Imperfect Knowledge, Inflation Expectations, and Monetary Policy

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Abstract

This paper investigates the role of imperfect knowledge regarding the structure of the economy on the formation of expectations, macroeconomic dynamics, and the efficient formulation of monetary policy. With imperfect knowledge, economic agents rely on an adaptive learning technology to form expectations and continuously update their beliefs regarding the dynamic structure of the economy based on incoming data. Perpetual learning introduces an additional layer of dynamic interactions between monetary policy and economic outcomes. As a result, efficient monetary policy needs to account for the influence of policy on learning, in addition to the traditional stabilization concerns present under rational expectations with perfect knowledge. Using a simple model, we show that policies that are efficient under rational expectations can be quite inefficient when knowledge is imperfect. However, significantly improved economic performance can generally be achieved by placing greater emphasis on controlling inflation. Policies emphasizing tight inflation control reduce the persistence of inflation and facilitate the formation of accurate inflation expectations. This enhances economic stability and mitigates the influence of imperfect knowledge.

KEYWORDS: Inflation targeting, policy rules, rational expectations, learning, inflation persistence.

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1 Introduction

The advent of rational expectations as a model for expectations formation by economic agents has provided an elegant framework that has dominated thinking about the dynamic structure of the economy and econometric policy evaluation over the past thirty or so years. The elegance afforded by adopting rational expectations, however, comes at a cost to realism. In practice, economic agents do not possess the knowledge endowed to them in model economies with rational expectations. As Sargent (1993) emphasizes, "rational expectations models impute much *more* knowledge to the agents within the model ... than is possessed by an econometrician, who faces estimation and inference problems that the agents in the model have somehow solved" (p. 3, emphasis in original).[1] Missing from such models, as Friedman (1979) points out, "is a clear outline of the way in which economic agents derive the knowledge which they then use to formulate expectations."

Acknowledgment of the role for learning and the presence of imperfections in the formation of expectations reveals an additional and potentially important margin to policy evaluation. In the presence of imperfections in the expectations formation process, efficient monetary policy needs to account for the influence of policy on learning, in addition to the traditional stabilization concerns present under rational expectations with perfect knowledge.

This added layer of interaction between monetary policy and economic outcomes has often been cited by policymakers over the past two decades as crucial for understanding the success and failures of monetary policy but has received much less attention in formal econometric policy evaluations. An important example is the contrast between the stubborn persistence of inflation expectations during the 1970s when policy placed relatively greater attention to countercyclical concerns and the much improved stability in both inflation and inflation expectations following the renewed emphasis on price stability in 1979. In

[1] To be sure, this does not reflect a criticism of the traditional use of the concept of "rationality" as reflecting the optimal use of information in the formation of expectations, taking into account an agent's objectives and resource constraints. The difficulty is that in Muth's (1961) original formulation, rational expectations are not "profit maximizing" in this sense. Thus, the issue is not that the "rational expectations" concept reflects too much rationality but rather that it imposes too little rationality in the expectations formation process. For example, as Sims (2001) has recently pointed out, optimal information processing subject to a finite cognitive capacity may result in fundamentally different processes for the formation of expectations than implied by rational expectations. To acknowledge this terminological tension, Simon (1978) suggested that a less misleading term for Muth's concept would be "model consistent" expectations (p. 2).

explaining the rationale for this shift in emphasis in 1979, Chairman Volcker highlighted the importance of learning in shaping the inflation expectations formation process:[2]

> It is not necessary to recite all the details of the long series of events that have culminated in the serious inflationary environment that we are now experiencing. An entire generation of young adults has grown up since the mid-1960's knowing only inflation, indeed an inflation that has seemed to accelerate inexorably. In the circumstances, it is hardly surprising that many citizens have begun to wonder whether it is realistic to anticipate a return to general price stability, and have begun to change their behavior accordingly. Inflation feeds in part on itself, so part of the job of returning to a more stable and more productive economy must be to break the grip of inflationary expectations.
> (Statement before the J.E.C., October 17, 1979.)

This historical episode provides a clear example when inflation expectations became unhinged from the intended policy objective.

Our objective in this study is to formalize imperfect knowledge with a process of learning that deviates only marginally from rational expectations and to investigate the consequences of acknowledging the presence of this imperfection for inflation dynamics and policy evaluation. In doing so, we draw elements and build on earlier work on learning, estimation and policy design. Our model is related to investigations examining the formation of inflation expectations when the policymaker's objective may be unknown, which has been the subject of extensive work, especially in the context of a transition following a shift in policy regime. (See, for example Bomfim et al (1997), Erceg and Levin (2001), Tetlow and von zur Muehlen (2001) and the references therein). Our focus, however, is on the role of learning in the context of a dynamic stochastic economy in which monetary policy is stable. The policymakers in our model account for the effect of their actions on learning, similar to the dual control problem in policy design investigated by Balvers and Cosimano (1994), Wieland (1998), and others. Those treatments emphasize the policymaker's own uncertainty regarding the model and how policy design may facilitate its resolution. Instead, we concentrate on the policymaker's recognition that his actions may facilitate learning by other agents in the model when expectations formed by those agents are important

[2] Indeed, we would argue that the shift in emphasis towards greater focus on inflation was itself influenced by the recognition of the importance of facilitating the formation of stable inflation expectations—which had been insufficiently appreciated earlier during the 1970s. See Orphanides (2001) for a more detailed description of the policy discussion at the time and the nature of the improvement in monetary policy since 1979. See also Christiano and Gust (1999) and Sargent (1999) for interpretations of the 1970s inflation emphasizing the role of inflation expectations and their formation.

determinants of economic outcomes. Finally, our work draws from recent explorations of economic dynamics when economic agents deviate from rational expectations, and instead rely on estimated models to form expectations, as in Sargent (1999) and Cogley and Sargent (2001), though in our formulation expectations deviate only marginally from the rational expectations benchmark.

As a laboratory for our experiment, we employ a simple linear model of the U.S. economy with characteristics similar to more elaborate models frequently employed to study optimal monetary policy. Using the model, we then examine the stochastic properties of the macroeconomy when policy has a fixed inflation target and a dual objective of price stability and stability of real economic activity, consistent with the mandate of the Federal Reserve. After describing the properties of policies that are optimal under rational expectations, we examine the performance of these policies when expectations instead reflect imperfect knowledge and compare the resulting outcomes with policies designed to take into account the presence of this imperfection.

To formalize imperfect knowledge, we endow economic agents with knowledge of the correct structure of the economy and the reduced-form expectations formation functions consistent with rational expectations. Rather than endowing them with complete knowledge of the parameters of these functions—as would be required by imposition of the rational expectations assumption–we posit that economic agents rely on finite memory least squares estimation to update these parameter estimates. This setting conveniently nests rational expectations as the limiting case corresponding to infinite memory least squares estimation and allows varying degrees of imperfection in expectations formation to be characterized by variation in a single parameter in the model.

Our results suggest that even marginal deviations from rational expectations in the direction of imperfect knowledge can have economically important effects on the stochastic behavior of our economy and policy evaluation. An interesting feature of the model is that the interaction of learning and control creates rich non-linear dynamics that can potentially explain both the shifting parameter structure of linear reduced form characterizations of the economy and the appearance of shifting policy objectives or inflation targets. As we explain, broad characteristics of the data can be potentially explained within our model as the outcome of a learning process in an uncertain environment with fixed policy objectives.

With regard to policy evaluation, we find that policies designed to be efficient under rational expectations can be quite inefficient when knowledge is imperfect. This deterioration in performance is particularly severe when policymakers put a high weight on stabilizing real economic activity versus price stability. However, as we show, economic performance can be improved significantly by placing greater emphasis on controlling inflation and inflation expectations. We find that policies emphasizing tight inflation control can facilitate learning and provide better guidance for the formation of inflation expectations. Such policies mitigate the negative influence of imperfect knowledge on economic stabilization and yield superior macroeconomic performance.

2 The Model Economy

We consider a stylized model that gives rise to a non-trivial inflation-output variability tradeoff and in which a simple one-parameter policy rule represents optimal monetary policy under commitment.[3] Under the assumption of rational expectations with perfect knowledge, the evolution of the model economy and optimal monetary policy can both be expressed in terms of only two variables, the inflation rate, π, and the central bank's inflation target, π^*. The gap between the inflation rate and its target value drives the formation of inflation expectations and the policy choice. These, together with serially uncorrelated shocks, determine output and inflation in the following period. In this section, we describe the model specification for inflation and output and the central bank's optimization problem; in the next two sections, we take up the formation of expectations by private agents.

