting tradition and proposed a new index, the measure of economic welfare, MEW. Subsequently many others have made additional criticisms of standard national income measures and proposed additional corrections to the conventional measure.[1] Indeed, Robert Solow [28] has remarked that "It is a commonplace that the national income and product accounts, as currently laid out, give a misleading picture of the value of a nation's economic activity ..."

Most critics of welfare indices based on national income accounts disapprove because of the omission of important environmental consequences of economic activity. Nordhaus and Tobin based their critique largely on external effects that detract from the quality of life, to which subsequent authors have added concerns about the impact on future generations of depletion of the environmental resource base, under the heading of sustainability (see Geoffrey Heal [16]). Growth theorists have recently added new insights, arguing that standard measures omit important variables related to the role that human skills and society's knowledge base and other intangibles play as assets (Robert Hall [12]).

Below we develop the theoretical case for a new welfare index with two attractive properties: it meets all criticisms of standard measures, those from the environmental and the growth communities, and has an intellectual rationale that is solidly founded in welfare economics. Called National Wealth (NW), this measure has the key property possessed by an ideal welfare or income measure in a static context: an increase in national wealth is a necessary and sufficient condition for a potential Pareto improvement, a property not shared by any other measure proposed in the literature. National wealth is the value of a consumption plan at supporting prices, a generalization of the static measure, although in the dynamic context it appears unfamiliar. Changes in NW can be inferred from observable contemporaneous data although the full measure depends on future variables that are not currently observable.

After explaining the concept and establishing its properties, we relate it to other proposals. It is particularly interesting that the World Bank's "genuine savings" measure (Kirk Hamilton and Michael Clemens [13]) represents changes in NW. Our results provide an interesting formal perspective on some remarks of Solow's [28] to the effect that "Properly defined and properly calculated, this year's net national product can always be regarded as this year's interest on society's total stock of

[1] See amongst others Weitzman [29], Maler [22], Dasgupta [8], Asheim [3] [2], Aronson et al. [1], Dasgupta et al. [9], Lozada [21] and Hartwick [15].

capital." Solow proffers this as a conjecture rather than a formal result, and Partha Dasgupta and Karl-Göran Mäler disagree [11], stating that this is true only "if well-being is linear and accounting prices are constant over time." So there is uncertainty, possibly even controversy, over this issue. Of course the idea of income as interest on wealth goes back at least to John Hicks [18], and is an integral part of our thinking about individual income, so it is important to clarify whether this relationship holds at the national level. We show that net national product is the return on national wealth under certain conditions, and that quite generally the change in net national product as a result of a policy perturbation is the return on the corresponding change in national wealth. The return here is computed at the real discount rate, a point fully anticipated by Solow when he refers to the need to use "the real discount rate implicit in the whole story." The argument works via the Hamiltonian of a dynamic optimization problem, which as noted by Martin Weitzman [29] has a fundamental connection to measures of national income and welfare. In defining an index number for welfare in a dynamic economy we are also establishing the relationship between income and wealth. Because the rate of return used to compute interest is a real rate, involving price changes, the changes in wealth and in income resulting from a perturbation need not have the same sign, so that the income change need not have the same sign as the welfare change. Wealth and welfare always change in the same direction, however.

In the next section we present a general dynamic economic model which we use as the analytical framework for the remainder of the paper. Then in section 2.1 we present some basic but fundamental results on measuring welfare and welfare changes in a dynamic economy, all based on the properties of the state valuation function. These result are well-known but we summarize them for convenience. In section 3 we introduce the concept of national wealth and develop its properties. Section 4 presents results relating wealth to income measures and also shows that an increase in the Hamiltonian or in other measures related to this, including net national product, does not necessarily imply a Pareto improvement or an increase in the welfare function of the economy. Only changes in wealth are unambiguously indicative of welfare changes. Section 5 considers extensions to non-utilitarian objectives.

Before we move to the full dynamic model that allows us to formalize these points, it may be worth devoting space to a simple and intuitive analysis of the basic issues. Consider a consumer whose utility depends on two consumption goods $u(c_1, c_2)$ and who maximizes utility subject to a budget of m. Then the change in utility from a small change in consumption is $\Delta u = u_1 \Delta c_1 + u_2 \Delta c_2$ using the obvious notation that $\frac{\partial u}{\partial c_i} = u_i$. Dividing by the marginal utility of income L (assumed constant over a small change) gives $\Delta u/L = p_1 \Delta c_1 + p_2 \Delta c_2$ where the ps are market prices. From this it follow that if the market value of the change is positive then it increases welfare independently of the exact form of the utility function. Also note that if c_1 and c_2 are present and future consumption then the ps are present value prices and an increase in the present value of consumption increases welfare. In the intertemporal case the budget constraint m is a present value constraint and measure of wealth, and an increase in wealth at market prices increases welfare. It is these very basic insights that this paper extends to a general intertemporal framework.

2 A general dynamic model

As is standard in this literature, we use a generalized Ramsey model [25] as the workhorse. Let the vector $c(t)$ be an m-vector of flows of goods consumed and giving utility at time t, and $s(t)$ be an n-vector of stocks at time t, also possibly but not necessarily sources of utility. Each stock $s_i(t)$ changes over time in a way which depends on the values of all stocks and of all flows:

$$\dot{s}_i(t) = d_i\left(c(t), s(t)\right), i = 1, ..., n \tag{1}$$

We begin with a relatively simple case in which the economy's preferences are represented by the discounted sum of utilities, a case, we should note, that many environmentalists regard as inappropriate in terms of the present-future balance that it strikes. We shall consider alternatives later. The economy's objective is to maximize the discounted (at discount rate δ) integral of utilities (2):

$$\max \int_0^\infty u\left(c(t), s(t)\right) e^{-\delta t} dt \tag{2}$$

subject to the rate-of-change equations (1) for the stocks. The utility function u is assumed to be strictly concave and the reproduction functions $d_i\left(c(t), s(t)\right)$ are assumed to be concave. This is a very general and flexible formulation, and in the following we shall frequently specialize it to simple and more familiar cases. This general formulation makes it possible to include human capital and other assets. Note that if $n = m = 1$ and $u = u\left(c\right)$ and $\dot{s} = f\left(s\right) - c$, then we have the Ramsey-Solow model. In the context of resource economics, when u does not depend on s and (1) takes the form $\dot{s}_i(t) = -c_i(t)$ then we have the Hotelling model. The general formulation we have chosen captures the possible contributions of environmental stocks to consumer utility and to productive efficiency. One issue that this formulation avoids is that of population growth: by not specifying population or labor force specifically, we are implicitly taking the population to be constant, and for simplicity we can take it to be one in perpetuity. Population growth, whether exogenous or endogenous, leads to complications that we do not explore here (see Solow [27]).

