Online Appendix for "Financing Through Asset Sales"

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A Proofs

Proof of Lemma 1

The IC condition (9) is stronger than the ND condition (7) if and only if

$$\frac{(C_H + A_H)A_L(1+\overline{k}) - \mathbb{E}\left[A\right](C_L + A_L)}{\mathbb{E}[A] - A_L(1+\overline{k})} < \frac{(C_H + A_H)\mathbb{E}[A] - (C_L + A_L)A_H(1+\overline{k})}{A_H(1+\overline{k}) - \mathbb{E}[A]}$$

This yields $(1+\overline{k}) < \frac{\mathbb{E}[A]}{\sqrt{A_H A_L}}$.

Proof of Lemma 2

$F^{EPE,IC}$ is greater than $F^{EPE,ND,H}$ if and only if

$$\frac{A_L\mathbb{E}[C+A](1+\underline{k}) - A_H(C_L + A_L)}{A_H - A_L(1+\underline{k})} > \frac{A_L(C_H + A_H) - A_H\mathbb{E}[C+A](1+\underline{k})}{A_H(1+\underline{k}) - A_L}$$

which becomes:

$$1+\underline{k} > \frac{A_H A_L}{\pi A_H^2 + (1-\pi)A_L^2} = \frac{A_H A_L}{\mathbb{E}[A^2]}.$$

Proof of Lemma 3

We start by analyzing the magnitudes of the cutoffs k_H^* and k_L^*; these results apply regardless of whether the SE is full or partial. We then derive conditions under which we have a FSE, or a PSE. From the cutoff equation (18), we have

$$\frac{A_L(1+k_L^*)}{E_L} = \frac{A_H(1+k_H^*)}{E_H} = \frac{\mathbb{E}[A|X=A]}{\mathbb{E}[E|X=E]}.$$

These equations mean that, in any SE, k_L^* and k_H^* obey the following relationship:

$$1+k_H^* = \lambda(F)(1+k_L^*), \tag{37}$$

where $\lambda(F) \equiv \frac{A_L}{A_H}\frac{E_H}{E_L}$ and is decreasing in F. If $F < (>) F^*$, then $\lambda > (<) 1$ so $k_H^* > (<) k_L^*$ from (37). To ascertain the sign of k_H^*, cross-multiplication of (18) shows that $k_H^* > 0$ if and only if

$$E_H\mathbb{E}[A|X=A] > A_H\mathbb{E}[E|X=E]. \tag{38}$$

We start with case (ia), i.e. $F < F^*$. Since $k_H^* > k_L^*$, there is a positive (negative) price reaction to asset (equity) sales, and so $\mathbb{E}[A|X = A] > \mathbb{E}[A]$ and $\mathbb{E}[E|X = E] < \mathbb{E}[E]$. Thus, a sufficient condition for (38) is $E_H\mathbb{E}[A] > A_H\mathbb{E}[E]$. This condition is equivalent to $F < F^*$, the condition required for case (ia) in the first place. Moving to case (ib), $k_H^* < 0$ if and only if (38) is violated. Since $k_H^* < k_L^*$, we now have $\mathbb{E}[A|X = A] < \mathbb{E}[A]$ and $\mathbb{E}[E|X = E] > \mathbb{E}[E]$. Thus, a sufficient condition is $E_H\mathbb{E}[A] < A_H(\mathbb{E}[E])$. This condition is equivalent to $F > F^*$, the condition required for case (ib) in the first place. For case (ic), we have $\lambda(F^*) = 1$, and so $k_H^* = k_L^*$. If both qualities follow the same cutoff strategies, assets and equity are valued at their unconditional expectations. Thus, the quantities on the RHS of (18) are both equal to one, implying that both cutoffs are equal to zero.

We now derive conditions under which FSE exists. We start with part (iia), where $F < F^*$. The ND condition for (H, k_H^*) is $1 + k_H^* = \frac{E_H}{A_H}\frac{\mathbb{E}[A|X=A]}{\mathbb{E}[E|X=E]}$. Given a pair of cutoff rules k_H^* and k_L^*, and associated valuations $\mathbb{E}[A|X = A]$ and $\mathbb{E}[E|X = E]$, for some H-firms to be willing to issue equity, we must have

$$1 + \overline{k} > \frac{E_H}{A_H}\frac{\mathbb{E}[A|X = A]}{\mathbb{E}[E|X = E]}. \tag{39}$$

The RHS is bounded below by $\frac{E_H}{A_H}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$ (since $k_H^* > k_L^*$, we have $\mathbb{E}[A] < \mathbb{E}[A|X = A]$ and $\mathbb{E}[E] > \mathbb{E}[E|X = E]$) and above by $\frac{E_H}{E_L}$. Thus, a sufficient condition for some H-firms to issue equity is $1 + \overline{k} \geq \frac{E_H}{E_L}$ and a necessary condition is $1 + \overline{k} > \frac{E_H}{A_H}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$. Intuitively, if F and $\overline{k}$ are too low, the certainty effect is sufficiently weak that the (certainty-adjusted) information asymmetry of equity is so much higher than that of assets, that even the H-firm with greatest synergies (i.e. $(H, \overline{k})$) will sell assets.

We now turn to the indifference condition for (L, k_L^*), which is

$$1 + k_L^* = \frac{E_L}{A_L}\frac{\mathbb{E}[A|X = A]}{\mathbb{E}[E|X = E]}. \tag{40}$$

We use the intermediate value theorem ("IVT") to derive necessary and sufficient conditions for (L, k_L^*) not to deviate. Suppose we specify a candidate pair of cutoffs k_L' and $k_H' \equiv \lambda(F)(1 + k_L') - 1$, where types (q, k_q') sell assets for $k < k_q'$ and issue equity for $k > k_q'$. Assuming $\overline{k} > k_H'$ so that k_H' is feasible, this constitutes an equilibrium if and only if (L, k_L') is indifferent between the two claims. The incentive of (L, k_L') to sell assets is a function continuous in k'_L:

$$f(k'_L) \equiv \frac{E_L}{\mathbb{E}[E|X=E]} - \frac{A_L(1+k'_L)}{\mathbb{E}[A|X=A]}.$$

If $f(k'_L) > (<)\, 0$, (L, k'_L) will sell assets (equity). Thus, k'_L is an equilibrium cutoff if and only if $f(k'_L) = 0$. Our proof strategy is the following: for a given $F < F^*$, we show that (L, k'_L) sells assets if $1 + k'_L = \frac{E_L}{A_L}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$, and equity if $1 + k'_L = \frac{1+\overline{k}}{\lambda(F)}$. (The latter is the highest possible k'_L given that k'_L and k'_H are related by (37), and k'_H is capped at $\overline{k}$.) Then, by the IVT, there exists a k^*_L between these two values of k'_L for which $f(k^*_L) = 0$ and so the firm is indifferent.

To show that (L, k'_L) sells assets if $1 + k'_L = \frac{E_L}{A_L}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$, we use the fact that $F < F^*$ implies $\lambda(F) > 1$ and so $k'_H > k'_L$. We thus have $\mathbb{E}[A|X = A] > \mathbb{E}[A]$ and $\mathbb{E}[E|X = E] < \mathbb{E}[E]$, which yields $f(k'_L) > 0$. In addition, $1 + k'_L = \frac{1+\overline{k}}{\lambda(F)}$ yields

$$f(k'_L) = \frac{E_L}{\mathbb{E}[E|X=E]} - \frac{A_H(1+\overline{k})}{\mathbb{E}[A|X=A]}\frac{E_L}{E_H},$$

and so $f(k'_L) < 0$ holds if and only if $1 + \overline{k} > \frac{E_H}{A_H}\frac{\mathbb{E}[A|X=A]}{\mathbb{E}[E|X=E]}$, which is the same condition as (39). Thus, the sufficient condition for H, $1 + \overline{k} \geq \frac{E_H}{E_L}$, is also sufficient for L, and so is sufficient for the SE to exist.

The analysis for part (iib) $(F > F^*)$ is analogous. The ND condition is now

$$1 + \underline{k} < \frac{E_H}{A_H}\frac{\mathbb{E}[A|X=A]}{\mathbb{E}[E|X=E]}. \tag{41}$$

With $F > F^*$ we now have $\mathbb{E}[A] > \mathbb{E}[A|X = A]$ and $\mathbb{E}[E] < \mathbb{E}[E|X = E]$, so now the RHS of (41) is bounded *above* by $\frac{E_H}{A_H}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$. Thus, a sufficient condition for some H-firms to sell assets is $1 + \underline{k} \leq \frac{A_L}{A_H}$ and a necessary condition is $1 + \underline{k} < \frac{E_H}{A_H}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$. Intuitively, if F and $\underline{k}$ are too high, the certainty effect is sufficiently strong that the (certainty-adjusted) information asymmetry of equity is so much lower than that of assets, that even the H-firm with greatest dissynergies (i.e. $(H, \underline{k})$) prefers to sell equity.

We now turn to the ND condition for (L, k^*_L), which remains (40), and again use the IVT. We can easily show that (L, k'_L) will deviate to equity at $1 + k'_L = \frac{E_L}{A_L}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$. A sufficient condition for (L, k'_L) to deviate to asset sales at $1 + k'_L = \frac{1+\underline{k}}{\lambda(F)}$ is $1 + \underline{k} \leq \frac{A_L}{A_H}$, which is the same as the sufficient condition for H, and so is sufficient for the SE to exist.

We finally turn to PSE. In case (iiia), all H-firms sell assets and L-firms choose an interior cutoff. Assets are priced at $\mathbb{E}[A|X = A] > \mathbb{E}[A]$ and equity is priced at E_L.

The ND condition for H-firms is:

$$1+\overline{k} \leq \frac{\mathbb{E}[A|X=A]}{A_H}\frac{E_H}{E_L} = \lambda(F)\frac{\mathbb{E}[A|X=A]}{A_L}. \tag{42}$$

A sufficient condition for (42) is $1+\overline{k} \leq \frac{E_H}{E_L}\frac{\mathbb{E}[A]}{A_H}$ and a necessary condition is $1+\overline{k} < \frac{E_H}{E_L}$.

The indifference condition for (L, k_L^*) yields

$$1+k_L^* = \frac{\mathbb{E}[A|X=A]}{A_L}, \tag{43}$$

and so $k_L^* > 0$: since asset sales are met with a positive price reaction (camouflage effect), L is willing to sell them even if they are synergistic. Combining (42) with (43), we have $\frac{\mathbb{E}[A|X=A]}{A_L} < 1+\overline{k} \leq \lambda(F)\frac{\mathbb{E}[A|X=A]}{A_L}$. Since $\lambda(F^*) = 1$ and $\lambda'(F) < 0$, both conditions can be simultaneously satisfied only if $F < F^*$.