Inflation is determined by a modified Lucas supply function that allows for some intrinsic inflation persistence,

$$\pi_{t+1} = \phi\pi^e_{t+1} + (1-\phi)\pi_t + \alpha y_{t+1} + e_{t+1}, \quad e \sim \text{iid}(0, \sigma^2_e), \tag{1}$$

where π denotes the inflation rate, π^e is the private agents' expected inflation rate based on time t information, y is the output gap, $\phi \in (0,1)$, $\alpha > 0$, and e is a serially uncorrelated innovation. As discussed by Clark et al (1999), Lengwiler and Orphanides (forthcoming), and others, this specification incorporates an important role for inflation expectations for

[3] Since its introduction by Taylor (1979), the practice of analyzing monetary policy rules using such an inflation-output variability tradeoff has been adopted in a large number of academic and policy studies.

determining inflation outcomes while also allowing for a realistic element of inflation persistence. This specification collapses to the Lucas supply function when ϕ equals one. We restrict ϕ to be less than unity, however, because some intrinsic inflation inertia is necessary for the model to yield a nontrivial inflation-output gap variability tradeoff.

The output gap—the percent deviation of real output from potential output, is determined by the real rate gap—the difference between the short-term real interest rate and the equilibrium real interest rate,

$$y_{t+1} = -\xi(r_t - r^*) + u_{t+1}, \quad u \sim \text{iid}(0, \sigma_u^2). \tag{2}$$

where r is the short-term real interest rate, r^* is the equilibrium real rate, and u is a serially uncorrelated innovation. Note that a monetary policy action at period t affects output in the following period, reflecting the lag in the monetary transmission mechanism.

The central bank's objective is to design a policy rule that minimizes the loss, denoted by $\mathcal{L}$, equal to the weighted average of the asymptotic variances of the output gap and of deviations of inflation from the target rate,

$$\mathcal{L} = \omega Var(\pi - \pi^*) + (1 - \omega)Var(y), \tag{3}$$

where $Var(x)$ denotes the unconditional variance of variable x, and $\omega \in (0, 1]$ is the relative weight on inflation stabilization.

The central bank sets its instrument, the short-term (ex ante) real interest rate r_t, after private agents set their expectations for inflation in period $t+1$, π^e_{t+1}, but before time $t+1$ innovations are observed. We assume that the central bank can and does commit to a policy rule; thus, we only consider solutions under commitment. We further assume that the central bank has perfect knowledge regarding the structural parameters of the model, α, ϕ, ξ, and r^*. With this assumption, we can reformulate the policy instrument in terms of the choice at time t of the intended level of output gap in period $t+1$, $x_t = -\xi(r_t - r^*)$.[4] Hence, the realization of the output gap in period $t+1$ equals the intended output gap plus the control error, u_{t+1},

$$y_{t+1} = x_t + u_{t+1}. \tag{4}$$

[4]Note that here we abstract from the important complications associated with the real-time measurement of the output gap and and the equilibrium real interest rate for formulating the policy rule. See Orphanides (1998) and Laubach and Williams (2001) for analyses of these issues.

This completes the description of the structure of the model economy, with the exception of the expectations formation process that we examine in detail below.

3 The Perfect Knowledge Benchmark

We begin by considering the "textbook" case of rational expectations with perfect knowledge in which private agents know the structure of the economy and the central bank's policy. In this case, expectations are rational in that they are consistent with the true data generating process of the economy (the model). In the following section, we use the resulting equilibrium solution as a "perfect knowledge" benchmark against which we compare outcomes under imperfect knowledge, in which case agents do not know the structural parameters of the model, but instead must form expectations based on estimated forecasting models.

Under the assumption of perfect knowledge, the evolution of the economy and optimal monetary policy can all be expressed in terms of two variables, the current inflation rate and its target level. These variables determine the formation of expectations and the policy choice, which, together with serially uncorrelated shocks, determine output and inflation in period $t+1$. Specifically, we can write the monetary policy rule in terms of the inflation gap,

$$x_t = -\theta(\pi_t - \pi^*), \tag{5}$$

where $\theta > 0$ measures the responsiveness of the real rate gap to the inflation gap.

Given this monetary policy rule, inflation expectations are given by:

$$\pi^e_{t+1} = \frac{\alpha\theta}{1-\phi}\pi^* + \frac{1-\phi-\alpha\theta}{1-\phi}\pi_t. \tag{6}$$

Inflation expectations depend on the current level of inflation, the inflation target, and the parameter θ measuring the central bank's responsiveness to the inflation gap. Substituting this expression for expected inflation into equation (1) yields the rational expectations solution for inflation for a given monetary policy,

$$\pi_{t+1} = \frac{\alpha\theta}{1-\phi}\pi^* + (1 - \frac{\alpha\theta}{1-\phi})\pi_t + e_{t+1} + \alpha u_{t+1}. \tag{7}$$

One noteworthy feature of this solution is that the first-order autocorrelation of the inflation rate, given by $1 - \frac{\alpha\theta}{1-\phi}$, is decreasing in θ and is invariant to the value of π^*. Note that the rational expectations solution can also be written in terms of the "inflation expectation gap"—the difference between inflation expectations for period $t+1$ from the inflation target, $\pi^e_{t+1} - \pi^*_t$,

$$\pi^e_{t+1} - \pi^*_t = \frac{1 - \phi - \alpha\theta}{1 - \phi}(\pi_t - \pi^*). \tag{8}$$

Equations (5) and (6) close the perfect knowledge benchmark model.

3.1 Optimal Monetary Policy under Perfect Knowledge

For the economy with perfect knowledge, the optimal monetary policy, θ^P, can be obtained in closed form and is given by[5]:

$$\theta^P = \frac{-\alpha\omega + \sqrt{4(1-\phi)^2(1-\omega)\omega + (\alpha\omega)^2}}{2(1-\phi)(1-\omega)} \quad \text{for} \quad 0 < \omega < 1. \tag{9}$$

In the limit, when ω equals unity (that is when the policymaker is not at all concerned with output stability), the policymaker sets the real interest rate so that inflation returns to its target in the next period in expectation, that is, the inflation expectation gap is zero. The optimal policy in this case is given by: $\theta^P = \frac{1-\phi}{\alpha}$, and the irreducible variance of inflation, owing to unpredictable output and inflation innovations, equals $\sigma^2_e + \alpha^2\sigma^2_u$. Differentiation of equation 9 shows that θ^P is decreasing in both α and ϕ. In particular, the optimal policy response is larger, the greater the degree of intrinsic inertia in inflation, measured by $1 - \phi$.

The greater the central bank's weight on inflation stabilization, the greater is the responsiveness to the inflation gap, and the smaller the first-order autocorrelation in inflation. Differentiation of equation (9) shows that the policy responsiveness to the inflation gap is increasing in ω, the weight the central bank places on inflation stabilization. As a result, the autocorrelation of inflation is decreasing in ω, with a limiting value approaching unity when ω approaches zero and zero when ω equals one. That is, if the central bank cares only about output stabilization, the inflation rate becomes a random walk, while if the central bank cares only about inflation stabilization, the inflation rate displays no serial correlation. And, as noted, this model yields a nontrivial monotonic tradeoff between the variability of inflation and the output gap, for all values of $\omega \in (0, 1]$. These results are illustrated graphically in Figure 1. The top panel of the figure shows the variability tradeoff

[5] See Clark, Goodhart and Huang (1999) and Orphanides and Wieland (2000), for examples of the method of solving for the optimal policy. Note that owing to the linear-quadratic structure of the model, the distributions of the innovations do not influence the equilibrium determination of the expectations and policy functions.

described by optimal policies for values of ω between zero and one. The lower panel plots the optimal values of θ against ω.

4 Imperfect Knowledge

As seen by the perfect knowledge solution, private inflation forecasts depend on knowledge of the structural model parameters and policymaker preferences. In addition, these parameters influence the expectations formation function nonlinearly. We now relax the assumption that private agents have perfect knowledge of all structural parameters and the policymaker's preferences. Instead, we posit that agents must somehow infer the information necessary for forming expectations by observing historical data—in essence acting like econometricians in possession of the correct specification of the economy but uncertain about the parameters of the model.

In particular, we assume that private agents update the coefficients of their model for forecasting inflation using least squares learning with finite memory. We focus on least squares learning because of its desirable convergence properties, straightforward implementation, and close correspondence to what real-world forecasters actually do.[6] Estimation with finite memory reflects the perpetual concern of subtle changes in the structural parameters of the economy. To focus our attention on the role of imperfections in the expectations formation process itself, however, we deliberately abstract from the introduction of the actual uncertainty in the structure of the economy which would justify such concerns in equilibrium.

We follow Sargent (1999) and Evans and Honkapohja (2001) by modeling finite memory or "perpetual learning" by assuming agents use a constant gain in their recursive least squares formula that places greater weight on more recent observations. This algorithm is equivalent to applying weighted least squares where the weights decline geometrically

[6] This method of adaptive learning is closely related to optimal filtering where the structural parameters are assumed to follow random walks. Of course, if private agents know the complete structure of the model—including the laws of motion for inflation, output, and the unobserved states and the distributions of the innovations to these processes—then with this knowledge they could compute efficient inflation forecasts that could outperform those based on recursive least squares. However, uncertainty regarding the precise structure of the time-variation in the model parameters is likely to reduce the real efficiency gains from a method optimized to a particular model specification relative to a simple method such as least-squares learning. Further, once we begin to ponder how economic agents could realistically model and account for such uncertainty precisely, we quickly recognize the significance of respecting (or absurdity of ignoring) the cognitive and computational limits of economic agents.

with the distance in time between the observation being weighted and the most recent observation. This approach is closely related to the use of fixed sample lengths or rolling-window regressions to estimate a forecasting model (Friedman, 1979). In terms of the mean "age" of the data used, a rolling-regression window of length l is equivalent to a constant gain κ of $2/l$. The advantage of the constant gain least squares algorithm over rolling regressions is that the evolution of the former system is fully described by a small set of variables, while the latter requires one to keep track of a large number of variables.

