More Perils of Taylor Rules
Work in Progress

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Motivation

- Sargent and Wallace (JPE, 1975): indeterminacy under interest rate pegs
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- Conventional wisdom: solve with active Taylor rules
- The ability of hitting the interest rate target is taken for granted
Our Main Point

- An interest-rate peg sets the relative price of bonds and money
- In (non-strategic) monetary models, Fisher equation ensures low interest rates $\implies$ low inflation
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- An interest-rate peg sets the relative price of bonds and money
- In (non-strategic) monetary models, Fisher equation ensures low interest rates $\implies$ low inflation
- When open-market operations are subject to bounds, the peg is subject to runs
- Taking such bounds into account reveals a strategic complementarity in the game induced by an interest rate rule
An Extreme Example

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- Average 1922-23 inflation (annual rate): 1,400,000%
- Fraction of T-Bills held by the Reichsbank in Nov 1923: 99.1%
A Less Extreme Example

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- Fed just announced that we will hold rates at 0-0.25% until mid-2013
- What if inflation increases? How long is this feasible?
- Can there be a run? What does it look like?
Outline of Talk

- Set up simple Cash-In-Advance economy
- Analyze environment using standard general equilibrium tools: low inflation
- Revisit in a game setting, including bounds (and measurability restrictions): multiple equilibria
- Discuss some extensions that get closer to reality
The Cast of Actors

- A continuum of households
- A government/central bank described as an automaton (rules)
Timing

1. Households enter period $t$ holding $w_{t-1}$ units of nominal assets (bonds and money).
2. Government pays off bonds with cash, and levies lump sum taxes $T_t$ (in terms of cash).
3. Central bank is a “bond vending machine”: sets (one-period) bond price $Q_t$. Get one bond out per $Q_t$ dollars put in.
4. Households now have $m_t = w_{t-1} - T_t - Q_t b_t$ dollars on hand.
5. Households split into workers and shoppers.
6. Worker produces $y_t$.
7. Shopper purchases $c_t$.
8. Shoppers face cash-in-advance constraint, $c_t P_t \leq m_t$.
9. Workers then produce $g_t$ for government (which needs $\bar{G}$), paid in cash or bonds.
Preferences

\[ \sum_{t=0}^{\infty} u(c_t) - (y_t + g_t) \]

Assumptions: RRA > 1 around intended equilibrium
General Equilibrium: Household Problem

- Taking \( \{Q_t, P_t, T_t\}_{t=0}^{\infty}, w_{-1} \) as given, households solve

\[
\max_{c_t, m_t, b_t, y_t, g_t} \sum_{t=0}^{\infty} \beta^t [u(c_t) - (y_t + g_t)]
\]

s.t.

\[
Q_t b_t + m_t + T_t \leq w_{t-1}
\]

\[
w_t = m_t + P_t (y_t + g_t - c_t) + b_t
\]

\[
P_t c_t \leq m_t
\]

and no-Ponzi condition.
General Equilibrium: Necessary Conditions from Household Optimization

\[ u'(c_t) = \frac{1}{Q_t} \]

\[ \frac{P_{t+1}}{P_t} = \frac{\beta}{Q_{t+1}} \]

\( (\text{Assume } Q_t < 1) \)

\[ P_t c_t = m_t \]
General Equilibrium: Government Policy

- A government policy is a sequence \( \{ Q_t, T_t \}_{t=0}^{\infty} \), as a function of the price sequence \( \{ P_t \}_{t=0}^{\infty} \) that satisfies

  \[ T_t \leq B_{t-1} + P_{t-1} G_{t-1} + M_{t-1} (1 - \beta/Q_t) \]

- “Ricardian” policy (sufficient condition): there exist \( \bar{b} \) and \( \alpha \in (0, 1) \) such that and

  \[ |B_{t-1}/P_{t-1}| \geq \bar{b} \implies T_t \geq \alpha B_{t-1} \]

- Assumptions rule out commodity money (FTPL).
Equilibrium Price Sequences

- Pretty remarkable. Still lots of equilibria (since \( P_0 \) not pinned down), but all of them have the same inflation rate for every date:

\[
\frac{P_{t+1}}{P_t} = \frac{\beta}{Q_{t+1}}
\]

- Same consumption and welfare too
- Thus, if the government wants price stability (\( P_{t+1} = P_t \) for all \( t \)), all it has to is be willing to borrow or lend at \( (1 - \frac{1}{\beta}) \)
Sunspots

- Yes, there can be sunspots if $Q_t \equiv \beta$
- Necessary condition becomes

$$E\left[ \frac{P_t}{P_{t+1}} | I_t \right] = 1$$

- Expected (inverse) inflation, welfare fixed
Back to the Reichsbank

- Was the Reichsbank just very unlucky with sunspots?
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- Was the Reichsbank just very unlucky with sunspots?
- Need a better model of trade (especially between central bank and households)
Environment as a Game

- Households enter period with $w_{t-1}$ money and/or bonds
- Gov’t pays off bonds in cash and imposes lump sum taxes (in cash)
- Households unable to pay taxes are “flogged”
- Households access bond vending machine subject to bounds
- Bound has to depend on information up to this point ($P_t$ is out)
- Interest rate $1/Q_t$ also must depend on info up to here
- Exact bound not so important. Assume $B_t \geq 0$. 

