CAPACITY UTILIZATION AND THE ASYMMETRIC EFFECTS OF MONETARY POLICY

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ABSTRACT. This paper investigates the role of variable capacity utilization as a source of asymmetries in the relationship between monetary policy and economic activity. The nature of the asymmetry is directly linked to the bottle-necks and stock-outs that emerge from the existence of capacity constraints in the real side of the economy. Within a dynamic stochastic general equilibrium framework, money has real effects due to the presence of rigidities in port-folio decisions. The model features variable capacity utilization rates across firms due to demand uncertainty. The overall message of this paper is that, depending on the rate of capacity utilization, similar monetary policy actions will have significantly different macroeconomic effects. These asymmetries are likely to have relevant implications in the conduct of monetary policy.

1. INTRODUCTION

What are the effects of central bank policy? Do they depend on the state of the economy? How should monetary policy be conducted in the short run? For many years, macroeconomists have grappled with these questions, but have not yet reached a consensus. From a theoretical point of view, the main difficulty has been to develop models that can generate the salient features of aggregate time series, which is the first step towards reliable policy analysis. Models of the transmission mechanism should generate a response of economic variables to a monetary policy shock consistent with those found in the data in, at least, three dimensions: sign, timing and magnitude. The literature has provided models that are able to replicate reasonably well the sign and timing of the transmission mechanism. However, models that can adequately account for the magnitude of the responses to monetary policy remain to be developed. A relevant aspect in this regard, and the focus of the present article, refers to the issue of asymmetries: depending on the state of the economy, similar policy actions will generate significantly different effects on the main macroeconomic variables.

Economists offer several explanations for the asymmetric response that mone-tary policy generates on economic activity. One of these arguments is known as

the capacity constraint hypothesis.[1] The idea is that some firms find it difficult to increase their capacity to produce in the short-run, giving rise to supply shortages and production bottlenecks. This is going to have important implications on one particular relation which is at the heart of monetary policy analysis, the Phillips curve. Under the capacity constraint hypothesis, when the economy experiences strong shifts in aggregate demand, driven, for instance, by changes in monetary policy, the impact on inflation will be greater when more firms are restricted in their ability to raise output in the short-run. Consequently, the short-run aggregate supply relation will display a convex shape. This is consistent with early empirical work on the Phillips curve, including Phillips (1958), which assumed that such a relationship was non-linear. This feature of the Phillips curve has relevant consequences for the performance of a monetary policy aimed at controlling inflation. In this sense, Nobay and Peel (2000) have shown that the analysis of optimal discretionary monetary policy under a non-linear Phillips Curve yields results that are in marked contrast with those obtained under the conventional linear paradigm.

Another implication of a convex Phillips curve is that the more stable output is, the higher the level of output will be in the economy, on average. The thinking behind this result is that given the lags in the effects of monetary policy, there is an incentive for pre-emptive tighten responses to inflationary pressure. Specifically, if central bankers act in this way, it will prevent the economy from moving too far up the level where inflation begins to rise more rapidly, thereby avoiding the need for a larger negative output gap in the future to reverse this large rise in inflation. Clark, Laxton and Rosen (2001) have analyzed this issue by means of stochastic simulations in a model with an asymmetric output-inflation nexus.

Recently, a great deal of research has been devoted to test empirically whether the effects of monetary policy on economic activity are asymmetric effects of monetary policy from the existence of a convex supply curve. In this vein, Cover (1992) and Karras (1996) provide evidence of asymmetries between positive and negative monetary shocks on output and prices. Weise (1999), making use of an econometric methodology that allows to test for the different types of asymmetries, finds that monetary shocks have dramatically different effects depending on the state of the economy. Similar results are obtained in Peersman and Smets (2001) for several european countries.

The objective of this paper is to develop an analytical framework consistent with the aforementioned features of the monetary transmission mechanism. Such a framework consists of a dynamic stochastic general equilibrium model which displays the non-neutralities of money needed to perform policy analysis in the short run, as well as the production inflexibilities that are able to generate the asymmetric dynamics of key macroeconomic variables documented in empirical research. The model developed in this article has two basic ingredients: (i) it incorporates a real side with production inflexibilities that result in variable rates of utilization across firms, and (ii) it considers portfolio constraints that create a short-run non neutrality of monetary policy. Regarding the first component, the model presented

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[1]Other arguments are based on "menu costs" and nominal wage rigidities. Dupasquier and Ricketts (1997) briefly survey some of the different sources of asymmetries in this regard. Another strand in the literature emphasizes the role credit market imperfections in the monetary transmission mechanism. See, for instance, Bernanke, Gertler and Gilchrist (1998).

here follows the formulation of Fagnart, Licandro and Portier (1999).[2] The issue of capacity utilization is modeled under explicit microfoundations, considering idiosyncratic demand uncertainty and a rich modeling of the production sector (firms heterogeneity and absence of an aggregate production function) within a monopolistic competitive environment. The bulk of this part of the model relies on three basic aspects: first, the limited possibilities of a short run substitutability between production factors; second, the presence of uncertainty at the time of capacity choices, which explains the presence of underutilized equipments; and third, the existence of idiosyncratic uncertainty which results in a nondegenerate distribution of utilization rates across firms. In equilibrium, a proportion of firms face demand shortages and have idle capacities, while others are at full capacity and are unable to serve any extra demand.

The second basic element of the model is a limited participation restriction, that allows monetary policy to have real effects in the short-run. Basic references in the literature on limited participation monetary models include Lucas (1990), Fuerst (1992) and Christiano (1991). In these models, the effects of an unexpected monetary policy action are firstly felt through the demand for money and the short term interest rate, the *liquidity effect*, which subsequently affects other macroeconomic variables such as investment, employment and output. The magnitude and persistence of such effects are clearly an important issue, as they capture a key non neutral effect of monetary policy. The liquidity effect can generate a strong real response to monetary policy by changing the financial costs of hiring factors of production. The existence of production inflexibilities, arising from firms' capacity utilization constraints, will condition the intensity of the liquidity and output effects, giving rise to a *capacity effect*.[3] Depending on the magnitude of these inflexibilities, the response of the economy to a monetary policy action will differ notably. These asymmetries are quantified within the theoretical framework presented here.

The outline of the article is as follows. Section 2 presents a formal description of the model's behavioral aspects. Section 3 offers a characterization of the general equilibrium of the economy and its qualitative properties. The implications for short-run dynamics are analyzed at this stage. Section 4 studies the quantitative dimension of the model, what involves the computation of impulse responses of the main variables in the model to a monetary policy shock and other numerical simulation exercises. Section 5 offers some concluding remarks and plausible lines for further research.

2. Modeling Capacity Within a Monetary Framework

The model economy consists of households, financial intermediaries, a central bank in charge of the conduct of monetary policy and two productive sectors: a competitive sector producing a final good and a monopolistic sector providing intermediate goods. These intermediate goods are the only inputs necessary for the production of the final good. The final good can be used either for consumption

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[2]Probably the first attempt to rationalize explicitly equipment idleness in modern quantitative macroeconomics is due to Cooley, Hansen and Prescott (1995). The interaction between resource underutilization and monetary policy performance has been analysed by Finn (1996) and Cook (1999). However, their description of the underutilization phenomenon, which follows Burnside and Eichenbaum (1996) depreciation in use models, is highly stylized.

[3]Christiano and Eichenbaum (1995) also analyse these margins, but their focus is on inflexibilities arising in the labor market.

or for investment purposes. Capital and labor are used in the production of intermediate goods by means of a putty-clay technology.[4] This specification of the production function allows for the introduction of a simple, but realistic, concept of capacity. Each input firm makes its investment, pricing and employment decisions under idiosyncratic demand uncertainty, that is, before knowing the exact demand for its production. This structure implies that intermediate goods firms can be either sales or capacity constrained; it also allows different firms to face different capacity constraints. Consequently, the idiosyncratic uncertainty is what explains the presence of heterogeneity between firms at equilibrium regarding the degree of utilization of their productive capacities. Intermediate goods firms have to borrow money from the financial intermediaries in order to pay the wage bill. The supply of loanable funds consists of the bank deposits made by the households at the beginning of the period, as well as cash injections made by the central bank. Hence, the production side particularities described above are embedded into an otherwise standard limited participation model.

Before proceeding to describe in more detail the different aspects that constitute the basis of the model economy, it is convenient to define the information sets that appear in the model.

$$\Omega_{0,t} = \text{economy-wide variables dated at time } t-1 \text{ and earlier}$$

$$\Omega_{1,t} = \text{includes } \Omega_{0,t} \text{ and period } t \text{ monetary policy shock}$$

2.1. **Final Good Firms.** At time t, a single final good, denoted by $\mathcal{Y}$, is produced by a representative firm which sells it in a perfectly competitive market. Such commodity can either be used for consumption or for investment. There is no fixed input, which implies that the optimization program of these firms remain purely static. The production activities are carried out by combining a continuum of intermediate goods, indexed by $j \in [0,1]$. The production technology is represented by a constant return-to-scale CES function defined as follows

$$(2.1) \qquad \mathcal{Y}_t = \left[\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} v_{j,t}^{\frac{1}{\epsilon}} \mathrm{d}j \right]^{\frac{\epsilon}{\epsilon-1}},$$

with $\epsilon > 1$ being the elasticity of substitution of inputs and where $Y_{j,t}$ is the quantity of input j used in production at date t. Here, $v_{j,t} \geq 0$ is a productivity parameter corresponding to input j. It is assumed to be drawn from a stochastic process i.i.d. distributed across time and input firms,[5] with a log normal distribution function $F(v)$ that has unit mean and is defined over the support $[\underline{v}, \bar{v}]$ with $0 < \underline{v} < 1 < \bar{v}$. The representative firm purchases inputs to intermediate good firms taking into account that the supply of each input j is limited to an amount $\bar{Y}_{j,t}$. Assuming a uniform non-stochastic rationing scheme, the optimization program of the final firm can be written as follows

$$(P1) \qquad \max_{\{\mathcal{Y}_t, Y_{j,t}\}} \mathbf{P}_t \mathcal{Y}_t - \int_0^1 P_{j,t} Y_{j,t} \mathrm{d}j,$$

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[4]Capital and labor are substitutes *ex ante*, i.e., before investing, but complement *ex post*, i.e., when equipment is installed. This implies that each firm makes a capacity choice when investing. Gilchrist and Williams (2000) show that the putty clay technology is able to generate the persistence and asymmetry observed in aggregate fluctuations.

[5]In order to keep the model tractable, it is assumed that the idiosyncratic shock is not serially correlated. Thus, its realization influences exclusively contemporary production and employment decisions, but not investment decisions.

subject to

$$Y_{j,t} \leq \bar{Y}_{j,t} \ \forall j \in [0,1] \ ,$$

where $\mathbf{P}_t$ is the price of the final good which is taken as given by the firm. When maximizing profits, the final firm faces no uncertainty: it knows the input prices $\{P_{j,t}\}$, the supply constraints $\{\bar{Y}_{j,t}\}$ and the productivity parameters $\{v_{j,t}\}$. It is important to notice that the inclusion of supply constraints in the problem above is due to the particular structure of the model, where input producing firms set their prices before the idiosyncratic shock is realized.