4.1 Least Squares Learning with Finite Memory

Under perfect knowledge, the predictable component of next period's inflation rate is a linear function of the inflation target and the current inflation rate, where the coefficients on the two variables are functions of the policy parameter θ and the other structural parameters of the model, as shown in equation 6. In addition, the optimal value of θ is itself a nonlinear function of the central bank's weight on inflation stabilization and the other model structural parameters. Given this simple structure, the least squares regression of inflation on a constant and lagged inflation,

$$\pi_i = c_{0,t} + c_{1,t}\pi_{i-1} + v_i, \tag{10}$$

yields consistent estimates of the coefficients describing the law of motion for inflation (Cf. Marcet and Sargent (1988) and Evans and Honkapohja (2001)). Agents then use these results to form their inflation expectations.

To fix notation, let X_i and c_i be the 2×1 vectors, $X_i = (1, \pi_{i-1})'$ and $c_i = (c_{0,i}, c_{1,i})'$. Using data through period t, the least squares regression parameters for equation (10) can be written in recursive form:

$$c_t = c_{t-1} + \kappa_t R_t^{-1} X_t(\pi_t - X_t' c_{t-1}), \tag{11}$$

$$R_t = R_{t-1} + \kappa_t(X_t X_t' - R_{t-1}) \tag{12}$$

where κ_t is the gain. With least squares learning with infinite memory, $\kappa_t = 1/t$, so as t increases, κ_t converges to zero. As a result, as the data accumulate this mechanism converges to the correct expectations functions and the economy converges to the perfect knowledge benchmark solution. As noted above, to formalize perpetual learning—as would be required in the presence of structural change—we replace the decreasing gain in the infinite memory recursion with a small constant gain, $\kappa > 0$.[7]

With imperfect knowledge, expectations are based on the perceived law of motion of the inflation process, governed by the perpetual learning algorithm described above. The model under imperfect knowledge consists of the structural equation for inflation (1), the output gap equation (2), the monetary policy rule (5), and the one-step ahead forecast for inflation, given by

$$\pi^e_{t+1} = c_{0,t} + c_{1,t}\pi_t, \tag{13}$$

where $c_{0,t}$ and $c_{1,t}$ are updated according to equations (11) and (12).

We emphasize that in the limit of perfect knowledge, (that is as $\kappa \to 0$) the expectations function above converges to rational expectations and the stochastic coefficients for the intercept and slope collapse to:

$$c_0^P = \frac{\alpha\theta\pi^*}{1-\phi},$$

$$c_1^P = \frac{1-\phi-\alpha\theta}{1-\phi}.$$

Thus, this modeling approach accommodates the Lucas critique in that expectations formation is endogenous and adjusts to changes in policy or structure (as reflected here by changes in the parameters θ, π^*, α and ϕ). In a sense, our model is one of "noisy rational expectations." As we show below, although expectations are imperfectly rational in that agents need to estimate the reduced form equations they employ to form expectations, they are nearly rational in that the forecasts are close to being efficient.

5 Perpetual Learning in Action

We use model simulations to illustrate how learning affects the dynamics of inflation expectations, inflation, and output in the model economy. First, we examine the the behavior of the estimated coefficients of the inflation forecast equation and evaluate the performance of inflation forecasts. We then consider the dynamic response of the economy to a persistent inflationary shock similar to the oil price shocks experienced twice in the 1970s, first during

[7] In terms of forecasting performance, the "optimal" choice of κ depends on the relative variances of the transitory and permanent shocks, similar to the relationship between the Kalman gain and the signal-to-noise ratio in the case of the Kalman filter. Here, we do not explicitly attempt to calibrate κ in this way, but instead examine the effects for a range of values of κ.

1973-74 and later during 1979-80. Specifically, we compare the outcomes under perfect knowledge and imperfect knowledge with least squares learning that correspond to three alternative monetary policy rules to illustrate the additional layer of dynamic interactions introduced by the imperfections in the formation of inflation expectations. For all of these simulations, we set $\kappa = 0.05$, $\alpha = 0.25$, $\phi = 0.75$, and $\sigma_e = \sigma_u = 1$.

The three alternative policies we consider correspond to the values of θ, $\{0.1, 0.6, 1.0\}$. These values represent the optimal policies under perfect knowledge for policymakers with preferences with a relative weight on inflation, ω, 0.01, 0.5, and 1, respectively. Hence, $\theta = 0.1$ corresponds to an "inflation dove" policymaker who is primarily concerned about output stabilization, $\theta = 0.6$ corresponds to a "moderate" policymaker who weighs inflation and output stabilization equally, and $\theta = 1$ corresponds to a "inflation hawk" policymaker who cares exclusively about inflation.

5.1 The Performance of Least-Squares Inflation Forecasts

Even absent shocks to the structure of the economy, the process of least squares learning generates time variation in the formation of inflation expectations, and thereby in the processes of inflation and output. The magnitude of this time variation is increasing in κ—which is equivalent to using shorter samples (and thus less information from the historical data) in rolling regressions. Table 1 reports summary statistics of the estimates of agents' inflation forecasting model based on stochastic simulations of the model economy. As seen in the table, the unconditional standard deviations of the estimates increase with κ. This dependence of the variation in the estimates on the rate of learning is portrayed graphically in Figure 2, which shows the steady-state distributions of the estimates of c_0 and c_1 for three values of κ, 0.025, 0.05, and 0.10, given the value of the policy parameter, For comparison, the vertical lines in each panel indicate the values of c_0 and c_1 in the corresponding perfect knowledge benchmark.

The median values of the coefficient estimates are nearly identical to the values implied by the perfect knowledge benchmark; however, the mean estimates of c_1 are biased downward slightly. There is nearly no contemporaneous correlation between estimates of c_0 and c_1. Each of these estimates, however, is highly serially correlated, with first-order autocorrelations just below unity. This serial correlation falls only slightly as κ increases.

Table 1: Least Squares Learning

	κ			
	0 (PK)	.025	.05	.10
$\theta = 0.1$				
$\bar{c}_0$	.00	.02	.01	-.01
SD(c_0)	–	(.37)	(.68)	(1.40)
$\bar{c}_1$	.90	.86	.83	.79
SD(c_1)	–	(.11)	(.17)	(.25)
Median c_1	.90	.89	.88	.87
$\theta = 0.6$				
$\bar{c}_0$	.00	.01	.01	.00
SD(c_0)	–	(.25)	(.38)	(.59)
$\bar{c}_1$	.40	.37	.35	.31
SD(c_1)	–	(.20)	(.27)	(.37)
Median c_1	.40	.39	.38	.36
$\theta = 1.0$				
$\bar{c}_0$	.00	.01	.01	.01
SD(c_0)	–	(.24)	(.35)	(.52)
$\bar{c}_1$	.00	-.02	-.03	-.06
SD(c_1)	–	(.21)	(.29)	(.39)
Median c_1	.00	-.02	-.03	-.06

Note that a more aggressive policy response to inflation reduces the variation in the estimated intercept, c_0, but increases the magnitude of fluctuations in the coefficient on the lagged inflation rate, c_1. In the case of $\theta = 1$, the distribution of estimates of c_1 is nearly symmetrical around zero. For $\theta = 0.1$ and 0.6, the distribution of estimates of c_1 is skewed to the left, reflecting the accumulation of mass around unity, but the absence of much mass above 1.1.

Finite-memory least squares forecasts perform very well in this model economy. As shown in Table 2, the mean-squared error of agents' one-step ahead inflation forecasts is only slightly above the theoretical minimum of 1.03. Not surprisingly, given that we do not include any shocks to the structure of the economy, agents' forecasting performance deteriorates somewhat as κ increases. Nonetheless, finite memory least-squares estimates perform better than those with infinite memory (based on the full sample). In an economy where inflation is in part determined by the forecasts of other agents who use finite-memory least squares, it is better to follow suit rather than to use estimates that would have better forecast properties under perfect knowledge (Cf. Evans and Ramey 2001).

