The Central Bank's Balance Sheet as an Instrument of Monetary Policy
Technical Appendix*

Vasco Cúrdia† — Federal Reserve Bank of New York

Michael Woodford‡ — Columbia University

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1 Equilibrium Conditions

This section describes the complete model of credit frictions.[1] The first subsection contains all the non-linear equations and objective welfare function, the second presents the steady state, and the third the log-linearized equations.

Full set of non-linear equilibrium conditions

The objective:

$$\tilde{U}_t = \pi_b \frac{\left(\lambda_t^b\right)^{1-\sigma_b} \bar{C}_t^b}{1-\sigma_b^{-1}} + \pi_s \frac{\left(\lambda_t^s\right)^{1-\sigma_s} \bar{C}_t^s}{1-\sigma_s^{-1}} - \frac{\psi}{1+\nu} \left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}} \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \Delta_t \tag{1.1}$$

The equations describing the economy are summarized below:

$$0 = \left(1+i_t^d\right)\left(1+\omega_t\right)\beta E_t \left[\left[\delta + (1-\delta)\pi_b\right] \frac{\lambda_{t+1}^b}{\Pi_{t+1}} + (1-\delta)(1-\pi_b) \frac{\lambda_{t+1}^s}{\Pi_{t+1}}\right] - \lambda_t^b \tag{1.2}$$

*The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

†*E-mail*: `vasco.curdia@ny.frb.org`

‡*E-mail*: `michael.woodford@columbia.edu`

[1] For details on the derivations please refer to Cúrdia and Woodford (2009) and its technical appendix.

$$0 = \left(1+i_t^d\right)\beta E_t\left[(1-\delta)\,\pi_b\frac{\lambda_{t+1}^b}{\Pi_{t+1}} + \left[\delta+(1-\delta)(1-\pi_b)\right]\frac{\lambda_{t+1}^s}{\Pi_{t+1}}\right] - \lambda_t^s \tag{1.3}$$

$$0 = \Lambda\left(\lambda_t^b,\lambda_t^s\right)\mu^p\left(1+\omega_y\right)\psi\mu_t^w\tilde{\lambda}\left(\lambda_t^b,\lambda_t^s\right)^{-1}\bar{H}_t^{-\nu}\left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} + \alpha\beta E_t\left[\Pi_{t+1}^{\theta(1+\omega_y)}K_{t+1}\right] - K_t \tag{1.4}$$

$$0 = \Lambda\left(\lambda_t^b,\lambda_t^s\right)(1-\tau_t)\,Y_t + \alpha\beta E_t\left[\Pi_{t+1}^{\theta-1}F_{t+1}\right] - F_t \tag{1.5}$$

$$\begin{aligned} 0 \;=\;& \pi_b\pi_s B\left(\lambda_t^b,\lambda_t^s,Y_t,\Delta_t;\xi_t\right) - \pi_b b_t^g \\ &+\delta\left[b_{t-1}\left(1+\omega_{t-1}\right)+\pi_b b_{t-1}^g\right]\frac{1+i_{t-1}^d}{\Pi_t} - \left(1+\pi_b\omega_t\right)b_t \end{aligned} \tag{1.6}$$

$$0 = \pi_b\bar{C}_t^b\left(\lambda_t^b\right)^{-\sigma_b} + \pi_s\bar{C}_t^s\left(\lambda_t^s\right)^{-\sigma_s} + \tilde{\Xi}_t\left(b_t-L_t^{cb}\right)^{\eta_p} + \tilde{\Xi}_t^+\left(b_t-L_t^{cb}\right) + \bar{\Xi}^{cb}\left(L_t^{cb}\right)^{\eta_{cb}} + G_t - Y_t \tag{1.7}$$

$$0 = \alpha\Delta_{t-1}\Pi_t^{\theta(1+\omega_y)} + (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta(1+\omega_y)}{\theta-1}} - \Delta_t \tag{1.8}$$

$$0 = \frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha} - \left(\frac{F_t}{K_t}\right)^{\frac{\theta-1}{1+\omega_y\theta}} \tag{1.9}$$

$$0 = (1+\varkappa)\,\tilde{\chi}_t\left(b_t-L_t^{cb}\right)^{\varkappa} + \tilde{\chi}_t^+ + \eta_p\tilde{\Xi}_t\left(b_t-L_t^{cb}\right)^{\eta_p-1} + \tilde{\Xi}_t^+ - \omega_t \tag{1.10}$$

