

Living with the Fear of Floating: An Optimal Policy Perspective¹

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Abstract

As documented in recent studies, developing countries (classified by the IMF as floaters or managed floaters) are extremely reluctant to allow for large nominal exchange rate fluctuations. This “fear of floating” is reflected in the fact that, in spite of being subject to larger shocks, developing countries exhibit lower exchange rate variability and higher reserve variability than developed countries. Moreover, there is a positive correlation between changes in the exchange rate and interest rates and a negative correlation between both changes in reserves and the exchange rate and changes in interest rates and reserves. We build a simple model that rationalizes these key features as the outcome of an optimal policy response to monetary shocks. The model incorporates three key frictions: an output cost of nominal exchange rate fluctuations, an output cost of higher interest rates to defend the currency, and a fixed cost of intervention.

1 Introduction

It has long been recognized that “pure” floating exchange rate regimes (defined as regimes in which the monetary authority does not intervene at all in foreign exchange markets) have rarely – if ever – existed in practice. More surprising, however, is the reluctance of developing countries (which claim to be floaters) to let the nominal exchange rate fluctuate in response to shocks, as convincingly documented by Calvo and Reinhart (2000).¹ To assess this phenomenon, consider, as a benchmark for a relatively pure floater, the cases of United States and Japan. As indicated in Table 1, the probability that the monthly variation in the nominal exchange rate falls within a plus/minus 2.5 percent band is 58.7 percent for the United States and 61.2 percent for Japan. In contrast, for developing countries classified by the IMF as free floaters (FL) or managed floaters (MF), the average probability is 77.4 percent. This is even the more remarkable considering that one would conjecture that developing countries are subject to larger and more frequent shocks.² Thus, the revealed preference for smoothing out exchange rate fluctuations – or “fear of floating” – is nothing short of remarkable.

How do emerging countries smooth out exchange rate fluctuations in practice? Not surprisingly, they do so by actively intervening in foreign exchange markets and engaging in an active interest rate defense of the currency. Again, for the United States and Japan, the probability that the monthly variation in international reserves falls within a plus/minus 2.5 percent band is 62.2 percent and 74.3 percent, respectively. The corresponding average for developing countries is 35 percent, indicating a much larger variability in

¹See also Sturzenegger and Levy (2000).

²As evidenced, for instance, by the fact that real output is between two and two and half times more volatile in developing countries than in the G-7 countries, while real private consumption is between three and four times more volatile (as shown by Talvi and Végh (2000) based on Hodrick-Prescott filtered data for 1970-1994).

international reserves. In the same vein, the probability that the monthly variation in nominal interest falls in a plus/minus 25 basis points band is 59.7 percent for the United States and 67.9 percent for Japan. The corresponding figure for emerging countries is 28.4 percent, suggesting a much more active interest rate defense of weakening currencies in developing countries.

In addition, based on contemporaneous correlations among residuals from a VAR analysis for individual episodes, Calvo and Reinhart (2000) conclude that in most instances, (i) the correlation between the exchange rate and interest rates is positive, (ii) the correlation between reserves and the exchange rate is negative, and (iii) the correlation between interest rate changes and reserves changes is negative. All three correlations seems to be consistent with the overall story told by Table 1.

This paper starts from the presumption that the policies just described reflect an optimal policy response to underlying shocks.³ In this light, this extreme “fear of floating” is puzzling since, even if nominal exchange rate fluctuations were costly, one would expect a monotonic relationship between nominal exchange rate variability and the size of the underlying shock (i.e., the larger are shocks, the larger is nominal exchange rate variability). At best, costly exchange rate fluctuations would explain a departure from a pure floating but not the fact that countries subject to larger shocks have less volatile exchange rates, as suggested by the data.. The theoretical challenge is thus to build a simple model which allows for an explicit welfare evaluation of alternative policies and analyze whether the optimal policy in the model roughly replicates the observed policies. This paper represents a first effort on our part to tackle this important question.⁴ We develop a simple theo-

³We consider the main alternative hypothesis (irrational policymakers) to be, by and large, factually wrong and theoretically uninteresting (as we do not have good theories of irrationality).

⁴Naturally, the choice of how much to intervene and/or raise interest rates in response to a negative shock that tends to weaken the domestic currency is related to the optimal

retical model in which, in response to monetary shocks, the optimal policy response replicates most of the key policy facts just described.⁵ In particular, the model predicts that the variability of the nominal exchange rate is a non-monotonic function of the underlying shock (i.e., for small shocks, the nominal exchange rate is an increasing function of the shock but, for large shocks, the nominal exchange rate is fully stabilized).

What are the main ingredients of our model? In the model, the “fear of floating” stems from the fact that exchange rate variability leads to output costs. In the presence of nominal wage rigidities, changes in the exchange rate lead to changes in the actual real wage, which in turn leads to “voluntary unemployment” (to use Barro and Grossman’s (1971) terminology) if the real wage falls below its equilibrium value or “involuntary unemployment” if the real wage rises above its equilibrium level. (Notice that exchange rate variability is costly regardless of whether the exchange rate depreciates or appreciates.)⁶ We model active interest rate defense of the currency along the lines of Calvo and Végh (1995) by assuming that it basically entails paying interest on some interest-bearing liquid asset.⁷ As in Lahiri and Végh

choice of exchange rate regimes. An important literature in the 1980’s emphasized the fact that the choice was not limited to fixed versus fully flexible exchange rates but rather entailed a choice of the optimal degree of foreign exchange market intervention (with fixed and flexible rates just being the extreme cases), as best captured by the classic contribution of Aizenman and Frenkel (1985).

⁵For analytical simplicity, we only focus on monetary shocks. As indicated in Table 1, monetary aggregates are much more volatile in developing countries, which is consistent with the idea that monetary shocks are larger.

⁶We should stress that this is just a convenient analytical way of capturing costs of exchange rate fluctuations. In practice, there may be other (and possibly more important) sources of costly exchange rate fluctuations (see Calvo and Reinhart (2000b)). Our focus is on analyzing the resulting optimal policy mix and *not* on providing sophisticated microfoundations for the cost of exchange rate fluctuations.

⁷This paper is therefore related to an incipient theoretical literature that analyzes the active use of interest rates to defend an exchange rate peg (see Drazen (1999a,b), Flood

(2000b), we incorporate into the model an output cost of raising interest rates. Hence, in our model, higher interest rates raise the demand for domestic liquid assets but at the cost of a fall in output. Finally, we assume that there is a fixed (social) cost of intervening in foreign exchange markets. While not explicitly modeled, we view this cost as capturing a fixed cost of portfolio adjustment for the private sector when it has to deal with the central bank (in the spirit of asset market segmentation stories a la Alvarez, Atkeson, and Kehoe (2000)).

In the context of such a model, consider a negative shock to real money demand. If the shock is small, the output costs entailed by the resulting depreciation of the exchange rate will also be small. It is thus optimal for policymakers not to intervene and let the exchange rate depreciate. Since exchange rate fluctuations are costly, however, it is optimal for policymakers to partially offset the shock to money demand by raising domestic interest rates. Hence, for small shocks to money demand, the exchange rate and domestic interest rates move in the same direction, while reserves do not change.

If the shock is large (i.e., above a well-defined threshold), the output costs resulting from exchange rate fluctuations would be too large relative to the cost of intervening. It thus become optimal to intervene and stabilize the exchange rate completely. Furthermore, since there are no output costs of exchange rate volatility, it is not optimal to raise domestic interest rates to prop up the currency. Hence, for large negative shocks, international reserves fall but the exchange rate and domestic interest rates do not change.

If we think of the real world as involving a sequence of monetary shocks (with developed countries facing mostly small shocks and emerging countries facing mostly large shocks), the model would predict the following. First, from a cross sectional point of view, (i) developing countries should exhibit

and Garber (2000) and Lahiri and Végh (2000a,b)).

low exchange rate variability and high reserve variability; and (ii) conversely, developed countries should exhibit high exchange rate variability and low reserve variability. Second, from a time-series point of view (i.e., in individual countries), (iii) the correlation between exchange rates and interest rates should be positive; (iv) the correlation between the exchange rate and reserves should be negative; and (v) the correlation between interest rates and reserves should be negative.⁸ The model thus captures some of the main features of the data described above and should therefore provide a useful conceptual framework to think about policy responses in a world in which policymakers live with the fear of floating (i.e., nominal exchange rate fluctuations are costly).

The paper proceeds as follows. Section 2 develops the model under flexible wages. Section 3 introduces sticky wages into the picture. Section 4 analyzes the optimal policy mix under costless intervention. Section 5 derives the main results of the paper. Section 6 concludes.

2 The Model

Consider an infinitely-lived, representative household model of a small open economy. The economy consumes and produces two goods – y and x – both of which are freely traded. The economy takes the world price of the two goods as given and the law of one price is assumed to hold for both goods. The foreign currency price of good y is assumed to be constant and, for convenience, normalized to unity. The world relative price of good x in terms of good y is p which is also assumed to be constant over time. The economy has access to perfectly competitive world capital markets where it

⁸Our model is not stochastic, so this characterization of the predictions is just based on the co-movement of variables in the model. A stochastic simulation of the model is left for future research.

can borrow and lend freely in terms of good y at the constant world interest rate r .

2.1 Households

The representative household derives utility from consuming the two goods and disutility from supplying labor. The lifetime welfare of the household is given by

$$W \equiv \int_0^{\infty} \frac{1}{1 - 1/\sigma} \{ [c_t - \zeta (l_t^s)^\nu]^{1-1/\sigma} - 1 \} e^{-\beta t} dt, \quad \sigma > 0, \quad \zeta > 0, \quad \nu > 1, \quad (1)$$

where

$$c = (c^y)^\rho (c^x)^{1-\rho} \quad (2)$$

is a consumption composite index.⁹ We use l_t^s to denote labor supply. Throughout the paper we maintain a notational distinction between labor supply and labor demand since, in the presence of nominal wage rigidities, labor supply will not necessarily equal labor demand at all times. In order to rule out secular consumption dynamics, we make the standard assumption that $\beta = r$ (i.e., the rate of time preference equals the world real rate of interest).

The household's flow budget constraint in terms of good y (or foreign currency) is given by

$$\dot{a}_t = ra_t + w_t l_t^s + \Omega_t^y + \Omega_t^x + \Omega_t^b + \tau_t - c_t^y - pc_t^x - I_t^d h_t - v(\hat{h}_t; \alpha), \quad (3)$$

where w denotes the wage rate in foreign currency (henceforth referred to as the real wage), h are demand deposits in terms of the foreign currency and

⁹We adopt these preferences for analytical convenience, as they imply that the labor supply decision becomes independent of wealth. Moreover, Correia, Neves, and Rebelo (1995) have shown that these preferences provide a better description of current account dynamics for small open economies than standard CES preferences.

$I^d \equiv i - i^d$ is the deposit spread with i^d being the interest paid on deposits. Ω^y and Ω^x are dividends received from firms in sectors y and x respectively, while Ω^b are dividends from commercial banks. Also, τ denotes lump sum transfers from the government to households, while $a = b + h$ represents net household assets in terms of foreign currency (or, equivalently, in terms of good y).