Table 2: Forecasting Performance: Mean-squared Error

	κ		
Forecast method	.025	.05	.10
Perfect knowledge	1.03	1.03	1.03
$\theta = 0.1$			
LS (finite memory)	1.04	1.05	1.08
LS (infinite memory)	1.05	1.06	1.12
Long-lag Phillips curve	1.05	1.06	1.08
$\theta = 0.6$			
LS (finite memory)	1.04	1.04	1.05
LS (infinite memory)	1.06	1.09	1.14
Long-lag Phillips curve	1.05	1.06	1.10
$\theta = 1.0$			
LS (finite memory)	1.04	1.04	1.05
LS (infinite memory)	1.06	1.10	1.18
Long-lag Phillips curve	1.05	1.07	1.10

With imperfect knowledge, the ability of private agents to forecast inflation depends on the monetary policy in place, with forecast errors on average smaller when policy responds more aggressively to inflation. The marginal benefit to tighter inflation control on agents' forecasting ability is greatest when the policymaker places relatively little weight on inflation stabilization. In this case, inflation is highly serially correlated, and the estimates of c_1 are frequently in the vicinity of unity. Evidently, the ability to forecast inflation deteriorates when inflation is nearly a random walk. As seen by comparing the cases of θ of 0.6 and 1.0, the marginal benefit of tight inflation control disappears once the first-order autocorrelation of inflation is well below one.

Finally, even though only one lag of inflation appears in the equations for inflation and inflation expectations, it is possible to improve on infinite-memory least squares forecasts by including additional lags of inflation in the estimated forecasting equation. This result is similar to that found in empirical studies of inflation, where relatively long lags of inflation help predict inflation (Staiger, Stock, and Watson 1997, Stock and Watson 1999, Brayton, Roberts, and Williams 1999). Evidently, in an economy where agents use adaptive learning, multi-period lags of inflation are a reasonable proxy for inflation expectations. This result may also help explain the finding that survey-based inflation expectations do not not appear to be "rational" using standard tests (Roberts 1997, 1999). With adaptive learning, inflation forecast errors are correlated with data in the agents' information set; the standard test for forecast efficiency only applies to stable economic environments in which agents' estimates of the forecast model have converged to the true values.

Table 3: Inflation Persistence: First-order Autocorrelation

	κ			
θ	0 (PK)	.025	.05	.10
0.1	.90	.97	.98	.99
0.6	.40	.48	.55	.66
1.0	.00	.03	.06	.12

5.2 Least Squares Learning and Inflation Persistence

The time variation in inflation expectations resulting from perpetual learning induces greater serial correlation in inflation. As shown in Table 3, the first-order unconditional autocorrelation of inflation increases with κ. In the case of the "inflation dove" policymaker ($\theta = 0.1$), the existence of learning raises the first-order autocorrelation from 0.9 to very nearly unity for values of κ of 0.05 and above. For the policymaker with moderate preferences ($\theta = 0.6$), the autocorrelation of inflation rises from 0.4 under perfect knowledge to 0.66 under imperfect knowledge and $\kappa = 0.1$. Thus, in a model with a relatively small amount of intrinsic inflation persistence, the autocorrelation of inflation can be very high, even with a monetary policy that places significant weight on inflation stabilization. Even for the "inflation hawk" policymaker whose policy under perfect knowledge result in no serial persistence in inflation, the perpetual learning taking place with imperfect knowledge generates positive serial correlation in inflation. As we discuss below, the rise in inflation persistence associated with perpetual learning in turn affects the optimal design of monetary policy.

5.3 The Economy Following an Inflationary Shock

Next, we consider the dynamic response of the model to a sequence of unanticipated inflationary shocks, similar in spirit to those arising from a sustained rise in imported oil prices as occurred in the 1970s.[8] The dashed lines in Figure 3 show the outcomes for inflation and the output gap assuming perfect knowledge, for the three different values of θ.[9] In the following simulations, we set $\kappa = 0.05$.

With perfect knowledge, the inflationary shock causes a temporary rise in inflation and a decline in the output gap. The speed at which inflation is brought back to target depends on the monetary policy response, with the more aggressive policy yielding relatively sharp, but short, decline in output and a rapid return of inflation to target. With the inflation hawk or moderate policymakers, the peak increase in inflation is no more than three percentage points and inflation has returned to its target within 10 periods. In contrast, with the inflation dove policymaker, the modest policy response avoids the sharp decline in output. As a result, inflation is allowed to peak about six percentage points above target, and the return to target is more gradual, with inflation still remaining one percentage point above target after 20 periods.

Learning amplifies and prolongs the response of inflation and output to the inflationary shock, and this effect is especially pronounced if the central bank places significant weight on output stabilization. As seen in the upper panel of the figure, the inflation hawk's aggressive response to inflation effectively keeps inflation from drifting significantly away from target. As a result, inflation expectations are well anchored to the policy objective and the resulting paths of inflation and the output gap differ only modestly from those under perfect knowledge. The solid lines in Figure 4 show the responses of the public's estimates of the intercept and the slope parameter of the inflation forecasting equation under imperfect knowledge. The serially correlated inflationary shocks cause some increase in both

[8] These simulations are for illustrative purposes and are not explicitly calibrated to the events of the 1970s. In ongoing research, using a estimated model based on that presented in this paper, we are examining the role of monetary policy and inflation expectations in response to the shocks of the 1970s.

[9] Note that under least squares learning, the model becomes non-linear so impulse response functions cannot be computed simply by simulating a deterministic version of the model. Here, the updating process depends on the path for the R matrix, which in turn depends on the transitory innovations to inflation and output, u and e, respectively. For this reason, the responses under least squares learning reported in this section are averages over 1000 simulations that include innovations to inflation and output as well as the specified shocks. The initial conditions for each simulation are drawn from the steady-state distribution of the states.

estimates, but the implied increase in the inflation target peaks at only 0.4 percentage point (not shown). These estimates converge back to their steady-state means gradually as the influence of the shocks wanes. Even for the moderate policymaker who accommodates some of the inflationary shock for a time, the perceived inflation target rises by just under one percentage point. The outcomes under the inflation dove, however, are dramatically different.

The inflation dove attempts to finesse a gradual reduction in inflation without incurring a large decline in output. Instead, the timid response to rising inflation causes the perceived process for inflation to become unhinged from the policymaker's objectives. In particular, the estimated persistence of inflation, already very high owing to the policymaker's desire to minimize output fluctuations while responding to inflation shocks, rises steadily, approaching unity. With inflation temporarily perceived to be a near-random walk with positive drift, agents expect inflation to continue to rise. The policymaker's attempts to constrain inflation are too weak to counteract this adverse expectations process. Despite the best of intents, the gradual disinflation prescription that would be optimal with perfect knowledge yields stagflation—the simultaneous occurrence of persistently high inflation and low output.

Interestingly, the final simulation appears to capture some key characteristics of the United States economy at the end of the 1970s following the oil shocks, and accords well with Chairman Volcker's assessment of the economic situation at the time:

> Moreover, inflationary expectations are now deeply embedded in public attitudes, as reflected in the practices and policies of individuals and economic institutions. After years of false starts in the effort against inflation, there is widespread skepticism about the prospects for success. Overcoming this legacy of doubt is a critical challenge that must be met in shaping–and in carrying out–all our policies.
>
> Changing both expectations and actual price performance will be difficult. But it is essential if our economic future is to be secure.
> (March 27, 1981)

In contrast to this dismal experience, the model simulations suggest that the rise in inflation—and the corresponding costs of disinflation—would have been much smaller if policy had responded more aggressively to the inflationary developments of the 1970s. Although this was apparently not recognized at the time, Chairman Volcker's analysis suggests that the stagflationary experience of the 1970s played a role in the subsequent recognition of the value of continued vigilance against inflation in anchoring inflation expectations.

6 Imperfect Knowledge and Monetary Policy

6.1 Naive Application of Rational Expectations Solution

We now turn to the design of efficient monetary policy under imperfect knowledge. We start by considering the experiment in which the policymaker sets policy under the assumption that the economy is in the perfect knowledge equilibrium when, in fact, private agents possess only imperfect knowledge and base their expectations on the perpetual learning mechanism described above. That is, policy follows (5) with the response parameter, θ computed using (9).

Figure 5 compares the variability pseudo-frontier corresponding to this equilibrium to the frontier from the perfect knowledge benchmark. The top panel shows the outcomes in terms of inflation and output gap variability where the model is parameterized as above. The bottom panel shows the results of the same experiment with a more forward looking specification for inflation, with $\phi = 0.9$. In both cases, the imperfect knowledge equilibrium shown is computed with $\kappa = 0.05$.

With imperfect knowledge, the perpetual learning mechanism that determines expectations formation introduces random errors in expectations formation, that is deviations of expectations from the values that would correspond to the same realization of inflation, π_t, and policy rule. These errors are a realistic manifestation of imperfect knowledge. They are, however, costly for stabilization and are responsible for the deterioration of performance shown in Figure 5.

This deterioration in performance is especially pronounced for the policymaker who places relatively low weight on inflation stabilization. As seen in the simulations of the inflation shocks, for such policies, the time variation in the estimated autocorrelation of inflation in the vicinity of unity associated with learning can be especially costly. Furthermore, the extent of the deterioration in performance, relative to the case of perfect knowledge benchmark, is greater the more important expectations are for driving the dynamics of inflation. With the higher value for ϕ note that if a policymaker's preference for inflation stabilization is too low, the resulting outcomes under imperfect knowledge are strictly dominated by the outcomes corresponding to the naive policy equilibrium for higher values of ω.

