

Exclusive Contracts and the Institution of Bankruptcy*

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Abstract

This paper suggests a motivation for the institution of bankruptcy: Whenever exclusive contracts cannot be enforced ex ante, e.g., a bank cannot monitor whether the borrower enters into contracts with other creditors, bankruptcy enables the enforcement of exclusivity ex post, and hence relaxes the incentive constraints. In general, though, while a bankruptcy institution improves on non-exclusive contractual relationships, it is not a perfect substitute for ex ante exclusivity.

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1 Introduction

In most market economies bankruptcy laws have an important role in organizing the restructuring or the liquidation of financially distressed or insolvent firms.¹ This role is usually not limited to ensuring the orderly functioning of markets via the enforcement of default provisions contained in private contractual agreements. Rather, bankruptcy laws specify general procedures that allow courts to actively intervene in the restructuring or liquidation process of insolvent firms. For instance in the U.S. both the liquidation (under Chapter 7 of the Bankruptcy Code) and the restructuring (under Chapter 11) of insolvent firms as well as the discharge or restructuring of the debt of insolvent individuals (under Chapter 7 and 13, respectively) are directly governed by courts (see Baird (1993) for an introduction to the Code).

In the canonical environment considered in the theory of optimal contracts, though, it is difficult to justify the need to set up bankruptcy procedures which do not coincide with the enforcement of private default provisions. Agents can specify the actions and payoffs of all parties in the contract following an insolvency state, and contractual *default* provisions would suffice to guarantee the efficient functioning of markets.²

As a consequence, most of the economic literature on bankruptcy studies environments in which bankruptcy is motivated by exogenous restrictions on the set of contracts which debtors and creditors can write ex ante to address potential conflicts arising between different creditors in case of insolvency ex post (i.e., incomplete contracts). Aghion, Hart and Moore (1992), for instance, introduce their classical analysis of bankruptcy institutions in an incomplete contracting framework as follows: “A debtor who borrows from a creditor could specify as part of the debt contract how his assets will be divided between various creditors (and the debtor himself) in the event of default or insolvency, who will supervise the division process, etc. Writing such contracts is likely to be difficult and costly, however, particularly since the debtor may acquire different types of assets and new creditors as time passes, and it may be very hard to specify how the division process should

¹See Berkovitch and Israel (1999) for a comparative analysis of bankruptcy laws.

²See Dubey, Geanakoplos and Shubik (1996) and Zame (1993) for a general equilibrium analysis of contracts with default provisions; and see Schwartz (1997) for an analysis of the distinction between contracts with default provisions and bankruptcy institutions.

change as a function of such developments” (pg. 6).³

In this paper we depart from most of the literature and model the difficulties involved in writing contracts when, in the spirit of the previous citation, “the debtor may acquire different . . . assets and new creditors . . .” directly. Specifically, we study economies in which bankruptcy provisions arise as part of optimal contracts when firms have potentially multiple contractual positions which are not observed by all parties and, thus, creditors cannot contractually restrict the acquisition of additional assets and liabilities by the debtor.⁴ We focus, again in contrast to most of the literature, on the effect of bankruptcy law on the equilibrium contractual relationships that borrowers and lenders enter into *ex ante*, rather than on the conflicts arising between the various lenders *ex post*, i.e., *after insolvency*.⁵

In a borrowing and lending contractual relationship in which the borrower has some private information, the inability of lenders to *ex ante* write contracts contingent on the total assets and liabilities of a borrower (due to the fact that the borrower can enter various contractual relationships) has striking effects on the borrower’s incentives. In particular, it substantially limits the borrower’s ability to borrow and insure under an optimal contract.

In such an environment a bankruptcy institution may allow lenders to observe the assets and liabilities of the borrower in case of insolvency *ex post* (and hence implicitly make the contract contingent on these). Thus, bankruptcy provisions may have the important effect of relaxing the constraints imposed on borrowing and lending contracts by the inability of the

³See also Berglöf, Roland and von Thadden (2000).

⁴In the literature such environments are usually referred to as characterized by *non-exclusive* contractual relationships. See Arnott and Stiglitz (1983) for a pioneering analysis in economies with hidden action and, more recently, Bisin and Guaitoli (1998), Bizer and DeMarzo (1992, 1999) and Kahn and Mookherjee (1998). Allen (1985), Fudenberg, Holmström and Milgrom (1990), and Cole and Kocherlakota (1999) study related environments with different forms of non-exclusivity. Also, see Berglöf and von Thadden (1994), Biais and Gollier (1997), Bolton and Scharfstein (1996), Dewatripont and Maskin (1995), Dewatripont and Tirole (1994), and Winton (1995), for models of multiple lending relationships. For evidence on multiple contractual relationships see, e.g., Petersen and Rajan (1997).

⁵Besides Aghion, Hart and Moore (1992), see also e.g. Bebchuk (1988) and Cornelli and Felli (1998) for models which focus on the attribution of the debtor’s assets to lenders in the event of insolvency. A notable exception is Berglöf, Roland and von Thadden (2000) who, like us, study the effect of bankruptcy law on the equilibrium capital structure of firms.

lender to enforce exclusive contracts with the borrowers ex ante. In other words, bankruptcy provisions can operate as a sort of ex post monitoring device. Importantly, it is not the outcome of the borrower's project that is being monitored but rather the composition of his assets and liabilities with respect to junior claimants. This monitoring device is combined with insurance provisions for the debtor in case the monitoring reveals that the debtor's assets have not been diverted to junior claimants.⁶

The rationale for bankruptcy law put forth here is consistent with the main principles underlying the U.S. Bankruptcy Code which governs corporate bankruptcy. In particular, under Chapter 11 the debtor is allowed to restructure, after disclosing the books to the court, and possibly proceed with the project. Along these lines, we interpret the frequent attribution of control rights to the insiders as well as the extensively documented deviations from the absolute priority rule (see, e.g., Franks and Torous (1989)⁷) as implicit insurance provisions.⁸ In our interpretation of bankruptcy law restructuring provisions which implicitly favor insiders are therefore an essential component of the law. Liquidation procedures, as in Chapter 7 of the U.S. Code, are seen mainly as a device to inhibit the diversion of assets away from senior claimants ex ante.⁹

This is important because the economic literature on bankruptcy, mo-

⁶We underplay the role of the classical argument in support of bankruptcy institutions, namely that they avoid uncoordinated actions on the part of creditors affecting the debtor's assets in case of his inability to repay (see e.g., Baird (1993)), in order to highlight the most novel components of our analysis. That argument does not require non-exclusivity of contractual relationships, but only the necessity of multilateral contracts, and implies that liquidation procedures should play a prominent role in bankruptcy law.

⁷Notice that the focus here is on violations of absolute priority between insiders and outsiders rather than between lenders with different seniority.

⁸In the case of individual bankruptcy, the U.S. Code literally specifies insurance provisions, in the form of the *fresh start* policy for individual debtors. Baird (1993), for instance, stresses that one of the main purposes of the Code is that it "allows creditors to scrutinize the debtor's affairs and, assuming no misbehavior is found, it provides the debtor with a fresh start" (pg. 32). While our analysis applies therefore closely also to individual bankruptcy, we prefer to refer explicitly to corporate bankruptcy throughout the paper.

⁹Consistently Baird (1993) argues that "[t]he purpose of allowing corporations to file Chapter 7 petitions is not so much to give creditors assets, as it is to assure creditors that the corporation has no assets" (pg. 15). See also Schwartz (1989) for related considerations on such a rationale for the Bankruptcy Code.

tivated by analysis focusing as noted above on conflicts between creditors arising in the event of insolvency ex post, has tended to undervalue the importance of restructuring procedures as opposed to liquidation procedures. Aghion, Hart and Moore's bankruptcy reform, for instance, calls for a variant of the liquidation procedure contained in Chapter 7 and the abandonment of the restructuring procedures of Chapter 11 (see also Hart et al. (1997)).

One of the main questions motivating our analysis is: Can the functions of bankruptcy, namely of providing senior claimants with an ex post monitoring device on the assets and liabilities of the debtor and of providing the debtor with insurance against the possibility of an unsuccessful project, be implemented merely through private contractual default agreements? Or is the institution of a bankruptcy law necessary?

In our set-up, bankruptcy essentially consists of two elements: A seniority rule and insurance provisions for the debtor in the form of fresh start and restructuring provisions. While it is clear that in an environment in which multiple contractual positions are possible and not observable ex ante the enforcement of the seniority rule requires more than just the enforcement of private contractual agreements, this is not the case for fresh start and restructuring provisions. It would appear that such provisions could be efficiently determined contractually by the debtor and the senior claimant. But in an economy in which multiple contractual relationships are necessary for the success of the entrepreneur's project, the contractual determination of the debtor's provision for consumption may in equilibrium suffer from the inefficiency associated with the lack of coordination on the part of the various claimants. The "generosity" of the fresh start and restructuring provisions could suffer, as a consequence, in equilibrium and the efficiency of bankruptcy as ex post monitoring would conflict with the insurance needs of the entrepreneurs, resulting in inefficient contractual arrangements. By providing a rationale for the efficiency of insurance provisions in the form of "generous" fresh start and restructuring provisions, as well as of a seniority rule, our analysis may then suggest a role for bankruptcy institutions as a mechanism enforcing the efficient functioning of markets in environments in which private default provision would not suffice.

