A Ramsey Theory of Low Interest Rates

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Abstract

Berndt, Lustig, and Yeltekin (2012) document that a large fraction of fiscal shocks are absorbed neither by taxes nor by an immediate drop in the value of government debt, but rather by continuing low rates of return in the years following the shock. This is similar to the findings documented by Reinhart and Sbrancia (2015). From the perspective of standard models of optimal taxation, this finding is puzzling: typically, the government should finance such shocks either through immediate contingent taxes, or by previously-issued state-contingent debt, or by increasing labor taxes, with only minor effects arising from intertemporal distortions. We study how this answer changes in the presence of financial frictions such that the entrepreneurs' net worth has a direct effect on their ability to invest. In this case, such net worth is affected by the government policy through changes in intertemporal prices. A new trade-off emerges between pursuing policies that relax the financial constraints and those that minimize the distortions of financing government spending, providing a potential avenue to reconcile the empirical findings with the theory.

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1 Introduction

How should governments finance bad fiscal shocks, such as wars or a recession? This question has attracted much interest in the past. From the perspective of the optimal-taxation models

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in Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1994), bad shocks should be accompanied by devaluations of government debt (possibly through inflation, as in Siu (2004)), so the low rates of inflation and high build-up of debt may seem surprising.

Other papers, such as Barro (1979) and Aiyagari, Marcet, Sargent, and Seppälä (2002), rule out state-contingent debt and capital taxes; in this case government debt acts as a shock absorber and increases following a bad shock. In this case, the bad shocks will be absorbed by future taxes.

From the perspective of this line of theoretical analysis, the empirical findings of Berndt, Lustig, and Yeltekin (2012) and Reinhart and Sbrancia (2015) are surprising. They find that, in the aftermath of bad fiscal shocks, such as World War II, interest rates on government debt remain persistently low and help in reducing the required future primary surpluses needed to stabilize government finances.

In this paper, we revisit this question in a model in which liquidity frictions feature prominently. We adopt an otherwise standard neoclassical growth model, but now investment is undertaken by entrepreneurs whose net worth affects their ability to access external sources of finance. This new feature provides two twists to the traditional answers of the Ramsey optimal taxation literature. First, government debt is now a source of liquidity in the economy. Second, the classic no-capital taxation result is no longer true when capital prices are endogenous and enter directly into the liquidity constraints of entrepreneurs.

In the model, private agents face *idiosyncratic* investment risks. Some of them have investment projects, while others do not. There is a government that has an exogenous stream of expenditures to finance. It can tax income from labor and capital, and it can issue (aggregate) state-contingent government debt with one period maturity.

When private agents have investment projects, they seek outside financing. But, because of the financial frictions, these agents are constrained in issuing new claims and/or reselling old claims to capital. Government bonds are fully liquid, but privately-issued claims are partially liquid. Before the realization of idiosyncratic investment risks, agents have thus a precautionary motive to accumulate government bonds.

If the government responds to a bad fiscal shock by imposing a large immediate devaluation of its liabilities, this would have severe repercussions on private investment. It can thus pay off to delay the fiscal adjustment and rely more on future taxes. However, the fact that the optimal tax mix involves capital as well as labor taxes makes it possible for the optimal solution to distort future capital accumulation as a way of alleviating the additional burden falling on public finances. We thus study how a government trades off reducing endogenous financial frictions and financing its expenditures in the least costly way. The portfolio of the national savings, in privately-issued assets (backed by capital stock) and in government bonds, is at the center stage of the analysis.

This paper has financial frictions in the form of liquidity frictions similar to Kiyotaki and Moore (2012) and Shi (2015)[1]. We are further exploring the role of endogenizing these frictions through the directed search framework of Cui and Radde (2016) and Cui (2016), so that the supply of government debt can affect the participation in asset markets.[2] Search frictions literally exist in many markets, such as markets for corporate bonds, IPO, and acquisition. They can also capture many aspects of frictional financial markets with endogenous market participation (see e.g., Vayanos and Wang, 2013; Rocheteau and Weill, 2011), while still keeping the simple structure of neoclassical macro framework. This tractability is crucial since one can use all the insight from a standard Ramsey plan. In particular, we use the "primal approach" (see e.g., Lucas and Stokey, 1983; Chari and Kehoe, 1999) to show the allocations chosen by a Ramsey planner.