To solve this problem we construct a Hamiltonian which takes the form

$$H(t) = u\left(c(t), s(t)\right) e^{-\delta t} + \sum_{i=1}^{n} \lambda_i(t) e^{-\delta t} d_i\left(c(t), s(t)\right) \tag{3}$$

where the $\lambda_i(t)$ are the shadow prices of the stocks. Two features of this formulation may be worth commenting upon. First, note that the Hamiltonian is in some sense a utility function. It suggests that there is a trade off between the current and the future; consumption "now" provides instant utility as measured by $u\left(c(t), s(t)\right)$, which is traded-off against a smaller net investment $\sum_{i=1}^{n} \lambda_i(t) e^{-\delta t} d_i\left(c(t), s(t)\right)$ in this period. Second, if we insert the Solow model in the Hamiltonian, we obtain a definition of net national income (in utility terms). If the utility function is linear in c, the correspondence to net national income is exact.

The first order conditions for optimality can be summarized as

$$\frac{\partial u\left(c(t), s(t)\right)}{\partial c_j} = -\sum_{i=1}^{n} \lambda_{i,t} \frac{\partial d_i\left(c(t), s(t)\right)}{\partial c_j} \tag{4}$$

and

$$\dot{\lambda}_i(t) - \delta\lambda_i(t) = -\frac{\partial u\left(c(t), s(t)\right)}{\partial s_i} - \sum_{k=1}^{n} \lambda_k(t) \frac{\partial d_k\left(c(t), s(t)\right)}{\partial s_i} \tag{5}$$

2.1 Measuring future welfare

In this section we present basic results about the present discounted value of welfare along the future of an optimal path, providing measures of the changes in this and of its average value. We make use of the state valuation function $V(s)$, which we define in the usual manner:

$$V(s_0) = \max_{\{c_t\}} \int_0^{\infty} u(c, s)\, e^{-\delta t} dt,\ \dot{s}_{i,t} = d_i\left(c_t, s_t\right), i = 1, ..., n,\ s_0 \text{ given}$$

This function gives the maximum present value utility obtainable from the initial stock vector s_0. By standard results,

$$\frac{\partial V}{\partial s_i} = \lambda_i \tag{6}$$

so that the shadow price of the $i-$th stock is the marginal social productivity of that stock. It immediately follows that

$$\frac{dV}{dt} = \sum_i \lambda_i \dot{s}_i \tag{7}$$

Note that (7) tells us something of interest: it states that the rate of change of the state valuation function along an optimal path is positive if and only if the value of stocks at shadow prices is increasing. Formally,

Proposition 1 Future welfare rises along an optimal path if and only if investment is positive at shadow prices.

This result is in Dasgupta and Mäler [10] and Pemberton and Alistair Ulph [24], as well as Geir Asheim and Martin Weitzman [3]. The precise origin of the result that $\frac{dV}{dt} = \sum_i \lambda_i \dot{s}_i$ is difficult to pin down; in general terms it has been known in the economics literature for several decades. We can obtain a second expression for the rate of change of the state valuation function by differentiating under the integral sign in the definition:

$$\frac{dV}{dt} = -u(c,s) + \delta \int_0^\infty u(c,s)\, e^{-\delta t} dt$$

Equating the two expressions for $\frac{dV}{dt}$ gives

$$\delta V = H \tag{8}$$

so that the Hamiltonian can be seen as "interest" on the state valuation function, where the interest rate is the discount rate. This result is a special case of the well-known Hamilton-Jacobi equation of classical physics, see Carson Haurie and Liezarowitz [5]. Given (8), it is tempting to think of the Hamiltonian H as income and the state valuation function V as wealth. Unfortunately the properties of H and V do not usually support these interpretations. Our task is to find definitions of income and wealth for which a relationship like (8) holds.

Let $\{c^*(t), s^*(t)\}$ be the solution to the problem of maximizing (2) subject to (1). Also let $CH(t)$ be the Hamiltonian corresponding to this problem, not discounted to time zero: $CH(t)$ is the current value Hamiltonian. Then we have the following generalization of Weitzman [29]:

Proposition 2 A utility stream from t to infinity of a constant value equal to the Hamiltonian evaluated on the optimal path at t has the same present value as the utility stream from t to infinity associated with a solution to the problem of maximizing (2) subject to (1). Formally, for any date t,

$$\int_t^\infty CH(t)\,(c^*(t), s^*(t))\, e^{-\delta(\tau - t)} d\tau = \int_t^\infty u(c^*(\tau), s^*(\tau))\, e^{-\delta(\tau - t)} d\tau$$

Proof. This result follows immediately from (8). Note that $V = H/\delta = \int_0^\infty H e^{-\delta t} dt$.[2] ■

The Hamiltonian is thus a measure of the "equivalent constant utility level" associated with the future of an optimal path, and, as many authors have noted, it is a natural candidate for a measure of Hicksian national income. It is sometimes referred to as a "sustainable" utility level, but this is in fact inaccurate (see Geoffrey Heal and

[2] Weitzman (1976 [29]) published this result for the case of linear utilities.Many others have since extended it, and it has become the foundation for a large number of papers on measuring national income in a dynamic economy. Although the derivation here is simple, this is deceptive: there is a lot of machinery behind it.