The solution to (P.1) determines the quantity demanded by the final good firm of the goods produced by each intermediate firm. Under deterministic quantity constraints and a uniform rationing scheme, effective demands are not well defined. Realized transactions can be derived, however. The quantity of inputs used will be determined by the corresponding idiosyncratic productivity level of each intermediate firm as described in the next result:

Lemma 1 (Realized Transactions). *The optimal allocation of inputs across intermediate good firms is given by the following system of equations*

$$(2.2) \qquad Y_{j,t} = \begin{cases} \mathcal{Y}_t v_{j,t} \left(\dfrac{P_{j,t}}{\mathbf{P}_t} \right)^{-\epsilon} & \text{if} & \underline{v}_{j,t} \leq v_{j,t} \leq \tilde{v}_{j,t} \\ \bar{Y}_{j,t} & & \text{otherwise} \end{cases}$$

with

$$(2.3) \qquad \tilde{v}_{j,t} = \frac{\bar{Y}_{j,t}}{\mathcal{Y}_t \left(\dfrac{P_{j,t}}{\mathbf{P}_t} \right)^{-\epsilon}} \ .$$

The variable $\tilde{v}_{j,t}$ determines the critical value of the productivity parameter $v_{j,t}$ for which the unconstrained demand equals the supply constraint $\bar{Y}_{j,t}$. The term $(P_{j,t}/\mathbf{P}_t)^{-\epsilon}$ appearing in the demand function of a firm with excess capacities represents, at given $\mathcal{Y}_t$, the positive spillover effects a firm with idle resources benefits from. As mentioned above, for tractability purposes it will be assumed that all intermediate firms are *ex-ante* equal. This symmetry means that input prices and capacities are the same across firms. Assuming that a law of large numbers applies in the present context, the final output supply can be expressed as follows

$$(2.4) \qquad \mathcal{Y}_t = \left[\int_{\underline{v}}^{\bar{v}} Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} v_{j,t}^{\frac{1}{\epsilon}} \mathrm{d}\mathrm{F}(v) \right]^{\frac{\epsilon}{\epsilon-1}}$$

or taking into account equation (2.2),

$$(2.5) \qquad \mathcal{Y}_t = \left\{ \left[\left(\frac{P_{j,t}}{\mathbf{P}_t} \right)^{-\epsilon} \mathcal{Y}_t \right]^{\frac{\epsilon-1}{\epsilon}} \int_{\underline{v}}^{\tilde{v}_t} v \, \mathrm{d}\mathrm{F}(v) + \bar{Y}_t^{\frac{\epsilon-1}{\epsilon}} \int_{\tilde{v}_t}^{\bar{v}} v^{\frac{1}{\epsilon}} \mathrm{d}\mathrm{F}(v) \right\}^{\frac{\epsilon}{\epsilon-1}} \ .$$

Recall that $\mathrm{F}(v)$ is the distribution function of idiosyncratic shocks; thus, for a proportion $\mathrm{F}(\tilde{v})$ of intermediate firms, the realized value of the productivity parameter is below $\tilde{v}$. Some manipulation of the previous expression allows one to write relative prices as a function of $\tilde{v}_t$, the proportion of firms with excess capacities

$$(2.6) \qquad \frac{P_{j,t}}{\mathbf{P}_t} = \left\{ \int_{\underline{v}}^{\tilde{v}_t} v \, \mathrm{d}\mathrm{F}(v) + \tilde{v}_t^{\frac{\epsilon-1}{\epsilon}} \int_{\tilde{v}_t}^{\bar{v}} v^{\frac{1}{\epsilon}} \mathrm{d}\mathrm{F}(v) \right\}^{\frac{1}{\epsilon-1}} \ .$$

The right hand side of this expression is increasing in $\tilde{v}$ and bounded above by one.[6] Consequently, the spillover term $(P_{j,t}/\mathbf{P}_t)^{-\epsilon}$ is larger than the unity. This term is going to play a significant role in the model's behavior, as will be stressed later.

2.2. Intermediate Good Firms.

In this sector, each intermediate good is produced by a monopolistically competitive firm making use of capital and labor, which are combined for production through a *putty-clay* technology. Intermediate firms start period t with a predetermined level of capacity. Such a production plan cannot be adapted to the needs of the firm within the period. Hence, investment achieved during period $t - 1$ becomes productive at date t. Investment consists of the design of a production plan by simultaneously choosing a quantity of capital goods $K_{j,t}$ and employment capacity $N_{j,t}$ according to the following Cobb-Douglas technology:

$$(2.7) \qquad \bar{Y}_{j,t} = K_{j,t}^{\alpha} N_{j,t}^{1-\alpha}$$

where $0 < \alpha < 1$. The variable $N_{j,t}$ represents the maximum number of available work-stations in the firm. Hence, the firm is at full capacity when all these work-stations are operating full-time. As it is common in models featuring a *putty-clay* technology, it is convenient to express investment decision as the choice of both $K_{j,t}$ and a capital-labor ratio $X_{j,t} \equiv K_{j,t}/N_{j,t}$. Consequently, the expression in (2.7) can be rewritten as

$$(2.8) \qquad \bar{Y}_{j,t} = X_{j,t}^{\alpha-1} K_{j,t},$$

from where the technical productivity of the installed equipments can be deduced. For the case of capital, it is given by $X_{j,t}^{\alpha-1}$, whereas $X_{j,t}^{\alpha}$ represents that of labor, so that this production function displays constant returns-to-scale in the within-period labor input. In particular, if the firm uses a quantity of labor $L_{j,t}^d$ smaller than $N_{j,t}$, it then produces $X_{j,t}^{\alpha} L_{j,t}^d$ units of intermediate good. Once the idiosyncratic (demand) shock $v_{j,t}$ is revealed, the firm instantaneously adjusts its labor demand $L_{j,t}^d$ to cover the needs of its production plan, $Y_{j,t}$, that is,

$$(2.9) \qquad L_{j,t}^d = \frac{Y_{j,t}}{X_{j,t}^{\alpha}} = \frac{1}{X_{j,t}^{\alpha}} \min\left\{ \mathcal{Y}_t v_{j,t} \left(\frac{P_{j,t}}{\mathbf{P}_t}\right)^{-\epsilon}, \bar{Y}_{j,t} \right\} .$$

In order to finance such productive activities, intermediate good firms must borrow the necessary amount of money from a financial intermediary since cash earnings do not arrive in time to finance the period wage bill. Specifically, firms rent labor at a wage W_t which is paid with cash obtained from the financial intermediary at an interest rate $R_t^L > 0$. At the end of the period, the firm pays back the loan and the interests: $W_t L_{j,t}^d (1 + R_t^L)$.

After observing the aggregate shocks, but before knowing the idiosyncratic one, input producing firms take their price decisions. Input prices are announced on the basis of (rational) expectations, before the exact value of the demand for their production is realized. This price-setting assumption has the advantage of giving a

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[6] To see this, notice that the marginal productivity of a supply-constrained input, $\partial \mathcal{Y}_t / \partial Y_{j,t}$, remains larger that its real marginal cost, $P_{j,t}/\mathbf{P}_t$, while they are equal for unconstrained inputs.

symmetric equilibrium in prices, avoiding in this manner price aggregation difficulties. [7] The price decision is static and the same rule will be followed by all firms given that, *ex-ante,* all of them are identical; that is, $P_t = P_{j,t}$. Consequently, each firm chooses a price in order to maximize its current period expected profits,

$$(2.10) \qquad P_t \equiv \arg\max E_v \left[P_t Y_{j,t} - \left(1 + R_t^L \right) W_t L_{j,t}^d \right]$$

where the expectation is defined with respect to the realization of the idiosyncratic demand shock. The expected demand for each firm is derived directly from expression (2.2), leading to

$$(2.11) \qquad E_v \left(Y_{j,t} \right) = \left(\frac{P_t}{\mathbf{P}_t} \right)^{-\epsilon} \mathcal{Y}_t \int_{\underline{v}}^{\tilde{v}_t} v \, \mathrm{dF}(v) + \bar{Y}_t \int_{\tilde{v}_t}^{\bar{v}} \mathrm{dF}(v).$$

Taking into account these considerations, the optimal price decision can be characterized by the following result:

Lemma 2 (Intermediate-Goods Pricing). *The price decision of any input firm j at date t adopts the following expression:*

$$(2.12) \qquad P_t = \left(1 - \frac{1}{\epsilon \pi \left(\tilde{v}_t \right)} \right)^{-1} \frac{\left(1 + R_t^L \right) W_t}{X_t^\alpha}$$

where $\pi \left(\tilde{v}_t \right)$ represents the probability of excess capacity in the economy, that is, $\pi \left(\tilde{v}_t \right)$ is a weighted measure of the proportion of firms for which demand is smaller than their productive capacity,

$$(2.13) \qquad \pi \left(\tilde{v}_t \right) = \frac{\left(\frac{P_t}{\mathbf{P}_t} \right)^{-\epsilon} \mathcal{Y}_t}{E_v \left(Y_{j,t} \right)} \int_{\underline{v}}^{\tilde{v}_t} v \, dF(v).$$

Notice that $\pi \left(\tilde{v} \right)$ depends only on $\tilde{v}$, as becomes clear from the combination of equations (2.11) and (2.3) above,

$$(2.14) \qquad \pi \left(\tilde{v}_t \right) = \frac{\int_{\underline{v}}^{\tilde{v}_t} v \, \mathrm{dF}(v)}{\int_{\underline{v}}^{\tilde{v}_t} v \, \mathrm{dF}(v) + \tilde{v}_t \int_{\tilde{v}_t}^{\bar{v}} \mathrm{dF}(v)}$$

The pricing mechanism resulting from (2.12) implies that intermediate firms set their price as a mark-up over the marginal cost.[8] The mark-up rate depends negatively on the (absolute) value of the price elasticity of expected sales, which is defined as the elasticity of expected sales to expected demand, $\pi \left(\tilde{v} \right)$, times the price elasticity of expected demand, ϵ. This means that when $\pi \left(\tilde{v} \right)$, the probability of a sales constraint, is large, that is, when more input firms are likely to produce under their full capacity level, firm's actual market power is reduced, implying a smaller mark-up rate. Notice that when no firm is producing at full capacity, that is, when $\pi \left(\tilde{v} \right) = 1$, the pricing rule implies a constant mark-up over the marginal cost as in the standard case. It is assumed that $\pi \left(\tilde{v} \right) > 1/\epsilon$ in order to prevent the input price being zero.

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[7]This assumption on the price behavior of input firms should not have important implications on the manner in which the economy responds to the monetary shock: since prices are announced at the time shocks are known, they are perfectly flexible in this sense.

[8]The derivation of this condition supposes that each monopolistic firm only considers the direct effect of its price decision on demand and neglects all indirect effects (e.g. the effects through $\mathcal{Y}_t$). This approximation is reasonable in a context where there is a continuum of firms.

The next step in the description of the behavior of intermediate goods firms corresponds to the choice of the productive capacity to be installed. This is a problem that has a dynamic nature. Firms choose a contingency plan $\{K_{t+1}, X_{t+1}\}_{t=o}^{\infty}$ to maximize the expected discounted value of the dividend flow

$$(2.15) \quad E_{\Omega_{1,0}} \left[\sum_{t=0}^{\infty} \Delta_{t+1} \left(P_t Y_{j,t} - W_t L_{j,t}^d (1 + R_t^L) - \mathbf{P}_t \left(K_{t+1} - (1 - \delta) K_t + \Psi \right) \right) \right],$$

with δ the capital's depreciation rate and Δ_{t+1} the stochastic discount factor, which corresponds to the representative household's relative valuation of cash across time:

$$(2.16) \qquad\qquad \Delta_{t+1} = \frac{\beta^{t+1} U_c \left(C_{t+1}, L_{t+1} \right)}{\mathbf{P}_{t+1}},$$

where β is the discount factor and U_c is the marginal utility for the household of consumption, as will be explained later. The solution to the firm's dynamic problem is characterized as follows:

Lemma 3 (Capacity Choice). *The optimal decision of investment in capital K_{t+1} and capital-labor ratio X_{t+1} is given, respectively, by the following Euler equations*
(2.17)

$$E_{\Omega_{1,0}} \left\{ \Delta_{t+1} \mathbf{P}_t - \Delta_{t+2} \mathbf{P}_{t+1} \left(1 - \delta \right) \right\} = E_{\Omega_{1,0}} \left\{ \Delta_{t+2} \left(1 - F\left(\check{v}_{t+1} \right) \right) \Phi_{t+1} \left(\frac{\bar{Y}_{t+1}}{K_{t+1}} \right) \right\}$$

and
(2.18)

$$E_{\Omega_{1,0}} \left\{ \Delta_{t+2} \Phi_{t+1} \left(\frac{\bar{Y}_{t+1}}{X_{t+1}} \right) \left[\left(\frac{\alpha \left(\epsilon - 1 \right)}{\check{v}_{t+1}} \right) \int_{\underline{v}}^{\check{v}_{t+1}} v \, dF(v) - \int_{\check{v}_{t+1}}^{\bar{v}} dF(v) \right] \right\} = 0$$

where

$$(2.19) \qquad\qquad \Phi_{t+1} \equiv P_{t+1} - \frac{\left(1 + R_{t+1}^L \right) W_{t+1}}{X_{j,t+1}^{\alpha}}.$$

The first equation states that the optimal capital stock is such that the expected user cost of capital is equal to its expected revenue, which is given by the discounted increase in profits generated by an additional unit of capital corrected by the probability of operating such unit. From the second equation one can observe the trade-off faced by the intermediate firm when choosing the optimal capital-labor ratio. When increasing the capital-labor ratio, the firm increases its labor productivity, which is given by X_t^{α}, something that has a favorable effect on its competitive position in case of excess capacities. However, increasing X_t means that the maximum level of employment available in period t will be lower, and likewise the maximum volume of sales of the firm. The optimal capital-labor ratio will be such that the two opposite effects on expected profits are equal in the margin.