The presence of liquidity constraints opens up the possibilities for government bonds or fiat money to circulate, as in Holmström and Tirole (1998). That is, if private liquidity is not enough, public liquidity can be added to improve efficiency.[3] In this paper, government debt provides liquidity service and has a "crowding-in" effect, similar to Woodford (1990).[4] This paper provides a novel channel in which public liquidity provision is costly due to distortionary taxation. Therefore, an optimal supply of public liquidity emerges.

Our work is complementary to Angeletos, Collard, Dellas, and Diba (2013), who study a model where non-state-contingent government bonds also may crowd in private investment. In their context, capital-income taxes are ruled out, and a secondary-market price for capital is not present. In their paper, the main link between interest rates and government debt goes from the former to the latter: shocks that decrease returns call for additional issuance. We analyze the opposite force, when shock to the fiscal needs of the government lead policy to optimally interfere with intertemporal prices, providing a possible justification for the empirical findings that motivate our research.

Azzimonti and Yared (2017, 2019) consider optimal supply of public liquidity with lump-

[1]Similar papers at least include Nezafat and Slavik (2010), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017), Ajello (2016), and Bigio (2012).

[2]Recent work by Lagos and Rocheteau (2008), Rocheteau (2011), and Cao and Shi (2014) also use search to endogenize liquidity and asset price, but not on the linkage between asset liquidity and asset price as in this paper. Endogenous liquidity gives rise to different degree of liquidity constraints and risk sharing.

[3]There are thus both fully liquid government issued assets and partially liquid private claims. Changing the portfolio compositions of the two assets can potentially affect the real economy. More recently, financial intermediations are added and the policy affects the asset compositions held by intermediaries. See, for example, Gertler and Karadi (2011) and Gertler and Kiyotaki (2010).

[4]This aspect is in contrast with Aiyagari and McGrattan (1998), where government debt is a perfect substitute to private assets (or capital stock). There, government debt relaxes agents' borrowing constraints but also crowds out capital accumulation.

sum taxes when agents differ in their income. The issuance of public debt is at the stage when agents know their types. Their framework also generates an incentive for the government to manipulate debt prices, keeping interest rate low and some agents liquidity constrained. Compared to their work, we consider the role of idiosyncratic risk and precautionary savings in determining investment. This opens up the discussion of the interaction between low interest rates, capital-income taxes, and capital accumulation.

Finally, Farhi (2010) adds capital to Aiyagari, Marcet, Sargent, and Seppälä (2002) and shows that the results from Aiyagari, Marcet, Sargent, and Seppälä (2002) extend to this case, so that no further major role emerges for distorting intertemporal prices. Our analysis emphasizes capital taxation and supply of public liquidity when *both* idiosyncratic risks and aggregate risks are present, which leads to rich precautionary motives for holding liquidity. The precautionary incentive interacts with distortionary taxation, which alters the classic results in Ramsey taxation. Contrary to Farhi (2010) and Aiyagari, Marcet, Sargent, and Seppälä (2002), we analyze a situation in which markets are complete with respect to aggregate shocks (that is, government debt can be contingent on the realization of shocks), but insurance is not possible against idiosyncratic risks.

2 The Two Period Model

We start our analysis with a two period model which abstracts from aggregate uncertainty. The provision of public liquidity as collateral is exogenous, and we analyze how liquidity frictions affect the choice of distorting interest rates, and how this choice in turn depends on the fiscal constraints faced by the government. Throughout the paper, we use lowercase variables for individual choices, and uppercase for aggregate allocations.

The economy is populated by a continuum of families, each of which is composed by a continuum of agents. In period 1, a fraction χ of agents from each household are revealed to be entrepreneurs and the remainder $1-\chi$ are workers. Entrepreneurs and workers are separated at the beginning of the period. They both start the period with K_0 units of initial capital per capita; the entrepreneurs have B_0^i units of government bonds per capita, whereas workers have B_0^n units per capita,[5] and we define total per capita bonds to be[6]

$$B_0 \equiv B_0^i + B_0^n.$$

[5]The per-entrepreneur level of initial bonds that entrepreneurs have is therefore B_0^i/χ, and the per-worker amount owned by workers is $B_0^n/(1-\chi)$.

[6]In multi-period versions, the identity of entrepreneurs will not be known ex ante and $B_0^i = B_0^n$. We keep the two initial conditions separate because it allows us to study how the problem changes as a function of the entrepreneurs' initial net worth.

