Abstract: Preference falsification, the act of misrepresenting one’s beliefs under social pressure, is widespread but not ubiquitous. We show that when individuals perceive a concave cost of deviating from their principles, ideological extremists are more likely to falsify preferences. Being in ideological minority, they cave-in on decisions they disagree with ideologically because once they deviate slightly, further deviations entail relatively little additional cost. To this result—ideological perfectionism—which is supported by recent lab experiments, we add the first evidence in a high-stakes field setting regarding which individuals, on an ideological scale, conform under social pressure and which stand their ground.

Keywords: Judicial decision making, group decision making, ideology, peer pressure.

JEL codes: D7, K0, Z1.
1 Introduction

Economics tends to gravitate towards the assumption that costs – be they economic, effort or cognitive – are convex. One rationale for this assumption is that it makes theoretical models analytically tractable. Another rationale is that it seems intuitively plausible. However, such intuition has proved fragile following a number of recent experiments suggesting that, when it comes to decision making on moral and ethical issues, individuals perceive a concave cost of deviating from what they believe is right (Kajackaite and Gneezy 2017; Hurkens and Kartik 2009; Gneezy et al. 2013; Gino et al. 2010). That is, already a small deviation entails a high cost while a larger deviation entails only a small additional cost. The question remains whether concave preferences have empirically observable implications for important real world decision situations, such as those requiring agreement between individuals with conflicting views.

Indeed, within politics, work-life and in most social contexts, individuals have to come to terms with others who disagree with them ideologically. They may thus be subject to norms or decisions they disagree with on an ideological ground. How would an individual’s behavior be affected by being placed in an environment where her peers hold opinions far from her own? Would that individual, who is in ideological minority, behave more or less confrontationally than her peers, that is, speak out more or less against decisions and norms? And does the answer depend on the curvature of the cost of deviation from one’s bliss point? While intuition may suggest that the one in strongest disagreement will be confronting the most, this paper presents empirical results from an ideologically-salient field setting (U.S. Circuit Courts) showing just the opposite—individuals with frequent and strong disagreement with their peers confront less than others. Our empirical analysis indicates that judges who are in strong ideological disagreement with their peers remain silent. In particular, we show that extremists rarely affect the majority decision, yet are on aggregate the least confrontational. This final

\footnote{Kajackaite and Gneezy (2017) show that once the incentives to lie are higher than the cost, subjects switch from telling the truth to lying to the full extent. Earlier research has also been suggestive of a concave cost of lying: The decision whether to lie is often insensitive to the outcome of lying once it is preferred over the outcome of being truthful (Hurkens and Kartik 2009) and so a maximal deviation from the truth will often be chosen by those deciding to lie (Gneezy et al. 2013). Likewise, using a dynamic setting, Gino et al. (2010) showed that once individuals are induced to cheat, they succumb to full-blown cheating.}
observation is arguably incompatible with a convex cost of deviating from principles, since
the more often one falsifies preferences, the more costly it would be, so we should expect
extremists to be the most confrontational.

To unravel the source of the apparent incompatibility, we develop a formal model with
the following features. There is a large number of cases. For each case, three randomly chosen
judges are assigned to a panel where they negotiate the ideological flavor of the majority
opinion (decision), with the median judge succeeding to set the opinion to match her own
ideology (as is indeed observed in our data). Each judge then decides whether to confront this
opinion by formally dissenting. Doing so is costly in terms of time and collegiality. But not
dissenting entails a personal cost: the more often a judge signs majority opinions she disagrees
with (and the more ideologically distant these opinions are from her own bliss point), the
worse she feels. We then show that extreme judges are the ones who falsify preferences if
and only if the ideological cost of bliss-point deviations is sufficiently concave.

The intuition for the result is as follows. A judge who is in ideological minority in
the greater group of peers (the court’s pool) will rarely be the median judge in a panel and
will mostly have to decide whether or not to sign opinions far from her ideological bliss
point. However, always dissenting on opinions she does not like would imply a very high
collegial pressure as she will be facing such opinions virtually all the time, and signing only
some opinions while dissenting against others helps little when the perceived cost of signing
few unfavorable opinions instead of many is almost the same due to the concave ideological
cost. Hence, facing a sufficiently high collegial pressure, such an extreme judge will tend
to sign virtually all opinions, thus being non-confrontational. In comparison, judges with
more consensual ideology will more often be the median of their panels hence will less often
need to decide whether to sign unfavorable opinions. So when they do face this problem they
dissent, since the cost of deviating from their ideological bliss point (due to concavity) is high
even if it is only rarely done. Such judges will therefore dissent from time to time. Thus,
overall, judges far from the mainstream ideology will be the least confrontational despite
having the most reasons to be just that (rarely determining the opinion and often facing
ideologically distant opinions). More precisely, our theoretical model predicts a hill-shaped
relationship between a judge’s dissent rate and how extreme she is relative to her peers in the pool of judges she interacts with over time (see visualization of raw data in Figure 2). That is, centrist judges rarely dissent, moderately ideological judges often dissent and extremely ideological judges rarely dissent. The above theoretical logic does not hold if the personal cost of bliss-point deviation is linear or convex, and we present necessary and sufficient conditions for the extent of concavity needed for the theory to align with the empirical observations.

The question of whether holding non-consensual views makes a person more or less prone to challenge norms and decisions was empirically open due to unobservability of individual ideology and due to endogeneity of the choice of whom to interact with. These two problems are resolved in our setting of U.S. Circuit Courts, where there exist commonly used, ex ante measures of individual ideology, and assignment of whom an individual interacts with is determined exogenously. Notably, our result is not about people with extreme ideology per se. Rather, it occurs when a judge is ideologically extreme relative to the current ideological composition of her pool, which we demonstrate by showing our result for extremists holds with judge fixed effects. This suggests that the interaction between peers who are in ideological disagreement is silencing those with opinions far from the mainstream. The implications of this finding are that it may hide undercurrents of dissatisfaction, distort the perception of the distribution of views and create false impressions of consensus where it is absent. In the conclusions (Section 4) we discuss the broader empirical and theoretical implications of our finding of a concave ideological cost.

2 Empirical Puzzle

2.1 The empirical setting

Our setting is the U.S. Circuit Courts (see Appendix A for a detailed description of the institutional setting and data). Each Circuit Court presides over 3–9 U.S. states and consists of a pool of 8–40 judges (depending on the circuit), three of whom are randomly drawn to serve as a panel for each case. The panel decides on a verdict (affirming or overturning the lower court verdict) and composes an “opinion” (i.e., a text) motivating the verdict. It is well documented that ideology plays an important role in forming the opinion (e.g., Epstein
et al. 2013; Sunstein et al. 2006; Berdejó and Chen 2014). Appointment to the pool of judges is done by the President and confirmed by the U.S. Senate. Strictly speaking, a judge may of course turn down such a job offer. But vacancies are rare and it is considered a great honor to serve in a Circuit Court. Importantly, given the life tenure of judges, once a judge is appointed she has no control over her peer group in practice since 96% of all judges serve until they retire or pass away.

The ideological score we use, which is a standard summary measure coming from the Judicial Common Space database (Epstein et al. 2007), leverages this political appointment process. It assumes that the appointing politicians take the opportunities they get to assign judges of their ideological liking while exploiting the norm of senatorial courtesy. Thus, it assigns to a judge an ideological score based on the observed ideology of the President and home-state senators.\(^2\) The score has two advantages. First, it is exogenous since it assigns the ideology of the judge before her behavior at the court is observed, which is key for identification. Second, it predicts well judges’ voting patterns in court, as visualized in Appendix Figure 2. The ideology score takes values in between roughly ±0.8.

### 2.2 Median Voter Theorem in U.S. Circuit Courts

We begin by verifying the median voter theorem holds in our setting. That is, we examine the effect of the ideology score of the judges in the panel on the ideology of the majority opinion. To conduct this analysis we regress the Opinion Ideology\(^3\) (from the U.S. Courts of Appeals Database Project) on a judge’s ideology score (Score Relative to Center of Judge Pool\(^4\)) and its interaction with whether the judge is the median of the panel in

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\(^2\)See Appendix A.2 for more details.

\(^3\)The U.S. Courts of Appeals Database Project hand-codes the opinions as being liberal or conservative.

\(^4\)For each Circuit at each year, each judge’s score was recalculated to reflect her location relative to the average ideology in the pool of judges in that circuit and year. That is, Score Relative to Center of Judge Pool measures the ideological disagreement between a judge and all the peers she regularly interacts with. In most specifications we use this score (rather than the “raw” ideology scores per se) for two reasons. First, because the ideological disagreement between a judge and the peers she regularly interacts with is the focus of our research. Second, because when the entire pool moves to the left or to the right, panelists’ scores move accordingly and become slightly correlated with the ideological content of the opinion even without actually affecting it. The pool is determined by the Circuit Court and year in which the panel occurs.
terms of ideology score:

\[
\text{Opinion Ideology}_{pcit} = \alpha + \gamma_1 \text{Score Relative to Center of Judge Pool}_{cit} + \\
\gamma_2 1 (i \text{ is median}_{pcit}) + \\
\gamma_3 \text{Score Relative to Center of Judge Pool}_{cit} * 1 (i \text{ is median}_{pcit}) + \nu_{pcit}
\]

for judge \(i\) on panel \(p\) in Circuit \(c\) and year \(t\). If the ideology of a judge influences the opinion, we should expect a positive relationship between the judge’s ideology score (where a high value means a very conservative judge) and the likelihood of a conservative opinion. Figure 1 visualizes the results and shows that the median judge is essentially single-handedly determining the ideological color of the opinion (Fact 1).\(^5\)

\(^5\)Appendix Table A.1 shows that only the median judge’s ideology score affects the opinion, and it does so positively. Appendix Table A.7 presents a robustness check of this result.
Intuitively, given Fact 1, one might expect that the more distant a judge is from the panel center, the more she will dissent (or concur, which is a milder form of disagreement). Indeed, we find that a judge is more likely to dissent when the panel median is ideologically far from her (Fact 2—see Appendix A.3.2 for details), and this is reported in Appendix Figure 3 and Appendix Table A.2. What about the effect of being distant from the pool center?

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A judge can disagree with the panel in one of two ways. A dissent objects to the (binary) verdict and the opinion. A concurrence agrees with the verdict but disagrees with the reasoning in the opinion.
2.3 Which judges are dissenting the most?

We now turn to examine which judges, in terms of their (relative) ideology score, are behaving most confrontationally (i.e., dissenting the most). Figure 2 presents a non-parametric visualization of Dissent Rate by the ideology Score Relative to Center of Judge Pool using a local polynomial regression.

Figure 2 reveals a surprising pattern: starting from the left, the most extreme judges rarely dissent, then there is a marked increase in dissent as judges become more moderate, followed by a decrease in dissent rates towards the center of the judge pool. A similar pattern appears on the right. We will refer to this pattern as a *spider pattern*, due to the figure’s resemblance of the body and legs of a spider. This is our *Fact 3*.

Appendix Table A.3 shows that a quadratic specification of Dissent Rate on Ideological Distance to Center of Judge Pool (the absolute value of Score Relative to Center of
Judge Pool) is statistically significant (at p<0.01). Appendix Figure 9 further shows that
the spider pattern is robust to residualizing by circuit and year fixed effects and presents
a similar graph when grouping judges into 15 separate bins according to Score Relative to
Center of Judge Pool. It also presents the Concurrence Rate of judges according to Score
Relative to Center of Judge Pool (local polynomial and bins). Notably, the pattern of the
spider is robust. Finding these patterns for dissents and concurrences separately of course
strengthens our confidence in these results. Additional robustness checks are reported in
Appendix Tables A.8 to A.10.

To verify that the spider pattern is not driven by some distortion in the ideology
score, we ran the same regression using an alternative ideology score—the party of appointing
President. This score, in its raw form, does not distinguish between judges nominated by
different Republican Presidents (all have a score of 1). Likewise it does not distinguish
between judges nominated by different Democrat Presidents (all have a score of 0). To create
a relative score based on this raw score we calculate the average ideology in the pool in a
circuit-year and calculate a judge’s distance to this average. Hence, the relative score reflects
whether a judge is in minority (a large absolute score) or in majority (a small absolute
score). Appendix Table A.4 shows the spider pattern is robust to using this alternative
score. Furthermore, this score provides a very straightforward reinterpretation of the spider
pattern. An “extremist” in this scoring system is simply a judge placed in a pool of judges
in which a large majority of the judges were nominated by a President from the other party.
Hence, the spider pattern when using this score clearly captures the tendency of judges to
stay silent when they are in ideological minority, while behaving the most confrontationally
when having a similar number of peers from their own party and from the other party.

The spider pattern we find implies that judges who are extreme relative to their
greater group of peers are less confrontational than more moderately-distanced judges. It is
novel to this paper and, to our knowledge, no paper in any domain has ever examined how
being in ideological minority affects one’s tendency to confront decisions of the majority.
In Appendix A.4.3 we show an important apparent implication of the spider pattern: the
silencing of extreme judges implies that they end up having a less ideologically-biased voting
pattern in court than moderates do.\textsuperscript{7} This result is particularly surprising given the median voter theorem holds in this setting (Fact 1) and given that, at the level of a single judge, confrontation is driven by ideological disagreement with the majority opinion (Fact 2). See also Appendix A.3.2.

It should be noted that the spider pattern is driven by a judge’s ideology relative to her peers—the same judges who exhibit the low dissent rate when they are extremists, exhibit a high dissent rate when they are moderates relative to their pool. To show this, Table I reports the coefficients of a regression of dissent rate on polynomials of Distance to Center of Judge Pool with judge fixed effects, using a subsample that contains all the judges who, at a certain point of their career, have Distance to Center of Judge Pool greater than 0.6 (the location of the hump of the quadratic regression that tests Fact 3, see Table A.3). Table I clearly shows that the spider pattern holds for that subsample of “extreme” judges, indicating that these judges do not remain silent when they are placed in a pool that is ideologically closer to their bliss point (as reflected in the increasing part of the spider).\textsuperscript{8} Moreover, as can be seen in Appendix Figure 10, the spider pattern is profoundly attenuated when considering the raw (i.e., not relative) Ideology Score. Hence, we can conclude that the non-confrontational behavior of extreme judges is not driven by their ideology per se or by some related personal characteristics but rather by regularly being in strong ideological disagreement with most of their peers.\textsuperscript{9}

\textsuperscript{7}Our main model (Section 3) can account for this finding too—see Appendix B.1.
\textsuperscript{8}For judges who were never “extreme” at some point of their career, using judge fixed effects renders more measurement error (see Greene, 2018) and less meaningful variance to estimate the spider pattern.
\textsuperscript{9}Recall also the reinterpretation of extremism as belonging to a partisan minority when using the alternative party-of-appointing-President score.
TABLE I
Dissent and Ideological Distance to Center of Judge Pool for “Extreme” Judges, Including Judge Fixed Effects

<table>
<thead>
<tr>
<th></th>
<th>Column 1</th>
<th>Column 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Dissent</td>
<td>0.0971***</td>
<td>0.102***</td>
</tr>
<tr>
<td></td>
<td>(0.0334)</td>
<td>(0.0291)</td>
</tr>
<tr>
<td>Distance to Center of Judge Pool</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concur</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.102***</td>
<td>0.105***</td>
</tr>
<tr>
<td></td>
<td>(0.0388)</td>
<td>(0.0303)</td>
</tr>
<tr>
<td>Distance^2</td>
<td>-0.102***</td>
<td>-0.105***</td>
</tr>
<tr>
<td></td>
<td>(0.0388)</td>
<td>(0.0303)</td>
</tr>
<tr>
<td>Circuit Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Judge Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>N</td>
<td>1519</td>
<td>1519</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.425</td>
<td>0.343</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Sample of judges who are extreme relative to their pool at some point of their career (Distance to Center of Judge Pool greater than 0.6 (the location of the hump of regression specification). Ideology scores come from the Judicial Common Space database. The main independent variable is (the absolute value of) ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The dependent variable is the judge’s dissent rate (column 1) or concurrence rate (column 2) in a Circuit-year. Fixed effects include year, circuit, and judge. Observations are weighted by the number of votes cast by the judge in the Circuit-year.

3 Theoretical Model

There is a large pool of judges (formally, a continuum of infinitesimal judges). Each judge has an ideology score $t$ which is public information. The judges’ ideologies are uniformly distributed in $[-1, 1]$. There is a long sequence (a continuum) of judicial cases. For each case, three judges ($t_1, t_2$ and $t_3$) are randomly and independently drawn from the pool of judges to sit together on a panel.

The timing of the actions within each case is as follows. First, the three judges in the panel bargain on the opinion $v \in \mathbb{R}$. Second, each judge decides whether to sign the opinion or not. The first stage (choosing $v$) is executed as follows. Judges make sequential offers and counter offers for up to $I$ periods. In the first period ($i = 1$), a randomly chosen judge offers an opinion, and afterwards, in periods $i = 2, 3, \ldots, I$, coalitions of (two or three) judges can make counter offers. If after some period $i < I$ there are no more counter offers, the last offer becomes the panel’s opinion $v$. Otherwise, the offer made at time $I$ is the panel’s opinion.
Once the opinion was decided in this way, the second stage begins. In this second stage, each judge has to decide whether to sign the panel’s opinion or not (with no obligation to sign the opinion if the judge was part of the coalition that offered it).\(^\text{10}\) Let \(s(v)\) be an indicator function that equals 1 if the judge chooses to sign opinion \(v\) and equals 0 if she dissents. We denote by \(V\) the (equilibrium) set of opinions and by \(k(v)\) its distribution (the pdf). Furthermore, we denote by \(V(t) \subset V\) the set of opinions faced by judge \(t\) during her judicial term and by \(k(v|t)\) its distribution. We assume (for simplicity) that the set of panel peers faced by judge \(t\) during her career is known in advance and is identical to the continuous distribution of two random judges drawn from \(t \sim U(-1,1)\).

A judge cares about the ideology of the opinion per se and also about whether she herself signs it or not. In particular, she wants the opinion to reflect her bliss point \((t)\). The (dis)utility associated with caring about the opinion per se is captured by \(O(V(t),t)\), which is a cost function that increases in the distance between each \(v \in V(t)\) and \(t\).\(^\text{11}\) The (dis)utility associated with signing unfavorable opinions is captured as follows. When signing an opinion \(v \neq t\), the judge bears an inner discomfort associated with actively approving it. This is captured by the cost function

\[
D = D(x)
\]

\[
x \equiv \int |v - t| \ k(v|t) \ s(v) \ dv.
\]

We assume that \(D\) increases in its argument, which means the ideological cost is increasing in the number of unfavorable opinions a judge signs and the more unfavorable each opinion she signs is. For tractability we model \(D\) as a power function

\[
(2) \quad D(x) = x^\alpha
\]

\(^{10}\)In the rules of the US Circuit Courts, it is sufficient that one judge signs the opinion. This can happen when two judges write two separate concurrences or when one judge concurs and the other dissents. In our theory we do not model concurrences and dissents separately.

\(^{11}\)More precisely, we assume that for any non-zero mass of verdicts \(\tilde{V} \subset V(t)\), if the distance between each \(v \in \tilde{V}\) and \(t\) strictly increases (ceteris paribus), then \(O(V(t),t)\) strictly increases.
where $\alpha > 0$. $D$ can be interpreted in two main ways: Either as a judge’s own perceived cost of being part of an opinion she disagrees with; or as a loss when not standing up for the ideology of one’s supporters or ideological faction. Putting the power $\alpha$ on the aggregate deviation $x$ instead of putting it on the term $|v - t|$ captures the idea that judges are optimizing over their whole career. Put differently, the judge’s reference point (or ambition) is signing only opinions she agrees with. Then she perceives a cost based on the total deviations from this reference point.\footnote{It should be noted that if a judge would optimize case by case (that is, the power would be applied to each deviation) then our main theoretical conclusions would still hold. However, an additional assumption would be needed, namely, that the collegial pressure (see below) is increasing in the extremeness of the dissenting opinion. We prefer to avoid this extra assumption and we also find the assumption of case-by-case optimization less realistic, but we briefly refer to this alternative in Appendix D.2.8.}

In total, a judge $t$ has the loss function

$$ L = O(V(t),t) + D\left(\int |v - t| k(v|t) s(v) dv\right) + WP(t), $$(3)

where

$$ P(t) \equiv \int (1 - s(v)) k(v|t) dv $$

is the judge’s rate of dissent, and the last term in the loss function represents the cost arising from collegial pressure. $W$ can also be interpreted as capturing the effort of writing a separate opinion. We let this cost be linear in the rate of dissent for tractability, thus $W$ is the constant marginal cost of dissent.

### 3.1 Model results

We will now present the model results and how they relate to the empirical facts. We focus on equilibria with stationary bargaining strategies, that is, where judges consider only the current panel composition when making offers and counter-offers in the first (bargaining) stage. Our explanations will be informal and draw on intuition. The formal proofs are in the appendix.

It is straightforward to show that Facts 1 and 2 are predicted by the model. First,
denoting the bliss point of the median judge of the panel by $t_m$. Fact 1 (the median of the panel is determining the opinion) follows from the striving of each judge to minimize $|v - t|$ during the bargaining process.

**Proposition 1 (Fact 1)** There exists an equilibrium with stationary bargaining strategies. In any such equilibrium, $v = t_m$ in all panels.

**Proof:** See appendix C.1. \( Q.E.D. \)

Second, given the bargaining outcome, it is clear that signing is optimal for the median judge in the panel. As for the other judges in the panel, they minimize (3), hence it follows that a judge will sign opinions that are close to her ideology and dissent against opinions that are far.

**Proposition 2 (Fact 2)** In any equilibrium with stationary bargaining strategies, every judge $t$ has a single cutoff $c(t)$ such that she signs opinion $v$ if and only if $|v - t| \leq c(t)$.

**Proof:** See Appendix C.2. \( Q.E.D. \)

Interpreting this empirically, the model predicts that a judge is more likely to dissent the farther away her bliss point is from that of the median in the panel (Fact 2). The probability of dissent is then given by $P(c(t); t)$.

We will now show that the model generates the spider pattern of Fact 3—a hill-shaped relationship between the Dissent Rate and the Distance to Center of Judge Pool.

**Proposition 3 (Fact 3)** Consider an equilibrium with stationary bargaining strategies. For any $\alpha < \frac{2}{3}$ there exist values of $W$ such that $P(c(t); t)$ has a spider pattern (is first increasing and then decreasing in $|t|$ in the range $[0, 1]$). If $\alpha \geq \frac{2}{3}$, a spider pattern cannot exist in equilibrium.

**Proof:** See Appendix C.3. \( Q.E.D. \)
The proposition states that the ideological cost has to be sufficiently concave for the spider pattern to hold. We will now explain why and, for this purpose, we abstract from the size of ideological deviations and concentrate only on the number of dissents as a proxy for the extent of ideological compromise.

Figure 3 depicts the optimal choices of each type of judge. On the horizontal axis we see the choice variable (number of dissents) and on the vertical axis the resulting utility loss, which is comprised of two parts. First, there is disutility from the collegial pressure, which is linearly increasing in the number of dissents (the gray dotted line). Second, there is ideological disutility, which equals zero if a judge dissents whenever she is not the median and then increases concavely (when $\alpha$ is small) as the judge lowers her rate of dissent by signing unfavorable opinions (the black dashed lines). Since judges differ in how often they are median in a panel—where more centrist judges are more likely to be median—they will differ in the number of times they have to dissent in order to fully stick to their ideology. In the figure this implies that the ideological cost (the dashed line) of an extremist is further to right than that of a moderate, which is further to the right than that of a centrist – the more extreme a judge is the more opportunities she has to dissent. Consequently, the ideological
disutility associated with *never* dissenting (the point where the dashed lines meet the Y axis) is the largest for an extremist and the smallest for a centrist, but the concavity of the ideological cost means they are not vastly different.\(^\text{13}\)

The optimal choices of the three types of judges are depicted in Figure 3 with stars. First note that since \(D\) (the dashed line) is concave, there is no point in signing only few unfavorable opinions. This is since once a judge has signed a few such opinions the ideological cost of signing more opinions is very low. In order to reduce the ideological cost to any meaningful degree the judge has to never sign any unfavorable opinion. We call this feature of the concave ideological cost “perfectionism”. This perfectionism drives judges to either always dissent when they disagree with the opinion (the point where the dashed line meets the horizontal axis), or to never dissent (the point where the dashed line meets the vertical axis).

Starting with the centrist judge, she optimizes when choosing to always stick to her ideology because this implies a small collegial pressure and no ideological cost, whereas any compromise she would make (by decreasing her number of dissents) would entail a large ideological cost due to perfectionism. This is true also for the moderate judge in the figure – the collegial pressure she endures for sticking to her ideology is still lower than the cost of never dissenting. However, since the moderate is less often the median of her panel, her choice to stick to her ideology implies she will have a higher dissent rate than the centrist will. This explains why, as a judge moves from being a centrist to a moderate, the dissent rate goes up (the increasing part of the spider leg). Finally, an extremist judge who would stick to her ideology would face a much larger collegial pressure (see the rightmost point on the dotted line). Hence, she would rather give up on her ideology and just bear the ideological cost associated with never dissenting. Thus, the dissent rate falls sharply as a judge becomes sufficiently extreme. Overall, we get a hill-shaped relationship between a judge’s dissent rate and how extreme she is relative to her peers in the pool of judges she interacts with over

\(^{\text{13}}\)In the corner case where the ideological cost is a step function, the ideological disutility associated with never dissenting is the same for all judges.
The above theoretical logic would not hold if the personal cost of bliss-point deviation were instead convex, as depicted in Figure 4. The reason is that in this case each judge chooses an inner solution to the optimization problem (see the stars in the figure), where it is very costly for an extremist to dissent as seldom as a moderate or a centrist judge given the increasing marginal cost of ideological deviation.