We now turn to a more detailed discussion of the results of this paper. We first characterize the effects of the inability of lenders to ex ante write contracts contingent on the total assets and liabilities of a borrower (i.e., non-

exclusivity of contractual relationships) in the benchmark economy without bankruptcy law. In other words, we compare the equilibrium allocation and the optimal contract offered by the bank in an environment in which banks can enforce exclusivity clauses (and therefore effectively render secondary credit markets inactive) and an environment in which such exclusivity clauses cannot be enforced because secondary credit markets cannot be monitored.

We show that the inability to enforce exclusivity clauses (i.e., to monitor secondary credit markets) has in general two effects. First, it reduces the insurance provided by the repayment schedule of the loan offered by the bank against the possibility that the project is unsuccessful. Second, it reduces the amount that the entrepreneur can borrow.

If a bankruptcy institution, i.e., a seniority rule and insurance provisions for the entrepreneur, is introduced when exclusive contracts are not otherwise enforceable, the bank can implicitly monitor the existence of contractual relationships between the entrepreneur and secondary lenders *ex post*. With bankruptcy, the bank can and will, in fact, choose a repayment schedule which has the property that the entrepreneur will declare bankruptcy if the project turns out not to be successful. Making the bank loan senior to any debt incurred in secondary markets has the effect that debt in secondary markets will in general not be repaid when the project financed is unsuccessful. This severely restricts the set of contracts that the entrepreneur can enter into in the secondary market. If the bankruptcy provision for the entrepreneur's consumption in case of insolvency is chosen appropriately, the institution of bankruptcy thus alleviates the incentive problem induced by the non-exclusivity of contractual relationships, i.e., it is welfare improving.

We then show that the institution of bankruptcy is not a complete substitute for the enforceability of exclusive contracts, however. While the set of contracts offered to the entrepreneur in the secondary market is considerably restricted by the introduction of bankruptcy procedures, the repayment schedule imposed by the bank must still be such that the entrepreneur has no incentive to choose any such contract. As a consequence, the optimal contract offered by the bank provides the entrepreneur with less insurance than provided by the optimal contract when exclusive contracts are enforceable. In other words, while bankruptcy in our model can be interpreted as operating as a monitoring device of the debtor's assets, such monitoring is costly. Monitoring costs derive implicitly from the fact that bankruptcy provisions are postulated to be state independent, which can be motivated, e.g.,

by informational constraints on what is observable by the courts.

These results contrast with those derived by Bizer and DeMarzo (1999) in an interesting paper closely related to ours. They study the problem of a principal employing an agent who can borrow and lend as well as default on loans. They show that there is an optimal intermediate range of bankruptcy protection levels that attain constrained efficiency, i.e., the optimal exclusive allocation can be implemented when bankruptcy is allowed. Two important properties differentiate the environment studied by Bizer and DeMarzo (1999) from ours and are instrumental in generating their efficiency result. First, the cash flows from the project accrue to the principal and thus the transfer between the principal and the agent is a wage payment *from* the principal *to* the agent which is unaffected by the bankruptcy procedure. In particular, the agent cannot default on the principal, but only on the “secondary” lenders. Second, only borrowing and lending contracts are considered in the secondary credit market and contracts that explicitly provide insurance are not allowed, which means that the feasible deviations are essentially one dimensional.

The final set of results of this paper concerns the characterization of the optimal contract offered by banks when exclusive contracts are not enforceable and bankruptcy is allowed for. These contracts have several features which match stylized properties of contracts observed in borrowing and lending markets. For instance, the repayment schedule of the loan offered by the bank can optimally be chosen to be state independent (i.e., pure debt is optimal). Moreover, the terms of the bank’s contract, in equilibrium, are such that bankruptcy is declared if and only if the entrepreneur’s project is unsuccessful. Bankruptcy provisions therefore have the role of insuring the consumption allocation of the entrepreneur.

Furthermore, the terms of the optimal contract offered by the bank are designed to constrain the debtor’s incentive to incur excessive liabilities with secondary lenders. In other words, the optimal contract offered by the bank has the property that if the borrower were to enter the secondary credit market he would take out additional loans. This is important because it is in contrast to the result in the standard principal agent analysis of borrowing and lending relationships (see e.g. Rogerson (1985)). There the optimal contract offered by the bank has to constrain the debtor’s incentive to save in

secondary markets, a result typically considered puzzling in the literature.¹⁰

The paper proceeds as follows. Section 2 describes the model. Section 3 characterizes the optimal contract depending on the environment. In Section 3.1 we characterize the optimal exclusive contract, i.e., the optimal contract with neither a secondary credit market nor bankruptcy. In Section 3.2 we study non-exclusive credit relationships when there is no bankruptcy institution. In Section 3.3 we consider non-exclusive credit relationships with a bankruptcy institution. Section 4 computes the optimal contract for the various environments in an example economy. Section 5 concludes.

2 The Model

We study economies in which an entrepreneur needs to borrow to engage in a productive endeavor, a project. The probability of success of the project depends on a costly unobservable action of the entrepreneur. A bank providing the necessary funds to finance the project schedules repayments, conditional on the outcome of the project, such that the entrepreneur has an incentive to take the action which maximizes the value of the project. We consider and compare three cases: *i)* The bank is the only lender, and the entrepreneur cannot declare bankruptcy. *ii)* The entrepreneur can raise funds in addition to those obtained from the bank from an outside source (which we refer to as “secondary lenders” throughout the paper), i.e. the entrepreneur has access to a secondary credit market, but cannot declare bankruptcy, and the bank cannot condition the repayment schedule of the loan on the total funds raised by the entrepreneur (the bank does not observe successive contractual relationships of the entrepreneur). *iii)* The entrepreneur can raise funds in addition to those obtained from the bank, the bank cannot condition the repayment schedule of the loan on the total funds raised by the entrepreneur, and the entrepreneur can declare bankruptcy. If he does declare bankruptcy, the bank debt is senior and the entrepreneur consumes a fixed, predetermined amount. Bankruptcy therefore, we stress, coincides in our analysis with a seniority rule and provisions to guarantee the consumption of the entrepreneur.

¹⁰See however Bizer and DeMarzo (1999) for an important exception.

While our analysis is necessarily abstract since it is designed to identify a new motivation for the institution of bankruptcy, we believe the environment we analyze captures several important elements of borrowing and lending markets.

The secondary credit market for instance does not have to be interpreted literally as borrowing from a second bank, although this is a possible interpretation. Trade credit, which is an important source of financing for firms in the United States (see e.g. Petersen and Rajan (1997)), can also be a secondary source of funds. So can other sources of working capital and specifically accounts payable, such as employees or the tax authority. The secondary source may also be credit card financing in addition to a primary bank loan (in particular for small firms) or unsecured, informal lending supplementing a first, formal loan.

Also, the focus of the paper is clearly on unsecured lending, i.e., loans that are not collateralized by specific assets. Unsecured lending is an important source of financing especially for small firms: The fraction of loans to firms with less than 500 employees which are not collateralized is 31% in data from the 1993 National Survey of Small Business Finances. Trade credit and credit card loans, mostly unsecured, also constitute a sizable form of financing for small businesses.

2.1 The Economy

We consider a simple economy with two dates, $t = 0, 1$, and one single consumption good that is populated by an *entrepreneur* and several agents which operate in *credit markets*. We will distinguish between an agent which will take the role of the main creditor of the entrepreneur, which, for concreteness, we call *bank*, and all the other potential creditors which populate what we call the *secondary credit market*.

The Entrepreneur. The entrepreneur is endowed with $\alpha < 1$ units of the good at time $t = 0$, and has the opportunity to engage in a productive endeavor, a project, which requires an investment of 1 unit of the consumption good at time $t = 0$. The project yields, at time $t = 1$, ω_H units of the consumption good with probability π_a and ω_L units of the consumption good with probability $1 - \pi_a$; a denotes an unobservable action of the entrepreneur, effort toward the success of the project, which affects the probability distrib-

ution of the outcome of the project, and takes two values, e and E . Without loss of generality, $\omega_H > \omega_L$, and $\pi_E > \pi_e$, so that ω_H takes the interpretation of ‘high’ realization of the output of the project, and E of ‘high’ effort.

The entrepreneur is also endowed with preferences represented by a Von Neumann-Morgenstern, continuous, smooth, strictly monotonic increasing, strictly concave utility function, $u : \mathfrak{R}_+ \rightarrow \mathfrak{R}$. If we denote with c_0 and c_1 , respectively, consumption at time 0 and 1, the entrepreneur’s lifetime utility is

$$U = u(c_0) + \mathbb{E}[u(c_1)|a] - v(a)$$

where $\mathbb{E}[\cdot|a]$ denotes the expectation conditional on action a . This specification of preferences assumes separability of consumption and effort and no discounting for tractability. Consistent with the interpretation of E as ‘high’ effort, we assume $v(E) > v(e)$.

The entrepreneur finances his project in the credit markets. All agents in the credit markets are identical, risk neutral and ready to invest in the entrepreneur’s project. The first agent the entrepreneur enters in a contractual relationship with will have the special role of the main/primary creditor, and will be called the bank.

The Bank. The bank provides the entrepreneur, at $t = 0$, with funds $I \in \mathfrak{R}$ (possibly negative) to operate the project and/or smooth consumption. It sets state contingent repayments at $t = 1$, (B_H, B_L) , to maximize profits. Profits are evaluated at the entrepreneur’s equilibrium action $a \in \{e, E\}$, and at the equilibrium repayment levels $(\tilde{B}_H, \tilde{B}_L)$ in case the entrepreneur decides not to fully repay his debt. Since we assume no discounting, we set the exogenous interest rate to 0, and the bank’s profits are:

$$\pi_a \tilde{B}_H + (1 - \pi_a) \tilde{B}_L - I.$$

The entrepreneur, after entering the contractual relationship with the bank, might trade in the secondary credit market, with secondary lenders.