In each period, a continuum of firms can operate a constant-returns to scale technology that uses capital and labor to produce output $F(K_{t-1}, L_t)$. We assume that F satisfies Inada conditions, so we can neglect corner solutions. Firms hire labor and rent capital in competitive markets at the wage rate w_t and the rental rate r_t. Workers supply labor to the firms. Entrepreneurs do not supply labor. Rather, they can turn one unit of the firms' output into one unit of new capital to be used in the subsequent period. This ability will only be used in the first period, since the economy ends after period 2. Entrepreneurs cannot sell the capital directly, but they can sell claims to the capital k_1 that they produce in a frictional competitive market, in the amount $s_1^i \geq 0$, subject to a financing constraint:

$$s_1^i \leq \phi_1 k_1, \tag{1}$$

where $\phi_1 < 1$ is for now an exogenous limit[7]. An entrepreneur's budget constraint in period 1 is given by

$$k_1 \leq r_1 K_0 + B_0^i/\chi + q_1^i s_1^i : \tag{2}$$

entrepreneurs can only borrow by selling claims to capital at the price q_1^i, and any left-over funds after investment has taken place are brought back to the family at the end of the period. We will typically be interested in equilibria where constraint ((2)) is binding and entrepreneurs use all of their available funds to undertake new investment.

Workers use some of their income to purchase new claims to capital from entrepreneurs and new government debt b_1^n, and return the remaining funds to the family. Their period-1 budget constraint is

$$q_1^n s_1^n + p_1^b b_1^n \leq r_1 K_0 + B_0^n/(1-\chi) + w_1 \ell_1, \tag{3}$$

where $s_1^n \geq 0$ is the end-of-period private claims on capital that they purchase, ℓ_t is their labor supply, q_1^n is the price at which claims to capital can be bought, and p_1^b is the price of government bonds, which are a risk-free claim to one unit of goods in period 2.

Claims to capital are subject to an intermediation cost. Intermediaries are competitive and their cost is η per unit of capital intermediated; therefore we have

$$q_1^n = \eta + q_1^i. \tag{4}$$

At the end of the first period, entrepreneurs and workers rejoin their family, pool their capital

[7] In a future version, we will analyze the implication of endogenous liquidity, as inCui and Radde (2016) and Cui (2016).

and their left-over funds, pay taxes, and consume. Their constraint is

$$c_1 + \tau_1^\ell w_1 \ell_1 (1-\chi) = w_1 \ell_1 (1-\chi) + r_1 K_0 + B_0 - (1-\chi) q_1^n s_1^n - p_1^b (1-\chi) b_1^n - \chi (k_1 - q_1^i s_1^i), \quad (5)$$

where c_t is the family's consumption in period t, and τ_t^ℓ is the tax rate on labor income. For the usual reasons, we constrain the government not to tax initial capital.

In period 1, the government budget constraint ensures that its revenues from labor-income taxation and new borrowing cover debt repayments that become due as well as any government spending G_1:[8]

$$G_1 + B_0 = p_1^b B_1 + \tau_1^\ell w_1 L_1. \quad (6)$$

The second and final period is similar to the first, except that no new investment takes place, so that entrepreneurs no longer have any role. We can then collapse the two subperiods, and simply write the joint family budget constraint

$$c_2 = (1-\tau_2^k) r_2 [\chi (k_1 - s_1^i) + (1-\chi) s_1^n] + (1-\tau_2^\ell) w_2 (1-\chi) \ell_2 + B_1, \quad (7)$$

and the government's

$$G_2 + B_1 = \tau_2^k r_2 K_1 + \tau_2^\ell w_2 L_2, \quad (8)$$

with G_2 being government spending in the second period.

Contrary to period 1, the government is allowed to tax (or subsidize) capital in the second period at a rate τ_2^k, and our goal is to study how this power is used in the presence of credit frictions.

The household preferences are represented by:

$$\sum_{t=1}^{2} \beta^t \left[u(c_t) - (1-\chi)^2 v\left(\frac{\ell_t}{1-\chi}\right) \right], \quad (9)$$

where u and v are strictly increasing and continuously differentiable functions, u is strictly concave, and v is strictly convex.[9]

In summary, the household maximizes (9), subject to (1), (2), (5), and (7).

[8]Note that the individual labor supply is normalized in per-worker terms, while the aggregate labor supply is in per-capita terms. So, an aggregate labor supply L_1 corresponds to a supply of $L_1/(1-\chi)$ for each worker. Similar normalizations occur for aggregate capital K_1, bonds B_1, and intermediated capital S_1^n.

[9]The particular choice of scale for the function v is a pure normalization that is convenient to obtain simpler expressions when studying the aggregate allocation.

