Sticky Prices and Inventories: Production Smoothing Reconsidered*

Andreas Hornstein† and Pierre-Daniel Sarte‡

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Abstract

We study the implications of nominal price rigidities in a model where firms use inventories to smooth production because of increasing marginal cost. Conventional criticisms of production smoothing models have focused on their inability to replicate the following two stylized facts: (1) inventory investment is positively correlated with sales and, therefore, production is more volatile than sales, (2) movements in inventory-sales ratios are persistent. In contrast, we show that a standard production smoothing model of inventory behavior is consistent with these facts when prices are sticky. Furthermore, these results hold irrespective of whether the economy is driven by nominal demand or real supply shocks. It has also been suggested that increasing short-run marginal cost at the firm level can make the effects of nominal shocks more persistent. We show that if firms can hold inventories nominal demand shocks will have lasting effects on sales, but not necessarily on production.

JEL: E13, E22, E31

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†Federal Reserve Bank of Richmond

‡Federal Reserve Bank of Richmond

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1. Introduction

According to a long-held view in macroeconomics, money is nonneutral because nominal prices are inflexible. The assumption of rigid nominal prices has only recently been incorporated in fully articulated general equilibrium models. In most of these models, firms possess some degree of market power and adjust their nominal prices only infrequently. The latter feature reflects the presence of fixed costs associated with nominal price adjustment. *A priori*, it is equally reasonable to assume that firms face increasing short-run marginal costs of production and, therefore, do not wish to vary production plans over time. If a firm's product is not perishable, then standard marginal cost smoothing arguments suggest that it will use inventories to smooth production relative to sales. In this paper, we study whether an economy where firms set nominal prices, face increasing short-run marginal cost, and use inventories to smooth production, is consistent with the stylized facts of inventory investment over the business cycle. We further study how the presence of inventories changes the economy's dynamic response to both monetary and technology shocks.

Production smoothing models have often been criticized on the grounds that their predictions concerning the behavior of sales, output, and inventory investment are inconsistent with business cycle observations. Specifically, these models predict that if the economy is predominantly driven by demand shocks, inventory investment is negatively correlated with sales and, therefore, production is less volatile than sales. For the U.S. economy, however, we note that the exact opposite is true. Inventory investment is positively correlated with sales and production is more volatile than sales. In principle, these stylized facts might well apply to the evaluation of sticky-price models. In particular, while these models are meant to provide a rationale for nominal demand disturbances, they also constitute a natural setting for the use of inventories as a means to smooth production. Hence, the inability of these models to reproduce the empirical behavior of aggregate production, sales, and inventory investment would represent an indirect rejection of the sticky-price framework.[1] In this paper, we show that quite the opposite holds.

For a standard parameterization of a model with staggered prices, Taylor (1980), we find that aggregate production, sales, and inventory investment conform to the stylized facts. These findings emerge irrespective of whether the economy is driven by monetary or productivity shocks. In addition, they occur as the result of an aggregation effect intrinsic to price staggering. Consider, for instance, the impact of a positive monetary shock. Firms that do not adjust their prices for some time observe higher sales today and, since the price level is to begin rising, higher future sales as well. Because these firms smooth production in the face

[1] Cost shocks can account for production being more volatile than sales in a production smoothing model, but the focus of inflexible price models has been on nominal demand disturbances.

of a rising sales path, they first increase production by *more* than the initial increase in sales. In contrast, firms that can adjust their price choose to increase their price in response to a monetary innovation and, therefore, experience falling sales contemporaneously. However, since they anticipate a rising sales path from then on, they do not necessarily decrease production, and may even increase it. In the aggregate, the initial sales increase is muted as the sales of price-adjusting firms fall. Aggregate production, therefore, rises by more than sales, and inventory investment and sales move together.

Sticky-price models have also been criticized because monetary disturbances do not have persistent effects on output. To address this issue, it has been suggested that increasing short-run marginal cost can lead to more persistent real effects of monetary disturbances.[2] However, earlier work on inflexible prices has failed to recognize that with increasing short-run marginal cost, firms also have an incentive to use inventories. We show that the introduction of inventories, to some degree, magnifies the "persistence problem." In essence, nominal demand shocks now generate persistent movements in aggregate sales but not necessarily aggregate production.

This paper is organized as follows. In section 2, we describe the problem of a firm which adjusts its nominal price only infrequently and uses inventories to smooth production. In section 3, we construct a partial-equilibrium industry model consisting of many such firms and study how the industry responds to both nominal demand and real supply shocks. In section 4, we then expand the model to a general equilibrium setting and describe the stochastic properties of aggregate variables when the economy is subject to these shocks. Finally, section 5 offers some conclusions and directions for future research.

2. Sticky Prices and Inventories: The Firm's Problem in a Stationary Environment

Suppose that a firm periodically sets its nominal price for a fixed number of periods while the general price level increases at a constant rate. Because its relative price falls as long as its nominal price remains unchanged, the firm's sales will increase over that time interval, everything else constant. This is the standard set-up in recent models of inflexible prices such as Chari, Kehoe, and McGrattan (2000). The usual assumption is that a firm will meet increased sales out of current production, but this is optimal only if the firm's technology is constant-returns-to-scale and marginal cost is constant. On the other hand, if some of the firm's factors of production are fixed in the short run, marginal cost is increasing and it is optimal for the firm to smooth production over time. In particular, in any period, a

[2] See Erceg (1997), Gust (1997), and Kim (1997).

firm will accumulate inventories whenever production exceeds sales. To make these notions transparent, we now formalize the problem of a firm which adjusts prices only infrequently and uses inventories to smooth production.

Consider a firm which is the monopoly producer of a good, and the demand for its good, q, is a declining function of the price it sets P,

$$q_t = \left(P_t/\mathcal{P}_t\right)^{-\theta} \tilde{q},\ \theta > 1, \tag{2.1}$$

where the firm takes as given the aggregate price level $\mathcal{P}_t$ and aggregate demand $\tilde{q}$. For simplicity, we suppose that the firm operates in a stationary environment so that aggregate demand is constant while the price level grows at a constant rate $\mu \geq 1$. By assumption, the firm cannot adjust its price in every period but instead sets a price P_t^* in periods $t = 0$, N, $2N$, etc... .

The firm uses labor, h, as the only input to a decreasing returns production technology, $y_t = zh_t^\alpha$, $0 < \alpha < 1$, where y_t denotes output and z is a technological shift parameter. The firm hires labor in a competitive labor market at the given constant real wage rate w, and its cost of production is

$$C(y_t) = w\left(y_t/z\right)^{1/\alpha}. \tag{2.2}$$

This cost function is strictly increasing as well as strictly concave. In other words, marginal cost is strictly increasing. The firm can store its own good but no other firm, nor the firm's customers, can store its good. The firm accumulates inventories according to

$$n_{t+1} = n_t + y_t - q_t, \tag{2.3}$$

where $n_t \geq 0$ denotes the beginning-of-period t inventory of the firm.

The firm maximizes the present value of profits, discounted at the rate $\beta < 1$,

$$\sum_{j=0}^{\infty} \beta^{Nj} \left\{ \sum_{\tau=0}^{N-1} \beta^{\tau} \left[\left(P_{Nj}^*/\mathcal{P}_{Nj+\tau}\right)^{1-\theta} \tilde{q} - w\left(y_{Nj+\tau}/z\right)^{1/\alpha} \right] \right\}, \tag{2.4}$$

subject to the inventory equation (2.3). The goal of this section is to show that there exists a stationary cycle for this firm's optimal policy where the decisions on output y_τ, sales q_τ, inventories n_τ, and the relative price $P^*/\mathcal{P}$ depend only on how much time, τ, has elapsed since the firm last changed its price. This sequence of decisions represents a relative price cycle.

From the expression for the present value of profits (2.4), it is clear that we can subdivide the profit maximization problem into a sequence of subproblems where each subproblem starts with the period in which the firm adjusts its price. Let n_0 denote the firm's inventory holdings at the beginning of the subproblem and suppose the firm plans to hold inventory n_0' after N periods when it is next able to adjust its price. This firm has to decide on its optimal relative price $P^*/\mathcal{P}$ in the current period, as well as production y_τ and inventory n_τ for this and the next $N-1$ periods. This is a well-defined concave optimization problem,

$$\begin{aligned} W(n_0, n_0') = \quad & \max_{P^*,\,\{y_\tau, n_\tau\}} \quad \textstyle\sum_{\tau=0}^{N-1} \beta^\tau \left[(P^*/\mathcal{P}\mu^\tau)^{1-\theta} \tilde{q} - w (y_\tau/z)^{1/\alpha} \right] \\ & \text{subject to} \quad n_{\tau+1} = n_\tau + y_\tau - (P^*/\mathcal{P}\mu^\tau)^{-\theta} \tilde{q} \geq 0, \\ & \qquad \text{for } \tau = 0, \ldots, N-1, \\ & \qquad n_0 \text{ and } n_N = n_0' \text{ given.} \end{aligned} \tag{P1}$$

Given our assumptions about demand and the cost function, the objective function is strictly concave and the constraints are convex in the choice variables. Therefore, there exists a unique solution. We note that the maximal value $W(n_0, n_0')$ is a strictly concave function of the two inventory stocks. The firm's complete profit maximization problem is then given by

$$\max_{\{n_{Nj}\}} \sum_{j=0}^{\infty} \beta^{Nj} W\left[n_{Nj}, n_{N(1+j)}\right] \text{ subject to } n_{Nj} \geq 0 \text{ and } n_0 \text{ given.}$$

Since W is strictly concave, there exists a unique solution to this problem and, in a steady state, $n_{Nj} = n_0$ for each j. Observe that an optimal policy is such that inventories will be zero at least once during a cycle. Otherwise, the firm could make the same sales but replace production in some period by the inventories which are never sold and thereby increase profits.

We now describe the salient features of the firm's stationary cycle in more detail. The firm's optimal pricing equation satisfies

$$\frac{P^*}{\mathcal{P}} = \left(\frac{\theta}{\theta-1}\right) \frac{\sum_{\tau=0}^{N-1} \beta^\tau \lambda_\tau \mu^{\tau\theta}}{\sum_{\tau=0}^{N-1} \beta^\tau \mu^{\tau(\theta-1)}}, \tag{2.5}$$

where λ_τ is the Lagrange multiplier associated with the firm's inventory accumulation equation in problem (P1). Alternatively, λ_τ is the marginal cost of producing y_τ. This last equation simply states that the optimal price is chosen so as to equate discounted marginal revenue to discounted marginal cost over the time interval in which P^* is to remain fixed. Once the firm has set its nominal price, its relative price will decline at rate μ and its sales, therefore, will rise at rate μ^θ over the cycle.