Real demand deposits held by the household are denoted by $\hat{h} = \frac{H}{P}$, where H denotes nominal demand deposits and P is the domestic currency price index of the composite consumption good c . Transactions costs incurred by the household are denoted by $v(\hat{h}; \alpha)$, where $\alpha > 0$ is a positive constant. As is standard, we assume that the function $v(\hat{h}; \alpha)$ is strictly convex in \hat{h} so that $v_{\hat{h}} < 0$ and $v_{\hat{h}\hat{h}} > 0$. Thus, the household can reduce transactions costs by holding more demand deposits in terms of the composite consumption good. The parameter $\alpha (> 0)$ is a shift parameter for money demand. In particular, we assume that $v_{\alpha} < 0$ and $v_{\hat{h}\alpha} < 0$. As will be clear below, this implies that money demand, \hat{h} , is an increasing function of the parameter α . Lastly, we will assume that v is a social cost, so that it will enter in the country's resource constraint.

Given (2), it is easy to establish that the domestic currency price index is given by

$$P = \frac{p^{1-\rho}}{\rho^\rho(1-\rho)^{1-\rho}} E \equiv \frac{E}{B}, \quad (4)$$

where E denotes the nominal exchange rate (domestic currency price of the foreign currency), while $B = \frac{\rho^\rho(1-\rho)^{1-\rho}}{p^{1-\rho}}$ is a positive constant.¹⁰ Since $h = H/E$, equation (4) implies that $\hat{h} = Bh$. Hence, transactions costs are given by $v(\hat{h}; \alpha) = v(Bh; \alpha)$. Since the relative price p is constant over time, it is also easy to see from (4) that the rate of inflation in this economy must equal the rate of currency depreciation at all points in time. Hence, we must have

¹⁰ P is the consumption-based price index, which is defined as the minimum expenditure required to purchase one unit of the composite consumption index $(c^y)^\rho (c^s)^{1-\rho}$.

$\frac{\dot{P}}{P} = \frac{\dot{E}}{E} = \varepsilon$. Perfect capital mobility then implies that the nominal interest rate must be given by $i = r + \varepsilon$.

Integrating the household's flow constraint subject to the transversality condition on a gives

$$a_0 + \int_0^\infty (w_t l_t^s + \Omega_t^y + \Omega_t^x + \Omega_t^b + \tau_t) e^{-rt} dt = \int_0^\infty [c_t^y + p c_t^x + I_t^d h_t + v(\hat{h}_t; \alpha)] e^{-rt} dt. \quad (5)$$

To simplify the analysis, it will be assumed that the transactions cost technology is quadratic. Formally,

$$v(\hat{h}, \alpha) = \hat{h}^2 - \alpha \hat{h} + \kappa, \quad \hat{h} \in [0, \frac{\alpha}{2}], \quad (6)$$

where α and κ are positive constants.

The household chooses time paths for c^y, c^x, l^s and \hat{h} to maximize (1) subject to (5) taking as given a_0 and the paths for $w, \tau, r, p, I^d, \Omega^f, \Omega^y$ and Ω^b . The first-order conditions for utility maximization imply that:

$$\rho c_t [c_t - \zeta (l_t^s)^\nu]^{-1/\sigma} = \lambda c_t^y, \quad (7)$$

$$(1 - \rho) c_t [c_t - \zeta (l_t^s)^\nu]^{-1/\sigma} = p \lambda c_t^x, \quad (8)$$

$$\nu \zeta (l_t^s)^{\nu-1} = B w_t, \quad (9)$$

$$\alpha - 2\hat{h} = \frac{I^d}{B}. \quad (10)$$

Equations (7) – (10) can be used to derive the following relationships:

$$\frac{1 - \rho}{\rho} \frac{c_t^y}{c_t^x} = p, \quad (11)$$

$$l_t^s = \left(\frac{B w_t}{\nu \zeta} \right)^{\frac{1}{\nu-1}}, \quad (12)$$

$$Bh_t = \hat{h}_t = \frac{\alpha}{2} - \frac{I^d}{2B}. \quad (13)$$

Equation (11) says that the marginal rate of consumption substitution between the two goods must equal their relative price. Equation (12) shows that households' labor supply is an increasing function of the real wage. Lastly, equation (13) says that real money demand in terms of good y must be falling in the cost of holding deposits I^d . Also, for a given I^d , a higher α implies that h must go up. Hence, the parameter α can be thought of as a shock to money demand.

2.2 Firms

Since there are two distinct sectors in this economy, there are two types of firms – those that produce good y and those that produce good x . Both sectors are assumed to be perfectly competitive.

2.2.1 Sector y firms

The industry producing good y is characterized by perfectly competitive firms which hire labor to produce the good using the technology

$$y_t = (l_t^d)^\eta, \quad \eta \in (0, 1], \quad (14)$$

where l^d denotes labor demand. Firms may hold foreign bonds, b^y . Thus, the flow constraint faced by the firm is

$$\dot{b}_t^y = rb_t^y + (l_t^d)^\eta - w_t l_t^d - \Omega_t^y. \quad (15)$$

Integrating forward equation (15), imposing the standard transversality condition, and using equation (??) yields

$$\int_0^\infty e^{-rt} \Omega_t^y dt = b_0^y + \int_0^\infty [(l_t^d)^\eta - w_t l_t^d] e^{-rt} dt. \quad (16)$$

The firm chooses a path of l^d to maximize the present discounted value of dividends, which is given by the right hand side of equation (16) taking as given the paths for w_t, I_t^l, r and the initial stock of financial assets b_0^f . The first order condition for this problem is given by

$$\eta(l_t^d)^{\eta-1} = w_t. \quad (17)$$

Equation (17) yields the firm's demand for labor:

$$l_t^d = \left(\frac{w_t}{\eta} \right)^{\frac{1}{\eta-1}}, \quad (18)$$

which shows that labor demand by firms is decreasing in the real wage (recall that $0 < \eta \leq 1$).

One should note that in the case of a linear production function (i.e., $\eta = 1$), the first order condition for profit maximization (equation (17)) reduces to

$$w_t = 1.$$

Hence, the labor demand schedule in this case is given by

$$l_t^d|_{\eta=1} = \begin{cases} 0 & \text{for } w_t > 1 \\ \infty & \text{for } w_t < 1 \end{cases} \quad (19)$$

In other words, labor demand is zero for any real wage above one and “infinite” for any real wage below one.

2.2.2 Sector x firms

Sector x is also characterized by perfectly competitive firms which produce good x . Firms in this sector use an imported input q to produce good x , according to the technology given by

$$x_t = q_t^\theta, \quad \theta \in (0, 1), \quad (20)$$

where q denotes the imported input. The world relative price of q in terms of good y is p^q which is assumed to be constant. To economize on notation and with no loss of generality, we assume $p^q = 1$. Sector- x firms are, however, dependent on bank loans for their working capital needs. In particular, we assume that firms face a credit-in-advance constraint to pay for the imported input:

$$n_t \geq \psi q_t, \quad \psi > 0, \quad (21)$$

where n denotes loans from commercial banks. This constraint introduces a demand for bank loans and, hence, a credit channel into the model. As is well known, this constraint will hold as an equality along all paths where the cost of loans, I^l , is positive. (In addition, we will also assume that it holds as an equality if $I^l = 0$.)

These firms may hold foreign bonds, b^x . Hence, the real financial wealth of the representative firm at time t is given by $a_t^x = b_t^x - n_t$. Using i^l to denote the lending rate charged by banks and letting $I^l \equiv i^l - i$ denote the lending spread, we can write the flow constraint faced by the firm as

$$\dot{a}_t^x = r a_t^x + p x_t - q_t - I_t^l n_t - \Omega_t^x. \quad (22)$$

Integrating forward equation (22), imposing the standard transversality condition, and using equations (20) and (21) yields

$$\int_0^\infty e^{-rt} \Omega_t^x dt = a_0^x + \int_0^\infty [p q_t^\theta - q_t (1 + \psi I_t^l)] e^{-rt} dt. \quad (23)$$

Note that the proportion of imports which have to be paid for using bank credit introduces an extra cost of inputs to the firm, given by ψI^l .

The firm chooses a path of q to maximize the present discounted value of dividends, which is given by the right hand side of equation (23) taking as given the paths for I_t^l , r and the initial stock of financial assets a_0^x . The representative firm's first order condition for profit maximization is

$$p \theta q_t^{\theta-1} = 1 + \psi I_t^l. \quad (24)$$

Equation (24) implies that the demand for bank loans is given by

$$q_t = \left(\frac{p\theta}{1 + \psi I_t^l} \right)^{\frac{1}{1-\theta}}. \quad (25)$$

Hence, the firm's demand for the imported input is decreasing in the lending spread. This captures the credit channel in our model. Lastly, the loan demand by sector- x firms can be determined from equation (25) as

$$n_t = \psi \left(\frac{p\theta}{1 + \psi I_t^l} \right)^{\frac{1}{1-\theta}}. \quad (26)$$

For later reference, it is useful to note $\frac{\partial n}{\partial I^l} < 0$ and $\frac{\partial^2 n}{\partial (I^l)^2} > 0$. Moreover, input demand q is also a decreasing and convex function of I^l .

2.3 Banks

The economy is assumed to have a perfectly competitive banking sector. We formalize the banking sector along the lines of Lahiri and Végh (2000b). The representative bank accepts deposits from consumers, lends to sector- x firms (n), and holds domestic government bonds (z^b).¹¹ The bank charges an interest rate of i^l to firms and earns i^g on government bonds. It also holds required cash reserves, m (high powered money). The bank pays depositors an interest rate of i^d . Thus, the balance sheet identity of the bank implies that $m_t + n_t + z_t^b = h_t$.¹²

¹¹Commercial bank lending to governments is particularly common in developing countries. Government debt is held not only as compulsory (and remunerated) reserve requirements but also voluntarily due to the lack of profitable investment opportunities in crisis-prone countries. This phenomenon was so pervasive in some Latin America countries during the 1980's that Rodriguez (1991) aptly refers to such governments as "borrowers of first resort". For evidence, also see Druck and Garibaldi (2000).

¹²Similar results would go through if we allowed banks to hold foreign bonds in world capital markets as long as banks face a cost of managing domestic assets (along the lines

Letting $I^g = i^g - i$ denote the interest rate spread from lending to the government, the flow constraint of the representative bank is

$$\Omega_t^b = I_t^l n_t + I_t^d h_t + I_t^g z_t^b - i_t m_t. \quad (27)$$

Note that since required reserves are non-interest bearing, the opportunity cost of holding required reserves for banks is the foregone nominal interest rate i . Lastly, we assume that the central bank imposes a reserve-requirement ratio $\delta > 0$. Since required reserves do not earn interest, at an optimum the bank will not hold any excess reserves. Hence, we must have

$$m_t = \delta h_t. \quad (28)$$

Thus, the representative commercial bank's balance sheet identity can be written as

$$(1 - \delta)h_t = n_t + z_t^b. \quad (29)$$

The bank maximizes profits by choosing sequences of n_t , z_t^b , h_t and m_t subject to equations (28) and (29) taking as given the paths of I^l , I^d , I^g , δ and i . The first order conditions for the banks' optimization problem are (assuming an interior solution)

$$(1 - \delta) I_t^l + I_t^d = \delta i_t, \quad (30)$$

$$(1 - \delta) I_t^g + I_t^d = \delta i_t. \quad (31)$$

Conditions (30) and (31) simply say that, at an optimum, the representative bank equates the marginal cost of deposits (RHS) to the marginal revenue

of Edwards and Végh (1997), Burnside, Eichenbaum, and Rebelo (1999), or Agenor and Aizenman (1999). Put differently – and as is well-known – some friction needs to exist at the banking level for banks to play a non-trivial role in the credit-transmission mechanism. We chose the specification with no foreign borrowing because it is analytically simpler. Moreover, it is not a bad description of commercial banking in most developing countries.

from an extra unit of deposits (LHS). Note that the marginal revenue from an additional unit of deposits has two components. The first, given by I_t^d , is due to the fact that borrowing from consumers is cheaper for banks (whenever $I_t^d > 0$) than borrowing in the open market. The second, given by either $(1 - \delta) I_t^l$ or $(1 - \delta) I_t^g$, captures the fact that banks can lend a fraction $1 - \delta$ of each additional unit of deposits to either firms or to the government.