The Secondary Lenders. Any secondary lender observes I, B_H, B_L , and then potentially provides the entrepreneur at time $t = 0$, with extra credit, $\gamma \in \mathfrak{R}$ (possibly negative), and a state contingent repayment scheme at time $t = 1$, (R_H, R_L) , also to maximize profits evaluated at the entrepreneur’s equilibrium action $a \in \{e, E\}$, and equilibrium repayment levels $(\tilde{R}_H, \tilde{R}_L)$:

$$\pi_a \tilde{R}_H + (1 - \pi_a) \tilde{R}_L - \gamma.$$

The Bankruptcy Institution. The entrepreneur might decide not to repay his debt with the bank and/or with the secondary lenders. In this case, *i)* the project is seized by the creditors; *ii)* the entrepreneur receives P units of consumption or all the output, whichever is less; *iii)* the bank's credit is senior. The bank receives the difference between the output of the project plus the positive payoffs of the entrepreneur's other claims (if any) and P , if less than the full credit amount, and the secondary lenders are partially reimbursed if and only if the bank is fully repaid.¹¹ We assume that the secondary lenders get equal shares of the proceeds in bankruptcy, up to the amount of their loan. As we discuss below, this specific assumption about the sharing rule in bankruptcy is not critical for our main conclusion. It simplifies the characterization of the solution by allowing us to essentially ignore the strategic interaction in the secondary market and proceed as if there were only one secondary lender.

Such a bankruptcy institution can be implemented by *courts* as follows. The contract between the bank and the entrepreneur is deposited in court, or in a *credit bureau* operated by the courts, and therefore registered and dated. Whenever bankruptcy is called, the courts verify the state of insolvency of the entrepreneur, and guarantee the seniority rule; in particular they guarantee the seniority of the bank's claim. All private contracts put forward to the courts in the state of insolvency, even those not registered in the credit bureau, are honored by the bankruptcy process. The bank holding the senior claim can (and will, without loss of generality) impose an exclusivity clause, therefore eliminating any possible seniority structure between potential secondary lenders, which then have no incentive to deposit their contracts with the credit bureau. As a consequence, we postulate that secondary lenders are repaid equal shares of what is left after reimbursing the bank holding the senior claim and providing the entrepreneur with P units of consumption.

2.2 Assumptions

We impose the following assumptions.

¹¹If the output of the project is sufficient to repay all of the entrepreneur's debt after guaranteeing himself at least P , there is no reason for him not to repay and declare bankruptcy.

Assumption 1 *Preferences satisfy $\lim_{c \rightarrow 0} u'(c) = \infty$, $\lim_{c \rightarrow 0} u(c) = -\infty$ and are extended to the entire real line with $u(c) = -\infty$ for $c < 0$.*

This assumption guarantees positivity of consumption in equilibrium. Furthermore, the assumption formally guarantees that an agent neither wants to enter into inconsistent contracts that result in negative consumption nor renege on, i.e., not honor, his promises (which we assume implies 0 consumption in the absence of bankruptcy protection).

Assumption 2 *Lower bound on the expected rate of return of the project:*

$$\pi_e \omega_H + (1 - \pi_e) \omega_L > 1.$$

The expected rate of return implied by the project, when operated at the low effort e , is $\pi_e \omega_H + (1 - \pi_e) \omega_L$, while the safe rate of return of the economy is 1. Assumption 2 requires production efficiency of the project at effort e . (Alternatively, the assumption requires that an agent would find it profitable to invest in the project, operate it at the low effort e , fully insure the outcome of the project at fair prices, and perfectly smooth his consumption at the safe rate of return 1, rather than not operating the project and smoothing his consumption, which is deterministic if the project is not operated.)

Let $c_e := \frac{1}{2}(\alpha - 1 + \pi_e \omega_H + (1 - \pi_e) \omega_L)$, the consumption allocation at each time and state of the world of an agent choosing $a = e$ in the case of perfect smoothing and full insurance.

Assumption 3 *Lower bound on the relative productivity of high effort E :*

$$\pi_E u(\omega_H - I_e) + (1 - \pi_E) u(\omega_L - I_e) - v(E) > u(c_e) - v(e)$$

where I_e is the level of borrowing or lending which supports consumption c_e at $t = 0$, i.e., $I_e := c_e - (\alpha - 1)$.

Assumption 3 requires that, when consuming c_e at $t = 0$, the agent would prefer to operate the project at the effort E without insurance rather than operating the project at the low effort e and fully insuring the outcome of the project.

3 Equilibrium Contracts

In equilibrium, the entrepreneur chooses the agent with which he enters into a primary contractual relationship, i.e. the bank he deals with; moreover the entrepreneur trades in the secondary credit market, thereby choosing the secondary lenders he deals with. Given the aggregate contractual position he has entered into, and given the bankruptcy institution, the entrepreneur chooses the effort he exerts toward the success of the project and a repayment strategy contingent on the state of the world (the outcome of the project). The bank chooses which contract to offer, anticipating the entrepreneur's trades in the secondary credit market, and anticipating the effort and repayment decisions of the entrepreneur. The secondary lenders choose which contract to offer, knowing the terms of the contract between the entrepreneur and the bank, and anticipating the effort and repayment decisions of the entrepreneur.

We do not explicitly describe the mechanism by which the entrepreneur chooses the agent which acts as the bank, or the agents to deal with in the secondary credit market. We instead postulate free entry in both the primary and the secondary credit markets, and hence will characterize contracts which maximize the entrepreneur's expected utility provided non-negative expected profits are guaranteed for the bank and the secondary lenders. This is just for simplicity. A simple mechanism which supports free entry can easily be constructed.¹²

Before studying the equilibrium in our economy with unobservable effort, secondary lenders, and bankruptcy, we analyze the equilibrium of a simple economy as a benchmark.¹³

¹²For instance, the following mechanism would do: Agents have the option to offer a primary credit contract to the entrepreneur in an exogenous order. The entrepreneur, accepting one such contract, stops the sequence of offers. The secondary market, in which agents make offers simultaneously, then opens, and the entrepreneur accepts as many offers as he pleases.

¹³Under *full information* (observable effort) the optimal contract is characterized by: $a = E$ and

$$c_0 = \pi_E c_H + (1 - \pi_E) c_L \quad (\text{perfect smoothing})$$

$$c_H = c_L \quad (\text{full insurance})$$

i.e., $c_0 = c_H = c_L = \frac{1}{2}(\alpha - 1 + \pi_E \omega_H + (1 - \pi_E) \omega_L)$.

3.1 Exclusive Contracts and No Bankruptcy

We consider here the case in which banks have the ability to impose exclusivity clauses on the entrepreneurs (perhaps because they can monitor their trades), and, as a consequence, secondary lenders do not operate. We also assume that no bankruptcy law is in effect in the economy. By “no bankruptcy law” we mean that in particular there is no bankruptcy protection and, thus, if an agent did not keep his promises, he would get consumption of zero. Hence, given our assumptions, agents keep their promises.

This is in fact the standard principal agent model of borrowing and lending, which we use as a benchmark (see Rogerson (1985)). Because we assume that many agents can act as banks (free entry), in equilibrium the bank will choose to offer the entrepreneur a contract which maximizes the entrepreneur’s utility, provided the contract guarantees the bank non-negative profits.

It is easy to show that Assumptions 1-3 imply that such an optimal contract is characterized by $a = E$ and $\alpha + I > 1$. To extract effort E from the entrepreneur, the contract must satisfy an incentive constraint. The optimal contract offered by the bank is the solution to the following maximization problem:

$$\max_{I, B_H, B_L} u(\alpha + I - 1) + \pi_E u(\omega_H - B_H) + (1 - \pi_E)u(\omega_L - B_L) - v(E) \quad (1)$$

subject to

$$\pi_E B_H + (1 - \pi_E)B_L \geq I \quad (2)$$

$$u(\alpha + I - 1) + \pi_E u(\omega_H - B_H) + (1 - \pi_E)u(\omega_L - B_L) - v(E) \geq \quad (3)$$

$$u(\alpha + I - 1) + \pi_e u(\omega_H - B_H) + (1 - \pi_e)u(\omega_L - B_L) - v(e).$$

The optimal contract maximizes the utility of the entrepreneur (1), subject to the zero-profit condition of the bank (2), and the incentive compatibility constraint (3). Let (c_0^*, c_L^*, c_H^*) denote the equilibrium allocations of the debtor (i.e., his consumption at the optimal contract). Similarly, let (I^*, B_L^*, B_H^*) denote the optimal contract offered by the bank.

The following proposition characterizes the consumption allocation of the entrepreneur at the optimal contract (see Rogerson (1985)).

Proposition 1 *Suppose Assumptions 1-3 hold, contractual relationships are exclusive, and there is no bankruptcy institution. Then,*

$$c_H^* := \omega_H - B_H^* > c_L^* := \omega_L - B_L^* \quad (\text{partial insurance}),$$

and

$$u'(c_0^*) < \pi_E u'(c_H^*) + (1 - \pi_E) u'(c_L^*) \quad (\text{imperfect smoothing}).$$

Moreover, if $1/u'$ is convex (i.e., the precautionary motive for savings is not too strong), then

$$c_0^* = \alpha + I^* - 1 > \pi_E c_H^* + (1 - \pi_E) c_L^*.$$

If $1/u'$ is concave, the last inequality is reversed.¹⁴

All proofs are in the Appendix unless noted otherwise. Notice that imperfect smoothing is (constrained) optimal independent of the precautionary savings effect driven by the curvature of marginal utility: In fact, with linear marginal utility (quadratic preferences), savings are restricted in equilibrium, and $c_0^* = \alpha + I^* - 1 > \pi_E c_H^* + (1 - \pi_E) c_L^*$.