Auxiliary equations and definitions:

$$\begin{aligned} B\left(\lambda_t^b,\lambda_t^s,Y_t,\Delta_t;\xi_t\right) \;\equiv\;& \bar{C}_t^b\left(\lambda_t^b\right)^{-\sigma_b} - \bar{C}_t^s\left(\lambda_t^s\right)^{-\sigma_s} \\ &-\left[\left(\frac{\lambda_t^b}{\psi_b}\right)^{\frac{1}{\nu}} - \left(\frac{\lambda_t^s}{\psi_s}\right)^{\frac{1}{\nu}}\right]\left(\frac{\tilde{\lambda}_t}{\psi}\right)^{-\frac{1+\nu}{\nu}}\mu_t^w\bar{H}_t^{-\nu}\left(\frac{Y_t}{Z_t}\right)^{1+\omega_y}\Delta_t \end{aligned} \tag{1.11}$$

$$\Lambda\left(\lambda_t^b,\lambda_t^s\right) \equiv \pi_b\lambda_t^b + \pi_s\lambda_t^s \tag{1.12}$$

$$\tilde{\lambda}\left(\lambda_t^b,\lambda_t^s\right) \equiv \psi\left[\pi_b\left(\frac{\lambda_t^b}{\psi_b}\right)^{\frac{1}{\nu}} + \pi_s\left(\frac{\lambda_t^s}{\psi_s}\right)^{\frac{1}{\nu}}\right]^{\nu} \tag{1.13}$$

$$\tilde{\Lambda}\left(\lambda_t^b,\lambda_t^s\right) \equiv \psi^{\frac{1}{1+\nu}}\left[\pi_b\psi_b^{-\frac{1}{\nu}}\left(\lambda_t^b\right)^{\frac{1+\nu}{\nu}} + \pi_s\psi_s^{-\frac{1}{\nu}}\left(\lambda_t^s\right)^{\frac{1+\nu}{\nu}}\right]^{\frac{\nu}{1+\nu}} \tag{1.14}$$

$$c_t^b = \bar{C}_t^b\left(\lambda_t^b\right)^{-\sigma_b} \tag{1.15}$$

$$c_t^s = \bar{C}_t^s\left(\lambda_t^s\right)^{-\sigma_s} \tag{1.16}$$

2 Optimal policy

The optimal policy problem is to maximize welfare (1.1) with respect to the variables listed in (**??**) subject to the laws of motion of the economy (1.2) - (1.10), with multipliers φ_t^i for $i = 1, ..., 9$, respectively, and two additional constraints to enforce non-negative central bank lending

$$L_t^{cb} \geq 0 \tag{2.1}$$

and non-negative interest rate

$$i_t^d \geq 0 \tag{2.2}$$

and the multiplier of the additional constraints, ζ_t and Υ_t.