Equations (30) and (31) imply that we must have

$$I_t^g = I_t^l. \quad (32)$$

This also implies that $i^l = i^g$, i.e., the lending rate to firms must equal the interest rate on government bonds. Intuitively, loans and government bonds are perfect substitutes in the bank's asset portfolio. Since the bank can get i^g by lending to the government, it must receive at least as much from firms in order to extend loans to them.

From equation (31), it is also easy to see that the deposit spread, I^d , is given by

$$I_t^d = i_t - (1 - \delta)i_t^g. \quad (33)$$

Since $I^d = i - i^d$, it follows immediately that we must have $i_t^d = (1 - \delta)i_t^g$ for all t . Thus, a rise in the domestic interest rate i^g must result in a higher deposit rate for consumers and, hence, an increase in demand deposits. Since i^g may be controlled by policymakers, the preceding shows that interest rate policy in this model effectively amounts to the government being able to pay interest on money.

Lastly, we will restrict attention to parameter ranges for which I^d and I^l are non-negative. Thus, we will confine attention to environments where $i^d \leq i \leq i^g$. This restriction is needed to ensure a determinate demand for both loans and demand deposits. Note that this amounts to restricting the relevant interest rates to be in the range $0 \leq i^g - i \leq \delta i^g$.

2.4 Government

The government in this economy is composed of the fiscal authority and the monetary authority (i.e., the central bank). The fiscal authority makes lump-sum transfers τ to the public and issues domestic bonds (Z), which are held either by the monetary authority or the commercial banks. Domestic bonds are interest bearing and pay i^g per unit. The monetary authority in this economy issues high powered money $m (= \delta h)$, holds real government debt z^g , and sets the reserve requirement ratio δ on deposits. The central bank also holds foreign exchange reserves R which bear the world rate of interest r . Thus, the *consolidated* government's flow budget constraint is given by

$$\dot{R}_t = rR_t + \dot{m}_t + \dot{z}_t^b + \varepsilon_t m_t + (\varepsilon_t - i_t^g) z_t^b - \tau_t. \quad (34)$$

where we have used the fact that the government's net liability to the private sector is $z^b = z - z^g$ (and z denotes the real stock of domestic bonds).

The central bank balance sheet identity (in terms of foreign currency) is given by

$$R_t + z_t - z_t^b = m_t. \quad (35)$$

Note that $z^g = z - z^b$ is the monetary authority's real domestic credit to the public sector. We assume that the fiscal authority keeps the *nominal* stock of outstanding government debt fixed at \bar{Z} .¹³ Hence,

$$\frac{\dot{Z}_t}{Z_t} \equiv \mu_t = 0, \quad Z_0 = \bar{Z}. \quad (36)$$

Substituting the central bank balance sheet in equation (34) and using (36) yields

$$\tau_t = rR_t + \varepsilon_t(m_t - z_t) + (\varepsilon_t - i_t^g) z_t^b. \quad (37)$$

¹³This is the natural assumption to make given that we will abstract from fiscal considerations and focus only on stationary equilibria involving constant nominal variables.

In effect, the lump sum transfers to the representative consumer consist of the inflation tax from issuing cash (net of real net domestic debt, z) and the inflation tax (*net* of interest payments) on government bonds held by the private sector (i.e., the banking system).

In this model, policymakers may choose to use i^g as a policy instrument. In that case, and for analytical convenience, we will think of I^g as the policy instrument (recall that, by definition, $I^g = i^g - i$). Given that, as shown below, $i_t = r$ for all t in this economy, the central bank can always set an i^g to implement the desired value of I^g . We shall also assume that the government lets fiscal transfers τ adjust endogenously so that equation (37) is satisfied.

It is useful at this stage to restate the two key effects of interest rate policy in the model. First, since government bonds and bank credit to firms are perfect substitutes in the banks' portfolio, a higher interest rate on government bonds leads to an increase in the lending rate. This reduces bank credit and causes an output contraction (see equations (25) and (32)). This effect will be referred to as the *output effect* of interest rate policy. Second, the higher interest rate on government bonds induces banks to also pay a higher rate on bank deposits (recall that $i^d = (1 - \delta)i^g$) and, as a result, increases the demand for bank deposits. We will refer to this as the *money demand effect*.

2.5 Resource constraint

By combining the flow constraints for the consumer, the firms in sector- x and sector- y , the bank and the government (equations (3), (15), (22), (27) and (34)) we get the economy's flow resource constraint:

$$\dot{k}_t = rk_t + y_t + px_t - c_t^y - pc_t^x - q_t - v(Bh_t; \alpha). \quad (38)$$

where $k = R + b + b^y + b^x$. Note that the right hand side of equation (38) is simply the economy's current account. Integrating forward subject to the

No-Ponzi game condition gives

$$k_0 + \int_0^{\infty} [y_t + px_t - c_t^y - pc_t^x - q_t - v(Bh_t; \alpha)]e^{-rt} dt = 0. \quad (39)$$

2.6 Policy regimes

Before proceeding to define the different policy regimes in this economy, notice that the rate of devaluation/depreciation (ε) will always be zero in this stationary economy. Under a fixed exchange rate, this is trivially true. Under a floating regime, this follows from the fact that (as shown below), real money balances will be constant along a perfect foresight equilibrium path.

In this economy, policymakers have, in principle, four different policy instruments: the exchange rate (E), international reserves (R), the domestic interest rate (I^g), and nominal domestic credit (Z^g). Only two of these four instruments, however, can be chosen independently. For any two instruments controlled by the central bank, the other two will adjust endogenously. To see this, consider the following equations that are the relevant ones for monetary policy purposes:

$$R + z^g = \delta h, \quad (40)$$

$$\bar{Z} = Ez^b + Z^g, \quad (41)$$

$$n + z^b = (1 - \delta)h, \quad (42)$$

where n is a function of I^g through the loan demand equation (26) while h is a function of I^d through the money demand equation (13), and I^g and I^d are linked through equation (31) (recall that $\varepsilon = 0$). Equation (40) is the central bank's balance sheet, equation (41) is the equilibrium condition in the government bond market, and equation (42) is the commercial bank's balance sheet. Equations (40)-(42) thus define a system of 3 equations in

5 unknowns (E , R , z^b , Z^g , and I^g). This implies that there are 2 policy variables that can be set by policymakers

For the purposes of the subsequent analysis, we can therefore define the following policy regimes:

1. **Fixed exchange rate:** policymakers fix E at a certain level and set Z^g . Both international reserves and I^g adjust endogenously.¹⁴ This regime is intended to capture a hard peg (à la Argentina or Hong Kong) in which the monetary authority maintains a constant backing of the monetary base with international reserves and thus forgoes completely active monetary policy (i.e., the monetary authority lets I^g be determined by market forces).
2. **Pure floating:** policymakers fix R at a certain level and set I^g . Both the exchange rate and Z^g adjust endogenously. This regime is intended to capture a floating regime in which policymakers actively engage in monetary policy by setting domestic interest rates.
3. **Dirty floating:** policymakers set R (and may change it in response to shocks) as well as I^g , while E and Z^g adjust endogenously.
4. **Fully sterilized intervention:** policymakers target a constant level of h and set the level of z^g (real domestic credit). In this case, both reserves and the exchange rate adjust endogenously.

2.7 Flexible wages equilibrium

We now characterize the perfect foresight equilibrium path (PFEP) for this economy under flexible wages and floating exchange rates (either the pure

¹⁴We will also refer to this case below as the “full intervention” case, since the central bank keeps the exchange rate fixed by intervening in the foreign exchange market.

floating or the dirty floating regimes, as defined above). In other words, we will assume that the central bank chooses to keep the time path of international reserves piecewise constant (i.e., $\frac{\dot{R}}{R} = 0$). However, we will allow the central bank to undertake *discrete* interventions in the foreign exchange market (dirty floating).

The labor market clearing condition dictates that labor demand must equal labor supply, i.e., $l_t^s = l_t^d$. Imposing this condition on equations (12), (17) and (26) yields the equilibrium labor and real wage for this economy (equilibrium values are denoted with a bar):

$$\bar{l} \equiv \left(\frac{B\eta}{\zeta\nu} \right)^{\frac{1}{\nu-\eta}}, \quad (43)$$

$$\bar{w} = \eta \left(\frac{B\eta}{\zeta\nu} \right)^{\frac{\eta-1}{\nu-\eta}}. \quad (44)$$

In other words, along a PFEP, both employment and the real wage are constant.

Next, notice that the evolution of real high powered money in the economy is given by $\frac{\dot{m}}{m} = \frac{\dot{M}}{M} - \varepsilon$ where M denotes nominal money and where we have used the fact that, by definition, $m = \frac{M}{E}$. Time-differentiating the central bank balance sheet identity to derive an expression for \dot{M}/M and substituting it in the expression for $\frac{\dot{m}}{m}$ derived above yields

$$\dot{m}_t = (r - i^d - B\alpha + 2B^2 h_t) z_t, \quad (45)$$

where $z = \bar{Z}/E$. Note that in deriving (45) we have used the household's first order condition for demand deposits, equation (10), to substitute out for ε . Equation (45) shows that the steady state level of demand deposits, \bar{h} , is given by

$$\bar{h} = \frac{1}{B} \left(\frac{\alpha}{2} - \frac{r - i^d}{B} \right). \quad (46)$$

It follows that the steady state level of h is increasing in both i^d and α .

It is easy to check that (45) is an unstable differential equation around the steady state.¹⁵ Hence, m ($= \delta h$) must always be equal to its steady-state level, \bar{m} , along a perfect foresight equilibrium path. In other words, the policy-controlled i^g implies a certain i^d , which determines the constant level of real money demand. This determines, through (42), the level of z^b (recall that n is also determined by i^g). For this level of z^b and a given R_0 , equations (40) and (41) determine the constant level of the exchange rate:

$$\bar{E} = \frac{\bar{Z}}{\bar{m} - R_0 + \bar{z}^b}, \quad (47)$$

where $\bar{m} = \delta \bar{h}$. Equation (47) shows that policymakers have two avenues for influencing the exchange rate. First, for a given R_0 , they can use interest rate policy to affect I^d and I^l . This will influence \bar{m} and \bar{z}^b directly and, hence, change E . Second, for a given \bar{m} and \bar{z}^b , they can intervene in the foreign exchange market and alter the level of R_0 and, hence, E . The determination of the optimal mix of these two policies is an issue that we will come back to later.