3.2 Non-exclusive Contracts and No Bankruptcy

We consider here the case in which contractual relationships in the credit market are non-exclusive: Secondary lenders, as well as a bank, can provide the entrepreneur with credit. The bank cannot condition the terms of its contract on the entrepreneur's trading in the secondary credit market. We consider though for the moment the case in which bankruptcy is not allowed. The entrepreneur cannot declare bankruptcy and default on his debt obligations.¹⁵

The determination of the equilibrium contractual relationships between the bank, the entrepreneur, and the secondary lenders, as well as the equilibrium allocation of the entrepreneur, is not straightforward. However, we

¹⁴The borderline case is thus logarithmic utility ($1/u'$ linear) for which consumption is a martingale: $c_0^* = \pi_E c_H^* + (1 - \pi_E) c_L^*$.

¹⁵Again, by “no bankruptcy law” we mean that if an agent did not keep his promises he would get zero consumption (there is no bankruptcy protection) and thus he would never choose to do so.

claim that, as in the case discussed in the previous section, the equilibrium allocation of the entrepreneur can be uniquely determined by the solution to a programming problem. We construct such a problem below.

First of all, note that the bank and any of the secondary lenders are identical in terms of objectives (they are risk neutral and maximize expected profits) and in terms of contract space. This implies that any agent can act as the bank, and free entry guarantees that the contract offered by the bank involves zero expected profits in equilibrium (after the entrepreneur's trading with secondary lenders). Furthermore, any allocation for the entrepreneur can be supported by a single contract with the bank, and no relationship with any of the secondary lenders. The set of contracts that the secondary creditors could offer the entrepreneur can also be offered by the bank *ex ante*.¹⁶

The equilibrium allocation for the entrepreneur can then be supported as an optimal contract restricted to one in which secondary lenders are inactive, i.e., $\gamma = R_H = R_L = 0$, provided the entrepreneur does not have any incentive to choose a joint deviation of supplying effort e and supplementing the bank's credit with secondary credit (an incentive compatibility constraint). In other words, the contract offered by the bank in equilibrium will be such that the entrepreneur has no reason to enter into a contract with one or several secondary lenders in order to create a portfolio which induces him to exert low effort, thereby reducing the expected repayment to the bank.

Finally, in the secondary market for credit, again because of free entry, the entrepreneur can guarantee himself a contract which maximizes his utility provided each secondary lender makes zero expected profits in equilibrium. (Note that this is essentially independent of the specific strategic interactions between secondary lenders in the market.)

Since Assumptions 1-3 guarantee that the optimal contract is characterized by $a = E$ and $\alpha + I > 1$, the optimal contract with secondary lenders and no bankruptcy is the solution to the following maximization problem:

$$\max_{I, B_H, B_L} u(\alpha + I - 1) + \pi_E u(\omega_H - B_H) + (1 - \pi_E)u(\omega_L - B_L) - v(E) \quad (4)$$

¹⁶There is thus no intrinsic role for financing by the secondary lenders in our model. It would be an interesting extension to study an environment in which secondary financing would play a non-degenerate role.

subject to

$$\pi_E B_H + (1 - \pi_E) B_L \geq I \quad (5)$$

$$u(\alpha + I - 1) + \pi_E u(\omega_H - B_H) + (1 - \pi_E) u(\omega_L - B_L) - v(E) \geq \quad (6)$$

$$u(\alpha + I + \gamma - 1) + \pi_e u(\omega_H - B_H - R_H) + (1 - \pi_e) u(\omega_L - B_L - R_L) - v(e)$$

for all (γ, R_H, R_L) which satisfy

$$\pi_e R_H + (1 - \pi_e) R_L \geq \gamma \quad (7)$$

$$R_H \geq \gamma. \quad (8)$$

The optimal contract maximizes the expected utility of the entrepreneur (4), subject to the zero-profit condition of the bank (5), and the incentive compatibility constraint (6). In specifying the incentive compatibility constraint (6), we restrict the set of contracts offered in the secondary market to those which satisfy (7-8). (8) is obviously more restrictive than necessary, as it requires restricting contractual relationships in the secondary market to those which involve positive insurance, and hence guarantees non-negative expected profits for secondary lenders even in the case where the entrepreneur chooses effort E . We shall prove (Lemma 1 in the Appendix) that this restriction is formally without loss of generality, i.e. the optimal contract is unaffected if we restrict the set of feasible contracts offered by secondary lenders to those satisfying (7-8).

Note that the incentive constraint when the secondary credit market is active, (6), is more restrictive than when it is not, (3), since by trading in this market the entrepreneur can optimally tailor his credit and insurance positions to the case in which he chooses the low effort $e = a$. In particular, the value to the entrepreneur of a joint deviation to effort e and to secondary credit increases with the present value of wealth after bank credit has been received, $\alpha + I - 1 + \pi_e(\omega_H - B_H) + (1 - \pi_e)(\omega_L - B_L)$, or, using the bank's zero profit condition, with $\alpha - 1 + \pi_e \omega_H + (1 - \pi_e) \omega_L + (\pi_E - \pi_e)(B_H - B_L)$, and hence it increases with $(B_H - B_L)$ (see Lemma 1).

Let $(c_0^{**}, c_L^{**}, c_H^{**})$ denote the equilibrium allocations of the debtor (i.e., his consumption at the optimal contract). Similarly, let $(I^{**}, B_L^{**}, B_H^{**})$ denote the optimal contract offered by the bank.

Proposition 2 *Suppose Assumptions 1-3 hold, contractual relationships are non-exclusive, and there is no bankruptcy institution. Then*

$$c_H^{**} := \omega_H - B_H^{**} > c_L^{**} := \omega_L - B_L^{**} \quad (\text{partial insurance}),$$

and

$$u'(c_0^{**}) = \pi_E u'(c_H^{**}) + (1 - \pi_E) u'(c_L^{**}) \quad (\text{perfect smoothing}).$$

Moreover, if u' is convex (the precautionary savings motive is present), then

$$c_0^{**} := \alpha + I^{**} - 1 < \pi_E c_H^{**} + (1 - \pi_E) c_L^{**}.$$

If u' is concave, the last inequality is reversed.¹⁷

Note that the incentive constraint with a secondary credit market implies that the bank cannot use imperfect smoothing to relax the incentive problem. (In fact, with quadratic preferences, and hence no precautionary savings motive, $c_0^{**} = \alpha + I^{**} - 1 = \pi_E c_H^{**} + (1 - \pi_E) c_L^{**}$.) As a consequence extracting the high effort E is more difficult. Hence, one would expect that it requires reducing the amount of insurance that can be offered to the entrepreneur at the optimal contract.

We now compare the allocation at the optimal contract with and without a secondary credit market to confirm this intuition. The main result of this section is that in general non-exclusivity of contractual relationships has the effect of reducing both the amount of borrowing and the insurance the bank can provide to the entrepreneur. More precisely,

Proposition 3 *Suppose Assumptions 1-3 hold, and there is no bankruptcy institution. Then:*

- (i) *If $1/u'$ is convex and $u''' > 0$, $c_0^{**} < c_0^*$ (non-exclusivity reduces borrowing).*
- (ii) *$c_H^{**} - c_L^{**} > c_H^* - c_L^*$ and $c_H^{**} > c_H^*$ (non-exclusivity reduces insurance).*

It is not true for general utility functions that the optimal contract when the secondary credit market is operating involves reduced borrowing, however. For instance, with quadratic preferences (no precautionary savings) a

¹⁷The borderline case is thus quadratic utility (u' linear) for which consumption is a martingale: $c_0^{**} = \alpha + I^{**} - 1 = \pi_E c_H^{**} + (1 - \pi_E) c_L^{**}$.

similar argument as the one in the proof of Proposition 3 shows that the optimal contract with a secondary credit market involves higher borrowing. For the special case of logarithmic utility Proposition 3 obviously implies that $c_0^{**} < c_0^*$. Moreover, in this case we have a sharper characterization of the insurance effect, namely $\frac{c_H^{**}}{c_L^{**}} > \frac{c_H^*}{c_L^*}$.

It is also of interest to characterize the equilibrium deviation of the entrepreneur at the optimal contract. In other words, if the entrepreneur would actively trade in the secondary credit market (remember that in equilibrium the entrepreneur is indifferent between trading and not trading in such markets), which contract would he trade? Would such a contract involve borrowing or lending by the entrepreneur?

More precisely, given the optimal contract offered by the bank in equilibrium, $(I^{**}, B_H^{**}, B_L^{**})$, the equilibrium deviation of the entrepreneur is the solution to the following maximization problem:

$$\max_{\gamma, R_H, R_L} u(\alpha + I^{**} + \gamma - 1) + \pi_e u(\omega_H - B_H^{**} - R_H) + (1 - \pi_e) u(\omega_L - B_L^{**} - R_L) - v(e),$$

subject to (7) and (8). I.e., the equilibrium deviation is the choice of (γ, R_H, R_L) at which the incentive constraint holds with equality.

