Sentiments

Slides by Marco Bassetto
paper by G.M. Angeletos and J. La'O

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(My) Motivation

- Studying business cycles in models of dispersed information
- Model with fully rational agents, yet has a flavor of behavioral economics
- Advantage: easier to think about policy, efficiency, no need to have paternalistic policies...

(Their) motivation

- Want to formalize the idea that "expectations," or "sentiments" (animal spirits?) matter
- Want to achieve this without having to rely on sunspots
  - With unique equilibrium, can do comparative statics
  - We can also do policy analysis

Key trick

- Shocks to beliefs
- It's not (first-order) beliefs about aggregates, but about higher orders
- More sophisticated version of Lucas (1972)
- Beliefs are not directly about aggregates, but about higher-order elements

"Simple" analytical model

- A continuum of islands
- Each island populated by many identical households (competitive pricing)
- One good per island, produced with labor and land (fixed factor)
- In each period, islands are randomly matched and trade pairwise

Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{it}}{1-\eta} \right)^{1-\eta} \left(\frac{c_{it}^*}{\eta} \right)^{\eta} - \frac{n_{it}^{\epsilon}}{\epsilon}$$

- c_{it}: domestically produced consumption
- c_{it}^*: consumption imported from matched island
- n_{it}: labor

Production

$$y_{it} = A_i (n_{it})^{\theta} (k_{it})^{1-\theta}$$

- Firms hire labor and land to produce the local good
- Island-specific TFP, fixed: $A_i \sim \mathcal{F}_A$ (+usual i.i.d.-like assumptions)
- k_{it} : land in use (in equilibrium must be K, fixed supply)

Timing and information

- Two subperiods
- First subperiod:
  - get signal $x_{it} \in \mathbb{R}^n$
  - Only aggregate random variable is $\xi_t \in \mathbb{R}^m$, not observed (unless it's part of x_{it})
  - Firms hire labor and capital and produce
- Second subperiod:
  - Matched with random island
  - Share information with the matched island
  - Trade consumption good with matched island

Keeping track of information

- ω_{it}: information available in the first subperiod
- z_{it}: information in the second subperiod
- $\omega_{it} = (z_{i,t-1}, x_{it})$
- $z_{it} = (\omega_{it}, \omega_{m_t(i),t})$
- $m_t(i)$: identity of island matched with island i in period t
- Record-keeping: $z_{it} \in \mathbb{R}^{n(2^{t+1}-1)}$, $\omega_{it} \in \mathbb{R}^{n(2^{t+2}-2)}$

Markets

- Spot market for labor
- Rental market for land
- (Optional) island-specific markets for land or borrowing/lending
- All households on island identical $\Longrightarrow$ no trade in asset market
- No information revelation from asset prices either (symmetric info for all islanders and also with the matched island)
- No cross-island asset markets: what is important is no centralized market
- Need to avoid revelation of aggregate information

(Symmetric) Equilibrium: elements

- Allocation $\{n_t, k_t, y_t, c_t, c_t^*\}$
- Price system $\{w_t, r_t, p_t, p_t^*\}$, where
  - w_t: wage rate
  - r_t: rental rate of land
  - p_t, p_t^*: price of domestic production and imports
  - Note: one price can be normalized, will normalize Lagrange multiplier
- $\{n_t, k_t, y_t, w_r, r_t\}$ are functions of ω_{it} and A_i
- $\{c_t, c_t^*, p_t, p_t^*\}$ are functions of $(\omega_{it}, \omega_{m_t(i),t}$ and $(A_i, A_{m_t(i)})$
  - Note: we have $z_{it} = (\omega_{it}, \omega_{m_t(i),t})$.
  - Written explicitly for the two to imply that the order matters because of previous choices
  - Order stops mattering after period t

Equilibrium: definition

- Given the price system (and expectations), the allocation is optimal for the firms and the households
- Markets clear (labor, land, imports/exports)
- (Expectations are rational)
- Free trade, relative price of goods the same within a match. Specifically:
-

$$\frac{p_t^*(\omega_{it}, \omega_{m_t(i),t}, A_i, A_{m_t(i)})}{p_t(\omega_{it}, \omega_{m_t(i),t}, A_i, A_{m_t(i)})} = \frac{p_t(\omega_{m_t(i),t}, \omega_{it}, A_{m_t(i)}, A_i)}{p_t^*(\omega_{m_t(i),t}, \omega_{it}, A_{m_t(i)}, A_i)}$$