Game (continued)

- Households split into a worker and a shopper, travel to separate islands
- Workers and shoppers are anonymous on the island
- Bonds cannot be transported to the island
- In each island, a Shubik market is present.
The Shubik Stage of the Game

- Shoppers bid $m_t$ (up to their holdings); aggregate bid: $M_t$
- Workers bid $y_t \geq \epsilon$; aggregate: $Y_t$
- Price is determined as $P_t = M_t/Y_t$
- Shopper receives $m_t Y_t / M_t = m_t / P_t$ unit of goods
- Worker receives $y_t M_t / Y_t = y_t P_t$ units of money
Back to the Center Island

- Government auctions $P_t \tilde{G}$ units of money on another Shubik market
- Households bid to produce for the government
The Intended Equilibrium

- Households act as price takers, solve the same problem as before
- Assuming that $B_t > 0$ in the desired equilibrium, it remains an equilibrium
The Reichsbank Equilibrium

- Suppose you believe that all other households will not hold bonds in period $t$
- Fed monetizes government debt
- High money growth, high inflation, nobody lends at low nominal rate
- Government policy becomes a (high) money growth rule, get GE equilibrium of a high money growth rule
The Reichsbank Equilibrium in Math

- HH Euler equation now says

\[
\frac{P_{t+1}}{P_t} \geq \frac{\beta}{Q_{t+1}}
\]

- Equality is necessary only if \( b_t > 0 \)

- New equilibrium:
  - \( B_t = 0 \),
  - \( M_t = M_{t-1} + B_{t-1} + P_{t-1} G_{t-1} - T_t \)
  - \( M_t / P_t = C_t \)
  - \( u'(C_t) = \frac{P_t}{\beta P_{t-1}} \)
Is this just about the Reichsbank?

- So far, two equilibria: intended equilibrium and hyperinflation
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- Many frictions can lead to runs with lower inflation:
  - Long-term bonds
  - Limited participation
  - Rational inattention
  - Cost of accessing the market (going to the bond vending machine)
Illustration: Limited Participation

- Same environment as before, except:
- Households divided into $T$ groups
- Each group can only produce every $T$ periods
New household necessary conditions

- $u'(c^j_t) = \beta \lambda^j_t$
- $1 = \lambda^j_{j+kT} P_{j+kT}$
- $Q_t \lambda^j_t \geq \beta \lambda^j_{t+1}, \quad \text{if } B^j_t > 0$
The Intended Equilibrium

- Borrowing constraint not binding
- $u'(C_t^j) = 1/\bar{Q}$
- $P_{t+1}/P_t = \beta/\bar{Q}$
- $P_t C_t^j = M_t^j$
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- $P_{t+1}/P_t = \beta/\bar{Q}$
- $P_tC^j_t = M^j_t$
- Requires right initial distribution of wealth, right initial price level
- (Otherwise, more in general) periodic allocation and $P_{t+T}/P_t = \beta/\bar{Q}^T$
Run in period $t$ only (Intuition)

- Only period-$t$ producers borrowing constrained
- Other households cut back on bond purchases, but less
- CB is not completely cornering the market, but selling pressure emerges and money increases
- Inflation more limited
Conclusion

- Interest rate rules are subject to runs just as exchange rate pegs
- Runs more severe if interest target is on a deep market
- How do we really achieve price stability?
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• Interest rate rules are subject to runs just as exchange rate pegs
• Runs more severe if interest target is on a deep market
• How do we really achieve price stability?
• Commodity money - fiscal policy? (back to Sargent)
Interpretation of Interest Rate Rules

Two interpretations of interest rate rules:

- “Prescribed guide for monetary policy conduct” (Svensson and Woodford, 2005)
  - Implementation is left to the wizards at the trading desk in NY
- Here: central bank strategy to achieve unique implementation of a desirable equilibrium.
  - We are muggles trying to make sense of the wizardry
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  - We are muggles trying to make sense of the wizardry
  - Of course, as muggles we fail