2.3. Money Supply and Financial Intermediation. Financial intermediaries or banks provide liquidity to input producing firms. Banks begin each period with assets and liabilities that consist solely of households' deposits, D_t. Competition among banks for these deposits determines the market-clearing gross interest rate, $\left(1 + R_t^D \right)$, which is payable at the end of the period. Banks finance their lending activities with household deposits, as well as with funds obtained from cash injections, $\mathcal{M}_t$, made by the monetary authority every period. The asset side of banks' balance sheet is composed by loans, B_t^S, that are supplied to intermediate firms.

The bank charges a gross lending rate equal to $\left(1 + R_t^L\right)$. Financial intermediation is assumed to be a costless activity. With no barriers to entry, competitive forces will ensure that the equilibrium interest rate on loans equals the rate paid on deposits, that is, $R_t^L = R_t^D$. Moreover, in equilibrium, the financial intermediaries will supply inelastically the total amount of loanable funds at their disposal:

$$(2.20) \qquad B_t^S = D_t + \mathcal{M}_t.$$

At the end of the period, financial intermediaries remit an amount of money equal to $\Pi_t^b = (1 + R_t)\,\mu_t M_t$ to the households in concept of dividends. The variable μ_t represents the growth rate of money, that is,

$$(2.21) \qquad \mu_t \equiv \frac{M_{t+1} - M_t}{M_t} = \frac{\mathcal{M}_t}{M_t}$$

which is assumed to follow a first order autoregressive stochastic process

$$(2.22) \qquad \mu_t = (1 - \rho_\mu)\,\mu + \rho_\mu \mu_{t-1} + \varepsilon_{\mu_t},$$

with $0 < \rho_\mu < 1$ and ε_{μ_t} is an i.i.d. shock to μ_t with zero mean and standard deviation σ_μ. The random variable ε_{μ_t} is assumed to be orthogonal to all other variables in the model. This shock represents exogenous monetary policy actions by the central bank.

2.4. **Households.** The economy is populated by a continuum of homogeneous households of unit measure. These agents value alternative stochastic streams of a (composite) consumption good C_t and labor L_t^s, according to the following lifetime expected utility function

$$(2.23) \qquad E \sum_{t=0}^{\infty} \beta^t U\left(C_t, 1 - L_t^s\right)$$

where $\beta > 0$ represents households' intertemporal discount factor. The representative household begins period t holding an amount M_t of liquid assets that represent the economy's stock of money. At this point in time, it decides how much money is going to be deposited in a saving account, D_t. Importantly, this portfolio decision takes place before the realization of the aggregate money supply shock. This is a limited participation constraint that generates the required non-neutralities of monetary policy. Households also face a cash-in-advance constraint, since they have to finance the purchases of the consumption good using money. This can be done with the funds not deposited in the bank account, $M_t - D_t$, and also with labor income, that is,

$$(2.24) \qquad P_t C_t \leq M_t - D_t + W_t L_t^s$$

where L_t^s represents the fraction of time actually devoted to work and W_t is the wage paid in the competitive labor market for each unit of time supplied.

Income for the household is derived from several sources: rents from bank deposits, labor payments, which is the only source of income available to finance current period transactions and profits from financial intermediaries Π^b as well as from the goods producing firms Π^f. Consequently, the stock of money in the hands of the household at the end of the period, M_{t+1}, is given by

$$(2.25) \qquad M_{t+1} \leq M_t - D_t + W_t L_t^s - P_t C_t + \left(1 + R_t^d\right) D_t + \Pi^f + \Pi^b.$$

The optimal behavior of the household is characterized as follows:

Lemma 4. *The optimal consumption and labor decisions are given by*

$$(2.26) \qquad -\frac{U_{L_t}}{U_{C_t}} = \frac{W_t}{\mathbf{P}_t}$$

and the optimal portfolio choice

$$(2.27) \qquad E_{\Omega_{0,t}} \left\{ \frac{U_{C_t}}{\mathbf{P}_t} - \frac{\beta\left(1 + R_t^d\right) U_{c,t+1}}{\mathbf{P}_{t+1}} \right\} = 0,$$

where U_c and U_L denote the partial derivatives of U with respect C and L respectively; from the Cash-in-Advance constraint, consumption is derived

$$(2.28) \qquad C_t = \frac{M_t - D_t + W_t L_t^s}{\mathbf{P}_t}.$$

The formulation and results in this section are rather standard within the literature of limited participation models. Equation (2.26) governs the household's consumption and labor decision. Equation (2.27) is associated with the household's portfolio decision. Note that the decision on deposits is made conditional on $\Omega_{0,t}$ which excludes the time period t shocks from the time t information set. Since households cannot immediately adjust their nominal savings, a monetary shock disproportionately affects banks reserves and, hence, the supply of loanable funds. This creates the liquidity effect.

3. Qualitative Properties

This section explores the main insights and qualitative implications derived from the model economy presented above, with their corresponding intuitions. In this regard, the dynamic equilibrium associated to this economy is characterized, introducing next the important concept of aggregate capacity utilization rate. This section concludes with an analysis of the implications that different values of this variable have on the magnitude of the short-run response of key macroeconomic variables to a monetary policy shock.

3.1. Dynamic General Equilibrium.

An equilibrium for this model can be defined in the usual way: given the initial productive equipments K_0 and X_0, the initial monetary growth rate μ_0 with its corresponding stochastic process (2.22), an equilibrium for the model economy described above can be stated as follows,

Definition 1 (Dynamic General Equilibrium). *The general equilibrium of the economy during any period $t \geq 0$ is determined by a stochastic process for prices $\{\mathbf{P}_t, P_t, R_t, W_t, \Delta_t\}_{t=0}^{\infty}$ a quantity vector $\{K_t, X_t, C_t, D_t, L_t, \mathcal{Y}_t\}_{t=0}^{\infty}$ and a proportion of firms $\{F(\tilde{v}_t)\}_{t=0}^{\infty}$ that result from the optimal choices (consistent with the available information) of the central bank, the households and the firms. In an equilibrium, these choices are required to be made under rational expectations and consistent with the following market-clearing conditions:*

$$\mathcal{Y}_t = C_t + K_{t+1} - (1-\delta)K_t + \Phi$$

$$L_t^s = L_t^d$$

$$W_t L_t^d = D_t + \mathcal{M}_t$$

$$M_t^d = M_t^s$$

which represent the goods, labor, loans and money markets, respectively.

In the previous definition, the aggregate allocation and pricing functions depend on the relevant state. In particular, deposits, D_t, are a function of the information set $\Omega_{0,t}$ whereas all other price and allocation rules are elements of $\Omega_{1,t}$, where $\Omega_{0,t}$ and $\Omega_{1,t}$ are defined as above. Moreover, at equilibrium, $\mathrm{F}(\tilde{v}_t)$ represents the proportion of firms that underuse their productive capacities, i.e., those for which $v_{j,t} \in [\underline{v}, \tilde{v}_t]$. The variable $\pi(\tilde{v}_t)$ weights this proportion of firms by the relative importance of their production in total output. An important feature of this equilibrium is its symmetry: all input firms j choose the same capacity level and take the same pricing decisions. With all prices identical, aggregate employment, denoted by L_t, is equal to individual expected employment levels (up to a scaling factor):

$$(3.1) \qquad L_t = \frac{\left(\frac{P_L}{\mathbf{P}_t}\right)^{-\epsilon} \mathcal{Y}_t}{X_t^\rho} \int_{\underline{v}}^{\tilde{v}_t} v_t \, \mathrm{dF}(v_t) + \frac{K_t}{X_t} \int_{\tilde{v}_t}^{\bar{v}} \mathrm{dF}(v_t)$$

where K_t and X_t stand for aggregate capital and capital/labor respectively at time $t-1$ and available at time t, and

$$(3.2) \qquad \tilde{v}_t = \frac{\bar{Y}_t}{\mathcal{Y}_t \left(\frac{P_L}{\mathbf{P}_t}\right)^{-\epsilon}}$$

represents the ratio of productive capacity to expected demand for intermediate inputs. Notice that, as $v_{j,t} < \tilde{v}_t$, the aggregate productive capacity is underutilized at equilibrium. The individual capacity utilization rates are given by:

$$(3.3) \qquad \mathcal{C}_{j,t} = \begin{cases} \left(\frac{P_L}{\mathbf{P}_t}\right)^{-\epsilon} \mathcal{Y}_t v_{j,t}/\bar{Y}_t & \text{if} \quad v_{j,t} \leq \tilde{v}_t \\ 1 & \text{if} \quad v_{j,t} > \tilde{v}_t \end{cases}$$

which introduced into (2.1) yields the aggregate capacity utilization rate,

$$(3.4) \qquad \mathcal{C}_t \equiv \frac{\mathcal{Y}_t}{\bar{Y}_t}.$$

For a given distribution $\mathrm{F}(v_t)$ and thus given σ_v^2, there is a decreasing relationship between the capacity utilization rate, $\mathcal{C}_t$, and the weighted proportion of firms with idle resources, $\pi(\tilde{v}_t)$, which subsequently determines the mark-up rate. The aggregate capacity utilization rate is directly linked to the proportion of firms that produce at full capacity, $(1 - \pi(\tilde{v}_t))$. At given price elasticity of demand, ϵ, this implies a positive relationship between the capacity utilization and mark-up rates

$$\text{Mark-Up} \equiv \left(1 - \frac{1}{\epsilon \pi(\tilde{v}_t)}\right)^{-1}.$$

3.2. Implications for Short Run Dynamics.

In order to understand why the response of key macroeconomic variables to a monetary policy shock depends on the degree of utilization of productive resources, a diagrammatic representation of the labor and final goods market equilibrium, at given capacity level, is presented.

Figure 1 corresponds to the short-run labor market, where the upward sloping curve represents the aggregate labor supply schedule, as given in equation (2.26). The other curve, concave and sloping downwards, represents the macroeconomic labor demand curve given in equation (3.1). In the very short run, at given capacity, the labor demand curve intersects both axes. The intersection with the horizontal axis is due to the fact that even at zero real wage rates, the short-run demand for

labor is bounded above by the maximum number of work stations corresponding to the full employment of installed capacities.

Notice that when $\tilde{v} \rightarrow \underline{v}$, equation (3.1) reduces to the following expression:

$$(3.5) \qquad\qquad L_t^d = \frac{K_t}{X_t}.$$

In the opposite case, when all firms have idle resources, and thus underutilize their productive capacities, the proportion of firms $\pi(\tilde{v}_t) = 1$ and the real wage rate given in (2.12) becomes,

$$(3.6) \qquad\qquad \frac{W_t}{P_t} = \left(1 - \frac{1}{\epsilon}\right) \frac{X_t^\alpha}{(1 + R_t^L)}.$$

Along the short-run labor demand curve there is a negative relationship between the demand elasticity of sales, $\pi(\tilde{v}_t)$, and employment, L_t. Also, a downward shift along the short-run labor demand curve increases the mark-up, since the proportion of firms at full capacity is larger and so is the spill-over effect from constrained to unconstrained firms. The implications of a monetary policy shock on the response of the labor market are shown in Figure 2. An unanticipated expansionary monetary policy shock leads to a reduction in the short-term nominal interest rate through the *liquidity effect*. This implies that the maximum feasible real wage rate increases. The short-run labor demand curve intersects now the vertical axis at a higher value. The labor supply curve is not affected by the monetary shock, since wages enter into the cash-in-advance constraint. As a result, the equilibrium in the labor market implies a rise in employment. The number of firms producing at full capacity also increases. This fact produces a positive spillover into the remaining firms that have idle resources. The market power of these firms naturally rises and hence does the mark-up in the economy.