FOC

FOC w.r.t. i_t^d

$$0 = \varphi_t^1 \lambda_t^b + \varphi_t^2 \lambda_t^s + \delta\beta E_t \left[\varphi_{t+1}^5 \left[b_t \left(1 + \omega_t \right) + \pi_b b_t^g \right] \frac{1 + i_t^d}{\Pi_{t+1}} \right] + \Upsilon_t \tag{2.3}$$

and complementary slackness

$$0 = \Upsilon_t i_t^d \tag{2.4}$$

FOC w.r.t. ω_t

$$0 = \varphi_t^1 \frac{\lambda_t^b}{1 + \omega_t} - \varphi_t^5 \pi_b b_t + \delta\beta E_t \left[\varphi_{t+1}^5 \frac{1 + i_t^d}{\Pi_{t+1}} b_t \right] - \varphi_t^9 \tag{2.5}$$

FOC w.r.t. l_t^{cb}

$$0 = \varphi_t^6 \left[\Xi_t^{cb\prime} \left(L_t^{cb} \right) - \Xi_t^{p\prime} \left(b_t - L_t^{cb} \right) \right] - \varphi_t^9 \left[\chi_t^{\prime\prime} \left(b_t - L_t^{cb} \right) + \Xi_t^{p\prime\prime} \left(b_t - L_t^{cb} \right) \right] + \zeta_t \tag{2.6}$$

and under the current functional forms this is equivalent to

$$\begin{aligned} 0 \quad = \quad & \varphi_t^6 \left[\eta_{cb} \bar{\Xi}^{cb} \left(L_t^{cb} \right)^{\eta_{cb}-1} - \eta_p \tilde{\Xi}_t \left(b_t - L_t^{cb} \right)^{\eta_p - 1} - \tilde{\Xi}_t^{+} \right] \\ & - \varphi_t^9 \left[\varkappa \left(1 + \varkappa \right) \tilde{\chi}_t \left(b_t - L_t^{cb} \right)^{\varkappa - 1} + \eta_p \left(\eta_p - 1 \right) \tilde{\Xi}_t \left(b_t - L_t^{cb} \right)^{\eta_p - 2} \right] \\ & + \zeta_t \end{aligned} \tag{2.7}$$

FOC w.r.t. b_t

$$
\begin{aligned}
0 &= -\varphi_t^5\left(1+\pi_b\omega_t\right)+\delta\beta E_t\left[\varphi_{t+1}^5\left(1+\omega_t\right)\frac{1+i_t^d}{\Pi_{t+1}}\right] \\
&\quad +\varphi_t^6\Xi_t^{p\prime}\left(b_t-L_t^{cb}\right)+\varphi_t^9\left[\chi_t''\left(b_t-L_t^{cb}\right)+\Xi_t^{p\prime\prime}\left(b_t-L_t^{cb}\right)\right]
\end{aligned} \tag{2.8}
$$

and with the current

$$
\begin{aligned}
0 &= -\varphi_t^5\left(1+\pi_b\omega_t\right)+\delta\beta E_t\left[\varphi_{t+1}^5\left(1+\omega_t\right)\frac{1+i_t^d}{\Pi_{t+1}}\right] \\
&\quad +\varphi_t^6\left[\eta_p\tilde{\Xi}_t\left(b_t-L_t^{cb}\right)^{\eta_p-1}+\tilde{\Xi}_t^+\right] \\
&\quad +\varphi_t^9\left[\varkappa\left(1+\varkappa\right)\tilde{\chi}_t\left(b_t-L_t^{cb}\right)^{\varkappa-1}+\eta_p\left(\eta_p-1\right)\tilde{\Xi}_t\left(b_t-L_t^{cb}\right)^{\eta_p-2}\right]
\end{aligned} \tag{2.9}
$$