**Figure 4.**— Why a convex cost cannot generate a spider pattern

![Figure 4](image)

### 4 Conclusions and discussion

We study a high-stakes field setting where decisions have an ideological element and judges are repeatedly randomly assigned into panels of three. In this setting, we present a consistent and robust set of evidence that together suggest that judges with non-consensual world views (“extremists”) are less confrontational, despite being less likely to determine the panel’s opinion on the case. Our findings further show that the results are not driven

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14It should be noted that the model and proof take into account not only that an extreme judge faces more opinions she dislikes, but also that she usually dislikes these opinions more than a moderate judge dislikes the opinions that *she* faces. When taking that into account we still get a spider pattern but the drop of the dissent rate when going from moderate to extreme judges is often more gradual than what our intuitive explanation suggests.
by having extreme ideology per se but rather by being extreme relative to the people one interacts with. Hence, interaction between individuals who disagree ideologically silences those who regularly find themselves far from the mainstream.

To rationalize our empirical observations, we present a simple model of judicial decision making in panels, where dissenting judges are subject to collegial pressure. This model suggests that the non-confrontational behavior of extreme judges is due to the concavity of ideological preferences which makes them cave in under peer pressure instead of just compromising a little bit, as would have been the case if the ideological cost were convex. Meanwhile, a concave ideological cost induces moderately ideological judges to stand their ground whenever they disagree with an opinion and hence appear more confrontational. Our theoretical finding thus suggests that the cost of deviating from one’s principles should be concave.\textsuperscript{15}

To the extent that the results generalize to other settings where ideology is salient, our findings may have important implications, e.g., for the expected behavior of immigrants vis-à-vis native society, for individual differences in confronting social and religious norms and for which factions of society are expected to publicly question the consensus. Concave ideological costs would have far-reaching implications for the empirical predictions of theoretical models in various domains. Concavity drives polarization in political platforms (Osborne 1995; Kamada and Kojima 2014). A concave ideological cost also affects socially optimal policy in that, when trying to bridge differences of opinion, disagreeing agents should each get to decide on a subset of issues rather than compromise within each debated issue (Eguia 2013). Given this wide array of applications, more empirical research into the curvature of moral and ideological costs is warranted, as are survey technologies that elicit these curvatures.

\textsuperscript{15}A large number of alternative models are further considered in Appendix D.2.
References


For Online Publication
A Empirical Appendix

A.1 Institutional background

The U.S. Federal Courts are a system of local level (District Court), intermediate level (Circuit Court), and national level (Supreme Court) councils. Members of these are appointed by the U.S. President and confirmed by the U.S. Senate. They are responsible for the adjudication of disputes involving federal law. Their decisions establish precedent for adjudication in future cases in the same court and in lower courts within its geographic boundaries. Each state has 1–4 District Courts. The 94 U.S. District Courts serve as trial courts with juries. The 12 U.S. Circuit Courts (Courts of Appeals), which are the empirical focus of this paper, take cases appealed from the District Courts. The Circuit Courts have no juries. Each Circuit Court presides over 3–9 states and consists of a pool of 8–40 judges (depending on the circuit). Figure 1 displays District Court boundaries in dotted lines and Circuit Court boundaries in solid lines.

Appendix Figure 1.— Geographical Boundaries of U.S. Federal Courts

Circuit Courts rule on the application of federal law, such as the constitutional validity of state laws, among other things. 98% of their decisions are final. Hence, they have a substantial impact on precedence, decision making and policy in the US.

In each of the 12 Circuit Courts, every judicial case gets three randomly assigned judges who are drawn from the court’s pool of judges. We refer to these three judges as the panel. The three judges in the panel decide a binary verdict (affirming or overturning the lower court verdict), where a majority of two judges is needed to set the verdict. They also compose an opinion (i.e., a text) motivating the verdict. The opinion serves as precedent for future cases and as such has a large impact on society and policy. Furthermore, being a text, the opinion can reflect the assertiveness of the panel and its ideological composition. A judge has to write a separate (minority) opinion if she either dissents (votes against the binary verdict) or concurs (votes for the verdict but for a different reason, as manifested in her minority
opinion). Both dissents and concurrences are costly in terms of time and collegiality and they cannot be cited as binding precedent. Note that, for a judge, dissenting and concurring are two mutually exclusive actions that both imply expressing dissatisfaction with the majority opinion – a form of confrontation.

Our empirical strategy rests on two key ingredients: 1) judges do not choose whom to interact with; and 2) our measure of their ideology is (reasonably) correct. Point 1 is motivated below and Point 2 in the next subsection.

Who a judge interacts with is determined by the pool of other judges serving on the same Circuit Court at the same time and by which panels a judge is assigned to. The assignments to the pool and to panels are both plausibly exogenous for a single judge. Starting with the assignment to the pool, appointment of judges to Circuit Courts is done by the President and confirmed by the U.S. Senate. Strictly speaking, a judge may of course turn down such a job offer and there is scarcely any way of knowing the extent to which this happens. But vacancies are rare and it is considered a great honor to serve in a Circuit Court—for most judges this would be the peak of their career. Hence, we do not find it likely that a judge will turn down an offer awaiting a better one. Importantly, given the life tenure of judges, once a judge is appointed she has no control over her peer group in practice. This is since 96% of all judges serve until they retire or pass away and since a vacant position on a circuit appears only when a judge retires (61% of vacancies in our sample), passes away (12%) or the number of seats is expanded (24%) while resignations are rare (3% of vacant positions). So, while it cannot be entirely ruled out that judges within a circuit pressure each other to quit, this is not a quantitatively important problem for what we do.

The assignment of judges into panels within a Circuit Court is random. Case assignments fall into two categories: 1) Once a case arrives, three randomly chosen judges are assigned to the case; 2) Once a year, the calendar is randomly set up in advance determining which judges will sit in which panels on which days in the upcoming year, and when a case comes up it gets assigned to the next panel. It is well established and has been thoroughly tested that both procedures are indeed random (see e.g. Chen and Sethi (2011), Berdejo and Chen (2016) and Chen (2016)).

A.2 Data and main variables

The data on dissents and concurrences come from Openjurist, which contains all cases from 1950 to 2007 (all summary statistics are presented in Appendix Table 4). The data was first digitized by one of

---

16 Given the binary nature of the verdict, at most one judge will dissent, but it can be the case that one judge dissents and another one concurs (where the third judge writes the “majority” opinion).
17 We analyze concurrences separately from dissents in order to show the robustness of our results and because they are legally distinct. However, as both require writing a separate minority opinion, we bind them together in our theoretical model and in the further robustness checks reported in the appendix, thus treating them as two alternative manifestations of the same thing: a judge’s decision to confront her panel’s opinion.
18 A very large literature, including judges’ writings about their own experience, documents a norm of consensus (see e.g. Edwards and Livermore 2008). Epstein et al. (2011) refer to this as “dissent aversion”. This literature attributes the peer pressure largely to collegiality concerns (Fischman 2011; Hettinger et al. 2007; Sunstein et al. 2006).
19 On average there is a vacancy once every 1.5 years for any single circuit (naturally, less often for smaller circuits and more often for larger ones). By then, the window of opportunity may have changed due to a change of President or senators or due to the arrival of competitors that better fit these politicians’ tastes.
20 New seats appeared, for instance, when the 5th Circuit was split into two in 1981.
21 Several recent papers employ random assignment of judges (e.g., Aizer and Doyle 2015; Shayo and Zussman 2011; Kling 2006; Belloni, Chernozhukov, and Hansen 2011; Lim, Silveira, and Snyder 2016).
22 See also Lim (2013) for empirical results from judicial courts, which shows that the best cost function to fit her data is one which is first convex (for small ideological deviations) and then concave (for large deviations). For tractability, our model utilizes power functions, whose curvature is fixed. However, in a numerical simulation in Appendix D.1, we show that an s-shaped function as considered by Lim (2013) can replicate our empirical findings. Hence, our theoretical finding is very much in line with Lim (2013) and further shows that what matters is that the cost function is concave when deviations become large – individuals do not differentiate between intermediate and large bliss-point deviations, as embodied by the notion of the what-the-hell effect.
the authors of the current paper (in Berdejó and Chen 2014) for whether there was a dissenting opinion and whether there was a concurring opinion. The current paper extracts the judge names and merges each judge with his/her ideology score. The ideology score we use is a standard summary measure coming from the Judicial Common Space database (Epstein et al. 2007) that was first coded by Giles et al. (2001). Many papers have used this score (e.g. Peresie (2005) and Kim (2009)). The general idea behind the score is leverage the political appointment process. That is, it assumes that—given that vacancies are rare and that Circuit Courts have a substantial impact on policy—the appointing politicians take the opportunities they get to assign judges of their ideological liking. It exploits the norm of senatorial courtesy by the President and is constructed as follows. If a judge is appointed from a state where the President and at least one home-state Senator are of the same party, the nominee is assigned the score of the home-state Senator (or the average of the home-state Senators if both members of the delegation are from the President’s party). 23 If neither home-state Senator is of the President’s party, the judge receives the score of the appointing President. The score thus assumes that the President does favors to senators from the same party while ignoring the preferences of senators from the other party. The score has two additional main advantages. First, it is exogenous since (unlike common measures of Supreme Court judges’ ideology) it assigns the ideology of the judge before her behavior at the court is observed, which of course is key since we are interested in how a judge’s behavior at the court is affected by her ideology. 24 The second main advantage of this score is its high ability to predict judges’ voting patterns in court, as clearly visualized in Figure 2 (see below for how the voting variable is constructed). The ideology score takes values in between roughly ±0.8 (see Appendix Figure 4 for a histogram of the distribution of ideology scores in our data). As a robustness check we use the party of appointing President as ideology score and find qualitatively the same results—see Section 2.3.

23 The scores of the Senators are located in a two-dimensional space on the basis of the positions that they take in roll-call votes, but only the first of the two dimensions is salient for most purposes. The ideology scores of Presidents are then estimated along this same dimension based on the public positions that they take on bills before Congress.

24 Other papers (e.g. Bailey 2016 and Jacobi and Sag 2009) with a different purpose than ours use non-exogenous ideology scores based on (Supreme Court) judges’ voting at the court (Martin and Quinn 2002).
APPENDIX FIGURE 2.— Vote Ideology and the Judicial Common Space database Ideology Score – local polynomial

Notes: x-axis: (Non-demeaned) Judicial Common Space database Ideology score of a judge, where more conservative scores are along the right on the x-axis. y-axis: Vote ideology, demeaned to be centered at zero. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is Vote Ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. The dashed lines depict the 95% confidence interval. Data come from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample).

To examine the ideological color of the majority and minority opinions and of the judge’s vote on each panel, we also employ the U.S. Courts of Appeals Database Project, a random sample of roughly 5% of appeals-courts decisions from 1925 to 2002. This database includes hand-coded information on the ideological content of each coded opinion (liberal = -1, conservative = 1, and mixed or unable to code = 0), to which we will refer as Opinion Ideology. This database also reports dissents and concurrences and codes the ideological content of their corresponding minority opinions.

Using the data from the Courts of Appeals database, we create a measure of the ideology of a judge’s voting pattern. This measure is set to equal the ideology of the majority opinion if the judge did not dissent or concur and to equal the ideology of her own minority opinion if the judge dissented or concurred. We call this variable Vote Ideology (this variable is, for instance, used on the y-axis of Figure 2).

Our sample contains 293,868 decisions in Openjurist and 18,686 decisions in the Courts of Appeals database. Overall, 8.5% of opinions in Openjurist have dissents (6.4% have concurrences) while in the Courts of Appeals database 7.9% of opinions have dissents (3.6% have concurrences).

To construct a measure of ideological disagreement between a judge and her peers on the panel of a specific case, we calculate the Score Relative to Panel Median, which is positive when the judge is more conservative than the median and negative when the judge is more liberal than the panel median. We also take the absolute value of this variable and refer to it as Distance to Panel Median.

26The Appeals Court Database Project states that for most issue categories, these will correspond to conventional notions of “liberal” and “conservative”. The directionality codes parallel closely the directionality codes in the Spaeth Supreme Court database.
27Any analysis requiring the panel median includes only panels where there are no tied or missing scores (panels with tied scores are excluded because the identity of the median judge is not uniquely determined). All results presented in the paper are robust to including also tied scores.
Next we construct a measure of ideological disagreement between a judge and her peers in the pool, whom she can expect to meet regularly. To do this we begin by calculating the average ideology score of the pool of judges for each Circuit and each year. This average score represents the center of the pool of judges available to be assigned at that Circuit-year, to which we refer as Center of Judge Pool. We then demean the ideology score of a judge by the Center of Judge Pool and get a measure we refer to as Score Relative to Center of Judge Pool. This measure can take both positive values (if the judge is more conservative than her peers) and negative ones (if the judge is more liberal than her peers) and serves as our main ideological measure. The value of this measure for a given judge may change during her service as the composition of her pool changes. A histogram of Score Relative to Center of Judge Pool is displayed in the center part of Appendix Figure 4. We refer to the absolute value of this measure as Distance to Center of Judge Pool.

Using the data from Openjurist we calculate the Dissent Rate and Concurrence Rate for each judge in each Circuit-year. These are essentially measures of a judge’s tendency to confront her peers.

A.3 Stylized Facts

A.3.1 Who affects the majority opinion?

Our first empirical result is this:

FACT 1 The median of the panel is determining the opinion.

Appendix Table A.1 reports the coefficients for the regression specified in (1).

<table>
<thead>
<tr>
<th></th>
<th>Opinion Ideology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score Relative to Center of Judge Pool</td>
<td>0.0166 (0.0125)</td>
</tr>
<tr>
<td>Panel Median</td>
<td>0.00118 (0.000775)</td>
</tr>
<tr>
<td>Score Relative to Center of Judge Pool</td>
<td>0.142*** (0.0409)</td>
</tr>
<tr>
<td>* Panel Median</td>
<td>23031</td>
</tr>
<tr>
<td>N</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data come from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is opinion ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal.

Fact 1 is of course not surprising given the pivotal role of the median in a three-judge panel and is consistent with many conventional bargaining models. This result also aligns with a recent experimental study of decision making within groups (Ambrus et al. 2015), finding that the median has the greatest impact on the group’s joint decision. Ambrus et al. (2015) further find that the most extreme individuals relative

---

28We use Circuit and year since this represents the ideology of the other judges a specific judge expects to sit with in a panel.

29As robustness checks, we calculate for each judge the dissent and the concurrence rates averaged over a 2-year bin and over a judge’s lifetime. In these calculations, a judge’s score is the judge’s Score Relative to Center of Judge Pool averaged over all the cases on which she was sitting over the corresponding period.
to the median have a particularly small effect on the joint outcome, a result that is largely echoed by our further investigation, see Appendix Figure 7 and description in Appendix A.5.1.

A.3.2 Does ideological disagreement drive dissent?

We proceed now to showing a within-judge property regarding when a judge will choose to dissent. Figure 3 presents a non-parametric visualization of the Dissent Rate by ideology Score Relative to Panel Median using a local polynomial regression (see Altman 1992 and Fan and Gijbels 1996). This figure reveals a clear pattern: the more a judge is distant from the panel median, the more likely she is to dissent. This holds both on the left and on the right. We present similar non-parametric visualizations for concurrences in Appendix Figure 8.

APPENDIX FIGURE 3.— Dissent and Ideology Score Relative to Panel Median – local polynomial

Notes: x-axis: Ideology score of a judge demeaned by the median of the panel of judges assigned on the case. y-axis: Rate of dissent. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is rate of dissent. The dashed lines depict the 95% confidence interval. Data come from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

To test it in a regression specification, we regress the Dissent and Concurrence rates of each judge-case combination on polynomials of the judge’s Distance to the Panel Median—an absolute value that captures the strength of ideological disagreement with her panel peers. We also add judge fixed effects ($I_i$) to ensure that the result is not driven by the ideology scores of judges per se. We also include Circuit ($C_c$) and year

---

A prior empirical examination of the role of the median judge in Circuit-Court panels (by Cross, 2007) did not find that the median judge was setting the court’s opinion, when including the score of the median judge and the sum of the scores of all the judges on the panel as two explanatory variables in the same regression. We replicate their specification in Appendix Table A.7 and show that if one controls for the average ideology of the circuit or if one breaks the sum of judges’ scores to the scores of each of the three judges separately (instead of including the median judge twice, as Cross practically does in his specification) then that corroborates our result that the median determines the opinion.

---
(\(T_i\)) fixed effects and cluster the standard errors by Circuit-year. The basic regression specification is:

\[
\text{Dissent Rate}_{pcit} = \gamma_1 \text{Distance to Panel Median}_{pcit} + \\
\gamma_2 \text{Distance to Panel Median}^2_{pcit} + I_i + C_c + T_t + \nu_{pcit}
\]

for judge \(i\) on panel \(p\) in Circuit \(c\) at year \(t\). Table A.2 indicates that the frequency of both dissents and concurrences increases in the distance to the panel median, implying that ideological disagreement is a driver of dissent.

### APPENDIX TABLE A.2

**Dissent and Ideological Distance to Median of Panel**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dissent</td>
<td>Concur</td>
</tr>
<tr>
<td>Distance to Median of Panel</td>
<td>0.00425***</td>
<td>0.00244***</td>
</tr>
<tr>
<td></td>
<td>(0.00119)</td>
<td>(0.000907)</td>
</tr>
<tr>
<td>Distance^2</td>
<td>-0.00142</td>
<td>-0.000868</td>
</tr>
<tr>
<td></td>
<td>(0.00154)</td>
<td>(0.00116)</td>
</tr>
<tr>
<td>Judge Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Circuit Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>N</td>
<td>541163</td>
<td>541163</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.411</td>
<td>0.414</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Ideology scores are demeaned by the actual center of the panel of judges assigned on a case. The dependent variable is a dummy for whether a judge dissented (column 1) or concurred (column 2) in the panel. Fixed effects include year, circuit, and judge.

Based on these results our second empirical finding is:

**Fact 2** A judge is more likely to dissent when the panel median is ideologically far from her.

This result, that for the individual judge ideological disagreement with her panel peers is an important determinant for when she will dissent, is in line with previous observations in the literature (e.g., Wahlbeck et al. 1999, Spriggs et al. 1999 and Hettinger et al. 2004).

To test it in a regression specification, we regress the dissent and concurrence rate of each judge on polynomials of her (absolute) distance to the center of the pool of judges in her Circuit-year:

\[
\text{Dissent Rate}_{cit} = \alpha + \gamma_1 \text{Distance to Center of Judge Pool}_{cit} + \\
\gamma_2 \text{Distance to Center of Judge Pool}^2_{cit} + C_c + T_t + \nu_{cit}
\]

for judge \(i\) in Circuit \(c\) and year \(t\). Circuit and year fixed effects are represented by \(C_c\) and \(T_t\) and standard errors are clustered by Circuit-year. Appendix Table A.3 indicates that the spider pattern is robust: according to the estimated linear and quadratic coefficients in the table, the maximum dissent rate is obtained for Distance to Center of Judge Pool of 0.6 (for dissents) and 0.46 (for concurrences) which are clearly within the bounds of our distribution, which goes from around -0.8 to +0.8.\(^{31}\)

\(^{31}\)See Appendix Figure 4 for the distribution of scores.
APPENDIX TABLE A.3

Dissent and Ideological Distance to Center of Judge Pool

<table>
<thead>
<tr>
<th>(1)</th>
<th></th>
<th>(2)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dissent</td>
<td>0.0404***</td>
<td>Concur</td>
<td>0.0285***</td>
</tr>
<tr>
<td>Distance to Center of Judge Pool</td>
<td>(0.00756)</td>
<td>Distance to Center of Judge Pool</td>
<td>(0.00570)</td>
</tr>
<tr>
<td>Distance</td>
<td>-0.0334***</td>
<td>Distance</td>
<td>-0.0313***</td>
</tr>
<tr>
<td></td>
<td>(0.0118)</td>
<td></td>
<td>(0.00862)</td>
</tr>
<tr>
<td>Circuit Fixed Effects</td>
<td>Y</td>
<td>Circuit Fixed Effects</td>
<td>Y</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Y</td>
<td>Year Fixed Effects</td>
<td>Y</td>
</tr>
<tr>
<td>N</td>
<td>10043</td>
<td>N</td>
<td>10043</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.109</td>
<td>R-sq</td>
<td>0.086</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. The main independent variable is (the absolute value of) ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The dependent variable is the judge’s dissent rate (column 1) or concurrence rate (column 2) in a Circuit-year. Fixed effects include year and circuit. Observations are weighted by the number of votes cast by the judge in the Circuit-year.

APPENDIX TABLE A.4

Dissent and Alternative Ideology Score

<table>
<thead>
<tr>
<th>(1)</th>
<th></th>
<th>(2)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dissent</td>
<td>0.0501***</td>
<td>Concur</td>
<td>0.0284***</td>
</tr>
<tr>
<td>Distance to Center of Judge Pool</td>
<td>(0.0157)</td>
<td>Distance to Center of Judge Pool</td>
<td>(0.0107)</td>
</tr>
<tr>
<td>Score Based on Party of Appointment</td>
<td>-0.0367**</td>
<td>Score Based on Party of Appointment</td>
<td>-0.0222*</td>
</tr>
<tr>
<td></td>
<td>(0.0164)</td>
<td></td>
<td>(0.0115)</td>
</tr>
<tr>
<td>Circuit Fixed Effects</td>
<td>Y</td>
<td>Circuit Fixed Effects</td>
<td>Y</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Y</td>
<td>Year Fixed Effects</td>
<td>Y</td>
</tr>
<tr>
<td>N</td>
<td>10033</td>
<td>N</td>
<td>10033</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.106</td>
<td>R-sq</td>
<td>0.085</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Ideology scores are simply the party of appointment (Republican or Democrat, coded as 1 and 0). The main independent variable is (the absolute value of) ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The dependent variable is the judge’s dissent rate (column 1) or concurrence rate (column 2) in a Circuit-year. Fixed effects include year and circuit. Observations are weighted by the number of votes cast by the judge in the Circuit-year.

\[ \text{For instance, a judge nominated by a Democrat President (that is, score 0) in a circuit-year consisting of a total of 8 Republican-nominated and 2 Democrat-nominated judges will get a relative score of } 0 - \frac{(8 \times (+1) + 2 \times 0)}{10} = -0.8. \text{ A Republican-nominated judge in that same pool will get a relative score of } 1 - \frac{(8 \times (+1) + 2 \times 0)}{10} = 0.2. \]
### APPENDIX TABLE A.5

**Summary Statistics**

<table>
<thead>
<tr>
<th>Vote-Level</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dissent</td>
<td>0.03</td>
<td>0.17</td>
</tr>
<tr>
<td>Concur</td>
<td>0.02</td>
<td>0.13</td>
</tr>
<tr>
<td>Score Relative to Center of Judge Pool</td>
<td>-0.01</td>
<td>0.30</td>
</tr>
<tr>
<td>Distance to Median of Panel</td>
<td>0.18</td>
<td>0.24</td>
</tr>
<tr>
<td>Ideology Score</td>
<td>0.01</td>
<td>0.34</td>
</tr>
<tr>
<td>N</td>
<td>541182</td>
<td></td>
</tr>
</tbody>
</table>

**Case-Level (Songer-Auburn sample)**

| Opinion Ideology (1 = Conservative) | 0.19   | 0.90   |
| Panel Median                       | 0.33   | 0.47   |
| Score Relative to Center of Judge Pool | -0.01  | 0.32   |
| Ideology Score                     | -0.03  | 0.34   |
| Center of Judge Pool Ideology Score | -0.02  | 0.16   |
| N                                 | 7677   |

**Circuit-Year Level**

| Distance to Center of Judge Pool | 0.28   | 0.17   |
| Distance to Center of Judge Pool based on Party of Appointment | 0.46   | 0.16   |
| Number of Votes Cast Per Judge   | 65.73  | 55.88  |
| N                                 | 10033  |

| Number of Judges per Circuit-Year | 16.95  | 9.65   |
| Number of Circuit-Years          | 667    |

Notes: Data on dissents and concurrences comes from OpenJurist (1950-2007). Data on opinion ideology come from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Sample includes three-judge panels where there are no tied or missing scores. Opinion ideology is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. Ideology scores come from the Judicial Common Space database (Epstein et al. 2007), which provides a summary measure using the voting patterns of the appointing President and home-state Senators.
**A.4.2 Distribution of ideological scores**

**Appendix Figure 4.**— Distribution of Ideology Scores

Notes: Ideology scores from the Judicial Common Space database (Epstein et al. 2007). Left panel: raw ideology score. Central panel: ideology score demeaned by the center of the judge pool in a circuit-year. Right panel: ideology score demeaned by the mean ideology score of members of the Supreme Court (the supreme-court ideology comes from Martin and Quinn (2002)).

**A.4.3 Which judges have the most ideological voting pattern?**

In this section we investigate the relationship between a judge’s (relative) ideology score and the ideological color of her votes (Vote Ideology) in the cases she is sitting on.