The capacity utilization rate also moves in the same direction. It is important to notice that the effects of the monetary disturbance are going to depend crucially on the state of the economy at the time of the shock, with the state determined by the capacity utilization rate. Hence, further reductions in the nominal interest rate achieved through expansionary policies will have less impact on employment and, as will be shown later, a higher effect on prices.

An shortcoming of the previous intuition about the short-run effects of a monetary shock is that it is based on an exogenous movement in the interest rate. However, the equilibrium rate of interest is determined jointly with other variables in the model such as employment and output. The results below aim at providing a general equilibrium insight into this issue.

Proposition 1. *The impact effect of an unanticipated monetary policy shock on employment is positive,*

$$L_{\mu,t} \equiv \frac{d \log L_t}{d \log \mu_t} \equiv \frac{dL_t}{d\mu_t} \frac{\mu_t}{L_t} > 0$$

as is the instantaneous correlation with output

$$\mathcal{Y}_{\mu,t} \equiv \frac{d \log \mathcal{Y}_t}{d \log \mu_t} \equiv \frac{d\mathcal{Y}_t}{d\mu_t} \frac{\mu_t}{\mathcal{Y}_t} > 0$$

Proof. Taking the ratio of the loan market-clearing condition, $W_t L_t = D_t + \mathcal{M}_t$ to the cash equation, equation (2.24), one obtains

$$(3.7) \qquad \Gamma_t = \frac{W_t}{P_t} \frac{L_t}{C_t} = \frac{D_t + \mathcal{M}_t}{M_t + \mathcal{M}_t}.$$

Notice that since $D_t < M_t$ and both variables are predetermined relative to $\mathcal{M}_t$, the response of Γ_t to an innovation in the rate of growth of money $\mu_t \equiv \mathcal{M}_t/M_t$ is positive, that is,

$$(3.8) \qquad \Gamma_{\mu,t} \equiv \frac{d\Gamma_t}{d\mu_t} = \frac{M_t - D_t}{[M_t(1 + \mu_t)]^2} > 0$$

which establishes, for example, that a monetary contraction creates a relative shortage of liquidity in the financial market. Now, introducing (2.26) into (3.7) one gets

$$(3.9) \qquad \Gamma_t = -\frac{U_{L_t}}{U_{C_t}} \frac{L_t}{C_t};$$

but from (4.1a)

$$(3.10) \qquad -\frac{U_{L_t}}{U_{C_t}} = \left(\frac{\gamma}{1-\gamma}\right) \frac{C_t}{(1-L_t)}$$

so that

$$(3.11) \qquad \Gamma_t = \left(\frac{\gamma}{1-\gamma}\right) \frac{L_t}{(1-L_t)}.$$

Differentiating implicitly the previous equation yields

$$(3.12) \qquad L_{\mu,t} \equiv \frac{d\log L_t}{d\log \mu_t} = \left(\frac{1-\gamma}{\gamma}\right) \frac{\mu_t \Gamma_{\mu,t}}{L_t/(1-L_t)^2} > 0$$

From (2.4) and (2.9), final output is a function that depends positively on employment

$$(3.13) \qquad \mathcal{Y}_t = \left[\int_{\underline{v}}^{\bar{v}} (X^\alpha L_t)^{\frac{s-1}{s}} v_t^{\frac{1}{s}} \, dF(v)\right]^{\frac{s}{s-1}}.$$

Hence, a positive (negative) monetary shock increases (reduces) output in the short run, that is,

$$(3.14) \qquad sign\left(\frac{d\log \mathcal{Y}_t}{d\log \mu_t}\right) = sign\left(\frac{d\log L_t}{d\log \mu_t}\right) > 0$$

$\square$

The previous result and the fact that, in the short-run, the level of installed equipment is fixed, imply a change in the capacity utilization rate in the economy. At the same time, the market power of those firms with idle resources is increased. This can be expressed, more formally, as follows:

Corollary 1. *In the short-run, an increase (decrease) in the equilibrium level of employment, due to an unanticipated change in the rate of growth of the money supply, rises (reduces) the mark-up, as well as the capacity utilization rate, whereas it decreases (increases) the price relation between intermediate and final goods.*

Proof. The capacity utilization rate was defined in equation (3.4) as

$$C_t \equiv \frac{\mathcal{Y}_t}{\bar{Y}_t}.$$

Given that in the short-run $\bar{Y}_t$, the capacity level in the economy, is fixed and since from the previous proposition final output increases, it follows immediately that the capacity utilization rate increases. From (2.3), the capacity utilization rate can be alternatively expressed as

$$(3.15) \qquad\qquad C_t = \frac{(P_t/\mathbf{P}_t)^\epsilon}{\tilde{v}_t}.$$

Substituting the price relation for its value given in (2.6), it follows that the capacity utilization rate depends only on $\tilde{v}_t$, the cut-off value of the idiosyncratic shock

$$(3.16) \qquad C_t \equiv C(\tilde{v}_t) = \frac{1}{\tilde{v}_t} \left\{ \int_{\underline{v}}^{\tilde{v}_t} v\,\mathrm{dF}(v) + \tilde{v}_t^{\frac{\epsilon-1}{\epsilon}} \int_{\tilde{v}_t}^{\bar{v}} v^{\frac{1}{\epsilon}}\,\mathrm{dF}(v) \right\}^{\frac{-\epsilon}{\epsilon-1}}.$$

Since, in the short-run, the capacity utilization rate increases after a positive monetary policy shock, the cut-off value $\tilde{v}_t$ must decrease (recall that the price relation $(P_t/\mathbf{P}_t)$, is a decreasing function of $\tilde{v}_t$). Notice that the weighted proportion of firms with idle resources, $\pi(\tilde{v}_t)$, depends positively on $\tilde{v}_t$, as becomes clear after rewriting (2.14) as

$$(3.17) \qquad\qquad \pi(\tilde{v}_t) = \left[1 + \frac{\int_{\tilde{v}_t}^{\bar{v}} \mathrm{dF}(v)}{v \int_{\underline{v}}^{\tilde{v}_t} v\,\mathrm{dF}(v)} \right]^{-1}.$$

Thus, the unanticipated monetary policy shock increases the mark-up since this variable, defined as,

$$(3.18) \qquad\qquad \text{Mark-Up} \equiv \left(1 - \frac{1}{\epsilon \pi(\tilde{v}_t)} \right)^{-1}$$

depends negatively on the proportion $\pi(\tilde{v}_t)$. $\qquad\square$

It is worthwhile stressing the highly non-linear relationship that exists between the mark-up and the capacity utilization rate. This means that in a high capacity economy, the effect on the mark-up of an extra increase in the capacity utilization rate due, for instance, to a monetary policy shock will be higher than the effect of the same policy in a low capacity economy. Next, the response of the nominal interest rate and the real wage rate to an unanticipated monetary policy shock is analyzed.

Proposition 2. *For a fixed level of investment and the utility function in (4.1a), the impact effect of an unanticipated monetary policy shock on the nominal interest rate is negative, while the real wage rate responds positively to the same shock. That is,*

$$R_{\mu,t} \equiv \frac{d\log(1+R_t)}{d\log\mu_t} < 0$$

and

$$(W/\mathbf{P})_{\mu,t} \equiv \frac{d\log(W_t/\mathbf{P}_t)}{d\log\mu_t} > 0.$$

Proof. From (2.26) and (3.10), the equilibrium real wage rate is given by

$$(3.19) \qquad \frac{W_t}{\mathbf{P}_t} = \frac{\gamma}{1-\gamma} \frac{C_t}{(1-L_t)}.$$

Taking into account the final-goods market clearing condition, this can be expressed as

$$(3.20) \qquad \frac{W_t}{\mathbf{P}_t} = \frac{\gamma}{1-\gamma} \frac{\mathcal{Y}_t - K_{t+1} + (1-\delta)K_t + \Phi}{(1-L_t)}$$

from where

$$(3.21) \qquad \frac{d \log (W_t/\mathbf{P}_t)}{d \log \mu_t} = \frac{\gamma}{1-\gamma} \left(\frac{d \log (\mathcal{Y}_t)}{d \log \mu_t} - \frac{d \log (1-L_t)}{d \log \mu_t} \right) > 0$$

which is positive since in the previous proposition it was proven that employment and output are positively related to an unanticipated monetary policy shock. Now, from (2.12) one can solve the gross nominal interest rate as a function of the mark-up, the ratio of intermediate to final-good prices and the wage rate as follows,

$$(3.22) \qquad (1 + R_t) = \left(1 - \frac{1}{\epsilon \pi (\tilde{v}_t)} \right) \left(\frac{P_t}{\mathbf{P}_t} \right) \frac{X_t^\alpha}{W_t/\mathbf{P}_t}.$$

Consequently, the effect on the nominal interest rate due to the unanticipated monetary shock acts through three channels, namely the mark-up, the price relation and the real wage rate:

$$(3.23) \qquad \frac{d \log (1+R_t)}{d \log \mu_t} = \frac{d \log (1 - 1/\epsilon \pi (\tilde{v}_t))}{d \log \mu_t} + \frac{d \log (P_t/\mathbf{P}_t)}{d \log \mu_t} - X_t^\alpha \frac{d \log (W_t/\mathbf{P}_t)}{d \log \mu_t}$$

The monetary shock rises employment and final output implying a reduction in the real wage rate. As discussed above, higher employment levels imply a higher capacity utilization rate and other related variables such as the mark-up and the proportion of firms producing at full capacity, $1 - \pi (\tilde{v}_t)$, so that,

$$sign \left(\frac{d \log (1 - 1/\epsilon \pi (\tilde{v}_t))}{d \log \mu_t} \right) < 0$$

Moreover, the price ratio decreases in response to an unanticipated monetary shock. This is so because from (2.6) this ratio depends only on the cut-off value $\tilde{v}_t$, which is negatively related with the equilibrium level of employment

$$sign \left(\frac{d \log (P_t/\mathbf{P}_t)}{d \log \mu_t} \right) < 0$$

From the discussion above, it follows that the derivative in (3.23) is negative.   □

Altogether, this version of the model displays the *liquidity effect* of a money supply shock, as well as the other features that characterize monetary economies. As noted above, the presence of capacity constraints as well as the monopolistic competitive environment provide a rich source of dynamics. In particular, the liquidity effect in (3.23) can be decomposed into three elements, the one corresponding to the change in the real wage, which also appears in standard limited participation models; the one corresponding to the change in the mark-up and the one corresponding to the change in the input-price relation. These last two elements are particular of the model presented here and provide the key determinant of the asymmetric

effects of the monetary policy shocks. The implications of this feature of the model is discussed in more detail below.