In order to determine steady state consumption, notice that equation (11) implies that the ratio $\frac{c^x}{c^y}$ is a constant. Hence, $\frac{c}{c^y}$ must also be constant. This, combined with the first order condition for consumption and the fact that the equilibrium level of employment \bar{l} is constant, implies that c^y , c^x , and c must all be constant. The country resource constraint then implies that the constant levels of consumption of the two goods are:

$$\bar{c}^y = \rho \left[rk_0 + \bar{l}^n + p \left(\frac{p\theta}{1 + \psi I^l} \right)^{\frac{\theta}{1-\theta}} - \left(\frac{p\theta}{1 + \psi I^l} \right)^{\frac{1}{1-\theta}} - v(Bh_t; \alpha) \right], \quad (48)$$

¹⁵Notice that since, by assumption, i^g is kept constant over time, i^d will also be constant over time.

$$p\bar{c}^x = (1 - \rho) \left[rk_0 + \bar{l}^n + p \left(\frac{p\theta}{1 + \psi I^l} \right)^{\frac{\theta}{1-\theta}} - \left(\frac{p\theta}{1 + \psi I^l} \right)^{\frac{1}{1-\theta}} - v(Bh_t; \alpha) \right] \quad (49)$$

2.8 Money demand shocks under flexible wages

As a benchmark case, consider an unanticipated and permanent fall in α (i.e., a negative money demand shock) under a pure floating rate and flexible wages. Since real money demand decreases, the nominal exchange rate depreciates instantaneously to accommodate the lower real money demand (see equation (47)). Furthermore, the nominal wage rises by the same proportion as the exchange rate. Thus, with an unchanged interest rate policy, the real side of the economy remains completely insulated. Consumption of both goods falls since the equilibrium level of transactions cost rises. Note that under a fixed exchange rate (i.e., full intervention), the economy would also adjust instantaneously as the central bank intervenes in the foreign exchange market (by selling international reserves), thus accommodating the fall in real money demand.

3 Nominal Wage Rigidities

3.1 Perfect foresight equilibrium path under flexible exchange rates

We now depart from the flexible wages paradigm by introducing a nominal wage rigidity into the model. We will examine first the case of flexible exchange rates.¹⁶ We assume that nominal wages cannot jump at any point in time. Hence, the labor market clearing condition $l^d = l^s = \bar{l}$ does not necessarily hold at all points in time. In particular, it is assumed that nominal

¹⁶As will become clear below, in the fixed exchange rate case, the real sector remains insulated from monetary shocks.

wages, W , adjust according to the following dynamic equation:

$$\dot{W}_t = \gamma(\bar{w} - \frac{W_t}{E_t}), \quad W_0 \text{ given}, \quad (50)$$

where $\gamma \in (0, \infty)$ captures the speed of adjustment towards the equilibrium real wage \bar{w} . Recall that \bar{w} is given by equation (44). The implication of introducing sticky nominal wages (as shown below) is that an exchange rate depreciation will now lead to a fall in the real wage and cause a temporary labor market disequilibrium and concomitant output losses in sector- y .

Using the previously shown result that along any PFEP with flexible exchange rates, E must already be at its steady state value \bar{E} at time $t = 0$, one can solve equation (50) to get

$$w_t = \bar{w} + e^{-\frac{\gamma}{\bar{E}}t} (w_0 - \bar{w}), \quad (51)$$

where $w_t = W_t/\bar{E}$ and $w_0 = W_0/\bar{E}$. As is easily checked, $\lim_{t \rightarrow \infty} w_t = \bar{w}$. Moreover, $\dot{w}_t \gtrless 0$ for $w_t \lesseqgtr \bar{w}$. Lastly, the equilibrium nominal wage is given by $\bar{W} = \bar{w}\bar{E}$.

As in standard disequilibrium models, it will be assumed that actual employment is given by the short end of the market. In other words, when the real wage is below (above) its equilibrium value, actual labor is determined by labor supply (demand). Notice that this disequilibrium model implies that only one of the two labor optimality conditions will hold. If the real wage is below its equilibrium value, the household's labor condition (equation (9)) will hold but the firm's (equation (18)) will not. Conversely, if the real wage is above its equilibrium value, the firm's first order condition will hold but the household's will not.

There are two potential cases of disequilibrium. For $w_0 < w_t < \bar{w}$ we have $l_t^a = l_t^s = \left(\frac{Bw_t}{\nu\zeta}\right)^{\frac{1}{\nu-1}}$. Substituting in for w_t from equation (51) and

simplifying the result yields the path for actual employment:

$$l_t^a = \bar{l} \left[1 + \left(\frac{w_0}{\bar{w}} - 1 \right) e^{-\frac{\gamma}{E}t} \right]^{\frac{1}{\nu-1}}, \quad (52)$$

with $l_0^a = \left(\frac{Bw_0}{\nu\zeta} \right)^{\frac{1}{\nu-1}}$. Analogously, for the case $w_0 > w_t > \bar{w}$ we have $l_t^a = l_t^d = \left(\frac{w_t}{\eta} \right)^{\frac{1}{\eta-1}}$. The path for actual employment is now given by

$$l_t^a = \bar{l} \left[1 + \left(\frac{w_0}{\bar{w}} - 1 \right) e^{-\frac{\gamma}{E}t} \right]^{-\frac{1}{1-\eta}}, \quad (53)$$

with $l_0^a = \left(\frac{w_0}{\eta} \right)^{\frac{1}{\eta-1}}$. Substituting these relations into equation (??) yields the time path of output of good y for each case.

It useful to note that, in both cases, $l^a < \bar{l}$ throughout the transition. Intuitively, any deviation of the real wage from its equilibrium value implies that the short end of the labor market determines actual employment. In the case of an unanticipated increase in the real wage, labor demand falls while labor supply goes up (relative to the equilibrium). Since labor demand is the short side of the market, actual employment equals labor demand. Hence, output of sector- y falls. Conversely, when the real wage is below the equilibrium, labor supply is smaller while labor demand is greater relative to the equilibrium. In this event, actual employment is supply-determined. Hence, employment falls and output of sector- y declines.¹⁷

This result is key to understanding the real effects of exchange rate fluctuations within this model. It implies that exchange rate appreciations and depreciations are *both* contractionary. This result stands in stark contrast to the standard Mundell-Fleming model with rigid prices in which depreciations are expansionary while appreciations are contractionary. The difference arises because the standard models in the Mundell-Fleming tradition postulate output to be demand-determined with demand being a function of the

¹⁷This case is exactly what Barro and Grossman (1971) called “voluntary unemployment” in their analysis of disequilibrium models.

real exchange rate. As this model shows, introduction of an explicit supply-side alters the implications quite dramatically.

The consumption dynamics along the adjustment path can be determined directly from the employment dynamics. Noting that λ is constant along a PFEP and c^x/c^y and c/c^y are both constants at all times, one can differentiate the first order condition (7) with respect to time to get:

$$\dot{c}_t = \zeta \nu (l_t^a)^{\nu-1} \dot{l}_t^a > 0, \quad (54)$$

which says that consumption rises along with employment during the transition. There is a unique time path of consumption that satisfies (54) and the intertemporal resource constraint. Given the paths for c and l^a , the values of c_0 and l_0^a would then determine the value of the multiplier through the first order condition (7). Clearly, welfare should be lower than it would be under flexible wages (and floating rates) because either firms in sector- y are forced to accept a path for labor that does not satisfy their first-order condition (17) or the first order condition for households, equation (9), is violated.¹⁸

3.2 The Menu of Policy Options

We can now describe the economy's response to a negative money demand shock (i.e., a fall in α) in the presence of sticky wages under the four policy regimes defined above. (Table 2 summarizes the outcome under these four different options.) Notice that, on the monetary side, the economy will always adjust instantaneously to this shock. On the real side, sector- x output will always adjust instantaneously as well. On the other hand, sector- y output will adjust gradually over time if the exchange rate deviates from its initial steady-state, along the lines described above.

¹⁸Notice that an important advantage of this framework over a model with demand-determined output is that welfare analysis in our model is well-defined.

1. Fixed exchange rate

Under a fixed exchange rate policymakers respond to the shock by keeping E and Z^g unchanged. Hence, real domestic credit, z^g , also remains unchanged. From (41), it follows that z^b will not change either. Since the negative money demand shock reduces real demand for deposits, the commercial bank's balance sheet (equation (42)) implies that loans, n , must fall. But this can only occur through a rise in I^g . In the new equilibrium, the fall in real money demand is smaller than the initial shock because the rise in the domestic interest rate partially offsets the money demand shock. International reserves decline endogenously to accommodate the lower level of base money.

Intuitively, the initial fall in real demand deposit induces a fall in the demand for government bonds by commercial banks. At unchanged levels of central bank holdings of government bonds z^g and the nominal exchange rate E , this implies an excess supply of government bonds. The central bank responds to this by raising domestic interest rates since this makes domestic bonds and demand deposits more attractive to the private sector.

On the real side, sector- y output remains unchanged at its equilibrium level. Since the exchange rate is fixed, the actual wage will not deviate from the equilibrium real wage, and there will be no disequilibrium in the labor market. In contrast, higher domestic interest rates extract an output cost in sector- x as banking credit becomes more expensive and banking lending falls. In addition, the fall in real money balances implies an increase in transaction costs.

2. Pure float

Under a pure float, policymakers respond to the negative money demand shock by keeping international reserves, R , and the domestic

interest rate, I^g , unchanged, while allowing the exchange rate and domestic credit to adjust endogenously. An unchanged domestic interest rate implies that base money falls by the full amount of the shock. Since R is unchanged, real domestic credit, z^g , must fall to accommodate the shock. The fall in demand deposits along with an unchanged lending rate (and hence loan demand) implies that the demand for government bonds by commercial banks, z^b , falls. The excess supply of government bonds implies that its price $1/E$ falls, i.e., the nominal exchange rate depreciates.

Under the pure float case, sector- x remains completely insulated from the shock since the domestic interest rate remains unchanged. However, the exchange rate depreciation implies a fall in the real wage. Hence, the labor market goes into disequilibrium on impact and returns to the steady state asymptotically, as shown by (51) and (52). Hence, output of sector- y remains below the steady state level throughout the adjustment period. Moreover, the policy also implies a contraction in real deposits, and, hence, higher transactions costs and lower consumption.

3. Dirty floating

Under a dirty floating, policymakers intervene in the foreign exchange market (by selling international reserves) to achieve a smaller fall in the exchange rate (i.e., a smaller depreciation) than under the pure floating case. Specifically, suppose that policymakers reduce R so as to maintain the stock of real domestic credit unchanged. Then, since I^g does not change, it follows from (42) that z^b will fall. This, in turn, implies from (41) that E rises. Notice that this rise in E will be less than in the pure floating case described above. The reason is that z^b falls by the same amount in either case, whereas z^g falls under a pure float but does not change under dirty floating. From (41), it follows

that E will rise by less.

Intuitively, starting from the pure floating case described above, policymakers intervene in foreign exchange markets by selling international reserves. Since the domestic interest rate is kept unchanged, the lower stock of international reserves will be reflected in a higher stock of real domestic credit. This implies that, at the level of the exchange rate prevailing under a pure floating, there is an excess demand for government bonds. Hence, their price ($1/E$) must increase, which implies that E must fall (relative to the pure floating case). The outcome is that the exchange rate depreciates by less than it does in the pure floating case, while international reserves fall.

Since the exchange rate depreciates by less under a dirty floating, the output losses in sector- y will be lower than under a pure floating. There are no output costs in sector x .