Intuitively, $\frac{\mathbb{E}[A|X=A]}{A_L} < 1+\overline{k} \leq \lambda(F)\frac{\mathbb{E}[A|X=A]}{A_L}$ shows that synergies must be so strong that $\left(L, \overline{k}\right)$ eschews the capital gain from selling overvalued assets and chooses to retain synergistic assets. However, synergies cannot be so strong as to induce $\left(H, \overline{k}\right)$ to deviate to equity. These conditions can simultaneously be satisfied because L considers the gain from selling overvalued *assets*, and H considers the loss from selling undervalued *equity*. Since equity exhibits higher information asymmetry, H will not deviate.

Moreover, for (43) to hold, we must have $1+\overline{k} > \frac{\mathbb{E}[A|X=A]}{A_L}$, for which $1+\overline{k} > \frac{A_H}{A_L}$ is a sufficient condition and $1+\overline{k} > \frac{\mathbb{E}[A]}{A_L}$ is a necessary condition. Intuitively, if $\overline{k}$ is sufficiently low, then all Ls would sell assets, even the type with the highest synergies, since they will get a capital gain of $\frac{\mathbb{E}[A|X=A]}{A_L}$ that is greater than the loss of synergies.

Finally, we need to show that a cutoff k_L^* actually exists at which the cutoff type (L, k_L^*) is indifferent between asset sales and equity (at which the equilibrium condition (43) holds). We again employ the IVT. If we specify a cutoff $1+k_L'$ equal to the necessary lower bound $\frac{\mathbb{E}[A]}{A_L}$ on $1+\overline{k}$, (L, k_L') deviates to asset sales. Meanwhile, if we specify $1+k_L' = \frac{A_H}{A_L}$, (L, k_L') deviates to equity. Thus, a pair of sufficient conditions for existence of the equilibrium is $1+\overline{k} \geq \frac{A_H}{A_L}$ and $1+\overline{k} \leq \frac{E_H}{E_L}\frac{\mathbb{E}[A]}{A_L}$.

In case (iiib), all H-firms issue equity and L-firms choose an interior cutoff. Assets are priced at A_L and equity is priced at $\mathbb{E}[E|X=E] > \mathbb{E}[E]$. The indifference condition for (L, k_L^*) yields

$$1+k_L^* = \frac{E_L}{\mathbb{E}[E|X=E]}, \tag{44}$$

and so $k_L^* < 0$. For (44) to hold, we must have $1+\underline{k} < \frac{E_L}{\mathbb{E}[E|X=E]}$, for which $1+\underline{k} \leq \frac{E_L}{E_H}$ is a sufficient condition and $1+\underline{k} < \frac{E_L}{\mathbb{E}[E]}$ is a necessary condition. Intuitively, if

$\underline{k}$ is sufficiently high, then all Ls would sell equity, even the type with the greatest dissynergies, since they will get a capital gain of $\frac{\mathbb{E}[E|X=E]}{E_L}$ that is greater than the avoidance of dissynergies.

The ND condition for H-firms is

$$1+\underline{k} \geq \frac{A_L}{A_H}\frac{E_H}{\mathbb{E}[E|X=E]} = \lambda(F)\frac{E_L}{\mathbb{E}[E|X=E]}. \tag{45}$$

A sufficient condition for (45) is $1+\underline{k} \geq \frac{A_L}{A_H}\frac{E_H}{\mathbb{E}[E]}$ and a necessary condition is $1+\underline{k} > \frac{A_L}{A_H}$. Combining (45) with (44), we have $\lambda(F)\frac{E_L}{\mathbb{E}[E|X=E]} \leq 1+\underline{k} < \frac{E_L}{\mathbb{E}[E|X=E]}$. Since $\lambda(F^*) = 1$ and $\lambda'(F) < 0$, both conditions can be simultaneously satisfied only if $F > F^*$.

Finally, we need to show that a cutoff k_L^* actually exists at which the cutoff type (L, k_L^*) is indifferent given the resulting equilibrium valuations. We again employ the IVT. If we specify a cutoff $1+k_L'$ equal to the necessary upper bound $\frac{E_L}{\mathbb{E}[E]}$ on $1+\underline{k}$, (L, k_L') deviates to equity. Meanwhile, if we specify $1+k_L' = \frac{E_L}{E_H}$, (L, k_L') deviates to asset sales. Thus, a pair of sufficient conditions for existence of the equilibrium is $1+\underline{k} < \frac{E_L}{E_H}$ and $1+\underline{k} > \frac{A_L}{A_H}\frac{E_H}{\mathbb{E}[E]}$.

Proof of Proposition 1

Parts (i), (ia), and (ib) follow from the discussion of the various equilibria in Lemmas 1-3. For (ic), we first prove $F^{EPE,IC} < F^* < F^{APE,IC}$. Suppose $F \leq F^{EPE,IC}$. This means that the IC is violated for EPE, so that $\frac{A_L(1+\underline{k})}{A_H} \geq \frac{E_L}{\mathbb{E}[E]}$. This implies $\frac{A_L}{A_H} > \frac{E_L}{E_H}$ and so $F < F^*$. Thus $F^{EPE,IC} < F^*$. Similarly, suppose $F \geq F^{APE,IC}$. This means that the IC is violated for APE, so that $\frac{E_L}{E_H} \geq \frac{A_L(1+\overline{k})}{\mathbb{E}[A]}$. This implies $\frac{E_L}{E_H} > \frac{A_L}{A_H}$, and so $F > F^*$. Thus $F^{APE,IC} > F^*$.

Next, we prove that $F^* \leq F^{APE,ND,H}$. $F^* \leq F^{APE,ND,H}$ if $F \geq F^{APE,ND,H}$ implies $F \geq F^*$; and the inequality is strict if $F \geq F^{APE,ND,H}$ implies $F > F^*$. Suppose that $F \geq F^{APE,ND,H}$, so that some H-firm would deviate under APE, i.e. $\frac{A_H(1+\overline{k})}{\mathbb{E}[A]} \geq \frac{E_H}{E_L}$. If $1+\overline{k} < \frac{\mathbb{E}[A]}{A_L}$, then $\frac{A_H(1+\overline{k})}{\mathbb{E}[A]} < \frac{A_H}{A_L}$ and thus $F > F^*$. If we only have $1+\overline{k} \leq \frac{\mathbb{E}[A]}{A_L}$, then $\frac{A_H(1+\overline{k})}{\mathbb{E}[A]} \leq \frac{A_H}{A_L}$ and thus $F \geq F^*$. Recall that $1+\overline{k} \leq \frac{\mathbb{E}[A]}{A_L}$ was a necessary condition for APE to be sustainable, from part (i). Thus, $F^* \leq F^{APE,ND,H}$ whenever APE is sustainable, and the inequality is strict except when $1+\overline{k}$ exactly equals $\frac{\mathbb{E}[A]}{A_L}$.

Finally, we prove $F^{EPE,ND,H} \leq F^*$. $F^{EPE,ND,H} \leq F^*$ if $F \leq F^{EPE,ND,H}$ implies $F \leq F^*$, and the inequality is strict if $F \leq F^{EPE,ND,H}$ implies $F < F^*$ Suppose $F \leq F^{EPE,ND,H}$, so that some H-firm weakly prefers to deviate under EPE, i.e. $\frac{E_H}{\mathbb{E}[E]} \geq \frac{A_H(1+\underline{k})}{A_L}$. If $1+\underline{k} > \frac{E_L}{\mathbb{E}[E]}$, then $\frac{E_H}{E_L} > \frac{A_H}{A_L}$ and thus $F < F^*$. If we only have $1+\underline{k} \geq \frac{E_L}{\mathbb{E}[E]}$, then $\frac{E_H}{E_L} \geq \frac{A_H}{A_L}$ and thus $F \leq F^*$. Recall that $1+\underline{k} \geq \frac{E_L}{\mathbb{E}[E]}$ was a necessary condition for EPE to be sustainable, from part (i). Thus, whenever EPE is sustainable, we have $F^{EPE,ND,H} \leq F^*$, and the inequality is strict except when $1+\underline{k}$ exactly equals $\frac{E_L}{\mathbb{E}[E]}$.

Taking these three points together, whenever both PEs are sustainable, $F^{EPE} \leq F^{APE}$. The inequality is strict unless $1+\overline{k} = \frac{\mathbb{E}[A]}{A_L}$ *and* $1+\underline{k} = \frac{E_L}{\mathbb{E}[E]}$.

Points (ii)-(v) also follow from the discussion of the equilibria in Lemmas 1-3.

It now remains to prove that there are no gaps in the necessary conditions for the various equilibria. We first start with the case of $F < F^*$. Then four of the equilibria stated in the Proposition are possible: APE, a PSE where H sells assets, a FSE with $k_H^* > k_L^*$, and EPE. We prove that there is no combination of parameters that simultaneously violates at least one necessary condition for the first three of these equilibria. Therefore, we are unable to rule out all of the first three equilibria (and so we cannot rule out all four equilibria).

One necessary condition for APE is (5), the ND condition for L, which is violated if $1+\overline{k} > \frac{\mathbb{E}[A]}{A_L}$. A second is given by (8), the ND condition for H. This condition is violated if $1+\overline{k} > \frac{\mathbb{E}[A]}{A_H}\frac{E_H}{E_L}$, which implies either $F > F^*$ or $1+\overline{k} > \frac{\mathbb{E}[A]}{A_L}$. The third is the necessary condition implied by (9), the IC. This condition is violated if $1+\overline{k} < \frac{\mathbb{E}[A]}{A_L}\frac{E_L}{E_H}$, but this in turn implies $F > F^*$ (or else the upper bound on $1+\overline{k}$ is less than 1). In sum, to violate at least one of the necessary conditions for APE given $F < F^*$, we require $1+\overline{k} > \frac{\mathbb{E}[A]}{A_L}$.

For PSE where H sells assets, from part (iiia) of the Proposition, we can rule out this equilibrium if $1+\overline{k} < \frac{\mathbb{E}[A]}{A_L}$ or if $1+\overline{k} > \lambda(F)\frac{\mathbb{E}[A|X=A]}{A_L}$ (note that $\lambda(F) > 1$ when $F < F^*$). Thus, we can rule out both APE and PSE by choosing $\overline{k}$ such that $1+\overline{k} > \lambda(F)\frac{\mathbb{E}[A|X=A]}{A_L}$. Finally, from part (iia), we can rule out FSE if $1+\overline{k} \leq \frac{E_H}{A_H}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$. The RHS is less than the lower bound $\lambda(F)\frac{\mathbb{E}[A|X=A]}{A_L}$ above, so there is no overlapping range of $\overline{k}$ that simultaneously violates at least one necessary condition for each of these three equilibria.