6.2 Efficient Simple Rule

Next we examine imperfect knowledge equilibria in which policy follows a rule in the class of optimal policies under the perfect knowledge "RE" benchmark but is aware of the imperfection in expectations formation and optimizes the responsiveness of the rule θ to attain the best possible outcome, subject to the imperfection. That is, the policymaker follows the rule:

$$x_t = -\theta^S(\pi_t - \pi^*).$$

where θ^s represents the efficient choice corresponding to the relative inflation stabilization ω. We note that this is obviously not the optimal unrestricted rule in this case, simply the restricted optimal within the family of rules that are optimal under rational expectations.

Figure 6 compares the efficient choices for θ and the corresponding loss to the policymaker with perfect and imperfect knowledge for different preferences ω. From the top panel note that re-optimization can significantly improve stabilization performance relative to outcomes obtained when the policymaker naively adopts policies deemed efficient under rational expectations with perfect knowledge. More importantly, as shown in the bottom panel, the efficient policy response with imperfect knowledge is to become more vigilant against inflation deviations from the policymaker's target, relative to the optimal response under rational expectations. And the increase in the optimal value of θ is especially pronounced when the policymaker places relatively little weight on inflation stabilization, that is when inflation would exhibit high serial correlation under perfect knowledge. In the presence of imperfect knowledge, it would be efficient for a policymaker to act as if he valued inflation stability more than he actually does—if policy were to be described in terms of what appears optimal under rational expectations with perfect knowledge.

6.3 Dissecting the Benefits of Vigilance

A useful analytical illustration of the interaction of imperfections in the formation of expectations and efficient policy can be obtained by concentrating on the stochastic properties of the parameters determining the formation of expectations. From equation (6) recall that expectation formation is driven by the stochastic coefficient expectations function:

$$\pi^e_{t+1} = c_{0,t} + c_{1,t}\pi_t \tag{14}$$

One way to examine the imperfection introduced by our perpetual learning mechanism in this setting is by modeling the stochastic coefficients $c_{0,t}$ and $c_{1,t}$ in terms of their deviations from the perfect knowledge benchmark: $c_{0,t} = c_0^P + v_{0,t}$ and $c_{1,t} = c_1^P + v_{1,t}$. Substituting expectations into the Phillips curve and rearranging terms, results in the following reduced form characterization of the dynamics of inflation in terms of the control variable x:

$$\pi_{t+1} = (1 + \phi v_{1,t})\pi_t + \frac{\alpha}{1-\phi}x_t + \alpha u_{t+1} + e_{t+1} + \phi v_{0,t} \tag{15}$$

From this, it becomes evident that if the stochastic process for v were exogenous, the stabilization problem would collapse to an optimal control problem with stochastic coefficients on the dynamics of the system. A particularly useful benchmark is the special case of v_0 and v_1 being drawn from independent zero mean normal distributions with a variances σ_0^2 and σ_1^2. As is well known, this special case is analytically tractable. The optimal policy with stochastic coefficients has the same linear structure as the optimal policy with fixed coefficients and perfect knowledge and the optimal policy response is monotonically increasing in the variance σ_1^2.[10]

Although informative, the analytical case examined above does not capture the most important effects of the variation of v_0 and v_1 on θ. Quantitatively, more important for the efficient choice of θ effect than the variability of v_0 and v_1 is their serial correlation. The efficient choice of θ cannot be written in closed form in this case, but a set of stochastic simulations is informative. Consider the efficient choice of θ for our benchmark economy with balanced preferences, $\omega = 0.5$. Under perfect knowledge, the optimal choice of θ is

[10]See Turnovsky (1977) and Craine (1979) for early applications of the well known optimal control results for this case. For our model, specifically, the optimal response can be written as:

$$\theta = \frac{\alpha(1-\phi)s}{(1-\phi)(1-\omega) + \alpha^2 s}$$

where s is the positive root of the quadratic equation:

$$0 = \omega(1-\omega)(1-\phi)^2 + (\omega\alpha^2 + (1-\omega)(1-\phi)^2\phi^2\sigma_1^2)s + (\phi^2\sigma_1^2 - 1)\alpha^2 s^2.$$

While the optimal policy response to inflation deviations from target, θ, is independent of σ_0^2, the variance of the $v_{0,t}$ differentiation reveals that is it is increasing in σ_1^2, the variance of $v_{1,t}$. As $\sigma_1^2 \to 0$, of course, this solution collapses to the optimal policy with perfect knowledge.

approximately 0.6. Instead, simulations assuming an exogenous autoregressive process for either c_0 or c_1 with a variance and autocorrelation matching our economy with imperfect knowledge suggest an efficient choice of θ approximately equal to 0.7—regardless of whether the variation is due to c_0 or due to c_1. For comparison, with the endogenous variation in the parameters in the economy with learning the efficient choice of θ is 0.75.

As noted earlier, for a fixed policy choice of policy responsiveness in the policy rule, θ, the uncertainty in the process of expectations formation with imperfect knowledge raises the persistence of the inflation process relative to the perfect knowledge case. This can be seen by comparing the solid and dashed lines in Figure 7 which plot the persistence of inflation when policy follows the RE-optimal rule and agents have perfect and imperfect knowledge, respectively. This increase in inflation persistence complicates stabilization efforts as it raises, on average, the output costs associated with restoring price stability when inflation deviates from its target.

The key benefit of adopting greater vigilance against inflation deviations from the policymaker's target in the presence of imperfect knowledge comes from reducing this excess serial persistence of inflation. More aggressive policies reduce the persistence of inflation, thus improving the inflation control. The resulting efficient choice of reduction in inflation persistence is reflected by the dash-dot line in Figure 7.

7 Conclusion

When knowledge is imperfect, expectation formation deviates from the idealized form associated with rational expectations with perfect knowledge. To formalize this imperfection, we posit that economic agents estimate the parameters of the expectations formation functions that would be consistent with rational expectations using finite memory least squares. The resulting perpetual learning mechanism, which nests rational expectations as a limiting case, introduces an additional layer of interactions between monetary policy and economic outcomes, highlighting the role of imperfect knowledge on the dynamics of the economy and efficient formulation of monetary policy.

Using a simple linear model, we show that although inflation expectations remain, on average, nearly efficient, imperfect knowledge raises the persistence of inflation and distorts the policymaker's tradeoff between inflation and output stabilization. As a result, policies that appear efficient under rational expectations can, in fact, be quite inefficient and result in economic outcomes significantly worse than would be expected by analysis based on the false presumption of perfect knowledge. However, significantly improved economic performance can be achieved by adjusting monetary policy so as to minimize the impact of imperfect knowledge on expectations formation. This can be achieved by placing greater emphasis on controlling inflation. Policies emphasizing tight inflation control reduce the persistence of inflation and facilitate the formation of accurate inflation expectations. This enhances economic stability and mitigates the influence of imperfect knowledge on the economy.

In this paper, our analysis focused on the design of monetary policy under imperfect knowledge in the context of a simple illustrative model. Rational expectations, however, serves as the dominant framework for modeling expectations formation in models designed to address a much broader array of issues in macroeconomics and finance. Based on our results and given the pervasive nature of imperfect knowledge, consideration of perpetual learning and its effect on the formation of expectations potentially has wider application than to the specific issues analyzed in this paper.