4. Fully sterilized intervention

In our definition, the case of a fully sterilized intervention corresponds to keeping the level of real money demand unchanged and targeting a higher level of real domestic credit.¹⁹ In this scenario, the domestic interest rate, the level of international reserves, and the exchange rate will adjust endogenously. For real demand deposits to remain unchanged, equation (13) implies that $\frac{\alpha}{2} - \frac{I^d}{2B}$ must remain unchanged. Hence, in response to a fall in α , I^d must fall. From equation (31), a fall in I^d implies a rise in I^g . Hence, loans n must fall while commercial bank holdings of government debt z^b rise by an offsetting amount. Since, by construction, z^g has gone up, the nominal exchange rate must

¹⁹Naturally, this scenario assumes that the initial level of \hat{h} is still technologically feasible after the shock. (Recall from (6) that the transactions technology imposes the restriction that $\hat{h} \leq \frac{\alpha}{2}$).

appreciate. International reserves fall one-to-one with the increase in real domestic credit.

Intuitively, under a fully sterilized intervention, the central bank reacts to a negative money demand shock by increasing domestic credit through a purchase of government bonds while raising the domestic interest rate in order to keep money demand unchanged. The resultant increase in the lending rate causes sector- x firms to reduce their loan demand. Commercial banks react to the lower demand for loans by increasing their demand for government bonds. Hence, the total demand for government bonds rises. Since the nominal supply of these bonds is fixed, the price of these bonds, $1/E$, must rise. Hence, the nominal exchange rate appreciates. The final outcome is a change in the composition of the central bank assets (lower international reserves and higher real domestic credit) with no change in the level.

While this policy succeeds in insulating domestic money demand from the negative shock (which implies that transaction costs fall by less than they would otherwise), the insulation comes at the expense of higher domestic interest rates and an exchange rate appreciation. The higher interest rate causes an output contraction in sector- x . The exchange rate appreciation, on the other hand, induces a rise in the real wage (recall that nominal wages are rigid) and a fall in labor demand and sector- y output.

In the next section we will show that the optimal policy mix (when intervention is costless) implies that neither of these extreme cases is optimal. The optimal policy mix will fall somewhere in the “interior” of these pure cases.

3.3 Real versus monetary shocks

We conclude this section by noting that this model reproduces the standard Mundell-Fleming results regarding the optimal exchange rate regime under fixed and flexible rates. Under fixed exchange rates, sticky wages do not make any difference to the adjustment path of the economy. The economy adjusts instantaneously under both flexible and sticky wages, as the central bank buys and sells reserves to keep E unchanged. Thus, relative to flexible exchange rates, fixed exchange rates are better for insulating the real side of the economy from monetary shocks.

To think about real shocks in this model, consider a shock to p – the relative price of good x . This shock changes the equilibrium real wage and, hence, requires a change in the market real wage. Under flexible rates, this would happen instantaneously through a change in the nominal exchange rate. Under fixed exchange rates and rigid nominal wages, the economy cannot adjust instantaneously because neither the nominal wage nor the nominal exchange rate can jump. The economy returns to the long run equilibrium only through a slow adjustment of the nominal wage which is accompanied by an output contraction in sector- y (unless, of course, there is a policy change in the exchange rate). Thus, for the purposes of insulating the real side of the economy from real shocks, flexible exchange rates are better than fixed exchange rates.

4 Optimal Stabilization Policy

Having described the adjustment of the economy to money demand shocks under different policy regimes, we now turn to the issue of the optimal policy response to such shocks. ²⁰For the purposes of solving for the optimal

²⁰For analytical convenience, this section solves for the optimal policy in the absence of the fixed cost of intervention (a key feature of the model to be introduced in the next

policy response to a monetary shock, we will view policymakers as optimally choosing the domestic interest rate, I^g , and the level of international reserves (which, if different from the pre-shock level, implies a discrete one-shot intervention in the foreign exchange market). It is clear from equations (40)-(42) that an optimal choice of I^g and R will imply a unique choice of E , Z^g , and h . Specifically, notice that, for a given α , a choice of I^g implies a unique level of h through (42). Given this level of h and the choice of R , equation (40) determines a unique level of z^g . Equation (41) then determines a unique level of E which, given z^g , will in turn determine the optimal level of Z^g .

To study the optimal policy response, start from a steady state with $\alpha = 1$, $E = \bar{E}$, $m = \bar{m}$ and $R = \bar{R}$, and consider an unanticipated fall in α at time $t = 0$. The policymaker's goal is to choose I_0^g and R_0 to maximize the welfare of the representative agent. Solving the optimal policy problem gets greatly simplified due to the following proposition:

Proposition 1 *Given any choice of I_0^g by the policymaker in response to an α shock, it can never be optimal for the central bank to choose an R_0 such that $E_0 \neq \bar{E}$.*

Proof. Recall that any $E_0 \neq \bar{E}$ implies that the market real wage $W_0/E_0 \neq \bar{w} = W_0/\bar{E}$ where \bar{w} is the equilibrium real wage. Hence, output and employment must fall on impact and then rise gradually back towards the long run steady state. The central bank can always choose R_0 such that $\bar{m} - \bar{R} + \bar{z}^b = m_0 - R_0 + z_0^b$. Such a choice of R_0 would imply that $E_0 = \bar{E}$, which would leave the labor market completely unaffected and hence, output of sector- y unchanged. Moreover, output of sector- x is independent of the size of the intervention. Since intervention is costless from the perspective of the country as a whole, country wealth is unaffected by the size of intervention (a larger R just corresponds to a lower private foreign bond holdings,

section). The next section solves for the optimal policy problem in the complete model.

b , leaving k unchanged). Hence, this particular choice of R_0 dominates any other post-shock choice of reserves. ■

Since the only source of dynamics in the model is the labor market friction, the preceding implies that, under costless intervention, the economy has to remain stationary along an optimal policy plan. In other words, the policymaker will respond to a monetary shock by always keeping the nominal exchange rate unchanged so as to insulate the economy from any labor market frictions. The problem thus reduces to one of choosing optimal real money balances in a stationary economy through an appropriate choice of I^g . Once m (and hence, z^b) is chosen, the optimal intervention involves choosing an R_0 such that $E_0 = \bar{E}$.

Note that the proposition 1 immediately eliminates the option of allowing the entire (or some of the) adjustment to occur through an adjustment of the nominal exchange rate from the set of optimal policies. Hence, it is already clear that neither the pure float nor the dirty floating regimes considered above will prove to be the optimal policy response.

The stationarity of the economy implies that lifetime welfare of the representative household is given by

$$W = \frac{1}{r \left(1 - \frac{1}{\sigma}\right)} \left[\{c - \zeta \bar{l}^\nu\}^{1 - \frac{1}{\sigma}} - 1 \right],$$

where we have used the fact that the policymaker will ensure that the labor market will always be in equilibrium. Hence, $l^a = \bar{l}$. For a stationary economy the country resource constraint given by equation (39) implies that

$$c^y + pc^x = rk_0 + \bar{l}^\eta + pq^\theta - q - \left(\hat{h}^2 - \alpha \hat{h} + \kappa\right).$$

Moreover, the first order conditions for consumption imply that $c^y + pc^x = c^y/\rho$ and $c/B = c^y/\rho$ where $B = \frac{\rho^\rho(1-\rho)^{1-\rho}}{p^{1-\rho}}$. Hence, the country budget constraint reduces to

$$c = B \left[rk_0 + \bar{l}^\eta + pq^\theta - q - \left(\hat{h}^2 - \alpha \hat{h} + \kappa\right) \right].$$

Since $c - \zeta l^\nu$ is constant along any perfect foresight equilibrium path while W is monotonically rising in $c - \zeta l^\nu$, the policymaker's problem reduces to choosing I^g , $I^g \in [0, \frac{\delta}{1-\delta}r]$, to maximize $c - \zeta \bar{l}^\nu (\equiv \hat{W}^{peg})$ subject to equations (13), (25) and (31) for a given α , k_0 and \bar{l} . Note that welfare in this case corresponds to welfare under a fixed exchange rate.

The country resource constraint implies that

$$\hat{W}^{peg} = Brk_0 + B\bar{l}^\eta - \zeta \bar{l}^\nu + B(pq^\theta - q) - B(\hat{h}^2 - \alpha \hat{h} + \kappa). \quad (55)$$

Differentiating \hat{W}^{peg} with respect to I^g gives

$$\frac{d\hat{W}^{peg}}{dI^g} \equiv \Gamma = -\frac{B\psi(p\theta)^{\frac{1}{1-\theta}}\psi I^g}{(1-\theta)} \left(\frac{1}{1+\psi I^g}\right)^{1+\frac{1}{1-\theta}} + \frac{(1-\delta)}{2B}I^d, \quad (56)$$

where we have used equation (32) to get $I^l = I^g$. In the following we shall use I_{peg}^g to denote the optimal value of I^g in the case where the policymaker keeps the exchange rate pegged at all times. I_{peg}^g is defined by the relation $\Gamma|_{I_{peg}^g} = 0$. It is easy to check from equation (56) that $\Gamma|_{I^g=0} > 0$ and $\Gamma|_{I^g=\frac{\delta}{1-\delta}r} < 0$. Hence, $I_{peg}^g \in (0, \frac{\delta}{1-\delta}r)$. In other words, the optimal domestic interest rate lies strictly in the interior of the permissible range. Note that $I^g = \frac{\delta}{1-\delta}r$ corresponds to $I^d = 0$ which is equivalent to implementing the Friedman rule within this model.

Equation (56) clearly shows the two key margins over which the policymaker chooses the optimal I^g . First, a higher I^g implies that I^l goes up. Hence, the cost of funds for sector- x firms goes up which implies that output (net of the import bill), and, hence, consumption falls. This effect is captured by the first term on the right hand side of (56). However, a higher I^g also implies a higher deposit rate for depositors and hence, a lower opportunity cost of holding deposits, I^d . This causes money demand to go up which, in turn, reduces transactions costs and thereby increases consumption. This is the positive money demand effect of higher domestic interest rates which is captured by the second term on the right hand side of (56). Note that the

Friedman rule ($I^d = 0$) emerges as the optimum when $\psi = 0$. When $\psi = 0$, higher lending rates do not have any output effect since firms do not rely on bank credit at all. Hence, it is optimal to raise the domestic interest rate all the way to $I^g = \frac{\delta}{1-\delta}r$ which implies that $I^d = 0$, thus achieving the lowest possible level of transaction costs.

For I_{peg}^g to be an optimum, we also need to ensure that the second order condition for a maximum is satisfied. The condition $\psi r \frac{\delta}{1-\delta} < 1 - \theta$ is sufficient (but not necessary) for the second order condition for the government's welfare maximization problem to be satisfied. Moreover, this condition implies that \hat{W}^{peg} is globally concave in I^g and, hence, the optimal solution, I_{peg}^g , is unique. We omit a detailed statement of the proof since it follows simply from differentiating Γ with respect I^g . In what follows we shall restrict attention to parameter ranges for which the second order condition is satisfied.

Of key interest to us is the behavior of the optimal domestic interest rate with respect to α . In particular,

$$\frac{\partial I_{peg}^g}{\partial \alpha} = -\frac{\partial \Gamma / \partial \alpha}{\partial \Gamma / \partial I^g} = 0,$$

as follows from the fact that $\partial \Gamma / \partial I^g < 0$ from the second order condition for welfare maximization, while $\partial \Gamma / \partial \alpha = 0$.