Equilibrium conditions: consumption

- From now on, use j for $m_t(i)$
- Budget constraint

$$p_{it} c_{it} + P_{it}^* c_{it}^* \leq w_{it} n_{it} + r_{it} K + \pi_{it}$$

- π_{it}: firm profits/losses (not zero because of uncertainty)
- From household optimality and market clearing:
  - $c_{it} = (1 - \eta) y_{it}$, $c_{it}^* = \eta y_{jt}$
  - $p_{it}^* / p_{it} = y_{it} / y_{jt}$
  - Normalize prices so that the Lagrange multiplier is 1 (ideal price index based on consumption preferences)
  - Get

$$p_{it} = (y_{it} / y_{jt})^{-\eta}$$

Equilibrium conditions: labor

- Household side:

$$n_{it}^{\epsilon-1} = w_{it}$$

- Firm side

$$w_{it} = \theta \frac{y_{it}}{n_{it}} E_{it} p_{it}$$

Equilibrium: Remaining observations

- No linkages across periods in equilibrium (no intertemporal decisions)
- Rental rate of land, profits irrelevant (determined as residual)
- Important that firms are owned by households on the island

Equilibrium: combining conditions

- $$p_t(\omega_t, \omega'_t, A_i, A_{m_t(i)}) = \left(\frac{y(\omega_t, A_i)}{y(\omega'_t, A_{m_t(i)})}\right)^{-\eta}$$
- $$y(\omega_t, A_i) = \left(\theta^{\vartheta} A_i K^{1-\theta}\right)^{1/(1-\vartheta)}$$
$$\cdot \left(\int p_t(\omega_t, \omega'_t, A_i, A_{m_t(i)}) d\mathcal{P}_t(\omega'_t, A_{m_t(i)}) | (\omega_t, A_i)\right)^{\frac{\vartheta}{1-\vartheta}}$$
$\vartheta := \theta/\epsilon$
- Using first equation into second, we see that output on island i depends (only!) on the expectation about the output of island $m_t(i)$

Complementarities

- Output of island $m_t(i)$ depends positively on the expectation of output of island i
- $\Longrightarrow$ Need to compute fixed point, and there are complementarities
- Complementarities because substitution effect dominates income effect (linear preferences in Cobb-Douglas aggregate)

Equilibrium: existence and uniqueness

- Rewrite equilibrium as

$$\log y(\omega_t, A_i) = (1-\alpha) f(A_i) + \alpha \mathbf{E}_{it}\Big(\log y(\omega_t', A_{m_t(i)})\Big)$$

- $$\mathbf{E}_{it}(x) := \log\Big(E_{it}[\exp(x)]\Big)$$

- Mapping above is a contraction (from the space of bounded continuous functions $\log y(.)$ into itself)
- Note importance of static nature of the problem

Some record-keeping

Define

- $$\log \mathcal{B}_t := E\left[E_{it}(\log y_{jt})\right]$$
- $$\log Y_t := E\left[\log y_{jt}\right]$$
- Note: E_{it}, undistorted expectation
- E is ex ante expectation (average over all realizations of (ω_t, A_i))

"Perfect communication" case

- No uncertainty within period: island i knows $y_{m_t(i)}$ at production stage
- Proposition 2:
  - $$\log Y_t = \log \mathcal{B}_t = \kappa + \frac{\int_0^1 \log A_i di}{1 - \vartheta}$$
  - Aggregate output is constant, independent of beliefs
  - True even with time-varying TFP (problem is independent across periods)

Proof of proposition 2

With no uncertainty

- $$\log y_{it} = (1-\alpha) f(A_i) + \alpha \log y_{jt}$$
- $$\log y_{jt} = (1-\alpha) f(A_j) + \alpha \log y_{it}$$
- Solve system for (y_{it}, y_{jt}) as a loglinear function of $f(A_i), f(A_j)$
- Take expected value across all matches, get a constant

Theorem 2

- "Along the unique equilibrium, aggregate output Y_t and the average expectation $\mathcal{B}_t$ can vary with the extrinsic shock ξ_t if and only if communication is imperfect."
- Translation:
    - Proposition 2
    - $+$ exist examples in which ξ_t matters

Imperfect communication: Economic intuition

- You might be surprised (I was)
- Perfect communication equilibrium did not require much information
- Why on earth is the perfect communication equilibrium not an equilibrium anymore?