Moving to $F > F^*$, again four equilibria are possible: EPE, a PSE where H sells equity, a FSE with $k_H^* < k_L^*$, and APE. Again, we prove that there is no combination of parameters that simultaneously violates at least one necessary condition for the first three of these equilibria. One necessary condition for EPE is (11), which is violated if $1+\underline{k} < \frac{E_L}{\mathbb{E}[E]}$. A second is given by (12), the ND condition for L. This condition is violated if either $F < F^*$ or $1+\underline{k} < \frac{E_L}{\mathbb{E}[E]}$. The third is (13), the IC, but to violate this requires $1+\overline{k} > \frac{E_L}{\mathbb{E}[E]}\frac{A_H}{A_L}$, which in turn implies $F < F^*$ (or else the lower bound on $1+\overline{k}$ is greater than 1). In sum, to violate at least one of the necessary conditions for EPE given $F > F^*$, we require $1+\overline{k} > \frac{\mathbb{E}[A]}{A_L}$. $1+\underline{k} < \frac{E_L}{\mathbb{E}[E]}$.

For the PSE where H sells equity, from part (iiib), we can rule out this equilibrium if $1+\underline{k} > \frac{E_L}{\mathbb{E}[E]}$ or if $1+\underline{k} < \lambda(F)\frac{E_L}{\mathbb{E}[E]}$ (note that $\lambda(F) < 1$ when $F > F^*$). Thus, we rule out both EPE and PSE by choosing $\underline{k}$ such that $1+\underline{k} < \lambda(F)\frac{E_L}{\mathbb{E}[E]}$. Finally, from part (iib), we can rule out FSE if $1+\underline{k} > \frac{\mathbb{E}[A]}{\mathbb{E}[E]}$. The RHS is greater than the upper bound $\lambda(F)\frac{E_L}{\mathbb{E}[E]}$ above, so there is no overlapping range of $\underline{k}$ that simultaneously violates at least one necessary condition for each of these three equilibria.

Proof of Proposition 2

After the derivation of Lemmas 4 and 5, it only remains to show the ordering $\omega^{APE,ND,L} < \omega^{EPE}$.

First, since synergies are zero, (24) becomes

$$\pi\left(\frac{A_L}{A_H} - \frac{E_L}{\mathbb{E}[E]}\right) > (1-\pi)\left(\frac{E_H}{\mathbb{E}[E]} - \frac{A_H}{A_L}\right)$$

or equivalently

$$\pi\frac{A_L}{A_H} + (1-\pi)\frac{A_H}{A_L} > (1-\pi)\frac{E_H}{\mathbb{E}[E]} + \pi\frac{E_L}{\mathbb{E}[E]}.$$

Since $\pi > \frac{1}{2}$, this inequality holds if the LHS exceeds 2, i.e.

$$A_L^2 + A_H^2 > 2A_H A_L$$
$$(A_L - A_H)^2 > 0.$$

Since (24) holds, we have $\omega^{EPE} = \omega^{EPE,IC}$. We thus need to prove that $\omega^{APE,ND,L} < \omega^{EPE,IC}$. Since $\pi \geq \frac{1}{2}$, it is sufficient to replace $1-\pi$ with π in the denominator of $\omega^{EPE,IC}$ and show that this new quantity is greater than $\omega^{APE,ND,L}$. These expressions only differ in the numerator, and the numerator of the $\omega^{APE,ND,L}$ is smaller if

$$\frac{A_L}{A_H} - \frac{E_L}{\mathbb{E}[E]} > \frac{A_L}{\mathbb{E}[A]} - 1,$$

which holds because $\frac{A_L}{A_H} > \frac{A_L}{\mathbb{E}[A]}$, and $\frac{E_L}{\mathbb{E}[E]} < 1$.

Proof of Lemma 6

To prove (i), the logic is as follows. We seek a pair of cutoffs (k_H^*, k_L^*) for which both types (q, k_q^*) are indifferent between the two financing sources. As before, we use k_q' to denote candidate cutoffs that may not be equilibria, in response to which we will derive the optimal action of the types.

First we show (under certain assumptions) that, given any candidate cutoff k_H', there will be a k_L' at which type (L, k_L') is indifferent, with this value of k_L' implicitly determined as a continuous function of k_H'. Then we consider candidate equilibria such that k'_L is chosen conditional on k'_H in this manner, and we show that there exists a k'_H where (H, k'_H) is indifferent as well. This method will show that an equilibrium exists.

To prove the first statement, we take as given a cutoff $k'_H > 0$, and we employ the IVT as before, showing that for a sufficiently low (high) k'_L, type (L, k'_L) will deviate to assets (equity). He deviates to asset sales if the difference in stock price between an asset seller and an equity issuer is greater than:

$$(1-\omega)F\left(\frac{A_L(1+k'_L)}{\mathbb{E}[A|X=A]} - \frac{E_L}{\mathbb{E}[E|X=E]}\right).$$

Recall that the difference in stock price is positive by assumption. If $1 + k'_L < \frac{E_L}{E_H}\frac{A_H}{A_L}$, then the above expression is negative, and (L, k'_L) will then deviate to asset sales. On the other hand, as we increase $k'_L \to k'_H > 0$, the relative share price reaction falls to a negative value (the difference in posterior probabilities $Pr(q = H|X = A) - Pr(q = H|X = E)$ falls to zero, and the expected synergy loss grows), while the above expression is positive and increasing. Thus, there will be values of k'_L high enough that type (L, k'_L) issues equity rather than sell assets. Note that both of these conclusions hold regardless of the value of k'_H. Thus, applying the IVT, and allowing sufficiently strong dissynergies that $1 + k'_L < \frac{E_L}{E_H}\frac{A_H}{A_L}$ is feasible, we conclude that for any candidate value of k'_H, there is a value of k'_L at which type (L, k'_L) is indifferent between asset sales and equity. Moreover, since there are no discontinuities in the model, the function implicitly determining this value is continuous.

Turning to the second statement, let us consider different candidate values k'_H, and choose k'_L such that (L, k'_L) is indifferent as described above. Type (H, k'_H) will deviate to asset sales if the (positive) stock price reaction to asset sales relative to equity is greater than

$$(1-\omega)F\left(\frac{A_H(1+k'_H)}{\mathbb{E}[A|X=A]} - \frac{E_H}{\mathbb{E}[E|X=E]}\right).$$

This expression is negative if $1 + k'_H < \frac{E_H}{A_H}\frac{\mathbb{E}[A|X=A]}{\mathbb{E}[E|X=E]}$. Since the RHS of this inequality is greater than 1, there will be values $k'_H > 0$ such that type (H, k'_H) deviates to asset sales. On the other hand, the above expression grows without bound in k'_H, while the difference in the stock price reactions to asset sales and equity is bounded above by $(C_H - C_L) - (A_L - A_H) - F\underline{k}$. Thus, after $\overline{k}$ crosses some threshold $\overline{k}^H$, there will be values of k'_H high enough that type (H, k'_H) issues equity rather than sell its highly-synergistic assets. (As described above, k'_L adjusts in both cases such that type (L, k'_L) remains indifferent.) We conclude that with synergies strong enough such that $k > \overline{k}^H$ and $1 + k < \frac{E_L}{E_H}\frac{A_H}{A_L}$ are both feasible, then there will be at least one pair of cutoff values k_q^* at which types (H, k_H^*) and (L, k_L^*) are both indifferent between equity and asset sales, giving rise to the existence of a FSE.

To prove (ii), it suffices to write out the ND conditions for both qualities, solve for ω, and state the bounds in terms of the type with the synergy value that is most likely to issue a different claim.

To prove (iii), first we examine H's ND condition, which is:

$$\omega\left(Pr(q = H|X = A)\left((C_H - C_L) - (A_L - A_H)\right) - F \times \mathbb{E}[k|k < k_q^*]\right)$$
$$> (1-\omega)F\left(\frac{A_H(1+\overline{k})}{\mathbb{E}[A|X = A]} - \frac{E_H}{E_L}\right)$$

This is relatively easy to satisfy, since the relative share price reaction to asset sales (the LHS of the inequality) is positive, and since the fundamental loss to asset sales relative to equity for H (the RHS of the inequality) is negative in the absence of synergies. In general, the condition is that ω be sufficiently high that even managers with the highest level of synergies cooperate with asset sales. To obtain a condition that is sufficient regardless of the equilibrium value of k_L^*, we consider the limiting case $k_L^* \to \overline{k}$ (the strictest possible condition on ω, where all L-firms are issuing equity). Then the bound on ω is

$$\omega \geq \frac{F\left((1+\overline{k} - \frac{E_H}{E_L}\right)}{((C_H - C_L) - (A_L - A_H)) - \frac{1}{2}F(\overline{k} + \underline{k}) + F\left((1 + \overline{k} - \frac{E_H}{E_L}\right)}$$

Note that this bound is identical to $\omega^{SE^q,H}$. In this limiting case, we require the same behavior of H as in the SE^q: all H-firms must cooperate with asset sales, which perfectly reveal their quality, while equity would perfectly "reveal" them to be L.

Next, we again apply the IVT to prove existence of an equilibrium. We first seek a candidate cutoff value k_L' at which (L, k_L') will deviate to asset sales, given the price reactions that result from this cutoff. This happens if the (positive) difference in stock price reactions between asset sales and equity is greater than

$$(1-\omega)F\left(\frac{A_L(1+k_L')}{\mathbb{E}[A|X = A]} - 1\right)$$

When $1 + k_L' = \frac{A_H}{A_L}$, the above expression is negative. Thus if $1 + \underline{k}$ is at least this low, there will be an L-firm that deviates to asset sales.

Finally, we must find a candidate cutoff value k_L' at which (L, k_L') will deviate to equity. Clearly, L will do this if k'_L is sufficiently high, and as we have imposed no upper bound on $\overline{k}$, we conclude that for sufficiently high $\overline{k}$ (along with the previously-imposed bounds on ω and $\underline{k}$), there will be values of k'_L such that L deviates to equity, allowing the equilibrium to exist. (Note that the lower bound on ω increases as we raise $\overline{k}$. This does not invalidate the equilibrium, as that lower bound is still strictly less than 1.)