Bengt Kriström [17]): it is an average utility level, but not necessarily a utility level that can be maintained for ever. This leaves open the issue of whether an increase in the Hamiltonian is an increase in welfare as measured by the objective function of the optimal growth problem, or whether it is a potential Pareto improvement. We return to this later, showing that in general neither of these prepositions is true: the welfare implications of an increase in the Hamiltonian are limited to those stated in proposition 2.

3 National wealth

We can now move to the main concept of this paper, a concept in the mainstream tradition of welfare economics established in the 1950s and 1960s. It draws on the standard theory of static welfare economics and extends it to a dynamic context via the time-dated commodity approach of Kenneth Arrow and Gerard Debreu: it does not address directly the task of computing a static equivalent to future consumption levels and so has no direct connection with the issues underlying the conventional interpretations of sustainability. Nevertheless it lends itself naturally to the discussion of long-run issues, in fact more so than Hamiltonian-based approach, and so in this sense is useful for answering questions about sustainability. There is an irony here: an approach devised specifically to grapple with sustainability and its interpretation is in the end less well-adapted to this than a more generic approach sanctioned by traditional theory.

In the context of an intertemporal economy where consumption bundles are given by functions of time, prices must likewise be functions of time for the entire infinite horizon. National wealth is a measure of national welfare based the separating hyperplane approach used in general equilibrium theory and welfare economics, but restated to the context of the general model in equations (1) and (2).

The use of arguments about separating hyperplanes in problems involving infinite time horizons is mathematically quite delicate, so we need to be precise about the framework to be used. We shall assume that the functions $d_i\left(c(t), s(t)\right), i = 1, ..., n$ are such that the set of feasible paths for $c_j(t)$ and $s_i(t)$ is bounded: reasonable conditions sufficient for this are presented for the specific models used here in Heal [16].[3] A hyperplane which supports the optimal path is one that separates the set of paths preferred to an optimum from those which are feasible (for a formal definition see the previous footnote and the appendix). This is a time path of prices for stocks

[3] Under this assumption, the paths of all variables, including utilities, are such that their integrals against a discount factor with a positive discount rate are finite. Formally, for any i and j, $\int_0^\infty c_j(t)e^{-\delta t}dt < \infty$, $\int_0^\infty s_i(t)e^{-\delta t}dt < \infty$ where $c_j(t)$ and $s_i(t)$ are real-valued functions of time. We can therefore regard the space of possible paths of consumptions levels and stocks as a weighted l_∞ space, with the norm $\|f(t)\| = \sup_t \left|f(t)\, e^{-\delta t}\right|$ and the inner product of two functions $f(t)$ and $g(t)$ being $\langle f, g\rangle = \int_0^\infty f(t)\, g(t)\, e^{-\delta t}dt$. A supporting hyperplane for a set S is then given by a function $h(t)$ such that everything in the set is above it in the sense of having at least as great a

and flows $p_{c,j}(t)$ and $p_{s,i}(t)$ which satisfies two conditions: any path at least as good as the optimum has a value at these prices at least as great as the optimal path, and any feasible path costs no more than the optimum.

Definition 3 A set of prices $p_{c,j}(t)$ and $p_{s,i}(t)$ supporting the optimal path will be called optimal prices and will be used to define national wealth as follows: national wealth along the optimal path is

$$\int_0^\infty \{\langle p_c(t), c^*(t)\rangle + \langle p_s(t), s^*(t)\rangle\} e^{-\delta t} dt.$$

Here $\langle p_c(t), c^*(t)\rangle$ represents the inner product of the price vector $p_c(t)$ at time t with the consumption vector $c^*(t)$ at time t. We want ultimately to establish that any small change which increases this measure is a welfare improvement. In the next proposition we characterize a set of optimal prices. These prices are quite intuitive: they are the marginal utilities of the stocks and flows along an optimal path. So price ratios are just marginal rates of substitution as usual. These marginal utilities define the marginal rates of substitution between the different arguments of the maximand $\int_0^\infty u(c,s) e^{-\delta t} dt$, and are natural candidates for the role of defining a separating hyperplane.

Proposition 4 The sequence of prices defined by the derivatives of the utility function along an optimal path, i.e.,

$$\{p_{c,j}(t), p_{s,i}(t)\} = \left\{\frac{\partial u(c^*(t), s^*(t))}{\partial c_j(t)}, \frac{\partial u(c^*(t), s^*(t))}{\partial s_i(t)}\right\} \ \forall j, i, t$$

form a set of optimal prices in the sense of definition 3.

Proof. See the appendix. ■

We have now established that the derivatives of the utility function with respect to stocks and flows on an optimal path can be used to define a hyperplane which separates the set of paths preferred to an optimal path from the set of feasible paths. They can therefore be used to define a price system at which national income in the national welfare sense can be computed. It is immediate that any small change in a path which has a positive present value at these optimal prices will increase welfare:

Corollary 5 A variation $\{\Delta c(t), \Delta s(t)\}_{t=0}^\infty$ on optimal path $\{c^*(t), s^*(t)\}_{t=0}^\infty$ has positive present value at the optimal prices $\{p_{c,j}(t), p_{s,i}(t)\}_{t=0}^\infty$ if and only if the implementation of this variation leads to an increase in welfare.

Proof. See the Appendix. ■

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value at the prices defining the hyperplane:

$$\langle s(t), h(t)\rangle = \int_0^\infty s(t) h(t) e^{-\delta t} dt \geq 0 \ \forall s(t) \in S.$$

If the function $s(t)$ is a $n-$vector-valued function defined on the real numbers, then likewise $h(t) : \Re \to \Re^n$ and $s(t) h(t)$ is interpreted as the inner product of two vectors in $\Re^n$.