**Appendix Figure 5.**— Vote Ideology and Ideology Score of Judge Relative to Center of Judge Pool – local polynomial

Notes: x-axis: Ideology score of a judge demeaned by the center of the pool of judges in a Circuit-year. y-axis: Vote ideology, demeaned to be centered at zero. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is Vote Ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. The dashed lines depict the 95% confidence interval. Data come from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample).

We test the relationship between a judge’s ideology Score Relative to Center of Judge Pool and her Vote Ideology using a local polynomial regression. The results are presented in Appendix Figure 5. As can be seen, the most ideological voting is obtained for moderately ideological judges and once a judge becomes
sufficiently extreme her voting becomes less ideological. To test the statistical significance of this result we run the regression:

\[
\text{Vote Ideology}_{pct} = \alpha + \gamma_1 \text{Score Relative to Center of Judge Pool}_{cit} + \\
\gamma_2 \text{Score Relative to Center of Judge Pool}^2_{cit} + \\
\gamma_3 \text{Score Relative to Center of Judge Pool}^3_{cit} + \nu_{pct}
\]

for judge \( i \) on panel \( p \) in Circuit \( c \) and year \( t \). The regression results are presented in Appendix Table A.6. It confirms (by the negative cubic term) that judges with moderate scores have the most ideological voting pattern while judges with extreme scores have a less ideological voting pattern, implying that being in ideological minority makes a judge behave less ideologically. According to the coefficients in the table, the strongest ideological voting pattern is obtained for judges with scores of \(-0.36\) and \(+0.47\) which are both well within the bounds of our distribution, which goes from around \(-0.8\) to \(+0.8\).

**APPENDIX TABLE A.6**

**Vote Ideology and Ideology Score of Judge Relative to Center of Judge Pool**

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<tr>
<th>Score Relative to Center of Judge Pool</th>
<th>Vote Ideology</th>
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<tr>
<td></td>
<td>0.180***</td>
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<tr>
<td></td>
<td>(0.0308)</td>
</tr>
<tr>
<td>Score^2</td>
<td>0.0614</td>
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<tr>
<td></td>
<td>(0.0659)</td>
</tr>
<tr>
<td>Score^3</td>
<td>-0.366***</td>
</tr>
<tr>
<td></td>
<td>(0.125)</td>
</tr>
<tr>
<td>N</td>
<td>23031</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* \( p < 0.10 \); ** \( p < 0.05 \); *** \( p < 0.01 \)). Data on cases comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Ideology scores come from the Judicial Common Space database. Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is Vote Ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. The independent variables are polynomials of the ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year.

The result is robust when using polynomials of a higher order and when using the lifetime average for each judge, splitting the sample according to whether the case affirmed the lower court decision, or using the alternative ideology score (the one using the appointing President’s party and the share of judges in the pool who are of the other party). These robustness checks are available upon request.

It is worth noting that, like Fact 3, also this result disappears when using raw ideology scores (i.e., not relative to the pool)—see Figure 2. Hence, what drives this result is that a judge is ideologically extreme relative to her peers—it is not about extreme ideology per se, but about the interaction between peers who disagree ideologically. It may be noted also that the regression in (6) does not contain fixed effects. Adding circuit and year fixed effects makes the results non significant, possibly due to the much smaller sample used here compared to Facts 1-3.\(^{34}\)

\(^{33}\)We use polynomial of the third degree in the regression to enable testing for a U-shape on the left and a hill-shape on the right.

\(^{34}\)Recall that in order to construct our measure of Vote Ideology we rely on the Opinion Ideology as coded by the U.S. Courts of Appeals Database Project, which consists of a random sample of only 5% of all cases. Recall also that each of the roughly 10000 observations reported in Table A.3 represents on average about 30 cases.
A.5 Robustness of empirical results

A.5.1 Robustness for Fact 1

In Column 1 of Table A.7, we reproduce the finding from Table 6.3 in Cross (2007). Our data sample is slightly smaller than that of Cross (2007) because we drop panels with judges whose scores are tied (adding ties does not change the results), but we are able to replicate the finding: the sum of the scores of the judges on the panel is correlated in this specification with opinion ideology, but the score of the median judge is not. In Column 2, we add the Center of Judge Pool and find that the sum of the scores of the judges is no longer significant. Hence, this suggests that the results of Cross (2007), that panel composition determines the opinion, is in fact driven by a variable he has omitted – the ideology of the circuit. In Column 3, we separate the sum of the scores into the score of the left judge and score of the right judge. Now we see that the score of the median judge is the main driver of opinion ideology, though the score of the right-most judge is also correlated with opinion ideology. In Column 4, we show what is arguably the best specification by including all of these measures and, importantly, controlling for the circuit’s ideology, and find that the median judge is the only judge affecting the decision. Appendix Figure 6 visualizes the results. Columns 3 and 4 are close to the specification in Ambrus et al. (2015), a recent experimental paper examining group-decision making. Controlling for Center of Judge Pool instead of subtracting it from the judge’s ideology score (like we do in Table A.1) also shows the relevance of the average ideology score of the pool of judges in each Circuit and each year.

<table>
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<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median of Panel Ideology Score</td>
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<td>0.0772</td>
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<td>0.121***</td>
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<td>(0.0607)</td>
<td>(0.0591)</td>
<td>(0.0308)</td>
<td>(0.0354)</td>
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<td>Sum of Panel’s Ideology Scores</td>
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<td>0.0437</td>
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<tr>
<td></td>
<td>(0.0347)</td>
<td>(0.0331)</td>
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<tr>
<td>Center of Judge Pool</td>
<td>0.249*</td>
<td></td>
<td>0.251*</td>
<td></td>
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<tr>
<td>Ideology Score</td>
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<td>(0.133)</td>
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<td>Left of Panel Ideology Score</td>
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<tr>
<td></td>
<td>(0.0717)</td>
<td>(0.0720)</td>
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<tr>
<td>Right of Panel Ideology Score</td>
<td>0.102*</td>
<td>0.0373</td>
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<td>(0.0527)</td>
<td>(0.0448)</td>
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<tr>
<td>N</td>
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<td>7677</td>
<td>7677</td>
<td>7677</td>
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<tr>
<td>R-sq</td>
<td>0.007</td>
<td>0.008</td>
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</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data come from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is ideology of opinion, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal.

35We are unaware of other studies (apart from ours and Cross (2007)) examining the median voter theorem in Circuit Courts. Many studies construct the relative position of judges, for example, Peresie (2005) sorts the judges on a panel, but presents a regression where judge votes are the unit of observation and the scores of the judge’s two colleagues (left and right of the pair) are included as regressors. This means that each judge score enters three times in the dataset, once for the score of the judge whose vote is the dependent variable, and twice more as the left score or right score or both.
Notes: x-axis: Ideology score of a judge. y-axis: Opinion ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. Each dot represents the average of all opinions in a bin of judges with similar ideology scores and with the same role within a panel (left, median or right). The lines represent the regression coefficients from Table A.1. As such, each set of dots is plotted after residualizing for the other variables in the regression. The y-axis is demeaned to be centered around zero. Data come from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Sample includes three-judge panels where there are no tied or missing scores.

Next, also as a form of robustness, we investigate the relationship between a judge’s ideology and the ideological color of the (majority) opinion produced by panels she is sitting in. If the median of the panel is the primary driver of the opinion, then extreme judges should rarely influence the outcome of the panel since they are seldom median. Appendix Figure 7 visualizes a local polynomial regression with a judge’s ideology as independent variable and the opinion ideology as a dependent variable. As can be seen, the most ideological opinion is obtained when moderates are involved. Put differently, extreme judges are not affecting the opinions. This result is robust to adding quartic terms and splitting the sample according to whether the lower court decision was affirmed or not and to using the lifetime average for each judge.
Appendix Figure 7.— Ideology of Opinion and Ideology Score of Judge Relative to Center of Judge Pool – local polynomial

Notes: x-axis: Ideology score of a judge demeaned by the center of the pool of judges available to be assigned in a Circuit-year. y-axis: Opinion ideology, demeaned to be centered at zero. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is Opinion Ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. The dashed lines depict the 95% confidence interval. Data come from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample).
A.5.2 Robustness for Fact 2

Appendix Figure 8 (local polynomial) shows that the result is robust to using concurrence instead of dissent.

APPENDIX FIGURE 8.— Concurrence and Ideology Score of Judge Relative to Median of Panel – local polynomial

Notes: x-axis: Ideology score of a judge demeaned by the median of the panel of judges assigned on the case. y-axis: Rate of concur. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is rate of concur. The dashed lines depict the 95% confidence interval. Data come from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.
A.5.3 Robustness for Fact 3

The upper left panel of Appendix Figure 9 shows the raw data of Dissent Rate by Score Relative to Judge Pool when grouping observations with similar relative ideology into separate bins. These scores were divided from left to right into 15 evenly-spaced bins, where for each bin we estimated the average dissent rate in that bin. We also present the 95% confidence interval around the average dissent rate. As can be seen the spider pattern appears clearly here too.

APPENDIX FIGURE 9.— Dissent or concur and Ideology Score Relative to Center of Judge Pool

Notes: Data on cases come from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. x-axis: Ideology score of a judge demeaned by the center of the pool of judges available to be assigned in a Circuit-year. Upper left panel: Dissent rate (y-axis) when the x-axis is divided into 15 evenly-spaced bins (the number denotes the midpoint of the bin); the y-axis shows the mean concurrence rate for each bin; the means are weighted averages over all judges and all Circuit-years, accounting for the number of times each judge actually appeared on cases in any given Circuit-year. Upper right panel: Concurrence rate (y-axis) when the x-axis is divided into 15 evenly-spaced bins. Lower left panel: Concurrence rate (y-axis) using a local polynomial regression with an Epanechnikov kernel. Lower right panel: Dissent rate (y-axis) using a local polynomial regression with an Epanechnikov kernel after residualizing by Circuit and Year fixed effects. The dashed lines in all panels depict the 95% confidence interval.

36 Note that the share of judges having the same score (or range of scores) has no effect on the pattern of dissent rate. The pattern depicts the average dissent rate for each score, regardless of how many judges have this score, hence there is no distorting effect of weighting by a variable number of judges.

37 The 95% confidence interval comes from a weighted regression of the dissent rate on a constant for each bin, with weights being the number of votes cast by a judge in a Circuit-year.
The upper right panel (bins) and the lower left panel (local polynomial) of Appendix Figure 9 show the spider pattern appears also when using concurrences instead of dissents, though less markedly on the right. The lower right panel of Appendix Figure 9 (local polynomial) shows the spider pattern appears also when residualizing by circuit and year fixed effects.

We further check if the spider is present under different weighting and scores. Appendix Table A.8 Column 1 repeats the main specification using (the absolute) Distance to Center of Judge Pool. Column 2 includes a cubic term (the quadratic and cubic terms jointly produce a spider). Column 3 weights each judge equally but excludes judges who vote less than 10 times. Column 4 does the same but presents a logit model. Column 5 uses a 2-year binned dissent and concur rates. Column 6 uses the lifetime average rate. Column 7 uses the Distance to the Supreme Court. Column 8 uses both Distance to Center of Judge Pool and Distance to the Supreme Court. As can be seen, the spider pattern is robust in these specifications. Column 9 shows that the spider is robust to including polynomials of the distance to panel median as controls, indicating that the spider pattern is not driven by interactions within particular panels. Finally, Column 10 randomly assigns Distance to Center of Judge Pool to a different judge to mitigate the concern of spurious significance or erroneous clustering level. As can be seen the result then disappears implying the result is not driven by spurious significance or by the chosen level of clustering.

We also check if the spider is robust to dropping one Circuit at a time. Appendix Table A.9 Column 1 repeats the main specification using (the absolute) Distance to Center of Judge Pool. Columns 2 to 13 drop one Circuit at a time. As can be seen the spider pattern is robust. This mitigates the concern that the pattern is driven by outliers.
### Appendix Table A.8.— Robustness to Alternative Scores

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<td><strong>Distance to Center of Judge Pool</strong></td>
<td>0.0664***</td>
<td>-0.0140</td>
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<td>4.074***</td>
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<td>Based on Ideology</td>
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<tr>
<td><strong>Distance to Center of Judge Pool</strong></td>
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<td>0.0678***</td>
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Notes: Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Absolute value of the distance to the center of the judge pool is the main independent variable. The dependent variable is the judge’s sum of dissent rate and concurrence rate in this Circuit-year (with the exception of column 5, which is the rate calculated over two years; column 6, which is the lifetime rate; and column 9, which is the decision to dissent or concur on this panel). Fixed effects include circuit and year (year of appointment for column 6). Observations are weighted by the number of votes cast by the judge in the time-unit of observation (with the exception of columns 3 and 4, which do not weight but exclude judges with less than 10 votes in a Circuit-year). Column 4 runs a logit model and all other columns run linear probability models. Distance to Panel Median is not an alternative score, but is presented as a rejection of judge-specific mechanisms. Resampled Distance to Center of Judge Pool is presented as a rejection of spurious significance, where judicial scores have been randomly reassigned. All columns use robust standard errors clustered at the Circuit-year level, except column 5, which clusters at the Circuit-2-year-bin level, and column 6, which clusters at the Circuit level (* p < 0.10; ** p < 0.05; *** p < 0.01).
Appendix Table A.9—Robustness to Dropping One Circuit at a Time

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<th>Drop Circuit</th>
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<th>Circuit Fixed Effects</th>
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<th>R-sq</th>
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<th>Scores based on Ideology</th>
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<td>Y</td>
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<td>0.124</td>
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<td>0.0103</td>
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<td>4</td>
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<td>0.123</td>
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<td>8881</td>
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<td>0.127</td>
<td>0.0705***</td>
<td>0.0106</td>
</tr>
<tr>
<td>11</td>
<td>Y</td>
<td>Y</td>
<td>9386</td>
<td>0.114</td>
<td>0.0645***</td>
<td>0.0105</td>
</tr>
<tr>
<td>12</td>
<td>Y</td>
<td>Y</td>
<td>9275</td>
<td>0.127</td>
<td>0.0763***</td>
<td>0.0104</td>
</tr>
</tbody>
</table>

Notes: Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Absolute value of the distance to the center of the judge pool is the main independent variable. The dependent variable is the judge’s sum of dissent and concurrence rate in this Circuit-year. Fixed effects include circuit and year. Observations are weighted by the number of votes cast by the judge in the time-unit of observation. Each one of the 12 columns drops one circuit from the sample. All columns use robust standard errors clustered at the Circuit-year level. *** p < 0.01, ** p < 0.10, * p < 0.05.
Appendix Table A.10 (columns 1 and 2) controls for biographical characteristics of the judge. These are controlled using dummy indicators for party of appointment, whether the judge and appointing President were of the same or different political parties, whether government (Congress and President) was unified or divided at the time of appointment, whether the judge was Protestant, Evangelical Protestant, Mainline Protestant, Catholic, Jewish, or non-religious, whether the judge was Black, non-white, or female, whether the judge received a law degree from a public institution, a bachelor’s degree from a public institution, a bachelor’s degree from within the state of appointment, or obtained further graduate studies in law (LLM or SJD), was born in the 1910s, 1920s, 1930s, 1940s, or 1950s, had previous experience as federal district judge, law professor, U.S. attorney, assistant U.S. attorney, Solicitor-General, mayor, state governor, Attorney-General, Deputy or assistant district/county/city attorney, Bankruptcy judge, U.S. Magistrate, Congressional counsel, District/County/City Attorney, Local/municipal court judge, Sub-cabinet secretary, Cabinet secretary, Special prosecutor, State lower court judge, State high court judge, or Local/municipal court judge, or had experience in City council, Department of Justice, Solicitor-General's office, or served as a member of the State house, State senate, U.S. House of Representatives, or had previous experience in private practice, in government, or in other federal capacity, or received an exceptional rating from the American Bar Association, and, in case the judge was elevated from the district courts, the party of the President who made the district bench appointment. As can be seen, the spider pattern is robust and is not driven by some special characteristics of extreme judges.

**APPENDIX TABLE A.10**

**DISSENT AND IDEOLOGY SCORE OF JUDGE RELATIVE TO CENTER OF JUDGE POOL**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance to Center of Judge Pool</td>
<td>0.0459***</td>
<td>0.0288***</td>
<td>0.0460***</td>
<td>0.0460***</td>
<td>0.0760***</td>
<td>0.0522***</td>
</tr>
<tr>
<td></td>
<td>(0.00784)</td>
<td>(0.00586)</td>
<td>(0.0107)</td>
<td>(0.0184, 0.0708)</td>
<td>(0.0109)</td>
<td>(0.0136)</td>
</tr>
<tr>
<td>Distance$^2$</td>
<td>-0.0403***</td>
<td>-0.0324***</td>
<td>-0.0433**</td>
<td>-0.0433**</td>
<td>-0.0750***</td>
<td>-0.0495**</td>
</tr>
<tr>
<td></td>
<td>(0.0125)</td>
<td>(0.00886)</td>
<td>(0.017)</td>
<td>[-0.0778, 0.0020]</td>
<td>(0.0167)</td>
<td>(0.0215)</td>
</tr>
<tr>
<td>Judge Characteristics</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Circuit Fixed Effects</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Sample</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>Affirmed</td>
<td>Not Affirmed</td>
</tr>
<tr>
<td>N</td>
<td>8692</td>
<td>8692</td>
<td>7744</td>
<td>7744</td>
<td>9577</td>
<td>9622</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.183</td>
<td>0.173</td>
<td>0.111</td>
<td>0.111</td>
<td>0.091</td>
<td>0.072</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Judicial characteristics come from Federal Judiciary Center/Attributes of U.S. Federal Judges Database and controlled for as binary indicators. The main independent variable is (the absolute value of) ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. Column 1: dependent variable is dissent rate, controlling for judges’ biographical characteristics. Column 2: dependent variable is concurrence rate, controlling for judges’ biographical characteristics. Column 3: dependent variable is dissent+concurrence rate, bootstrapped standard errors clustered at the Circuit-year level. Column 4: dependent variable is dissent+concurrence rate, wild bootstrap. Column 5: dependent variable is dissent+concurrence rate, subsample of decisions that affirmed the lower court opinion. Column 6: dependent variable is dissent+concurrence rate, subsample of decisions that did not affirm the lower court opinion.

Appendix Table A.10 (columns 3 and 4) presents the main specification but using two forms of bootstrapping. As can be seen, the spider result is robust and the p-values similar as in the main specification.

Appendix Table A.10 (columns 5 and 6) presents the main specification but splits the sample according to whether the decision affirmed the lower court verdict. The sample size differs slightly when there are no affirmances or all affirmances for a judge in a Circuit-year. As can be seen, the spider result is robust.

Finally, Appendix Figure 10 shows the raw data (local polynomial) using the non-demeaned ideology
score as the independent variable. As can be seen, the spider pattern is then strongly attenuated (compared to with the equivalent Figure 2 using relative scores), which indicates that the spider result is driven by the interaction of peers who disagree ideologically rather than by extreme ideology per se.

APPENDIX FIGURE 10.— Dissent and (non-relative) Ideology Score of Judge – local polynomial

![Graph](image)

Notes: x-axis: Non-demeaned Ideology score of a judge. y-axis: Rate of dissent. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is rate of dissent. The dashed lines depict the 95% confidence interval. Data come from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

---

38 Figure 10 has a slightly more narrow range on the x-axis compared to the equivalent figures using relative ideology scores. This is since the distribution of raw scores has a more narrow support than for the relative score (see Appendix Figure 4).
B Further theoretical predictions

B.1 Predicting the S-shaped voting pattern

Here we show that our main model produces also the empirical observation presented in Section A.4.3—an S-shaped voting pattern where the moderately ideological judges have the most ideological voting. To explain this observation analytically we construct a measure of the ideology \( I(t; v) \) of a vote of judge \( t \) as follows:

\[
I(t; v) = \begin{cases} 
  v & \text{if } s(v; t) = 1 \\
  t & \text{if } s(v; t) = 0
\end{cases}
\]

In equilibrium, \( v = t_m \), hence, by signing the majority opinion, a judge in practice votes for the median’s bliss point. Meanwhile, the minority opinion of a dissenting judge is assumed to equal her own bliss point \( (t) \) because, in this case, she composes a separate opinion in which she can express whatever she really thinks.\(^{39}\) This way, \( E_V(t) [I(t; v)] \) captures the “ideological bias” of judge \( t \)’s voting pattern.

**Proposition 4** There exists an \( \hat{\alpha} (\approx 0.295) \) such that, for each \( \alpha < \hat{\alpha} \), there exists a range of values of \( W \) for which \( |E[I(t; v)]| \) is maximized for an intermediate value of \( |t| \) (i.e., \( \arg\max_t |E[I(t; v)]| \notin \{-1, 0, 1\} \)).

**Proof:** See Appendix C.4.

Q.E.D.

The proposition expresses a sufficient condition for when moderately ideological judges will have the most ideological voting—\( D \) has to be sufficiently concave. The intuition is straightforward. As explained above, centrists and moderately ideological judges will dissent virtually whenever they are not the median, hence will have \( I = t \) in almost all cases. This implies that among them we will observe an increase in ideological bias as \( |t| \) increases. Conversely, extreme judges almost never dissent, hence their voting pattern mostly reflects the majority opinions they sign, which are determined by the median of their panels, hence tend to be less ideological than the moderates’ voting pattern.\(^{40}\) Hence, a concave ideological cost can rationalize the empirical finding reported in Appendix A.4.3.\(^{41}\)

B.2 Predicting the effect of collegial pressure

Here we derive two additional predictions that we will use in Section D.2.16 for testing our model against an alternative model (detailed in Section D.3). The additional predictions relate to the effect of collegial pressure on the pattern of dissent. In our main model we have the following equilibrium property.

**Proposition 5** Consider the equilibrium described for \( \alpha < \hat{\alpha} \). Then: (i) if \( W = 0 \), \( P(t) \) is monotonically increasing in \( |t| \); and (ii) \( \arg\max_t P(t) \) is decreasing in \( W \).

\(^{39}\)In the data, the ideology of a vote equals the majority opinion if the judge signs it and otherwise equals the ideology of her own text (the minority opinion), which was coded separately.

\(^{40}\)The result of Proposition 4 holds also for other values of \( \alpha < 2/3 \) but this is harder to show analytically since the pattern of dissent is more complex then (see for instance the summary paragraph of appendix C.3.1).

\(^{41}\)Kajackaite and Gneezy (2017) show that once the incentives to lie are higher than the cost, subjects switch from telling the truth to lying to the full extent. Earlier research has also been suggestive of a concave cost of lying: The decision whether to lie is often insensitive to the outcome of lying once it is preferred over the outcome of being truthful (Hurkens and Kartik 2009) and so a maximal deviation from the truth will often be chosen by those deciding to lie (Gneezy et al. 2013). Likewise, using a dynamic setting, Gino et al. (2010) showed that once individuals are induced to cheat, they succumb to full-blown cheating.
PROOF: See Appendix C.5.

Q.E.D.

Part (i) says that, should collegial pressure become very small, the spider pattern will disappear and the dissent rate will be a purely increasing function of a judge’s extremeness. The intuition for this result is that, without collegial pressure, it becomes possible also for extreme judges to be ideologically picky and hence dissent when they dislike the opinion even slightly. Then, given that they are rarely the median in their panels, they will dissent more often than everyone else. Part (ii) of the proposition expresses that, as collegial pressure increases, the range of judges who dissent whenever they are not the median shrinks. This is of course natural since strictly adhering to one’s morals becomes more costly under high peer pressure, hence less judges will do so. In Appendix D.2.16 these predictions are corroborated empirically.

C Proofs

C.1 Proof of Proposition 1

C.1.1 Single-peaked preferences

LEMMA 1 If all judges have stationary bargaining strategies then, in each case, a judge t prefers v1 over v2 if and only if |v1 − t| < |v2 − t|.

PROOF: Suppose all judges have stationary strategies when bargaining about the opinion and consider a judge t with a signing strategy s(v). Now consider this judge’s choice in a particular case while taking the outcome (opinion and signing) in all other cases as given.42 In the particular case, the judge’s strategy w.r.t. signing opinions v1 and v2 may imply either 1) signing none of them (s(v1) = s(v2) = 0); or 2) signing both (s(v1) = s(v2) = 1); or 3) signing one but not the other. 1) If s(v1) = s(v2) = 0, then the marginal loss implied by v2 being the panel’s opinion rather than v1 is larger if and only if |v1 − t| < |v2 − t| because O(V(t),t) is increasing in the distance between each v ∈ V(t) and t. Hence judge t prefers v1 over v2 if and only if |v1 − t| < |v2 − t|. 2) If s(v1) = s(v2) = 1, then the marginal loss of v2 is larger than the marginal loss of v1 iff |v1 − t| < |v2 − t| because both O(V(t),t) and D(·) are increasing in the distance between each v ∈ V(t) and t. 3) Under the strategy of signing one of the opinions but not the other, suppose, w.l.o.g., that s(v1) = 1 while s(v2) = 0. Then, since the judge is minimizing L, it must be that (i) the increase in D(·) due to signing v1 is smaller than W which is the increase in L had the judge chosen instead s(v1) = 0; (ii) the increase in D(·) due to signing v2 is larger than W. Then (i) and our supposition that s(v2) = 0 imply that L is smaller under v1, which is thus preferred over v2; and (i) and (ii) together are consistent with D(·) being increasing in |v − t| iff |v1 − t| < |v2 − t|. Thus we get that judge t prefers v1 over v2 if and only if |v1 − t| < |v2 − t|.