The firm's first order conditions associated with problem (P1) also imply that

$$y_{\tau+1} = (1/\beta)^{\alpha/(1-\alpha)}\, y_\tau \text{ for } \tau = 0, ..., N-2, \tag{2.6}$$

so that optimal production grows at a constant rate. Equation (2.6) reflects the production smoothing motive implicit in the firm's problem. With no discounting, the firm would simply choose a constant production volume.

In a steady state, inventories at the beginning and at the end of the cycle are identical. Therefore, total sales equal total production over the cycle and we have

$$y_0 \sum_{\tau=0}^{N-1} (1/\beta)^{\tau\alpha/(1-\alpha)} = (P^*/\mathcal{P})^{-\theta}\, \tilde{q} \sum_{\tau=0}^{N-1} \mu^{\tau\theta}. \tag{2.7}$$

Suppose that $\mu^\theta \geq (1/\beta)^{\alpha/(1-\alpha)}$ so that sales grow faster than production over the cycle. Then equation (2.7) directly implies that beginning-of-cycle production is greater than beginning-of-cycle sales. In this case, inventories first monotonically increase and then monotonically decline. Furthermore, since the inventory stock is the same at the beginning as it is at the end of the cycle, holding zero beginning-of-cycle inventories is feasible and optimal. This is shown in Figure 1.[3]

3. Staggered Price Setting and Inventories: Industry Dynamics with Demand and Cost Shocks.

In this section, we study an industry equilibrium in which firms set their nominal price for a fixed number of periods and use inventories to smooth production. We first define a recursive industry equilibrium for this environment and characterize the stationary equilibrium without aggregate uncertainty. We then investigate the industry's response to both demand and cost shocks. We show that with staggered prices, and independently of the shock, inventory investment is positively correlated with sales and, therefore, output is more volatile than sales at the industry level. Earlier work on production smoothing obtains this result only for cost shocks.[4] Moreover, relative to an industry without inventories, the response of aggregate output to shocks exhibits less persistence.

[3] By a similar argument, if $\mu^\theta < (1/\beta)^{\alpha/(1-\alpha)}$ so that production grows faster than sales, then by equation (2.7) beginning-of-cycle production is less than beginning-of-cycle sales. Therefore, inventories first decline and then increase over the cycle. To satisfy feasibility and optimality, beginning-of-cycle inventories are such that the minimum value of inventories over the cycle is zero.

[4] In linear-quadratic models of inventory investment, demand shocks can induce positive comovement between inventory investment and sales if one assumes the existence of an inventory-sales target which can be missed only at a cost (see Ramey and West 1999 for instance).

3.1. The Industry Equilibrium without Aggregate Uncertainty.

Consider an economy in which there is a final good that is produced in a competitive sector. The technology is constant-returns-to-scale and uses a continuum of intermediate goods as inputs, $j \in [0,1]$. In particular, the production function for the final good $\bar{q}$ is of the constant-elasticity-of-substitution variety,

$$\tilde{q}_t = \left[\int_0^1 q_t\left(j\right)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)}, \theta > 1,$$

where the intermediate goods $q_t\left(j\right)$ are produced by the kind of monopolistically competitive firms described in the previous section. Given nominal prices $\{P_t\left(j\right) : j \in [0,1]\}$ for the intermediate goods, cost minimization defines the unit cost function or price index,

$$\mathcal{P}_t = \left[\int_0^1 P_t(j)^{1-\theta} dj \right]^{1/(1-\theta)}. \tag{3.1}$$

A solution to the cost minimization problem also defines the conditional demand functions for producers of intermediate goods, which take the form of (2.1). In this section, we assume that aggregate demand is given by real balances,

$$\tilde{q}_t = M_t/\mathcal{P}_t, \tag{3.2}$$

where the growth rate of money, M_t, is given by μ.

The technology and labor market conditions characterizing intermediate goods production are the same as in the previous section. That is, all producers face the same wage and productivity. As before, producers of intermediate goods can adjust their price only once every N periods. In addition, we assume that in each period a fraction $1/N$ of these producers choose a new price. Each producer is indexed according to how much time, τ_t, has elapsed as of time t since it last changed its price. Put another way, we can think of τ_t as indexing producer types, where $\tau_t \in \{0, 1, ..., N-1\}$, so that there are N types of firms. We restrict our attention to symmetric outcomes in which all producers within a group are identical. Specifically, they all hold the same inventory and charge the same price. Thus, we denote by n_{τ_t} the time t inventory holdings of a producer who, τ_t periods ago, set its price to P_{τ_t}. This simplifies the definition of an equilibrium since the aggregate state of the economy, denoted $\mathbf{x}_t$, can be summarized by the money stock, the vector of prices charged by intermediate goods producers who do not adjust their prices, inventory holdings of all intermediate goods producers, and the exogenous state variables, wage and productivity: $\mathbf{x}_t = (M_t, P_{1t}, \ldots, P_{N-1,t}, n_{0t}, n_{1t}, \ldots, n_{N-1,t}, w_t, z_t)$. In a recursive equilibrium, the aggregate state evolves according to some law of motion $\mathbf{x}' = G(\mathbf{x})$, and the price level will be a function of the aggregate state, $\mathcal{P} = H(\mathbf{x})$.

Given the recursive nature of the environment, we can define the dynamic programming problem of an intermediate goods producer as follows,

$$
\begin{aligned}
V(P,n,\tau;\mathbf{x}) = \max_{P^*,\,y,\,n} \quad & \left\{ (P^*/\mathcal{P})^{1-\theta}\,\tilde{q} - w\,(y/z)^{1/\alpha} + \beta E\left[V(P',n',\tau';\mathbf{x}')\,|\mathbf{x}\right] \right\} \\
\text{subject to} \quad & P^* = P \text{ if } \tau > 0 \text{ and } P^* \geq 0 \text{ if } \tau = 0 \\
& P' = P^* \text{ and } n' = n + y - (P^*/\mathcal{P})^{-\theta}\,\tilde{q}, \\
& \tau' = 0 \text{ if } \tau = N-1 \text{ and } \tau' = \tau + 1 \text{ if } \tau \geq 0 \\
& \mathbf{x}' = G(\mathbf{x}) \text{ and } \mathcal{P} = H(\mathbf{x})\,.
\end{aligned}
\tag{P2}
$$

Let $x = (P,n,\tau)$ denote the state of an individual producer. We define the optimal decision rules for prices, output, sales, and inventories as $P = g_p(x;\mathbf{x})$, $y = g_y(x;\mathbf{x})$, $q = g_q(x;\mathbf{x})$, and $n' = g_n(x;\mathbf{x})$, respectively.

Definition 1. *A recursive equilibrium is a collection of functions (g,G,H,V) such that, (i) given the aggregate law of motion G, and the equilibrium price function H, the decision rules g and value function V solve the functional equation defined by (P2), and (ii) g and (G,H) are consistent, that is*

$$
\begin{aligned}
G_{n_0} &= g_n(P_\tau,n_\tau,\tau;\mathbf{x}) \text{ for } \tau = N-1, \\
G_{n_{\tau+1}} &= g_n(P_\tau,n_\tau,\tau;\mathbf{x}) \text{ for } \tau = 0,\ldots,N-2, \\
G_{p_{\tau+1}} &= g_p(P_\tau,n_\tau,\tau;\mathbf{x}) \text{ for } \tau = 0,\ldots,N-2, \\
H(\mathbf{x}) &= \left[(1/N)\textstyle\sum_{\tau=0}^{N} g_p(P_\tau,n_\tau,\tau;\mathbf{x})^{1-\theta}\right]^{1/(1-\theta)}.
\end{aligned}
$$

Without aggregate uncertainty, there exists a stationary equilibrium with an invariant distribution of inventory holdings, production, sales, and relative prices over intermediate goods producers' types. By construction, this distribution corresponds to the stationary cycle described in the previous section. In other words, in each time period there always exist N types of firms, and each firm type is at a different stage in the stationary inventory cycle described in the previous section. Hence, in this stationary equilibrium, aggregate production, sales, and inventory, are constant for the industry and given by $\bar{y} = (1/N)\sum_{\tau=0}^{N-1} y_\tau$, $\bar{q} = (1/N)\sum_{\tau=0}^{N-1} q_\tau$, and $\bar{n} = (1/N)\sum_{\tau=0}^{N-1} n_\tau$, respectively. Note that aggregate inventory investment is zero so that $\bar{q} = \bar{y}$. Finally, all nominal variables grow at the constant money growth rate, μ, in the stationary equilibrium. In what follows, all nominal variables are detrended accordingly so that the detrended money stock, for instance, is defined as $\tilde{M}_t = M_t/\mu^t$.

3.2. The Response to Shocks in the Presence of Flexible and Sticky Prices.

In this section, we illustrate the importance of infrequent price adjustment for the behavior of industry production, sales, and inventories. To emphasize the role of nominal rigidities, we first make clear that without preset prices, firms will not hold inventories. We then demonstrate that for production to be more volatile than sales, it is enough that prices be set one period ahead. In the next section, we provide a quantitative analysis of the complete dynamics of production, sales, and inventories for various durations of preset prices.

For production to be more volatile than sales when the environment is one of monopolistic competition, it is important that some prices be fixed. To see this, consider the industry model described above, but assume that all prices are flexible ($N = 1$). In this case the firm's optimization problem is

$$\begin{aligned} \max \quad & E_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \left(P_{0t}/\mathcal{P}_t\right)^{1-\theta} \tilde{q}_t - w_t y_t^{1/\alpha} \right\} \right] \\ \text{subject to} \quad & \left(P_{0t}/\mathcal{P}_t\right)^{-\theta} \tilde{q}_t + n_{t+1} = y_t + n_t, \text{ and } y_t,\ P_{0t},\ n_t \geq 0. \end{aligned} \tag{3.3}$$

Denoting the Lagrange multiplier associated with the constraint in (3.3) by λ_t, the associated first order conditions are,

$$\begin{aligned} P_{0t}/\mathcal{P}_t \quad &: \quad (\theta - 1)\left(P_{0t}/\mathcal{P}_t\right)^{-\theta} \tilde{q}_t = \theta \lambda_t \left(P_{0t}/\mathcal{P}_t\right)^{-\theta-1} \tilde{q}_t \\ y_t \quad &: \quad \lambda_t = w_t y_t^{(1-\alpha)/\alpha}/\alpha \\ n_{t+1} \quad &: \quad \lambda_t \geq \beta E_t\left[\lambda_{t+1}\right] \text{ with equality for } n_{t+1} > 0. \end{aligned} \tag{3.4}$$

Since all firms are identical, the equilibrium relative price is $P_{0t}/\mathcal{P}_t = 1$, and it follows from the first order conditions that $\lambda_t = (\theta - 1)/\theta > \beta(\theta - 1)/\theta$ since $\theta > 1 > \beta$. Therefore, $n_{t+1} = 0$ and $q_t = y_t = \tilde{q}_t$. We summarize this result in the following Lemma.