Proposition 2 *The optimal domestic interest rate policy, I_{peg}^g , is independent of the money demand parameter α . Hence, along any optimal policy plan, a negative (positive) money demand shock (fall in α) leaves I_{peg}^g unchanged.*

This proposition says that a social welfare maximizing policymaker, who keeps the exchange rate fixed by freely intervening in the foreign exchange market, should not alter the domestic interest rate in response to money demand shocks. Intuitively, at an optimum, the marginal benefit in terms of

reducing transactions costs, given by the last term on the RHS of equation (56), is independent of α . There is therefore no reason for the optimal domestic interest rate to change. Since $\frac{I^d}{2B} = \frac{\alpha}{2} - \hat{h}$, a change in α just induces a corresponding parallel shift up or down in money demand but leaves unchanged the marginal benefit of changing the domestic interest. Hence, both \hat{h} and h fall in response to a negative money demand shock.

The preceding analysis is sufficient for us to tie down the behavior of all the endogenous variables in the model in response to a money demand shock. We summarize them in the following proposition:

Proposition 3 *Along an optimal policy path with costless intervention, an unexpected fall (rise) in α causes real money balances to fall (rise). The central bank responds to the shock by intervening in the foreign exchange market by selling (buying) international reserves in order to keep the nominal exchange rate unchanged at the pre-shock level. The domestic interest rate I^9 is kept unchanged. The unchanged domestic interest rate and exchange rate implies that output of both sectors remains unaffected. Furthermore, real domestic credit increases (falls) while international reserves fall (increase).*

Notice how, in response to a negative money demand shock, the optimal policy response involves elements of the four regimes described above. Specifically, the fall in real money demand is accommodated fully through foreign exchange market intervention (i.e., selling reserves), without letting the exchange rate change, as would happen under a fixed exchange rate. In addition, the domestic interest rate is kept unchanged (i.e., monetary policy is not tightened in response to the shock) as occurs under either a pure or dirty float. Finally, the optimal response involves an increase in real domestic credit as would occur if policymakers were attempting to (partially) sterilize the fall in real money demand. We should note that the existence of nominal wage rigidities is key for generating this insulation. Absent a

nominal wage rigidity, exchange rate fluctuations are costless. In that event, the central bank has no incentive to intervene, which implies that a free float is optimal.^{21,22}

Having this (costless intervention) case as an important analytical benchmark, the next section completes the specification of the general model by incorporating costly intervention.

5 Costly Interventions

This section completes the specification of the general model by incorporating costly intervention and derives the optimal policy response in such a case. We proceed in two steps. We first study the optimal policy contingent on no intervention and then contingent on intervention and we then tackle the question of when it will be optimal for policymakers to intervene or not.

5.1 Optimal policy under intervention

As before, we analyze the effects of a negative money demand shock. In particular, starting from a steady state with $\alpha = 1$, we study the effects of

²¹Within the context of the model, without nominal wage rigidities the policymaker is essentially indifferent between intervening and not intervening since allocations are independent of the level of the exchange rate. However, even an infinitesimal cost of intervening would imply that the optimal policy would be a free float and no foreign exchange market interventions at all.

²²We should note that the strict independence of the optimal interest rate policy from the money demand shock is due to the quadratic transactions cost technology. In a more general set-up for transactions costs, say $\alpha v(\hat{h})$, it is easy to show that the optimal response to a negative money demand shock is to raise the domestic interest rate to partially offset the effect of the shock on money demand. Hence, optimal policy, in general, would entail a mix of interest rate and intervention policy in order to insulate the nominal exchange rate from nominal shocks.

an unanticipated fall in α . To simplify notation and without loss of generality, we also choose initial conditions such that $\bar{E} = 1, \bar{R} = 0$. For $\bar{R} = 0$ this corresponds to an initial situation where $\bar{Z} = h - n$. To see this one can rewrite the central bank balance sheet as $E = \frac{\bar{Z}}{z^b + m - R}$. But the commercial bank balance sheet implies that $z^b = h - m - n$. Hence, for $R = 0$, the expression for E reduces to

$$E = \frac{\bar{Z}}{h - n}. \quad (57)$$

Lastly, we also assume that in the initial steady state the domestic interest rate policy is given by the solution to the optimal policy problem under costless intervention. Hence, $\bar{I}^g = I_{peg}^g$ while n and h are given by their corresponding levels under I_{peg}^g .

For simplicity, we assume that the central bank incurs a fixed cost $\phi > 0$ in the event that it intervenes in the foreign exchange market. Moreover, this fixed cost is symmetric, i.e., it applies to both increasing or decreasing the stock of reserves²³. Clearly, if $\phi = 0$, the model reduces to the one analyzed earlier where the optimal response is to fully insulate the exchange rate from all money shocks. Under this general specification, the resource constraint for the economy now becomes:

$$k_0 - \phi + \int_0^\infty \left[(l_t^a)^\eta + pq_t^\theta - c_t^y - pc_t^x - q_t - (\hat{h}^2 - \alpha \hat{h} + \kappa) \right] e^{-rt} dt = 0. \quad (58)$$

It is useful to start by noting that since the intervention cost is fixed and independent of the size of intervention, there can only be two potential outcomes to the policymaker's problem. Either the monetary authority will

²³We take this fixed cost to be a highly heuristic representation of a world with asset market segmentation in which agents need to pay a fixed cost to transfer money between the goods market and the asset market, along the lines of Alvarez, Atkeson, and Kehoe (1999). Formalizing this channel is far from trivial (since the main exercise involves performing comparative statics for the optimal policy) and is left for future research.

pay the fixed cost of intervention and intervene by the full amount necessary to keep the nominal exchange rate unchanged, or it will not intervene at all.²⁴

In the event that the policymaker intervenes, optimal policy will coincide with that under costless intervention (which was derived above). This follows from the fact that the cost of intervention is independent of the size of intervention. Hence, none of the marginal conditions for optimal policy are affected. The policymaker would thus respond to a negative money demand shock by keeping the domestic interest rate and the nominal exchange rate unchanged. Money demand would fall by the full amount of the shock while the central bank would sell reserves by the full amount necessary to keep the nominal exchange rate unchanged. Hence, under full intervention, output of both sectors remains completely invariant to changes in α .

Noting that welfare of the representative household in this event is proportional to $\hat{W}^I \equiv c - \zeta \bar{l}^\nu$, we have

$$\hat{W}^I = Brk_0 + B\bar{l}^\eta - \zeta \bar{l}^\nu + B(pq^\theta - q) - B(\hat{h}^2 - \alpha \hat{h} + \kappa) - Br\phi \quad (59)$$

where we have used the resource constraint (equation (58)) to substitute out for c . The only difference between this expression and the right hand side of equation (55) is the cost of intervention term, $Br\phi$, which is independent of all endogenous variables. This establishes our assertion that, if it is optimal to intervene, optimal policy in this instance must coincide with optimal policy under the costless intervention case. For future reference it is also useful to note that in this case,

$$\frac{dW^I}{d\alpha} = \frac{B\alpha}{2} > 0, \quad (60)$$

where we have used the first order condition (10), the money demand equa-

²⁴Note that partial intervention would imply that the exchange rate must change which, in turn, would imply output losses in sector- y . Since these losses are costless to avoid through an appropriate intervention (the fixed cost implies that the marginal intervention is costless), partial intervention can never be an optimal policy choice.

tion (13), as well as the fact that the optimal domestic interest rate is independent of α . Hence, the smaller the money demand parameter α (i.e., the bigger the shock), the lower is welfare.

5.2 Optimal policy under no intervention

If the policymaker chooses not to pay the cost of intervention, then there will be no intervention at all. Hence, reserves will remain unchanged in response to the money demand shock. In this event, there emerges a role for interest rate policy for domestic macroeconomic management. To see this, consider the case where the central bank reacts to the negative money demand shock by not only not intervening but also by keeping domestic interest rates unchanged. Money demand falls by the full amount of the shock while domestic loans (and, hence, sector- x output) remain unchanged. With an unchanged nominal stock of government bonds, \bar{Z} , equation (57) implies that the nominal exchange rate must depreciate in proportion to the fall in demand deposits h . The nominal depreciation along with the nominal wage rigidity implies a fall in the real wage. This causes a contraction of labor supply and output of sector- y which returns to the steady state level only asymptotically.

Now suppose that the central bank raised the domestic interest rate marginally in response to the shock. This would lower loans while reducing the fall in money demand. From equation (57) it is easy to see that relative to the previous case of unchanged interest rates, the nominal exchange rate depreciation induced by the shock must be smaller. Hence, the fall in the real wage must be lower which, in turn, implies a smaller contraction of sector- y . Of course, this benefit comes at the cost of an output contraction in sector- x since the credit cost of the imported input is greater. It is clear though that, at the margin, the policymaker should be trading off these two margins in

determining the optimal domestic interest rate.

To formalize the government's problem, notice that welfare is proportional to $c - \zeta(l^a)^\nu (\equiv W^{NI})$, where we have used the fact that, for negative money demand shocks, actual employment must equal labor supply, i.e., $l^a = l^s$. In the event of no intervention, the country resource constraint implies that

$$\hat{W}^{NI} = Brk_0 + B(pq^\theta - q) - B(\hat{h}^2 - \alpha\hat{h} + \kappa) + rY, \quad (61)$$

where $Y = \int_{t=0}^{\infty} (B(l_t^s)^\eta - \zeta(l_t^s)^\nu) e^{-rt} dt$ is the present discounted value of sector- y output net of the disutility from labor supply.

At this stage it is easy to see that the optimal policy problem becomes more complicated than in the costless intervention case. In the event of an exchange rate depreciation the nominal wage rigidity implies that the economy displays intrinsic output dynamics since the labor market goes into disequilibrium. In order to make analytical progress, we thus simplify the model by setting $\eta = 1$ and $\nu = 2$. Effectively, these parameter values make the production function of sector- y linear in labor while also making the marginal disutility from labor supply linear in labor. This simplification allows us to derive closed-form analytical solutions even in the presence of intrinsic output dynamics.

Linear production in sector- y implies that the equilibrium real wage in this economy is unity, i.e., $\bar{w} = 1$. Since the initial value of the nominal exchange rate is unity, this implies that the nominal wage in the initial steady state must be unity as well. Hence, on impact, the real wage in this economy is given by $w_0 = E_0^{-1} < 1$. Moreover, under negative money demand shocks we are restricting attention to $w_0 \in [0, 1]$. From equation (57) it follows that

$$w_0 = \frac{h - n}{\bar{Z}}. \quad (62)$$

Since $h - n$ is increasing in I^g it is obvious that the initial real wage is an increasing function of the domestic interest rate. In particular, $\frac{\partial w_0}{\partial I^g} =$

$\left(\frac{1}{Z}\right) \left(\frac{1-\delta}{2B^2} - \frac{\partial n}{\partial I^g}\right) > 0$ while $\frac{\partial^2 w_0}{\partial (I^g)^2} = -\left(\frac{1}{Z}\right) \frac{\partial^2 n}{\partial (I^g)^2} < 0$. Hence, the initial real wage is an increasing and concave function of the domestic interest rate.

As we showed above, an unexpected depreciation of exchange rate implies that labor supply is the short end of the labor market. Hence, actual employment is given by labor supply. Under our assumptions on η and ν , the actual path of employment given by equation (52), reduces to

$$l_t^a = \bar{l} [1 + (w_0 - 1)e^{-\gamma w_0 t}]. \quad (63)$$

Since employment and labor supply are linear in the initial real wage while $Y = \int_{t=0}^{\infty} (Bl_t^a - \zeta(l_t^a)^2) e^{-rt} dt$, one can differentiate Y with respect to w_0 to get

$$Y_w = \frac{B\bar{l}(1 - w_0)}{(r + 2\gamma w_0)^2} [r + \gamma(1 + w_0)] \geq 0, \quad (64)$$

where we have used equations (9) and (63) to integrate out over t . For later reference it useful to note that $Y_w|_{w_0=0} = \frac{B\bar{l}}{r^2} (r + \gamma) > 0$ and $Y_w|_{w_0=1} = 0$. It is straightforward to check that $Y_{ww} < 0$ for $w_0 \in [0, 1]$.