Q.E.D.

C.1.2 Proving Proposition 1

The proof is based on backward induction. Suppose all judges have stationary bargaining strategies and consider a particular case with any given panel composition. Suppose that, within this case, in period i = I − 1 the offer put forward by a coalition of judges is vT−1 ≠ tm, and suppose w.l.o.g. that vT−1 = tm + x for some x > 0. Then, in period I, a coalition of tm and the judge to her left can put on the table a counter offer vT ∈ (tm − x, tm], which will therefore become the final verdict and is preferred by both of them to vT−1 (given their single-peaked and symmetric preferences, as established in Lemma 1). Foreseeing this scenario, the judge on the right of tm will strive to minimize x and form a coalition that offers vT−1 = tm. This can be

42We can treat the judge’s choice in a particular case in isolation given that all other judges have stationary bargaining strategies and because each judge, including judge t, sits in a continuum of panels whose distribution is known in advance.
easily done by collaborating with \( t_m \). Thus, unless—on the equilibrium path—the offer made in period \( i = I \) is \( v = t_m \), an offer of \( v = t_m \) will be made already in period \( i = I - 1 \) beforehand. Once an offer of \( v = t_m \) is made, no coalition has any reason to make a counter offer: \( t_m \) gets her bliss point and at least one of the two other judges is better off with \( v = t_m \) than with any competing opinion. Hence, under any equilibrium with stationary bargaining strategies, \( v = t_m \) in all cases (which proves the second sentence of the proposition). Furthermore, since the opinion \( v \) depends only on the current median, we have established that no judge has a bargaining strategy that depends on panel compositions other than the current one, which shows that our initial supposition (of stationary bargaining strategies) holds in equilibrium. This proves the first sentence of the proposition, about existence.

C.2 Proof of Proposition 2

Suppose all judges have stationary bargaining strategies. Then Proposition 1 applies and it is clear that signing is optimal for the median judge of the panel. As for the other judges in the panel, note first that \( O(V(t), t) \) is independent of signing or not. Therefore, the first argument in the loss function, \( O(V(t), t) \), does not affect any signing decision made by a judge. Each of these judges therefore minimizes

\[
(7) \quad l(s(v); t) \equiv D \left( \int |v - t| k(v|t) s(v) dv \right) + WP(t).
\]

We will now show that if a judge \( t \) signs an opinion \( v_2 \) then she will also sign any opinion \( v_1 \) for which \( |v_1 - t| < |v_2 - t| \). This is equivalent to the judge \( t \) having a single cutoff \( c(t) \) such that she signs an opinion \( v \) if and only if \( |v - t| \leq c(t) \).

Suppose by negation that a judge who signs \( v_2 \) does not sign \( v_1 \). Taking the outcome (opinion and signing) of all other cases the judge sits in as given,\(^{43}\) the extra marginal loss implied by signing \( v_2 \) but not \( v_1 \) is an increase in \( D(\cdot) \) due to signing \( v_2 \) and an increase of \( W \) due to not signing \( v_1 \). However, since \( D(\cdot) \) is increasing in \( |v - t| \) it follows that the increase in \( D(\cdot) \) due to signing \( v_1 \) would have been smaller than the increase due to signing \( v_2 \). At the same time, switching the signatures from \( v_2 \) to \( v_1 \) does not affect the loss \( W \) due to collegial pressure. Hence, the judge has a profitable deviation to signing \( v_1 \) instead of \( v_2 \). This proves that, for any signed \( v_2 \), the judge will also sign any \( v_1 \) for which \( |v_1 - t| < |v_2 - t| \). The maximal value of \( v_2 \) the judge signs is her cutoff \( c(t) \).

C.3 Proof of Proposition 3

Lemma 3 is proven in Section C.3.2. The proposition itself is proven:

- For \( \alpha > 1/2 \) in Section C.3.4.
- For \( \alpha < 1/2 \) in Section C.3.5.
- For \( \alpha = 1/2 \) in Section C.3.6.

Unless stated otherwise, the distribution of judges is assumed to be uniform in \([-1, 1]\), and w.l.o.g we consider judges with \( t \geq 0 \). We occasionally refer to \( t \) as the judge’s type.

**Lemma 2** Let \( F(\cdot) \) denote any cumulative distribution of judges’ ideology scores. The probability that judge \( t \) is the median is denoted by \( P_m \) and given by

\[
(8) \quad P_m = 2 F(t) [1 - F(t)].
\]

\(^{43}\)Since each judge sits in a continuum of panels whose opinion distribution equals the distribution of \( t_m \) this judge is facing (as established in Proposition 1), we can treat the judge’s actions in any two particular cases in isolation.
Proof: Judge $t$ is the median when one other judge is to her left, which happens with probability $F(t)$, and the other is to her right, which happens with probability $1 - F(t)$. As judge’s types are i.i.d. and the order of the assignment of judges is irrelevant, we get (8).

Q.E.D.

C.3.1 Deriving the probability of dissent

Given Proposition 2, the choice of a judge of whether to sign the opinion $v$ boils down to choosing her cutoff $c$. In the proof of Proposition 2 it was shown that the signature strategy of a judge is not affected by the $O(\cdot)$ argument of the loss function. Rewriting the loss function in (7) while excluding the $O(\cdot)$ argument, the minimization problem for judge $t$ is given by

\[
\min_c l = \min_c \left\{ D \left( \int_{t-c}^{t+c} |v - t| k(v|t) \, dv \right) + WP(c; t) \right\},
\]

where $P(c; t)$ is the probability of facing opinions beyond judge $t$’s cutoff.

We turn now to explicitly express the conditional opinion distribution $k(v|t)$ in terms of the cumulative distribution of judges’ ideology scores $F(\cdot)$ and the corresponding density function $f(\cdot)$. For a given judge $t$, the probability of having an opinion to her left is $Pr(v < t) = Pr(t_m < t) = [F(t)]^2$, as this happens if and only if both other judges have bliss points below $t$. Similarly, $Pr(v > t) = Pr(t_m > t) = [1 - F(t)]^2$. In the remaining cases, judge $t$ is the median, in which case $|v - t| = 0$. More generally, the probability that a judge $t$ faces an opinion $v$ that is smaller than some $v' < t$ is $Pr(t_m < v') = [F(v')]^2$ and the probability of facing an opinion $v$ that is larger than some $v' > t$ is $Pr(t_m > v') = [1 - F(v')]^2$. Differentiating $Pr(t_m < v')$ and $Pr(t_m > v')$, the probability density of judge $t$ encountering opinion $v'$ is $k(v'|t) = 2F(v')f(v')$ in the range $v' < t$ and $k(v'|t) = 2[1 - F(v')]f(v')$ in the range $v' > t$.

Consider now a judge $t$ using a cutoff $c$. Since the median in each panel determines the opinion, it follows that $t$ will dissent if and only if both other judges are at a distance $c$ or more from $t$ (on the same side of $t$). Hence, the probability of dissent for judge $t$ who uses a cutoff $c$ is given by

\[
P(c; t) = Pr(t_m < t - c) + Pr(t + c < t_m) = [F(t - c)]^2 + [1 - F(t + c)]^2.
\]

Using the expression for $D$ in equation (9), we get that

\[
D(x) = D\left( \int_{t-c}^{t} (t - v) 2F(v) f(v) \, dv + \int_{t}^{t+c} (v - t) 2(1 - F(v)) f(v) \, dv \right).
\]

Using (10) and (11) in (9), the problem of judge $t$ is thus to choose a cutoff $c$ to minimize

\[
D\left( \int_{t-c}^{t} (t - v) 2F(v) f(v) \, dv + \int_{t}^{t+c} (v - t) 2(1 - F(v)) f(v) \, dv \right) + WP\left[ [F(t - c)]^2 + [1 - F(t + c)]^2 \right] \]

The probability that judge $t$ dissents is determined by two main factors. The first is the chosen cutoff value. For any given judge $t$, a larger $c$ implies less dissent. The second factor is the probability density of opinions outside this cutoff. In particular, under a uniform distribution of judges, a judge at the tail of the distribution is bound to encounter more panels where both other judges are on the same side of her and outside a given cutoff $c$, compared to a judge with the same cutoff but whose bliss point is at the center of the distribution.\footnote{One may note that both expressions for \(k(v|t)\) are independent of \(t\). This is since the role of \(t\) here is restricted to determining the switching point between the two expressions.} The equilibrium function $c(t)$ has implications for the probability of dissent of each judge as follows.

\footnote{Note that this holds under any single-peaked distribution, see the proof of Lemma 3.}
LEMMA 3  If \( c(t) \) is locally weakly decreasing in \(|t|\) then \( P(c(t); t) \) is locally strictly increasing in \(|t|\).

PROOF: Follows from Lemma 4 in Appendix C.3.2.

Q.E.D.

The lemma expresses the notion that a judge who is both more extreme and has a smaller cutoff (hence is pickier) will dissent more. The formal proof is in the appendix but the intuition follows directly from the two factors discussed above: A judge who is more extreme encounters more panels in which the median is beyond her cutoff, and more so if the cutoff is smaller. For consistency with Fact 3 it is thus necessary that \( c(t) \) in equilibrium will not be decreasing toward the edges of the distribution of judges.

C.3.2  Properties of dissent as a function of the cutoff \( c(t) \)

Following the derivations above, the probability of dissent for judge \( t \) whose optimal cutoff is \( c(t) \) is

\[
P(c(t); t) = [F(t - c(t))]^2 + [1 - F(t + c(t))]^2.
\]

We will occasionally refer to this probability simply as \( P(t) \).

Differentiating by \( t \) yields

\[
P'(c(t); t) = 2F(t - c)f(t - c) \left(1 - \frac{dc}{dt}\right) - 2 \left[1 - F(t + c)\right]f(t + c) \left(1 + \frac{dc}{dt}\right).
\]

The pattern of a spider is defined as follows.

DEFINITION 1  A spider pattern is when \( P(t) \) is first increasing and then decreasing.

Proof of Lemma 3

Lemma 3 holds more broadly than only for a uniform distribution. Hence its proof here will cover also any distribution of judges that is single-peaked and symmetric around 0.

LEMMA 4  Let the distribution of judges be uniform in \([-1, 1]\) or single-peaked and symmetric around 0. If \( c(t) \) is locally weakly decreasing in \(|t|\) then \( P(c(t); t) \) is locally strictly increasing in \(|t|\).

PROOF: That \( P(c(t); t) \) is locally weakly increasing in \(|t|\) follows from (13) while noting that, for \( t \geq 0 \), 1) symmetry around 0 implies that \( F(t - c) \geq [1 - F(t + c)] \), 2) a single-peaked or a uniform distribution that is symmetric around 0 satisfies \( f(t - c) \geq f(t + c) \), and 3) \( 1 - \frac{dc}{dt} \geq 1 + \frac{dc}{dt} \) when \( c(t) \) is locally weakly decreasing in \( t \). Then, given that \( F(t - c) \geq [1 - F(t + c)] \) holds with strict inequality for any \( t > 0 \), we get that for any non-zero range of values of \( t \), \( P(c(t); t) \) is locally strictly increasing in \(|t|\).\(^{46}\)

Q.E.D.

The following two lemmas apply to certain ranges of a uniform distribution of judges and will be useful later on.

\(^{46}\)Technically, if either \( F(t - c) = 0 \) or \( f(t - c) = 0 \) (where these two conditions necessarily happen simultaneously for a single-peaked or a uniform distribution), we get that \( P'(c(t); t) = 0 \) hence \( P(c(t); t) \) is not strictly increasing in \( t \). However, for this to be the case, the cutoff should be strictly increasing in \( t \) (because in this case \( c \) is the distance from \( t \) to the lower limit of the distribution), which contradicts the initial condition in the lemma that \( c(t) \) is locally weakly decreasing.
Lemma 5  Let the distribution of judges be uniform in \([-1,1]\), and suppose the cutoff \(c\) is constant at a certain range of types for whom \(t \geq 0\), and for each type \(t\) at this range \(c < 1 - t\). Then the probability of dissent is increasing in \(t\) in that range.

Proof: Differentiating \(P(t)\) and using the properties of a uniform distribution yields \(P'(t) = 2F(t-c)f(t-c) - 2[1 - F(t+c)] f(t+c) \geq 0\), because symmetry implies that \(F(t-c) \geq [1 - F(t+c)]\) and the uniform distribution implies that \(f(t-c) = f(t+c)\) when \(c < 1 - t\). Moreover, given that \(F(t-c) \geq [1 - F(t+c)]\) holds with strict inequality for any \(t > 0\), we get that the probability of dissent is strictly increasing in the given range of \(t\) (except exactly at \(t = 0\)).

Q.E.D.

Lemma 6  Let the distribution of judges be uniform in \([-1,1]\), and suppose the cutoff \(c(t)\) satisfies the condition \(1 - t < c(t) < 1 + t\) for a certain range of types for whom \(t \geq 0\). Then the probability of dissent decreases if and only if \(\frac{dc}{dt} > 1\) (when \(\frac{dc}{dt}\) is well defined).

Proof: Noting that \(1 - t \leq c\), hence the dissent occurs only for opinions to the left of \(t\), we get from (13) that \(P'_i(c(t); t) = \frac{1}{2} (1 + t - c) (1 - \frac{dc}{dt})\), i.e., \(P'_i(c(t); t) < 0\) iff \(\frac{dc}{dt} > 1\).

Q.E.D.

C.3.3 Analyzing the loss as a function of the cutoff, \(L(c)\)

Denoting

\[
(14) \quad z \equiv \int_{t-c}^{t} (t-v)2F(v)f(v)dv + \int_{t}^{t+c} (v-t)2 [1 - F(v)] f(v)dv
\]

we can express the loss of judge \(t\) with cutoff strategy \(c\) as follows

\[
(15) \quad L(c) = \left[ \int_{t-c}^{t} (t-v)2F(v)f(v)dv + \int_{t}^{t+c} (v-t)2 [1 - F(v)] f(v)dv \right]^{\alpha} + WP(t) = z^\alpha + WP(t)
\]

Differentiating by \(c\) yields \(\frac{dL}{dc} = \alpha z^{\alpha-1} \frac{dz}{dc} + W \frac{dP(t)}{dc} \rightarrow \)

\[
(16) \quad \frac{dL}{dc} = 2cM \alpha z^{\alpha-1} - 2WM = 2M [\alpha c z^{\alpha-1} - W],
\]

where

\[
(17) \quad M \equiv F(t-c)f(t-c) + [1 - F(t+c)] f(t+c).
\]

\(M\) is positive since \(F \in [0,1]\) and \(f \geq 0\).

Given that the distribution of judges is \(U(-1,1)\), we have \(f(t) = 0.5, F(t) = \frac{1+t}{2}\). Then, for \(t \geq 0\), we have three possible regions for \(c\):

**Case 1: \(c \geq 1 + t\)**

\[
(18) \quad z = \frac{1}{2} \left[ \int_{t-1}^{t} (t-v) (1+v) dv + \int_{t}^{1} (v-t) (1-v) dv \right] = ...
\]

\[
(19) \quad z = \frac{1}{2} \left[ t^2 + \frac{1}{3} \right]
\]

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Note that $M = 0$ (in 17) since $F(t - c) = f(t + c) = 0$ under a uniform distribution $c \geq 1 + t$. Hence (from (16)) $\frac{dL}{dc} = 0$.

**Case 2:** $1 - t \leq c < 1 + t$

\[
z = \int_{t-c}^{t} (t-v) 2F(v)f(v) dv + \int_{t}^{t+c} (v-t) 2(1 - F(v)) f(v) dv = \int_{t-c}^{t} (t-v) \frac{1+v}{2} dv + \int_{t}^{t} (v-t) \frac{1-v}{2} dv = ...
\]

\[
= \frac{1}{2} \left[ \frac{(t-c)^3}{3} - (t-1) \frac{(t-c)^2}{2} + \left( c - \frac{1}{2} \right) t + \frac{1}{6} \right]
\]

\[
\Rightarrow \frac{dL}{dc} = 2M \left[ \alpha c \left( \frac{1}{2} \left[ \frac{(t-c)^3}{3} - (t-1) \frac{(t-c)^2}{2} + \left( c - \frac{1}{2} \right) t + \frac{1}{6} \right] \right)^{-1} - W \right].
\]

Substituting $M = \frac{1}{2} \left[ \frac{1+t-c}{2} \right]$ into $\frac{dL}{dc}$ yields

\[
\frac{dL}{dc} = \left( \frac{1+t-c}{2} \right) \left( 2^{1-\alpha} \alpha c \left[ \frac{(t-c)^3}{3} - (t-1) \frac{(t-c)^2}{2} + \left( c - \frac{1}{2} \right) t + \frac{1}{6} \right]^{-1} - W \right).
\]

We will interchangeably use different formulations for $z$:

\[
2z = \frac{(t-c)^3}{3} - (t-1) \frac{(t-c)^2}{2} + \left( c - \frac{1}{2} \right) t + \frac{1}{6} = ...
\]

\[
2z = -\frac{1}{3} c^3 - \frac{1}{6} t^3 + \frac{1}{2} t c^2 + \frac{1}{2} t^2 + \frac{1}{2} c^2 - \frac{1}{2} t + \frac{1}{6} = ...
\]

\[
2z = \frac{1}{6} (1-t)^3 - \frac{1}{3} c^3 + \frac{1}{2} t c^2 + \frac{c^2}{2}.
\]

**Case 3:** $c \leq 1 - t$

\[
z = \int_{t-c}^{t} (t-v) 2F(v)f(v) dv + \int_{t}^{t+c} (v-t) 2(1 - F(v)) f(v) dv
\]

\[
= \int_{t-c}^{t} (t-v) \frac{1+v}{2} dv + \int_{t}^{t+c} (v-t) \frac{1-v}{2} dv = ... = c^2 \left[ \frac{1}{2} - \frac{1}{3} c \right]
\]

\[
\Rightarrow \frac{dL}{dc} = 2M \left[ \alpha c \left( c^2 \left[ \frac{1}{2} - \frac{1}{3} c \right] \right)^{-1} - W \right]
\]

\[
= 2M \left[ \alpha c^{2\alpha-1} \left[ \frac{1}{2} - \frac{1}{3} c \right]^{-1} - W \right].
\]

Using the uniform distribution in equation (17) for $M$

\[
M = \frac{1}{2} \left[ \frac{1+t-c}{2} + \frac{1-t-c}{2} \right] = \frac{1}{2} (1-c)
\]

and substituting into (23) yields

\[
\frac{dL}{dc} = (1-c) \left[ \alpha c^{2\alpha-1} \left[ \frac{1}{2} - \frac{1}{3} c \right]^{-1} - W \right],
\]

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(25) \[ \lim_{c \to 0} \frac{dL}{dc} = \begin{cases} +\infty & \text{if } \alpha < 1/2 \\ 2^{-1/2} - W & \text{if } \alpha = 1/2 \\ -W & \text{if } \alpha > 1/2 \end{cases}. \]

**Lemma 7** \( \frac{dL}{dc} \) is continuous everywhere.

**Proof:** It is immediate that \( L \) is continuous within each range so we only need to check the transitions. At \( c = 1 - t \):

\[
\frac{dL}{dc} = \left( \frac{1 + t - c}{2} \right) \left( 2^{1-\alpha c} \left[ \frac{(t-c)^3}{3} - \frac{(t-1)(t-c)^2}{2} + \left( c - \frac{1}{2} \right) t + \frac{1}{6} \right]^{\alpha - 1} - W \right)
\]

\[
= (1-c) \left( 2^{1-\alpha c} \left[ -\frac{2}{3} c^3 + c^2 \right]^{\alpha - 1} - W \right) = (1-c) \left( 2^{1-\alpha c} \left[ 1 - \frac{2}{3} c^3 \right]^{\alpha - 1} - W \right)
\]

which equals \( dL/dc \) at case 3 (see equation 24). At \( c = 1 + t \):

\[
\frac{dL}{dc} = \left( \frac{1 + t - c}{2} \right) \left( 2^{1-\alpha c} \left[ \frac{(t-c)^3}{3} - \frac{(t-1)(t-c)^2}{2} + \left( c - \frac{1}{2} \right) t + \frac{1}{6} \right]^{\alpha - 1} - W \right) = 0
\]

which equals \( dL/dc \) at case 1 (see equation 24).

\( Q.E.D. \)

**Analyzing \( dL/dc \)**

In order to investigate \( \frac{dL}{dc} \), define the function \( g(c,t) \) as follows:

(26) \[ g(c,t) \equiv \begin{cases} c^{2\alpha - 1} \left[ 1 - \frac{2}{3} c^3 \right]^{\alpha - 1} & \text{if } 0 < c < 1 - t \\ c \left[ -\frac{1}{6} c^3 - \frac{1}{2} t c^2 + \frac{1}{2} c^2 + \frac{1}{2} t - \frac{1}{2} + \frac{1}{6} \right]^{\alpha - 1} & \text{if } 1 - t \leq c \leq 1 + t \end{cases}. \]

**Lemma 8** The sign of \( \frac{dL}{dc} \) equals the sign of \( 2^{1-\alpha} g(c,t) - W \).

**Proof:** Follows immediately from substituting the values of \( g(c,t) \) for the ranges \( 0 < c < 1 - t \) and \( 1 - t \leq c < 1 + t \) in the corresponding expressions of \( \frac{dL}{dc} \).

\( Q.E.D. \)

**Lemma 9** The sign of \( \frac{\partial g(c,t)}{\partial c} \) is determined by the sign of

(27) \[ E \equiv \begin{cases} \left[ 2\alpha (1-c) - 1 + \frac{1}{3} c \right] & \text{if } 0 < c < 1 - t \\ \left[ \frac{1}{6} c^3 - \frac{1}{2} t c^2 + \frac{1}{2} c^2 + \frac{1}{2} t - \frac{1}{2} + \frac{1}{6} + \alpha \left( tc^2 + c^2 - c^3 \right) \right] & \text{if } 1 - t \leq c < 1 + t \end{cases}. \]
Lemma 8 implies that, in inner solutions, \( W = \alpha 2^{1-\alpha} g(c(t'), t') \). For any \( t < t' \) we have \( g(c, t) = g(c, t') \) at the range \( c \in [0, 1 - t] \), hence \( W = \alpha 2^{1-\alpha} g(c(t'), t') \) where \( c(t') \leq 1 - t' \) implies that \( c(t') < 1 - t \). In other words, \( t \) has an inner solution at \( c(t') \) as well. 

\[ Q.E.D. \]

\[ \frac{\partial g(c, t)}{\partial c} = \frac{\partial}{\partial c} \left\{ \begin{array}{ll} c^{2\alpha-1} \left[ 1 - \frac{2}{3} c \right] \frac{1}{\alpha - 1} & \text{if } 0 < c < 1-t \\ \frac{1}{6} c^3 + \frac{1}{2} c^2 + \frac{1}{6} \right. \times \\
\]
Lemma 11  Let \( h(t) \equiv g(1 + t, t) = (1 + t) \left( \frac{1}{3} + t^2 \right)^{\frac{\alpha - 1}{\alpha}} \). Then \( h(t) \) is monotonically increasing if \( \alpha \geq \frac{2}{3} \) but has a unique max point at

\[
(29) \quad t_{\max} = \frac{1 - \alpha - \sqrt{(\alpha - 2)(\alpha - \frac{2}{3})}}{2\alpha - 1}
\]

if \( \alpha \in \left[ \frac{1}{2}, \frac{2}{3} \right] \).

Proof:  Corollary (25) implies a type has an inner solution iff

\[
if \begin{cases}
\alpha > \frac{2}{3}, & \text{it must be that all types } \begin{cases}
h \in [0, 1] & \text{with cutoffs } c \in [1 - t, 1 + t] \text{ and such that among them the probability of dissent is decreasing in } t.
\end{cases}
\end{cases}
\]

Lemma 12  Let \( \alpha \geq \frac{2}{3} \). A necessary condition for getting the pattern of a spider is to have a range of types \([t, 1]\) with cutoffs \( c \in [1 - t, 1 + t] \) and such that among them the probability of dissent is decreasing in \( t \).

Proof:  Corollary (25) implies a type has an inner solution iff \( W < (2 - \alpha)g(1 + t, t) \). Lemma 11 says that, when \( \alpha \geq \frac{2}{3} \), \( h(t) \equiv g(1 + t, t) \) is monotonically increasing. Hence if some type \( t' \) has an inner solution \( c \), then any type \( t > t' \) has an inner solution too. If \( W \) is sufficiently large so that no type has an inner solution, it follows that no type ever dissents, and this is a degenerate case with no spider. Otherwise, the pattern of a spider requires that among types with sufficiently large \( t \) the probability of dissent will be decreasing. To complete the proof, we will show that a decrease in dissent cannot happen if the inner solutions are such that \( c(t) < 1 - t \). To see that, note that if a given type \( t'' \) has an inner solution \( c(t'') = \hat{c} < 1 - t'' \), then it must be that all types \( t < t'' \) have \( c(t) = \hat{c} \) as their solution too, because \( \forall t \leq t'' \) we have \( \hat{c} < 1 - t \) and because the part of \( g(c, t) \) at the range \( c \in [0, 1 - t''] \) is identical for all these types (see equation 26 in the first range). This implies by Lemma 5 that at the range \( t \in [0, t''] \) the probability of dissent is increasing in \( t \). Hence, a necessary condition for getting the pattern of a spider is a decrease in the probability of dissent among a range of types whose cutoffs are such that \( 1 - t < c < 1 + t \).