Lemma 1. *If prices are flexible (N=1), firms do not hold inventories and production is equal to sales.*

Observe that this result relies on the monopolistic competition structure of our model. If the industry consisted of a single monopolist, $P_{0t} = \mathcal{P}_t$ would not be an equilibrium condition, and the monopolist might then want to hold inventories (see Ramey and West 1999).

We now argue that for production to be more volatile than sales it is enough that some prices be set one period ahead. For this purpose, consider the industry model above with $N = 2$ and, without loss of generality, impose the restriction that a firm can carry inventories only during the times in which its price is fixed. In other words, all firms that adjust their price start out with no inventories, $n_{0t} = 0$. In this case, the firm's optimization problem simplifies to

$$\begin{array}{lll} \max_{\substack{P_{0t},q_{0t},y_{0t},n_{t+1},\\ q_{1,t+1},y_{1,t+1}}} & (P_{0t}/\mathcal{P}_t)\, q_{0t} - w_t y_{0t}^{1/\alpha} + \beta E_t \left[(P_{0t}/\mathcal{P}_{t+1})\, q_{1,t+1} - w_{t+1} y_{1,t+1}^{1/\alpha} \right] & \\ \text{subject to} & q_{0t} = (P_{0t}/\mathcal{P}_t)^{-\theta} (M_t/\mathcal{P}_t) = y_{0t} - n_{t+1}, & \\ & q_{1,t+1} = (P_{0t}/\mathcal{P}_{t+1})^{-\theta} (M_{t+1}/\mathcal{P}_{t+1}) = y_{1,t+1} + n_{t+1}, & (3.5) \\ & P_{0t},\ q_{0t},\ y_{0t},\ n_{t+1},\ q_{1,t+1},\ y_{1,t+1} \geq 0. & \end{array}$$

The equilibrium conditions are

$$\begin{array}{l} 1 = (1/2) \left[(P_{0t}/\mathcal{P}_t)^{1-\theta} + (P_{0t-1}/\mathcal{P}_t)^{1-\theta} \right], \\ \mathcal{P}_{t+1}/\mathcal{P}_t = \mu_{t+1} (\mathcal{P}_{t+1}/M_{t+1}) (M_t/\mathcal{P}_t) \text{ and} \\ \mathcal{P}_t/\mathcal{P}_{t-1} = \mu_t (\mathcal{P}_t/M_t) (M_{t-1}/\mathcal{P}_{t-1}). \end{array} \quad (3.6)$$

Aggregate production and sales are given by $\overline{q}_t = (1/2)[q_{0t} + q_{1t}]$ and $\bar{y}_t = (1/2)[y_{0t} + y_{1t}]$, respectively.

An analytical characterization of the full rational expectations equilibrium cannot be derived easily for this economy. Nevertheless, by confining the analysis to that of a temporary equilibrium, we can readily obtain the model's basic insight regarding the relative responses of production and sales to a demand shock.

Definition 2. *A temporary equilibrium is a vector* $(P_{0t}, q_{0t}, y_{0t}, n_{t+1}, q_{1,t+1}, y_{1,t+1}, y_{1t}, \mathcal{P}_t, \mathcal{P}_{t+1}, M_t/\mathcal{P}_t)$ *such that, conditional on* $(n_t, P_{0,t-1}, M_{t-1}/\mathcal{P}_{t-1}, M_{t+1}/\mathcal{P}_{t+1})$*: (i) for firms that set prices, the vector* $(P_{0t}, q_{0t},\ y_{0t}, n_{t+1}, q_{1,t+1}, y_{1,t+1})$ *solves problem (3.5), (ii) for firms with preset prices,* $y_{1,t} = (P_{0,t-1}/\mathcal{P}_t)^{-\theta} (M_t/\mathcal{P}_t) - n_t$*, and (iii) markets clear, that is the equations defined in (3.6) are satisfied.*

When the duration for which prices are fixed is short (β close to 1) and the inflation rate is low (μ close to 1), we obtain the following Lemma for $N = 2$, proved in the Appendix.

Lemma 2. *In a temporary equilibrium with one-period sticky prices (N=2), aggregate production responds more strongly than aggregate sales to a small money growth shock in a neighborhood of* $\beta = 1$ *and* $\mu = 1$.

In the proof of the lemma, we show that in response to a positive money growth shock, all firms increase production in the temporary equilibrium. However, the sales response depends on whether a firm is able to adjust its price at the time of the shock. Firms that are unable to adjust their price see their sales rise contemporaneously as their relative price falls. In contrast, firms that can increase their price in response to the shock tend to see their sales fall. At the aggregate level, therefore, the sales response is less pronounced than that of production.[5]

The above lemma shows that production is more volatile than sales when inflation is close to zero. However, it does not indicate how close to zero inflation has to be. To acquire some idea of the quantitative importance of our result, we have plotted the percentage response of output relative to the percentage response of sales for a fixed labor income share of $\alpha = 0.7$ and $\beta = 0.99$ in Figure 2. As we can see, aggregate output responds more strongly than aggregate sales for $\mu \in [1.0025, 1.05]$ and $\theta \in [2, 15]$, and this relative response is increasing in the demand elasticity θ and decreasing in the money growth rate μ. Below, we characterize numerical approximations of the economy's rational expectations equilibrium for values of $N = 2$ and higher, and we shall see that our results for the temporary equilibrium are consistent with the full rational expectations equilibrium.

3.3. Price Stickiness and the Behavior of Production and Sales

We now study the economy's response to a transitory money growth shock and a transitory productivity shock for various degrees of price rigidity. For this purpose, we linearize the economy around its nonstochastic steady state. We find that aggregate production, sales, and inventory investment move together. In the case of a productivity shock, we further find that aggregate output and inventory investment follow sales with a lag. Irrespective of the source of the shock as well as the duration for which prices are fixed, aggregate output responds more strongly than aggregate sales, and the response of aggregate output is less persistent than the response of aggregate sales.

Before proceeding with the quantitative analysis of the industry's response to demand and cost shocks, we first address the issue of calibration. The parameter values of the economy are selected along the lines of other quantitative studies on business cycles. A time period represents a quarter, and we set $\beta = 0.96^{1/4}$. The labor elasticity of output, α, equals 2/3 and firms' demand elasticity, θ, is set to 10 which implies a static markup ratio of 1.11. Because firms set their price as a markup over marginal cost, the wage income share is lower than the labor elasticity of output, $w \sum_\tau n_\tau / \sum_\tau p_\tau y_\tau = 0.59$. For the postwar U.S. economy, the average growth rate of M1, μ, is $1.05^{1/4}$. Furthermore, M1 growth is well represented by an AR(1) process whose autocorrelation coefficient, ρ_M, is 0.6.[6] A standard specification for productivity assumes an AR(1) process with autocorrelation coefficient $\rho_z = 0.95$. This specification reflects the properties of the Solow residual for the postwar U.S. economy (see

[5] One can also show that aggregate production responds more strongly than aggregate sales to a wage/productivity shock for standard production smoothing reasons.

[6] Christiano, Eichenbaum, and Evans (1998) argue for an MA(2) specification of monetary shocks. We obtain very similar results for their specification of monetary shocks.

Prescott 1986).[7] To study the effect of predetermined prices, we vary the price adjustment period from every other quarter ($N = 2$) to every other year ($N = 8$).

For our quantitative analysis, we have to somewhat modify the firm's inventory problem. This is done for purely computational reasons, and we will argue that it does not affect the qualitative features of our results. Recall that in the stationary equilibrium without aggregate uncertainty, the nonnegativity constraint on inventories is binding at some stage in the steady state cycle. In particular, given the above choices for β and μ, so that in this case $\mu^\theta > (1/\beta)^{\alpha/(1-\alpha)}$, firms that adjust their price enter the period with zero inventories, $n_0 = 0$. This implies that in the economy with aggregate uncertainty, the nonnegativity constraint on inventories may or may not be binding for these firms. Unfortunately, this feature of the model prohibits the use of standard methods that linearize the dynamics of the economy around its stationary equilibrium. At the same time, however, we wish to use these linearization techniques as the state space of our problem is quite large, and techniques that can handle occasionally binding constraints cannot be used for a large state space.

A complete analysis of the firm's problem under uncertainty and with inventories restricted to being nonnegative will deliver an optimal policy where inventories are frequently positive, even when a firm adjusts its price. Kahn (1987) has shown that firms hold inventories in order to avoid stock-outs when sales are characterized as an exogenous stochastic process. In essence, firms hold inventories for precautionary reasons. We approximate the precautionary motive for holding inventories through a stockout avoidance cost. Because the precautionary motive matters for the inventory decision only in the period prior to that in which a firm adjusts its price, we introduce the stockout avoidance cost only at that stage. Consequently, the cost function for a type $\tau = N - 1$ firm is altered to

$$C(y_{N-1,t}, n_{0,t+1}) = w\left(y_{N-1,t}/z\right)^{1/\alpha} + (\phi/2)\left(n_{0,t+1} - \tilde{n}\right)^2, \tag{3.7}$$

where $\tilde{n}$ denotes the target precautionary inventory stock. The cost functions of all other types of firms remain unchanged as in (2.2). With this specification in hand, we no longer have to worry about the nonnegativity constraint on inventories for the linearized version of the economy with aggregate risk, provided that the value of $\tilde{n}$ is large enough. Since $\widetilde{n}$ is a constant, the model's linearized dynamics are independent of the target precautionary inventory stock. Thus, for the questions we are interested in, the choice of $\tilde{n}$ represents only

[7] This specification is not quite appropriate for our economy since it is derived from productivity measures where one assumes that goods markets are competitive, production is constant returns to scale, and there are no externalities. In our economy, the last two assumptions are satisfied but not all markets are competitive. Productivity measures that try to account for possible deviations from these assumptions usually find the same degree of serial correlation in the productivity process, but they differ in the estimated standard deviations of productivity innovations (see Hornstein 1993).

an arbitrary normalization.[8]

3.3.1. Dynamic Effects of Demand Shocks

Figure 3 displays the response of aggregate production, sales, the production-sales ratio, and inventory stocks to a 1% innovation in money growth for various degrees of price stickiness, $N = 2, 4, 6, 8$. The literature on Taylor-type models usually assumes $N = 4$, which we treat as a benchmark. Figure 4 displays the associated responses of individual firms' output and sales to the same disturbance for $N = 4$.

Observe that in response to an aggregate demand shock both output and sales increase, but output increases by more than sales (i.e. $\Delta\bar{y}_t/\bar{y} - \Delta\bar{q}_t/\bar{q} > 0$ and the production-sales ratio rises). Furthermore, inventory stocks also increase. In this model, industry production is more volatile than sales and inventory investment is positively correlated with sales.