3.1 Illustration 1

To illustrate the workings and the essential simplicity of this concept, consider briefly the present value of national wealth in the case of the classical formulation due to Harold Hotelling [19]. In this case, we seek to:

$$\max \int_0^\infty u(c)\, e^{-\delta t} dt \text{ subject to } \int_0^\infty c dt = s_0.$$

Denoting the optimal path of consumption by an asterisk, the present value national welfare would in this case be measured by $NW = \int_0^\infty c^* u^0(c^*)\, e^{-\delta t} dt$. Noting that $u^0(c^*)\, e^{-\delta t}$ is a constant by the first order conditions characterizing an optimal path, equal to the initial value of the shadow price λ_0, this is simply the initial stock of the resource multiplied by the initial shadow price:

$$NW = \lambda(0) s(0)$$

This is an extremely simple and natural measure of wealth: the welfare the economy can attain depends on its stock and the social value of this.[4] It follows that the change in NW resulting from a change in the stock is clearly $\Delta NW = \lambda_0 \Delta s_0$.

4 Capital, income and wealth

The next issue is the relationship between changes in capital stocks and changes in the Hamiltonian. Take a linear approximation to changes in the current value Hamiltonian as a result of changes in its arguments and call this approximation ΔH.

$$\Delta H = \sum_{j=1}^{m} \Delta c_j \left\{ \frac{\partial u}{\partial c_j} + \sum_{i=1}^{n} \lambda_i \frac{\partial d_i}{\partial c_j} \right\} + \sum_{i=1}^{n} \Delta s_i \left\{ \frac{\partial u}{\partial s_i} + \sum_{k=1}^{n} \lambda_k \frac{\partial d_k}{\partial s_i} \right\} \tag{9}$$

We are now in a position to establish a simple but fundamental relationship between changes in the linearized Hamiltonian and the stocks in the economy and the returns available on them. The change in the linearized Hamiltonian is just the real return on the change in the nation's stocks: it is the sum of the shadow values of stock changes each multiplied by the real rate of return on the stock. From now on we maintain the following assumption, which is a regularity assumption:

The Hamiltonian (3) has non-zero derivatives with respect to all stock variables on an optimal path. (R)

[4] This result is closely related to the Hotelling Valuation Principle, which has been used to test the Hotelling model empirically, see Jeffrey Krautkraemer [20] for a survey of this literature.

Proposition 6 Consider a small variation about an optimal path of the problem of maximizing (2) subject to (1). The change in the linearized current value Hamiltonian is a return on the change in the economy's stocks: it is equal to the changes in the value of the stocks in the economy at time t, valued at the shadow prices at time t, multiplied by the discount rate minus the rate of appreciation of the shadow prices at time t. Formally,

$$\Delta H = \sum_{i=1}^{n} \Delta s_i \lambda_i \left\{ \delta - \frac{\dot{\lambda}_i}{\lambda_i} \right\} \tag{10}$$

Proof. Immediate from (4) (5) and (9). ■

The rate of return to be applied to the value of a stock equals the real rate of return on this stock, and equals the discount rate in a stationary state. To see that it is the real rate of return, consider the expression for the change in the shadow price, which from equation (5) above is given by

$$\frac{\dot{\lambda}_i}{\lambda_i} + \frac{1}{\lambda_i} \left\{ \frac{\partial u\left(c,s\right)}{\partial s_i} + \sum_{k=1}^{n} \lambda_k \frac{\partial d_k\left(c,s\right)}{\partial s_i} \right\} = \delta \tag{11}$$

The left hand side here is the total return on a unit of the $i-$th. stock: the first term is the capital gain and the second represents the contribution made by an extra unit of the stock to utility and to the growth of all stocks, multiplied by their shadow prices. This second term is the real return: its component terms represent the return to an increment of the stock used in the economy. In the long run, with full adjustment of stocks to their appropriate levels, one would expect the real return to equal the discount rate: indeed in a stationary state they are equal. This is just the familiar no arbitrage condition, in a format slightly more general than usual because we extend it to cover non-market goods. The return applied in (10) to the value of stocks at each point in time, $\delta - \dot{\lambda}_i/\lambda_i$, is equal to the real return, and at a stationary state this equals the discount rate. Note that the Hotelling rule $\dot{\lambda}_i/\lambda_i = \delta$ is obtained as a special case when $\frac{\partial u(c,s)}{\partial s_i} = \frac{\partial d_k(c,)}{\partial s_i} = 0$, the traditional assumptions behind the pure Hotelling case.

It will prove convenient to rewrite the RHS of (10) as

$$\Delta H = \Delta s. \left|\lambda\right| .r$$

where Δs is a vector of stock changes, $|\lambda|$ is a diagonal matrix whose non-zero elements are the shadow prices λ_i, and r is a vector of real returns whose i-th component is $\left\{ \delta - \frac{\dot{\lambda}_i}{\lambda_i} \right\}$.

The linearized Hamiltonian is equivalent to the Hicksian concept of income as a return on stocks, with a generalization that deals with non-steady-state behavior, and expressed in terms of first differences rather than levels: the change in the linearized Hamiltonian equals the return $(\delta - \dot{\lambda}_i/\lambda_i)$ on the change in the value of the stocks $(\Delta s_i \lambda_i)$.

If a consumption path changes slightly, then the change in the Hamiltonian, or equivalently in Hicksian income (9) or (10), is the real return on the change in national wealth. To understand this point, recall that $\Delta H = \Delta s. |\lambda| .r$. And let $(\Delta W) = \Delta s. |\lambda|$: (ΔW) is a vector of changes in stocks each valued at the corresponding shadow price — $(\Delta W) = (\Delta s_1 \lambda_1, ..., \Delta s_n \lambda_n)$. If we add the components of (ΔW), the result is clearly ΔV, i.e. $\Delta V = (\Delta W).I$ where I is a vector of ones. And clearly $\Delta V = \Delta NW$, as NW is nothing more than a linear approximation to V. So $\Delta NW = \Delta W.I = \Delta s. |\lambda| .I$. This implies that $\Delta H = (\Delta NW).r$ where $(\Delta NW) = \Delta W = (\Delta s_1 \lambda_1, ..., \Delta s_n \lambda_n)$.