Q.E.D.

Lemma 13  Let \( \alpha > \frac{1}{2} \) and suppose there exists a range of types with inner solutions \( c \in [1 - t, 1 + t] \). Then, in this range of inner solutions, \( \frac{dc}{dt} \leq 0 \) if and only if \( \alpha \geq 1 \).
PROOF: For type \( t \) with an inner solution at \( 1 - t < c < 1 + t \) the first order condition (19) must hold:

\[
2^{1-\alpha}c \left[ \frac{(t-c)^3}{3} - (t-1)(t-c)^2 + \left( c - \frac{1}{2} \right) t + \frac{1}{6} \right]^{\alpha-1} - W = 0.
\]

Using the implicit function theorem we get

\[
\frac{dc}{dt} = -\frac{\frac{d}{dc} \left[ \left( \frac{1}{3} c^3 - \frac{1}{6} t^3 + \frac{1}{2} t^2 + \frac{1}{2} c^2 - \frac{1}{2} t + \frac{1}{6} \right)^{\alpha-1} \right]}{\frac{d}{dc} \left[ (1-\alpha) \left( \frac{1}{3} t^2 + \frac{1}{2} c^2 + t - \frac{1}{2} \right) \right]} \rightarrow \ldots
\]

(31)

\[
\frac{dc}{dt} = \frac{c}{3} c^3 - \frac{1}{6} t^3 + \frac{1}{2} t^2 - \frac{1}{2} c^2 - \frac{1}{2} t + \frac{1}{6} + \alpha (tc^2 + c^2 - c^3).
\]

Since we look at the case of \( \alpha > \frac{1}{2} \), and noting that

\[
tc^2 + c^2 - c^3 = c^2 (t + 1 - c) > 0,
\]

it is enough to show that \( \left[ \frac{2}{3} c^3 - \frac{1}{6} t^3 - \frac{1}{2} t^2 + \frac{1}{2} c^2 - \frac{1}{2} t + \frac{1}{6} + \alpha (tc^2 + c^2 - c^3) \right] \) is positive at \( \alpha = \frac{1}{2} \) (since the expression increases in \( \alpha \)).\(^{48}\) In order to conclude that the denominator is positive. Indeed, this expression with \( \alpha = 1/2 \) can be shown to equal \( \frac{1}{2} \left[ \frac{3}{4} c^3 + (1-t)^3 \right] > 0 \). It thus follows that for \( \alpha \in [1/2, 1] \) the sign of \( \frac{dc}{dt} \) equals the sign of \( -\frac{1}{2} t^2 + \frac{1}{2} c^2 - t - \frac{1}{2} \) while for \( \alpha > 1 \) the sign of \( \frac{dc}{dt} \) is the opposite of the sign of

\[
-\frac{1}{2} t^2 + \frac{1}{2} c^2 - t - \frac{1}{2} = ... \geq \frac{1}{2} t^2 + \frac{1}{2} (1-t)^2 + t - \frac{1}{2} = ... = -\frac{1}{2} t^2 + \frac{1}{2} - t + \frac{1}{2} t^2 + t - \frac{1}{2} = 0.
\]

It thus follows that \( \frac{dc}{dt} \leq 0 \) if \( \alpha \geq 1 \) and \( \frac{dc}{dt} > 0 \) if \( \alpha \in [1/2, 1] \).

Q.E.D.

**LEMMA 14**: Suppose there exists a range of types with inner solutions \( c \in [1-t, 1+t] \). Then \( \frac{dc}{dt} > 1 \) iff

(33) \[ G \equiv c^2 \left[ (3\alpha - 1) c - (2\alpha - 1) 3 (1 + t) \right] - (1 - t)^2 \left[ (1 - t) + (1 - \alpha) 3c \right] > 0. \]

**PROOF**: Using \( \frac{dc}{dt} \) from (31), \( \frac{dc}{dt} > 1 \) holds iff

\[
c^2 \left[ \frac{1}{3} c^3 - \frac{1}{6} t^3 + \frac{1}{2} t^2 + \frac{1}{2} c^2 - \frac{1}{2} t + \frac{1}{6} + \alpha (tc^2 + c^2 - c^3) \right] \geq 1 \Leftrightarrow ...
\]

\[
c^2 \left[ (3\alpha - 1) c - (2\alpha - 1) 3 (1 + t) \right] - (1 - t)^2 \left[ (1 - t) + (1 - \alpha) 3c \right] > 0.
\]

Q.E.D.

**Convexity and linearity** \( \alpha \geq 1 \)

**PROPOSITION 6**: There cannot be a spider pattern when \( \alpha \geq 1 \)

**PROOF**: Lemma 12 implies that a necessary condition for getting the pattern of a spider is a decrease in the probability of dissent at the range \( 1 - t < c < 1 + t \). It thus follows from Lemmas 13 and 6 that there cannot be a spider when \( \alpha \geq 1 \).

Q.E.D.

\(^{48}\)Since \( tc^2 + c^2 - c^3 = c^2 (1 + t - c) > 0 \) when \( c < 1 + t \).
Very weak concavity $\alpha \in \left[\frac{2}{3}, 1\right[$

**Proposition 7** There cannot be a spider pattern when $\alpha \in \left[\frac{2}{3}, 1\right[$.

**Proof:** Lemma 12 implies that a necessary condition for getting the pattern of a spider is a decrease in the probability of dissent at the range $1 - t < c < 1 + t$, which together with Lemma 6 implies that $G$ (defined in equation (33)) must be strictly positive somewhere in the range to maintain the possibility of a spider. Investigating $G$, note first that if $\left[\left(3\alpha - 1\right)c - (2\alpha - 1)3\left(1 + t\right)\right] \leq 0$ then $G \leq 0$. Otherwise, if $\left[\left(3\alpha - 1\right)c - (2\alpha - 1)3\left(1 + t\right)\right] > 0$, then it gets its max value for $c = 1 + t$, while the part that is deducted from it, $(1 - t)^2 \left[(1 - t) + (1 - \alpha)3c\right]$, is positive and increases in $c$ hence is minimal when $c = 1 - t$. By plugging these values correspondingly we get that $G < (1 + t)^3 \left[2 - 3\alpha\right] - (1 - t)^3 \left[4 - 3\alpha\right] \leq 0$, when $\alpha \in \left[\frac{2}{3}, 1\right[$. Hence there cannot be a spider pattern.

\[Q.E.D\]

Mildly weak concavity $\alpha \in \left]\frac{1}{2}, \frac{2}{3}\right[$

We will show that for any $\alpha \in \left]\frac{1}{2}, \frac{2}{3}\right[$ the following pattern of spider exists (though other kinds of spider can be generated too): most types, including those close to 0 and 1, never dissent, while there exists a non-empty range of types in-between that do dissent sometimes.

**Proposition 8** For any $\alpha \in \left]\frac{1}{2}, \frac{2}{3}\right[$, there exist values of $W$ for which dissent has the pattern of a spider.

**Proof:** From Lemma 11 we know that $h(t) = g\left(1 + t, t\right)$ has a hill-shape for any $\alpha \in \left]\frac{1}{2}, \frac{2}{3}\right[$ with a peak at $t_{\text{max}}$ (defined in equation (29)). We will prove the proposition for $W = \alpha 2^{1-\alpha} \left[g\left(1 + t_{\text{max}}, t_{\text{max}}\right) - \varepsilon\right]$, where $\varepsilon$ is very small. In this case, since $W < \alpha 2^{1-\alpha} g\left(1 + t_{\text{max}}, t_{\text{max}}\right)$, Corollary 1 says inner solutions exist for a small range of types $t \in (t_{\text{max}} - \delta_1, t_{\text{max}} + \delta_2)$, with $c$ very close to $1 + t_{\text{max}}$. For a given $\alpha$, they also depend on $\varepsilon$ as a parameter. From corollary 1, we know that all other types, including those close to 0 or close to 1, choose a corner solution $c = 1 + t$ (always dissent). To get the pattern of a spider we need to verify that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases. Given $W$ has been set such that type $t = 0$ has a corner solution, Lemma 10 implies that any type $t$ in the range of types with inner solutions has a solution at the range $c \in \left[1 - t, 1 + t\right]$. In this range, we know from Lemma 6 that the probability of dissent decreases if and only if $\frac{dW}{dt} > 1$.

Let us denote the inner min point of $L(c)$ for type $t \in (t_{\text{max}} - \delta_1, t_{\text{max}} + \delta_2)$ by $c_0\left(t, \varepsilon\right)$. Then we can expand $c_0\left(t, \varepsilon\right)$ into Taylor series in $\varepsilon$ and $t$ around $\varepsilon = 0$ and $t = t_{\text{max}}$ as follows:

\[
(34) \quad \Delta c \equiv c_0\left(t, \varepsilon\right) - (1 + t_{\text{max}}) = \left(\frac{\partial c_0}{\partial t}\right)_{c_0(0) = t_{\text{max}}} \Delta t + \frac{1}{2} \left(\frac{\partial^2 c_0}{\partial t^2}\right)_{c_0(0) = t_{\text{max}}} \Delta t^2 + \left(\frac{\partial c_0}{\partial \varepsilon}\right)_{c_0(0) = t_{\text{max}}} \varepsilon + \ldots,
\]

where $\Delta t \equiv t - t_{\text{max}}$. We know, since $t_{\text{max}}$ is the peak of $h\left(t\right)$, that at $t_{\text{max}}$ and $c = 1 + t_{\text{max}}$ we have

\[
\frac{dh}{dt} = \frac{\partial g\left(c, t\right)}{\partial t} + \frac{\partial g\left(c, t\right)}{\partial c} \frac{d}{dt} \left(1 + t\right) = \frac{\partial g\left(c, t\right)}{\partial t} + \frac{\partial g\left(c, t\right)}{\partial c} = 0,
\]

hence at $t_{\text{max}}$ we have (from the implicit function theorem)

\[
(35) \quad \left(\frac{\partial c_0}{\partial t}\right)_{c_0(0) = t_{\text{max}}} = -\left(\frac{\partial g\left(c, t\right)}{\partial t}\right)_{c = 1 + t_{\text{max}}, t = t_{\text{max}}} \left/ \left(\frac{\partial g\left(c, t\right)}{\partial c}\right)_{c = 1 + t_{\text{max}}, t = t_{\text{max}}} = 1\right.
\]

To find the other derivatives in equation (34) we will expand into series the defining equation

\[g\left(c_0\left(t\right), t\right) - g\left(1 + t_{\text{max}}, t_{\text{max}}\right) = -\varepsilon.\]
The expansion will be
\[
\left( \frac{\partial g}{\partial t} \right)_{t=\tau_{\text{max}}} \Delta t + \left( \frac{\partial g}{\partial c} \right)_{c=1+\tau_{\text{max}}} \Delta c + \frac{1}{2} \left( \frac{\partial^2 g}{\partial t^2} (\Delta t)^2 + 2 \frac{\partial^2 g}{\partial t \partial c} (\Delta t)(\Delta c) + \frac{\partial^2 g}{\partial c^2} (\Delta c)^2 \right)_{t=\tau_{\text{max}}} + \ldots
\]
\[
= \left( \frac{\partial g}{\partial t} \right)_{t=\tau_{\text{max}}} \Delta t + \left( \frac{\partial g}{\partial c} \right)_{c=1+\tau_{\text{max}}} \left[ \Delta t + \frac{1}{2} \left( \frac{\partial^2 g}{\partial t^2} \right)_{\varepsilon=0} (\Delta t)^2 + \left( \frac{\partial g}{\partial \varepsilon} \right)_{\varepsilon=0} \varepsilon \right] + \frac{1}{2} \left( \frac{\partial^2 g}{\partial t^2} (\Delta t)^2 + 2 \frac{\partial^2 g}{\partial t \partial c} (\Delta t)(\Delta c) + \frac{\partial^2 g}{\partial c^2} (\Delta c)^2 \right)_{t=\tau_{\text{max}}} + \ldots
\]
\[= -\varepsilon \]

Equation (35) implies that \( \Delta c \simeq \Delta t \) hence we can write\(^49\)
\[
\left( \frac{\partial g}{\partial t} \right)_{t=\tau_{\text{max}}} \Delta t + \left( \frac{\partial g}{\partial c} \right)_{c=1+\tau_{\text{max}}} \left[ \Delta t + \frac{1}{2} \left( \frac{\partial^2 g}{\partial t^2} \right)_{\varepsilon=0} (\Delta t)^2 + \left( \frac{\partial g}{\partial \varepsilon} \right)_{\varepsilon=0} \varepsilon \right] + \frac{1}{2} \left( \frac{\partial^2 g}{\partial t^2} (\Delta t)^2 + 2 \frac{\partial^2 g}{\partial t \partial c} (\Delta t)(\Delta c) + \frac{\partial^2 g}{\partial c^2} (\Delta c)^2 \right)_{t=\tau_{\text{max}}} + \ldots
\]
\[= -\varepsilon \]

Given that the RHS is independent of \( \Delta t \) or \((\Delta t)^2\), and that the coefficient of \( \Delta t \) in the LHS,
\[
\left( \frac{\partial g}{\partial t} \right)_{t=\tau_{\text{max}}} + \left( \frac{\partial g}{\partial c} \right)_{c=1+\tau_{\text{max}}} = \left( \frac{\partial h}{\partial t} \right)_{t=\tau_{\text{max}}}
\]
is zero, we can conclude from the coefficient of \((\Delta t)^2\) that
\[
\left( \frac{\partial g}{\partial c} \right)_{c=1+\tau_{\text{max}}} \left( \frac{\partial^2 g}{\partial t^2} \right)_{\varepsilon=0} = - \left( \frac{\partial^2 g}{\partial t^2} + 2 \frac{\partial^2 g}{\partial t \partial c} + \frac{\partial^2 g}{\partial c^2} \right)_{t=\tau_{\text{max}}}
\]
hence
\[
\left( \frac{\partial^2 g}{\partial t^2} \right)_{\varepsilon=0} = \frac{\left( \frac{\partial g}{\partial \varepsilon} \right)_{c=1+\tau_{\text{max}}} - \left( \frac{\partial g}{\partial \varepsilon} \right)_{c=1+\tau_{\text{max}}}}{\left( \frac{\partial^2 g}{\partial t^2} + 2 \frac{\partial^2 g}{\partial t \partial c} + \frac{\partial^2 g}{\partial c^2} \right)_{t=\tau_{\text{max}}}}
\]
The denominator is positive because \( g(c, t) \) increases in \( c \) (even for \( c = 1 + t \) - see Lemma 16 below, which applies to any \( \alpha \)). The numerator is the explicit expression of \( \left( \frac{\partial h}{\partial t} \right)_{t=\tau_{\text{max}}}, \) which is negative given that \( t_{\text{max}} \) is a max point of \( h(t) \). It thus follows that
\[
\left( \frac{\partial^2 g}{\partial t^2} \right)_{\varepsilon=0} > 0
\]
hence it follows from a Taylor expansion of \( \frac{\partial g}{\partial c} \) around \( t_{\text{max}} \), using equation (35), that
\[
\left( \frac{\partial g}{\partial t} \right)_{t=\tau_{\text{max}}} = 1 + \left( \frac{\partial^2 g}{\partial t^2} \right)_{\varepsilon=0} (\Delta t + \ldots) \geq 1 \text{ for } \Delta t \geq 0.
\]

Hence, by Lemma 6, the dissent is first increasing and then decreasing as \( t \) passes \( t_{\text{max}} \). We get a spider of the following kind: when \( t \) goes from 0 to 1 the probability of dissent is first 0, then it jumps to some strictly positive probability, then it first increases and then decreases, and finally the probability of dissent decreases abruptly back to 0 and stays there.

\(^{49}\)Note that we plug in \( \Delta c = \Delta t \) only in the expressions for \((\Delta t)(\Delta c)\) and \((\Delta c)^2\), where elements of size \( \varepsilon \Delta t \) or \( \varepsilon^2 \) in the expansion can be ignored.
C.3.5 Strong concavity $\alpha < \frac{1}{2}$

We start with some first useful results. When $\alpha < \frac{1}{2}$ we have by equation (25) $\lim_{c \to 0} \frac{dL}{dc} = \infty$, implying that $\forall t, c = 0$ is a potential solution. Since $\frac{dL}{dc}$ is continuous everywhere, there can be an inner solution only if $\frac{dL}{dc} = 0$ more than once. In what follows we study some properties of $g(c,t)$ before splitting the strict concavity case into two sub-cases.

The shape of $g(c,t)$

In this subsection we study the properties of $g(c,t)$ as a function of $c$.

**Lemma 15** The sign of $\left. \frac{\partial g(c,t)}{\partial c} \right|_{c \to 1-t}$ equals the sign of $\left. \frac{\partial g(c,t)}{\partial c} \right|_{c \to 1-t}$.

**Proof:** We start by revisiting Lemma 9 which showed that the sign of $\frac{\partial g(c,t)}{\partial c}$ is solely determined by $E$ (defined in equation 27) which we rewrite as follows

$$E = \begin{cases} 
2\alpha(1-c) - 1 + \frac{3}{2} c & \text{if } 0 < c < 1 - t \\
c^2 \left[ (\frac{2}{3} - \alpha) c + (\alpha - \frac{1}{2}) (1 + t) \right] + \frac{1}{6} (1-t)^3 & \text{if } 1 - t \leq c < 1 + t 
\end{cases}$$

Thus, the sign of $\left. \frac{\partial g(c,t)}{\partial c} \right|_{c \to 1-t}$ equals the sign of $(1-t)^2 \left[ (\frac{2}{3} - \alpha) (1 - t) + (\alpha - \frac{1}{2}) (1 + t) \right] + \frac{1}{6} (1-t)^3 = ... = (1-t)^2 \left[ \frac{1}{2} - \frac{2}{3} t + 2\alpha t \right]$, where the bracket equals $E \mid_{c \to 1-t}$. Hence both limits have the same sign.

Q.E.D.

**Lemma 16** $\left. \frac{\partial g(c,t)}{\partial c} \right|_{c \to 1-t} > 0$.

**Proof:** Using Lemma 9 and equation (36) the sign of $\left. \frac{\partial g(c,t)}{\partial c} \right|_{c \to 1-t}$ equals the sign of $$(1 + t)^2 \left[ (\frac{2}{3} - \alpha) (1 + t) + (\alpha - \frac{1}{2}) (1 + t) \right] + \frac{1}{6} (1-t)^3 = \frac{1}{6} (1 + t)^3 + \frac{1}{6} (1-t)^3 > 0.$$ Q.E.D.

**Lemma 17** $\frac{\partial g(c,t)}{\partial c}$ has at most one local min point (with respect to $c$) at the range $[1-t,1+t]$.

**Proof:** By Lemma 9 follows that $\frac{\partial g(c,t)}{\partial c}$ has a local min point only if $dE/dc = 0$. Differentiating equation (36) by $c$ yields

$$E' = \begin{cases} 
\frac{1}{3} - 2\alpha & \text{if } 0 < c < 1 - t \\
c \left[ (2 - 3\alpha) c + (2\alpha - 1) (1 + t) \right] & \text{if } 1 - t \leq c < 1 + t 
\end{cases}$$

To learn the sign of $\left. \frac{\partial g(c,t)}{\partial c} \right|_{c \to 1-t}$ throughout the range $1 - t \leq c < 1 + t$, we differentiate the expression in (37) at this range, yielding

$$E'' = 2 (2 - 3\alpha) c + (2\alpha - 1) (1 + t) > 0$$
Thus, equation (37) implies that for \( c > 0 \), since \( \alpha < 1/2 \), \( E \) which by Lemma 9 determines the sign of \( \frac{\partial g(c,t)}{\partial c} \) at the range \( 1-t \leq c < 1+t \) has at most one local extremum, at

\[
(39) \quad c = \frac{(1-2\alpha)(1+t)}{2-3\alpha},
\]

and equation (38) implies that this is a min point.

\[Q.E.D.\]

**Lemma 18** \( g(c,t) \) does not have a local max point at \( 0 < c < 1-t \)

**Proof:** We know from equation (37) Lemma 9 that \( \frac{\partial g(c,t)}{\partial c} \) cannot turn from positive to negative at the range \( 0 < c < 1-t \) (since \( \alpha < 1/2 \)), hence \( g(c,t) \) cannot have a local max point there.

\[Q.E.D.\]

**Lemma 19** If \( c_b = \frac{1-2\alpha}{\frac{3}{2}-2\alpha} > 1-t \), then \( g(c,t) \) has a U-shape at the range \([0,1+t]\)

**Proof:** By Lemma 9 we know \( \frac{\partial g(c,t)}{\partial c} = 0 \) when \( E = 0 \). Setting \( E = 0 \) in equation (36) yields

\[
\left\{ \begin{array}{l}
c = \frac{1-2\alpha}{\frac{3}{2}-2\alpha} \text{ if } 0 < c < 1-t \\
c^2 \left[ (\frac{3}{3} - \alpha) c + (\alpha - \frac{1}{2}) (1+t) \right] = -\frac{1}{6} (1-t)^3 \text{ if } 1-t \leq c < 1+t
\end{array} \right.
\]

Starting with the first region \( (c < 1-t) \), we get by (27) that \( \lim_{c \to 0} E = 2\alpha - 1 < 0 \) (when \( \alpha < 1/2 \)). Hence (by Lemma 9) the function \( g(c,t) \) is decreasing initially. Then it keeps on decreasing until \( c = c_b \) at which \( \frac{\partial g(c,t)}{\partial c} = 0 \). Thus, if \( c_b \equiv \frac{1-2\alpha}{\frac{3}{2}-2\alpha} > 1-t \), we get that \( \frac{\partial g(c,t)}{\partial c} \) stays negative throughout the first region, and from Lemma 15 we know that it does not change signs at \( c = 1-t \). Then, Lemmas 17 and 16 imply that \( \frac{\partial g(c,t)}{\partial c} \) changes sign exactly once, from negative to positive, and so, overall, \( g(c,t) \) has a U-shape at the range \([0,1+t]\).

\[Q.E.D.\]

**Lemma 20** \( g(c,t) \) may have a local max point at the range \( 1-t \leq c < 1+t \) only if \( \alpha \in \left[ \frac{1}{3}, \frac{1}{2} \right] \) and \( t \in \left[ \frac{1-\alpha}{3-5\alpha}, \frac{1}{4-6\alpha} \right] \)

**Proof:** If \( c_b \leq 1-t \) (as defined in Lemma 19), then \( g(c,t) \) has a local min point at \( c_b \) and \( \frac{\partial g(c,t)}{\partial c}{|}_{c \to 1-t} \geq 0 \). In that case, Lemmas 17 and 16 imply that, at the range \( 1-t \leq c < 1+t \), it might be that \( g(c,t) \) first increases, then decreases, and then increases again. In this case \( g(c,t) \) would have a local max point at this range. A necessary condition for \( g(c,t) \) to have a local max point is that

\[
c_b \leq 1-t \Rightarrow t \leq 1 - \frac{1-2\alpha}{\frac{3}{2}-2\alpha} = \frac{1}{4-6\alpha}
\]

Moreover, the min point \( (c = \frac{(1-2\alpha)(1+t)}{2-3\alpha}) \), as given by equation (39), must exist within the range \([1-t,1+t]\). Otherwise the fact that both \( \frac{\partial g(c,t)}{\partial c}{|}_{c \to 1-t} \) and \( \frac{\partial g(c,t)}{\partial c}{|}_{c \to 1+t} \) are positive would imply that \( g(c,t) \) increases throughout the range \( 1-t \leq c < 1+t \) and so cannot have a local max point. The condition \( \frac{(1-2\alpha)(1+t)}{2-3\alpha} < 1+t \) is indeed fulfilled, given that

\[
\frac{1-2\alpha}{2-3\alpha} = 1 - \frac{1-\alpha}{2-3\alpha} < 1
\]
when $\alpha < 1/2$. The condition \( \frac{(1-2\alpha)(1+t)}{2-3\alpha} > 1-t \) can be rewritten as \((1-2\alpha)(1+t) > (1-t)(2-3\alpha) \) \( \iff \)
\[ t > \frac{1 - \alpha}{3 - 5\alpha}. \]
For \( c \in [1-t, 1+t] \) we thus get the necessary condition \( \frac{1 - \alpha}{3 - 5\alpha} \leq \frac{1}{4 - 6\alpha} \) \( \iff \alpha \in \left[ \frac{1}{3}, \frac{1}{2} \right) \).

**Q.E.D.**

**g \((1+t, t)\) as a function of \( t \)**

We now turn to study further the properties of \( g(c, t) \) at \( c = 1 + t \), as implied by the function \( h\ (t) \) defined in Lemma 11.

**Lemma 21** Let \( \alpha \in \left] 0, \frac{1}{2} \right[ \). Then, as defined in Lemma 11, \( h\ (t) = g(1 + t, t) = (1 + t) \left( \frac{1}{3} + t^2 \right)^{\alpha-1} \) has a unique inner global max point at \( t_{\text{max}} = \frac{1 - \alpha - \sqrt{\alpha(\alpha - 2)(\alpha - \frac{2}{3})}}{2\alpha - 1} \).