To prove (iv), we first examine the ND condition for L:

$$\omega\left(1 - Pr(q = H|X = E))((C_H - C_L) - (A_L - A_H)) - \frac{1}{2}F(\underline{k} + k^*_H)\right)$$
$$\leq (1-\omega)F\left(\frac{A_L(1+\underline{k})}{A_H} - \frac{E_L}{\mathbb{E}[E|X = E]}\right)$$

Note that this is more difficult to satisfy than the ND condition for H in (ii). This equilibrium requires all L-firms, and some H-firms, to pool on equity despite the negative share price reaction relative to asset sales (whereas the previous equilibrium required pooling on asset sales, which is encouraged by the price reaction). To satisfy this, we require ω to be sufficiently low. Consider the limiting case $k^*_H \to \overline{k}$. If

$$\omega \leq \frac{F\left(\frac{A_L(1+\underline{k})}{A_H} - 1\right)}{((C_H - C_L) - (A_L - A_H)) - \frac{1}{2}F(\overline{k} + \underline{k}) + F\left(\frac{A_L(1+\underline{k})}{A_H} - 1\right)}$$

then all L-firms will cooperate with equity issuance. The bound on ω is identical to $\omega^{SE^q,L}$: In this limiting case, we require the same behavior of L as in SE^q: all L-firms must cooperate with equity issuance even though it perfectly reveals their quality, while asset sales would perfectly "reveal" them to be H.

Note also that we must also have $1 + \underline{k} > \frac{A_H}{A_L}$ for this to be possible, the reverse of the condition that was imposed in (ii) to ensure that some L-firms sell assets.

Given these conditions, we proceed as before. We find candidate cutoffs k'_H at which (H, k'_H) deviates to asset sales and to equity, and then apply the IVT to conclude that an equilibrium cutoff k^*_H exists between them. H will deviate to asset sales if the positive stock price incentive to sell assets is greater than

$$(1-\omega)F\left((1 + k'_H) - \frac{E_H}{\mathbb{E}[E|X = E]}\right)$$

Since $E_H > \mathbb{E}[E|X = E]$, the above expression is negative, and the inequality holds, for any $k'_H \leq 0$.

Finally, H will deviate to equity if the opposite is true:

$$\omega\Bigg(1 - Pr(q = H|X = E))((C_H - C_L) - (A_L - A_H)) - \frac{1}{2}F(\underline{k} + k_H^*)\Bigg)$$
$$\leq (1 - \omega)F\left((1 + k'_H) - \frac{E_L}{\mathbb{E}[E|X = E]}\right)$$

With no upper bound imposed on synergies, we can choose $\overline{k}$ sufficiently high that there will be values of k'_H satisfying this inequality.

Proof of Lemma 7

The IC condition holds if:

$$F\left(\mathbb{E}[A](1 + r_L) - A_L(1 + r_H)\right) \leq A_L(C_H + A_H) - \mathbb{E}[A](C_L + A_L). \qquad (46)$$

The contrast with the core model ((9)) is similar to the ND conditions. If $\frac{1+r_H}{1+r_L} \geq \frac{\mathbb{E}[A]}{A_L}$, (46) holds for all F. If instead $\frac{1+r_H}{1+r_L} < \frac{\mathbb{E}[A]}{A_L} \leq \frac{r_H}{r_L}$, the upper bound on F becomes looser than in the core model since the information asymmetry of the investment increases L's incentives to deviate and be revealed as H, since he will receive a capital gain on the investment value R as well as the core asset value C. However, if $\frac{\mathbb{E}[A]}{A_L} > \frac{r_H}{r_L}$, then the bound becomes tighter. As with the ND condition, this holds if $r_L > r_H$ (as is intuitive) but can also hold even if $r_H \geq r_L$.

We first consider the case of $\frac{1+r_H}{1+r_L} > \frac{\mathbb{E}[A]}{A_L}$. Since $\frac{\mathbb{E}[A]}{A_L} > \frac{A_H}{\mathbb{E}[A]}$, this case implies that the LHS of both (34) and (36) are negative so they are trivially satisfied, and so the upper bound on F is ∞. If $\frac{A_H}{\mathbb{E}[A]} < \frac{1+r_H}{1+r_L} < \frac{\mathbb{E}[A]}{A_L}$, the ND upper bound is ∞ (the LHS of (34) is negative) but the IC upper bound is finite and as stated in the Lemma. Finally, if $\frac{1+r_H}{1+r_L} < \frac{A_H}{\mathbb{E}[A]}$, both upper bounds are nontrivial (less than ∞). From (32), $\frac{A_H}{\mathbb{E}[A]} < \frac{C_H+A_H}{C_L+A_L}$ and so $\frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}$. This is a sufficient condition for the IC to be stronger.

Proof of Proposition 3

It only remains to show that $F^{EPE,I} < F^{APE,I}$ (i.e. the equilibria overlap) if and only if $\frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}$. When the APE bound is not trivial ($\frac{1+r_H}{1+r_L} < \frac{\mathbb{E}[A]}{A_L}$), the relevant bound is always given $F^{APE,IC}$, as explained in the proof of Lemma 7. We first wish to show that, when the IC bound is also the relevant bound for EPE, $F^{APE,IC,I} > F^{EPE,IC,I}$ where $F^{EPE,IC,I} = \frac{A_L\mathbb{E}[C+A]-A_H(C_L+A_L)}{A_H(1+r_L)-A_L(1+\mathbb{E}[r_q])}$ (see Appendix D). This inequality is equivalent to:

$$(C_H + A_H)(1 + r_L)[\pi A_L \mathbb{E}[A] - A_H A_L + (1-\pi)A_L^2]$$
$$> (C_L + A_L)(1 + r_H)[\pi A_L \mathbb{E}[A] - A_H A_L + (1-\pi)A_L^2]$$

The bracketed term is positive, so the above inequality is equivalent to $\frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}$, which holds since the IC bound is the relevant one for EPE.

When ND is the relevant bound for EPE, we need to compare $F^{APE,IC,I}$ and $F^{EPE,ND,I}$. The proof of Lemma 9 will later show that ND is the relevant bound ($F^{EPE,ND,I} > F^{EPE,IC,I}$) when $\frac{1+r_H}{1+r_L} \geq \frac{C_H+A_H}{C_L+A_L}$, and the paragraph above showed that, if $\frac{1+r_H}{1+r_L} \geq \frac{C_H+A_H}{C_L+A_L}$, $F^{EPE,IC,I} > F^{APE,IC,I}$. Thus, $F^{EPE,I} < F^{APE,I}$ if and only if $\frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}$.

Proof of Proposition 4

For part (i), we start with SE, which is similar to Proposition 3. L-firms will not deviate to doing nothing, as they are enjoying a fundamental gain and exploiting a desirable investment opportunity. A high-quality equity issuer will not deviate to doing nothing if

$$1 + r \geq \frac{E_H}{\mathbb{E}\left[E|X = E\right]}, \tag{47}$$

i.e. the capital loss from selling undervalued equity is less than the value of the growth opportunity. Similarly, a high-quality asset seller will not deviate if

$$1 + r > \frac{A_H\left(1 + k_H\right)}{\mathbb{E}\left[A|X = A\right]}.$$

Since k_H^* is defined by $\frac{E_H}{\mathbb{E}[E|X=E]} = \frac{A_H\left(1+k_H^*\right)}{\mathbb{E}[A|X=A]}$, we have $\frac{A_H(1+k_H)}{\mathbb{E}[A|X=A]} < \frac{E_H}{\mathbb{E}[E|X=E]}$ for all asset sellers (because $k_H \leq k_H^*$). Thus, (47) is necessary and sufficient for no firm to deviate and is the condition given in the Proposition.

For the APE of Lemma 1, the additional condition is

$$\frac{A_H\left(1 + \overline{k}\right)}{\mathbb{E}\left[A\right]} < 1 + r,$$

where the LHS is the per-dollar loss suffered by type $\left(H, \overline{k}\right)$ the type that loses the most, and the RHS is the per-dollar gain from raising capital. Similarly, for the EPE of Lemma 2, the additional condition is

$$\frac{E_H}{\mathbb{E}\left[E\right]} < 1 + r.$$

For the PSE of Lemma 3, where all H-firms sell assets, the additional condition is

$$\frac{A_H\left(1+\overline{k}\right)}{\mathbb{E}\left[A|X=A\right]}<1+r,$$

and for the PSE where all L-firms sell assets, the additional condition is

$$\frac{E_H}{\mathbb{E}\left[E|X=E\right]}<1+r.$$

Turning to part (ii), we start by considering the case of interior cutoffs. The definitions of k_H^* and k_L^* in the Proposition are given by the indifference conditions. Since $1=\frac{A_L\left(1+k_L^*\right)}{\mathbb{E}[A|X=A]}$, we have $k_L^*>0$. L-firms will not deviate to doing nothing, as they are enjoying a (weakly positive) fundamental gain and exploiting a desirable investment opportunity. A H-firm doing nothing will not deviate to equity if

$$1+r<\frac{E_H}{E_L},$$

i.e. the capital loss from selling undervalued equity exceeds the value of the growth opportunity. If the above is satisfied, it is easy to show that a high-quality asset seller will not deviate either to doing nothing or to issuing equity.

Combining $1+r=\frac{A_H\left(1+k_H^*\right)}{\mathbb{E}[A|X=A]}$ and $1=\frac{A_L\left(1+k_L^*\right)}{\mathbb{E}[A|X=A]}$ yields

$$(1+r)\frac{A_L}{A_H}=\frac{1+k_H^*}{1+k_L^*}.$$

When r is high (specifically, $1+r>\frac{A_H}{A_L}$), we have $k_H^*>k_L^*$: H is more willing to sell assets than L because, if it switches to doing nothing, it loses the valuable growth opportunity (whereas L continues to exploit the growth opportunity if it does not sell assets, since it issues equity instead). When $r\leq\frac{A_H}{A_L}-1$, we have $k_H^*\leq k_L^*$: H is less willing to sell assets than L, because they are undervalued. Note that r is bounded above, since $1+r<\frac{E_H}{E_L}$ for this equilibrium to hold. Thus, we have

$$\begin{aligned}1+r&=\frac{1+k_H^*}{1+k_L^*}\frac{A_H}{A_L}\\ \frac{E_H}{E_L}&>\frac{1+k_H^*}{1+k_L^*}\frac{A_H}{A_L}.\end{aligned}$$

If $\frac{E_H}{E_L}<\frac{A_H}{A_L}$ in Proposition 3, we had $k_H^*<k_L^*$; we similarly have $k_H^*<k_L^*$ here. If

$\frac{E_H}{E_L} > \frac{A_H}{A_L}$ in Proposition 3, we had $k_H^* > k_L^*$. However, $k_H^* > k_L^*$ does not necessarily follow here, since it is possible to have $k_H^* < k_L^*$. As is intuitive, giving the firms the option to do nothing makes H relatively less willing to sell assets, as he has the outside option of doing nothing.

Finally, if $1+r < \frac{A_H(1+\underline{k})}{A_L}$, then all H-firms do nothing: we have a boundary cutoff. The investment opportunity is sufficiently unattractive, and dissynergies are sufficiently weak, that no H-firm wishes to sell its high-quality assets for a low price.