Proposition 3. *The magnitude of the response of employment (output) and the real wage rate to an unanticipated change in the growth rate of money is negatively related with the capacity utilization rate at the time of the shock, whereas the opposite is true for the nominal rate of interest.*

Proof. The strategy of the proof is the following: first it is shown that the magnitude of the response of employment to the monetary shock depends negatively on the level of employment at the time of the shock; next it is proven that in a high (low) capacity economy employment will be higher (lower). Recall that the ratio of funds passing through the loan market to funds passing through the goods market, defined in (3.7), can be expressed as a function of the of labor to leisure ratio,

$$\Gamma_t = \left(\frac{\gamma}{1-\gamma}\right)\frac{L_t}{(1-L_t)}.$$

Thus, the larger is the employment level, the larger is the corresponding ratio of funds Γ_t. This implies that for a given rate of growth of money, $\mu_t \equiv M_t/\mathcal{M}_t$ the ratio D_t/M_t is also high since,

$$\Gamma_t = \frac{D_t + \mathcal{M}_t}{M_t + \mathcal{M}_t} = \frac{(D_t/M_t) + \mu}{1+\mu}.$$

But a high value of Γ_t implies a low value of $(M_t - D_t)$ so that the change in the pool of funds passing through the financial intermediary that are lent to firms

(3.24)
$$\frac{d\Gamma_t}{d\mu_t} = \frac{M_t - D_t}{[M_t(1+\mu_t)]^2}$$

will be low. Next, it remains to be shown that in a high capacity utilization rate economy, employment is higher that in a low capacity utilization rate. To do this, notice that the capacity utilization rate, C_t, was defined in equation (3.4) as the ratio of current output, $\mathcal{Y}_t$, to maximum output, $\bar{Y}_t$, that is,

$$C_t \equiv \frac{\mathcal{Y}_t}{\bar{Y}_t}.$$

Since in the short-run the maximum level of output is fixed, a high level of capacity is obtained with a more intensive use of existing resources. This implies that current output will be higher. But this can be achieved only with a higher level of employment, since current output is given by

(3.25)
$$\mathcal{Y}_t = \left[\int_{\underline{v}}^{\bar{v}} (X^\alpha L_t)^{\frac{\epsilon-1}{\epsilon}} v_t^{\frac{1}{\epsilon}}\, dF(v)\right]^{\frac{\epsilon}{\epsilon-1}}.$$

Altogether, in the short run, a high level of capacity utilization rate implies a high level of employment, and thus the effect of the monetary shock on employment will be lower. The impact on the other macro variables follows from applying the results in the previous proposition.   □

Next, it is shown how final prices and output respond asymmetrically to the unanticipated monetary policy shock. The result is illustrated in Figure 3 and the argument is as follows: assume first that the economy is in the equilibrium point (Y, P) and that an unanticipated monetary policy shock takes place. The

monetary shock shifts the short-run supply curve to the right from SS to $S'S'$, since it reduces production costs by driving down the equilibrium rate of interest. Notice that the supply curve is vertical at the maximum output level, Y^*. The money injection increases, at the same time, aggregate demand of the final good through the household's cash-in-advance constraint, from DD to $D'D'$. The new equilibrium is reached at (Y',P'). If a new monetary injection occurs, the supply moves to $S''S''$ and demand to $D''D''$. The intersection of both curves determines the equilibrium value of output and price level at (Y'',P''). The increase in prices will be higher than the corresponding increase in production when comparing the two equilibrium allocations and prices. This intuition is proved more formally in the following result.

Proposition 4. *The short-run response of the price level to an unanticipated monetary policy shock depends positively on the capacity utilization rate at the time of the shock.*

Proof. Combining the cash-in-advance constraint in equation (2.24), evaluated at equality, and the loan market-clearing condition, $W_t L_t = D_t + \mathcal{M}_t$, one obtains

$$P_t C_t = M_t + \mathcal{M}_t.$$

Making use of the goods market-clearing condition, consumption can be substituted out, which yields

$$P_t \left(\mathcal{Y}_t - K_{t+1} + (1 - \delta)K_t - \Phi \right) = M_t + \mathcal{M}_t$$

assuming that capital is kept constant, since the focus is on the intra-temporal response of the variables and noting that $\mu_t \equiv \mathcal{M}_t / M_t$, the previous equation becomes

$$(3.26) \qquad\qquad\qquad P_t \mathcal{Y}_t = 1 + \mu_t.$$

Taking logarithms in the previous equation and differentiating it with respect to the gross rate of monetary growth, it follows that

$$(3.27) \qquad\qquad \frac{d\log(\mathbf{P}_t)}{d\log(1 + \mu_t)} + \frac{d\log(\mathcal{Y}_t)}{d\log(1 + \mu_t)} = 1.$$

Now, from the previous results the response of output was shown to be positive and negatively related to the capacity utilization rate of the economy. Hence, the response of prices is larger the smaller is the effect on output.   □

Up until now, the short-run or impact effects of a monetary shock have been explored. However, in order to study the dynamic response of the model to a monetary policy action, it is necessary to determine the equilibrium laws of motion by means of a numerical approximation algorithm. Precisely, this is the objective of the next section, where the quantitative properties of the model are evaluated and simulation exercises are performed as well.

4. QUANTITATIVE ANALYSIS

This section contains a description of the quantitative properties of the model economy. The objective is to illustrate the interactions between capacity utilization and mark-up rate changes by analyzing numerically the dynamic behavior of some key macroeconomic variables in response to a monetary shock. One of the results pursuit here is to show how the same shock can have significantly different short

run effects depending on the characteristics of the economy at the time the shock occurs. The variable of reference is the level of the capacity utilization rate. In order to compute the impulse response functions, the model has to be solved numerically. The solution method adopted is based on a linear approximation of the equilibrium policy rules about the non-stochastic steady state.

4.1. **Parameter Values.** The model's parameters are chosen in order to match the long-run properties of the post-war US time-series with the non-stochastic steady state values of the model. The parameterization follows, in some extent, Christiano, Eichenbaum and Evans (1998) and Fagnart, Licandro and Portier (1999). The time period is one quarter. The parameter for preferences and technology are assigned values that are standard in the business cycle literature. Table 1 summarizes the values of the calibrated parameters which are described in the sequel. The discount factor is set at $(\beta) = (1.03)^{-0.25}$; the utility function $U(\cdot)$ is given by

$$(4.1a) \qquad U(C_t, L_t^s) = \frac{\left[C_t^\gamma (1 - L_t^s)^{1-\gamma}\right]^{1-\sigma} - 1}{1 - \sigma}.$$

The consumption expenditure share in the utility function $(\gamma) = 0.35$ so that one third of the time endowment in the steady state corresponds to labor; the relative risk aversion $(\sigma) = 2$. Model calibration requires that capital's share on aggregate income $(\alpha) = 0.3485$; the annual depreciation rate of 10% implies a value $(\delta) = 0.018$; the elasticity of intermediate goods is chosen to obtain a markup ratio of 1.7 so that $(\epsilon) = 8.7364$; the fixed cost that assures zero monopolistic profits is $\Phi = 0.1057$. The calibration of the aggregate uncertainty component, the exogenous monetary reaction function, is performed following the common practice in the related research. Specifically, it is assumed that the growth rate of the aggregate money stock follows an AR(1) process. The mean growth rate of money $(\mu) = 0.016$, a value that corresponds to the mean quarterly growth rate of the monetary base in the U.S. as obtained in Cook (1999) for the period 1970:1-1995:1. The persistence of the monetary shock $(\rho_\mu) = 0.32$ with the standard deviation of $(\sigma_\mu) = 0.0038$.

Finally, the variance of the idiosyncratic shock, σ_v^2, is chosen in order to reproduce different scenarios regarding the degree of utilization of the economy's aggregate resources. In particular, setting $\sigma_v^2 = 1.75$ gives rise to a situation of low capacity utilization (65%), while fixing $\sigma_v^2 = 0.25$ results in a high rate of aggregate utilization (95%). In this manner, it is possible to analyze the quantitative importance of real side constraints as a source of asymmetry in the dynamic behavior of the economy.

4.2. **Dynamic Properties.** Recall that the main objective of this paper is to provide a formal theoretical background to the recently documented asymmetric responses of key macroeconomic variables to unanticipated monetary policy shocks. In this sense, it is studied whether the level of utilization of the productive capacity of the economy alters the dynamic properties of the model. An important characteristic for a good model to have is its ability to reproduce real world's response to simple monetary policy experiments. This section reports the dynamic responses of selected variables in the model to a one percent increase in the gross rate of monetary growth. The economy's impulse responses under the two parameterized scenarios described in the previous section are compared.

A number of results are worth noting here. First, the model is able to reproduce the stylized facts of monetary policy claimed in many studies of the identified VAR literature. In particular, an expansive money supply shock leads to an increase of employment, aggregate output and real wages and to a decrease of nominal interest rates. A liquidity effect is found in the model in that the monetary shock leads to a decrease in nominal interest rates and an increase in capital and labor. The capital/labor ratio also increases after the shock, thus, the maximum level of employment available in the period after the shock decreases, and likewise the maximum volume of sales of input firms. This negative effect is compensated by an increase in firms' labor productivity, something that has a favorable effect on their competitive position in case of excess capacities. The liquidity effect also causes output to rise immediately. Employment and investment respond to the policy shock much like output. Another important feature of the result is that real wages rise after a positive money shock. The real wage rate exerts upward pressure on the marginal cost of hiring labor, which had declined in the impact period because of the lower interest rates. Lower production costs push input prices downwards leading to an increase in input demands and inducing to a more extensive use of productive capacities in all the firms with idle productive resources. Reflecting the dynamics of output, prices initially rise and later decrease to slowly return to its non-stochastic steady state value.[9] The mark-up and the capacity utilization rates increase, whereas the weighted proportion of firms underusing their resources falls. The dynamics of these variables, in particular the increase in mark-ups induced by the higher capacity utilization, will partially offset the reduction in input prices. Consequently, the magnitude of the response of variables such as prices, output an employment, will depend crucially on the magnitude of the response of the mark-up. Recall that the response of this variable is closely related to the proportion of firms producing at full capacity. When capacity is high, the spill-over effect described above is high and thus is the market power and mark-up. Consequently, in situations of high capacity, and implied high mark-up's, the *liquidity effect* is to some extent augmented by a *capacity effect*. This is the source of asymmetry that is found in the responses of the main macroeconomic variables of this model.

The important result of this exercise is that the responses of the endogenous variables to the monetary shock depend crucially on the extent to which real resources are used. Panel (a) of Figures 4 to 13 show the impulse responses to an unanticipated shock in a situation of high and low utilization. Panel (b) shows the impact effect of the same shock for a level of capacity utilization rate ranging, at the time of the monetary policy action, from 65% to 95%. It is noticiable that the response of output, labor, real wages and investment is stronger when the capacity utilization rate is low. Intuitively, when the economy experiences a low level of capacity utilization, an expansive monetary policy shock will lead to a strong increase in output since less firms are producing at full capacity. Thus, in the low capacity case, the constraints that are associated to the predetermined level of equipment are less restrictive for a large set of input producing firms. The resulting expansion in output is achieved with the subsequent increase in employment. Under this same environment, the response of the nominal interest rate is higher due to a strong

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[9]In this version of the model, the dynamic response functions of the endogenous variables lack persistence. For instance, output and employment do not display the delayed hump shape response that the estimated response functions exhibit as reported in Chirstiano et al. (1998).

liquidity effect. The equilibrium interest rate at which firms will accept the new currency is much lower when more firms cannot increase their production due to the existence of capacity utilization constraints. Notice also the highly non-linear path that the impact response of this variable traces when the capacity utilization rate, at the time of the shock, increases. This result is, somehow, related to that of Cook (1999) who develops a model in which firms cannot transfer capital across sectors.

As expected, output prices react more in a high capacity economy, provided that investment adjusts sluggishly. The monetary shock also produces an impact change on some other important endogenous variables. The response of the capital-labor ratio is significantly different depending on the capacity utilization rate of the economy. The capacity utilization rate increases more and the price relation decreases more, the lower is the capacity at the time of the shock. The mark-up increases more in a high capacity economy, and the weighted proportion of firms producing while having idle resources also decreases more in a high capacity economy. The dynamics of these variables also display a remarkably non-linear shape. It must be pointed out that the monetary shock does not produce a dynamic response in these capacity-related variables. This reflects the intraperiod nature of the real frictions embedded in the model. As a result, the qualitative characteristics of the contemporaneous impact do not extend beyond the period of the monetary policy shock. The asymmetric dynamics are not kept along time. Hence, a rather interesting extension of the model presented here is to achieve more persistency in the response of the endogenous variables to the monetary shock, for instance, by extending the intratemporal nature of the idiosyncratic shock towards an intertemporal dimension.

Finally, Figure 14 represents a Pseudo Phillips curve. Each point in this Figure corresponds to a cumulated increase in the capacity and inflation gap due to a series of 1% unanticipated monetary policy shocks. It is relevant to see the non-linearity in such a relationship. For low levels of capacity utilization rate, the monetary policy shock exerts more pressure on real economic activity than on prices. As the economy moves toward a situation with a higher rates of aggregate capacity utilization, the effects of subsequent monetary policy shocks are comparatively more intense on prices.