**Proof:** The analysis of \( h\ (t) \) follows the same steps as in Lemma 11 (which was performed for the case of \( \alpha > 1/2 \)), up to the analysis of the two roots of the square brackets in (30), which determine the sign of \( h\ (t) \),
\[(2\alpha - 1) t^2 + 2(\alpha - 1) t + \frac{1}{3} \]
Then, when \( \alpha \in \left] 0, \frac{1}{2} \right[ \), we get that the first root \( t_1 = \frac{1 - \alpha + \sqrt{\alpha(\alpha - 2)(\alpha - \frac{2}{3})}}{2\alpha - 1} < 0 \), hence \( h\ (t) \) has a max point at \( t_2 = t_{\text{max}} = \frac{1 - \alpha - \sqrt{\alpha(\alpha - 2)(\alpha - \frac{2}{3})}}{2\alpha - 1} \) if this value falls within the range \([0, 1]\). We thus have
\[ \frac{1 - \alpha - \sqrt{\alpha(\alpha - 2)(\alpha - \frac{2}{3})}}{2\alpha - 1} < 1 \iff (2\alpha - 1) \left( \frac{\alpha - \frac{2}{3}}{3} \right) > 0 \]
which holds for any \( \alpha \in \left] 0, \frac{1}{2} \right[ \). Finally \( t_{\text{max}} > 0 \) iff \( \frac{1 - \alpha - \sqrt{\alpha(\alpha - 2)(\alpha - \frac{2}{3})}}{2\alpha - 1} > 0 \) which can be verified to hold for \( \alpha < 1/2 \).

**Q.E.D.**

**Lemma 22** \( t_{\text{max}} = \frac{1 - \alpha - \sqrt{\alpha(\alpha - 2)(\alpha - \frac{2}{3})}}{2\alpha - 1} \) increases in \( \alpha \) at the range \( \alpha \in \left] 0, \frac{1}{2} \right[ \).

**Proof:** The statement holds iff
\[ \frac{dt_{\text{max}}}{d\alpha} = \frac{d}{d\alpha} \left\{ \frac{1 - \alpha - \sqrt{(\alpha - 2)(\alpha - \frac{2}{3})}}{2\alpha - 1} \right\} > 0 \]
\[ \iff 2 \left( 1 - \alpha - \sqrt{(\alpha - 2)(\alpha - \frac{2}{3})} \right) < (1 - 2\alpha) \left( 1 + \frac{\alpha - 4/3}{\sqrt{(\alpha - 2)(\alpha - 2/3)}} \right) \]
\[ \iff 2 \left( (\alpha - 2) \sqrt{(\alpha - 2)(\alpha - \frac{2}{3})} - (\alpha - 2) \left( \alpha - \frac{2}{3} \right) \right) < (1 - 2\alpha) \left( \sqrt{(\alpha - 2)(\alpha - 2/3)} + (\alpha - 4/3) \right) \]
\[ \iff \sqrt{(\alpha - 2)(\alpha - \frac{2}{3})} < (1 - 2\alpha) (\alpha - 4/3) + 2(\alpha - 2) \left( \alpha - \frac{2}{3} \right) = \frac{4 - 5\alpha}{3}. \]
Since both sides are positive, this amounts to proving $\alpha^2 - \frac{8}{3}\alpha + \frac{4}{3} < \frac{25\alpha^2 - 40\alpha + 16}{9} \iff 0 < 16\alpha^2 - 16\alpha + 4 = 4(2\alpha - 1)^2$ which is evident since $\alpha < 1/2$.

Q.E.D.

**Lemma 23** Let $\alpha \in [0, \frac{1}{2}]$. Then $t_{\text{max}} \leq \frac{1}{3}$.

**Proof:** Lemma 22 implies that $t_{\text{max}} = \frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1}$ reaches its max value for $\alpha \in [0, \frac{1}{2}]$ when $\alpha = \frac{1}{2}$. Using L'Hopital we get

$$
\lim_{\alpha \to \frac{1}{2}} \frac{1 - \alpha - \sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1} = \lim_{\alpha \to \frac{1}{2}} \left\{ -\frac{1}{2} - \frac{4}{3} \sqrt{(\alpha-2)(\alpha-\frac{2}{3})} \right\} = \frac{1}{3}
$$

Q.E.D.

Splitting the strong concavity case into two sub-cases

**Lemma 24** Let $\alpha \in [0, \frac{1}{2}]$ and define $\Omega(t_{\text{max}}(\alpha), \alpha) \equiv \frac{\alpha(1+t_{\text{max}})}{1+3t_{\text{max}}}$. Then $\frac{d\Omega(t_{\text{max}}(\alpha), \alpha)}{d\alpha} > 0$.

**Proof:** By construction (see Lemma 21) $t_{\text{max}}$ is the solution to $\frac{dh}{dt} = 0$. Setting equation (30) to zero and solving for $\alpha$ yields

$$
\alpha(t_{\text{max}}) = \frac{\left(t_{\text{max}} + 1 + 2/\sqrt{3}\right) \left(t_{\text{max}} + 1 - 2/\sqrt{3}\right)}{2t_{\text{max}}(1 + t_{\text{max}})}.
$$

Using this in $\Omega(t_{\text{max}}(\alpha), \alpha)$ yields

$$
\Omega(t_{\text{max}}) = \frac{\left(t_{\text{max}} + 1 + 2/\sqrt{3}\right) \left(t_{\text{max}} + 1 - 2/\sqrt{3}\right)}{2t_{\text{max}}(1 + 3t_{\text{max}}^2)}
$$

where $t_{\text{max}} \in [2/\sqrt{3} - 1, 1/3]$ (the lower limit follows from substituting $\alpha = 0$ in $t_{\text{max}} = \frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1}$ while noting that $t_{\text{max}}$ is increasing in $\alpha$, as shown in Lemma 22, and the upper limit follows from Lemma 23, and from ). We will now show that $\Omega(t_{\text{max}})$ increases in $t_{\text{max}}$, which will imply (by Lemma 22) that $\Omega(t_{\text{max}}(\alpha), \alpha)$ increases in $\alpha$.

$$
B(t_{\text{max}}) \equiv \frac{d\ln\Omega(t_{\text{max}})}{dt_{\text{max}}} = B_1(t_{\text{max}}) + B_2(t_{\text{max}}) + B_3(t_{\text{max}}) + B_4(t_{\text{max}})
$$

for

$$
B_1(t_{\text{max}}) = \frac{1}{t_{\text{max}} + 1 - 2/\sqrt{3}},
$$

$$
B_2(t_{\text{max}}) = \frac{1}{t_{\text{max}} + 1 + 2/\sqrt{3}},
$$

$$
B_2'(t_{\text{max}}) = -\frac{1}{(t_{\text{max}} + 1 + 2/\sqrt{3})^2},
$$

$$
B_2''(t_{\text{max}}) = \frac{2}{(t_{\text{max}} + 1 + 2/\sqrt{3})^3} > 0,
$$

$$
B_3(t_{\text{max}}) = -\frac{1}{t_{\text{max}}},
$$

$$
B_3'(t_{\text{max}}) = \frac{1}{t_{\text{max}}^2},
$$

$$
B_3''(t_{\text{max}}) = -\frac{2}{t_{\text{max}}^3} < 0,
$$

$$
B_4(t_{\text{max}}) = \frac{-6t_{\text{max}}}{(1 + 3t_{\text{max}}^2)},
$$

$$
B_4'(t_{\text{max}}) = \frac{6 - 3t_{\text{max}}}{(1 + 3t_{\text{max}}^2)^2},
$$

$$
B_4''(t_{\text{max}}) = \frac{108t_{\text{max}}(1 - t_{\text{max}}^2)}{(1 + 3t_{\text{max}}^2)^2} > 0.
$$

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According to the signs of the second order derivatives, and designating $t_{\text{low}} \equiv 2/\sqrt{3} - 1$, $t_{\text{high}} \equiv 1/3$, we have for $t_{\max} \in [t_{\text{low}}, t_{\text{high}}]$

\[
B_2 (t_{\max}) \geq B_2 (t_{\text{low}}) + B'_2 (t_{\text{low}})(t_{\max} - t_{\text{low}}) = \frac{\sqrt{3}}{4} - \frac{3}{16} (t_{\max} - 2/\sqrt{3} + 1)
\]

\[
B_3 (t_{\max}) \geq B_3 (t_{\text{low}}) + \frac{B_3 (t_{\text{high}}) - B_3 (t_{\text{low}})}{t_{\text{high}} - t_{\text{low}}} (t_{\max} - t_{\text{low}}) = -3 - 2\sqrt{3} + 3 \left(3 + 2\sqrt{3}\right) (t_{\max} - 2/\sqrt{3} + 1)
\]

\[
B_4 (t_{\max}) \geq B_4 (t_{\text{low}}) + B'_4 (t_{\text{low}})(t_{\max} - t_{\text{low}}) = -\sqrt{3}/2 - \frac{3}{4} \left(3 + 2\sqrt{3}\right) (t_{\max} - 2/\sqrt{3} + 1).
\]

Let us designate $z \equiv t_{\max} - 2/\sqrt{3} + 1$. Then $B (t_{\max}) \geq N \left(z\right) = 1/z - (3 + 2.25\sqrt{3}) + (6.5625 + 4.5\sqrt{3}) z$ for $z \in [0, 4/3 - 2/\sqrt{3}]$. Since $N' \left(z\right) < 0$ for $z < 4/3 - 2/\sqrt{3}$ and $N \left(4/3 - 2/\sqrt{3}\right) > 0$ it follows that $N \left(z\right) > 0$ for all $z$ involved, and so $B \left(t_{\max}\right) > 0$ implying $\Omega \left(t_{\max}\right)$ increases in $t_{\max}$ and so finally $\Omega \left(t_{\max} \left(\alpha\right), \alpha\right)$ increases in $\alpha$.

Q.E.D.

Lemma 25. The functions $\alpha^{2^{1-\alpha}} h \left(t_{\max}\right)$ and $2 \left(1/6\right)^{\alpha}$ have one intersection point, denoted by $\alpha^*$, at the range $\alpha \in \left]0, \frac{1}{2}\right]$. Furthermore we have

\[
\left\{\begin{array}{ll}
\alpha^{2^{1-\alpha}} h \left(t_{\max}\right) < 2 \left(1/6\right)^{\alpha} & \text{if } \alpha \in \left(0, \alpha^*\right) \\
\alpha^{2^{1-\alpha}} h \left(t_{\max}\right) > 2 \left(1/6\right)^{\alpha} & \text{if } \alpha \in \left(\alpha^*, \frac{1}{2}\right)
\end{array}\right.
\]

Proof: Using $h \left(t\right)$ as defined in Lemma 11 we get

\[
\alpha^{2^{1-\alpha}} h \left(t_{\max}\right) = 2 \left(1/6\right)^{\alpha} \Leftrightarrow \alpha \left(1 + t_{\max}\right) \left(\frac{1}{6} + \frac{1}{2} t_{\max}^2\right)^{\alpha - 1} = \frac{1}{3} \left(1/6\right)^{\alpha - 1} \Leftrightarrow
\]

\[
3 \left(1 + 3 t_{\max}^2\right)^{\alpha - 1} = \frac{1}{\alpha \left(1 + t_{\max}\right)} \Leftrightarrow 3 \left(1 + 3 t_{\max}^2\right)^{\alpha} = \frac{1 + 3 t_{\max}^2}{\alpha \left(1 + t_{\max}\right)}
\]

The RHS is the inverse of $\Omega \left(t_{\max} \left(\alpha\right), \alpha\right)$ as defined in Lemma 24, and we know from that lemma that $\Omega \left(t_{\max} \left(\alpha\right), \alpha\right)$ increases in $\alpha$ hence the RHS decreases in $\alpha$.

Analyzing the LHS:

Let

\[
\Phi \left(t_{\max} \left(\alpha\right), \alpha\right) \equiv \left(1 + 3 t_{\max}^2\right)^{\alpha}
\]

In Lemma 22 we showed that $t_{\max}$ increases in $\alpha$. To show that $\Phi \left(t_{\max} \left(\alpha\right), \alpha\right)$ is increasing in $\alpha$ it is therefore enough to show that the two partial derivatives of $\Phi \left(t_{\max} \left(\alpha\right), \alpha\right)$ with respect to its two arguments, $t_{\max} \left(\alpha\right)$ and $\alpha$, are both positive.

\[
\frac{\partial \Phi \left(t_{\max} \left(\alpha\right), \alpha\right)}{\partial \alpha} = \left(1 + 3 t_{\max}^2\right)^{\alpha} \ln \left(1 + 3 t_{\max}^2\right) > 0
\]

\[
\frac{\partial \Phi \left(t_{\max} \left(\alpha\right), \alpha\right)}{\partial t_{\max}} = 6 t_{\max} \alpha \left(1 + 3 t_{\max}^2\right)^{\alpha - 1} > 0
\]

Thus, the LHS increases in $\alpha$ while the RHS decreases in $\alpha$, implying that there is a unique intersection point $\alpha^*$. To find which of the functions $\alpha^{2^{1-\alpha}} h \left(t_{\max}\right)$ and $2 \left(1/6\right)^{\alpha}$ is larger below and above $\alpha^*$ we can plug in specific values of $\alpha$. When $\alpha = 0$ the former function goes to 0 (recall that $h$ is bounded) while the latter equals 2 hence is larger. When $\alpha \rightarrow \frac{1}{2}$ we know from Lemma 23 that $t_{\max}$ approaches $\frac{1}{2}$ hence the former function approaches

\[
\frac{1}{2} \alpha^{1/2} \frac{4}{3} \left(\frac{4}{9}\right)^{-1/2} = \frac{4}{3} \left(\frac{9}{8}\right)^{1/2} = \sqrt{2}
\]

while the latter equals $2 \left(1/6\right)^{1/2} = \sqrt{2}/3$ hence is smaller. This also implies $\alpha^* \in \left]0, 1/2\right]$.

Q.E.D.
We will show the pattern of a spider separately for \( \alpha \in (0, \alpha^*) \) and \( \alpha \in (\alpha^*, \frac{1}{2}) \). The value of \( \alpha^* \) can be numerically calculated to be \( \approx 0.3 \).

**Very strong concavity \( \alpha \in [0, \alpha^*] \)**

**Lemma 26** \( \) If \( \alpha \in [0, 1/3] \) then \( g(c,t) \) has exactly one local min point with respect to \( c \).

**Proof:** We have two cases to consider. Case i) is where \( c_b > 1 - t \) (\( c_b \) is defined in Lemma 19). Lemma 19 then states that \( g(c,\cdot) \) has a U-shape, that is, exactly one local min point. Case ii) is where \( c_b \leq 1 - t \). Then (by the proof of Lemma 19) \( g(c,\cdot) \) has one local min point at \( c_b \leq 1 - t \), and \( \frac{\partial g(c,t)}{\partial c} \bigg|_{c=1-t} \geq 0 \). Continuity of \( \frac{\partial g(c,t)}{\partial c} \) at \( c = 1 - t \) (see Lemma 15) then implies \( \frac{\partial g(c,t)}{\partial c} \bigg|_{c=1-t} \geq 0 \). Lemma 16 states that \( \frac{\partial g(c,t)}{\partial c} \bigg|_{c=1+t} \geq 0 \). Finally, from Lemma 20 we know that, when \( \alpha < 1/3 \), \( g(c,\cdot) \) does not have a local max point in the range \( c \in [1-t,1+t] \), hence it must be that \( \frac{\partial g(c,t)}{\partial c} \geq 0 \) in this range. Thus, in case ii) there is one local min point.

**Q.E.D.**

**Lemma 27** Type \( t \) has an inner solution only if \( h(t) > \frac{W}{2^{1-\alpha}} \).

**Proof:** We (from Lemma 26) know that \( g(c,t) \) has a U-shape for all types (because \( \alpha^* < \frac{1}{3} \)). Remembering (from Lemma 8) that \( \frac{dL}{dc} > 0 \Leftrightarrow g(c,t) > \frac{W}{2^{1-\alpha}} \) and that (by equation 25) \( \lim_{c \rightarrow 0} \frac{dL}{dc} = +\infty \), we get that in order for type \( t \) to have an inner solution it must be that \( h(t) = g(1+t,t) > \frac{W}{2^{1-\alpha}} \).

**Q.E.D.**

We will now show the spider for the range of \( W \) that satisfy the following conditions:

(i) \( t = 0 \) prefers \( c = 0 \) over \( c = 1 + t \):

\[
L(0,0) = \frac{1}{2} W < \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^\alpha = L(t+1,0)
\]

\[
W < 2 \left( \frac{1}{6} \right)^\alpha
\]

(ii) \( t = 1 \) prefers \( c = 1 + t \) over \( c = 0 \):

\[
L(c) = z^\alpha = \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^\alpha = (2/3)^\alpha < W
\]

(iii) \( \forall t \in [0,1], \frac{dL}{dt} \bigg|_{c=1+t} < 0 \). From Lemma 8 this is equivalent to:

\[
\forall t \in [0,1], \; h(t) < \frac{W}{2^{1-\alpha}} \Rightarrow W > \alpha 2^{1-\alpha} \max_t h(t) \Rightarrow \{ \text{by Lemma 21} \} \; W > \alpha 2^{1-\alpha} h(t_{\text{max}})
\]

\[
\Rightarrow W > \alpha 2^{1-\alpha} \left( 1 + t_{\text{max}} \right) \left( \frac{1}{3} + t_{\text{max}}^2 \right)^{-\alpha^{-1}}
\]

**Lemma 28** For any \( \alpha \in [0, \alpha^*] \) there exists a range of \( W \) that satisfy conditions (i)-(iii)

**Proof:** The range of \( W \) that satisfy conditions (i)-(iii) is the intersection of the range of \( W \) that satisfy conditions (i) and (ii) and the range of \( W \) that satisfy conditions (i) and (iii) so it is enough to show that, for any \( \alpha \in [0, \alpha^*] \), none of these ranges is empty. Starting with conditions (i) and (ii), we note that

\[
\frac{(2/3)^\alpha}{2 \left( \frac{1}{6} \right)^\alpha} = \frac{4^\alpha}{2} < 1 \; \forall \alpha < \frac{1}{2}
\]

hence the range of \( W \) that satisfy conditions (i) and (ii) is not empty. Next, the fact that the range of \( W \) that satisfy conditions (ii) and (iii) is not empty when \( \alpha < \alpha^* \) follows directly from Lemma 25.
If $\frac{dL}{dc}|_{c=1+t} < 0$ for type $t = 0$ then $\Delta L \equiv L(0) - L(1 + t)$ increases in $t$.

**Proof:** From (18) and (15) we get

$$\Delta L = L(0) - L(1 + t) = W \left[ 1 - P_m \right] - \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^\alpha$$

{by 8} $$= W(1 - 2F(t)[1 - F(t)]) - \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^\alpha$$

{by $t \sim U(-1,1)$} $$= W \left[ 1 - \frac{1}{2} (1 + t)(1 - t) \right] - \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^\alpha$$

$$= \frac{1}{2} W \left( 1 + t^2 \right) - \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^\alpha$$

$$\frac{d}{dt} \Delta L = \left[ W - \alpha \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^{\alpha-1} \right] t$$

The term $\left[ W - \alpha \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^{\alpha-1} \right]$ increases in $t$ (since $\alpha < 1$) and at $t = 0$ it equals $W - 6\alpha \left( \frac{1}{6} \right)^\alpha$. If $\frac{dL}{dc}|_{c=1+t} < 0$ for type $t = 0$ then

$$\frac{dL}{dc}|_{c=1+t=1} < 0 \Rightarrow \{\text{Lemma 8} \} \Rightarrow$$

$$2^{1-\alpha} g(1,t) < \frac{W}{\alpha} \Rightarrow W > \alpha \left( \frac{1}{6} \right)^{\alpha-1} = 6\alpha \left( \frac{1}{6} \right)^\alpha$$

Thus, $\left[ W - \alpha \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^{\alpha-1} \right] > 0 \ \forall t \in [0,1]$ implying that $\Delta L$ increases in $t$.

**Q.E.D.**

**Lemma 30** Condition (iii) implies that $\Delta L$ increases in $t$.

**Proof:** Follows directly from applying condition (iii) to $t = 0$ and using Lemma 29.

**Q.E.D.**

**Proposition 9** For any $\alpha \in ]0,\alpha^*]$ there is a spider for any $W$ at the range of values that satisfy conditions (i)-(iii).

**Proof:** Lemma 27 and Condition (iii) imply that every type has a corner solution, either at $c = 0$ or at $c = 1 + t$. Then, conditions (i) and (ii) and Lemma 30 imply that there exists a unique switching point at the range $[0,1]$ such that types below it choose $c = 0$ while types above it choose $c = 1 + t$. Finally, Lemma 5 implies that dissent rate is increasing in $t$ when $t$ is below the switching point and then it abruptly falls to 0 at the switching point and stays there.

**Q.E.D.**

**Mildly strong concavity** $\alpha \in ]\alpha^*, \frac{1}{2}[$

**Lemma 31** Suppose that $g(c,t)$ has a local max point at some $c \in [1 - t, 1 + t]$. Then $c$ is strictly smaller than 1.
Lemma 20 implies that $g(c, t)$ has a local max point only if $t \leq \frac{1}{4 - 6\alpha}$. Noting that

$$\frac{1}{4 - 6\alpha} < \frac{1 - \alpha}{1 - 2\alpha} \Leftarrow \ldots \Leftarrow 6\alpha^2 - 8\alpha + 3 > 0,$$

which can be verified to hold for all $\alpha$, we indeed get that $t \leq \frac{1}{4 - 6\alpha}$ implies that $t < \frac{1 - \alpha}{1 - 2\alpha}$, hence the local max point is strictly smaller than 1.

Q.E.D.

Lemma 32 Suppose that $g(c, t)$ has a local max point at some $c_m \in [1 - t, 1 + t]$. Then the value of $g(c, t)$ at the local max point is strictly smaller than $g(1 + t, t)$.

Proof: Lemma 20 implies that if $g(c, t)$ has a local max point for some $c_m \in [1 - t, 1 + t]$ then $\alpha \in \left[\frac{1}{3}, \frac{1}{2}\right]$ and $t \in \left[\frac{1 - \alpha}{3 - 5\alpha}, \frac{1}{4 - 6\alpha}\right]$. So we focus on these values in the remainder of the proof. Lemma 31 further implies that $c_m < 1$. We will prove that the value of $g(c_m, t) < g(1, t)$. This is sufficient because, if this holds, then together with the fact that $c_m < 1$ it implies that at the range $c > 1$ the function $g(c, t)$ must be increasing and so $g(1, t) < g(1 + t, t)$. Focusing on the range $1 - t \leq c < 1 + t$, where (by equation (26))

$$g(c, t) = c \left[\left(-\frac{1}{3} c + \frac{1}{2} t + \frac{1}{2}\right) c^2 + \frac{1}{6} (1 - t)^3\right]^\alpha,$$

let

$$X(c; \alpha, t) \equiv [g(c)]^{\frac{1}{\alpha + t}} = c^{\frac{1}{\alpha + t}} \left[-\frac{1}{3} c^3 + \frac{1}{2} (t + 1) c^2 + \frac{1}{6} (1 - t)^3\right]$$

be defined in the domain $(\alpha, t)$ which corresponds to $\alpha \in \left[\frac{1}{3}, \frac{1}{2}\right]$ and $t \in \left[\frac{1 - \alpha}{3 - 5\alpha}, \frac{1}{4 - 6\alpha}\right]$. Noting that

$$t = \frac{1}{4 - 6\alpha} \Rightarrow \alpha = \frac{2}{3} - \frac{1}{6t},$$

$$t = \frac{1 - \alpha}{3 - 5\alpha} \Rightarrow t = \frac{2/5}{3 - 5\alpha} + \frac{1}{5} \Rightarrow \alpha = \frac{3}{5} - \frac{2}{25t - 5},$$

the domain can alternatively be described as $t \in \left[\frac{2}{5}, 1\right]$ and $\alpha \in [\alpha_1(t), \alpha_2(t)]$, where $\alpha_1(t) = \frac{2}{5} - \frac{1}{25t}$ and $\alpha_2(t) = \frac{3}{5} - \frac{2}{25t - 5}$. We will show that $X(c; \alpha, t)$ has an inner min point exactly where $g(c, t)$ has an inner max point, and that $X(c; \alpha, t)$ at this min point is larger than $X(1; \alpha, t)$, which is equivalent to showing that the value of $g(c, t)$ at the inner max point of $g(c, t)$ is smaller than $g(1, t)$. Holding $t$ constant, denoting the value of $c$ at the inner min point of $X(c; \alpha, t)$ by $c_0$ (if it exists) and exploiting the fact that $\frac{\partial X}{\partial c} = 0$ at

To see why it must be increasing, note that $g$ has at most one inner local max point (this follows from Lemma 17 since when there is one local min point of $\partial g/\partial c$ there can be at most two inner extrema where only one is a max point). Thus, as $c_0$ is the only inner max point of $g$ and $g(c_m, t) < g(1, t)$ then $g$ must be increasing at $c = 1$ and beyond.

Since in the expression for $X$ the power $\frac{1}{\alpha + t} < 0$. 