The policy choices at time 0 are $w_0 (= 1/E_0)$ and I^g . However, equation (62) makes clear that only one of these two can be freely chosen. We shall assume that the policymaker chooses I^g . Since \bar{Z} is given exogenously while $R_0 = \bar{R} = 0$, a given choice of I^g implies a value of h and n . These two variables allow us to uniquely determine w_0 from equation (62). Moreover, all private sector behavior can be expressed as functions solely of E_0 and I^g . Hence, the government's problem can be formalized as choosing I_0^g to maximize the right hand side of equation (61) subject to equations (13), (25), (32), (33), (62), and (63). The first order condition for this problem

is²⁵

$$\frac{d\hat{W}^{NI}}{dI^g} \equiv \Gamma^{NI} = -\frac{B\psi(p\theta)^{\frac{1}{1-\theta}}\psi I^g}{(1-\theta)} \left(\frac{1}{1+\psi I^g}\right)^{1+\frac{1}{1-\theta}} + \frac{(1-\delta)}{2B}I^d + rY_w\frac{\partial w_0}{\partial I^g}. \quad (65)$$

In the following we shall denote the optimal interest rate for the no intervention case by I_{NI}^g . Specifically, $\Gamma^{NI}|_{I^g=I_{NI}^g} = 0$. Three results follow directly from the first order condition. First, for $\alpha = 1$ the optimal domestic interest rate continues to be I_{peg}^g . This can be seen from the fact that $\Gamma^{NI} = 0$ for $I^g = I_{peg}^g$ and $\alpha = 1$. Note that for $I^g = I_{peg}^g$ and $\alpha = 1$ we must have $w_0 = 1$. Since $Y_w|_{w_0=1} = 0$, the last term on the right hand side (RHS) of (65) drops out. The result then follows from the fact that the first two terms on the RHS are just $d\hat{W}^{NI}/dI^g$ which is zero for $I^g = I_{peg}^g$ (see equation (56)). Hence, absent a money demand shock, $I_{NI}^g = I_{peg}^g$.²⁶

Second, for $\alpha < 1$ and $I^g = I_{peg}^g$ it is easy to check that $\Gamma^{NI} > 0$. Hence, it is optimal for the policymaker to respond to a negative money demand shock by raising the domestic interest rate, i.e., $I_{NI}^g > I_{peg}^g$. Third, in the case of $\alpha < 1$, it is never optimal for the policymaker to raise the domestic interest all the way to the point where $w_0 = E_0 = 1$. Since $Y_w|_{w_0=1} = 0$, this

²⁵It is important to note that we will only focus on values of ψ for which there are interior solutions when $\alpha < 1$. In general, however, there will be corner solutions. In fact, it is easy to show that for $\psi = 0$ and $\alpha < 1$, there will be a corner solution at the I^g corresponding to the Friedman rule (i.e., $I^d = 0$). The intuition is clear: if $\psi = 0$ and $\alpha < 1$, “increasing” I^g (if it were possible) above the Friedman rule would have no first-order effects on transactions costs but would have a first-order positive effect on sector- y output as the exchange rate appreciates (i.e., $\Gamma^{NI} > 0$, when evaluated at $I^d = 0$ for $\psi = 0$ and $\alpha < 1$). By continuity, therefore, there will also be corner solution for very small values of ψ . For larger values of ψ , interior solutions exist (as we have established using numerical examples).

²⁶As in the case of costless intervention studied earlier, it can be shown that the condition $\psi r \frac{\delta}{1-\delta} < 1 - \theta$ continues to be sufficient (but not necessary) for the second order condition to be satisfied.

follows from equation (56), which says that the sum of the first two terms on the RHS of (65) is negative for $I^g > I_{peg}^g$.

A key issue of interest to us is the relationship between the optimal interest policy and the size of the money demand shock. To determine this, we start by noting that from the implicit function theorem $\frac{\partial I_{NI}^g}{\partial \alpha} = -\frac{\Gamma_{I^g}^{NI}}{\Gamma_{I^g}^{NI}}$. Since $-\Gamma_{I^g}^{NI} > 0$ from the second order condition, it follows that the signs of $\frac{\partial I_{NI}^g}{\partial \alpha}$ and $\Gamma_{I^g}^{NI}$ are the same. Partially differentiating equation (65) with respect to α gives $\Gamma_{\alpha}^{NI} = \frac{r}{2B\bar{Z}} \frac{\partial w_0}{\partial I^g} Y_{ww} < 0$. Hence,

$$\frac{\partial I_{NI}^g}{\partial \alpha} = \frac{\frac{r}{2B\bar{Z}} \frac{\partial w_0}{\partial I^g} Y_{ww}}{-\Gamma_{I^g}^{NI}} < 0, \quad (66)$$

which says that the optimal domestic interest rate should be rising in the size of the negative money demand shock, i.e., the smaller the α the higher is I_{NI}^g .

The next issue of interest is the behavior of the nominal exchange rate as a function of the money demand shock along the optimal policy path. This is not immediately obvious since there are two potentially offsetting effects. A fall in α directly reduces money demand h and, hence, all else equal reduces the real wage by increasing E . (Recall that $w_0 = 1/E_0 = \bar{Z}^{-1}(h - n)$.) However, the fact that the central bank raises interest rates in response to a bigger money demand shock implies that, for a given α , money demand rises which appreciates the exchange rate and raises w_0 . If the latter effect is strong enough then the nominal exchange rate could appreciate in response to bigger and bigger shocks.

To shed light on this issue we totally differentiate equation (62) to get, after some rearrangement,

$$\frac{dw_0}{d\alpha} = \frac{1}{2\bar{Z}B\Gamma_{I^g}^{NI}} \left[\Gamma_{I^g}^{NI} - rY_{ww} \left(\frac{\partial w_0}{\partial I^g} \right)^2 \right],$$

where $\Gamma_{I^g}^{NI}$ is the partial derivative of equation (65) with respect to I^g . As

we noted above, $\psi r \frac{\delta}{1-\delta} < 1 - \theta$ is a *sufficient* condition for the second order condition for welfare maximization, i.e., $\Gamma_{I^g}^{NI} < 0$, to be satisfied. It is straightforward to check that this sufficiency condition is also a sufficient condition for $\Gamma_{I^g}^{NI} < rY_{ww} \left(\frac{\partial w_0}{\partial I^g}\right)^2$.²⁷ Hence, $\psi r \frac{\delta}{1-\delta} < 1 - \theta$ is a *sufficient* condition for $\frac{dw_0}{d\alpha} > 0$ which implies that as α becomes smaller (i.e., the money demand shock gets larger), the initial real wage w_0 becomes progressively smaller. Since $E_0 = 1/w_0$, this implies that along an optimal policy path with no intervention, the nominal exchange rate is a decreasing function of α – the bigger the negative money demand shock the larger the exchange rate depreciation.

Lastly, it is useful to characterize the behavior of welfare around the optimum under the no-intervention regime. Totally differentiating equation (61) with respect to α gives

$$\frac{d\hat{W}^{NI}}{d\alpha} = \frac{B\alpha}{2} + \frac{r}{2B\bar{Z}}Y_w > 0, \quad (67)$$

where we have used the fact that, at an optimum, the first order condition for welfare maximization (equation (65)) says that $\frac{\partial \hat{W}^{NI}}{\partial I^g} \frac{\partial I_{NI}^g}{\partial \alpha} = 0$. We collect these results in the following proposition:

Proposition 4 *Along an optimal policy path with no foreign exchange market intervention, the central bank responds to a given negative money demand shock by raising the domestic interest rate while allowing some depreciation of the nominal exchange rate to occur. Moreover, the larger the negative money demand shock, the larger the optimal domestic interest rate, the larger the nominal exchange rate depreciation, and the lower is welfare.*

²⁷To see this define $N = pq^\theta - q$. Hence, $\Gamma^{NI} = B \frac{\partial N}{\partial I^g} + \frac{1-\delta}{2B} I^d + rY_w \frac{\partial w_0}{\partial I^g}$ and $\Gamma_{I^g}^{NI} = B \frac{\partial^2 N}{\partial (I^g)^2} - \frac{(1-\delta)^2}{2B} + rY_{ww} \left(\frac{\partial w_0}{\partial I^g}\right)^2 + rY_w \frac{\partial^2 w_0}{\partial (I^g)^2}$. Moreover, $\Gamma_{I^g}^{NI} - rY_{ww} \left(\frac{\partial w_0}{\partial I^g}\right)^2 = B \frac{\partial^2 N}{\partial (I^g)^2} - \frac{(1-\delta)^2}{2B} + rY_w \frac{\partial^2 w_0}{\partial (I^g)^2}$. Since Y_{ww} and $\frac{\partial^2 w_0}{\partial (I^g)^2}$ are both negative, a sufficient condition for both $\Gamma_{I^g}^{NI} < 0$ and $\Gamma_{I^g}^{NI} < rY_{ww} \left(\frac{\partial w_0}{\partial I^g}\right)^2$ is $\frac{\partial^2 N}{\partial (I^g)^2} < 0$. It is easy to check that $\frac{\partial^2 N}{\partial (I^g)^2} < 0$ for $\psi r \frac{\delta}{1-\delta} < 1 - \theta$.

5.3 To Intervene or Not to Intervene

Having described the behavior of optimal interest rate policy contingent on the intervention regime (i.e., intervention or no intervention), we now turn to the determination of the optimal intervention regime itself. For a given α , it is straightforward to see that the optimal intervention strategy is determined by

$$\begin{aligned} \text{Do not intervene if } \hat{W}^{NI} \Big|_{I^g=I_{NI}^g} &> \hat{W}^I \Big|_{I^g=I_{peg}^g} \\ \text{Intervene if } \hat{W}^{NI} \Big|_{I^g=I_{NI}^g} &\leq \hat{W}^I \Big|_{I^g=I_{peg}^g} \end{aligned}$$

Notice first that around $\alpha = 1$, $\hat{W}^{NI} - \hat{W}^I = Br\phi$. This follows from the facts that for $\alpha = 1$, $I_{NI}^g = I_I^g = I_{peg}^g$ and $w_0 = 1$ while $l^s = l^a = \bar{l}$. Intuitively, around $\alpha = 1$, the only difference between the two regimes is the cost of intervention while the nominal exchange rate and the domestic interest rates are identical. Hence, welfare under intervention is lower.

From equations (60) and (67) it is also easy to see that $\frac{d\hat{W}^{NI}}{d\alpha} - \frac{d\hat{W}^I}{d\alpha} = \frac{r}{2BZ}Y_w > 0$. This says that while bigger money demand shocks (or lower α 's) cause welfare to decline under both regimes, the fall in welfare under the no-intervention regime is faster than that under the full intervention regime. Intuitively, the direct effect of the money demand shock on the transactions costs faced by the household is symmetric under the two regimes. However, under the no-intervention regime the direct effect of α induces the real wage to progressively decline with α due to the nominal wage rigidity. This extracts an extra output cost from sector- y .