Proposition 4 *Suppose Assumptions 1-3 hold, contractual relationships are non-exclusive, there is no bankruptcy institution and u' is not too convex. Then at the equilibrium deviation the entrepreneur would lend to the secondary market, i.e., $\gamma < 0$.*

The proof is left to the reader. A similar result holds for economies in which the bank can enforce exclusive contracts, a direct implication of Proposition 1 (see Rogerson (1985)). In the literature this implication of principal agent models of borrowing and lending markets is generally considered puzzling, since banks seem to be mostly concerned with the possibility of debtors incurring excessive unobserved liabilities in secondary markets, rather than with the possibility of unobserved saving. However, a less literal interpretation of secondary markets can in our opinion reconcile this result with what is observed in credit markets; informal lending to kin or business friends, as well as investments in unrelated projects or, e.g., junk bonds could in principal damage the position of the bank. In any case, we will show in the next section that this result is reversed when we allow for bankruptcy, since in that case the entrepreneur at the optimal contract would consider borrowing in secondary markets.

3.3 Non-exclusive Contracts and Bankruptcy

We now consider the case in which there exists a secondary market which can provide the entrepreneur with credit in addition to what is already provided by the bank, and in which the entrepreneur is allowed to default on his debt obligations and declare bankruptcy. In case of bankruptcy, the project is seized by the creditors, the entrepreneur receives P units of consumption or all the output, whichever is less; the bank's credit is senior (and hence the bank receives the difference between the output of the project plus the positive payoffs of the entrepreneur's other claims (if any) and P , if less than the full credit amount), and the secondary lenders are partially reimbursed if and only if the bank is fully repaid. We have indicated in Section 2 how such a bankruptcy procedure can be implemented by the institution of *courts*.

Note that we do not allow P to be state contingent. This reflects the implicit way in which we make bankruptcy and the enforcement of exclusivity costly. (If it were feasible to make P state contingent, exclusivity could be enforced for free in all states and we would be back to the exclusive contracting environment.) This formulation of implicit monitoring costs is the natural one in economies in which courts do not observe the realization of the state, for instance because this realization can be obscured by the entrepreneur. Alternatively, we could revert to a costly state verification environment in which the cost of monitoring contractual relationships is explicitly modeled, with similar results.

We restrict attention to parameters such that $\omega_L > P$, which simplifies notation. Alternatively, we could assume that the bank guarantees the entrepreneur a consumption of $\min\{\omega_L, P\}$ in bankruptcy or that the bank makes a payment to the entrepreneur in bankruptcy if $\omega_L < P$.¹⁸

As in the previous case we will transform the equilibrium analysis of such economies into the solution of a programming problem. We will show that at an equilibrium the bank's contract is set to induce the entrepreneur to choose the high effort E and to declare bankruptcy in state L only: Such a contract exists, since in the relevant range of contracts the bank can always

¹⁸We proceed as if the choice of P is part of the contract offered by the bank. Implicitly, we are thus determining the optimal bankruptcy protection level. More generally we should think of the bankruptcy provisions as an ex ante ("political") choice encompassing many or, in fact, all contracts, but it makes no difference in the simple economy we consider here.

set the repayment in state L , B_L , to guarantee that the entrepreneur will be insolvent in that state, and only in that state, even after taking into account his possible access to the secondary credit market. This bankruptcy pattern which is unilaterally induced by the design of the bank's contract restricts the class of contracts offered to the entrepreneur in the secondary market, since such contracts must guarantee non-negative profits to the secondary lenders. In particular, as a consequence of the fact that bankruptcy is always induced in state L , the contracts offered by the secondary lenders will not specify any repayment (neither positive nor negative) in that state. The problem will be effectively reduced at this point into a constrained optimization program, as desired.

We proceed now with the details of the analysis leading to the construction of the optimal contracting problem characterizing the equilibrium.

Notice that the optimal contract offered by the bank and the equilibrium allocation in the economy with non-exclusive contractual relationships and no bankruptcy, derived in the previous section, are sustainable in the present environment with bankruptcy: It is sufficient to set bankruptcy provisions P to 0.

Also, this equilibrium allocation can be supported by a contract which induces the insolvency of the entrepreneur in state L , with appropriate bankruptcy provisions. Such a contract, (I, B_H, B_L, P) , will specify a high enough repayment rate in state L , B_L , so that the entrepreneur will be insolvent, declare bankruptcy and consume P , which can be set equal to the equilibrium allocation in state L which we want to support, c_L^{**} . We still need to check that the set of contracts which can be offered to the entrepreneur in the secondary market is not enlarged as a consequence of the introduction of the bankruptcy institution, so that in equilibrium the secondary market will remain inactive. This could only happen if the entrepreneur were offered a contract which he would accept and which would induce him to change the pattern of bankruptcy declaration (for instance would induce him to declare bankruptcy in state H and not in state L). The entrepreneur will declare bankruptcy in each state of the world $s \in \{H, L\}$ in which $\omega_s - B_s - R_s < P$. It is easy to see that, by controlling B_L , the bank can make it prohibitively costly for secondary markets to change the pattern of bankruptcy declaration without affecting the equilibrium allocation of the entrepreneur (who never in fact repays B_L since he declares bankruptcy in state L).

The equilibrium allocation $(c_0^{**}, c_L^{**}, c_H^{**})$ guarantees the entrepreneur utility u^{**} . Could the entrepreneur reach higher utility with contracts with a different bankruptcy declaration pattern? We argue that the answer is no. Consider the three possible alternative bankruptcy declaration patterns in turn:

First, suppose the entrepreneur goes bankrupt in both state H and state L . Then he is fully insured, $c_H = c_L = P$, and chooses the low effort, e . As a consequence this contract is (weakly) dominated by the contract providing the entrepreneur with full insurance and perfect smoothing and the bank with non-negative profits. But such a contract, under Assumptions 1-3, is in turn strictly dominated by the contract which implements utility u^{**} , which we just showed is feasible and incentive compatible under bankruptcy in L .

Second, suppose the entrepreneur never goes bankrupt, neither in state H nor in state L . Then, if the utility for the entrepreneur associated with this contract is at least u^{**} , P and B_L can be chosen so that the entrepreneur is forced into bankruptcy in state L , while leaving his allocation and hence the utility associated with the contract unchanged.

Finally, suppose the entrepreneur goes bankrupt only in state H . In this case the consumption allocation in state L must necessarily be greater than the bankruptcy provision (and hence the allocation in the bankruptcy state H): $c_L \geq P$. It follows that the entrepreneur can never be induced to choose the high effort E . Again all possible allocations are then dominated by the allocation guaranteeing utility u^{**} , which is feasible and incentive compatible under bankruptcy in L .

We have therefore demonstrated that in equilibrium the entrepreneur will declare bankruptcy in state L only. We can then restrict the set of contracts offered by the bank without loss of generality to those which induce bankruptcy only in state L . As a consequence, any contract offered by the secondary market must then be such that it does not require any repayment in state L , neither positive nor negative. Since the entrepreneur goes bankrupt in state L , he will not honor liabilities with secondary lenders, and all of his assets will be assigned to the senior lender, i.e., the bank, as repayment.

More specifically, we can without loss of generality restrict the contract offered by the bank to one which satisfies

$$\omega_H - B_H > P \tag{9}$$

$$B_L \text{ high enough and in particular such that } > \omega_L - P \tag{10}$$

and thus we can also without loss of generality restrict the contracts offered in the secondary market to satisfy:

$$R_L = 0, \quad \pi_e R_H = \gamma \quad (11)$$

$$\omega_H - B_H - R_H \geq P. \quad (12)$$

To specify the optimal contracting problem solved by the bank, therefore, we need to characterize the entrepreneur's utility associated with a deviation to effort e and (optimal) active trading in the secondary credit market for any contract offered by the bank.

Given any contract offered by the bank, (I, B_H, B_L, P) , which satisfies properties (9-10), the utility that the entrepreneur can reach by optimally deviating, that is by choosing effort e and trading in the secondary market, is

$$u^D(I, B_H, P) := \max_{R_H, \gamma} u(\alpha + I + \gamma - 1) + \pi_e u(\omega_H - B_H - R_H) \quad (13)$$

$$+ (1 - \pi_e)u(P) - v(e)$$

subject to

$$\pi_e R_H \geq \gamma$$

$$P \leq \omega_H - B_H - R_H.^{19}$$

¹⁹More precisely, utility $u^D(I, B_H, P)$ represents only an upper bound because there is no guarantee that the equilibrium in the secondary credit market will actually include a set of contracts which in the aggregate provide the entrepreneur with the amount of credit γ and the repayment scheme (R_H, R_L) which solves the maximization problem in (13). It is easy to show though that, if the repayment scheme (R_H, R_L) which solves the maximization problem in (13) in fact does not induce bankruptcy in state H , then any equilibrium in the secondary credit market will in fact in the aggregate provide the entrepreneur with the amount of credit γ and the repayment scheme (R_H, R_L) . (Given our assumption that secondary lenders share the proceeds in bankruptcy equally, it is not possible for yet another lender to enter and offer an additional loan that induces bankruptcy, earns non-negative profits and makes the entrepreneur better off. The additional lender could only make the entrepreneur better off, while ensuring non-negative profits for himself, if he had an externality on the secondary lenders. But if $R_H < 1/2(c_H^+ - P)$, then the secondary lenders get fully repaid even if the borrower declares bankruptcy and thus there is no externality. But since the aggregate repayment R_H can be implemented by loans from more than one lender, say n lenders, and we can choose n such that $R_H < n/(n+1)(c_H^+ - P)$, it is possible to implement any aggregate repayment R_H while avoiding externalities from additional lenders.) In this case then $u^D(I, B_H, P)$ actually represents the maximum utility, given the contract offered by the bank (I, B_H, P) , which the entrepreneur can reach by trading in the secondary market.