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- Cool trick # 1: make everybody think that they will meet a better (or worse) than average island

Imperfect communication: Economic intuition

- You might be surprised (I was)
- Perfect communication equilibrium did not require much information
- Why on earth is the perfect communication equilibrium not an equilibrium anymore?
- Cool trick # 1: make everybody think that they will meet a better (or worse) than average island
- Even cooler trick # 2: do not take trick 1, make everybody think that their partner thinks that they will meet a better (or worse) than average island
- Neat properties:
  - Can do in a fully rational model
  - Endogenous amplification: optimistic expectations $\Longrightarrow$ produce more $\Longrightarrow$ others want to produce more

Constructing an example where beliefs matter

- $\log A_t \sim N(0, \sigma_A^2)$
- In period t, get advance signal of productivity of partner

$$x_{it}^1 = \log A_j + u_{it}^1$$

- Get advance signal of the signal of the partner

$$x_{it}^2 = x_{jt}^1 + \xi_t + u_{it}^2$$

- Usual normality assumptions

Nature of the equilibrium

- $$\log y_{it} = \phi_0 + \phi_a \log A_i + \phi_1 x_{it}^1 + \phi_2 x_{it}^2 \qquad \text{(guess)}$$
- $$E_{it} \log p_{it} = \psi_0 - \psi_a \log A_i + \psi_1 x_{it}^1 + \psi^2 x_{it}^2$$
- Since x_{it}^2 has an aggregate component ξ_t, aggregate output increases in ξ_t
- Proof: guess and verify
  - Plug (guess) on the right-hand side of the fixed-point equation
  - verify that the resulting RHS is a linear function of the same variables as in (guess)
  - solve for the fixed point of the vector ϕ
  - Verify that $\phi_1 > 0$, $\phi_2 > 0$

Higher-order beliefs

- The same trick applies to beliefs of any order k
- We could also have multiple shocks to different-order beliefs
- Remember that fixed-point equation is

$$\log y(\omega_t, A_i) = (1-\alpha) f(A_i) + \alpha \mathbf{E}_{it}\Big(\log y(\omega_t', A_{m_t(i)})\Big)$$

All that matters is $\mathbf{E}_{it} \log y_{jt}$.

- Different shocks $\xi_t^1, \xi_t^2, \xi_t^3$ that have the same impact on this expectation are observationally equivalent

An extension where correlation happens through communication patterns

- 2 types of islands, in the same amount, labeled N and S
- Productivity is the same for all islands of the same type
- A_N, A_S drawn from log-normal distribution
- Now we have aggregate TFP movements, but focus is on movements driven by "sentiments" alone

Information structure

- Three stages: "uninformed," "partially informed," and "fully informed."
- Uninformed only know productivity of own type
- Partially informed get a signal about first and second-order beliefs, so a N island gets

$$x_N^1 = \log A_S + \epsilon_N, \quad x_N^2 = x_S^1 + \xi$$

- Note: all shocks are the same conditional on type and constant over time
- Fully informed know both A_N and A_S

Trading structure

- Uninformed trade only with firms of their type, either partially informed or uninformed
- If uninformed trade with a partially informed, they become partially informed
- Partially informed trade with uninformed of their type or partially informed of the other type
- Partially informed that meet with other type learn everything, become fully informed
- Fully informed trade only with fully informed of the same group [not sure why it's important that it's the same group]
- Type of the island with which you trade known at production stage (but not information)

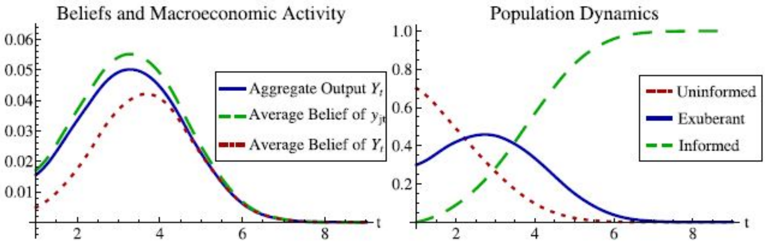
Comparison with previous model: the boring part

- The problem is still period by period, no linkages
- Consider first uninformed meeting uninformed: same type, know each other productivity, no uncertainty
- Uninformed meeting partially informed (of the same type): same as before, the uninformed will learn something, but for the current period productivity of the pair is common knowledge among the two parties (full symmetry)
- Fully informed: same as before (full symmetry as before)

Comparison with previous model: the action part

- Only interesting meeting: Partially informed islands of the two types meet
- Math is exactly the same as before, responsiveness computed in the same way
- important that you know at the production stage that you will be in this situation (to recycle math)
- Key question: how many meetings of this type will there be?