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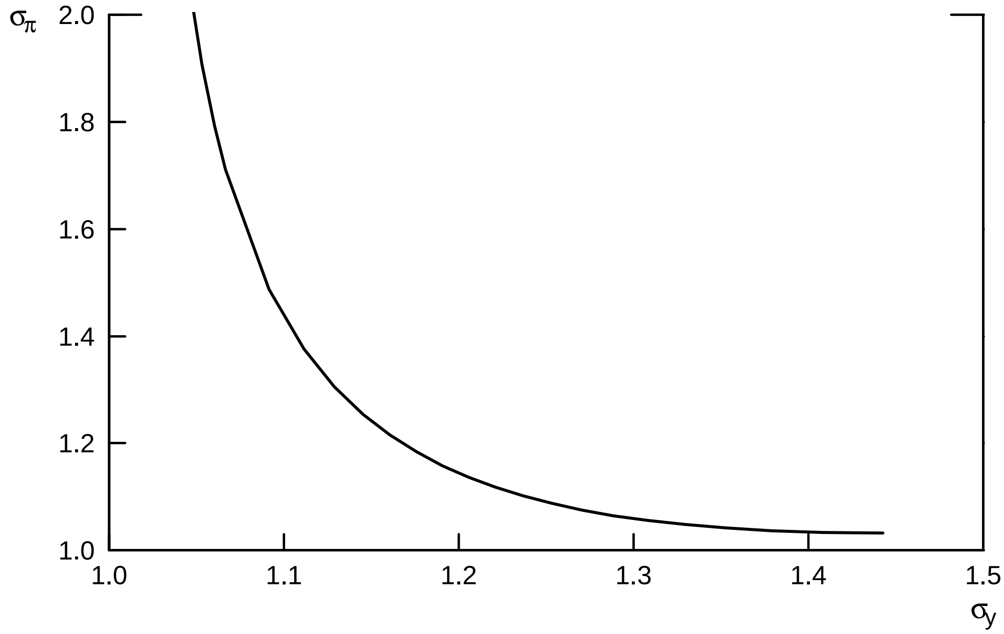
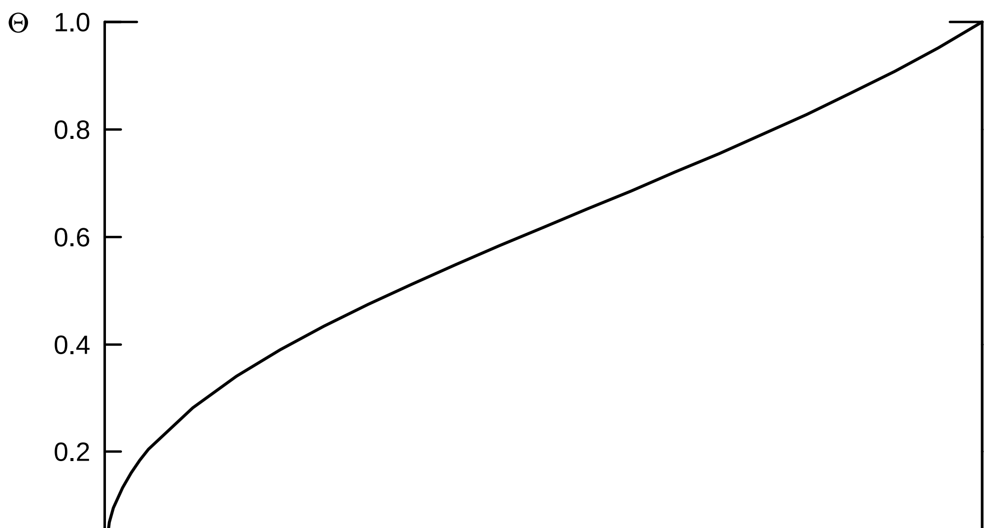
Figure 1

Efficient Policy Frontier with Perfect Knowledge



Optimal Policy Response to Inflation



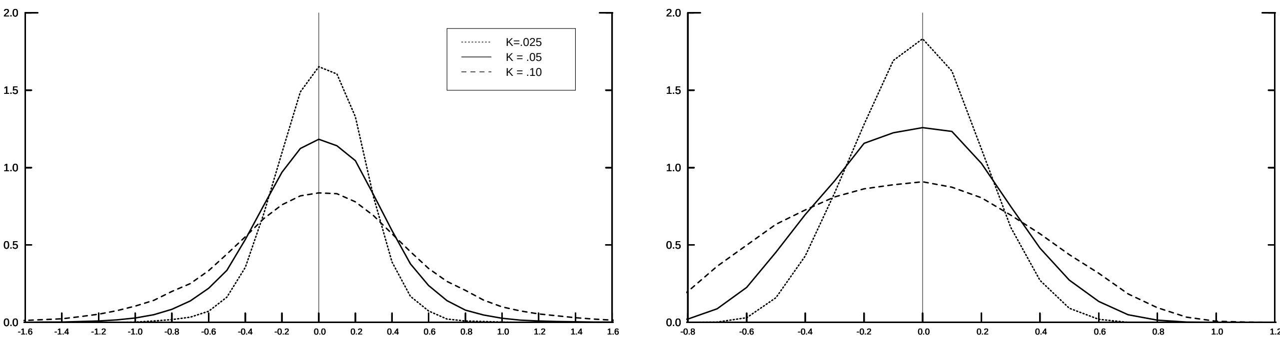
Notes: The top panel shows the efficient policy frontier corresponding to optimal policies for different values of the relative preference for inflation stabilization ω. The bottom panel shows the optimal response to inflation corresponding to the alternative weights ω.

Figure 2

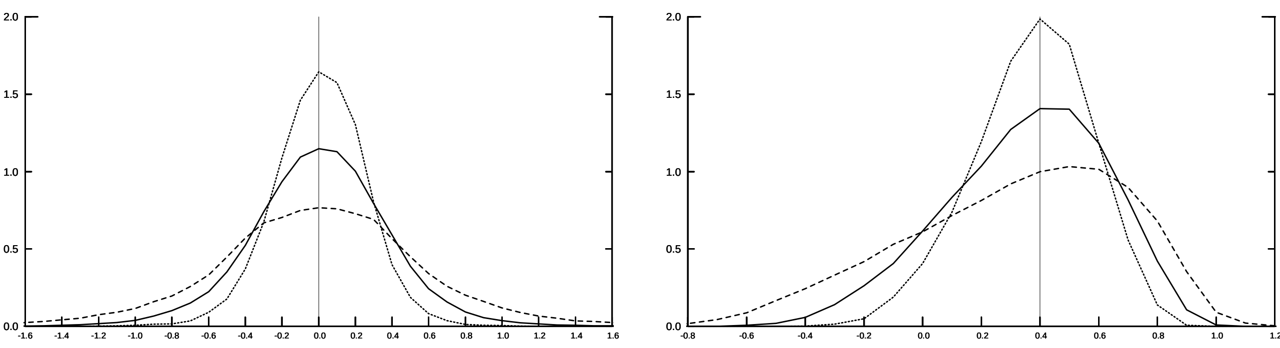
Estimated Expectations Function Parameters

Intercept   Slope

Inflation Hawk: $\theta = 1$



Balanced Preferences: $\theta = .6$



Inflation Dove: $\theta = .1$

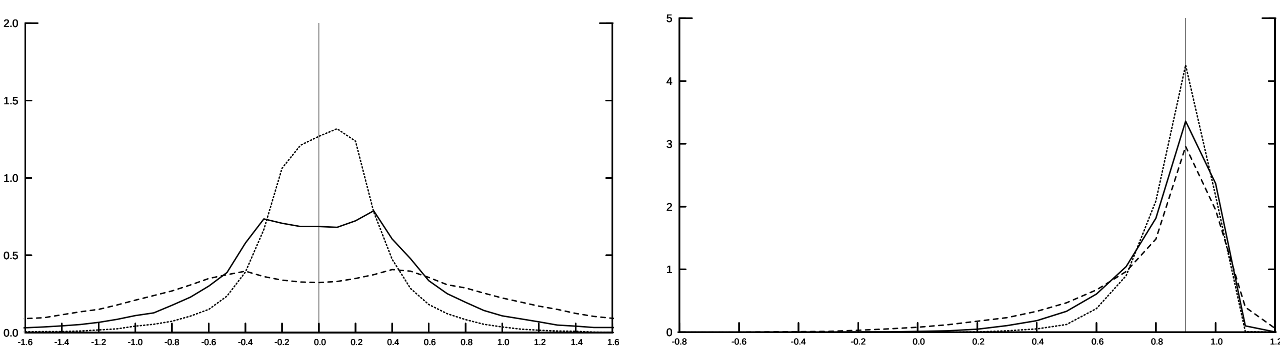
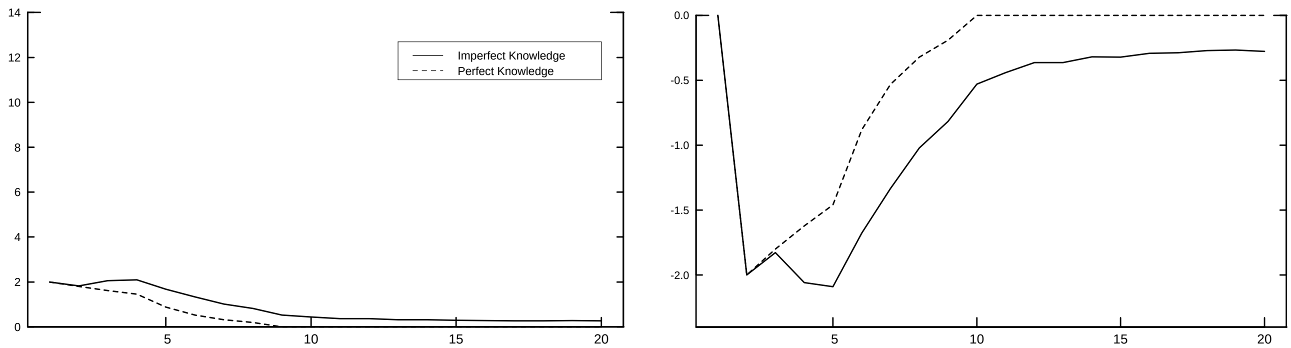