The optimal contracting problem can be written as:

$$\max_{I, B_H, P} u(\alpha + I - 1) + \pi_E u(\omega_H - B_H) + (1 - \pi_E)u(P) - v(E) \quad (14)$$

subject to

$$\pi_E B_H + (1 - \pi_E)(\omega_L - P) \geq I \quad (15)$$

and the incentive constraint:

$$u(\alpha + I - 1) + \pi_E u(\omega_H - B_H) + (1 - \pi_E)u(P) - v(E) \geq u^D(I, B_H, P), \quad (16)$$

where $u^D(I, B_H, P)$ is defined by the solution to the maximization problem (13).

We can now characterize the equilibrium allocation of the entrepreneur, the implied utility attained, the structure of the optimal contract offered by the bank, and the contract which constitutes the optimal deviation for the entrepreneur in the secondary credit market.

Let u^* denote the utility associated with the equilibrium allocation of the economy with exclusive contracts and no bankruptcy institution, u^{**} denote the utility associated with the equilibrium allocation of the economy with non-exclusive contracts and no bankruptcy institution, and u^+ the utility associated with the equilibrium allocation of the economy with non-exclusive contracts and a bankruptcy institution.

Proposition 5 *Suppose Assumptions 1-3 hold. Then the entrepreneur prefers the optimal contract with non-exclusive contracts and bankruptcy to the optimal contract with non-exclusive contracts and no bankruptcy; but he prefers the optimal contract with exclusive contracts and no bankruptcy to the optimal contract with a secondary credit market and bankruptcy:*

$$u^{**} < u^+ < u^*. \quad (17)$$

In the case in which the secondary credit market is active and exclusive contracts are not enforceable, the possibility of bankruptcy relaxes the incentive constraints. Bankruptcy allows the bank to monitor ex post the existence of a contractual relationship between the entrepreneur and the secondary credit market. The bank chooses a repayment in state L which has the property that (i.e. which is sufficiently high so that) the entrepreneur will

declare bankruptcy in that state. Since the bank debt is senior to the debt in the secondary credit market, the secondary lenders will not be repaid in that state. This restricts the set of contracts that the secondary lenders can offer by making non-negative profits to those that require full repayment in state H . As a consequence, the incentive constraints are relaxed and the optimal contract when bankruptcy is allowed for is preferred to the optimal contract when bankruptcy is not allowed for.

The institution of bankruptcy does not completely substitute for the enforceability of exclusive contracts. In fact, the repayment schedule imposed by the bank must be such that the entrepreneur has no incentive to choose effort e and supplement the contract offered by the bank with those offered by secondary lenders which require repayment in state H . Such a restriction can only be satisfied if the contract offered by the bank provides the entrepreneur with less insurance than provided by the optimal contract in the case in which a secondary credit market is not available (exclusive contracts are enforceable).

We have analyzed the problem with bankruptcy under the assumption that secondary lenders share the proceeds in bankruptcy equally. It is important however to notice that our main result, namely Proposition 5, is independent of that assumption. Clearly, given that the institution of bankruptcy is welfare improving taking our assumption about the sharing rule as given, it would be welfare improving if we were allowed to choose the sharing rule in the secondary markets. But, no matter what the sharing rule, there cannot be an equilibrium in which the existence of the secondary markets does not impose any constraints on the set of contracts which can be offered by the bank. If the bank offers the optimal exclusive contract, it is never an equilibrium for the secondary market to be inactive. Thus, the exclusive contract cannot be implemented no matter what the sharing rule in the secondary market. How to design the optimal sharing rule in the secondary market is an interesting open question.

Proposition 6 *Suppose Assumptions 1-3 hold and assume that contractual relationships are non-exclusive. Denote the optimal contract offered by the bank in the economy with bankruptcy by (I^+, B_H^+, B_L^+) . The optimal contract necessarily involves bankruptcy in state L only. Furthermore, a pure debt contract is optimal: The bank can choose the (notional) repayments so that $B_L^+ = B_H^+$.*

The fact that the optimal contract involves bankruptcy in state L has been established above. To see why a pure debt contract is optimal notice that by setting $B_L^+ \geq B_H^+$ the bank makes non-negative profit even if the entrepreneur does not declare bankruptcy in state L . Thus, the entrepreneur cannot profit from avoiding bankruptcy in state L . Setting B_L^+ equal to B_H^+ is hence sufficient. The fact that a pure debt contract is optimal is important since it reinforces our interpretation of the bankruptcy mechanism we construct as one which could in principle be implemented with courts which do not observe the realization of the state, thereby justifying the state independence of the provision P .

The possibility of bankruptcy, we argued, relaxes the incentive constraints. As a consequence, the optimal contract in this case provides more insurance to the entrepreneur. Furthermore, the amount of credit that the borrower gets from the bank under the optimal contract with bankruptcy exceeds the amount he gets if there is no bankruptcy institution.

Proposition 7 *Suppose Assumptions 1-3 hold and assume that contractual relationships are non-exclusive. Then:*

- (i) *If $u''' > 0$, then $c_0^+ > c_0^{**}$.*
- (ii) *$c_H^{**} - c_L^{**} > c_H^+ - c_L^+$ and $c_H^{**} > c_H^+$.*

We now characterize the equilibrium deviation, that is the best possible contract which can be offered in equilibrium in the secondary market, the solution to the maximization problem (13).

Proposition 8 *Suppose Assumptions 1-3 hold and assume that contractual relationships are non-exclusive. In the economy with bankruptcy, the equilibrium deviation of the entrepreneur with the secondary market satisfies $\gamma = \pi_e R_H > 0$.*

The proof is left to the reader. The entrepreneur thus considers taking out an additional loan in the secondary market. Again, this is in contrast to the standard result in the literature which implies that the agent would consider saving an extra amount. The intuition for our result is that since the debtor declares bankruptcy in the low state and additional assets are seized, there is no point in carrying extra resources into that state. The debtor is

hence considering deviations involving time 0 and the high state at time 1 only. Given that, he is tempted to borrow more against the good state at time 1.

4 Examples

In this section we compute a number of examples with the objective of clarifying the structure of contracts and allocations which arise in equilibrium in the different informational (exclusive vs. non-exclusive) and institutional (bankruptcy vs. no bankruptcy) environments analyzed in the previous sections.

Observable Effort. As a benchmark it is useful to notice that when effort is observable there is full insurance and the analytical solution is the following:

$$c_0 = c_H = c_L = \frac{1}{2}(\pi_E \omega_H + (1 - \pi_E) \omega_L - (1 - \alpha)) \quad (18)$$

and $I = \frac{1}{2}(\pi_E \omega_H + (1 - \pi_E) \omega_L + (1 - \alpha))$. The agent's consumption in each of the 2 periods is 1/2 of the net value of the agent's resources.

No Secondary Credit Market. When effort is not observable but there is neither a secondary credit market nor bankruptcy we can solve the problem analytically in the case of logarithmic utility. The solution is as follows:

$$c_0 = \frac{1}{2}(\pi_E \omega_H + (1 - \pi_E) \omega_L - (1 - \alpha)) \quad (19)$$

and $I = \frac{1}{2}(\pi_E \omega_H + (1 - \pi_E) \omega_L + (1 - \alpha))$, while

$$c_H = c_0 \left(1 + \frac{\pi_E - \pi_e}{\pi_E} \tau \right) \quad (20)$$

and

$$c_L = c_0 \left(1 - \frac{\pi_E - \pi_e}{1 - \pi_E} \tau \right), \quad (21)$$

where

$$\tau = \frac{\exp\left(\frac{v(E) - v(e)}{\pi_E - \pi_e}\right) - 1}{(\pi_E - \pi_e) \left(\frac{1}{\pi_E} + \frac{1}{1 - \pi_E} \exp\left(\frac{v(E) - v(e)}{\pi_E - \pi_e}\right) \right)}. \quad (22)$$

Table 1: Loan Size (I), Repayments (B_H and B_L) and Consumption (c_0 , c_H and c_L) depending on the Environment.

	No Secondary Credit Market	Secondary Credit Market	Secondary Credit Market with Bankruptcy	Riskless Borrowing and Lending
c_0	0.6250	0.6041	0.6271	0.6084
c_H	0.6932	0.7280	0.6911	0.7156
c_L	0.4204	0.3999	0.4184	0.4197
I	1.1250	1.1041	1.1271	1.1084
B_H	1.3068	1.2720	1.3089	1.2844
B_L	0.5796	0.6001	0.5816	0.5803

Thus, consumption at time 0 and the size of the loan coincide with the values under observable effort. So does the expected consumption at time 1.

Secondary Credit Market. When effort is not observable and there is a secondary credit market (but no bankruptcy) we do not have an analytical solution. We compute an example with logarithmic utility and the following choice of parameters: $\alpha = 1/2$, $\omega_H = 2$, $\omega_L = 1$, $\pi_E = 3/4$, $\pi_e = 1/4$, $v(E) = 1/4$ and $v(e) = 0$. The numerical solution for this case as well as for the case of no secondary credit market and the additional specifications of the environment discussed below are displayed in Table 1. Consumption at date 0 is lower than in the absence of a secondary credit market, which reflects the desire to save at the optimal contract in the absence of secondary lenders (see Rogerson (1985)).