Following an increase in nominal demand, Figure 4 shows that firms that do not change their price face an initial rise in sales which, as long as their price remains unchanged, is followed by further increases in sales. Since these firms wish to smooth production over the remainder of their pricing cycle, *they initially increase production by more than sales*, after which output grows at a slower rate than sales. These firm, therefore, see their inventory holdings rise on impact. In contrast, firms that do adjust their price at the time of the shock set their price high enough that their sales initially fall. However, because these firms face a rising sales path from then on, they do not vary their production plans initially which also results in higher inventory investment. In the aggregate, both the production and sales pattern of firms with fixed prices dominate. Therefore, aggregate production increases more than sales on impact, and inventory investment and sales move together.

In Figure 3, we can see that the response of aggregate sales is much more persistent than that of aggregate output. Since inventories allow firms to separate their production paths from their sales paths, aggregate production is not much affected by the presence of frictions: once all firms have adjusted their price in response to an unforeseen shock, aggregate production is essentially back to its steady state. In contrast, the deviations of aggregate sales from their steady state are much more persistent. In effect, the persistence in output which one expects when prices are sticky and marginal cost is increasing (see Chari et

[8] The model's dynamics depend, to a limited extend, on the elasticity of the stock-out avoidance cost parameter ϕ. As one would expect, the inventory decisions of type $N - 1$ firms depend crucially on ϕ: the higher ϕ is, the less n_0 deviates from its target $\tilde{n}$. Aggregate variables, however, do not depend very much on the choice of ϕ. In particular, the qualitative features of the impulse response functions do not depend on ϕ. The only quantitatively significant effect is that with a higher value of ϕ, output responds somewhat less strongly, but even this effect is quite small. The results we present below are based on a value of $\phi = 100,000$ so that inventories, when a firm adjusts its price, are essentially set to $\tilde{n}$ instead of zero. Consistent with the U.S. economy, we select $\tilde{n}$ such that the inventory-sales ratio is 0.80.

al. 2000, and Kim 1997) is now transferred to sales. Since aggregate production reacts more strongly than sales on impact, but aggregate sales are more persistent, we need to verify that the unconditional standard deviation of production is indeed higher than sales. We do this in the next section through simulations of a general equilibrium version of our model.

The qualitative properties of the impulse response functions in Figure 3 are robust to the duration for which prices are fixed, $N = 2, 4, 6, 8$. Observe, however, that as the degree of price rigidity increases, the response of aggregate sales becomes hump-shaped. Because of the autocorrelation in the money growth process, an unforeseen increase in money growth today implies additional increases in the following periods. Thus, as firms are unable to adjust their price for longer and longer periods, increases in aggregate sales continue to build up leading to the hump-shaped response. Furthermore, since aggregate sales become more persistent relative to output for higher values of N, we expect that the volatility of aggregate sales relative to that of production also increases with N.

3.3.2. Dynamic Effects of Cost Shocks

Figures 5 and 6 display the response of the variables we have just examined to a 1% innovation in productivity. In many ways, the industry response to such a productivity increase is similar to its response to a demand shock. Production responds more strongly than sales, and the response of production is less persistent than that of sales. The only difference is that, on impact, output and inventory investment move together but in the opposite direction of sales.

A productivity increase implies a lower marginal cost of production, and firms that change their price at the time of the shock lower their price and increase sales. Since marginal cost is increasing and these firms anticipate decreasing sales in future periods, they choose to increase production less than sales on impact. Because adjusting firms lower their price, both the aggregate price level and sales initially fall for firms that cannot change their price (i.e. their relative price rises). The latter firms anticipate further declines in sales in future periods and, on impact, *reduce their production by more than sales decline.*

In summary, the overall behavior of aggregate variables is determined by two different aggregation effects in this case. On the one hand, aggregate sales rise initially as the sales increase experienced by adjusting firms more than compensates for the lower sales of firms with fixed prices. On the other hand, aggregate output contemporaneously declines as the firms that do not adjust prices also reduce production. During the transition, however, aggregate output increases as more firms set new prices. In terms of absolute magnitude, Figure 5 clearly suggests that production is more volatile than sales. Finally, aggregate inventory investment initially declines because all firms are drawing down their inventories at first. However, because inventory investment rises during most of the transition back to the steady state, one still expects sales and inventory investment to be positively correlated.

As was the case for demand shocks, the qualitative features of the aggregate impulse response functions do not depend much on the degree of nominal price rigidity. For a short duration of fixed prices ($N = 2$), both aggregate output and sales behave in a way that is remarkably close to that of output in a frictionless economy, with the exception of the initial fall in production. As the degree of price rigidity increases, inventories make it possible for aggregate production to remain close to the frictionless output path. However, the sales response becomes more and more muted. This suggests that the volatility of sales relative to output decreases with N in this case, and we shall confirm this conjecture in the next section.

4. Production, Sales, and Inventories in General Equilibrium.

Thus far, we have studied the implications of a partial equilibrium model in which individual firms fix their nominal price and use inventories to smooth increasing marginal cost over time. We have shown that, in this economy, aggregate production, sales, and inventory investment behave in a manner consistent with observations on the U.S. business cycle. Furthermore, these results hold regardless of whether the economy is driven by demand or cost shocks. In this section, we study the statistical properties of aggregate production, sales, and inventories in a general equilibrium version of the economy described in section 3.

We consider the following three modifications. First, we allow for constant long-run marginal cost. In the previous section, we argued that firms face increasing marginal cost because they cannot adjust their capital stock in the short run. We now proceed with the less extreme assumption that firms can adjust their capital stock at some cost. In the limit, when this cost is zero and production is constant-returns-to-scale in capital and labor, marginal cost is independent of production and a firm has no incentive to hold inventories. Second, we allow the real wage to depend on aggregate employment. From previous work on price rigidities, we know that higher sales by firms that do not adjust prices lead to increases in the demand for labor and real wages. This effect, in turn, implies higher price increases by firms that can adjust their price. Allowing for endogenous real wages, therefore, creates a feedback mechanism which dampens the effects of a nominal shock on employment and output. Third, estimates of the money demand function show money demand to be interest elastic. To account for this observation, we follow Chari et al. (2000) and introduce money through the notion that households have preferences defined over real balances.

4.1. The general equilibrium model

There is an infinitely lived representative household with preferences of the form,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[\eta c_t^{\nu} + (1-\eta) \left(M_t/\mathcal{P}_t \right)^{\nu} \right]^{(1-\sigma)/\nu} / (1-\sigma) - \psi h_t \right\}, \tag{4.1}$$

where c_t denotes consumption of the homogeneous final good, h_t represents labor supply, $M_t/\mathcal{P}_t$ are real balances, and $0 < \beta < 1$ is the time discount factor. The labor specification follows Hansen's (1985) indivisible labor model. The household receives a real wage w_t in return for its labor supply. Moreover, it owns the producers of intermediate goods and receives their profits, Π_t, as dividends. The household also trades one-period state contingent claims, and we denote by $Q_t(\mathbf{x}_{t+1})$ the price of a claim which pays one unit of the consumption good when the next period's aggregate state is $\mathbf{x}_{t+1}$. Therefore, the household's period budget constraint is given by

$$c_t + M_t/\mathcal{P}_t + \int Q_t \left(\mathbf{x}_{t+1} \right) a_t \left(\mathbf{x}_{t+1} \right) d\mathbf{x}_{t+1} \leq w_t h_t + a_{t-1} \left(\mathbf{x}_t \right) + M_{t-1}/\mathcal{P}_t + \Pi_t + T_t/\mathcal{P}_t, \tag{4.2}$$

where $a_t \left(\mathbf{x}_{t+1} \right)$ is the number of state contingent claims owned by the household, and T_t denotes the nominal value of lump-sum government transfers.

The homogeneous final good can be used either for consumption or investment. As in the previous section, the final good is produced using a fixed measure of intermediate inputs. Producers of intermediate goods can also be described as in the previous section, except that production now uses capital and labor as inputs, and capital can be adjusted at a cost. The production function is constant returns-to-scale in labor and capital, $y_t = z_t k_t^{\alpha} h_t^{1-\alpha}$, and gross investment, i_t, changes the capital stock as follows,

$$k_{t+1} = (1-\delta) k_t + i_t - (\gamma/2) \left(i_t/k_t - \delta \right)^2. \tag{4.3}$$

An intermediate goods producer's profits in period t are $\Pi_t = P_t q_t/\mathcal{P}_t - w_t h_t - i_t$. In contrast to the partial equilibrium analysis, future profits are now discounted using the price of state contingent claims. Thus, the definition of an intermediate goods producer's problem (P2) has to be appropriately modified.

Equilibrium in both the final goods and labor market requires that $c_t + (1/N) \sum_{\tau=0}^{N-1} i_{\tau,t} = \tilde{q}_t$ and $(1/N) \sum_{\tau=0}^{N-1} h_{\tau,t} = h_t$, respectively. As in the previous sections, an intermediate goods producer is indexed by how much time has elapsed since it last changed its price. We can define a recursive equilibrium in this general equilibrium setting in a manner analogous to that described in section 3.

The calibration of the economy is standard for studies in quantitative general equilibrium theory and follows Chari et al. (2000), unless otherwise noted. Thus, we set the intertemporal elasticity of substitution such that $\sigma = 2$. From the optimality condition for real balances, we can derive a log-log demand function for real balances of the form

$$\log\left(M_t/\mathcal{P}_t\right) = -\frac{1}{1-\nu}\log\frac{\eta}{1-\eta} + \log c_t - \frac{1}{1-\nu}\log\left(1-\frac{1}{R_t}\right), \tag{4.4}$$

where R_t is the rate of return on a nominal bond from period t to period $t+1$. We set $\eta = 0.73$ and $\nu = 19.5$ so that the interest elasticity is -0.054. The quarterly depreciation rate δ is 0.025, and we set the capital stock adjustment cost parameter to match the volatility of investment relative to output. For the postwar U.S. economy, investment is about twice as volatile as output. Our model generates this relative volatility for a γ of 6.5. The remaining parameters β, α, θ, and ϕ are chosen as in the previous section. We select the innovations to both the money and productivity process so as to match output volatility in the model when $N = 4$ to that which emerges in the postwar U.S. economy. This procedure yields $\sigma_M = 0.0022$ and $\sigma_z = 0.01$.[9]

4.2. Results

We now discuss the statistical properties of the model we have presented. Our discussion focuses on the cyclical behavior of inventories, sales, and production, and we compare our results with those of the U.S. economy.[10] Specifically, we focus on two statistics that have received widespread attention in the literature on inventories. First, inventory investment and sales are positively correlated and, therefore, output is more volatile than sales. Second, one observes that inventory-sales ratios are highly persistent processes.[11] The statistics for the U.S. economy are based on quarterly series from 1959:1 to 1997:4. Output is GDP of the nonfarm business sector, excluding housing; consumption includes nondurable goods and services; investment is private fixed capital investment, excluding residential investment; and inventories are taken from the nonfarm business sector. We use final sales of the domestic business sector. All series are in chain-weighted 1992 dollars. For the model economy, GDP is

[9] The estimated value for the standard deviation of money innovations is $\sigma_M = 0.006$. Prescott (1986) estimates the standard deviation of productivity innovations as $\sigma_z = 0.0072$ and, given our values of the markup, we would adjust this estimate downward by 10 percent (see Hornstein 1993). Since we are mainly interested in the relative volatility of production and sales for any one shock, the absolute value of either σ_M or σ_z is not important.