The preceding implies that the relative welfare comparison between the two regimes reduces to a trade-off between the fixed cost of intervention and the output cost associated with not intervening. Crucially, the output costs under the no-intervention regime get bigger as the shock gets bigger while the corresponding cost of intervention is independent of the size of the

shock. Hence, the welfare differential between the two regimes shifts in favor of intervention as the shock gets bigger. Since $\hat{W}^{NI} - \hat{W}^I = Br\phi > 0$ around $\alpha = 1$, this implies that for a given ϕ there must exist a threshold $\hat{\alpha} < 1$ such that $\hat{W}^{NI}\big|_{\alpha=\hat{\alpha}} = \hat{W}^I\big|_{\alpha=\hat{\alpha}}$. Further, for all $\alpha < \hat{\alpha}$ we must have $\hat{W}^{NI} < \hat{W}^I$. These features of the optimal policy problem are captured in Figure 1 which depicts welfare under the two regimes as functions of α .

Using the results detailed above, one can now completely characterize the optimal policy response in this economy to money demand shocks (see Table 2). For small money demand shocks, i.e., $\alpha \in (\hat{\alpha}, 1)$, it is optimal for the policymaker not to intervene and, instead, allow the currency to float but raise domestic interest rates to fight the currency depreciation in order to insulate the real side of economy. As we saw earlier, this also implies that in this range, the larger the money demand shock, the bigger is the nominal exchange rate depreciation that the policymaker allows to occur. However, for larger money demand shocks, i.e., $\alpha < \hat{\alpha}$, it is optimal for the policymaker to keep domestic interest rates unchanged but pay the intervention cost and intervene fully in order to prevent the nominal exchange rate from fluctuating at all. Figure 2 depicts the behavior of the nominal exchange rate as a function of α .

The last result worth noting is that the threshold value of the shock parameter $\hat{\alpha}$ is a decreasing function of the fixed cost of intervention ϕ . This result follows from noting that $\frac{d\hat{W}^{NI}}{d\alpha} - \frac{d\hat{W}^I}{d\alpha} = \frac{r}{2BZ}Y_w$ is independent of ϕ . But a smaller ϕ implies that $\hat{W}^{NI} - \hat{W}^I = Br\phi$ is smaller. In terms of Figure 1, a smaller ϕ causes a parallel shift up of the \hat{W}^I schedule leaving its slope with respect to α unaffected. Hence, the threshold $\hat{\alpha}$ must be larger, i.e., it must be closer to unity. In the limit when $\phi = 0$ the two schedules coincide for $\alpha = 1$ with \hat{W}^I exceeding \hat{W}^{NI} for all $\alpha < 1$.

We summarize the preceding results in the following proposition:

Proposition 5 *The optimal policy response to a negative money demand*

shock is a function of the size of the shock. For small shocks, it is optimal for the policymaker not to intervene in the foreign exchange market but instead raise the domestic interest rate and let the exchange rate depreciate. Moreover, in this range, the larger the shock the higher are both the nominal exchange rate and the domestic interest rate. For large shocks, however, it is optimal for the central bank to intervene by the full amount necessary to keep the exchange rate and domestic interest rates unchanged. Crucially, the smaller the fixed cost of intervening, the smaller is the threshold size of the shock for which the full intervention policy becomes optimal.

To assess how well the model might explain the key stylized facts outlined in the introduction, let us perform the following conceptual experiment. Suppose that this economy were subject to a sequence of (stochastic) monetary shocks. Assume, further, that developing countries were hit, on average, by larger shocks than industrial countries. The outcome would be a series of changes in the endogenous policy variables, as captured by the last two rows in Table 2. From a cross-sectional point of view, the model would predict that developing countries (which face mostly large shocks) would exhibit low exchange rate variability and high reserve variability, while developed countries (which face mostly small shocks) would exhibit high exchange rate variability and low reserve variability. From a time-series perspective, we would observe an average response (since countries are hit by both small and large shocks) that would consist (for, say, negative monetary shocks) of falling reserves, more depreciated exchange rates, and higher interest rates. Hence, the correlation between changes in exchange rate and interest rates would be positive, while the correlation between reserves and exchange rates and between reserves and interest rates would be negative. All these predictions match the stylized policy facts described in the introduction.

6 Conclusions

The starting point for this paper has been the observation that, in spite of suffering larger shocks, developing countries (classified as floaters or managed floaters) exhibit lower exchange rate variability and higher reserve variability than developed countries which float. This extreme “fear of floating” is puzzling since, even if nominal exchange rate fluctuations are costly, one would still expect that larger shocks would lead to larger changes in the nominal exchange rate.

This paper has developed a simple and highly stylized theoretical model that is capable of explaining this puzzle. In particular, the model predicts that for small negative money shocks, policymakers find it optimal to let the exchange rate adjust while partly offsetting the shock by raising domestic interest rates. For large shocks, policymakers find it optimal to completely stabilize the exchange rate by intervening in the foreign exchange market. The model thus predicts a non-monotonic relationship between the nominal exchange rate and the size of the shock. If we identify small shocks with developed countries and large shocks with developing countries, the model predicts that developing countries should exhibit low exchange rate variability and high reserve variability while the converse is true for developed countries.

While we view this as a useful first step towards an understanding of the “fear of floating” puzzle, there are at least two directions in which this line of research should be taken. For starters, we would like to endogeneize the fixed cost of intervention, which is of course key to our results. A natural avenue for doing this would be to consider a set-up with asset market segmentation a la Alvarez, Atkeson, and Kehoe (2000). While this would be a major undertaking (since our focus is on optimal policies, which makes the problem already much more complicated from a formal point of view),

it would certainly be worthwhile to pursue. Second, it would be useful to develop a stochastic version of this model, calibrate it for some representative developing country, and try to match the observed correlations. Developing richer models along these lines would be extremely useful for both understanding the actual responses observed in the data as well as for devising implementable and usable policy rules for central bankers.

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TABLE 1: Exchange rate, foreign reserves and nominal money fluctuations

| Country | Regime | Period | E2.5 | R2.5 | I.25 | M2 |
|-----------------------------------|--------|------------------------------|-------|------|------|------|
| Bolivia | FL | September 1985-December 1997 | 93.9 | 19.6 | 16.3 | 33.8 |
| India | FL | March 1993-April 1999 | 93.4 | 50.0 | 6.4 | 53.4 |
| Kenya | FL | October 1993-December 1997 | 72.2 | 27.4 | 19.6 | 31.4 |
| Mexico | FL | December 1994-April 1999 | 63.5 | 28.3 | 5.7 | 22.7 |
| Nigeria | FL | October 1986-March 1993 | 74.5 | 12.8 | 89.7 | 18.0 |
| Peru | FL | August 1990-April 1999 | 71.4 | 48.1 | 24.8 | 34.3 |
| Phillipines | FL | January 1988-April 1999 | 74.9 | 26.1 | 22.1 | 27.9 |
| S. Africa | FL | January 1983-April 1999 | 66.2 | 17.4 | 35.6 | 75.0 |
| Uganda | FL | January 1992-April 1999 | 77.9 | 32.9 | 11.6 | 25.9 |
| Venezuela | FL | March 1989-June 1994 | n.a. | 35.9 | 0.0 | 25.0 |
| Indonesia | FL | July 1997-April 1999 | 14.3 | 29.9 | 13.3 | 9.5 |
| Korea | FL | November 1997-April 1999 | 17.7 | 5.6 | 4.6 | 12.5 |
| Thailand | FL | July 1997-April 1999 | 38.1 | 40.9 | 62.5 | 45.4 |
| Bolivia | MF | January 1998-April 1999 | 100.0 | 12.5 | 31.1 | 0.0 |
| Brazil | MF | July 1994-December 1998 | 94.3 | 51.8 | 11.1 | 27.8 |
| Chile | MF | October 1982-April 1999 | 83.8 | 48.2 | 5.0 | 53.8 |
| Colombia | MF | January 1979-April 1999 | 86.8 | 54.2 | 50.6 | 40.5 |
| Egypt | MF | February 1991-December 1998 | 98.9 | 69.4 | 78.9 | 52.6 |
| India | MF | February 1979-February 1993 | 84.5 | 36.7 | 49.7 | 46.8 |
| Indonesia | MF | November 1978-June 1997 | 99.1 | 41.5 | 30.6 | 33.9 |
| Israel | MF | December 1991-April 1999 | 90.9 | 43.8 | 26.4 | 39.8 |
| Kenya | MF | January 1988-April 1999 | 70.6 | 14.3 | 28.6 | 71.4 |
| Korea | MF | March 1980-October 1997 | 97.6 | 37.7 | 31.1 | 24.1 |
| Malaysia | MF | December 1992-September 1998 | 81.2 | 55.7 | 66.7 | 47.2 |
| Mexico | MF | January 1989-November 1994 | 95.7 | 31.9 | 8.3 | 27.8 |
| Pakistan | MF | January 1982-April 1999 | 92.8 | 12.1 | 34.8 | 40.2 |
| Singapore | MF | January 1988-April 1999 | 88.9 | 74.8 | 51.9 | 51.5 |
| Turkey | MF | January 1980-April 1999 | 36.8 | 23.3 | 3.4 | 21.7 |
| Uruguay | MF | January 1993-April 1999 | 92.0 | 36.5 | 2.7 | 39.5 |
| Venezuela | MF | April 1996-April 1999 | 93.9 | 29.4 | n.a. | 23.5 |
| USA | FL | February 1973-April 1999 | 58.7 | 62.2 | 59.7 | 67.2 |
| Japan | FL | February 1973-April 1999 | 61.2 | 74.3 | 67.9 | 41.9 |
| Average (excluding USA and Japan) | | | 77.4 | 35.0 | 28.4 | 35.2 |

NOTES: 1. FL -- Floating exchange rate regime; MF: Managed float regime

2. Column X_i is the probability that the monthly change in variable X is within a band of plus/minus i percent.

3. E is the nominal exchange rate, R are international reserves, and M is the nominal money base.

Source: Calvo and Reinhart (2000a).

Table 2. Response to a negative money demand shock

| Policy regime | R | E | I^g | z^g | h |
|------------------------------|-----|-----|-------|-------|-----|
| Fixed exchange rate | ↓ | → | ↑ | → | ↓ |
| Floating exchange rate | → | ↑ | → | ↓ | ↓ |
| Dirty floating | ↓ | ↑ | → | → | ↑ |
| Full sterilization | ↓ | ↓ | ↑ | ↑ | → |
| Optimal policy (small shock) | → | ↑ | ↑ | ↓ | ↓ |
| Optimal policy (large shock) | ↓ | → | → | ↑ | ↓ |

Note: Under dirty floating, the increase in E is smaller than under a pure floating.

Figure 1: Welfare comparison

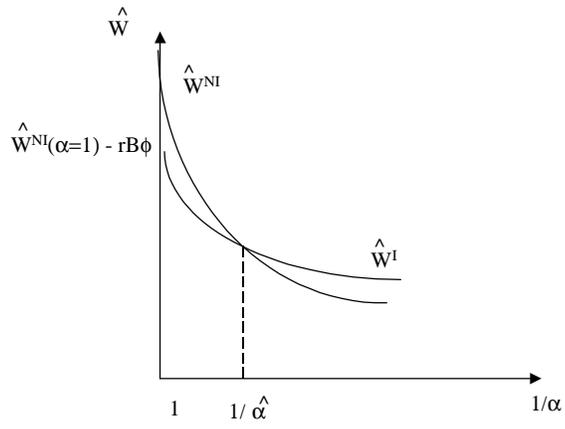


Figure 2: Optimal Exchange Rate