2.1 Competitive equilibrium

We next characterize a competitive equilibrium.

As a first step, we note that, in any equilibrium in which $q_1^i < 1$, there would be no sales of capital. To see this, consider a family whose entrepreneurs are selling capital. By reducing investment and capital sales one for one, the family can simultaneously relax the constraints (1), (2), and (5). The last budget constraint is necessarily binding, since families would otherwise increase their consumption, hence the original plan cannot be optimal.

With this observation, we can limit our analysis to $q_1^i \geq 1$ without loss of generality: for any competitive equilibrium in which $q_1^i < 1$, there exists a competitive equilibrium with the same allocation and the same prices, except for $q_1^i = 1$ and $q_1^n = 1 + \eta$.

In any competitive equilibrium, market clearing implies that $S_1 \equiv S_1^n = S_1^i$, where S_1 is the per-capita level of intermediated capital.

From the intermediaries' and firms' optimality conditions, we obtain (4),

$$w_t = F_L(K_{t-1}, L_t) \tag{10}$$

and

$$r_t = F_K(K_{t-1}, L_t). \tag{11}$$

From the families' necessary and sufficient first-order conditions we obtain:

- Period-1 labor supply:
$$(1 - \tau_1^\ell) w_1 u'(C_1) = v'(L_1); \tag{12}$$

- Period-2 labor supply:
$$(1 - \tau_2^\ell) w_2 u'(C_2) = v'(L_2); \tag{13}$$

- Demand for government bonds:
$$p_1^b = \frac{\beta u'(C_2)}{u'(C_1)}; \tag{14}$$

- Demand for claims on capital:
$$q_1^n \geq \frac{\beta u'(C_2)}{u'(C_1)} (1 - \tau_2^k) r_2, \tag{15}$$

  with equality if $S_1 > 0$;

- Investment and supply of claims:

$$u'(C_1) \leq \beta u'(C_2)(1-\tau_2^k)r_2, \tag{16}$$

with equality if $S_1 = 0$, and

$$q_1^i = \max\left\{1, \frac{K_1 - (\chi r_1 K_0 + B_0^i)}{\phi_1 K_1}\right\}. \tag{17}$$

A competitive-equilibrium allocation must satisfy the resource constraints:

$$F(K_0, L_1) = C_1 + K_1 + S_1\eta + G_1 \tag{18}$$

and

$$F(K_1, L_2) = C_2 + G_2. \tag{19}$$

In addition, the budget constraints (5) and (7) must be satisfied as an equality by the aggregate allocation (chosen by the representative family). The government budget constraint holds by Walras' law, and (3) does not bind.

If $q_1^i > 1$, then both the financing constraint and the entrepreneurs' budget constraint (1) and (2) bind, whereas if $q_1^i = 1$ (1) is certainly slack and (2) may or may not bind.

3 Optimal Policy

3.1 Forming the policy problem

We study the Ramsey outcome, that is, the best competitive equilibrium. To do so, we follow the primal approach, deriving a set of necessary and sufficient conditions for an allocation to be part of a competitive equilibrium, without reference to prices and tax rates. These conditions include a restriction that allows us to derive intermediated capital S_1 given the other variables (equation (20) below), and it is thus convenient to also substitute out this variable from the policy problem.

Given any allocation, we can ensure that (10) and (11) hold by setting factor prices w_t and r_t to the appropriate marginal product. Similarly, we can ensure that (12) and (13) hold with a suitable choice of τ_t^ℓ, $t = 1, 2$; (14) holds for the appropriate choice of p_1^b.

Next, in order for (2) and (1) to hold and for S_1 to be optimally chosen, we must have

$$S_1 = \begin{cases} 0 & \text{if } K_1 \leq (1-\phi_1)K^*, \\ K_1 - (1-\phi_1)K^* & \text{if } K_1 \in ((1-\phi_1)K^*, K^*], \\ \phi_1 K_1 & \text{if } K_1 > K^*, \end{cases} \tag{20}$$

where

$$K^* := \frac{\chi F_K(K_0, L_1)K_0 + B_0^i}{1-\phi_1}.$$

K^* is the maximum level of investment that entrepreneurs can finance when $q_1^i = 1$, and $(1-\phi_1)K^*$ is the maximum that they can finance using internal funds only.

τ_2^k can then be chosen so that either (15) or (16) hold as an equality, depending on whether S_1 is 0 or positive, with the remaining of the two equations holding as the appropriate inequality. Finally, equation (17) can be used to determine q_1^i and (4) to determine q_1^n.