[10] The model replicates the properties of standard RBC models: investment is substantially more volatile than consumption, and all components move with aggregate output (GDP). Expenditure categories are more strongly correlated with output for money shocks than they are for productivity shocks. We report these statistics in an earlier version of this paper, Hornstein and Sarte (1998).

[11] These two facts are widely cited in the inventory literature (see the surveys of Blinder and Maccini 1991, and Ramey and West 1999). Ramey and West argue that the persistence of inventory-sales ratios is related to the slow speeds of adjustment estimated for linear-quadratic inventory models with target inventories.

the sum of consumption, capital investment, and inventory investment. With the exception of inventory investment, the log of all variables is detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. For inventory investment, we detrend the level.

In Table 1, we display the statistics discussed above for an economy with money shocks only. In Table 2, we display the same statistics when the economy is driven by productivity shocks. We can see that for both shocks, inventory investment is positively correlated with sales. Since production is the sum of sales and inventory investment, it then follows that production is more volatile than sales. The contemporary correlation between inventory investment and sales is weaker for productivity shocks than it is for money shocks. In addition, as in the U.S. economy, inventory investment lags output when productivity shocks drive the model. The last two features are related to the fact that the initial effect of a productivity shock is to raise sales and lower inventory investment (recall Figure 5). However, the subsequent comovement of inventory investment and sales is still enough to induce a positive contemporaneous correlation between these two variables.[12] The model captures the persistence of the inventory-sales ratio quite well. For both shocks, the autocorrelations of inventory-sales ratios displayed in Tables 1 and 2 are close to those in the U.S. economy.

The basic statistical properties of production, sales, and inventory investment do not depend much on the extent of price rigidity. However, in our discussion of the impulse response functions, we indicated that an increase in the degree of price stickiness has a larger effect on the response of aggregate sales than that of production. In the case of a money growth shock, an increase in N allows increases in sales to cumulate for firms whose prices are fixed and, therefore, induces a hump-shaped response in aggregate sales. In contrast, inventories attenuate and draw out the sales response to a productivity shock as they allow aggregate production to behave as if in a frictionless economy. Tables 1 and 2 confirm these observations quantitatively for both money and productivity shocks. As N increases, the volatility of sales relative to production increases for money shocks and falls for productivity shocks.[13]

The effects of price rigidity we have just described suggest a rather indirect method of evaluating the relative importance of monetary shocks and productivity shocks. Suppose that we rank industries according to the average duration for which prices are fixed. To this end, we use the measures provided by Caucutt et al. (1999) who update and confirm previous work by Carlton (1986). One might then ask whether the degree of price stickiness is positively or negatively correlated with the relative volatility of sales to output across

[12] The model's predictions concerning the procyclicality of inventory investment is robust with respect to both the magnitude of stock-out avoidance costs and the elasticity of demand. This is shown in a previous working paper version (see Hornstein and Sarte 1998).

[13] We have confirmed these results for values of $\rho_Z \in [.35, .999]$ and $\rho_M \in [.6, .9]$.

industries. In Figure 7, we graph Caucutt et al.'s measure of price stickiness against the relative volatility of sales by industry, and a weak positive correlation is apparent.[14] This finding suggests that monetary shocks may play an important role over the business cycle.

As pointed out earlier, the literature on nominal rigidities has argued that increasing short-run marginal cost (at the firm level) renders the effects of monetary shocks on output more persistent.[15] In this paper, however, we have stressed that firms facing increasing marginal costs also have an incentive to separate sales from production. Moreover, the partial equilibrium analysis suggested that the increased persistence in output might now show up in sales, regardless of the shocks considered. In Tables 1 and 2, we display the first two autocorrelation coefficients for output and final sales. In the U.S. economy, there exists no substantial difference in the persistence of sales or output. By contrast, in the model economy, we note that sales are indeed more persistent than production if prices are adjusted at most every year ($N \leq 4$). We conclude that a consistent analysis of an environment with both nominal rigidities and firm specific increasing marginal costs weakens current results regarding the persistence of monetary shocks.

Finally, we note that in our framework production is substantially more volatile than sales when compared with the U.S. economy. This result should not overly concern us. In contrast to the U.S. economy, all intermediate goods are storable in the model. In fact, the concept of an intermediate good is somewhat artificial and unrelated to the National Income Accounts. If we were to assume that only a fraction of the intermediate goods are storable, as in the data, the model economy would behave as a weighted average of an economy with inventories and one without. The relative volatility of output and sales, therefore, would be closer to that actually observed.

5. Conclusions and directions for future research

In this paper, we have argued that in an economy where inventories are used to smooth production and prices are sticky the behavior of inventory investment is consistent with stylized facts. In our environment, firms adjust their nominal prices in a staggered fashion as in Taylor (1980). Because prices are sticky, shocks result in persistent sales movements

[14] We construct industry production and sales from data on shipment and inventories for two digit SIC industries in the manufacturing sector. A description of the data is included in Figure 7. We should point out that we cannot use all of the 2-digit manufacturing industries for two reasons. First, Caucutt et al.'s (1999) industry definitions are not exactly the same as in the SIC, and we cannot always find a counterpart in the SIC for one of their industries. Second, we construct output as the sum of inventory investment and sales, and therefore need a common deflator for the inventory and sales series. We use the shipment deflator, but this deflator is not available for all industries. The industries we have to exclude are apparel (23), lumber (24), furniture (25), printing (27), leather (31), and miscellaneous manufacturing (39).

[15] See Erceg (1997), and Chari et al. (2000).

and it is optimal for firms to smooth production when short-run marginal cost is increasing. We find that inventory investment is pro-cyclical and that production is more volatile than sales.

We have not shown that our results obtain in any economy with rigid prices and increasing short-run marginal cost. Consider two other popular models of sticky prices. In models based on Calvo (1983), the degree of nominal price rigidity is random. In every period, a firm is allowed to change its price with some probability. We conjecture that the behavior of inventory investment in this type of model would be very similar to that in our environment. Indeed, following a shock, a given firm's path of expected sales would be essentially the same as in our framework. It is difficult to verify this conjecture in a general equilibrium setting. In particular, individual firms would have to be indexed not only by their price but also by their inventory holdings. The state of the economy, therefore, would involve a nontrivial distribution of firms across inventories and prices. In our environment, this problem is simplified since, for any individual firm, we need only keep track of how much time has elapsed since it last changed its price. The other class of sticky-price models introduces an increasing cost of adjusting nominal prices into a standard representative firm model (see Rotemberg 1982). We conjecture that for these models the behavior of inventory investment will not conform to the stylized facts. Following an increase in nominal demand, for instance, a firm would find it optimal to spread the cost of adjusting prices over time. This postponed price adjustment implies that sales would first increase and then decrease. As a result, a cost-minimizing firm with short-run increasing marginal costs would still smooth production, but in this case increase production by *less* than sales initially.

Finally, it is important to emphasize that our framework conforms to the stylized facts on production, sales, and inventories as the result of an aggregation effect intrinsic to price staggering. At the firm level, however, inventories serve only as a means to smooth production. It would be informative, therefore, to explore whether the evidence regarding the greater volatility of aggregate production relative to sales is consistent with production smoothing by individual firms. While industry data on production and sales is readily available, firm data is often proprietary and cannot be collected easily. Nevertheless, we hope that future empirical work can make progress in that direction and provide additional evidence on the mechanisms we have outlined in this paper.

Appendix

Proof of Lemma 2

As defined in the text, a temporary equilibrium is characterized by the following system of equations for $(P_{0t}/\mathcal{P}_t, y_{0t}, y_{1t}, y_{1,t+1}, q_{0t}, q_{1,t+1}, n_{t+1}, M_t/\mathcal{P}_t)$, conditional on $(P_{0t-1}/\mathcal{P}_{t-1}, n_t, M_{t-1}/\mathcal{P}_{t-1}, M_{t+1}/\mathcal{P}_{t+1}, \mu_t, \mu_{t+1}, w_t, w_{t+1})$,

$$(\theta - 1)\left(\frac{P_{0t}}{\mathcal{P}_t}\right)\left\{\frac{M_t}{\mathcal{P}_t} + \beta E_t\left[\left(\frac{\mathcal{P}_{t+1}}{\mathcal{P}_t}\right)^{\theta-1}\frac{M_{t+1}}{\mathcal{P}_{t+1}}\right]\right\} = \theta\left\{\lambda_{0t}\frac{M_t}{\mathcal{P}_t} + \beta E_t\left[\lambda_{1,t+1}\left(\frac{\mathcal{P}_{t+1}}{\mathcal{P}_t}\right)^{\theta}\frac{M_{t+1}}{\mathcal{P}_{t+1}}\right]\right\}$$

$$\begin{aligned}
\lambda_{0t} &= w_t y_{0t}^{(1-\alpha)/\alpha}/\alpha \\
\lambda_{1,t+1} &= w_{t+1} y_{1,t+1}^{(1-\alpha)/\alpha}/\alpha \\
y_{1,t+1} &= \left(P_{0t}/\mathcal{P}_{t+1}\right)^{-\theta}\left(M_{t+1}/\mathcal{P}_{t+1}\right) + \left(P_{0t}/\mathcal{P}_t\right)^{-\theta}\left(M_t/\mathcal{P}_t\right) - y_{0t} \\
\lambda_{0t} &= \beta E_t\left[\lambda_{1,t+1}\right] \\
1 &= (1/2)\left[\left(P_{0t}/\mathcal{P}_t\right)^{1-\theta} + \left(P_{0,t-1}/\mathcal{P}_t\right)^{1-\theta}\right] \\
\mathcal{P}_{t+1}/\mathcal{P}_t &= \mu_{t+1}\left(M_t/\mathcal{P}_t\right)\left(\mathcal{P}_{t+1}/M_{t+1}\right) \\
\bar{y}_t &= (1/2)\left[y_{0t} + y_{1,t}\right] \\
\bar{q}_t &= (1/2)\left[\left(P_{0t}/\mathcal{P}_t\right)^{-\theta} + \left(P_{0,t-1}/\mathcal{P}_t\right)^{-\theta}\right]\left(M_t/\mathcal{P}_t\right),
\end{aligned}$$

where $y_{1,t} = \left(P_{0t-1}/\mathcal{P}_t^{-\theta}\right)\left(M_t/\mathcal{P}_t\right) - n_t$ and $\mathcal{P}_t/\mathcal{P}_{t-1} = \mu_t\left(M_{t-1}/\mathcal{P}_{t-1}\right)\left(\mathcal{P}_t/M_t\right)$.