Proposition 7 Consider a small variation about an optimal path of the problem of maximizing (2) subject to (1). Then the resulting changes in the Hamiltonian (Hicksian national income) and in national wealth are related as follows:

$$\Delta H = (\Delta NW).r$$

that is, the change in Hicksian national income is the real return on the change in national wealth.

This result can also shed light on a question posed earlier, namely whether an increase in the Hamiltonian implies an increase in the level of social welfare as measured by the objective of the optimal growth problem. Assume first that there is only one stock variable, so that we can write

$$\Delta H = \Delta s \lambda \left(\delta - \frac{\dot{\lambda}}{\lambda} \right)$$

This tells us that if the real return is positive then the sign of a change in wealth resulting from a policy change is the same as the sign of the change in the Hamiltonian or in Hicksian income. An increase in the Hamiltonian does indicate an increase in welfare in this case. In many though not all one good models the real return is always positive (see Heal and Kriström [17]). However, when we have more than one stock variable this is no longer the case. We may have $\Delta NW = \sum \Delta s_i \lambda_i > 0$ and all components of $r > 0$ yet $(\Delta NW).r < 0$ as some stock changes are negative. So the signs of ΔH and ΔNW are no longer the same. In a steady state, of course, we have $\left(\delta - \frac{\dot{\lambda}}{\lambda} \right) = \delta > 0$ so that the sign of ΔH is the same as that of ΔNW. In general therefore:

Remark 8 An increase in the Hamiltonian does not imply an increase in welfare in the sense of an increase in the discounted integral of utilities or a potential Pareto improvement, except in a stationary state.

This seems to reduce the appeal of the Hamiltonian as a welfare or income measure.

We can relate this analysis to the concept of net national product, NNP, often proposed as a welfare-related measure of income (Dasgupta and Mäler [10], Mäler [22]). There are variations in how this is defined but the most widely-use definition is that it equals the value of consumption plus the value of net investment (Dasgupta and Mäler [10]). In the present context this means that

$$NNP = \sum_i c u_{c_i} + \sum_i s u_{s_i} + \sum_i \lambda_i d_i(s, c)$$

Consider as above a small variation about an optimal path: then using the first order conditions the change in NNP is given by

$$\Delta NNP = \sum_{i=1}^{n} \Delta s_i \left\{ \frac{\partial u}{\partial s_i} + \sum_{k=1}^{n} \lambda_k \frac{\partial d_k}{\partial s_i} \right\} = \Delta H$$

The change in NNP is the same as the change in the Hamiltonian, so that we can invoke the previous proposition to state that $\Delta NNP = (\Delta NW).r$, i.e. the change in net national product is the real return on the change in wealth. In a stationary state, matters are much simpler. Stocks are constant so that $\sum_i \lambda_i d_i(s, c) = 0$. Consumption is also constant so that $NW = \int_0^\infty (p_c c + p_s s) e^{-\delta t} dt$ where all variables in the parentheses are constant, so $NW = \frac{(p_c c + p_s s)}{\delta}$. In this case NNP is just $(p_c c + p_s s)$ so that $NNP = \delta NW$. Income is return on wealth. To sum up,

Proposition 9 The change in net national product NNP resulting from a policy variation is given by $\Delta NNP = (\Delta NW).r$: it is the real return on the resulting change in wealth. In a stationary state, income is the return on wealth. Except in a stationary state, the sign of the change in NNP need not be the same as the sign of the change in national wealth so that a rise in NNP does not imply an increase in social welfare.

4.1 Illustration 2

The following example illustrates the relationship between NNP and NW and indeed shows that in some cases it is stronger than proposition 9 implies. We take a standard neoclassical growth model with a logarithmic utility function and a linear production function:[5]

$$u(c) = \log c,\ \ f(k) = Ak,\ \ \max \int_0^\infty \log c e^{-\delta t} dt \tag{12}$$

and solve for NW and NNP. It is routine, using the first order conditions for optimality in problem (12), to show that $NW = \frac{1}{\delta}$ and $NNP = \frac{A}{\delta}$. It also follows from

[5] We are grateful to Martin Weitzman for this example.

the first order conditions that $\dot{\lambda}/\lambda = (\delta - A)$. Putting these statements together and letting $r = \left(\delta - \dot{\lambda}/\lambda\right)$ we have

$$NNP = rNW$$

so that net national product is real interest on national wealth, even outside of a stationary state. In this case we have a result stronger than that of proposition 9, which refers only to changes in NNP and NW and not to their levels. We are not clear what conditions are needed for this stronger result to hold: it is immediate that the stronger result always holds along stationary paths. The neatness of this example depends on both the linearity of the production function and the choice of a logarithmic utility function, but some of its properties survive with a general utility function. Consider now the case

$$f(k) = Ak, \;\; \max \int_0^\infty u(c)\, e^{-\delta t} dt \tag{13}$$

with an arbitrary strictly concave utility. In this case using the first order conditions

$$NNP = u_c c + \lambda \dot{k} = u_c c + \lambda (Ak - c) = \lambda Ak = \lambda k(\delta - \dot{\lambda}/\lambda)$$

so that net national product equals the real return on the capital stock valued at shadow prices. In the case of a linear technology considered here, the capital stock valued at shadow prices is equal to national wealth, i.e. $\lambda k = \int_0^\infty u_c c e^{-\delta t} dt = NW$. Recall that this was also the case in the Hotelling model of Illustration 1. In summary,

Proposition 10 When the economy is given by a Ramsey-Solow model where the production technology is linear (i.e.$\dot{k} = Ak - c$) then net national product is always interest on national wealth.