The remaining conditions that characterize a competitive equilibrium are the following:

- The resource constraints (18) and (19); and
- The household budget constraints evaluated at the aggregate allocation, (2), (5), (7).[10]

Substituting prices and tax rates from the first-order conditions, we can aggregate the household budget constraints into the following implementability constraint:

$$\sum_{t=1}^{2} \beta^{t-1}[u'(C_t)C_t + v'(L_t)L_t] - u'(C_1)(F_K(K_0, L_1)K_0 + B_0)$$
$$= \begin{cases} 0 & \text{if } K_1 \leq (1-\phi_1)K^* \\ (1-\phi_1)u'(C_1)\left(\eta \max\{K_1, K^*\} + (1/\phi_1)\max\{0, K_1 - K^*\}\right) & \text{otherwise.} \end{cases}, \tag{21}$$

The implementability constraint has three branches, corresponding to the three possible types of equilibria in our economy. When the entrepreneurs are sufficiently wealthy, all of the investment is financed internally and rates of return reflect the entrepreneurs' intertemporal trade-off. As they become more constrained, they start issuing claims to capital, but the financing constraint remains slack and $q_1^i = 1$. In this case, the family's net worth in the intertemporal budget constraint goes up if more wealth belongs to the entrepreneurs rather than the workers, and the right-hand side of (21) reflects this extra value of the initial resources in the hands of entrepreneurs. Finally, when the financing constraint binds, this

[10] The financing constraint holds by construction when (17) holds.

effect is magnified in that additional investment can only take place if the price of capital q_1^i (and q_1^n) increases.

We now derive the first-order conditions that must hold if the Ramsey plan is interior. However, it is possible that the plan will be at one of the two corners, which need to be checked separately. We are particularly interested in studying comparative statics when the financing constraint is binding and Tobin's q responds to investment, which will be the case when the entrepreneurs' wealth is sufficiently low relative to the return on capital and the government's resources are sufficiently scarce relative to its spending.

Let $\beta^{t-1}\lambda_t$ be the Lagrange multiplier on the resource constraint, and Φ be the Lagrange multiplier on the implementability constraint.

The planner's first-order conditions for consumption C_1 and C_2 are

$$\begin{aligned}&u'(C_1)(1+\Phi)+\Phi u''(C_1)C_1-\lambda_1-\Phi u''(C_1)(F_K(K_0,L_1)K_0+B_0)\\&=\begin{cases}0 & \text{if } K_1<(1-\phi_1)K^*\\ \Phi(1-\phi_1)u''(C_1)\left(\eta\max\{K_1,K^*\}+(1/\phi_1)\max\{0,K_1-K^*\}\right) & \text{if } K_1>(1-\phi_1)K^*.\end{cases},\end{aligned} \tag{22}$$

$$u'(C_2)(1+\Phi)+\Phi u''(C_2)C_2=\lambda_2 \tag{23}$$

The planner's first-order conditions for labor supply L_1 and L_2 are

$$\begin{aligned}&v'(L_1)(1+\Phi)+\Phi v''(L_1)L_1-\lambda_1F_L(K_0,L_1)\\&=\begin{cases}0 & \text{if } K_1<(1-\phi_1)K^*\\ \Phi u'(C_1)\eta\chi F_{KL}(K_0,L_1)K_0 & \text{if } K_1\in((1-\phi_1)K^*,K^*)\\ \Phi u'(C_1)\,(1/\phi_1)\,\chi F_{KL}(K_0,L_1)K_0 & \text{if } K_1>K^*\end{cases},\end{aligned} \tag{24}$$

$$v'(L_2)(1+\Phi)+\Phi v''(L_2)L_2=\lambda_2F_L(K_1,L_2) \tag{25}$$

The first-order conditions for capital K_1 is

$$\lambda_1-\beta\lambda_2F_K(K_1,L_2)=\begin{cases}0 & \text{if } K_1\le(1-\phi_1)K^*\\ -\lambda_1\eta & \text{if } K_1\in((1-\phi_1)K^*,K^*)\\ -\lambda_1\phi_1\eta-\Phi u'(C_1)(1-\phi_1)\,(1/\phi_1+\eta) & \text{if } K_1>K^*\end{cases} \tag{26}$$