Complementary slackness

$$
0=\zeta_t L_t^{cb} \tag{2.10}
$$

FOC w.r.t. Π_t

$$
\begin{aligned}
0 &= -\beta^{-1}\varphi_{t-1}^1\left(1+i_{t-1}^d\right)\left(1+\omega_{t-1}\right)\beta\frac{\left[\left[\delta+\left(1-\delta\right)\pi_b\right]\lambda_t^b+\left(1-\delta\right)\left(1-\pi_b\right)\lambda_t^s\right]}{\Pi_t} \\
&\quad -\beta^{-1}\varphi_{t-1}^2\left(1+i_{t-1}^d\right)\beta\frac{\left[\left(1-\delta\right)\pi_b\lambda_t^b+\left[\delta+\left(1-\delta\right)\left(1-\pi_b\right)\right]\lambda_t^s\right]}{\Pi_t} \\
&\quad +\varphi_{t-1}^3\alpha\theta\left(1+\omega_y\right)\Pi_t^{\theta(1+\omega_y)}K_t+\varphi_{t-1}^4\alpha\left(\theta-1\right)\Pi_t^{\theta-1}F_t \\
&\quad -\varphi_t^5\delta\left[b_{t-1}\left(1+\omega_{t-1}\right)+\pi_b b_{t-1}^g\right]\frac{1+i_{t-1}^d}{\Pi_t} \\
&\quad +\varphi_t^7\alpha\theta\left(1+\omega_y\right)\left[\alpha\Delta_{t-1}\Pi_t^{\theta(1+\omega_y)}-\left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta(1+\omega_y)}{\theta-1}-1}\Pi_t^{\theta-1}\right] \\
&\quad -\varphi_t^8\frac{\alpha}{1-\alpha}\left(\theta-1\right)\Pi_t^{\theta-1}
\end{aligned} \tag{2.11}
$$

FOC w.r.t. K_t

$$
0=-\varphi_t^3+\varphi_{t-1}^3\alpha\Pi_t^{\theta(1+\omega_y)}+\varphi_t^8\frac{\theta-1}{1+\omega_y\theta}\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}K_t^{-1} \tag{2.12}
$$

FOC w.r.t. F_t

$$0 = -\varphi_t^4 + \varphi_{t-1}^4 \alpha \Pi_t^{\theta-1} - \varphi_t^8 \frac{\theta-1}{1+\omega_y\theta} \frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha} F_t^{-1} \tag{2.13}$$

FOC w.r.t. Δ_t

$$\begin{aligned} 0 \quad = \quad & -\frac{\psi}{1+\nu}\left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}} \bar{H}_t^{-\nu}\left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} - \varphi_t^7 + \alpha\beta E_t\left[\Pi_{t+1}^{\theta(1+\omega_y)}\right] \\ & -\varphi_t^5 \pi_b \left(1-\pi_b\right)\left[\left(\frac{\psi}{\psi_b}\frac{\lambda_t^b}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}} - \left(\frac{\psi}{\psi_s}\frac{\lambda_t^s}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}\right]\tilde{\lambda}_t^{-1}\psi\mu_t^w \bar{H}_t^{-\nu}\left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \end{aligned} \tag{2.14}$$

FOC w.r.t. Y_t

$$\begin{aligned} 0 \quad = \quad & -\left(1+\omega_y\right)\frac{\psi}{1+\nu}\left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}} \bar{H}_t^{-\nu}\left(\frac{Y_t}{Z_t}\right)^{1+\omega_y}\Delta_t + \varphi_t^4\Lambda_t\left(1-\tau_t\right)Y_t - \varphi_t^6 Y_t \\ & +\varphi_t^3\left(1+\omega_y\right)\Lambda_t\mu^p\left(1+\omega_y\right)\psi\mu_t^w\tilde{\lambda}\left(\lambda_t^b,\lambda_t^s\right)^{-1}\bar{H}_t^{-\nu}\left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \\ & -\varphi_t^5\left(1+\omega_y\right)\pi_b\left(1-\pi_b\right)\left[\left(\frac{\psi}{\psi_b}\frac{\lambda_t^b}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}} - \left(\frac{\psi}{\psi_s}\frac{\lambda_t^s}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}\right]\tilde{\lambda}_t^{-1}\psi\mu_t^w\bar{H}_t^{-\nu}\left(\frac{Y_t}{Z_t}\right)^{1+\omega_y}\Delta_t \end{aligned} \tag{2.15}$$