Figure 3

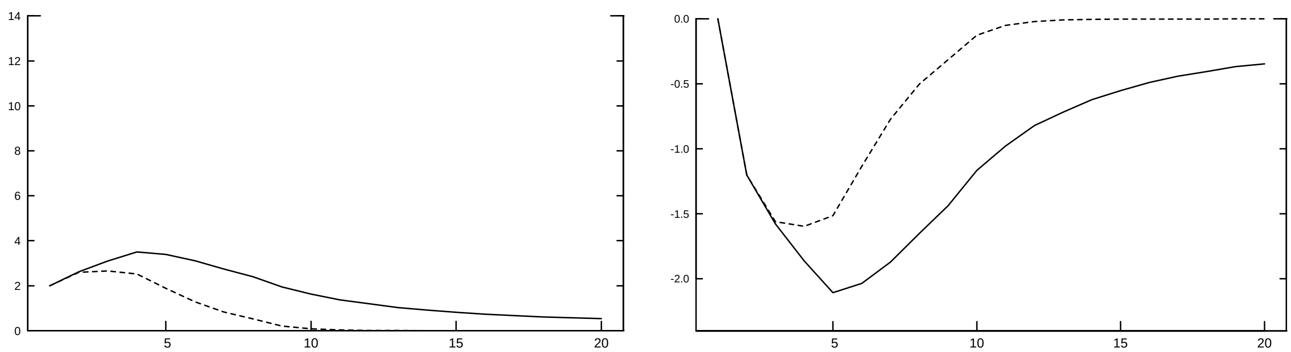
Evolution of Economy Following Inflation Shocks

Inflation | Output

Inflation Hawk: $\theta = 1$



Balanced Preferences: $\theta = .6$



Inflation Dove: $\theta = .1$

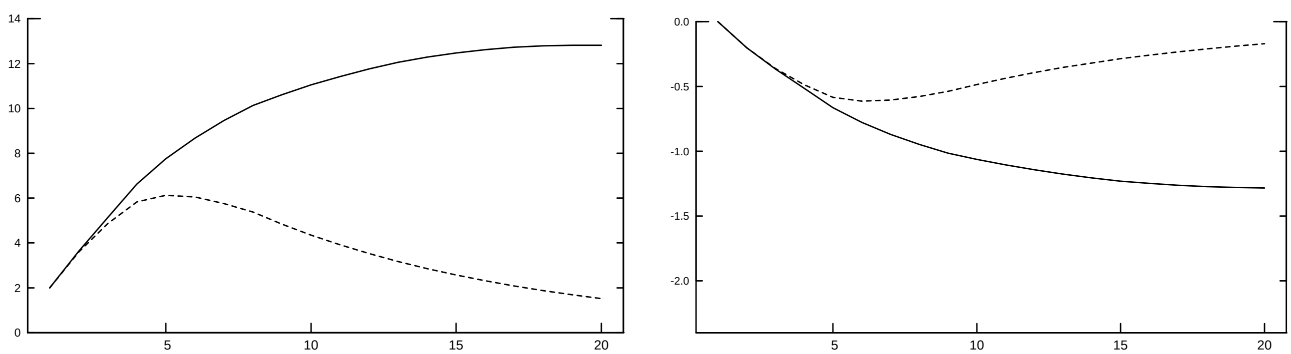
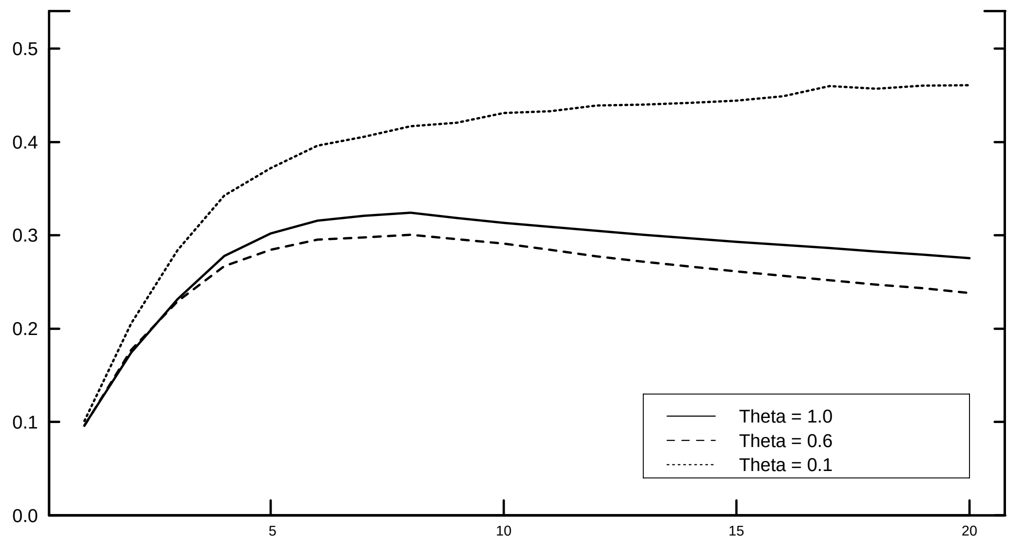


Figure 4

Estimated Intercept Following Inflation Shocks



Estimated Slope Following Inflation Shocks

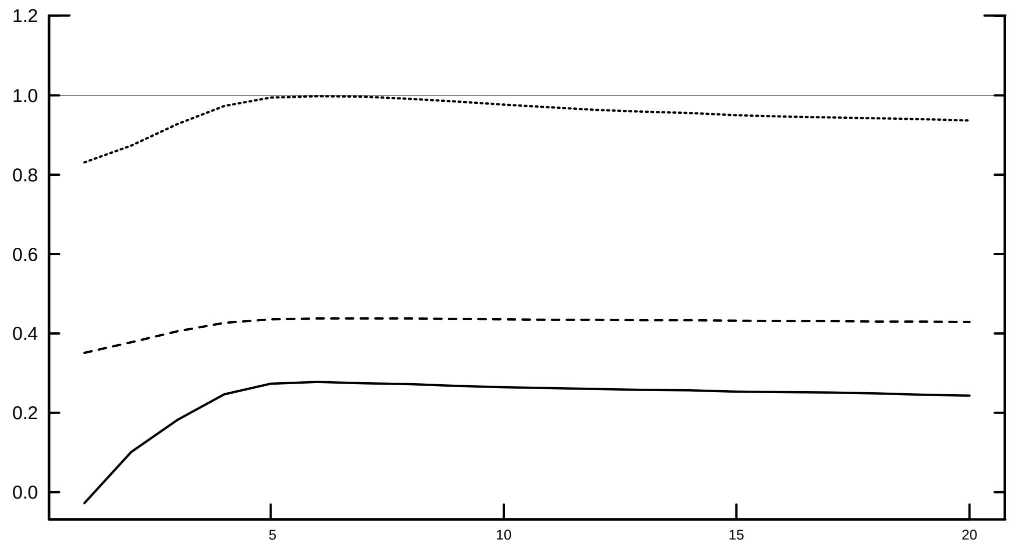
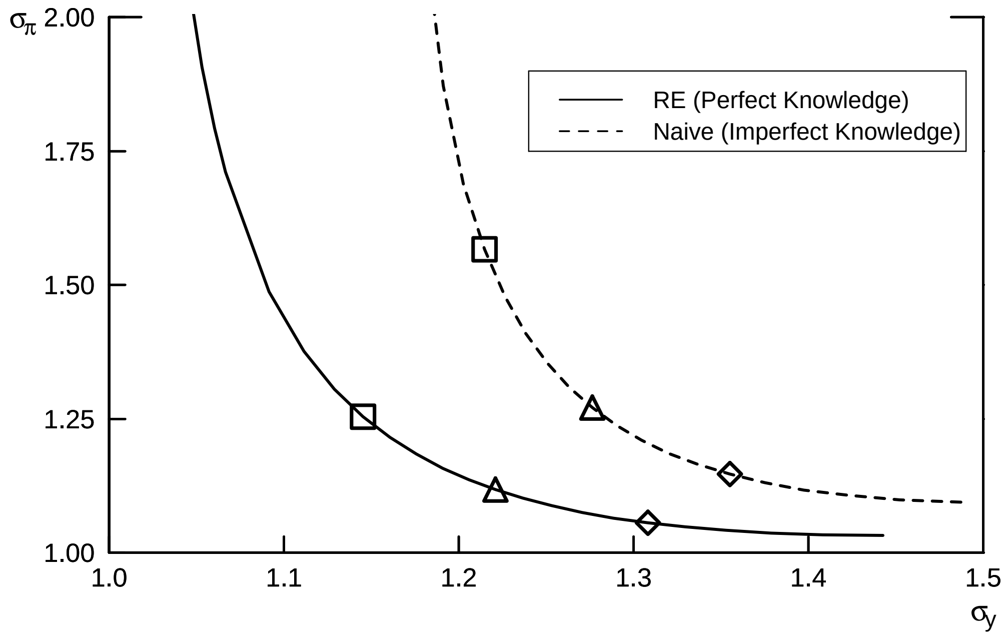
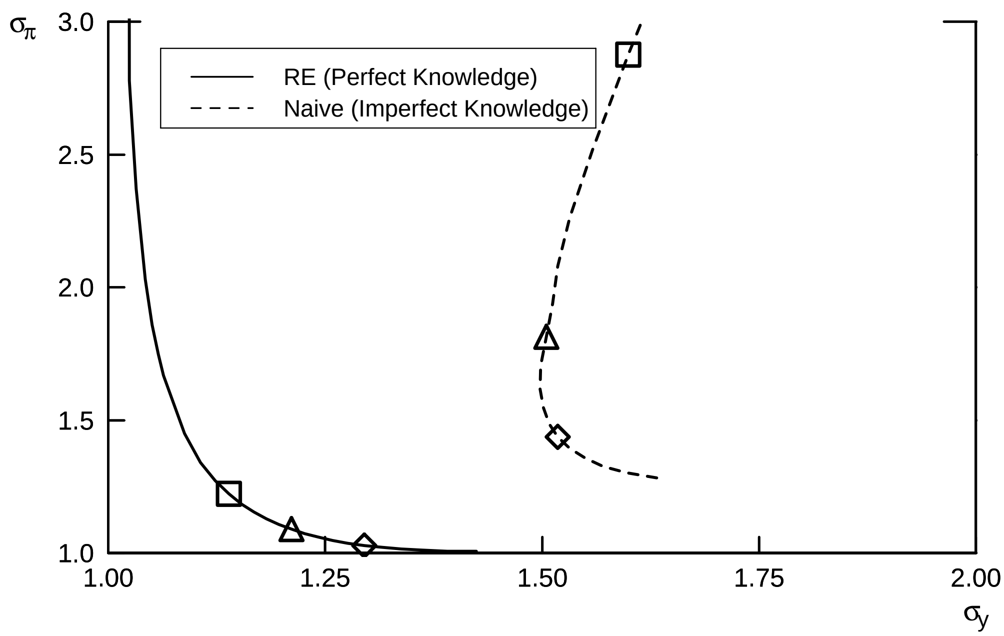