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this min point, we get
\[
\frac{dX(c; \alpha, t)}{dx} \bigg|_{x=c_\alpha} = \frac{\partial X}{\partial c} \frac{\partial c}{\partial x} + \frac{\partial X}{\partial \alpha} = \frac{\partial X}{\partial \alpha} > 0,
\]
implying that \(X(c_0; \alpha, t)\) for given \(t\) reaches its minimum at \(\alpha = \alpha_1(t)\). Hence, given that \(X(1; \alpha, t)\) is independent of \(\alpha\), it is necessary and sufficient to show that \(X(c_0; \alpha, t) > X(1; \alpha, t)\) for \(\alpha = \alpha_1(t)\).

The partial derivative \(\frac{\partial X(c; \alpha, t)}{\partial c}\) is
\[
\frac{\partial X(c; \alpha, t)}{\partial c} = -c^{-\frac{2-\alpha}{1-\alpha}} \left\{ c^2 \left[ \left( \frac{2}{3} - \alpha \right) c + \left( \alpha - \frac{1}{2} \right) (1 + t) \right] + \frac{1}{6} (1 - t)^3 \right\},
\]
where the expression in the curly brackets is the one determining the sign of \(\frac{\partial g(c; t)}{\partial c}\) (see equation (36)), hence the sign of \(\frac{\partial X(c; \alpha, t)}{\partial c}\) is opposite to that of \(\frac{\partial g(c; t)}{\partial c}\), and so indeed \(X(c; \alpha, t)\) has an inner min point exactly where \(g(c, t)\) has an inner max point. Substituting \(\alpha = \alpha_1(t)\) into the expression for \(\frac{\partial X(c; \alpha, t)}{\partial c}\) and equating to zero to find a local min point we get
\[
\left( \frac{2}{3} - \frac{2}{3} + \frac{1}{6t} \right) c^3 + \left( \frac{2}{3} - \frac{1}{6t} - \frac{1}{2} \right) (1 + t) c^2 + \frac{1}{6} (1 - t)^3 = 0 \iff ...
\]
whose roots in increasing order are
\[
c_{1,2,3} = \frac{1}{2} t (1 - t) \left[ 1 - \sqrt{1 + \frac{4}{t}} \right], \quad 1 - \frac{1}{2} t (1 - t) \left[ 1 + \sqrt{1 + \frac{4}{t}} \right].
\]
It can be verified that at \(c_2 = 1 - t\) the sign of \(\frac{\partial g(c; t)}{\partial c}\) turns from positive to negative,\(^{52}\) hence this is a local max point of \(g(c, t)\) and a local min point of \(X(c; \alpha, t)\). Let us now substitute \(c = c_2\) back into \(X(c; \alpha, t)\):
\[
X(c = 1 - t; \alpha_1(t), t) = \frac{1}{3} (1 - t)^{\frac{2}{1-t}} (2t + 1).
\]
We need to show that this is larger than
\[
X(c = 1; \alpha_1(t), t) = -\frac{1}{3} + \frac{1}{2} (t + 1) + \frac{1}{6} (1 - t)^3 = \frac{2 + 3t^2 - t^3}{6}.
\]
Let
\[
Y(t) = 3[X(1 - t; \alpha_1(t), t) - X(1; \alpha_1(t), t)] = (1 - t)^{\frac{2}{1-t}} (2t + 1) - \frac{2 + 3t^2 - t^3}{2}.
\]
Then the proof boils down to showing that \(Y(t) > 0\) \(\forall t \in \left[ \frac{1}{2}, 1 \right]\). Define
\[
y = -\frac{2 - 2t}{2t + 1} \ln (1 - t)
\]
Then \(y \geq 0\) for \(t \in \left[ \frac{1}{2}, 1 \right]\) and the Taylor-Lagrange formula for \(e^{-y}\) implies that \(e^{-y} \geq 1 - y\). Hence
\[
Y(t) \geq Y_1(t) \equiv (2t + 1) \left[ 1 + \frac{2 - 2t}{2t + 1} \ln (1 - t) \right] - \frac{2 + 3t^2 - t^3}{2} = 2t + 1 + 2(1 - t) \ln (1 - t) - \frac{2 + 3t^2 - t^3}{2}.
\]
Let us now investigate \(Y_1(t)\). We have
\[
Y_1'(t) = -2 \left[ 1 + \ln (1 - t) \right], \quad Y_1''(t) = \frac{2}{1 - t} > 0,
\]
\(^{52}\)Plugging \(c = 1 - t\) and the value of \(\alpha_1(t)\) into equation (37), which is the derivative of the expression determining the sign of \(\frac{\partial g(c; t)}{\partial c}\), yields \(\frac{1}{6} (1 - t)^{\frac{2}{1-t}} [1 - 2t]\), which is negative for \(t \in \left[ \frac{1}{2}, 1 \right]\), implying that the sign of \(\frac{\partial g(c; t)}{\partial c}\) turns from positive to negative at \(c = 1 - t\).
and it can be verified that \( Y_1 (1/2) < 0 \) while \( Y_1 (5/9) > 0 \), which implies that \( Y_1 (t) > 0 \ \forall t \geq 5/9 \), implying also that \( Y (t) > 0 \ \forall t \geq 5/9 \). So the only thing left to show is that \( Y (t) > 0 \ \forall t \in \left[ \frac{1}{2}, \frac{5}{9} \right] \). In the interval \( t \in \left[ \frac{1}{2}, \frac{5}{9} \right] \) we have

\[
Y (t) \geq Y_2 (t) = (2t + 1) (1-t) 1/2 - \frac{2 + 3t^2 - t^3}{2}
\]

Changing variables as follows: \( q = \sqrt{1-t} \), we get

\[
Y_2 (q) = q [1 + 2 (1-q^2)] - \frac{2 + 3 (1-q^3)^2 - (1-q^2)^3}{2} = -2 + 3q + \frac{3}{2} q^2 - 2q^3 + q^6
\]

for \( q \in \left[ 2/3, \sqrt{1/2} \right] \). Now, we have \( Y_2 (2/3) > 0 \) and \( Y_2' (2/3) > 0 \), so it is sufficient to show that \( Y_2'' (q) > 0 \). Indeed,

\[
Y_2'' (q) = 3 - 12q + 30q^4 > 0 \quad \text{for} \quad q \in \left[ 2/3, \sqrt{1/2} \right],
\]

because at the range \( q \in \left[ 2/3, \sqrt{1/2} \right] \) we have

\[
Y_2''' (q) = 12 (10q^3 - 1) \geq 12 \left( 10 \left( \frac{2}{3} \right) ^3 - 1 \right) > 0
\]

and

\[
Y_2'' \left( \frac{2}{3} \right) = 3 - 12 \left( \frac{2}{3} \right) ^4 + 30 \left( \frac{2}{3} \right) ^4 > 0.
\]

We have thus showed that \( Y_2 (q) > 0 \) for \( q \in \left[ 2/3, \sqrt{1/2} \right] \) and hence \( Y (q) > 0 \) for \( q \in \left[ 2/3, \sqrt{1/2} \right] \).

\[Q.E.D.\]

**Lemma 33** \textit{For any} \( \alpha \in ]\alpha^*, \frac{1}{2} [ \) \textit{and any} \( t \in [0, 1] \), the loss function \( L (c) \) \textit{does not have a local min point if} \( W \geq \alpha 2^{1-\alpha} h (t) \)

\[ \text{PROOF:} \quad \text{Lemma 8 implies that the sign of} \ \frac{dL}{d\alpha} \text{is determined by the sign of} \ 2^{1-\alpha} \alpha g (c, t) - W. \ \text{Recalling that} \ \lim_{\alpha \rightarrow 0} \frac{dL}{d\alpha} = \infty, \ \text{this implies that} \ L (c) \ \text{may have a local min point only if there are at least two different values of} \ c \in [0, 1 + t[ \ \text{for which} \ 2^{1-\alpha} \alpha g (c, t) = W. \ \text{Turning now the focus to} \ g (c, t), \ \text{the fact that} \ \lim_{\alpha \rightarrow 0} \frac{dL}{d\alpha} = \infty \ \text{implies further that} \ 2^{1-\alpha} \alpha g (0, t) > W, \ \text{while it is given that} \ 2^{1-\alpha} \alpha h (t) \leq W. \ \text{We thus get that a necessary (though insufficient) condition for} \ L (c) \ \text{to have a local min point is for} \ g (c, t) \ \text{to have a local max point in which its value exceeds that of} \ h (t). \ \text{This condition does not hold at the range} \ c \in [0, 1 - t[ \ \text{because} \ g (c, t) \ \text{does not have a local max point there (see Lemma 18), and it also does not hold at the range} \ c \in [1 - t, 1 + t[ \ \text{because Lemma 32 states that the value of} \ g (c, t) \ \text{at the local max point, if it exists, is strictly smaller than} \ h (t). \ \text{Q.E.D.}} \]

We will show the spider for the range of \( W \) that satisfy the following conditions (where the inequalities in I-III follow from Lemma 33):

(I) \text{There exist types with a potential inner solution (i.e. local min point):} \ W < \alpha 2^{1-\alpha} h (t_{\text{max}})
(II) The type $t = 0.34$ does not have an inner solution:
$$\alpha^{2^{t\alpha}h(0.34)} \leq W$$

(III) The type $t = 0$ does not have an inner solution:
$$\alpha^{2^{t\alpha}h(0)} < W$$

(IV) The type $t = 0$ strictly prefers $c = 1 + t$ over $c = 0$ which implies (by plugging the corner options into equation (15)):
\[
\frac{1}{2}W > \left(\frac{1}{2}\left(t^2 + \frac{1}{3}\right)\right)^\alpha
\]
\[
W > 2(1/6)^\alpha
\]

(V) There is no type $t$ for whom $g(c, t)$ has a local max point in which $\alpha^{2^t\alpha}g(c, t) > W$: Let
\[
W_c \equiv \alpha^{2^t\alpha}\max_t \{g(c, t) | c \text{ is local max of } g(c, t)\}
\]

Then we require $W > W_c$.

**Lemma 34** For any $\alpha \in ]\alpha^*, \frac{1}{2}[$ there exist a range of $W$ that satisfy conditions (I)-(V)

**Proof:** Condition (I) sets an upper bound on $W$ while the other four conditions set lower bounds, hence we need to show that the intersections of condition (I) and each of the other conditions are not empty. Conditions (I) and (II): Lemma 23 shows that $t_{max}$ is weakly smaller than $1/3$, hence $h(t_{max}) > h(0.34)$. Conditions (I) and (III): Lemma 21 establishes that $t_{max} > 0$, hence $h(t_{max}) > h(0)$. Conditions (I) and (IV): Lemma 25 implies that $2(1/6)^\alpha < \alpha^{2^t\alpha}h(t_{max})$ at the range $\alpha \in ]\alpha^*, \frac{1}{2}[$. Conditions (I) and (V): Let $t_c$ be a type for whom $W_c = \alpha^{2^t\alpha}g(c, t)$ at the local max point of $g(c, t)$. Then Lemma 18 implies that $g(c, t_c)$ reaches that local max point at some $c \in [1 - t, 1 + t]$, and from Lemma 32 we get that the value of $g(c, t_c)$ at that local max point is strictly smaller than $h(t_c) \leq h(t_{max})$ hence $W_c < \alpha^{2^t\alpha}h(t_{max})$.

Q.E.D.

**Lemma 35** Let $\alpha \in ]\alpha^*, \frac{1}{2}[$ and let $W$ satisfy conditions (I-V). Then there exists a (non-singleton) neighborhood of $t_{max}$ s.t. types at this neighborhood are choosing an inner solution while any other type is choosing $c = 1 + t$.

**Proof:** Lemma 33 implies that any $t$ for whom $W \geq \alpha^{2^t\alpha}g(1 + t, t)$ has a corner solution to the minimization problem, while condition (V) implies that any $t$ for whom $W < \alpha^{2^t\alpha}h(t)$ has at most one local inner min point.\(^{53}\) Condition (IV) states that type $t = 0$ strictly prefers $c = 1 + t$ over $c = 0$, and condition (III) states that $\alpha^{2^t\alpha}h(0) < W$, hence $\frac{dL}{dt}|_{c=1+t} < 0$ for $t = 0$ (Lemma 8), which by Lemma 29 implies that $\Delta L$ increases in $t$, hence all types prefer $c = 1 + t$ over $c = 0$. Thus any type with a corner solution chooses $c = 1 + t$, while all the types for whom $W < \alpha^{2^t\alpha}h(t)$ (which by condition (I) contain more than a singleton) choose their unique local min point as a solution because Lemma 8 implies that $\frac{dL}{dt}|_{c=1+t} > 0$ for these types hence $c = 1 + t$ cannot be their global min point. Finally, Lemma 21 implies that $h(t)$ has a hill shape with a peak at $t_{max}$, hence the types for whom $W < \alpha^{2^t\alpha}h(t)$ form a neighborhood around $t_{max}$.

Q.E.D.

**Lemma 36** Let $\alpha \in ]\alpha^*, \frac{1}{2}[$ and suppose condition (V) holds and some type $t$ has an inner solution $c(t') \leq 1 - t'$. Then any $t < t'$ has an inner solution $c(t') \leq 1 - t'$ too.

\(^{53}\)In inner solutions $\alpha^{2^t\alpha}g(c, t) = W$ and $g$ is increasing (so that $\alpha^{2^t\alpha}g(c, t)$ crosses the $W$-line from below), $g(c, t)$ can be either U-shaped or double-U-shaped. In the latter case $\alpha^{2^t\alpha}g(c, t)$ may cross the $W$-line from below twice. Condition (V) ensures that such a crossing from below happens at most once.
Proof: Lemma 8 implies that, in inner solutions, \( W = \alpha 2^{1-\alpha} g(c(t'),t') \). For any \( t < t' \) we have (by equation (26)) \( g(c,t) = g(c,t') \) at the range \( c \in [0,1-t'] \), hence \( W = \alpha 2^{1-\alpha} g(c(t'),t) \) where \( c(t') \leq 1-t' \) implies that \( c(t') < 1-t \). In other words, \( t \) has a local min point of \( L \) at \( c(t') \) as well. Condition (V) ensures that this local min point is unique.\(^{54} \) Furthermore, at an inner solution, \( \frac{dc}{dt} \) switches sign from negative to positive, hence \( g(c,t) \) is increasing (by Lemma 8). Condition (V) ensures that this increasing part of \( g(c,t) \) continues until \( c = 1+t \), implying that \( W < \alpha 2^{1-\alpha} h(t) \) hence \( \frac{dW}{dc} \big|_{c=1+t} > 0 \) and so the local min point is also the global min point.

\[ Q.E.D. \]

Proposition 10 For any \( \alpha \in \left[ \alpha^*, \frac{1}{2} \right] \) there is a spider for any \( W \) at the range of values that satisfy conditions (I)-(IV)

Proof: Lemma 34 established that the conditions imply a non-empty set of \( W \). Lemma 35 established that there exists a non-singleton neighborhood of \( t_{\text{max}} \) where types have inner solutions, while types outside that neighborhood choose \( c = 1 + t \) thus do not dissent. Condition (III) implies that type \( t = 0 \) is among these latter types, and condition (II) implies the same for all types with \( t \geq 0.34 \). Thus we know that the neighborhood of \( t_{\text{max}} \subset [0,1] \). Since \( P = 0 \) for all types who do not dissent, to get the pattern of a spider we need to verify that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases. First note that condition (III) implies that type \( t = 0 \) has no inner solution, and so condition (V) and Lemma 36 imply that any type \( t \) in the range of types with inner solutions has a solution at the range \( c \in [1-t,1+t] \). In this range, we know from Lemma 6 that the probability of dissent decreases if and only if \( \frac{dc}{dt} < 0 \). Equation (31) gives us the expression of \( \frac{dc}{dt} \),

\[
\frac{dc}{dt} = \frac{2}{3} c^3 - \frac{1}{6} t^3 - \frac{1}{2} t c^2 + \frac{1}{2} t^2 - \frac{1}{2} c^2 - \frac{1}{2} t + \frac{1}{6} + \alpha (tc^2 + c^2 - c^3),
\]

where equation (32) implies that the numerator is positive. The denominator is the expression that determines the sign of \( \frac{dc}{dt} \) (by Lemma 9) at the range \( c \in [1-t,1+t] \), hence is positive too given that at an inner solution \( \frac{dc}{dt} \) switches signs from negative to positive, which by Lemma 8 implies that \( g(c,t) \) must be increasing. It thus follows that \( \frac{dc}{dt} > 0 \). Furthermore

\[
\frac{dc}{dt} > 1 \iff \{\text{as established in Lemma 14}\}
\]

\[
G = c^2 \left( (3\alpha - 1) c - (2\alpha - 1) 3 (1+t) \right) - (1-t)^2 (1-t) + (1-\alpha) 3c > 0.
\]

To show the pattern of the spider it is thus sufficient to show that

\[
\frac{dG}{dt} = \frac{\partial G}{\partial t} + \frac{\partial G}{\partial c} \frac{dc}{dt} \geq 0,
\]

which, given that \( \frac{dc}{dt} > 0 \), holds if both partial derivatives, \( \frac{\partial G}{\partial t} \) and \( \frac{\partial G}{\partial c} \), are positive. Indeed,

\[
\frac{\partial G}{\partial t} = 3 \left[ (1-t)^2 + 2 (1-\alpha) c (1-t) - (2\alpha - 1) c^2 \right] > 0
\]

for \( \alpha \leq 1/2 \).

\[
\frac{\partial G}{\partial c} = 3 \left[ (3\alpha - 1) c^2 - 2 (2\alpha - 1) (1+t) c - (1-t)^2 (1-\alpha) \right]
\]

Fixing \( t \) and analyzing the behavior of \( \frac{\partial G}{\partial c} \) as a function of \( c \), we get

\[
\frac{\partial^2 G}{\partial c^2} = 6 \left[ (3\alpha - 1) c - (2\alpha - 1) (1+t) \right] = 0
\]

\[
\Rightarrow \ c = \frac{(2\alpha - 1) (1+t)}{(3\alpha - 1)}
\]

\(^{54}\)See proof of Lemma 35 for explanation.
If $\alpha \in [1/3, 1/2]$ then $\frac{\partial G}{\partial c}$ is U-shaped with min point at $c = \frac{(2\alpha - 1)(1 + t)}{(3\alpha - 1)} < 0$ (i.e., outside the permissible range), implying that at the range $c \in [1 - t, 1 + t]$ it reaches its min at $c = 1 - t$ where it equals

$$
\frac{\partial G}{\partial c} = 3 \left[(3\alpha - 1)(1 - t)^2 - 2(2\alpha - 1)(1 + t)(1 - t) - (1 - t)^2(1 - \alpha)\right] = -12(2\alpha - 1)(1 - t) t > 0
$$

Alternatively, if $\alpha < 1/3$, then $\frac{\partial G}{\partial c}$ is hill-shaped and

$$
c = \frac{1 - 2\alpha}{1 - 3\alpha} (1 + t) > 1 + t
$$

is a max point, implying again that at the range $c \in [1 - t, 1 + t]$ the function $\frac{\partial G}{\partial c}$ reaches its min at $c = 1 - t$ where it was just shown to be positive for any $\alpha < 1/2$. Thus, for any $\alpha \in ]\alpha^*, \frac{1}{2}[$, $G$ is increasing, implying that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases, as required for getting the pattern of a spider. Overall, we get a spider of the following kind: when $t$ goes from 0 to 1 the probability of dissent is first 0, then jumps to some strictly positive probability, then it either increases or decreases, or first increases and then decreases, and finally the probability of dissent decreases abruptly to 0 and stays there.

Q.E.D.

C.3.6 The special case of $\alpha = \frac{1}{3}$

Proposition 11 Suppose $\alpha = \frac{1}{3}$. Then there exists a non-empty set of $W$ such that there is a spider pattern.

Proof: We will prove a spider pattern of the following kind exists for some $W$: when $t$ goes from 0 to 1 the probability of dissent is first 0, then jumps to some strictly positive probability, then it either increases or decreases, or first increases and then decreases, and finally the probability of dissent decreases abruptly to 0 and stays there. By equation (16) we have

$$
\frac{dL}{dc} = 2M \left[c^{\frac{1}{2}} z^{-1/2} - W\right],
$$

and by 25

$$
\lim_{c \to 0} \frac{dL}{dc} = \frac{\sqrt{2}}{2} - W.
$$

Next, note that by (26)

$$
g(c, t) = \begin{cases} 
\left[1 - \frac{2}{3} c\right]^{-1/2} > 0 & \text{if } 0 < c < 1 - t \\
\left(-\frac{4}{3} c + \frac{1}{2} t + \frac{1}{2}\right) c^2 + \frac{1}{6} (1 - t)^3 & \text{if } 1 - t \leq c < 1 + t
\end{cases}
$$

$$
\frac{\partial g(c, t)}{\partial c} = \begin{cases} 
\frac{1}{3} \left[1 - \frac{2}{3} c\right]^{-3/2} & \text{if } 0 < c < 1 - t \\
\frac{1}{6} \left[1 - (1 - t)^3 - \frac{1}{3} c^3 + \frac{1}{2} t c^2 + \frac{1}{2} c^2\right]^{-3/2} & \text{if } 1 - t \leq c < 1 + t
\end{cases}
$$

We know the expression is positive since $\left[\frac{1}{6} (1 - t)^3 - \frac{1}{3} c^3 + \frac{1}{2} t c^2 + \frac{1}{2} c^2\right] = 2z \geq 0$ (since $z$ by definition (14) is a sum of two positive integrals). So $g(c, t)$ increases everywhere. If $W < \frac{\sqrt{2}}{2}$ so that $\lim_{c \to 0} \frac{dL}{dc} > 0$, then $L(c)$ is always increasing in $c$ (by Lemma 8 and since $\frac{\partial g(c, t)}{\partial c} > 0$) and so all types choose $c = 0$ hence dissent rate is monotonically increasing – no spider. If $W > \frac{\sqrt{2}}{2}$ so that $\lim_{c \to 0} \frac{dL}{dc} < 0$, then $L(c)$ is at least initially decreasing and so all types have either a unique (by Lemma 8 and since $\frac{\partial g(c, t)}{\partial c} > 0$) inner solution or a corner solution at $c = 1 + t$. Moreover, since $g(c, t)$ increases everywhere, we know from Lemma 8 that
type $t$ has an inner solution if and only if
\[ h(t) = g(1 + t, t) > \frac{W}{2^{1-n_{\alpha}}} = \sqrt{2}W \]

Using $h(t) = g(1 + t, t) = (1 + t) \left( \frac{1}{3} + t^2 \right)^{-1/2}$ (see Lemma 11), we get
\[
\frac{dh}{dt} = \left( \frac{1}{3} + t^2 \right)^{-1/2} - t(1 + t) \left( \frac{1}{3} + t^2 \right)^{-3/2} = ... = \left( \frac{1}{3} - t \right) \left( \frac{1}{3} + t^2 \right)^{-3/2}.
\]

So $h(t)$ is hill-shaped with a peak at $t_{\max} = 1/3$, where it equals $\frac{4}{3} \left( \frac{1}{3} \right)^{-1/2} = 2$. Thus, by setting $W < \sqrt{2}$ we can guarantee that at least types close to $t_{\max}$ have inner solutions. The hill-shape of $h(t)$ further implies that we can set $W$ to be strictly greater than
\[
\max \left\{ \frac{\sqrt{2}}{2} h(0), \frac{\sqrt{2}}{2} h(1) \right\} = \sqrt{3/2}
\]

yet strictly smaller than $\sqrt{2}$, so that, by corollary 1, types close to 0 or to 1 choose $c = 1 + t$ while there exists a non-empty range of types in-between with inner solutions. To get the pattern of a spider we need to verify that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases. As $W$ has been set such that type $t = 0$ has no inner solution, Lemma 10 implies that any type $t$ in the range of types with inner solutions has a solution at the range $c \in [1 - t, 1 + t]$. In this range, we know from Lemma 6 that the probability of dissent decreases if and only if $\frac{dc}{dt} > 1$. At the range $c \in [1 - t, 1 + t]$ (setting $g(c, t) = 0$ for inner solutions and applying the implicit function theorem) we have
\[
\frac{dc}{dt} = -\frac{\partial g}{\partial c} \cdot \frac{dt}{dc} = \left[ -\frac{1}{3}c^3 - \frac{1}{6}t^3 + \frac{1}{2}tc^2 + \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6} \right]^{-1/2} \cdot \left( -\frac{1}{2} \left[ -\frac{1}{2}t^2 + \frac{1}{2}c^2 + t - \frac{1}{3} \right] - \frac{1}{3}c^3 - \frac{1}{6}t^3 - \frac{1}{2}tc^2 + \frac{1}{2}t^2 - \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6} + \frac{1}{3} (tc^2 + c^2 - c^3) \right).
\]

This expression was shown before to be positive (from equation (32) we know the numerator is positive, and the denominator is positive (by Lemma 9) because it has the sign of $\frac{\partial g}{\partial c}$ which is positive), hence $\frac{dc}{dt} \geq 0$, implying the possibility of a spider.