We construct a log-linear approximation of the equilibrium conditions, and solve for production and sales as a function of the money growth shock. To simplify notation, we suppress the terms of all other exogenous variables (i.e. these are assumed constant). The solutions for the log-deviations of $P_{0t}/\mathcal{P}_t$ and $M_t/\mathcal{P}_t$ from their steady state values are

$$\Delta\log\left(P_{0t}/\mathcal{P}_t\right) = e_p\Delta\log\mu_t \text{ and } \Delta\log\left(M_t/\mathcal{P}_t\right) = e_m\Delta\log\mu_t,$$

where

$$\begin{aligned}
e_p &= -\Psi/\Omega \text{ and } e_m = \left[1 + \theta\left(1-\alpha\right)/\alpha\right]/\Omega, \\
\Omega &= 1 + \theta\left(1-\alpha\right)/\alpha - \mu^{1-\theta}\Psi, \\
\Psi &= \frac{1 + (\theta-1)\beta\mu^{\theta-1}}{1 + \beta\mu^{\theta-1}} - \frac{1}{\alpha}\frac{1+\theta\mu^{\theta}}{1+\mu^{\theta}}.
\end{aligned}$$

Substituting these expressions into the equations for the log-deviations of production and sales yields

$$\begin{aligned}
\Delta \log y_{0,t}/\Delta \log \mu_t &= \left\{\theta \frac{1+(\theta-1)\beta\mu^{\theta-1}}{1+\beta\mu^{\theta-1}} - (\theta-1)\frac{1+\theta\mu^{\theta}}{1+\mu^{\theta}}\right\}/\Omega, \\
\Delta \log y_{1,t}/\Delta \log \mu_t &= 1, \\
\Delta \log q_{0,t}/\Delta \log \mu_t &= \frac{1+\theta(1-\alpha)/\alpha+\theta\Psi}{1+\theta(1-\alpha)/\alpha-\mu^{1-\theta}\Psi}, \\
\Delta \log q_{1,t}/\Delta \log \mu_t &= \frac{1+\theta(1-\alpha)/\alpha-\theta\mu^{1-\theta}\Psi}{1+\theta(1-\alpha)/\alpha-\mu^{1-\theta}\Psi},
\end{aligned}$$

and aggregate production and sales are

$$\begin{aligned}
\Delta \log \bar{y}_t &= (1-\gamma)\Delta \log y_{0,t} + \gamma \Delta \log y_{1,t}, \\
\Delta \log \bar{q}_t &= \omega \Delta \log q_{0,t} + (1-\omega)\Delta \log q_{1,t},
\end{aligned}$$

where $\omega = 1/\left(1+\mu^{\theta}\right)$ is the steady state share in total sales of a firm which adjusts prices, and $\gamma = \beta^{\alpha/(1-\alpha)}/\left[1+\beta^{\alpha/(1-\alpha)}\right]$ is the steady state share in total production of a firm which does not adjust prices.

We cannot sign Ψ unambiguously, but for reasonable parameter values, $\Psi < 0$ which implies $\Omega > 0$. With $\Psi < 0$, the sales of a firm which cannot adjust its price increase unambiguously. Even with $\Psi < 0$, we cannot unambiguously sign the output and sales response of a firm which can adjust prices. We can get a better idea of the sign and relative magnitudes of output and sales responses for the limiting case, $\mu = \beta = 1$. The expression for Ψ simplifies to

$$\Psi = [\theta(1-1/\alpha) - 1/\alpha]/2 < 0.$$

and for the output and sales elasticities we have

$$\begin{aligned}
\Delta \log y_{0,t}/\Delta \log \mu_t &= \alpha/\bar{\Omega} > 0, \\
\Delta \log y_{1,t}/\Delta \log \mu_t &= 1, \\
\Delta \log q_{0,t}/\Delta \log \mu_t &= -\{\theta[(1-\alpha)\theta-1]+2\alpha(\theta-1)\}/\bar{\Omega}, \\
\Delta \log q_{1,t}/\Delta \log \mu_t &= \left\{\theta[1+2(1-\alpha)]+\theta^2(1-\alpha)+2\alpha\right\}/\bar{\Omega} > 0, \\
\Delta \log \bar{y}_t/\Delta \log \mu_t &= 0.5\left[(1-\alpha)\theta^2+(3-2\alpha)\theta+3\alpha\right]/\bar{\Omega} > 0, \\
\Delta \log \bar{q}_t/\Delta \log \mu_t &= 2[\alpha+(1-\alpha)\theta]/\bar{\Omega} > 0, \\
\bar{\Omega} &= 1+2\alpha+3(1-\alpha)\theta > 0.
\end{aligned}$$

Now, output of a price-adjusting firm unambiguously increases with a money growth shock, and sales decrease if $\theta > [\alpha/(1-\alpha)][(\theta-2)/(\theta-1)]$. In any case, aggregate production and aggregate sales increase with a money growth shock. Furthermore, aggregate production responds more strongly than aggregate sales for

$$\theta (1-\alpha)(\theta - 1) + \alpha (2\theta - 1) > 0.$$

This last condition is always satisfied when demand is elastic, $\theta > 1$, and marginal cost is increasing, $\alpha < 1$.

References

[1] Blinder, Alan S. and Maccini, Louis J. 1991. Taking stock: A critical assessment of recent research on inventories. *Journal of Economic Perspectives*, v.5 no.1, 73-96.

[2] Carlton, Dennis W. 1986. The rigidity of prices. *American Economic Review* v.76, 637-58.

[3] Caucutt, Elizabeth M., Gosh, Mrinal, Kelton, Christina M.L. 1999. Durability versus concentration as an explanation for price inflexibility. *Review of Industrial Organization* v.14, 27-50.

[4] Chari, V.V., Kehoe, Patrick, and McGrattan, Ellen. 2000. Sticky price models of the business cycle: Can the contract multiplier solve the persistence problem? *Econometrica* v.68, no.5, 1151-79.

[5] Calvo, Guillermo A. 1983. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics,* v.12, 383-98.

[6] Christiano, Lawrence J., Eichenbaum, Martin, and Evans, Charles L. 1998. Modeling money. NBER working paper 6371.

[7] Erceg, Christopher. 1997. Nominal wage rigidities and the propagation of aggregate demand disturbances. Board of Governors of the Federal Reserve System. mimeo.

[8] Gust, Christopher J. 1997. Staggered price contracts and factor immobilities: The persistence problem revisited. Northwestern University. mimeo.

[9] Hansen, Gary D. 1985. Indivisible Labor and the Business Cycle. *Journal of Monetary Economics* v.16, no.3, 309-27

[10] Hornstein, Andreas. 1993. Monopolistic competition, increasing returns to scale, and the importance of productivity shocks. *Journal of Monetary Economics*, v.31, 299-316.

[11] Hornstein, Andreas, and Sarte, Pierre-Daniel. 1998. Staggered prices and inventories: Production smoothing reconsidered. Federal Reserve Bank of Richmond Working Paper 98-8.

[12] Kahn, James A. 1987. Inventories and the volatility of production. *American Economic Review*, v.77, 667-79.

[13] Kim, Jinill. 1997. Monetary policy in a stochastic equilibrium model with real and nominal rigidities. Federal Reserve Board Working Paper.

[14] Prescott, Edward C. 1986. Theory ahead of measurement. Federal Reserve Bank of Minneapolis Quarterly Review.

[15] Ramey, Valerie A. and West, Kenneth D. 1999. Inventories. In John B. Taylor and Michael Woodford (eds.), *Handbook of Macroeconomics* 1B, 863-923, Elsevier Science: Amsterdam.

[16] Rotemberg, Julio J. 1982. Sticky prices in the United States. *Journal of Political Economy*, v.90, 1187-211.

[17] Taylor, John B. 1980. Aggregate dynamics and staggered contracts. *Journal of Political Economy*, v.88, 1-23.

Table 1.

Inventory Statistics for Money Shocks Only

N		2		4		6		8		**U.S.**
$\sigma_{\bar{y}}$		0.87	(0.06)	1.50	(0.14)	1.96	(0.20)	2.27	(0.27)	**1.50**
$\sigma_{\bar{q}}/\sigma_{\bar{y}}$		0.40	(0.02)	0.48	(0.01)	0.52	(0.02)	0.57	(0.01)	**0.81**
Corr$(\bar{q}_t, \Delta\bar{n}_{t+s})$										
	$s = -1$	0.16	(0.09)	0.53	(0.06)	0.64	(0.06)	0.69	(0.06)	**0.31**
	$s = \ \ 0$	0.70	(0.03)	0.74	(0.03)	0.81	(0.04)	0.87	(0.03)	**0.27**
	$s = +1$	-0.59	(0.04)	0.07	(0.02)	0.34	(0.04)	0.51	(0.05)	**0.40**
Corr$(\bar{n}_t/\bar{q}_t, \bar{n}_{t+s}/\bar{q}_{t+s})$										
	$s = 1$	0.38	(0.09)	0.76	(0.03)	0.84	(0.03)	0.86	(0.03)	**0.64**
	$s = 2$	0.09	(0.09)	0.31	(0.10)	0.52	(0.09)	0.60	(0.06)	**0.31**
	$s = 3$	-0.04	(0.10)	-0.02	(0.11)	0.16	(0.12)	0.28	(0.10)	**0.00**
	$s = 4$	-0.12	(0.09)	-0.18	(0.11)	-0.11	(0.13)	0.00	(0.13)	**-0.28**
Corr$(\bar{q}_t, \bar{q}_{t-s})$										
	$s = 1$	0.14	(0.09)	0.55	(0.06)	0.66	(0.06)	0.70	(0.06)	**0.72**
	$s = 2$	-0.04	(0.08)	0.15	(0.10)	0.33	(0.09)	0.40	(0.09)	**0.43**
Corr$(\bar{y}_t, \bar{y}_{t-s})$										
	$s = 1$	-0.23	(0.08)	0.46	(0.06)	0.64	(0.05)	0.70	(0.05)	**0.72**
	$s = 2$	-0.09	(0.09)	-0.04	(0.09)	0.27	(0.08)	0.40	(0.08)	**0.40**

Note: For each variable, except GDP, the first column displays the variable's volatility (standard deviation) relative to GDP, and the second column denotes the variable's contemporaneous correlation with GDP. The model statistics are the mean values calculated from 200 simulations of samples with 120 observations each. In square brackets are the standard deviations of the sample statistics.

Table 2.

Inventory Statistics for Productivity Shock Only.