5. CONCLUDING REMARKS AND EXTENSIONS

This paper provides a formal approximation to the issue of asymmetries within the quantitative monetary macroeconomics literature. The overall message arising from the present analysis is that the same policy actions may have significantly different effects depending on the extent to which productive resources are being used in the economy. These results have been obtained under a framework that considers the interaction of endogenous capacity utilization (derived from productive constraints and firm heterogeneity) and market power, together with a monetary structure that assumes a 'limited participation' constraint.

The source of the asymmetry is directly linked to the bottlenecks and stockouts that emerge from the existence of capacity constraints in the real side of the economy. These constraints act as a source of amplification of monetary shocks and generates asymmetries in the response of key macroeconomic variables. These effects interact additionally with those emerging from the imperfectly-competitive

environment that characterizes the intermediate-good sector through optimal mark-up changes. Within the structure of the model presented here, a non-walrasian pricing behavior in line with New Synthesis models à la Goodfriend and King (1997) could easily be incorporated. In this manner, one could follow the results of recent empirical evaluation exercises of dynamic stochastic general equilibrium models, such as those of Christiano, Eichenbaum and Evans. (1997), who claim that a combination of limited participation with sticky-price behavior could successfully account for the basic stylized facts observed in the data.

The empirically plausible asymmetry of the Phillips curve, due to the fact that some firms find it difficult to increase their capacity to produce in the short run, is going to have important implications for the conduct of monetary policy. In this sense, recent studies have shown that the analysis of optimal discretionary monetary policy under a non-linear Phillips Curve yields results that are in marked contrast with those obtained under the conventional linear paradigm. All these particularities, that are likely to offer interesting insights into the monetary transmission mechanism, are worthwhile exploring using the analytical framework developed here.

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TABLES

TABLE 1: PARAMETER VALUES

Consumption/leisure share	γ	0.35
Risk Aversion	σ	2
Intertemporal discount rate	β	$(1.03)^{-.25}$
Capital share	α	0.3485
Elasticity intermediate goods	ϵ	8.7364
Depreciation rate	δ	0.018
Fixed costs	Φ	0.1057
Autocorrelation monetary growth rate	ρ_μ	0.32
Mean monetary growth rate	μ	0.016
Standard deviation monetary shock	σ_μ	0.0038
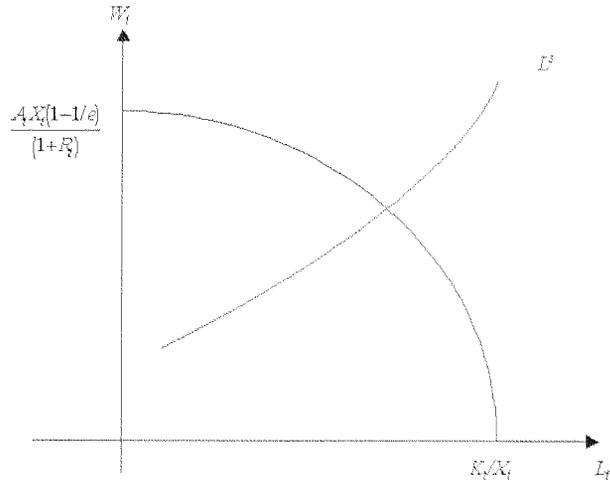
FIGURES

Figure1: Short-Run Labor Market Equilibrium
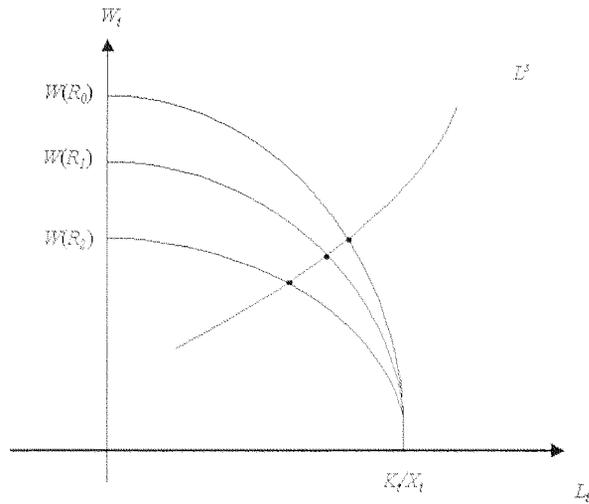
Figure 2: Short Run Effect of Monetary Policy Shock
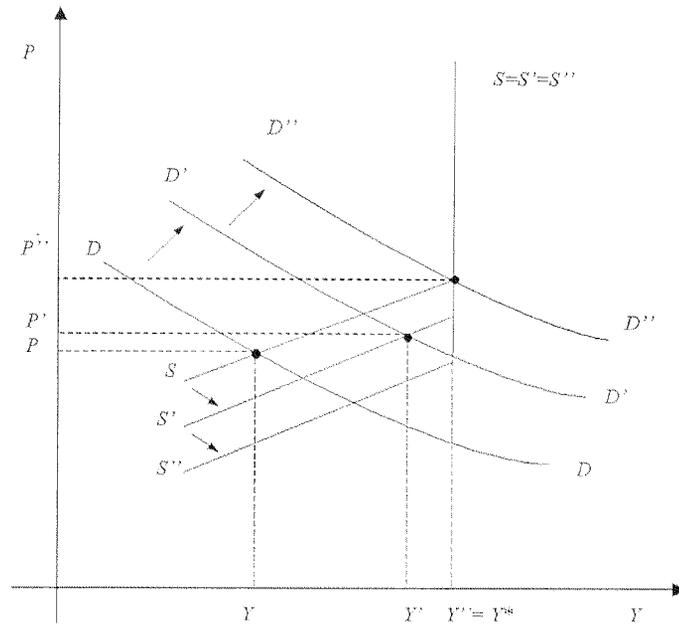
Figure 3: Price and Output Response to Monetary Shock
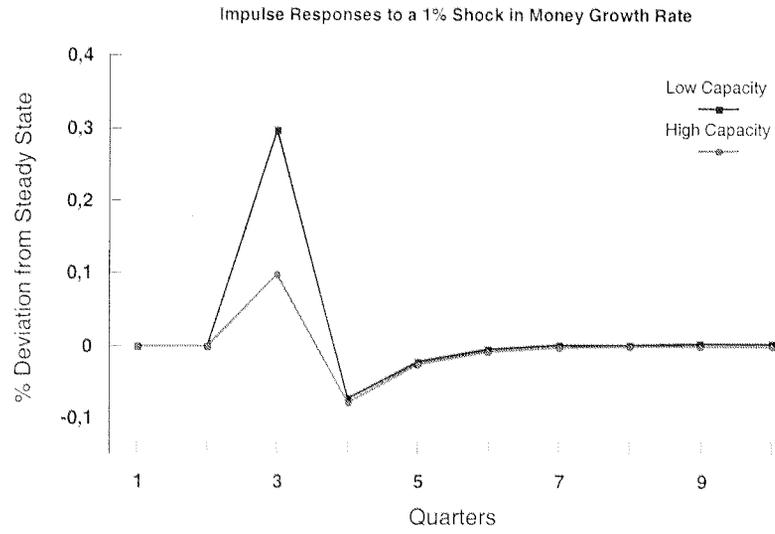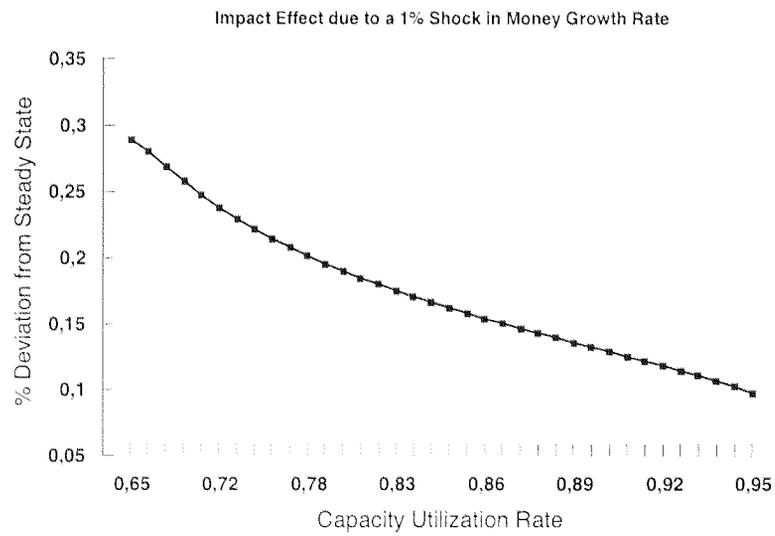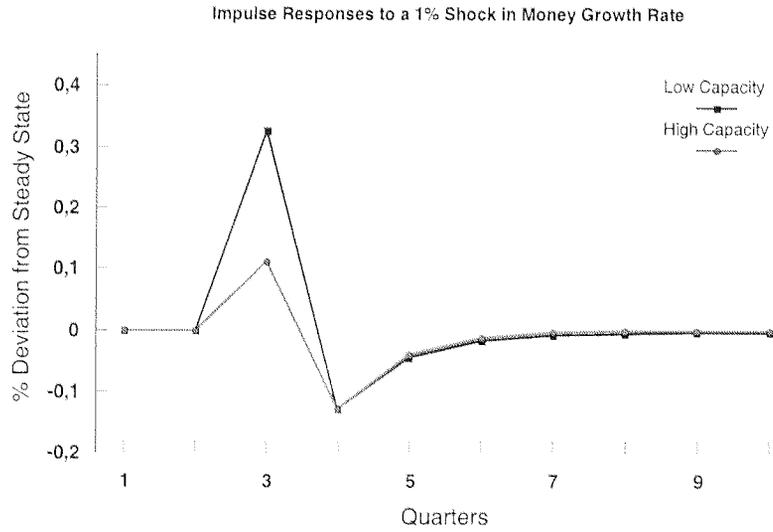
Impulse Responses to a 1% Shock in Money Growth Rate

Figure 4a: Final Output

Impact Effect due to a 1% Shock in Money Growth Rate

Figure 4b: Final Output

Impulse Responses to a 1% Shock in Money Growth Rate

Figure 5a: Employment

Impact Effect due to a 1% Shock in Money Growth Rate

Figure 5b: Employment

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Impulse Responses to a 1% Shock in Money Growth Rate