In the case with a secondary credit market, the consumption under the equilibrium deviation at the optimal contract c is 0.5430 (see also Table 2). This would imply the following contract with the secondary lenders: $\gamma = -0.0611$, $R_H = 0.1850$ and $R_L = -0.1431$. Thus, the agent's net position would be such that the agent would lend to the secondary lenders (i.e., save) and get repaid only if the project is unsuccessful.

Secondary Credit Market with Bankruptcy. Suppose there is both a secondary credit market and bankruptcy. The optimal solution is displayed in Table 1. Notice that the consumption at date 0 is higher than in

Table 2: Equilibrium Deviation (γ , R_H and R_L) and Consumption given the Deviation (\hat{c}_0 , \hat{c}_H and \hat{c}_L) depending on the Environment.

	No Secondary Credit Market	Secondary Credit Market	Secondary Credit Market with Bankruptcy	Riskless Borrowing and Lending
\hat{c}_0	0.6250	0.5430	0.6399	0.5395
\hat{c}_H	0.6932	0.5430	0.6399	0.7845
\hat{c}_L	0.4204	0.5430	0.4184	0.4886
γ	n.a.	-0.0611	0.0128	-0.0689
R_H	n.a.	0.1850	0.0512	-0.0689
R_L	n.a.	-0.1431	n.a.	-0.0689

the absence of a secondary credit market (and *a fortiori* higher than in the case with a secondary credit market but no bankruptcy). This also means that the loan size is higher in this case and both repayments are larger. Consumption in both states at date 1 is hence lower than without a secondary credit market. On the other hand, the variability of consumption is reduced relative to the case with a secondary credit market but no bankruptcy. In fact, the variability is almost as low as in the absence of a secondary credit market.

The equilibrium deviation in this case is to *borrow* more from the secondary lenders and repay only if the output is high at date 1 (see Table 2). This is of course in contrast to the case of a secondary credit market without bankruptcy, where the best deviation of the agent implies *lending* to the creditors in the secondary market! The intuition for the result is that the agent is using the secondary credit market to smooth his consumption. In the absence of bankruptcy the agent sells claims on consumption in the high output state, buys claims on consumption in the low output state and sells claims on consumption at date 0 since the present value of his allocation given low effort is lower. With the institution of bankruptcy however the agent can be prevented from varying consumption in the low state which implies that smoothing is only possible between date 0 and the high output state. But $c_H > c_0$ implies that the agent wants to borrow from the high state. Hence, the equilibrium deviation given bankruptcy is to take out an additional loan from the secondary lenders that is repaid only if bankruptcy

is avoided. Secondary loans are thus risky and in fact riskier than the bank loan.

Riskless Borrowing and Lending. To facilitate the comparison to the classic result in the literature where agents have access to a riskless technology only, we compute the allocation under this assumption as well. The solution with riskless borrowing and lending lies between the solution without a secondary credit market and the solution with a secondary credit market but no bankruptcy (see Table 1). Under the equilibrium deviation (see Table 2) the agent *lends* at the riskless rate. The amount of lending exceeds the amount of lending in the case with a secondary credit market, but only by a relatively small amount. The agent wants to smooth consumption, but given access to riskless borrowing and lending only, additional savings transfer resources not just into the low output state but into the high output state as well. Hence, it is costly. Notice that the consumption implied by the equilibrium deviation is considerably different in this case.

5 Conclusion

We provide a rationale for the institution of bankruptcy, i.e., bankruptcy law, as opposed to contractual default provisions. We argue that in the absence of enforceability of exclusive contracts *ex ante*, the institution of bankruptcy enables (conditional) enforcement of exclusivity *ex post* and thus relaxes the constraints imposed by enforceability. We show, however, that while a bankruptcy institution improves on non-exclusive contractual relationships, it is not a perfect substitute for exclusivity. More generally, the paper studies a principal agent problem in which the agent can enter into “side contracts,” but such side contracting can be monitored. We show that the ability to monitor is valuable and characterize the implications for the optimal contract and the side contracts that an agent would consider. In particular, we show that in our model, in contrast to most of the literature, the agent would consider taking out additional loans.

The paper proceeds formally as if the level of bankruptcy protection is part of the contract and hence in a sense determines the optimal bankruptcy law. However, the essential difference between bankruptcy law and contractual default provisions is that the former applies to all contracts an agent

enters into. This distinction, of course, becomes important only when the contractual relationships of all parties in a contract are not observable, i.e., only in the absence of enforceability of exclusive contracts ex ante. This is in fact also the reason why bankruptcy provisions limit the constraints imposed by the lack of enforceability.

Indeed, one of the crucial features of bankruptcy law that needs to be explained is the fact that agents cannot “opt out” of bankruptcy law. The problem with the incomplete contracts or transaction costs explanations is that agents should typically be allowed to “opt out” of rules provided by the law if alternative rules are more appropriate. We argue that mandatory bankruptcy provisions can be explained when other contractual relationships of the parties in a contract are unobservable, i.e., when exclusivity is not enforceable. Specifically, we can interpret the possibility of state contingent repayments as a form of opting out of bankruptcy law ex ante. Notice that in our setup agents would not want to opt out of bankruptcy law and restrict themselves to state contingent repayments only.

Appendix

Proof of Proposition 1. First of all note that Assumptions 2-3 guarantee that the set of (I, B_H, B_L) which satisfy the set of constraints, equations (1-2), is non-empty. The constraint set is compact, the objective function continuous, and hence a solution exists. Letting μ and τ denote the Lagrange multiplier associated with equation (2) and (3), respectively, the necessary first order conditions of the problem are (2), (3) and

$$u'(\alpha + I - 1) = \mu \quad (23)$$

$$u'(\omega_H - B_H) \left(1 + \tau \frac{\pi_E - \pi_e}{\pi_E}\right) = \mu \quad (24)$$

$$u'(\omega_L - B_L) \left(1 - \tau \frac{\pi_E - \pi_e}{1 - \pi_E}\right) = \mu. \quad (25)$$

Since $u'(c)$ is decreasing, by concavity of $u(c)$, (23-25) readily imply that $c_H > c_L$.

Simple algebraic manipulation of the first order conditions, (23-25), implies:

$$\frac{1}{u'(c_0)} = \pi_E \frac{1}{u'(c_H)} + (1 - \pi_E) \frac{1}{u'(c_L)}.$$

Thus, expected consumption is decreasing if $1/u'$ is convex and increasing otherwise. The above equality also implies that $u'(c_0) < \pi_E u'(c_H) + (1 - \pi_E) u'(c_L)$. \square

Lemma 1 *Constraint (6) is more restrictive than (3). Also, it can be replaced in the optimal contracting problem by:*

$$u(\alpha + I - 1) + \pi_E u(\omega_H - B_H) + (1 - \pi_E) u(\omega_L - B_L) - v(E) \geq 2u(c) - v(e) \quad (26)$$

where

$$c := \frac{1}{2}(\alpha + I - 1 + \pi_e(\omega_H - B_H) + (1 - \pi_e)(\omega_L - B_L)). \quad (27)$$

Proof. Constraints (3) and (6) coincide if (6) is evaluated at $\gamma = R_H = R_L = 0$. Evaluating (6) at $\gamma = R_H = 0$, $R_L = -\frac{\pi_e R_H}{1 - \pi_e}$, one has that the derivative of the right hand side of (6), $u(\alpha + I + \gamma - 1) + \pi_e u(\omega_H - B_H -$

$R_H) + (1 - \pi_e)u(\omega_L - B_L - R_L) - v(e)$ with respect to R_H is positive, which proves that (6) is more restrictive than (3); and also that (8) is never binding.

Since (6) must hold, for all (γ, R_H, R_L) which satisfy (7-8), it is sufficient to impose it for (γ, R_H, R_L) which solve:

$$\max_{\gamma, R_H, R_L} u(\alpha + I + \gamma - 1) + \pi_e u(\omega_H - B_H - R_H) + (1 - \pi_e)u(\omega_L - B_L - R_L)$$

subject to (7-8). It is easy to see that the argmaximum of this problem is reached at (γ, R_H, R_L) which solve

$$\begin{aligned} \alpha + I + \gamma - 1 &= \omega_H - B_H - R_H = \omega_L - B_L - R_L \\ &= \frac{1}{2}(\alpha + I - 1 + \pi_e(\omega_H - B_H) + (1 - \pi_e)(\omega_L - B_L)) \end{aligned}$$

which proves the statement. \square

Proof of Proposition 2. Let μ and τ denote the Lagrange multipliers associated with equation (5) and (26), respectively. The first order conditions of the maximization problem are (5), (26), (27) and

$$u'(\alpha + I - 1) = \tilde{\mu} \tag{28}$$

$$u'(\omega_H - B_H) + u'(c)\tilde{\tau}\frac{\pi_E - \pi_e}{\pi_E} = \tilde{\mu} \tag{29}$$

$$u'(\omega_L - B_L) - u'(c)\tilde{\tau}\frac{\pi_E - \pi_e}{1 - \pi_E} = \tilde{\mu} \tag{30}$$

where

$$\tilde{\mu} \equiv \frac{\mu}{1 + \tau} + \frac{\tau}{1 + \tau}u'(c) \tag{31}$$

and $\tilde{\tau} \equiv \tau/(1 + \tau)$. As a consequence, $u'(\omega_H - B_H) < u'(\alpha + I - 1) < u'(\omega_L - B_L)$, and hence $\omega_H - B_H > \alpha + I - 1 > \omega_L - B_L$. Also, simply manipulating the first order conditions:

$$u'(\alpha + I - 1) = \pi_E u'(\omega_H - B_H) + (1 - \pi_E)u'(\omega_L - B_L). \tag{32}$$

The statement on precautionary savings now simply follows from (32). \square

Proof of Proposition 3. Part (i): The present (expected) value of consumption, $c_0 + \pi_E c_H + (1 - \pi_E)c_L$, is constant across the two cases (with

or without the secondary credit market operating). But, Proposition 1 (and Rogerson's (1985) result) imply that in the absence of a secondary credit market $c_0^* \geq \pi_E c_H^* + (1 - \pi_E) c_L^*$. On the other hand, Proposition 2 implies that when there is a secondary credit market $c_0^{**} < \pi_E c_H^{**} + (1 - \pi_E) c_L^{**}$. The consumption at time $t = 0$ must then be lower when there is a secondary credit market, and so is I .