Proof. We need to show that the following equation holds for any T: $\lambda_T k_T = \int_T^\infty u_c c e^{-\delta(t-T)} dt$. We claim that it does if the technology is linear, $f(k) = Ak$.Consider the time derivative of the left hand side: using the first order conditions this is $\lambda f(k) - \lambda c + \lambda k(\delta - f_k)$. The derivative of the right hand side is $-\lambda c + \delta \int_T^\infty u_c c e^{-\delta(t-T)} dt$. The difference between the time derivatives of the two sides is $\lambda f(k) - \lambda k f_k + \delta \lambda k - \delta \int_T^\infty u_c c e^{-\delta(t-T)} dt$ which is $\delta \lambda_T k_T - \delta \int_T^\infty u_c c e^{-\delta(t-T)} dt$ when the technology is linear (and only when it is linear). So if the proposition holds at $T = 0$ then it aways holds. Note that the difference between the two measures satisfies the differential equation $d/dt \left(\lambda_T k_T - \int_T^\infty u_c c e^{-\delta(t-T)} dt\right) = \lambda_T k_T - \int_T^\infty u_c c e^{-\delta(t-T)} dt$ From this it follows that if the two ever differ then in the limit they differ by an infinite amount. As both are bounded this is impossible, proving the point. ■

The equivalent of a linear technology for a multi-factor production function is presumably constant returns to scale. Dasgupta and Mäler [11] work with a definition of wealth that is the shadow value of capital stocks, so that their definition differs from ours unless the technology is linear.

4.2 Summary

Proposition 9 and illustration 2 address the issue raised by quotation from Solow in the introduction to this paper - "Properly defined and properly calculated, this year's net national product can always be regarded as this year's interest on society's total stock of capital." Recall that Dasgupta and Mäler disagreed, stating that this is true only "if well-being is linear and accounting prices are constant over time." Solow's statement is true quite generally if applied to changes rather than levels: it is true in levels for stationary states, for the example and presumably for other cases that we have not yet delineated. The example has linearity but not in the welfare function, which contradicts Dasgupta and Mäler. But constancy of prices is sufficient, though not as they suggest necessary, for NNP to be interest on wealth. The idea that income is interest on wealth of course goes back at least to Hicks [18], and is an integral part of our thinking about individual income. So it is important to clarify the nature of this relationship at the aggregate level.

We are fully in agreement with Dasgupta and Mäler on one important point, which is that wealth not net national product is the right measure of welfare. They, like us in proposition 9, observe that "A country's NNP can rise even while its wealth declines." However their definition of wealth differs from ours: they use as wealth the shadow value of capital stocks. As noted in the last footnote, this is the same as our concept of wealth if the technology is linear: this is also true for the Hotelling model (Illustration 1). The present value of future consumption seems to be a more fundamental concept, and has neater relationships with other concepts.

We summarize the relationships between the Hamiltonian H, its linear approximation ΔH, income, wealth NW, net national product NNP and the state valuation function V for convenience:

$$\Delta NW = \Delta V = \sum_i \Delta s_i \lambda_i$$

$$\Delta H = \sum_i \lambda_i \Delta s_i \left(\delta - \frac{\dot{\lambda}_i}{\lambda_i} \right) = (\Delta NW) .r \text{ so in general } sign \Delta H \neq sign \Delta NW$$

$$\Delta H = \Delta NNP = (\Delta NW).r \text{ and } NNP = rNW \text{ in a stationary state.}$$

The correct welfare measure for small changes is ΔV (or ΔNW) so that $\sum_i \Delta s_i \lambda_i$ is the preferred instantaneous welfare index, as it indicates the sign of changes in NW. This is referred to by the World Bank as "genuine savings" [13], and is equal to the change in NW. The change in NNP can also indicate welfare changes when shadow prices are constant.

5 Extensions

The analysis so far has addressed a conventional case - utilitarian preferences with exponential discounting and no exogenous technical change. For many of the concepts suggested as measures of national welfare, this conventional case is the limit of their validity. For example, the Weitzman 1976 result in proposition 2 depends heavily on the fact that the Hamiltonian is autonomous, which implies that there is no exogenous technical change and that discounting is exponential. Our results on national wealth are valid more generally. The concept of national wealth can be defined for a generalization of the model of section 2 where we have any type of technical progress and of discounting - hyperbolic or logarithmic discounting and technical change pose no obstacle to the definition of national wealth or to the results of section 3 about supporting prices. Easy generalizations are available for these cases.

National wealth can also be defined for non-utilitarian objectives. We illustrate this briefly in the context of the "sustainable preferences" introduced by Graciela Chichilnisky [6], which involve using as the objective the weighted average of an integral of utilities and a term depending on long-run or sustainable utility levels. We shall focus on the case of exhaustible resources. Consider the optimal use problem in the simplest version of this case:

$$\max \alpha \int_0^\infty u\left(c_t, s_t\right) e^{-\delta t} dt + (1-\alpha) \lim_{t \to \infty} u\left(c_t, s_t\right), \; \dot{s}_t = -c_t \text{ and } s_t \geq 0 \forall t.$$

where $0 < \alpha < 1$. This problem can not be solved by standard Hamiltonian-based techniques (see Heal [16]). Let (c_t^*, s_t^*) be the optimal path for this problem. Then using an obvious generalization from the utilitarian case national welfare is now defined as

$$\left.\begin{array}{c} NW = \int_0^\infty \left\{c_t u_c\left(c_t^*, s_t^*\right) + s_t^* u_s\left(c_t^*, s_t^*\right)\right\} e^{-\delta t} dt \\ + \lim\limits_{t \to \infty} \left\{c_t u_c\left(c_t^*, s_t^*\right) + s_t u_s\left(c_t^*, s_t^*\right)\right\} \end{array}\right\} \tag{14}$$

Marginal utilities are again used as prices. The definition of wealth now contains two elements: one the integral term, as in the case of the discounted utilitarian approach, and an extra element arising from the value placed by the objective on the limiting utility level. In this the limiting stock values and consumption levels are valued at limiting prices. The value assigned to a path depends both on the time path over finite horizons, via the present value term, and also on the limiting or sustainable values along the path.[6]

With this definition of optimality, the price system contains undiscounted terms because of the limiting term in the definition. So national welfare is measured in (14) as a present value plus a term reflecting long run or sustainable welfare. This term

[6] Formally, we are now defining the value of a sequence of consumption and stock levels $(c(t), s(t))$ at prices $p_c(t), p_s(t)$, or equivalently defining the inner product of the consumption and stock sequences with the price sequences.