FOC w.r.t. λ_t^b

$$\begin{aligned}
0 &= -\sigma_b \pi_b \bar{C}_t^b \left(\lambda_t^b\right)^{-\sigma_b} \\
&+\frac{\psi}{\nu} \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \Delta_t \left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}} \pi_b \left(\frac{\psi}{\psi_b}\frac{\lambda_t^b}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}} \left[\left(\lambda_t^b\right)^{-1} - \tilde{\lambda}_t^{\frac{1}{\nu}}\right] \\
&-\varphi_t^1 + \varphi_{t-1}^1 \left[\delta + (1-\delta)\pi_b\right] \frac{1+i_{t-1}^d}{\Pi_t} (1+\omega_{t-1}) + \varphi_{t-1}^2 (1-\delta)\pi_b \frac{1+i_{t-1}^d}{\Pi_t} \\
&+\varphi_t^3 \pi_b \mu^p (1+\omega_y) \psi \mu_t^w \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \tilde{\lambda}_t^{-1} \left[1 - \Lambda_t \left(\frac{\psi}{\psi_b}\frac{\lambda_t^b}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}} \left(\lambda_t^b\right)^{-1}\right] \\
&+\varphi_t^4 \pi_b (1-\tau_t) Y_t - \varphi_t^5 \sigma_b \pi_b (1-\pi_b) \bar{C}_t^b \left(\lambda_t^b\right)^{-\sigma_b - 1} \\
&-\varphi_t^5 \frac{1}{\nu} \pi_b (1-\pi_b) \psi \mu_t^w \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \Delta_t \times \\
&\times \left(\frac{\psi}{\psi_b}\frac{\lambda_t^b}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}} \tilde{\lambda}_t^{-1} \left(\lambda_t^b\right)^{-1} \left[(1+\nu)\left(\frac{\psi}{\psi_s}\frac{\lambda_t^s}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}} - \nu\right] \\
&-\varphi_t^6 \sigma_b \pi_b \bar{C}_t^b \left(\lambda_t^b\right)^{-\sigma_b - 1}
\end{aligned} \tag{2.16}$$

FOC w.r.t. λ_t^s

$$\begin{aligned}
0 &= -\sigma_s (1-\pi_b) \bar{C}_t^s \left(\lambda_t^s\right)^{-\sigma_s} \\
&+\frac{\psi}{\nu} \left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}} \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \Delta_t (1-\pi_b) \left(\frac{\psi}{\psi_s}\frac{\lambda_t^s}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}} \left[\left(\lambda_t^s\right)^{-1} - \tilde{\lambda}_t^{\frac{1}{\nu}}\right] \\
&+\varphi_{t-1}^1 (1-\delta)(1-\pi_b) \frac{1+i_{t-1}^d}{\Pi_t} (1+\omega_{t-1}) \\
&-\varphi_t^2 + \varphi_{t-1}^2 \left[\delta + (1-\delta)(1-\pi_b)\right] \frac{1+i_{t-1}^d}{\Pi_t} \\
&+\varphi_t^3 (1-\pi_b) \mu^p (1+\omega_y) \psi \mu_t^w \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \tilde{\lambda}_t^{-1} \left[1 - \Lambda_t \left(\frac{\psi}{\psi_s}\frac{\lambda_t^s}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}} \left(\lambda_t^s\right)^{-1}\right] \\
&+\varphi_t^4 (1-\pi_b)(1-\tau_t) Y_t + \varphi_t^5 \pi_b (1-\pi_b) \sigma_s \bar{C}_t^s \left(\lambda_t^s\right)^{-\sigma_s - 1} \\
&+\varphi_t^5 \frac{1}{\nu} \pi_b (1-\pi_b) \psi \mu_t^w \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \Delta_t \times \\
&\times \frac{1}{\pi_b} \left(\frac{\psi}{\psi_s}\frac{\lambda_t^s}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}} \left(\lambda_t^s\right)^{-1} \tilde{\lambda}_t^{-1} \left[1 + \nu(1-\pi_b) - (1+\nu)(1-\pi_b)\left(\frac{\psi}{\psi_s}\frac{\lambda_t^s}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}\right] \\
&-\varphi_t^6 \sigma_s (1-\pi_b) \bar{C}_t^s \left(\lambda_t^s\right)^{-\sigma_s - 1}
\end{aligned} \tag{2.17}$$