For part (iii), the cutoff k_H^* is defined by the synergy level at which H is indifferent between selling assets and doing nothing. We thus have

$$F = F\left(1+k_H^*\right)\frac{A_H}{\mathbb{E}\left[A\right]}$$

which yields

$$k_H^* = \frac{\mathbb{E}\left[A\right]}{A_H} - 1 < 0.$$

Similarly, we have

$$F = F\left(1+k_L^*\right)\frac{A_L}{\mathbb{E}\left[A\right]}$$

which yields

$$k_L^* = \frac{\overline{A}}{\mathbb{E}\left[A\right]} - 1 > 0.$$

Proof of Proposition 5

This proof is a special case of the proof of Proposition 4, part (ii), with $r = 0$.

B Selling the Core Asset

This section verifies robustness of the results of the core model to allowing the firm to sell the core asset. For simplicity we consider the case of no synergies, and thus check robustness of the certainty and correlation effects.

B.1 Positive Correlation

In an APE, assets are sold for $\mathbb{E}\left[A\right] = \pi A_H + (1-\pi) A_L$. An issuer of another claim is inferred as L. Thus, the core asset is sold for C_L, and equity is sold at $C_L + A_L + F$.

As in the core model, H's capital loss is $\frac{F(1-\pi)(A_H-A_L)}{\mathbb{E}[A]}$ from pooling on assets and

$\frac{F(C_H - C_L + A_H - A_L)}{C_L + A_L + F}$ from deviating to equity. Thus, H does not deviate to equity if:

$$F \leq \frac{(C_H + A_H)\mathbb{E}[A] - (C_L + A_L)A_H}{A_H - \mathbb{E}[A]}.$$

If it sells the core asset, its capital loss is $\frac{F(C_H - C_L)}{C_L}$. Thus, to prevent deviation to the core asset, we require:

$$\frac{(1-\pi)(A_H - A_L)}{\mathbb{E}[A]} \leq \frac{C_H - C_L}{C_L}.$$

The OEPB that a seller of the core asset is L satisfies the IC if $\frac{C_L}{C_H} \leq \frac{A_L}{\mathbb{E}[A]}$, which is weaker than the above condition.

In an EPE: equity is sold for $\mathbb{E}[C + A] + F = \pi(C_H + A_H) + (1-\pi)(C_L + A_L) + F$. Core assets are sold for C_L, and non-core assets are sold for A_L. Quality H will not deviate to selling the non-core asset if:

$$F \geq \frac{A_L(C_H + A_H) - A_H \mathbb{E}[C + A]}{A_H - A_L},$$

and the OEPB that a seller of the non-core asset is of quality L satisfies the IC if:

$$F \geq \frac{A_L \mathbb{E}[C + A] - A_H[C_L + A_L]}{A_H - A_L}.$$

Analogously, H will not deviate to selling the core asset if:

$$F \geq \frac{C_L(C_H + A_H) - C_H \mathbb{E}[C + A]}{C_H - C_L},$$

and the OEPB that a seller of the core asset is of quality L satisfies the IC if:

$$F \geq \frac{C_L \mathbb{E}[C + A] - C_H[C_L + A_L]}{C_H - C_L}.$$

As expected, H is more likely to deviate to whichever asset (core or non-core) exhibits the least information asymmetry: this will be the tighter lower bound. More interesting is that equity may be sustainable even though it does not exhibit the least information asymmetry (absent the certainty effect). One of the assets (core or non-core) will exhibit more information asymmetry than the other, and so the information asymmetry of equity will lie in between. It may therefore seem (from MM) that the sale of one asset will always dominate equity, since one of the assets will have lower information asymmetry. However, even though equity is never the safest claim, its issuance may still be sustainable, if F is sufficiently large, due to the certainty effect.

Finally, we now consider a core-asset-pooling equilibrium (CPE). The core asset is sold for $\mathbb{E}[C] = \pi C_H + (1-\pi) C_L$, the non-core asset is sold for A_L, and equity is sold at $C_L + A_L + F$. As in the core model, F does not deviate to equity if:

$$F \leq \frac{(C_H + A_H)\mathbb{E}[C] - (C_L + A_L)C_H}{C_H - \mathbb{E}[C]},$$

and he does not deviate to the non-core asset if:

$$\frac{(1-\pi)(C_H - C_L)}{\mathbb{E}[C]} \leq \frac{A_H - A_L}{A_L}.$$

The IC conditions are trivially satisfied.

Comparing CPE and APE, the former is harder to sustain if

$$\frac{(C_H + A_H)\mathbb{E}[C] - (C_L + A_L)C_H}{C_H - \mathbb{E}[C]} < \frac{(C_H + A_H)\mathbb{E}[A] - (C_L + A_L)A_H}{A_H - \mathbb{E}[A]}$$
$$\frac{A_H}{A_L} < \frac{C_H}{C_L}.$$

Thus, as is intuitive, if the core asset exhibits greater information asymmetry, it is more difficult to sustain its sale. This result is a natural extension of MM and is not the main contribution of the paper. More important is that one of the main insights – the certainty effect and thus the importance of F – remains robust to allowing sales of the core asset.

B.2 Negative Correlation

In this extension, the core (non-core) asset is positively (negatively) correlated with firm value. Thus, the firm is able to choose the correlation of the asset that it sells (whereas in the main model, we either have the positive correlation case or the negative correlation case).

In an APE, H will automatically not deviate. L's objective function is

$$\omega E[C+A] + (1-\omega)\left(C_L + A_L + F - F\frac{A_L}{\mathbb{E}[A]}\right).$$

As in the main paper, if he deviates to equity, his objective function is $C_L + A_L$ and so we require

$$\omega \geq \frac{F\left(\frac{A_L - A_H}{\mathbb{E}[A]}\right)}{\pi((C_H - C_L) - (A_L - A_H)) + F\left(\frac{A_L - A_H}{\mathbb{E}[A]}\right)}.$$

If L deviates to selling the core asset, his objective function is also $C_L + A_L$ and so we have the same condition. This is intuitive: regardless of whether he deviates to the core asset or equity, the claim he issues is fairly priced as he is revealed L, and so his objective function ends up the same. The IC condition that a seller of the core asset or equity is of quality L is trivially satisfied.

In an EPE, L will automatically not deviate. H's objective function is

$$\omega\mathbb{E}[C + A] + (1 - \omega)\left(C_H + A_H + F - F\frac{C_H + A_H + F}{\mathbb{E}\left[C + A\right] + F}\right).$$

If he deviates to non-core assets, his objective function becomes:

$$\omega(C_L + A_L) + (1 - \omega)\left(C_H + A_H + F - F\frac{A_H}{A_L}\right),$$

and if he deviates to core assets, his objective function becomes:

$$\omega(C_L + A_L) + (1 - \omega)\left(C_H + A_H + F - F\frac{C_H}{C_L}\right).$$

H will always deviate to non-core assets than core assets, since $\frac{A_H}{A_L} < 1 < \frac{C_H}{C_L}$. Thus, we have the same ND condition as before:

$$\omega \geq \frac{F\left(\frac{C_H + A_H + F}{\mathbb{E}[C+A]+F} - \frac{A_H}{A_L}\right)}{\pi\left((C_H - C_L) - (A_L - A_H)\right) + F\left(\frac{C_H + A_H + F}{\mathbb{E}[C+A]+F} - \frac{A_H}{A_L}\right)}.$$

The IC condition that a seller of the core asset is of quality L is again trivially satisfied; the IC condition that a seller of the non-core asset is of quality L is satisfied if:

$$\omega \geq \frac{F\left(\frac{A_L}{A_H} - \frac{E_L}{\mathbb{E}[E]}\right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F\left(\frac{A_L}{A_H} - \frac{E_L}{\mathbb{E}[E]}\right)}.$$

In a CPE, L will automatically not deviate. H's objective function is

$$\omega\mathbb{E}\left[C + A\right] + (1 - \omega)\left(C_H + A_H + F - F\frac{C_H}{\mathbb{E}\left[C\right]}\right).$$

If he deviates to equity, his objective function becomes:

$$\omega(C_L + A_L) + (1-\omega)\left(C_H + A_H + F - F\frac{C_H + A_H + F}{C_L + A_L + F}\right),$$

and if he deviates to non-core assets, his objective function becomes:

$$\omega(C_L + A_L) + (1-\omega)\left(C_H + A_H + F - F\frac{A_H}{A_L}\right).$$

He will always prefer to deviate to non-core assets, since $\frac{A_H}{A_L} < 1 < \frac{C_H+A_H+F}{C_L+A_L+F}$. The ND condition is

$$\omega \geq \frac{F\left(\frac{C_H}{\mathbb{E}[C]} - \frac{A_H}{A_L}\right)}{\pi\left((C_H - C_L) - (A_L - A_H)\right) + F\left(\frac{C_H}{\mathbb{E}[C]} - \frac{A_H}{A_L}\right)}.$$

The IC condition that a seller of equity is of quality L is again trivially satisfied. The IC condition that a seller of the non-core asset is of quality L is satisfied if:

$$\omega \geq \frac{F\left(\frac{A_L}{A_H} - \frac{C_L}{\mathbb{E}[C]}\right)}{(1-\pi)((C_H - C_L) - (A_L - A_H)) + F\left(\frac{A_L}{A_H} - \frac{C_L}{\mathbb{E}[C]}\right)}$$

This condition is stronger than the APE lower bound if and only if:

$$\pi\left(\frac{A_L}{A_H} - \frac{C_L}{\mathbb{E}[C]}\right) > (1-\pi)\left(\frac{A_L - A_H}{\mathbb{E}[A]}\right)$$
$$\pi\left(\frac{C_L}{\mathbb{E}[C]}\right) < \frac{\pi A_L \mathbb{E}[A] - (1-\pi)\, A_H\,(A_L - A_H)}{A_H \mathbb{E}[A]}$$

Since $C_L < \mathbb{E}[C]$, it is sufficient that

$$\pi A_H \mathbb{E}[A] < \pi A_L \mathbb{E}[A] - (1-\pi)\, A_H\,(A_L - A_H)$$
$$0 < (A_L - A_H)\,[\pi\mathbb{E}[A] - (1-\pi)A_H]$$

which is true since $\pi > 1 - \pi$ and $\mathbb{E}[A] > A_H$.

Thus, the APE is easier to sustain than the CPE. This is a simple extension of the camouflage effect of the core model. A deviation from APE to either the core asset or equity is relatively unattractive, since the firm suffers a "lemons" discount on both the security being issued and the rest of the firm as a whole. This is because both the core asset and equity are positively correlated with the value of the firm. In contrast, a deviation from either CPE or EPE to selling the non-core asset is harder to rule out: even if a high price is received for the non-core asset, this does not imply a high valuation for the firm as a whole, and so it is difficult to satisfy the IC.