We define a supporting hyperplane for a set S of paths $(c(t), s(t))$ of consumption and of the is not discounted: apart from this it has the same form as the other terms, namely stocks and flows evaluated at prices given by marginal valuations along an optimal path. The presence of this extra term is important, because it gives a reason for using in the measurement of national welfare prices which relate to the distant future yet are nevertheless not discounted. This possibility has been discussed by several authors including William Cline [7] and John Broome [4].

We now want to establish that any small change which increases this welfare measure is a welfare improvement.

Proposition 11 Let a small variation $\{\Delta c_t, \Delta s_t\}$ about an optimal path $\{c_t^*, s_t^*\}$ increase NW as defined in (14). Then the implementation of this variation leads to an increase in welfare.

In summary, the definition of national income implied by Chichilnisky's criterion of intertemporal optimality involves the use of prices to assign value to a path of the economy: the value will have two components, one a present value computed at a discount rate in the conventional fashion, and one an undiscounted value associated with the very long run properties of the path.

6 Conclusions

National wealth is a good index of economic welfare for a society. It is a natural index and has the essential property that it increases if and only if there is a potential Pareto improvement. No other indicator proposed has this property, and in particular this property is not shared by various Hamiltonian-based indices. Following Weitzman's 1976 results and extensions of these we know that Hamiltonian-based indices do have interesting properties - correctly constructed they can indicate the average future welfare level, an interesting and surprising property. But this is not the same as indicating a welfare increase or decrease.

The formal definition of national wealth involves future prices and quantities, which are not naturally observable today. It is therefore important that we have indices that move with national wealth and depend only on current variables. On such indicator - net investment at shadow prices - has a natural interpretation and has been proposed and used by Hamilton and Clemens [13]). The other - the change in NNP - is very close to indicators proposed by several other authors on different grounds (see e.g. Dasgupta [8] and Dasgupta and Mäler [10]. In order to apply these ideas, we need to estimate the indicated shadow prices. There is now a significant

resource stock as functions $p_c(t), p_s(t)$ such that

$$\int_0^\infty \{c(t)p_c(t) + s(t)p_s(t)\} e^{-\delta t} dt + \lim_{t\to\infty} \{c(t)p_c(t) + s(t)p_s(t)\} \geq 0 \forall (c_t, s_t) \in S$$

body of literature on non-market valuation that can be tapped. For the most part, valuation methods such as contingent valuation are tailored for project-level analysis, focusing on consumer surplus measures. It is an open question if existing empirical evidence, mostly project-level based estimates on willingness-to-pay, can be used to approximate the shadow prices relevant at the level of a nation. If so, then the theories outlined above lends themselves to empirical application quite readily. The World Bank's empirical application of the Genuine Savings measure is a case in point. More work, however, is certainly needed on tying together application and theories of welfare measurement in dynamic economies.

Another advantage of the national wealth concept is that it can be applied in cases that cause problems for other concepts. It does not require an autonomous system and so can easily be extended to cases involving technical change and non-exponential discounting. It also covers some non-utilitarian specifications of the economy's objectives.

7 Appendix

Definition 12 Formally a separating hyperplane satisfies the following conditions (here an asterisk denotes the value of a variable along an optimal path):

$$\begin{aligned} \int_0^\infty u(c_t, s_t) e^{-\delta t} dt &\geq \int_0^\infty u(c_t^*, s_t^*) e^{-\delta t} dt \Rightarrow \qquad (15) \\ \int_0^\infty \{\langle p_c(t), c_t\rangle + \langle p_s(t), s_t\rangle\} e^{-\delta t} dt &\geq \int_0^\infty \{\langle p_c(t), c_t^*\rangle + \langle p_s(t), s_t^*\rangle\} e^{-\delta t} dt \end{aligned}$$

and

$$\begin{aligned} \{c_t, p_t\} \text{ feasible } &\Rightarrow \qquad (16) \\ \int_0^\infty \{\langle p_c(t), c_t^*\rangle + \langle p_s(t), s_t^*\rangle\} e^{-\delta t} dt &\geq \int_0^\infty \{\langle p_c(t), c_t\rangle + \langle p_s(t), s_t\rangle\} e^{-\delta t} dt \end{aligned}$$

where $\langle p_c(t), c_t\rangle$ denotes the inner product of the price vector $p_c(t)$ with the consumption vector c_t.

Proof of Proposition 4. We need to show that these prices satisfy (15) and (16). Consider a path $\{c_t, s_t\}$ such that

$$\int_0^\infty \{u(c_t, s_t) - u(c_t^*, s_t^*)\} e^{-\delta t} dt \geq 0 \qquad (17)$$

We need to show that in this case

$$\int_0^\infty \{\langle p_c, (t) c_t\rangle + \langle p_s, (t) s_t\rangle\} e^{-\delta t} dt \geq \int_0^\infty \{\langle p_c, (t) c_t^*\rangle + \langle p_s, (t) s_t^*\rangle\} e^{-\delta t} dt$$

Take a linear approximation to $u(c_t, s_t)$ about (c_t^*, s_t^*) along an optimal path, and use the concavity of the utility function:

$$\begin{aligned} u(c_t, s_t) - u(c_t^*, s_t^*) &< \sum_j \frac{\partial u(c_t^*, s_t^*)}{\partial c_{j,t}}(c_{j,t} - c_{j,t}^*) + \sum_i \frac{\partial u(c_t^*, s_t^*)}{\partial s_{i,t}}\left(s_{i,t} - s_{i,t}^*\right) \\ &= \sum_j p_{c,j}(t)(c_{j,t} - c_{j,t}^*) + \sum_i p_{s,i}(t)\left(s_{i,t} - s_{i,t}^*\right) \end{aligned}$$

Together with (17), this establishes the inequality needed, i.e., (15).