Numerical illustration



Quantitative setup: production

- What was consumption becomes final goods production

$$y_{it} = \left(\frac{h_{it}}{1-\eta}\right)^{1-\eta} \left(\frac{h^*_{it}}{\eta}\right)^{\eta}$$

  h_{it} is usage of intermediate goods (imported if *)
- Intermediate goods produced similarly to output before:

$$q_{it} = A_i (e_{it} k_{it})^{1-\theta} n_{it}^{\theta}$$

- k_{it} is now reproducible capital:

$$k_{i,t+1} = (1 - \Delta(e_{it}))k_{it} + i_{it}$$

- i_{it} investment (out of the final good)
- e_{it}: capital utilization
- $y_{it} = c_{it} + i_{it}$

More on capital utilization

- Sentiments shocks act in a similar way to news shocks
- With good news, production goes up, but TFP does not
- With fixed capital, production $\uparrow \Longrightarrow$ MPL $\downarrow$
- Production and wages would be negatively correlated
- Solution:

$$q_{it} = A_i (e_{it} k_{it})^{1-\theta} n_{it}^{\theta}$$

  $e_{it} \uparrow$ increases MPL
- Functional form: $\Delta(e) = (\delta/\mu) e^{\mu}$

Preferences

$$E_{i0} \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{it}^{1-\gamma}}{1-\gamma} - \frac{n_{it}^{\epsilon}}{\epsilon} \right]$$

Equilibrium conditions

- Market clearing
- Intratemporal optimality (labor/consumption):

$$V'(n_{it}) = \theta\zeta E_{it}\left[U'(c_{it})\frac{y_{it}}{n_{it}}|\omega_{it}\right]$$

- Capital utilization:

$$\Delta'(e_{it})e_{it} = (1-\theta)\zeta E_{it}\left[U'(c_{it})\frac{y_{it}}{k_{it}}|\omega_{it}\right]$$

- Euler equation:

$$U'(c_{it}) = \beta E_{it}\left[U'(c_{it\,t+1})\left(1+(1-\theta)\zeta\frac{\mu}{1+\mu}\frac{y_{i\,t+1}}{k_{i\,t+1}}\right)|k_{it}\right]$$

- $$\zeta := \left(\frac{1}{1-\eta}\right)^{1-\eta}\left(\frac{1}{\eta}\right)^{\eta}$$

Beliefs

- First-order belief:
$$x_{it} = \log A_{jt} + \epsilon_{it}$$
$\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$
- Second-order belief: $\epsilon_{jt} \sim N(\xi_t, \sigma_\epsilon^2)$
- ξ_t persistent Markov process commonly known

Standard parameters

- $\beta = 0.99$ (quarterly model)
- $\gamma = \epsilon = 2$
- $\theta = 0.65$
- $\mu = 2$

Other parameters

- $\xi_{t+1} = 0.98\xi_t + \nu_t$
- $\eta, \sigma_A, \sigma_\epsilon, \sigma_x i$ only matter in one dimension: how large is the output response to ξ_t
- Normalize 3, pick remaining one to hit (HP-filtered) output volatility

Results

TABLE I

BUSINESS-CYCLE STATISTICS OF OUR MODEL ALONG WITH THOSE OF THE U.S. ECONOMY[a]

	The Model		U.S. Data	
	std. dev	corr(X, Y)	std. dev	corr(X, Y)
Y	1.67	1.00	1.67	1.00
N	1.41	1.00	1.43	0.87
C	1.21	0.98	1.27	0.80
I	4.14	0.96	5.48	0.82
Y/N	0.26	0.99	0.82	0.51
LW	4.95	−1.00	4.98	−0.82

[a] All quantities are in quarterly frequency and HP-filtered. See Appendix B for details.

Labor wedge

- $MPL \neq MRS_{c,n}$
- Negative ξ_t shock: output down, consumption down, labor down, MRS down
- Labor productivity MPL down, not as much
- Labor wedge emerges