3 Zero inflation steady state

We assume that

$$\bar{L}^{cb} = 0, \tag{3.1}$$

$$\bar{\Pi} = 1.$$

We set, without loss of generality,

$$\bar{Y} = 1, \tag{3.2}$$

$$\psi = 1. \tag{3.3}$$

We further calibrate $\bar{b}/\bar{Y}$, $\bar{b}^g/\bar{Y}$ and the following ratios

$$s_c \equiv \pi_b s_b + \pi_s s_s, \tag{3.4}$$

$$\sigma_{bs} \equiv \sigma_b/\sigma_s, \tag{3.5}$$

$$\bar{\sigma} \equiv \pi_b s_b \sigma_b + \pi_s s_s \sigma_s, \tag{3.6}$$

$$\psi_{bs} \equiv \psi_b/\psi_s, \tag{3.7}$$

$$\bar{\tilde{\chi}} = 0, \tag{3.8}$$

$$\bar{\tilde{\chi}}^+ = 0, \tag{3.9}$$

$$\bar{\tilde{\Xi}}^+ = 0. \tag{3.10}$$

Further consider the following definitions

$$s_g \equiv \bar{G}/\bar{Y} \tag{3.11}$$

$$s_b \equiv \bar{c}^b/\bar{Y}, \tag{3.12}$$

$$s_s \equiv \bar{c}^s/\bar{Y}, \tag{3.13}$$

$$s_{\Xi^p} \equiv \Xi^p\left(\bar{b}\right)/\bar{Y}. \tag{3.14}$$

For the interest rate we have:

$$1 + \bar{r}^d = \beta^{-1}\frac{(\delta+1) + \bar{\omega}\left[\delta + (1-\delta)\,\pi_b\right] - \sqrt{\left\{(\delta+1) + \bar{\omega}\left[\delta + (1-\delta)\,\pi_b\right]\right\}^2 - 4\delta\,(1+\bar{\omega})}}{2\delta\,(1+\bar{\omega})}. \tag{3.15}$$

(Note that if $\bar{\omega} = 0$, this reduces to $1 + \bar{r}^d = \beta^{-1}$.) We use this steady-state relation to calibrate β, given assumed values for δ, π_b, $\bar{\omega}$ and $\bar{r}^d$.

We can also write

$$1 + \bar{\imath}^d = 1 + \bar{r}^d. \tag{3.16}$$

The steady state inflation will determine the steady state price dispersion:

$$\bar{\Delta} = 1 \tag{3.17}$$

We assume that the steady state spread is due solely to intermediation costs of the convex type, hence

$$\bar{\tilde{\Xi}}^p = \frac{\bar{\omega}}{\eta_p \bar{b}^{\eta_p - 1}},$$

and the fraction of intermediation costs to output is

$$s_{\Xi^p} = \frac{\bar{\omega}}{\eta_p} \rho_b. \tag{3.18}$$

In this case the FOC w.r.t. l_t^{cb} implies

$$\bar{\zeta} = \bar{\varphi}^9 \left(\eta_p - 1\right) \bar{b}^{-1} \bar{\omega} + \bar{\varphi}^6 \bar{\omega} - \bar{\varphi}^6 \Xi^{cb\prime}(0). \tag{3.19}$$

Furthermore we can write, from one of the Euler equations:

$$\bar{\lambda}^b = \bar{\Omega} \bar{\lambda}^s, \tag{3.20}$$

where

$$\bar{\Omega} \equiv \frac{1 - \left(1 + \bar{r}^d\right) \beta \left[\delta + (1 - \delta)(1 - \pi_b)\right]}{\left(1 + \bar{r}^d\right) \beta (1 - \delta) \pi_b}. \tag{3.21}$$

Given the assumption that we calibrate ψ_{bs} and ψ, we can then write

$$\psi_s = \psi \left[\pi_b \psi_{bs}^{-\frac{1}{\nu}} + \pi_s\right]^{\nu}, \tag{3.22}$$

and $\psi_b = \psi_{bs} \psi_s$.