Figure 6a: Investment

Impact Effect due to a 1% Shock in Money Growth Rate

Figure 6b: Investment

Impulse Responses to a 1% Shock in Money Growth Rate
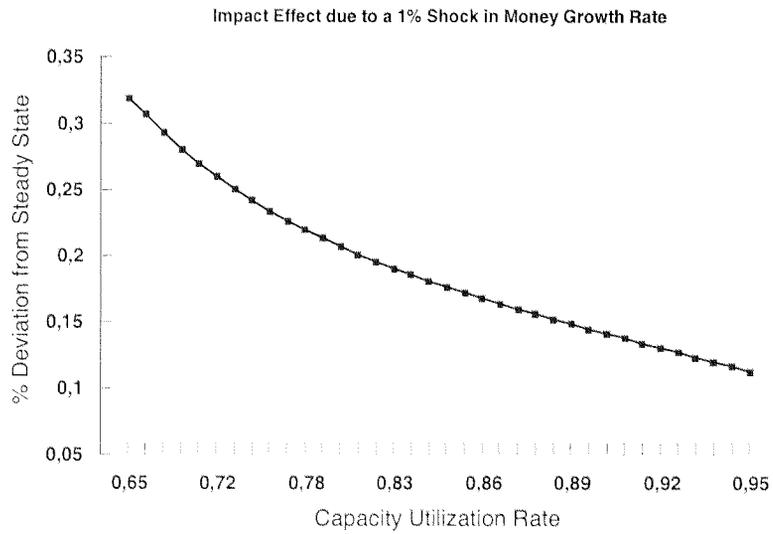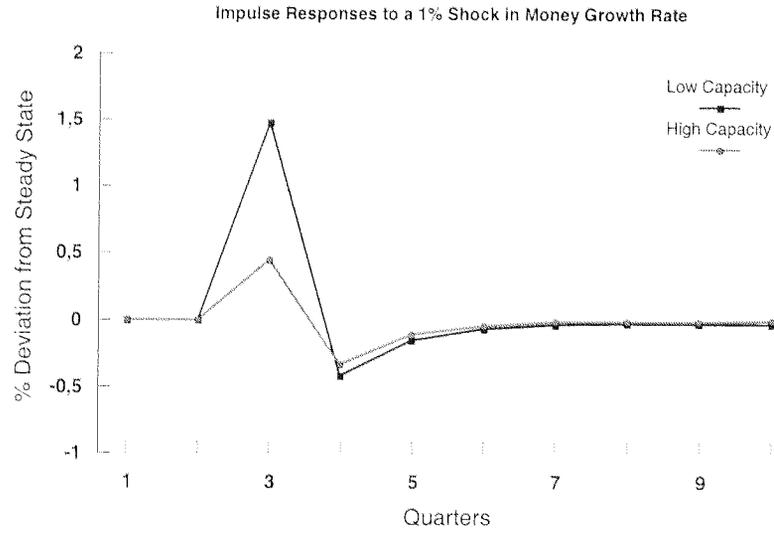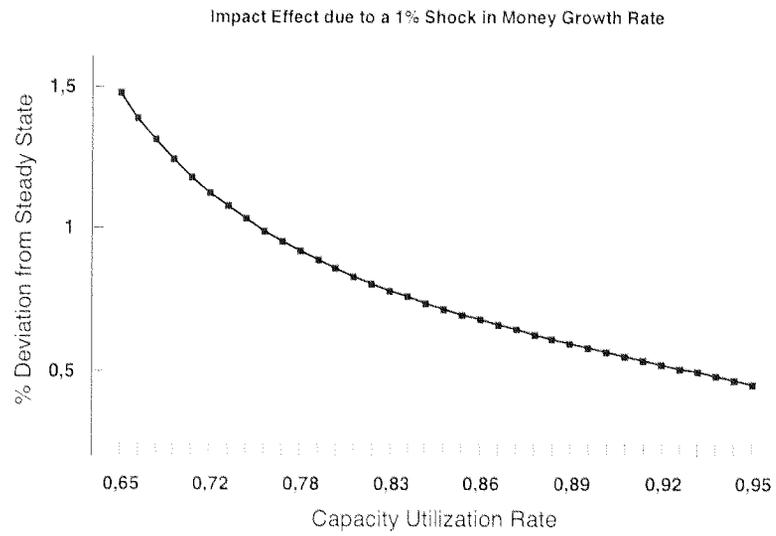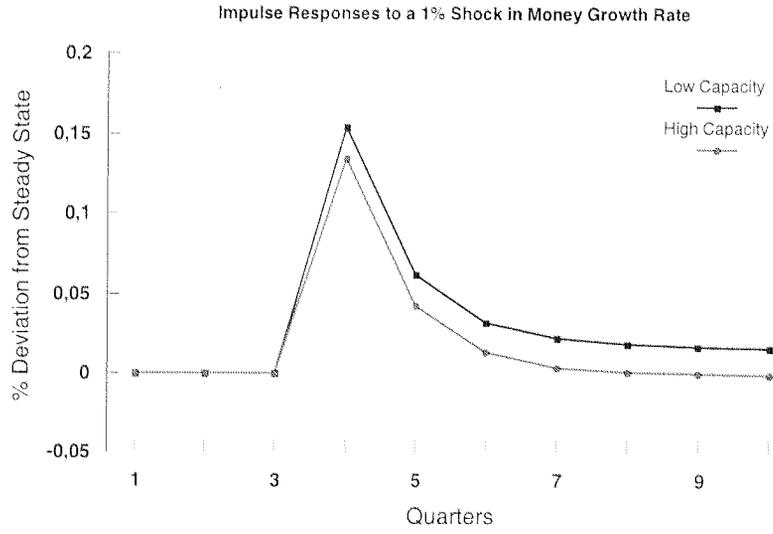
Figure 7a: Capital-Labor Ratio

Impact Effect due to a 1% Shock in Money Growth Rate
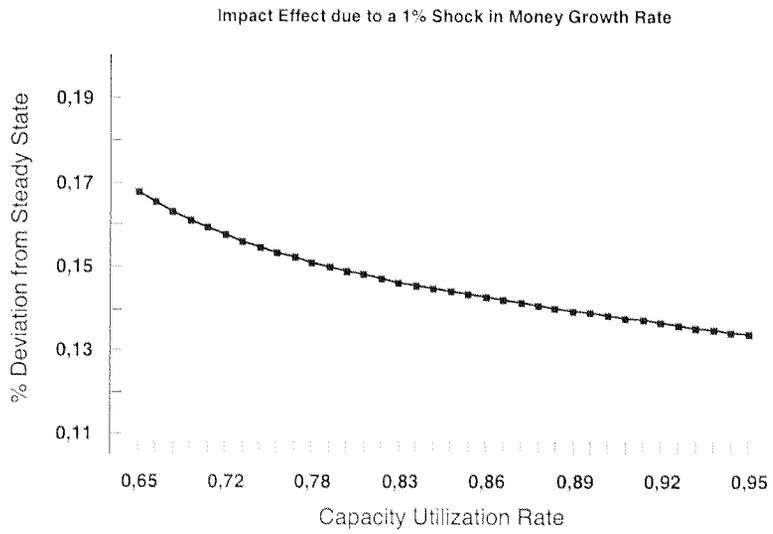
Figure 7b: Capital-Labor Ratio

Impulse Responses to a 1% Shock in Money Growth Rate
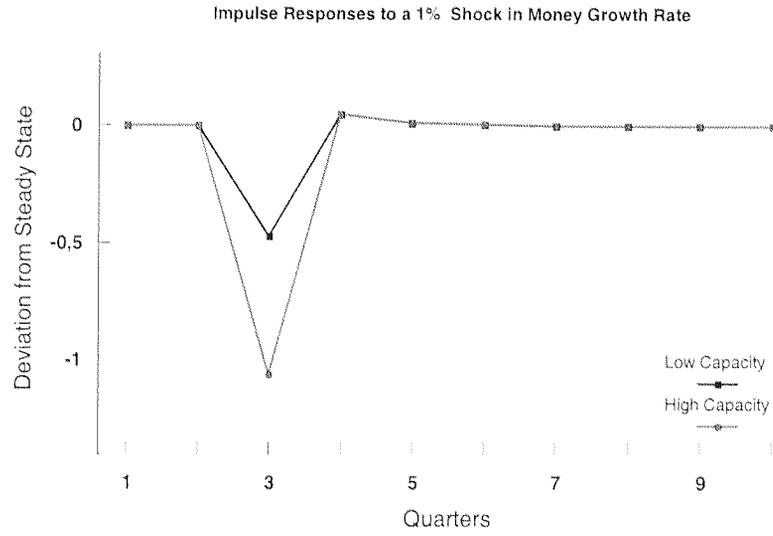
Figure 8a: Nominal Interest Rate

Impact Effect due to a 1% Shock in Money Growth Rate
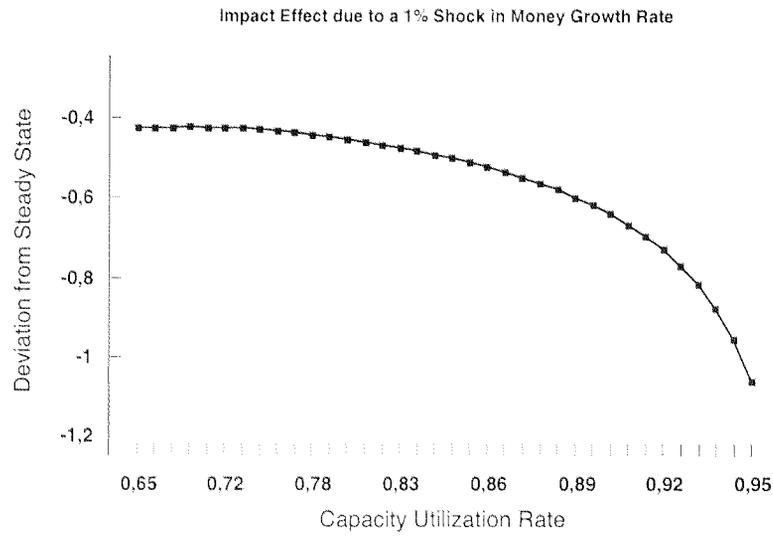
Figure 8b: Nominal Interest Rate

Impulse Responses to a 1% Shock in Money Growth Rate

Figure 9a: Deposits

One-period-ahead Effect due to a 1% Shock in Money Growth Rate

Figure 9b: Deposits

Impulse Responses to a 1% Shock in Money Growth Rate
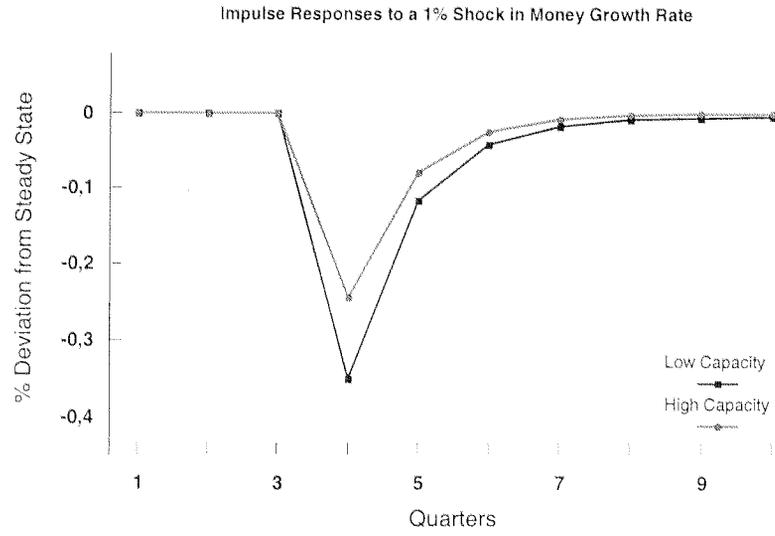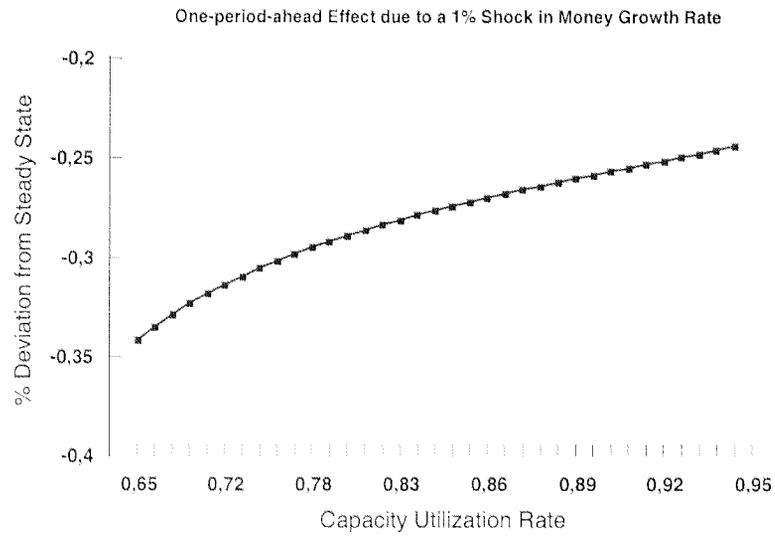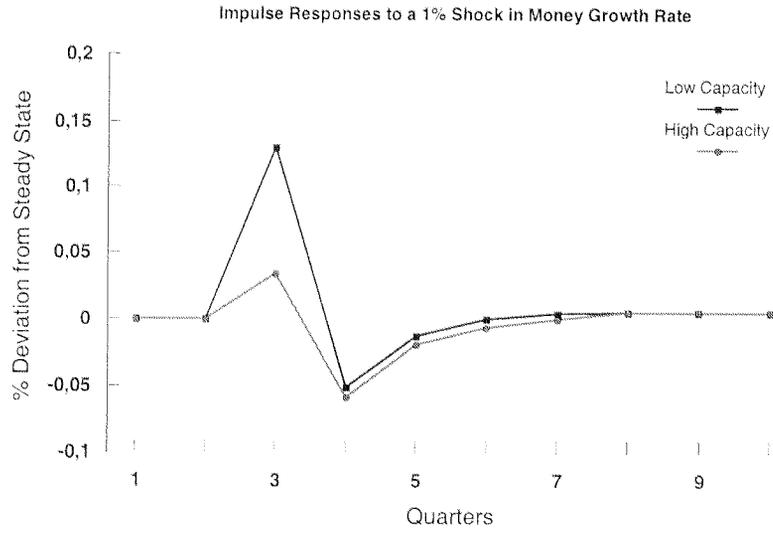
Figure 10a: Real Wage Rate