N		2		4		6		8		**U.S.**
$\sigma_{\bar{y}}$		1.45	(0.15)	1.50	(0.17)	1.45	(0.19)	1.35	(0.20)	**1.50**
$\sigma_{\bar{q}}/\sigma_{\bar{y}}$		0.76	(0.04)	0.60	(0.05)	0.57	(0.04)	0.60	(0.03)	**0.81**
$\text{Corr}(\bar{q}_t, \Delta\bar{n}_{t+s})$										
	$s = -1$	0.03	(0.03)	0.30	(0.05)	0.45	(0.07)	0.50	(0.09)	**0.31**
	$s = \ \ 0$	0.30	(0.04)	0.59	(0.05)	0.74	(0.04)	0.75	(0.05)	**0.27**
	$s = +1$	0.29	(0.04)	0.59	(0.05)	0.75	(0.05)	0.83	(0.04)	**0.40**
$\text{Corr}(\bar{n}_t/\bar{q}_t, \bar{n}_{t+s}/\bar{q}_{t+s})$										
	$s = 1$	0.64	(0.07)	0.81	(0.03)	0.88	(0.02)	0.90	(0.02)	**0.64**
	$s = 2$	0.39	(0.10)	0.43	(0.09)	0.60	(0.07)	0.67	(0.06)	**0.31**
	$s = 3$	0.20	(0.11)	0.08	(0.13)	0.26	(0.11)	0.38	(0.10)	**0.00**
	$s = 4$	0.04	(0.11)	-0.11	(0.13)	-0.03	(0.14)	0.09	(0.13)	**-0.28**
$\text{Corr}(\bar{q}_t, \bar{q}_{t-s})$										
	$s = 1$	0.86	(0.03)	0.89	(0.02)	0.89	(0.03)	0.86	(0.03)	**0.72**
	$s = 2$	0.58	(0.02)	0.66	(0.07)	0.68	(0.07)	0.66	(0.07)	**0.43**
$\text{Corr}(\bar{y}_t, \bar{y}_{t-s})$										
	$s = 1$	0.53	(0.08)	0.73	(0.04)	0.81	(0.03)	0.86	(0.03)	**0.72**
	$s = 2$	0.31	(0.10)	0.31	(0.11)	0.51	(0.08)	0.63	(0.07)	**0.40**

Note: See Table 1.

Figure 1.
Production, Sales, and Inventory Cycles.

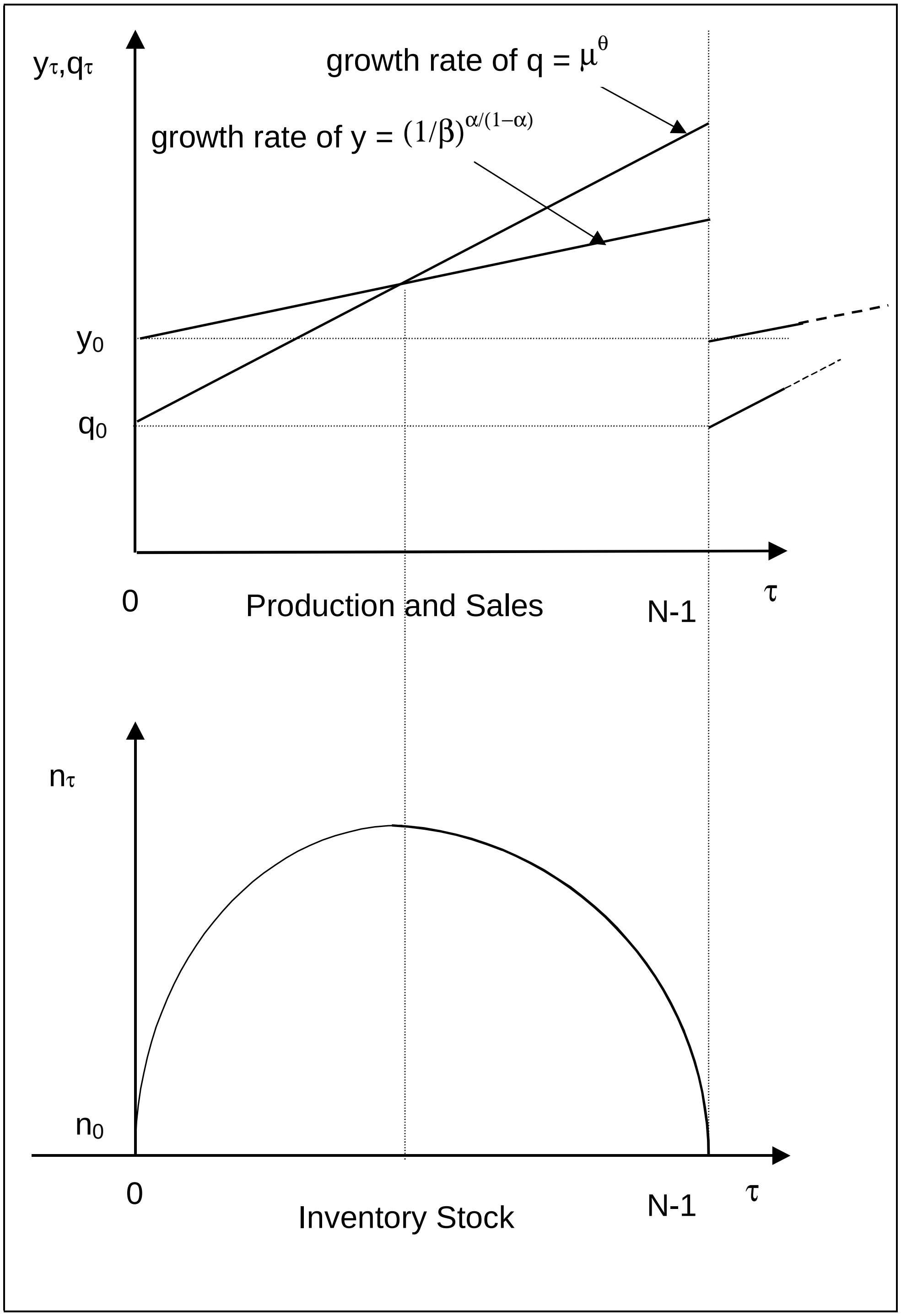
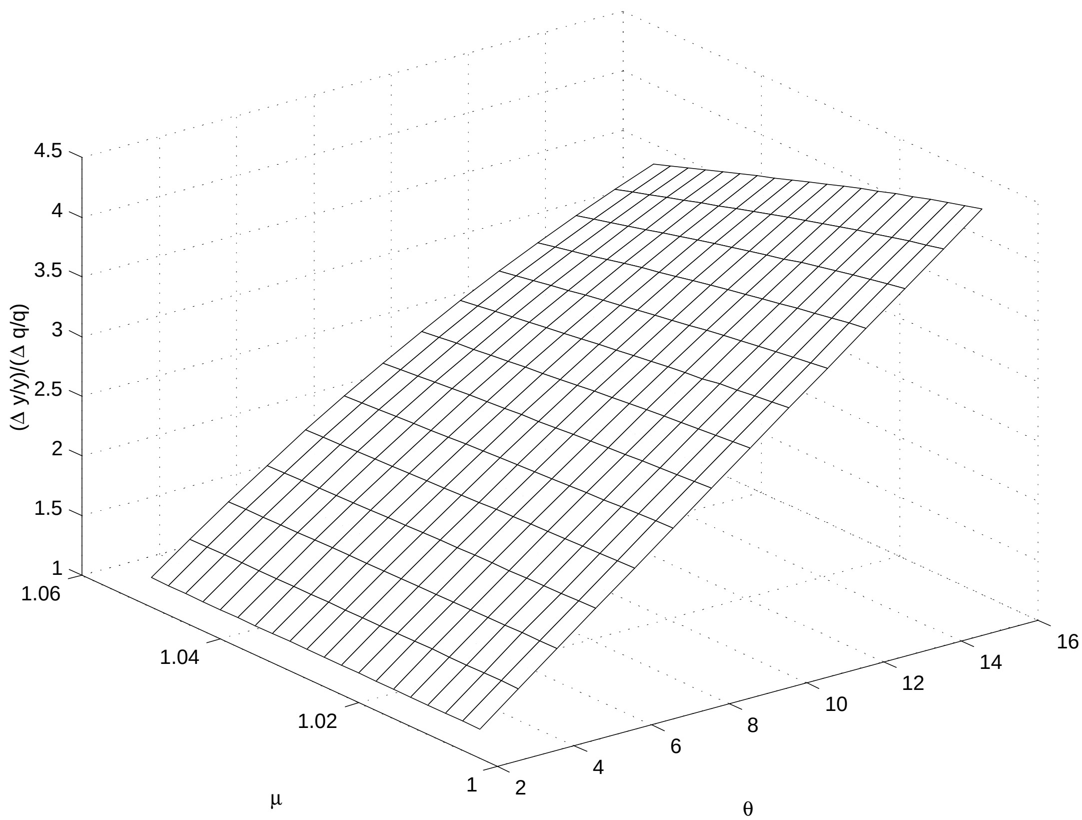


Figure 2.

The Relative Output-Sales Response to a Money Growth Shock.



Note: μ is the steady state money growth rate, θ is the demand elasticity, y is total production, and q is total sales.

Figure 3. Aggregate Response to a Money Growth Shock.

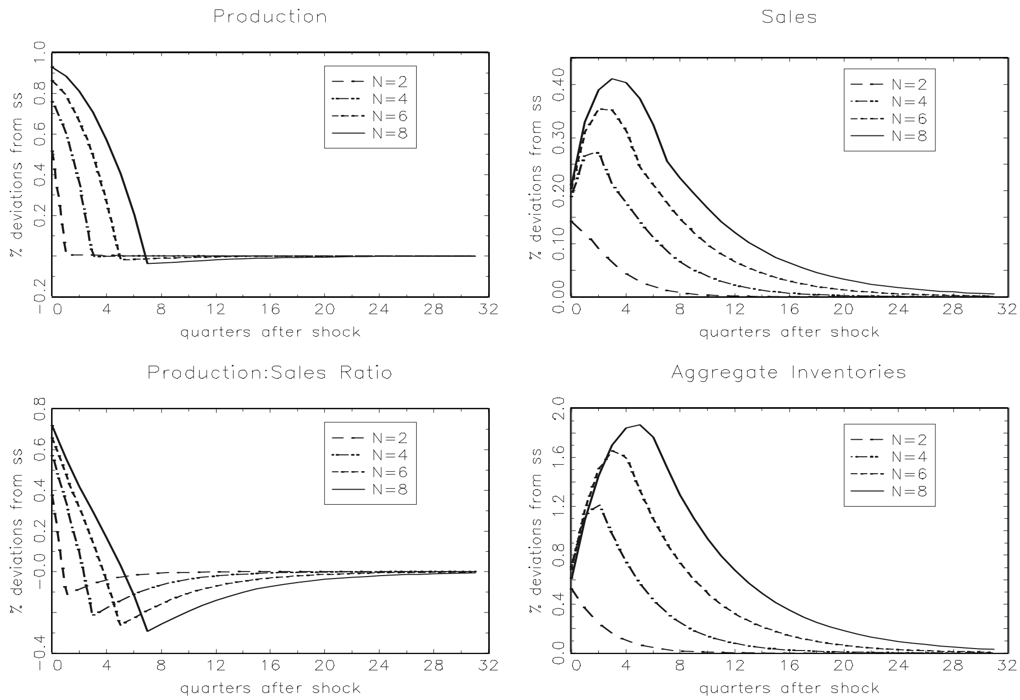


Figure 4. Individual Firm Response to a Money Growth Shock.