Part (ii): The proof is in 3 steps. Step 1: We claim that $(c_H^{**}, c_L^{**}) \not\leq (c_H^*, c_L^*)$. Suppose instead that $(c_H^{**}, c_L^{**}) \leq (c_H^*, c_L^*)$ and hence $c_0^{**} \geq c_0^*$ since the present value of both consumption allocations has to be the same. But then

$$u'(c_0^*) \geq u'(c_0^{**}) = \pi_E u'(c_H^{**}) + (1 - \pi_E) u'(c_L^{**}) \geq \pi_E u'(c_H^*) + (1 - \pi_E) u'(c_L^*),$$

which contradicts Proposition 1. Step 2: In the case of no secondary credit market the incentive compatibility constraint is binding at an optimal solution and implies that

$$u(c_H^*) - u(c_L^*) = \frac{v(E) - v(e)}{\pi_E - \pi_e}.$$

When there is a secondary credit market, the incentive compatibility constraint evaluated at an optimal solution implies

$$\begin{aligned} & 1/2u(c_0^{**}) + \pi_E/2u(c_H^{**}) + (1 - \pi_E)/2u(c_L^{**}) - 1/2v(E) \\ & \geq u(1/2c_0^{**} + \pi_e/2c_H^{**} + (1 - \pi_e)/2c_L^{**}) - 1/2v(e) \\ & > 1/2u(c_0^*) + \pi_e/2u(c_H^*) + (1 - \pi_e)/2u(c_L^*) - 1/2v(e) \end{aligned}$$

and thus

$$u(c_H^{**}) - u(c_L^{**}) > \frac{v(E) - v(e)}{\pi_E - \pi_e} = u(c_H^*) - u(c_L^*). \quad (33)$$

Now suppose that $c_H^{**} \leq c_H^*$ and thus $c_L^{**} \geq c_L^*$ by step 1. This contradicts equation (33) and hence $c_H^{**} > c_H^*$, i.e., the second assertion of part (ii) is established. Step 3: $c_H^{**} > c_H^*$ together with equation (33) imply the first assertion. \square

Proof of Proposition 5. To show that $u^+ < u^*$ it suffices to show that

$$\frac{\partial \left(u(c_0^* + \gamma) + \pi_e u(c_H^* - \frac{\gamma}{\pi_e}) + (1 - \pi_e) u(c_L^*) \right)}{\partial \gamma} > 0$$

which we leave to the reader to check.

To show that $u^+ > u^{**}$, consider the optimal contracting problem with a secondary credit market and no bankruptcy. Lemma 1 and Proposition 2 imply that the incentive compatibility constraint of that problem is binding for some $\gamma > 0$, $R_H > 0$, $R_L < 0$. Take $P = c_L^{**}$, and $B_L > \omega_L - P$, $\pi_e B_H + (1 - \pi_e)B_L \geq I$, and B_H and I supporting c_0^{**} , c_H^{**} . As we argued, the incentive constraint of problem (14) requires $R_L = 0$, and hence is not binding at such a choice of (I, B_H, B_L, P) : $u^{**} > u^D(I, B_H, B_L, P)$. Hence there exists some feasible and incentive compatible contract (with $P > c_L^{**}$) which supports an entrepreneur's utility u^+ such that $u^+ > u^{**}$. \square

Proof of Proposition 7. We will first prove part (ii) and then part (i) of the proposition.

Part (ii): Ignoring the constraint that $\omega_H - B_H - R_H \geq P$ for now, we have the following explicit expression for u^D :

$$\begin{aligned} u^D(I, B_H, B_L, P) &= (1 + \pi_e)u\left(\frac{1}{1 + \pi_e}(\alpha + I - 1) + \frac{\pi_e}{1 + \pi_e}(\omega_H - B_H)\right) \\ &+ (1 - \pi_e)u(P) - v(e). \end{aligned}$$

The first order conditions for the problem with bankruptcy are thus:

$$\begin{aligned} (1 + \tau)u'(\alpha + I - 1) &= \mu + \tau u'(c) \\ (1 + \tau)u'(\omega_H - B_H) &= \mu + \tau \frac{\pi_e}{\pi_E} u'(c) \\ (1 + \tau)u'(P) &= \mu + \tau \frac{1 - \pi_e}{1 - \pi_E} u'(P) \end{aligned}$$

where $c = 1/(1 + \pi_e)(\alpha + I - 1) + \pi_e/(1 + \pi_e)(\omega_H - B_H)$ and μ and τ are the multipliers on the constraints. Clearly, $u'(\omega_H - B_H) < u'(\alpha + I - 1)$ and thus $u'(c) < u'(\alpha + I - 1)$. Hence, $u'(\alpha + I - 1) < (1 + \tau)u'(\alpha + I - 1) - \tau u'(c) = \mu$. But then the last first order condition implies that $u'(P) > \mu$ and thus $c_H^+ > c_0^+ > c_L^+$. Setting $u'(\alpha + I - 1)$ equal to a weighted sum of the second two first order conditions shows that

$$u'(c_0^+) < \pi_E u'(c_H^+) + (1 - \pi_E)u'(c_L^+).$$

An argument analogous to the one in the first step of the proof of Proposition 3 establishes that $(c_H^{**}, c_L^{**}) \not\leq (c_H^+, c_L^+)$.

The incentive compatibility constraint of the problem with bankruptcy implies

$$\begin{aligned} u(c_0^+) + \pi_E u(c_H^+) + (1 - \pi_E)u(c_L^+) - v(E) = \\ (1 + \pi_e)u(1/(1 + \pi_e)c_0^+ + \pi_e/(1 + \pi_e)c_H^+) + (1 - \pi_e)u(c_L^+) - v(e), \end{aligned}$$

and the incentive compatibility constraint of the problem without bankruptcy implies

$$\begin{aligned} u(c_0^{**}) + \pi_E u(c_H^{**}) + (1 - \pi_E)u(c_L^{**}) - v(E) = \\ 2u(1/2c_0^{**} + \pi_e/2c_H^{**} + (1 - \pi_e)/2c_L^{**}) - v(e) \\ > (1 + \pi_e)u(1/(1 + \pi_e)c_0^{**} + \pi_e/(1 + \pi_e)c_H^{**}) + (1 - \pi_e)u(c_L^{**}) - v(e), \end{aligned}$$

and thus

$$\begin{aligned} (1 + \pi_e)u(1/(1 + \pi_e)c_0^+ + \pi_e/(1 + \pi_e)c_H^+) + (1 - \pi_e)u(c_L^+) > \\ (1 + \pi_e)u(1/(1 + \pi_e)c_0^{**} + \pi_e/(1 + \pi_e)c_H^{**}) + (1 - \pi_e)u(c_L^{**}). \end{aligned} \quad (34)$$

Denote the present value of a consumption allocation given action a by PV_a . Recall that $PV_E^+ = PV_E^{**}$. Note that $PV_e = PV_E - (\pi_E - \pi_e)(c_H - c_L)$. Suppose that, by contradiction, $c_H^{**} - c_L^{**} < c_H^+ - c_L^+$ and hence $PV_e^{**} > PV_e^+$. Equation (34) then implies that

$$1/(1 + \pi_e)c_0^{**} + \pi_e/(1 + \pi_e)c_H^{**} > 1/(1 + \pi_e)c_0^+ + \pi_e/(1 + \pi_e)c_H^+ > c_L^+ > c_L^{**},$$

since otherwise a lottery with lower expected value would never be preferred. But then $c_H^{**} > c_H^+$, a contradiction.

Part (i): Recall that the present (expected) value of consumption conditional on the high effort is the same in the two cases and that

$$u'(c_0^{**}) = \pi_E u'(c_H^{**}) + (1 - \pi_E)u'(c_L^{**})$$

and

$$u'(c_0^+) < \pi_E u'(c_H^+) + (1 - \pi_E)u'(c_L^+).$$

If $u''' > 0$, then $c_0^{**} < \pi_E c_H^{**} + (1 - \pi_E)c_L^{**}$. Given part (ii) established above, $c_H^{**} - c_L^{**} > c_H^+ - c_L^+$ and $c_H^{**} > c_H^+$. Suppose $c_0^+ < c_0^{**}$ and hence $c_L^+ > c_L^{**}$. Then $\pi_E c_H^+ + (1 - \pi_E)c_L^+ > \pi_E c_H^{**} + (1 - \pi_E)c_L^{**}$ and thus

$$\begin{aligned} u'(c_0^{**}) &= \pi_E u'(c_H^{**}) + (1 - \pi_E)u'(c_L^{**}) \\ &> \pi_E u'(c_H^+) + (1 - \pi_E)u'(c_L^+) > u'(c_0^+). \end{aligned}$$

But this implies $c_0^+ > c_0^{**}$, a contradiction. \square

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