Impact Effect due to a 1% Shock in Money Growth Rate
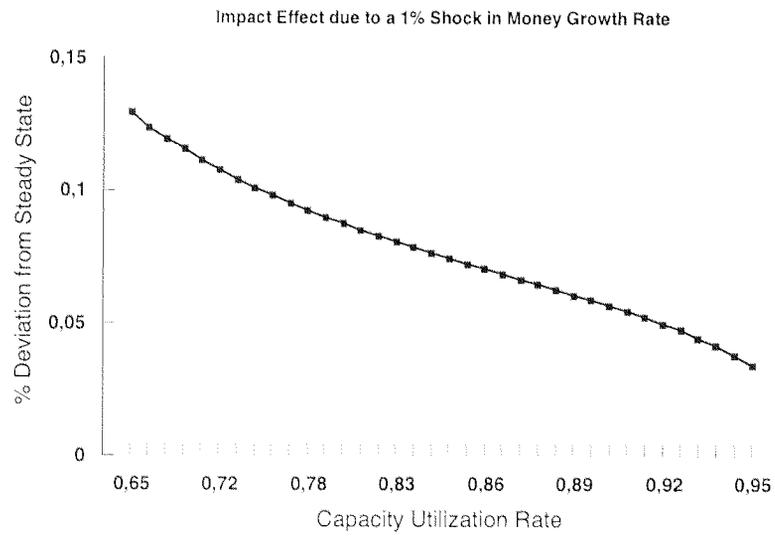
Figure 10b: Real Wage Rate

Impulse Responses to a 1% Shock in Money Growth Rate
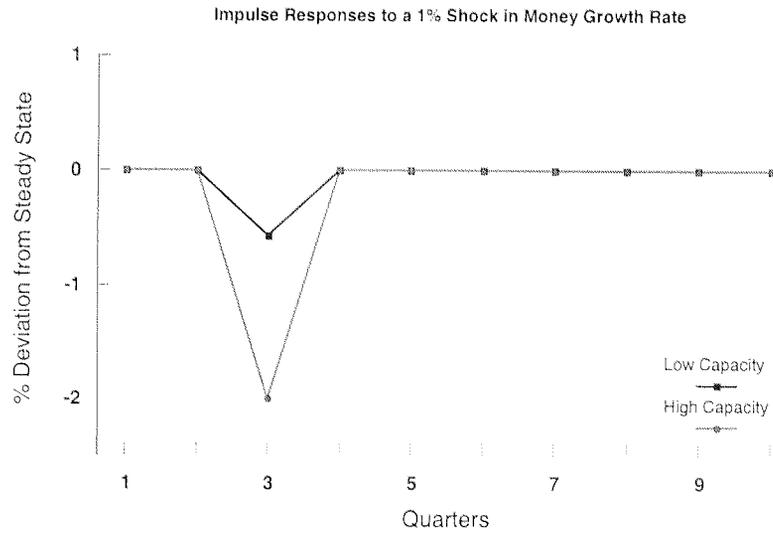
Figure 11a: Underused Resources

Impact Effect due to a 1% Shock in Money Growth Rate
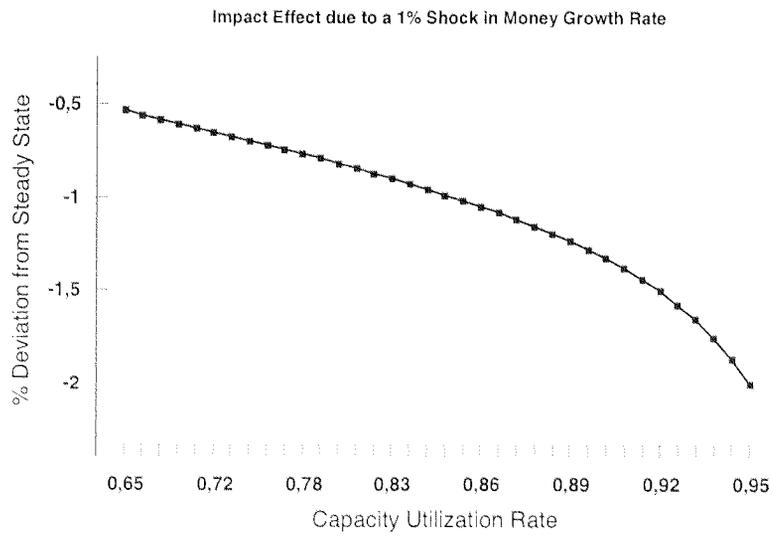
Figure 11b: Underused Resources

Impulse Responses to a 1% Shock in Money Growth Rate

Figure 12a: Mark Up

Impact Effect due to a 1% Shock in Money Growth Rate

Figure 12b: Mark Up

Impulse Responses to a 1% Shock in Money Growth Rate
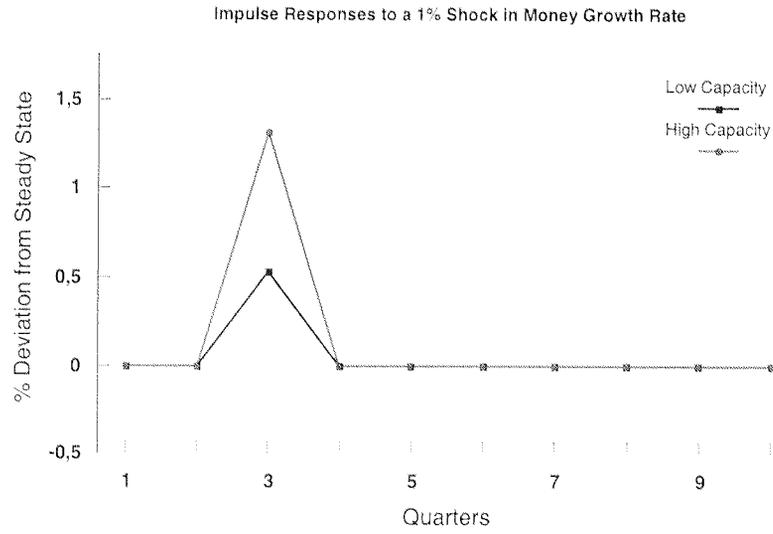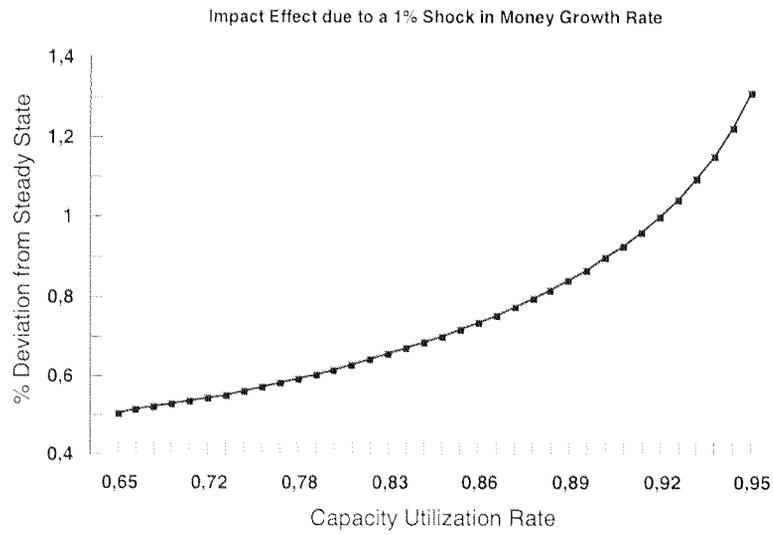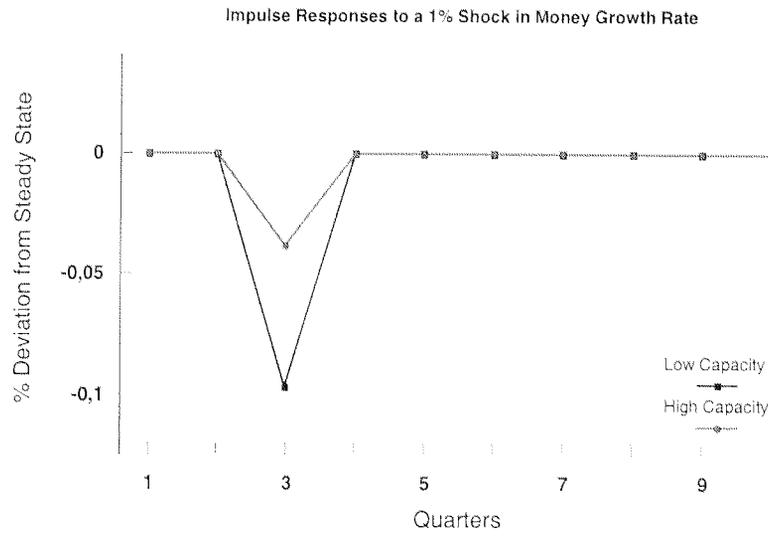
Figure 13a: Price Relation

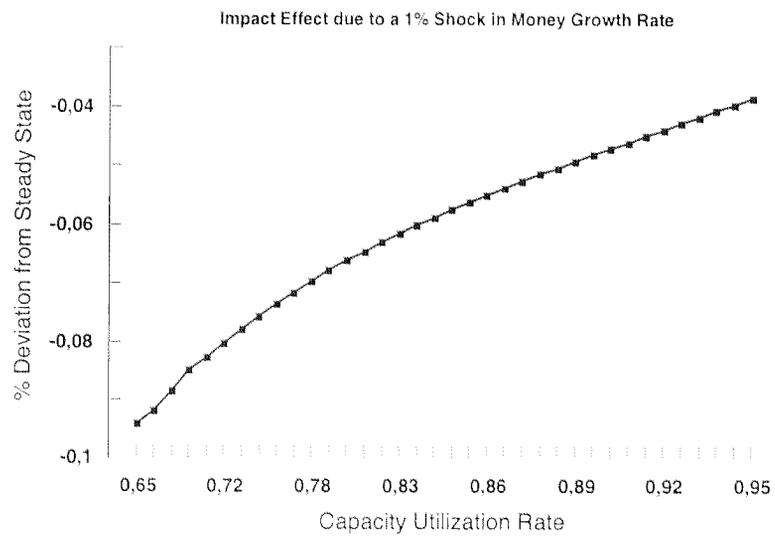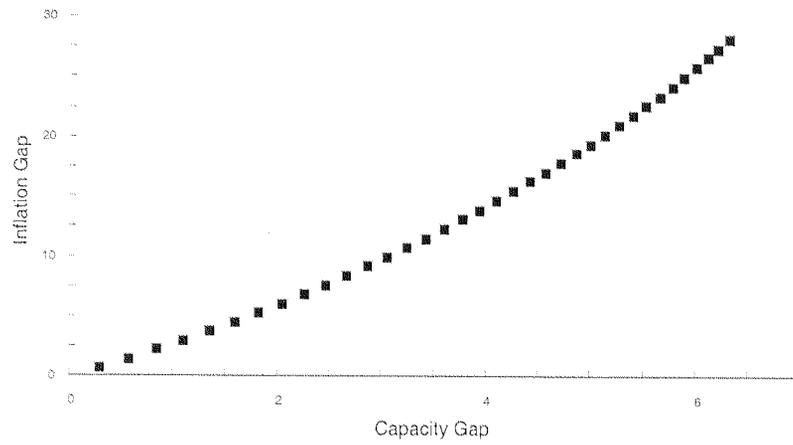Impact Effect due to a 1% Shock in Money Growth Rate

Figure 13b: Price Relation

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Note: Each point in the figure represent the cumulated increase in equilibriu
values of the capacity and inflation gap that results from a series of 1%
monetary policy shocks.

Figure 14: Pseudo Phillips Curve