The SEs are very similar to the core model. As in the core model, there is a SE where H sells non-core assets and L issues equity. There is also a SE where H sells non-core assets and L sells core assets. The conditions for this equilibrium to hold are exactly the same as in the core model. In both equilibria, by deviating, L's stock price increases but his fundamental value falls by $\frac{F(A_L - A_H)}{A_H}$. Regardless of whether L sells equity or core assets in the SE, deviation involves him selling his highly-valued non-core assets and thus suffering the loss. There is no SE where H sells core assets and L sells equity, or when H sells equity or L sells the core asset, since L will mimic H in both cases. The only possible SE is where H sells non-core assets, as L will not wish to mimic him as this will involve selling assets at a fundamental loss.

B.3 A Three-Asset Model

The previous sub-section showed that, in the case of negative correlation, it is easier to sustain an equilibrium in which all firms sell the non-core asset than one in which all firms sell the core asset. While this result is suggestive of the correlation effect, it may also arise from the fact that the non-core asset exhibits less information asymmetry, because $A_L - A_H < C_H - C_L$. If we reversed this assumption, then firm value would be higher for L than H, and so we would have the same model but with reversed notation. Since firm value is higher for L, then L is effectively H. Since A is positively correlated with firm value, A is effectively C and C is effectively A. We will obtain the result that it is easier to sustain CPE than APE, but this would be because C exhibits less information asymmetry rather than C being negatively correlated.

Thus, to allow for both positively and negatively correlated assets, and also for either asset to exhibit higher information asymmetry, we need to move to a 3-asset model. Let the three assets be C, P, and N. Asset C cannot be sold as it is the core asset, but assets P and N can be. Asset P is the positively correlated asset ($P_H \geq P_L$) and asset N is the negatively correlated asset ($N_H \leq N_L$). We allow for both $P_H - P_L > N_L - N_H$ and $P_H - P_L < N_L - N_H$: either asset may exhibit more information asymmetry. We only assume $C_H + P_H + N_H > C_L + P_L + N_L$: the existence of the third asset C means that H has a higher firm value than L, even if N exhibits more information asymmetry than P. Let $A = P + N$ be the total value of the two non-core assets.

A firm can either sell P, N, or equity. In a NPE, all firms sell the negatively-correlated asset and any firm that sells P or equity is inferred as being L. L has the greatest incentive to deviate and his objective function is

$$\omega\left(\mathbb{E}\left[C+A\right]\right)+(1-\omega)\left(E_L-F\left(\frac{N_L}{\mathbb{E}\left[N\right]}\right)\right).$$

If L deviates to equity (or to P), it becomes

$$\omega\left(C_L+A_L\right)+(1-\omega)\left(E_L-F\right).$$

The ND condition is:

$$\begin{aligned}\omega\pi\left(E_H-E_L\right) &\geq (1-\omega)\,F\left(\frac{N_L}{\mathbb{E}\left[N\right]}-1\right)\\ \omega &\geq \frac{F\left(\frac{N_L}{\mathbb{E}[N]}-1\right)}{\pi\left(E_H-E_L\right)+F\left(\frac{N_L}{\mathbb{E}[N]}-1\right)}.\end{aligned} \tag{48}$$

The IC conditions that L will deviate to P or equity if it were revealed good are trivially satisfied. L would make a capital gain on selling low-quality P or low-quality equity, compared to its capital loss on selling high-quality N, and enjoy a higher stock price.

In a PPE, all firms sell P and any firm that sells N or equity is inferred as being L. H has the greatest incentive to deviate and his objective function is

$$\omega\left(\mathbb{E}\left[C+A\right]\right)+(1-\omega)\left(E_H-F\left(\frac{P_H}{\mathbb{E}\left[P\right]}\right)\right).$$

H is strictly better off by deviating to N than to equity, as he will make a capital gain on selling low-quality N rather than a capital loss on selling high-quality equity. If H deviates to N, his objective function becomes

$$\omega\left(C_L+A_L\right)+(1-\omega)\left(E_H-F\left(\frac{N_H}{N_L}\right)\right).$$

Note that $\frac{N_H}{N_L}<1$: due to the correlation effect, H makes a capital gain from selling the non-core asset. The ND condition is:

$$\omega\pi\left(E_H - E_L\right) \geq \left(1-\omega\right) F\left(\frac{P_H}{\mathbb{E}\left[P\right]} - \frac{N_H}{N_L}\right)$$

$$\omega \geq \frac{F\left(\frac{P_H}{\mathbb{E}[P]} - \frac{N_H}{N_L}\right)}{\pi\left(E_H - E_L\right) + F\left(\frac{P_H}{\mathbb{E}[P]} - \frac{N_H}{N_L}\right)} \tag{49}$$

The IC condition that L would deviate to N if it were revealed good is:

$$\omega \geq \frac{F\left(\frac{N_L}{N_H} - \frac{P_L}{\mathbb{E}[P]}\right)}{\left(1-\pi\right)\left(C_H - C_L + A_H - A_L\right) + F\left(\frac{N_L}{N_H} - \frac{P_L}{\mathbb{E}[P]}\right)} \tag{50}$$

and the IC condition that L would deviate to equity if it were revealed good is:

$$\omega \geq \frac{F\left(\frac{E_L}{E_H} - \frac{P_L}{\mathbb{E}[P]}\right)}{\left(1-\pi\right)\left(C_H - C_L + A_H - A_L\right) + F\left(\frac{E_L}{E_H} - \frac{P_L}{\mathbb{E}[P]}\right)} \tag{51}$$

Note that if (50) is satisfied, (51) will be trivially satisfied since $\frac{N_L}{N_H} > 1 > \frac{E_L}{E_H}$, so we ignore (51).

The IC condition for PPE ((50)) is stronger than the ND condition for NPE ((48)) if and only if

$$\pi\left(\frac{N_L}{N_H} - \frac{P_L}{\mathbb{E}[P]}\right) > \left(1-\pi\right)\left(\frac{N_L}{\mathbb{E}[N]} - 1\right).$$

This always holds, since$\pi > 1-\pi$, and $\frac{N_L}{N_H} > \frac{N_L}{\mathbb{E}[N]}$, and $\frac{P_L}{\mathbb{E}[P]} < 1$. Thus, it is easier to sustain an equilibrium in which all firms sell negatively-correlated assets than one in which all firms sell positively-correlated assets, due to the correlation effect.

C Semi-Separating Equilibria: Additional Material

For the case of positive correlation, Lemma 3 analyzed PSEs where all H-firms issue one claim, and L-firms mix. This section considers the opposite equilibria where all L-firms issue one claim, and H-firms mix.

Lemma 8. *(Positive correlation, partial semi-separating equilibria where L-firms pool):*

(i) If $F < F^*$, a SE where all L-firms sell equity ($k_L^* = \underline{k}$) and H-firms strictly separate ($\overline{k} < k_H^* < \overline{k}$) is sustainable if $\underline{k} = 0$, $1 + \overline{k} > \frac{E_H}{E_L}$, and π is sufficiently close to

1.

(ii) If $F > F^*$, a SE where all L-firms sell assets ($k_L^* = \overline{k}$) and H-firms strictly separate ($\overline{k} < k_H^* < \overline{k}$) is sustainable if $\overline{k} = 0$, $1 + \underline{k} < \frac{A_L}{A_H}$, and π is sufficiently close to 1.

Proof. In case (i), assets are priced at A_H and equity is priced at $\mathbb{E}[E|X = E] < \mathbb{E}[E]$. The no-deviation condition for L is $\frac{A_L(1+\underline{k})}{A_H} > \frac{E_L}{\mathbb{E}[E|X=E]}$, or equivalently

$$\left(1 + Pr(q = H|X = E)\frac{E_H - E_L}{E_L}\right)(1 + \underline{k}) > 1 + \frac{A_H - A_L}{A_L}$$

Note that $\frac{E_H - E_L}{E_L} > \frac{A_H - A_L}{A_L}$ if and only if $F < F^*$. Then the inequality is satisfied if $F < F^*$, $\underline{k} = 0$ and $Pr(q = H|X = E)$ is sufficiently high. $Pr(q = H|X = E)$ approaches 1 from below as $\pi \rightarrow 1$ (in the limit, there are only H types remaining), so for π sufficiently close to 1 we can satisfy the inequality and all L types will cooperate with the equilibrium. It remains to show that there is an equilibrium k_H^* at which type (H, k_H^*) is indifferent between selling assets and issuing equity. As usual, we apply the intermediate value theorem. First we show that there is a candidate value k'_H at which (H, k'_H) deviates to selling assets: this happens if $1 + k'_H < \frac{E_H}{\mathbb{E}[E|X=E]}$, which will be satisfied in particular if $k'_H = 0$. Next we find a candidate k'_H at which (H, k'_H) deviates to issuing equity: this happens if $1 + k'_H > \frac{E_H}{\mathbb{E}[E|X=E]}$. A sufficient condition for such a k'_H to exist is that potential synergies be very high: if $1 + \overline{k} > \frac{E_H}{E_L}$, then we can specify a k'_H that will deviate to equity issuance regardless of the price reaction. Then an equilibrium k_H^* exists between 0 and $\frac{E_H}{E_L}$, allowing the equilibrium to exist. The proof for case (ii) is analogous. Assets are priced at $\mathbb{E}[A|X = A] < E[A]$ and equity is priced at E_H. The no-deviation condition for L is $\frac{A_L(1+\overline{k})}{\mathbb{E}[A|X=A]} < \frac{E_L}{E_H}$, or equivalently

$$\left(1 + \frac{E_H - E_L}{E_L}\right)(1 + \overline{k}) < 1 + Pr(q = H|X = A)\left(\frac{A_H - A_L}{A_L}\right)$$

This will be satisfied if $F > F^*$, $\overline{k} = 0$, and π is sufficiently close to 1 so that $Pr(q = H|X = A)$ is also close to 1. It remains to show that an equilibrium k_H^* exists. Type (H, k'_H) will deviate to asset sales if $1 + k'_H < \frac{\mathbb{E}[A|X=A]}{A_H}$, A sufficient condition for such a k'_H to exist is $1 + k'_H < \frac{A_L}{A_H}$. Type (H, k'_H) will deviate to equity issuance if $1 + k'_H > \frac{\mathbb{E}[A|X=A]}{A_H}$, which is satisfied for $k'_H = 0$. Thus all the conditions stated in the lemma are sufficient for the equilibrium to exist. ■

The intuition behind the sustainability of the PSEs where L pools is similar to the other cases. When L pools, he chooses the security with the most information asymmetry, in contrast to H: he thus pools on equity for $F < F^*$ and on asset sales for $F > F^*$. This reflects the desire of L firms to profit through the camouflage effect, by pooling with the actions of some H-firms.