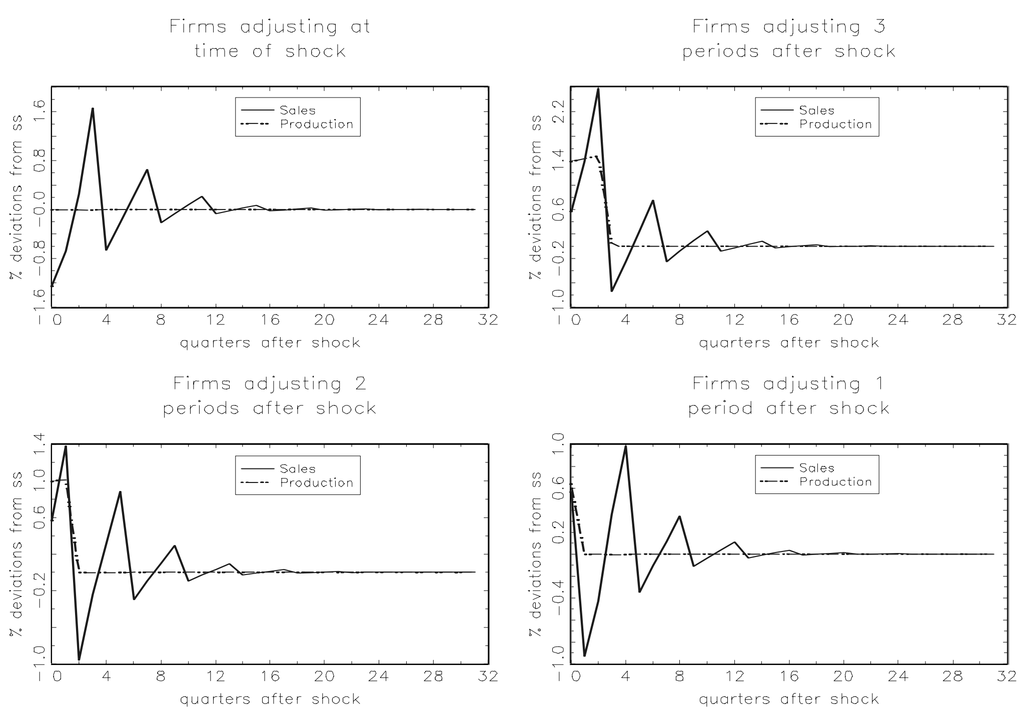


Figure 5. Aggregate Response to a Productivity Shock.

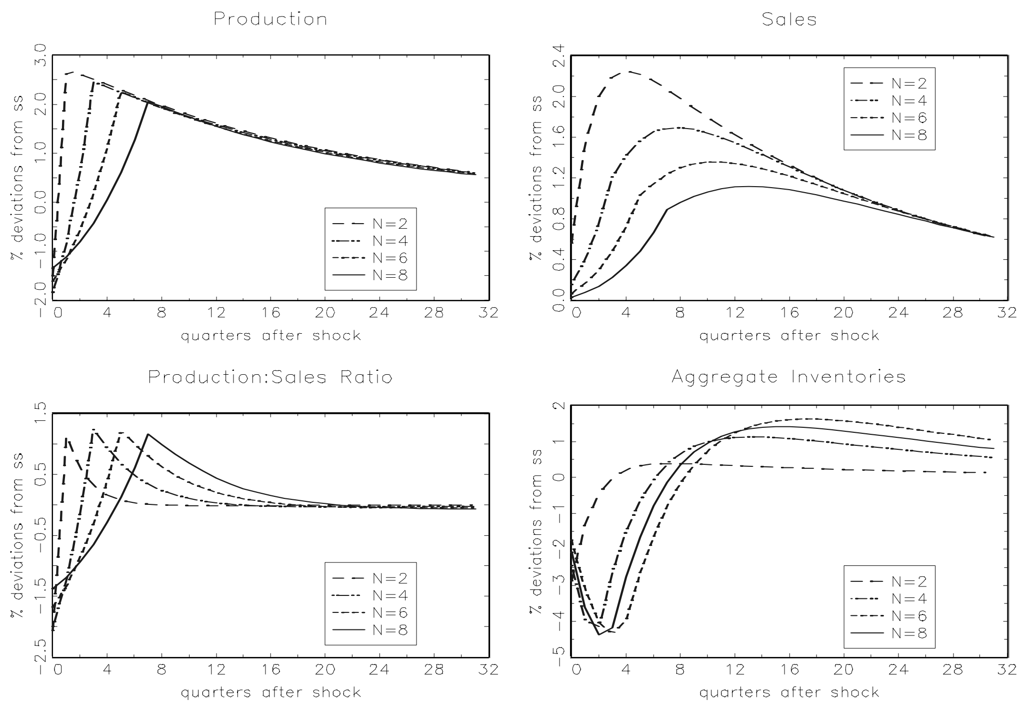


Figure 6. Individual Firm Response to a Productivity Shock.

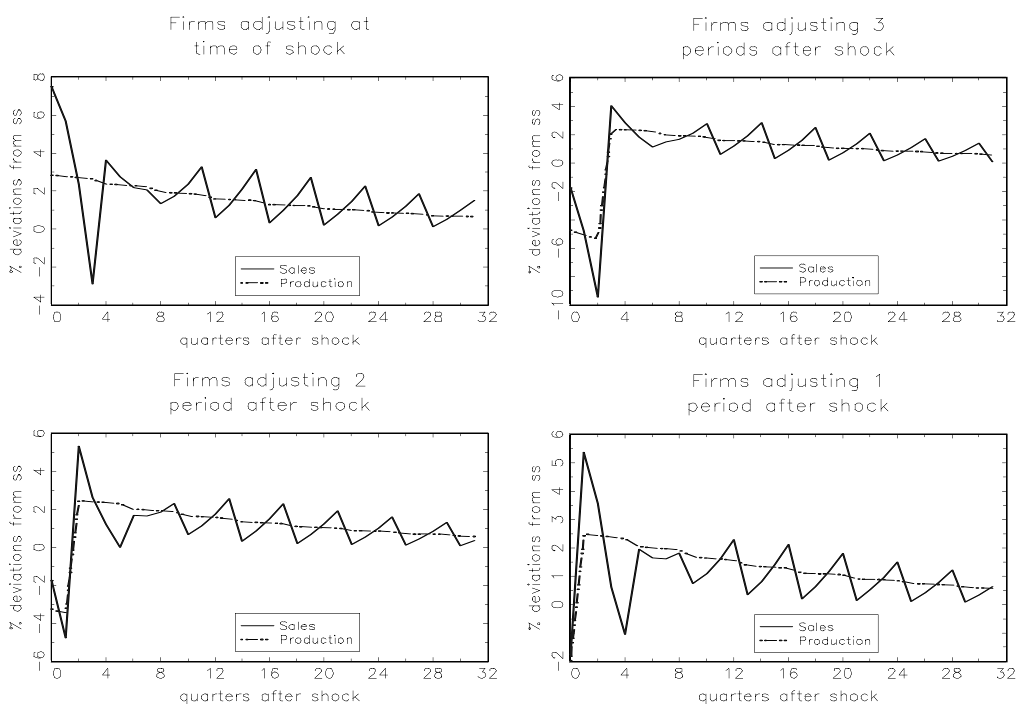
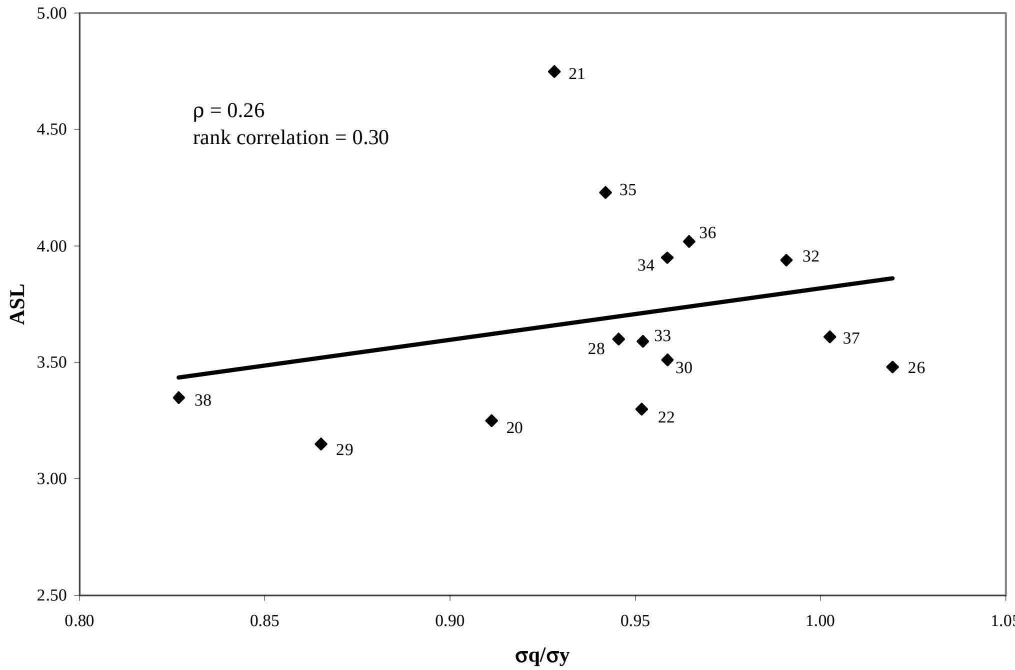


Figure 7.
Price-Rigidity vs. the Relative Volatility of Sales to Output.



Note: An industry's relative volatility of sales to output σ_q/σ_y is the ratio of the standard deviations of the HP filtered cyclical component of quarterly sales and output series from 1982:1 to 1994:2. Output for each industry is the sum of constant dollar sales (shipments) and the finished goods inventory investment. Constant dollar series are obtained by deflating an industry's nominal inventories and sales with the industry's shipments price deflator. The data on shipments and inventories are from the Bureau of Economic Analysis, Census Department. An industry's average duration of price rigidity (ASL) is from Caucutt et al. (1999). The industries included, with their SIC code in parentheses, are: food (20), tobacco (21), textile (22), paper (26), chemicals (28), petroleum (29), rubber (30), stome, clay, and glass (32), primary metals (33), fabricated metals (34), industrial machinery (35), electronic equipment (36), transportation equipment (37), and instruments (38).