Now we need to establish the inequality (16). Consider the problem of choosing a program to maximize present value at the prices $\{p_{c,j}(t), p_{s,i}(t)\}$:

$$\max \int_0^\infty \{p_c(t)\, c_t + p_s(t)\, s_t\}\, e^{-\delta t} dt$$

$$\text{subject to } \dot{s}_{i,t} = d_i(c_t, s_t), i = 1, ..., n$$

The corresponding Hamiltonian is

$$H = \{\langle p_c, (t)\, c_t\rangle + \langle p_s, (t)\, s_t\rangle\}\, e^{-\delta t} + \sum_{i=1}^n \mu_{i,t} e^{-\delta t} d_i(c_t, s_t)$$

and a solution satisfies

$$p_{c,j}(t) = -\sum_{i=1}^n \mu_{i,t} \frac{\partial d_i(c_t, s_t)}{\partial c_j}$$

and

$$\dot{\mu}_{i,t} - \delta \mu_{i,t} = -p_{s,i}(t) - \sum_{k=1}^n \mu_{k,t} \frac{\partial d_k(c_t, s_t)}{\partial s_i}$$

Now note that given the definition of the optimal prices, these conditions are precisely the same as the conditions (4) and (5) which characterize a solution to the general optimization problem of maximizing (2) subject to (1). Hence a path which solves the overall optimization problem also solves the problem of maximizing the present value of the path at the optimal prices. This completes the proof. ■

Proof of Corollary 5. By assumption

$$\int_0^\infty \{\langle \Delta c_t, p_c(t)\rangle + \langle \Delta s_t, p_s(t)\rangle\} e^{-\delta t} dt > 0 \tag{18}$$

Let $\{\Delta c_t + c_t^*, \Delta s_t + s_t^*\}$ be the path resulting from implementing the variation in the optimal path. The welfare associated with this path is

$$\int_0^\infty \{u(c_t^*, s_t^*) + \sum_j \frac{\partial u(c_t^*, s_t^*)}{\partial c_{j,t}} \Delta c_{j,t} + \sum_i \frac{\partial u(c_t^*, s_t^*)}{\partial s_{i,t}} \Delta s_{i,t}\} e^{-\delta t} dt$$

which by (18) and the definition of the optimal prices is greater than that on the optimal path, as required. The proof of the converse is immediate. ■

Definition 13 A hyperplane which separates the set of paths preferred to an optimum from those which are feasible is a time path of prices for stocks and flows $p_s(t)$ and $p_c(t)$ which must satisfy the following conditions (here an asterisk denotes the value of a variable along an optimal path):

$$\left.\begin{array}{c} \int_0^\infty u(c_t, s_t)\, e^{-\delta t} dt + \lim_{t\to\infty} u(c_t, s_t) \geq \\ \int_0^\infty u(c_t^*, s_t^*)\, e^{-\delta t} dt + \lim_{t\to\infty} u(c_t^*, s_t^*) \Rightarrow \\ \langle (c_t, s_t), (p_c(t), p_s(t))\rangle \geq \langle (c_t^*, s_t^*), (p_c(t), p_s(t))\rangle \end{array}\right\} \tag{19}$$

and

$$\left.\begin{array}{c} \{c_t, p_t\} \text{ feasible implies} \\ \langle (c_t^*, s_t^*), (p_c(t), p_s(t))\rangle \geq \langle (c_t, s_t), (p_c(t), p_s(t))\rangle \end{array}\right\} \tag{20}$$

where $\langle p_c(t), c_t\rangle$ denotes the inner product of the price vector $p_c(t)$ with the consumption vector c_t. ■

Proof of Proposition 10. We need to show that these prices satisfy (19) and (20). Consider a path $\{c_t, s_t\}$ such that

$$\int_0^\infty \{u(c_t, s_t) - u(c_t^*, s_t^*)\}\, e^{-\delta t} dt + \lim_{t\to\infty} \{u(c_t, s_t) - u(c_t^*, s_t^*)\} \geq 0 \tag{21}$$

We need to show that in this case

$$\begin{array}{l} \int_0^\infty \{p_c(t)\, c_t + p_s(t)\, s_t\}\, e^{-\delta t} dt + \lim_{t\to\infty} \{p_c(t)\, c_t + p_s(t)\, s_t\} \geq \\ \int_0^\infty \{p_c(t)\, c_t^* + p_s(t)\, s_t^*\}\, e^{-\delta t} dt + \lim_{t\to\infty} \{p_c(t)\, c_t^* + p_s(t)\, s_t^*\} \end{array}$$

Take a linear approximation to $u(c_t, s_t)$ about (c_t^*, s_t^*) along an optimal path, and use the concavity of the utility function:

$$\begin{aligned} u(c_t, s_t) - u(c_t^*, s_t^*) &< \sum_j \frac{\partial u(c_t^*, s_t^*)}{\partial c_{j,t}} (c_{j,t} - c_{j,t}^*) + \sum_i \frac{\partial u(c_t^*, s_t^*)}{\partial s_{i,t}} \left(s_{i,t} - s_{i,t}^*\right) \\ &= \sum_j p_{c,j}(t)\, (c_{j,t} - c_{j,t}^*) + \sum_i p_{s,i}(t) \left(s_{i,t} - s_{i,t}^*\right) \end{aligned}$$

Together with (21), this establishes the inequality needed, i.e., (19).

The inequality (20) can now be established using a very minor variation of the argument used for proposition 1. ■

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