For these equilibria to be sustainable, we require particularly strong synergies in one direction, so that there will some measure of H-firms that choose each action. For example, when $F < F^*$, H-firms are inclined to sell assets ($k_H^* > 0$) as they exhibit less information asymmetry than equity. However, if $\overline{k}$ is sufficiently high (note that the sufficient condition is higher than that for any other equilibrium), then some H-firms have such strong synergies that the synergy motive swamps information asymmetry conditions so that they retain their assets and issue equity. Once this measure of H-firms is issuing equity, L can pool with them and benefit through the camouflage effect. Note that, by deviating to asset sales, L will be inferred as H, as only H-firms are selling assets in this equilibrium. We thus also require dissynergies $\underline{k}$ to be weak, otherwise some L-firms have operational as well as capital gains motives for selling assets, and will deviate.

When $F > F^*$, the reverse logic holds throughout. H wishes to issue equity, but with sufficiently strong dissynergies, some H-firms will sell assets instead. This allows L to camouflage themselves by pooling with these H-firms. Furthermore, we require synergies $\overline{k}$ to be weak, otherwise some L-firms will have operational motives for issuing equity in addition to the capital gains motives of being inferred as H, and will deviate.

We finally turn to the analysis of Section 4.2, where capital raising is a choice. Part (i) of Proposition 4 stated that the equilibria in the core model continue to hold when capital raising is a choice, with an additional condition that is a lower bound on r. Here we study whether the PSEs where L pools continue to hold when capital raising is a choice.

The equilibrium in part (i) of Lemma 8, where all L-firms issue equity, requires an additional condition to hold. It is clear that L will not deviate to doing nothing, as L is enjoying a capital gain from raising financing, plus exploiting an investment opportunity. However, H may deviate to doing nothing. The H-firms that are issuing equity will only refrain from doing so if the growth opportunity is sufficiently attractive to outweigh their capital loss, i.e. $\frac{E_H}{\mathbb{E}[E|X=E]} < 1 + r$. Similarly, the H-firms that are selling assets will not do so if $1 + k_H < 1 + r$. Since $1 + k_H \leq \frac{E_H}{\mathbb{E}[E|X=E]}$ for $k \leq k_H^*$, with equality for $k = k_H^*$, $r > k_H^*$ is the additional necessary condition to deter all H-firms from deviating to doing nothing.

However, the equilibrium in part (ii) of Lemma 8, where all L-firms sell assets, does not require any additional condition to hold. As above, it is automatic that no L will deviate to doing nothing. H-firms that are issuing equity will not deviate to doing nothing, since they are making zero capital loss from issuing equity (which is priced at E_H) and exploiting an investment opportunity. The H-firms that are selling assets will be less inclined to deviate to doing nothing than to equity for the same reason: both deviations lead to zero capital loss, but the latter will allow exploitation of the investment opportunity. Thus, the indifference condition $1 + k_H^* = \frac{\mathbb{E}[A|X=A]}{A_H}$ that ensures that quality-H asset sellers do not issue equity also will ensure that they will not do nothing. Essentially, giving the firms the option not to raise capital has no effect: L firms will not exploit this option as they are already enjoying gains from capital raising, and H firms will not exploit this option as they will always prefer to issue equity than do nothing. Thus, no additional condition is required.

D Cash Used For Investment: Additional Material

This section provides additional material relevant to Section 4.1.

D.1 Positive Correlation, Positive-NPV Investment, EPE

We first start by analyzing EPE, for the case of positive correlation and positive-NPV investment. The effect of using cash for investment is similar to the APE case of the core model. Intuitively, it may seem that this usage will always make EPE harder to satisfy because the volatility of the investment reduces the certainty effect. However, if r_H is close to r_L, this volatility effect is outweighed by the fact that the investment is positive-NPV. The equilibrium is given in Lemma 9 below.

Lemma 9. *(Positive correlation, pooling equilibrium, all firms sell equity, cash used for investment.) Consider a pooling equilibrium where all firms sell equity ($X = E$) and an asset seller is inferred as quality L. The prices of assets and equity are A_L and $\pi(C_H + A_H) + (1-\pi)(C_L + A_L) + F$ respectively. This equilibrium is sustainable if*

$$F\left[A_H(1 + \mathbb{E}[r_q]) - A_L(1 + r_H)\right] \geq A_L(C_H + A_H) - A_H\mathbb{E}[C + A] \tag{52}$$

$$F\left(A_H(1 + r_L) - A_L(1 + \mathbb{E}[r_q])\right) \geq A_L\mathbb{E}[C + A] - A_H(C_L + A_L). \tag{53}$$

where $\mathbb{E}[r_q] = \pi r_H + (1-\pi)r_L$.

(i) If $\frac{1+r_H}{1+r_L} > \frac{A_H - (1-\pi)A_L}{\pi A_L}$, the equity-pooling equilibrium is not sustainable for any F.

(ii) If $\frac{1+r_H}{1+r_L} < \frac{A_H-(1-\pi)A_L}{\pi A_L}$ *and* $\frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}$*, the equity-pooling equilibrium is sustainable if* $F \geq F^{EPE,IC,I} = \frac{A_L\mathbb{E}[C+A]-A_H(C_L+A_L)}{A_H(1+r_L)-A_L(1+\mathbb{E}[r_q])}$*. Compared to the case where cash remains on the balance sheet (Lemma 2):*

(a) If $\frac{\mathbb{E}[r_q]}{r_L} < \frac{A_H}{A_L}$*, the lower bound on* F *is looser and the equity-pooling equilibrium is sustainable across a greater range of* F

(b) If $\frac{\mathbb{E}[r_q]}{r_L} \geq \frac{A_H}{A_L}$*, the lower bound on* F *is weakly tighter and the equity-pooling equilibrium is sustainable across a smaller range of* F

(iii) If $\frac{1+r_H}{1+r_L} < \frac{A_H-(1-\pi)A_L}{\pi A_L}$ *and* $\frac{1+r_H}{1+r_L} \geq \frac{C_H+A_H}{C_L+A_L}$*, the equity-pooling equilibrium is sustainable if* $F \geq F^{EPE,ND,I} = \frac{A_L(C_H+A_H)-A_H\mathbb{E}[C+A]}{A_H(1+\mathbb{E}[r_q])-A_L(1+r_H)}$*. Compared to the case where cash remains on the balance sheet (Lemma 2):*

(a) If $\frac{r_H}{\mathbb{E}[r_q]} < \frac{A_H}{A_L}$*, the lower bound on* F *is tighter and the equity-pooling equilibrium is sustainable across a smaller range of* F*,*

(b) If $\frac{r_H}{\mathbb{E}[r_q]} \geq \frac{A_H}{A_L}$*, the lower bound on* F *is weakly looser and the equity-pooling equilibrium is sustainable across a larger range of* F*.*

Proof. We start with the ND condition. By pooling, type H's fundamental value is

$$C_H + A_H + R_H - F\left(\frac{C_H + A_H + R_H}{\mathbb{E}[C + A + R]}\right).$$

By deviating, it becomes:

$$C_H + A_H + R_H - F\left(\frac{A_H}{A_L}\right).$$

Thus, he will not deviate if:

$$F\left[A_H(1 + E\left[r_q\right]) - A_L(1 + r_H)\right] \geq A_L(C_H + A_H) - A_H\mathbb{E}[C + A]$$

where

$$\mathbb{E}[r_q] = \pi r_H + (1 - \pi)r_L.$$

We now move to the IC condition. By pooling, type L's fundamental value is

$$C_L + A_L + R_L - F\left(\frac{C_L + A_L + R_L}{\mathbb{E}[C + A + R]}\right).$$

By deviating to asset sales and being inferred as type H, it becomes:

$$C_L + A_L + R_L - F\left(\frac{A_L}{A_H}\right).$$

Thus, he will deviate if:

$$F\left[A_H(1+r_L) - A_L\left(1+E\left[r_q\right]\right)\right] \geq A_L\mathbb{E}[C+A] - A_H(C_L+A_L).$$

This completes the derivation of conditions (52) and (53), the ND and IC upper bounds respectively. Note that (32) implies that the RHS of both conditions is positive. If $\frac{1+r_H}{1+\mathbb{E}[r_q]} > \frac{A_H}{A_L}$, the LHS of both conditions is negative, so the equilibrium will not be sustainable for any F (the lower bound on F is ∞). If $\frac{1+\mathbb{E}[r_q]}{1+r_L} > \frac{A_H}{A_L} > \frac{1+r_H}{1+\mathbb{E}[r_q]}$, the left side of (52) becomes positive but the LHS of (53) is still negative, so the equilibrium still is not sustainable. Only if $\frac{A_H}{A_L} > \frac{1+\mathbb{E}[r_q]}{1+r_L}$ does it become possible to satisfy both conditions. To facilitate comparison with Lemma 7, this can be rewritten as follows:

$$\frac{A_H}{A_L} > \frac{1+\mathbb{E}[r_q]}{1+r_L} \quad \Longleftrightarrow \quad \frac{1+r_H}{1+r_L} < \frac{\pi A_L + (A_H - A_L)}{\pi A_L}$$

This bound is greater than the corresponding quantity $\frac{\mathbb{E}[A]}{A_L}$ in Lemma 7. To derive the bound $((1+r_H)/(1+r_L)) < ((C_H+A_H)/(C_L+A_L))$ in this case, consider the case in which neither condition is automatically violated, and compare the two conditions: We have $F^{EPE,IC,I} > F^{EPE,ND,I}$ if and only if

$$(C_L+A_L)(1+r_H)(A_HA_L-\pi A_H^2-(1-\pi)A_L^2) > (C_H+A_H)(1+r_L)(A_HA_L-\pi A_H^2-(1-\pi)A_L^2)$$

The common term on both sides is less than zero, so the condition is equivalent to $((1+r_H)/(1+r_L)) < ((C_H+A_H)/(C_L+A_L))$. Thus the lower bound will be as stated in the Lemma, depending on whether this condition is satisfied. ■

D.2 Positive Correlation, Positive-NPV Investment, Relaxing Assumption (32)

We now relax assumption (32), under which assets are not sufficiently volatile that EPE is always sustainable in the core model regardless of F. Assumption (32) was sufficient for the RHS of equations (36), (52), and (53) to be positive. However, if (32) does not hold, i.e. assets exhibit sufficiently high information asymmetry compared to equity, it may be that the RHS of some of these equations becomes negative. In the APE, if the RHS of equation (36) is negative, then if $\frac{\mathbb{E}[A]}{A_L} < \frac{1+r_H}{1+r_L}$, the LHS is positive and so the APE is never sustainable for any F, just as in the core model when (32) is violated. Intuitively, if assets exhibit higher information asymmetry than both equity and the new investment, then no portfolio of equity and the new investment will have greater information asymmetry than assets, and so APE cannot be sustained. However, if $\frac{\mathbb{E}[A]}{A_L} \geq \frac{1+r_H}{1+r_L}$, the LHS is also negative and so we now have a lower bound, $F > \frac{\mathbb{E}[A](C_L+A_L)-A_L(C_H+A_H)}{A_L(1+r_H)-\mathbb{E}[A](1+r_L)}$. If the new investment has high information asymmetry, the portfolio of equity plus the new investment will also have high information asymmetry (allowing APE to hold) if the weight on the new investment is sufficiently high.