3.2 A Special Case

To clarify the role of different distortions, we consider the special case of a Cobb-Douglas production function, $F(K_{t-1}, L_t) = AK_{t-1}^{\alpha} L_t^{1-\alpha}$ and preferences given by

$$u(c_t, \ell_t) = c_t - \frac{\ell_t^{1+\psi}}{1+\psi}.$$

These preferences are convenient because they abstract from the usual incentive to distort intertemporal prices and devalue the families' initial claims, as emphasized by Armenter (2008). We can then instead focus on one of the new channels of intertemporal distortions that arise from financial frictions. In this case, the first-order conditions are given by

$$1 + \Phi = \lambda_1, \tag{27}$$

$$1 + \Phi = \lambda_2, \tag{28}$$

$$\begin{aligned} &L_1^{\psi}[1 + \Phi(1)(1+\psi)] - \lambda_1 A(1-\alpha)(K_0/L_1)^{\alpha} \\ &= \begin{cases} 0 & \text{if } K_1 < (1-\phi_1)K^* \\ \Phi\eta\chi A(1-\alpha)\alpha(K_0/L_1)^{\alpha} & \text{if } K_1 \in ((1-\phi_1)K^*, K^*) \\ \Phi\,(1/\phi_1)\,\chi A(1-\alpha)\alpha(K_0/L_1)^{\alpha} & \text{if } K_1 > K^* \end{cases}, \end{aligned} \tag{29}$$

$$L_2^{\psi}[1 + \Phi(1+\psi)] = \lambda_2 A(1-\alpha)(K_1/L_2)^{\alpha}, \tag{30}$$

and

$$\lambda_1 - \beta\lambda_2 A\alpha(K_0/L_1)^{\alpha-1} = \begin{cases} 0 & \text{if } K_1 \leq (1-\phi_1)K^* \\ -\lambda_1\eta & \text{if } K_1 \in ((1-\phi_1)K^*, K^*) \\ -\lambda_1\phi_1\eta - \Phi(1-\phi_1)\,(1/\phi_1 + \eta) & \text{if } K_1 > K^* \end{cases}. \tag{31}$$

Comparing the planner's optimality condition for capital (31) with the household optimality conditions (15) and (16), we can thus establish the following:

- If the allocation is such that the financing constraint of the entrepreneurs is not binding, then capital-income taxes are optimally set to zero, independently of the tightness of the government budget constraint (captured by the multiplier Φ). In this, case, we recover the standard result that it is not optimal to tax capital, which is an intermediate input. This case can arise either because entrepreneurs have enough wealth to finance investment internally, in which case the private cost of investment is 1 and the social cost is $1+\Phi$, or when they need to sell part of their capital, but not to the point at which q^i need to exceed 1; in this case the private cost of investment (through purchases of capital by the workers) is $1+\eta$ and the social cost is $(1+\eta)(1+\Phi)$. In both cases, the private reward in the second period is βr_2 and the social reward is $\beta r_2(1+\Phi)$. Thus, while the intertemporal trade-off is different and the equilibrium interest rate is different across the two cases, in both of them private and social costs are proportional to each other and capital-income taxes are zero; moreover, in both cases the trade-off coincides with the marginal rate of transformation coming from technology alone, taking into account the costs of intermediation.

- When entrepreneurs are sufficiently poor that the financing constraint binds, we obtain a very different result. In this case, in the absence of capital taxes or subsidies, the private rate of return does not coincide with the marginal rate of transformation. Furthermore, changes in the level of investment have an effect on the price of capital, and a higher price of capital tightens in turn the implementability constraint, forcing the government to raise more funds through distortionary taxes.[11] If the government has abundant resources and $\Phi \approx 0$, comparing (31) and (15) (taking into account $K_1 > K^*$) we can see that the optimal policy calls for a capital subsidy, which encourages workers to buy more capital even though its market price q^n exceeds the marginal rate of transformation $1+\eta$. However, as the cost of public funds Φ increases, the capital subsidy is reduced, and it eventually becomes ambiguous whether a government strapped for cash would subsidize or tax capital.

  By assuming linear preferences, we automatically imposed from equation (14) that $p_b^1 = \beta$, that is, the government choice of taxes or subsidies has no effect on the rate of return on government debt. A further channel at work when preferences are not linear is that a capital-income tax reduces the after-tax return on capital, and hence further favors government debt, which is a further beneficial force in the case of a constrained government. This effect appears on the right-hand side of equation (22) and we will analyze it in a more complete version of the paper.

[to be completed]

[11] The Lagrange multiplier Φ can be viewed as the cost to the planner of starting with an extra unit of government debt in period 0.

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