This implies that, given ψ_b and ψ_s, we get

$$\Lambda\left(\bar{\lambda}^b, \bar{\lambda}^s\right) = \left[\pi_b \bar{\Omega} + \pi_s\right] \bar{\lambda}^s, \tag{3.23}$$

$$\tilde{\lambda}\left(\bar{\lambda}^b,\bar{\lambda}^s\right)=\psi\left[\pi_b\bar{\Omega}^{\frac{1}{\nu}}\psi_b^{-\frac{1}{\nu}}+\pi_s\psi_s^{-\frac{1}{\nu}}\right]^{\nu}\bar{\lambda}^s, \tag{3.24}$$

$$\tilde{\Lambda}\left(\bar{\lambda}^b,\bar{\lambda}^s\right)=\psi^{\frac{1}{1+\nu}}\left[\pi_b\psi_b^{-\frac{1}{\nu}}\bar{\Omega}^{\frac{1+\nu}{\nu}}+\pi_s\psi_s^{-\frac{1}{\nu}}\right]^{\frac{\nu}{1+\nu}}\bar{\lambda}^s. \tag{3.25}$$

Using the inflation equation, that implies $\bar{F}=\bar{K}$,

$$(1-\bar{\tau})=\mu^p\left(1+\omega_y\right)\psi\bar{\mu}^w\tilde{\lambda}\left(\bar{\lambda}^b,\bar{\lambda}^s\right)^{-1}\frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}},$$

hence

$$\bar{\lambda}^s=\frac{\mu^p\left(1+\omega_y\right)\mu_t^w\frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}}}{(1-\bar{\tau})\left[\pi_b\bar{\Omega}^{\frac{1}{\nu}}\psi_b^{-\frac{1}{\nu}}+\pi_s\psi_s^{-\frac{1}{\nu}}\right]^{\nu}}. \tag{3.26}$$

The resources constraint implies

$$1-s_c-s_g=s_{\Xi}. \tag{3.27}$$

which determines s_g given s_c and s_{Ξ}.

The debt equation is

$$\left[1+\pi_b\bar{\omega}-\delta\left(1+\bar{\omega}\right)\left(1+\bar{r}^d\right)\right]\frac{\bar{b}}{\bar{Y}}=\pi_b\pi_s\frac{B\left(\bar{\lambda}^b,\bar{\lambda}^s,\bar{Y},\bar{\Delta};0\right)}{\bar{Y}}-\pi_b\frac{\bar{b}^g}{\bar{Y}}\left[1-\delta\left(1+\bar{r}^d\right)\right],$$

with

$$\frac{B\left(\bar{\lambda}^b,\bar{\lambda}^s,1,1;0\right)}{\bar{Y}}=s_b-s_s-\frac{\bar{\Omega}^{\frac{1}{\nu}}\psi_b^{-\frac{1}{\nu}}-\psi_s^{-\frac{1}{\nu}}}{\pi_b\bar{\Omega}^{\frac{1}{\nu}}\psi_b^{-\frac{1}{\nu}}+\pi_s\psi_s^{-\frac{1}{\nu}}}\frac{1-\bar{\tau}}{\mu^p\left(1+\omega_y\right)},$$