Similar intuition applies to the case of EPE. If equity exhibits low information asymmetry *and* investment exhibits low information asymmetry, the portfolio of equity and investment has low information asymmetry (allowing EPE to hold) if the weight on the new investment is sufficiently low. Thus, we now have an upper bound on F.

D.3 Negative Correlation, Positive-NPV Investment

This section considers the negative correlation case of Section 3 where the cash raised is used to finance investment. The results are very similar to the core model. In the absence of synergies, the only semi-separating equilibrium is SE^q, where H sells assets and L issues equity. This equilibrium is unchanged. In the absence of synergies, (30) is satisfied: H has no incentive to deviate as he will suffer a capital loss on undervalued equity and a lower stock price. The ND condition for L is achieved by plugging in $\underline{k} = 0$ into (31):

$$\omega \leq \omega^{SE} = \frac{\frac{F(A_L-A_H)}{A_H}}{\frac{F(A_L-A_H)}{A_H} + (C_H - C_L) - (A_L - A_H)}.$$

The new parameters for the investment return only matter when equity is misvalued, but this deviation condition involves either fairly-valued equity or undervalued assets.

Similarly, for APE, the ND condition for L is unchanged from (21):

$$\omega \geq \omega^{APE,ND} = \frac{F\left(\frac{A_L-A_H}{\mathbb{E}[A]}\right)}{(C_H - C_L) - (A_L - A_H) + F\left(\frac{A_L-A_H}{\mathbb{E}[A]}\right)}.$$

Again, the ND condition involves either fairly-valued equity or undervalued assets, and so is unaffected by the return parameters. As in the core model, it is automatic that H will not deviate, and the intuitive criterion will be satisfied.

The equity-pooling equilibrium does change, and the results are given by Lemma 10 below:

Lemma 10. *(Negative correlation, pooling equilibrium, all firms sell equity, cash used for investment.) Consider a pooling equilibrium where all firms sell assets* ($X_H =$

$X_L = A$) and an equity issuer is inferred as quality L. The prices of assets and equity are $\pi A_H + (1-\pi)A_L$ and $C_L + A_L + F$ respectively. This equilibrium is sustainable if

$$\omega \geq \omega^{EPE,IC,I} = \frac{F\left(\frac{A_L}{A_H} - \frac{C_L+A_L+F(1+r_L)}{\mathbb{E}[C+A]+F(1+\mathbb{E}[r_q])}\right)}{(1-\pi)((C_H - C_L) - (A_L - A_H)) + F\left(\frac{A_L}{A_H} - \frac{C_L+A_L+F(1+r_L)}{\mathbb{E}[C+A]+F(1+\mathbb{E}[r_q])}\right)} \tag{54}$$

where $\mathbb{E}[r_q] = \pi r_H + (1-\pi)r_L$. Compared to the case where cash remains on the balance sheet (Lemma 5):

(i) If $\frac{\mathbb{E}[r_q]}{r_L} < \frac{\mathbb{E}[C+A]+F}{C_L+A_L+F}$, the lower bound on ω is looser and the equity-pooling equilibrium is sustainable across a larger range of ω.

(ii) If $\frac{\mathbb{E}[r_q]}{r_L} \geq \frac{\mathbb{E}[C+A]+F}{C_L+A_L+F}$, the lower bound on ω is tighter and the equity-pooling equilibrium is sustainable across a smaller range of ω;

Proof. As in the core model, it is automatic that L will not deviate. Following similar steps to the core model, H will not deviate if:

$$\omega \geq \frac{F\left(\frac{C_H+A_H+F(1+r_H)}{\mathbb{E}[C+A]+F(1+E[r_q])} - \frac{A_H}{A_L}\right)}{\pi((C_H - C_L) - (A_L - A_H)) + F\left(\frac{C_H+A_H+F(1+r_H)}{\mathbb{E}[C+A]+F(1+E[r_q])} - \frac{A_H}{A_L}\right)}$$

and the IC condition is satisfied if:

$$\omega \geq \frac{F\left(\frac{A_L}{A_H} - \frac{C_L+A_L+F(1+r_L)}{\mathbb{E}[C+A]+F\times\mathbb{E}[1+r_q]}\right)}{(1-\pi)((C_H - C_L) - (A_L - A_H)) + F\left(\frac{A_L}{A_H} - \frac{C_L+A_L+F(1+r_L)}{\mathbb{E}[C+A]+F\times\mathbb{E}[1+r_q]}\right)}$$

Using similar steps to the proof of Proposition 5, the IC condition is stronger than the ND condition. ■

As in Lemma 5, there is a lower bound on ω to ensure that L will be willing to deviate to asset sales if he is inferred as H. Intuitively, it would might that, if $r_H \geq r_L$, using cash for volatile investment would increase the lower bound and make the equilibrium harder to sustain, but this intuition only holds if investment is sufficiently volatile, i.e. $\frac{\mathbb{E}[r_q]}{r_L} > \frac{\mathbb{E}[C+A]+F}{C_L+A_L+F}$ (similar to the results in Section 4.1).

The comparison of equilibria is given by Proposition 6, and is analogous to Proposition 2.

Proposition 6. *(Negative correlation, cash used for investment, comparison of pooling equilibria.) An asset-pooling equilibrium is sustainable if $\omega \geq \omega^{APE,ND,L}$ and an equity-pooling equilibrium is sustainable if $\omega > \omega^{EPE,IC,I}$, where $\omega^{APE,ND}$ and $\omega^{EPE,IC,I}$ are* *given by (21) and (54) respectively and* $\omega^{APE,ND,L} < \omega^{EPE,IC,I}$*. Thus, if:*

(i) $0 < \omega < \omega^{APE,ND}$*, neither pooling equilibrium is sustainable,*

(ii) $\omega^{APE,ND,L} < \omega < \omega^{EPE,IC,I}$*, only the asset-pooling equilibrium is sustainable,*

(iv) $\omega^{EPE,IC} \leq \omega < 1$*, both the asset-pooling and equity-pooling equilibria are sustainable.*

The thresholds $\omega^{APE,ND,L}$ *and* $\omega^{EPE,IC,I}$ *are both increasing in* F*.*

D.4 Negative-NPV Investment

We now consider the case in which the funds raised are used to finance a negative-NPV investment, i.e. there are agency problems. We now specify $-1 < r_L < 0$ and $-1 < r_H < 0$ but, as in the core model, allow for both $r_L \leq r_H$ and $r_L > r_H$.

We start with the positive correlation case, and consider the ND condition in APE, which is

$$F\left[A_H\left(1+r_L\right) - \mathbb{E}\left[A\right]\left(1+r_H\right)\right] \leq \mathbb{E}\left[A\right]\left(C_H + A_H\right) - A_H\left(C_L + A_L\right).$$

As in the core model, this condition is always satisfied if $\frac{1+r_H}{1+r_L} \geq \frac{A_H}{\mathbb{E}[A]}$: if the value of the investment is sufficiently less negative for H, this exacerbates the information asymmetry of the assets in place and makes equity less desirable. For $\frac{1+r_H}{1+r_L} < \frac{A_H}{\mathbb{E}[A]}$, we have a nontrivial upper bound. One might think that, if $r_H < r_L$ (i.e. investment is more value-destructive in H than L), this would mitigates H's superior assets in place and reduce the volatility of equity, making APE harder to sustain. However, this is not necessarily the case: the upper bound on F tightens only under the stronger condition of $\frac{A_H}{\mathbb{E}[A]} < \frac{r_H}{r_L}$ (note that this inequality is in the opposite direction to the positive-NPV case, since r_H and r_L are now negative). As in the positive-NPV case, there are two effects of using cash for negative-NPV investment, which can be seen by the following decomposition of investment returns:

$$R_L = F\left(1+r_L\right)$$
$$R_H = F\left(1+r_L\right) + F\left(r_H - r_L\right).$$

The first, intuitive effect is the $F\left(r_H - r_L\right)$ term which appears in the R_H equation only: if $r_H < r_L$, H's inferior use of invested cash mitigates its superior assets in place and strengthens the certainty effect. However, there is a second effect, captured by the $F\left(1+r_L\right)$ term which is common to both qualities. This weakens the certainty effect: since the investment is negative-NPV, it means that an equity investor now has a claim to a smaller certain value: $F(1+r_L)$ rather than F. Only if $\frac{r_H}{r_L} > \frac{A_H}{\mathbb{E}[A]} > \frac{1+r_H}{1+r_L}$ does the first effect dominate, leading to a decrease (tightening) of the upper bound for APE to be sustained. If $\frac{r_H}{r_L} < \frac{A_H}{\mathbb{E}[A]}$, the upper bound loosens and APE becomes easier to sustain. Equity investors now have a claim to a smaller certain value, which makes equity less attractive.

Turning to the IC condition in APE, the effect of investment now being negative-NPV is analogous to the ND condition. The condition for the bound to be trivially satisfied is exactly the same is in the positive-NPV case: $\frac{1+r_H}{1+r_L} \geq \frac{\mathbb{E}[A]}{A_L}$. If $\frac{1+r_H}{1+r_L} < \frac{\mathbb{E}[A]}{A_L}$, we have a non-trivial upper bound which now tightens if $\frac{r_H}{r_L} > \frac{\mathbb{E}[A]}{A_L} > \frac{1+r_H}{1+r_L}$, whereas it loosened in the positive-NPV case. The lower bound loosens if $\frac{r_H}{r_L} < \frac{\mathbb{E}[A]}{A_L}$. In sum, when cash is used for a negative-NPV investment, APE becomes easier to sustain: since equity investors now have a claim to a smaller certain value, the certainty effect weakens. Only if H destroys sufficiently more value than L does APE become harder to sustain.

The effect on EPE is analogous: the lower bound tightens unless r_H is sufficiently lower than r_L ($r_H < r_L$ is not sufficient). Similarly, for the negative correlation case, the inequalities reverse directions under the case of negative-NPV investment, and the economics are similar to above.