Figure 5

Outcomes with RE-policy, ($\phi = 0.75$)



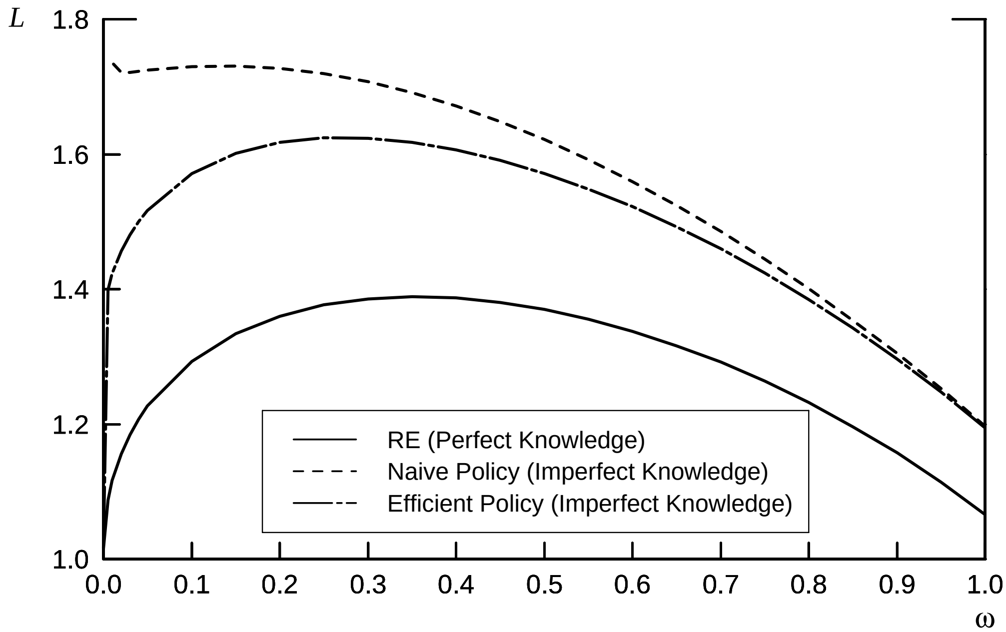
Outcomes with RE-policy, ($\phi = 0.9$)



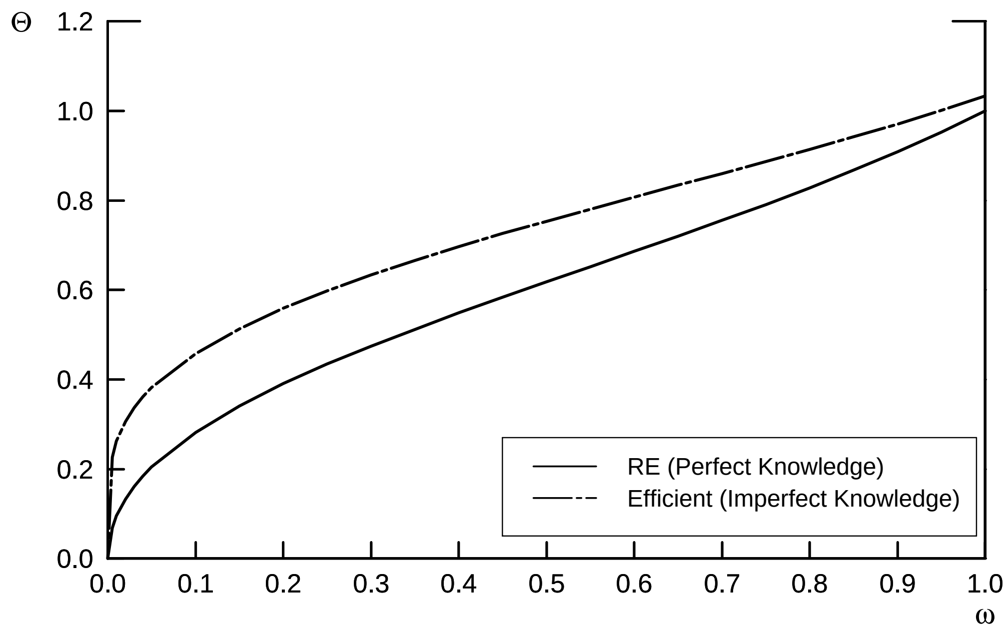
Notes: Each panel shows the efficient frontier with perfect knowledge, and corresponding outcomes when the RE-optimal policies are adopted when, in fact, knowledge is imperfect. The square, triangle and diamond correspond to preference weights $\omega = \{0.25, 0.5, 0.75\}$ respectively.

Figure 6

Policymaker Loss



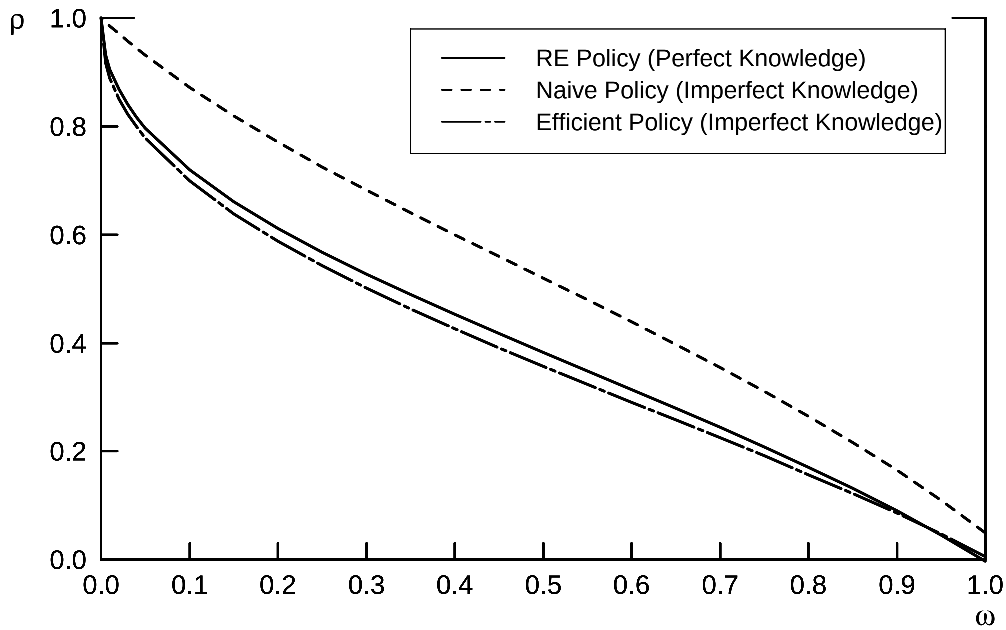
Efficient Policy Response to Inflation



Notes: The two panels show the loss and optimal policies corresponding to alternative values of the relative preference for inflation stabilization ω. The solid line presents the RE-optimal choice of θ and corresponding loss. The dash line shows the loss associated with the same policy choice when knowledge is imperfect. The dash-dot line shows the efficient choice of θ and loss with imperfect knowledge.

Figure 7

Inflation Persistence



Notes: The figure shows the population first-order autocorrelation of inflation corresponding to policies based on alternative inflation stabilization weights ω. For each value of ω, the solid line shows the inflation persistence in the benchmark case of rational expectations with perfect knowledge. The dashed line shows the corresponding persistence when policy follows the RE-optimal solution but knowledge is imperfect. The dash-dot line shows the persistence associated with the efficient one-parameter rule with imperfect knowledge.