implying that

$$\frac{\bar{b}}{\bar{Y}}=\frac{\pi_b\left(1-\pi_b\right)\left(s_b-s_s-\frac{\bar{\Omega}^{\frac{1}{\nu}}\psi_b^{-\frac{1}{\nu}}-\psi_s^{-\frac{1}{\nu}}}{\pi_b\bar{\Omega}^{\frac{1}{\nu}}\psi_b^{-\frac{1}{\nu}}+\pi_s\psi_s^{-\frac{1}{\nu}}}\frac{1-\bar{\tau}}{\mu^p(1+\omega_y)}\right)-\pi_b\frac{\bar{b}^g}{\bar{Y}}\left[1-\delta\left(1+\bar{r}^d\right)\right]}{1+\pi_b\bar{\omega}-\delta\left(1+\bar{\omega}\right)\left(1+\bar{r}^d\right)}. \tag{3.28}$$

Given that we calibrate ρ_b we can use this equation to determine s_b-s_s,

$$\begin{aligned} s_b-s_s &= \frac{\left[1+\pi_b\bar{\omega}-\delta\left(1+\bar{\omega}\right)\left(1+\bar{r}^d\right)\right]\frac{\bar{b}}{\bar{Y}}+\pi_b\frac{\bar{b}^g}{\bar{Y}}\left[1-\delta\left(1+\bar{r}^d\right)\right]}{\pi_b\pi_s} \\ &+\frac{\gamma_b-\pi_b}{\pi_b\pi_s}\frac{\psi}{\bar{\bar{\lambda}}}\frac{\left(1-\bar{\tau}\right)/\mu^p}{1+\omega_y} \end{aligned} \tag{3.29}$$

with

$$\gamma_b \equiv \pi_b \left(\frac{\psi}{\psi_b} \frac{\bar{\lambda}^b}{\bar{\tilde{\lambda}}} \right)^{\frac{1}{\nu}}. \tag{3.30}$$

Given our calibration of s_c we can then write

$$s_s = s_c - \pi_b \left(s_b - s_s \right), \tag{3.31}$$

and

$$s_b = s_c + \pi_s \left(s_b - s_s \right). \tag{3.32}$$

Finally,

$$\bar{C}^b = s_b \left(\bar{\lambda}^b \right)^{\sigma_b}, \tag{3.33}$$

$$\bar{C}^s = s_b \left(\bar{\lambda}^s \right)^{\sigma_s}, \tag{3.34}$$

$$\bar{K} = \bar{F} = \frac{\bar{\Lambda} \left(1 - \bar{\tau} \right)}{1 - \alpha\beta}. \tag{3.35}$$

If we further set ψ_b/ψ_s such that the labor supply is the same in steady state, which implies that

$$\frac{\bar{\lambda}^b}{\psi_b} = \frac{\bar{\lambda}^s}{\psi_s} \Leftrightarrow \frac{\bar{\lambda}^b}{\bar{\lambda}^s} = \frac{\psi_b}{\psi_s} \Rightarrow \frac{\psi_b}{\psi_s} = \bar{\Omega}, \tag{3.36}$$

then

$$\psi_s = \left[\pi_b \bar{\Omega}^{-\frac{1}{\nu}} + \pi_s \right]^{\nu}, \tag{3.37}$$

$$\psi_b = \bar{\Omega} \psi_s, \tag{3.38}$$

$$\bar{\lambda}^s = \frac{\mu^p \left(1 + \omega_y \right) \mu^w \frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}}}{\left(1 - \bar{\tau} \right) \left[\pi_b \bar{\Omega}^{\frac{1}{\nu}} \psi_b^{-\frac{1}{\nu}} + \pi_s \psi_s^{-\frac{1}{\nu}} \right]^{\nu}}, \tag{3.39}$$

$$\bar{\lambda}^b = \bar{\Omega} \bar{\lambda}^s, \tag{3.40}$$

$$\Lambda \left(\bar{\lambda}^b, \bar{\lambda}^s \right) = \pi_b \bar{\lambda}^b + \pi_s \bar{\lambda}^s. \tag{3.41}$$

References

Cúrdia, V. and M. Woodford (2009). Credit frictions and optimal monetary policy. *Unpublished*.