The probability of dissent ($P(t)$) decreases in the range of types with inner solutions iff $\frac{dc}{dt} \geq 1$ (see Lemma 6).
\[
\frac{dc}{dt} = -\frac{1}{3}c^3 - \frac{1}{6}t^3 + \frac{1}{2}tc^2 + \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{6} + \frac{1}{3} (tc^2 + c^2 - c^3) \geq 1
\]
\[
\Leftrightarrow \quad c^3 \geq (1 - t)^2 \left[ 2(1 - t) + 3c \right]
\]

Let
\[
H(t, c) \equiv c^3 - (1 - t)^2 \left[ 2(1 - t) + 3c \right].
\]

Then
\[
\frac{\partial H(t, c)}{\partial t} = 6(1 - t) \left[ (1 - t) + c \right] \geq 0
\]
and
\[
\frac{\partial H(t, c)}{\partial c} = 3 \left[ c^2 - (1 - t)^2 \right] \geq 0,
\]
(since we established earlier in the proof that $c > 1 - t$ in inner solutions) and so
\[
\frac{dH}{dt} = \frac{\partial H}{\partial t} + \frac{\partial H}{\partial c} \frac{dc}{dt} \geq 0,
\]
implying that at the range of inner solutions the probability of dissent is either monotonic or first increases
and then decreases, as required for getting the pattern of a spider.

Q.E.D.

C.4 Proof of Proposition 4

Lemma 37 The functions \(\alpha 2^{1-\alpha} h(t_{\text{max}})\) and \(1.8 (2/9)^{\alpha}\) have one intersection point, denoted by \(\hat{\alpha}\), at the range \(\alpha \in ]0, \frac{1}{2}[\). Furthermore we have

\[
\begin{align*}
\alpha 2^{1-\alpha} h(t_{\text{max}}) < 1.8 (2/9)^{\alpha} & \quad \text{if} \quad \alpha \in (0, \hat{\alpha}), \\
\alpha 2^{1-\alpha} h(t_{\text{max}}) > 1.8 (2/9)^{\alpha} & \quad \text{if} \quad \alpha \in (\hat{\alpha}, \frac{1}{2})
\end{align*}
\]

Proof: Using \(h(t)\) as defined in Lemma 11 we get

\[
\begin{align*}
\alpha 2^{1-\alpha} h(t_{\text{max}}) &= 1.8 (2/9)^{\alpha} \iff \alpha 2^{1-\alpha} (1 + t_{\text{max}}) \left(\frac{3}{4} + \frac{9}{\sqrt{2}} t_{\text{max}}\right)^{\alpha-1} = \frac{9}{5} (\frac{2}{9})^{\alpha} \\
& \iff \alpha (1 + t_{\text{max}}) \left(\frac{3}{4} + \frac{9}{\sqrt{2}} t_{\text{max}}\right)^{\alpha-1} = \frac{2}{5} (\frac{\alpha}{1 + t_{\text{max}}})^{\alpha-1} \\
& \iff \frac{5}{2} \left(\frac{3}{4} + \frac{9}{\sqrt{2}} t_{\text{max}}\right)^{\alpha} = \frac{3}{4} \left(\frac{1 + 3t_{\text{max}}^2}{\alpha(1 + t_{\text{max}})}\right)^{\alpha-1} \\
& \iff \frac{10}{3} \left(\frac{3}{4} + \frac{9}{\sqrt{2}} t_{\text{max}}\right)^{\alpha} = \frac{1 + 3t_{\text{max}}^2}{\alpha(1 + t_{\text{max}})}
\end{align*}
\]

The RHS is the inverse of \(\Omega(t_{\text{max}}(\alpha), \alpha)\) which is defined in Lemma 24, where it is also shown to be increasing in \(\alpha\), hence the RHS decreases in \(\alpha\).

Analyzing the LHS. Let

\[
\xi(t_{\text{max}}(\alpha), \alpha) = \left(\frac{3}{4} + \frac{9}{\sqrt{2}} t_{\text{max}}\right)^{\alpha}.
\]

In Lemma 22 we showed that \(t_{\text{max}}\) increases in \(\alpha\). To show that \(\xi(t_{\text{max}}(\alpha), \alpha)\) is increasing in \(\alpha\) it is therefore sufficient to show that the two partial derivatives of \(\xi(t_{\text{max}}(\alpha), \alpha)\) with respect to its two arguments, \(t_{\text{max}}(\alpha)\) and \(\alpha\), are both positive.

\[
\frac{\partial \xi(t_{\text{max}}, \alpha)}{\partial \alpha} = \left(\frac{3}{4} + \frac{9}{\sqrt{2}} t_{\text{max}}\right)^{\alpha} \ln \left(\frac{3}{4} + \frac{9}{\sqrt{2}} t_{\text{max}}\right) > 0
\]

where the inequality follows since the smallest possible \(t_{\text{max}}\) is \(\frac{5}{4} \sqrt{2} \sqrt{3} - 1\) (to see this recall that \(t_{\text{max}}\) increases \(\alpha\) and hence plug in \(\alpha = 0\) in equation 29) which implies \(\frac{3}{4} + \frac{9}{\sqrt{2}} t_{\text{max}} > 1\).

\[
\frac{\partial \xi(t_{\text{max}}, \alpha)}{\partial t_{\text{max}}} = 4.5t_{\text{max}} \alpha \left(\frac{3}{4} + \frac{9}{\sqrt{2}} t_{\text{max}}\right)^{\alpha-1} > 0
\]

Thus, the LHS increases in \(\alpha\) while the RHS decreases in \(\alpha\), implying that there is a unique intersection point \(\hat{\alpha}\). To find which of the functions \(\alpha 2^{1-\alpha} h(t_{\text{max}})\) and \(1.8 (2/9)^{\alpha}\) is larger below and above \(\hat{\alpha}\), we can plug in specific values of \(\alpha\). When \(\alpha = 0\) the former function goes to 0 (recall that \(h\) is bounded) while the latter equals 1.8 hence is larger. When \(\alpha \to \frac{1}{2}\) we know from Lemma 23 that \(t_{\text{max}}\) approaches \(\frac{1}{2}\) hence the former function approaches

\[
\frac{1}{2} 2^{1/2} \frac{4}{3} \left(\frac{4}{9}\right)^{-1/2} = \frac{4}{3} \left(\frac{9}{8}\right)^{1/2} = \sqrt{2}
\]

while the latter equals \(1.8 (2/9)^{1/2} = \frac{3}{5} \sqrt{2}\) hence is smaller. This also implies \(\hat{\alpha} \in ]0, 1/2[\). The value of \(\hat{\alpha}\) can be numerically calculated to be \(\approx 0.295\).

Q.E.D.
C.4.1 Proof of the proposition

We prove the proposition for $t \geq 0$. Equivalent statements can be made for $t \leq 0$.

The average vote of type $t$, denoted by $J(t) \equiv E_{V(t)}[I(t; v)]$, is a weighted sum of her own type, with probability $P_m + P(t)$, and the median of the judges’ panel otherwise.

Let

$$J(t) = [P_m + P(t)] t + \left[ \int_{t-c}^{t} 2vF(v) f(v) dv + \int_{t}^{t+c} 2v (1 - F(v)) f(v) dv \right]$$

We prove the proposition for $c \geq 1 + t$

$$Z = \frac{1}{2} \left[ \int_{t-1}^{t} v (1 + v) dv + \int_{t}^{1} v (1 - v) dv \right] = \frac{1}{2} \left[ \int_{t-1}^{t} (v + v^2) dv + \int_{t}^{1} (v - v^2) dv \right] = ...$$

$$= \frac{t^3}{3}$$

(41) \quad \Rightarrow J(t) = \frac{1}{2} \left( 1 - t^2 \right) t + \frac{t^3}{3} = \frac{1}{2} t - \frac{1}{6} t^3$$

$1 - t \leq c < 1 + t$

$$Z = \int_{t-c}^{t} \frac{v + v}{2} dv + \int_{t}^{1} \frac{v - v}{2} dv$$

$$= \frac{1}{2} \left[ \int_{t-1}^{t} (v + v^2) dv + \int_{t}^{1} (v - v^2) dv \right] = ...$$

$$= \frac{t^3}{6} + \frac{1}{12} \left( 1 - t^2 \right) + \frac{1}{2} \left( 1 + t - c \right)^2 \left( t + \frac{t^3}{6} + \frac{1}{12} t^2 - \frac{1}{4} t c - \frac{1}{2} c \right)$$

$$\Rightarrow J(t) = \left( \frac{1}{2} \left( 1 - t^2 \right) + \frac{1}{2} \left( 1 + t - c \right)^2 \right) \left( t + \frac{t^3}{6} + \frac{1}{12} t^2 - \frac{1}{4} t c - \frac{1}{2} c \right)$$

$$= ... = t + \frac{1}{12} (1 - t)^3 + c^2 \left( \frac{1}{6} c - \frac{1}{4} t - \frac{1}{4} \right)$$

$c < 1 - t$

$$Z = \int_{t-c}^{t} 2vF(v) f(v) dv + \int_{t}^{t+c} 2v (1 - F(v)) f(v) dv$$

$$= \frac{1}{2} \left[ \int_{t-1}^{t} (v + v^2) dv + \int_{t}^{1} (v - v^2) dv \right] = ...$$

$$= tc (1 - c)$$
(42) \[ J(t) = \left\{ \begin{array}{ll}
\frac{1}{2} (1 - t^2) + \left[ \frac{1}{2} (1 + t - c) \right]^2 + \left[ \frac{1}{2} (1 - t - c) \right]^2 & \text{if } t \leq \tilde{t} \\
\frac{1}{2} t - \frac{1}{6} t^3 & \text{if } t > \tilde{t}
\end{array} \right. \]

Note: the corner solution of \( c = 0 \) implies that the judge never signs \( v \neq t \) hence \( J(t) = t \). The corner solution of never dissenting \( (c = 1 + t) \) implies that the judge always votes according to the median of the panel, which equals \( \frac{1}{2} t - \frac{1}{6} t^3 \).

An equivalent statement of the proposition is that \( \text{argmax}(J(t)) \notin \{0, 1\} \). In Proposition 9 we showed that, for \( \alpha < \alpha^* \) (where \( 0.3 \approx \alpha^* > \tilde{\alpha} \approx 0.295 \)), there exists a unique switching point at the range \([0, 1]\) such that types below it never dissent whenever they are not the median of their panel \( (c = 0) \) while types above it never dissent \( (c = 1 + t) \) under the conditions

(43) \[ W < 2 (1/6)^\alpha \]
(44) \[ (2/3)^\alpha < W \]
(45) \[ \alpha 2^{1-\alpha} h(t_{\text{max}}) < W \]

(see (29) for a definition of \( t_{\text{max}} \)) and that these conditions hold for a non-empty set of \( W \). This dissent pattern implies (by (42) and (41)) that there exists some \( \tilde{t} < 1 \) such that

\[ J(t) = \left\{ \begin{array}{ll}
t & \text{if } t \leq \tilde{t} \\
\frac{1}{2} t - \frac{1}{6} t^3 & \text{if } t > \tilde{t}
\end{array} \right. \]

and hence (since \( t \leq 1 \)) that \( J(t) \) is first increasing for \( t \leq \tilde{t} \), then drops sharply at \( \tilde{t} \), and finally increases again for \( t > \tilde{t} \). To show that \( \text{argmax}(J(t)) \notin \{0, 1\} \) it is thus necessary and sufficient that

\[ J(\tilde{t}) > J(1) \iff \tilde{t} < \frac{1}{3}, \]

hence that the type \( t = 1/3 \) prefers \( c = 0 \) over \( c = 1 + t \):

\[ L(c = 0; t = 1/3) < L(c = 1 + 1/3; t = 1/3) \iff \]

using (12) with a uniform distribution, (15) and (18)

\[ W \left( \frac{1 + t^2}{2} + \left[ 1 - \frac{1 + t}{2} \right]^2 \right) < \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^\alpha \iff \]

(46) \[ W < \frac{9}{5} \left( \frac{2}{9} \right)^\alpha. \]

First we note that \( \frac{9}{5} \left( \frac{2}{9} \right)^\alpha < 2 (1/6)^\alpha \) for any \( \alpha < \tilde{\alpha} \), hence we must show that condition (46) holds along with conditions (44) and (45), i.e., that the set of \( W \) fulfilling the conditions is non-empty. Conditions (46) and (44) yield a non-empty set if

\[ (2/3)^\alpha < \frac{9}{5} \left( \frac{2}{9} \right)^\alpha \iff \]

\[ \alpha < \frac{\ln \left( \frac{2}{9} \right)}{\ln \left( \frac{5}{9} \right)} \approx 0.54, \]

which is fulfilled since \( \tilde{\alpha} \approx 0.295 \). Conditions (46) and (45) hold together for any \( \alpha < \tilde{\alpha} \) by Lemma 37.

**C.5 Proof of Proposition 5**

We prove the proposition for \( t \geq 0 \). Equivalent statements can be made for \( t \leq 0 \). In Proposition 9 we showed that, for \( \alpha < \alpha^* \) (where \( 0.3 \approx \alpha^* > \tilde{\alpha} \approx 0.295 \)), there exists a unique switching point at the range \([0, 1]\) such that types below it never dissent whenever they are not the median of their panel \( (c = 0) \) while types above it never dissent \( (c = 1 + t) \) under a non-empty set of \( W \).

Proof of part (i): When \( W = 0 \) the objective function for all types becomes \( \min D \) which clearly is achieved by dissenting whenever not being median \( (c = 0) \). Using (12) with a uniform distribution and \( c = 0 \) we get \( P(t) = \left[ \frac{1 + t}{2} \right]^2 + \left[ 1 - \frac{1 + t}{2} \right]^2 = \frac{t^2 + 1}{2} \) which clearly increases in \( t \).

Proof of part (ii): The dissent pattern just described implies that \( \tilde{t} = \text{argmax} P(t) \) equals the largest \( t \) that dissents when not being median. Since under these conditions the alternative is to never dissent, this type must be indifferent between these two options: \( L(c = 1 + t; \tilde{t}) = L(c = 0; \tilde{t}) \). Using (12) with a uniform
distribution together with (15) and (18) yields:

\[
\left( \frac{1}{2} \left[ \tilde{t}^2 + \frac{1}{3} \right] \right)^\alpha = W \left( \left[ \frac{1+t}{2} \right]^2 + \left[ 1 - \frac{1+t}{2} \right]^2 \right) = W \left( \frac{\tilde{t}^2 + 1}{2} \right),
\]

hence

\[
dW \over dt = \left( \frac{1}{2} \left[ \tilde{t}^2 + \frac{1}{3} \right] \right)^\alpha \over \left( \tilde{t}^2 + 1 \right) \over \alpha = \left( \frac{1}{2} \right)^{\alpha-1} \frac{2\tilde{t}}{\left( \tilde{t}^2 + 1 \right)^{\alpha}} \left[ \alpha - \left( \tilde{t}^2 + \frac{1}{3} \right) \right].
\]

It thus follows that for any \( \alpha < \tilde{\alpha} \approx 0.295 \) we have \( \frac{dW}{dt} < 0 \), hence \( \tilde{t} \) decreases in \( W \).

## D Appendix: Alternative Explanations

How can these puzzling empirical observations be explained? In our quest to understand the observations we have considered 17 separate explanations representing the “usual suspects” within research on judicial and group decision making.\(^{55}\) After analyzing each explanation separately we are left with two separate explanations (or models) that can explain all three facts and do not violate some simple auxiliary empirical observations. One model is about judges using dissents instrumentally to awake the interest of the supreme court who may then reverse the verdict and the other model is about judges using dissents to morally or emotionally distance themselves from majority opinions they do not agree with ideologically. We take these two models and derive additional empirical predictions by which they differ and test them empirically (see Appendix D.2.16).\(^{56}\) These tests favor one of the models (the moral/emotional distancing), which is hence presented in the main text, while the other model (instrumental dissents) is presented with great detail in Appendix D.3. We wish to emphasize that, while we have considered a large number of explanations and mechanisms, we cannot conclude that our main contender is the only or the true explanation for the empirical patterns observed. Just like when matching any theory to data, regardless of how many theories one has considered, one cannot exclude the possibility that yet another theory which is consistent with the investigated empirical pattern exists.

Let us briefly describe some of the other 15 explanations (for more details see next Appendix D.2). These explanations can be divided into a few categories (many explanations fit under more than one category and indeed most fail for two, three or even four reasons). One category consists of mechanisms that seem reasonable but ultimately fail to explain the main Fact 3, part of this fact or some of the other facts. It should be noted that our aim here is not just to explain why extremists dissent seldom but the full hill-shaped pattern of dissent when going from centrist to extreme and also to fit the other two facts. Mechanisms that fit into this category are, for instance, that extremists wish to signal that they are not extreme (D.2.1), that moderates wish to signal that they are extreme to please some extreme electorate (D.2.2) or that it is considered unreasonable to dissent against majority opinions that are close to one’s own preferences (D.2.3).

Another category consists of explanations that are inconsistent with other empirical observations. In particular, any explanation that suggests that extreme judges do not dissent because they manage to pull the ideological color of the opinion in their direction is inconsistent with auxiliary data. This includes, for instance, mechanisms saying that extremists, by bargaining well within the panel (D.2.4) or by getting co-panelists to agree with them in later cases (D.2.5) or by including facts that tie the hands of the supreme court (D.2.6), manage to affect the opinion disproportionally. Such explanations are inconsistent with observations presented in Appendix Figure 7 showing that, at both ends of the ideological spectrum, there is a negative correlation between a judge’s ideology and the majority opinion. Another refuted mechanism is that moderates dissent more than the others since they are spokesmen representing the party line hence are more vocal (D.2.7).

Yet another category consists of mechanisms that, upon closer inspection, either resemble or necessitate the mechanism of our main-contender theory in order to be able to explain the empirical facts. These mechanisms include that the peer pressure when dissenting is higher if the minority opinion is extreme

\(^{55}\) We are grateful to the many researchers, seminar audiences and practitioners who have suggested explanations to consider.

\(^{56}\) We use wartime and retired judges for these auxiliary checks.
(D.2.8), that extreme judges face a higher risk of impeachment (D.2.9) or that extreme minority opinions get fewer citations hence are not worth the effort of writing (D.2.10).

Finally, we also consider potential threats to our empirical variables or results. Many of these are also described as part of the robustness checks in the previous (empirical) section, but in the appendix (D.2.11-D.2.15) we discuss further grounds for why some empirical concerns are not likely to be the reason for our empirical patterns.

### D.1 Simulating an alternative functional form

This appendix aims to show that the spider-pattern (Fact 3) can be explained by our main theoretical model (Section 3) when replacing the ideological cost function (inner discomfort) in equation (2) by the following logistic cost function

\[
D(x) = \frac{1}{1 + e^{-kx}}.
\]

This function is convex for small \( x \) and concave for large \( x \). Appendix Figure 11 presents simulation results when using this functional form with \( k = 7 \) and \( W = 1.1 \) in the main model.\(^{57}\) As can be seen, the spider pattern appears with such a function as well (upper panel). The lower panel shows the cutoff \( c(t) \). Running the same simulation but varying \( k \) and \( W \) in a large number of ways, suggests the dissent rate is either constant at zero, increasing in \( |t| \), decreasing in \( |t| \), or indeed spider-shaped.

**APPENDIX FIGURE 11.— Simulation of a logistic \( D \) function**

![Logistic D-function](image)

Notes: Upper schedule: Dissent rate per type. Lower schedule: cutoff per type. The simulation uses the functional form in 47 with parameter \( k = 7 \) and peer-pressure parameter \( W = 1.1 \).

\(^{57}\)The program code is available upon request.
D.2 Refuting of alternative theories

This section will discuss in brief a number of alternative theories and why they cannot be the explanation for the patterns we find empirically. Among these alternative explanations only one can explain our empirical patterns. This is the instrumental model mentioned in the introduction. We treat this model (judges use dissent as an instrument in trying to reverse the majority opinion using the supreme court) in great detail at the end of the list and in the next section by developing a proper theoretical model (Section D.3), deriving predictions by which it differs from our main model (of peer pressure with a concave cost of bliss-point deviations) and testing these predictions against each other (Section D.2.16). We find that the instrumental model is refuted by the data.

The other stories can be divided into four categories (though many stories can fit under more than one category). The first group (see D.2.1-D.2.3) consists of mechanisms that seem reasonable but ultimately fail to explain the main Fact 3, part of this fact or some of the other facts. The second category (D.2.4-D.2.7) consists of explanations that are inconsistent with some other aspects of the data. The third group (D.2.8-D.2.10) consists of theories that can explain the empirical patterns only if judges have a concave cost of bliss point deviations, hence are basically variants of our main model. The final category (D.2.11-D.2.15) consists of potential threats to our empirical variables or results. Many of these are rebutted by the robustness checks in the empirical section (Section 2) but here we discuss further grounds for why some empirical concerns are not likely to be the reason for our empirical patterns. While this, of course, is not a final proof that our main model is correct, the long list of alternative explanations is meant to show that the “usual-suspect” mechanisms can be refuted as explanations for our observations (which of course does not mean that these mechanisms are not valid in general).

D.2.1 Signaling for conformity.

**Mechanism:** A model where dissent is a signal of being an extremist (which is supposedly a bad thing) and where peer pressure is applied to judges based on the type they are perceived to be (in expectation in equilibrium) would supposedly imply extreme judges dissent less. **Refutation:** Bernheim (1994) shows this kind of model will produce dissent (non-conformity in his model) that is increasing in the extremeness of the type, hence cannot explain why extreme judges dissent less.

D.2.2 Signaling for extremeness.

**Mechanism:** Judges aim to please voters or party members or those who selected them. Moderately ideological judges need to signal their ideological belonging while everyone knows already that extremists are extreme hence they do not need to use dissent to signal ideology. **Refutation:** i) This does not explain why centrists dissent less than moderately ideological judges. Alternatively, it assumes that centrists, unlike moderates, do not want to signal they are extreme. ii) As explained under D.2.11 our results are driven by ideology relative to Center of Judge Pool and not by ideology per se and they hold also when using the alternative score based on party of appointing President (where a judge cannot be more “extreme”, only more or less in minority).

D.2.3 Extreme judges dissenting on moderate cases are viewed as unreasonable.

**Mechanism:** A judge who dissents against an opinion that is close to her type is viewed as unreasonable. Since there are more moderate cases than there are extreme cases, the extreme judge dissents less than moderate judges do. **Refutation:** Under this mechanism, moderate judges would also be viewed as unreasonable when dissenting on slightly unfavorable cases. Considering this, the logic of the mechanism is incorrect since it disregards the opinions on the whole political spectrum. Considering all opinions (on the same side and the opposite side of the political spectrum), an extreme judge will statistically need to more often consider signing or dissenting against very unfavorable opinions than a moderate does and would therefore dissent more under this mechanism.
D.2.4 Bargaining.

**Mechanism:** Through the bargaining process, extreme judges are more successful than moderately ideological judges in pulling the majority opinion in their direction hence have less reasons to dissent. **Refutation:** This description is inconsistent with Appendix Figure 7 which, on the contrary, shows that at large distances from the center of the pool of judges there is a negative correlation between judges' ideology and the ideology of the majority opinion. It is also very implausible when considering the alternative ideological score (party of appointing President), because it would suggest that a Republican-nominated judge would have more bargaining power the larger is the share of Democrat-nominated judges in the pool.

D.2.5 Log-rolling.

**Mechanism:** An extreme judge joins an unfavorable opinion in return for getting the other judges to agree on the extremist’s view in the next case. **Refutation:** This description is inconsistent with Appendix Figure 7 which, on the contrary, shows that at large distances from the center of the pool of judges there is a negative correlation between judges' ideology and the ideology of the majority opinion. If extreme judges refrain from dissenting in return for getting to determine future opinions, then this influence should take away that negative correlation.

D.2.6 Tie the hands of the supreme court.

**Mechanism:** Extreme judges join the majority opinion in order to be able to add facts to it and thereby tie the hands of the supreme court. **Refutation:** i) If extremists do this, then moderates may want to do this too. ii) The facts added should affect the color of the opinion as the sample becomes large but this is inconsistent with Appendix Figure 7 which, on the contrary, shows that at large distances from the center of the pool of judges there is a negative correlation between judges’ ideology and the ideology of the majority opinion.

D.2.7 Moderates as official party spokesmen.

**Mechanism:** Moderate judges are the modal judges in their corresponding (liberal or conservative) group and are very close to their party leadership, while centrist and extremists are located at the tails of their party of affiliation. The moderates thus tend to signal the official party line and act accordingly by being vocal (i.e. dissent more and signal the party position on the issue). **Refutation:** i) Figure 2 shows a clear monotonic relationship between ideology of voting and a judge’s ideology score that is not demeaned by the Center of Judge Pool. This is an indication that, abstracting from the censoring effect of the environment, extreme judges adopt a more ideological line than moderate judges. ii) The judge fixed effects exercise in Section 2.3 shows that extreme judges are dissenting more when the pool changes so they become “moderates”. So it cannot be that one type of judge is designated to be the main defender of the party line since (by the judge fixed effects exercise) this person changes behavior when the environment changes. iii) Our results are robust to using party of appointing President as ideology score. Here, all judges in the pool who are from the same party get the same score, and “moderate” judges are simply (all) the judges who happen to sit in a well-balanced pool.

D.2.8 Peer pressure increases in extremeness of the dissenting opinion.

**Mechanism:** Extreme minority opinions (dissents or concurrences) are sanctioned by other judges in the pool more heavily than moderate minority opinions are. Hence, an extremist judge will find it harder to express her true view by dissenting. **Refutation:** Extreme judges can always imitate moderate ideologists, thus dissent at least as much as (instead of less than) the moderates do. To make the extremists prefer to dissent strictly less than the moderates, judges must have a concave personal cost of bliss-point deviations also in this mechanism. This way, moderate judges, who are under small peer pressure, would choose to
dissent and avoid the personal cost, while extreme judges, who are under severe social pressure, would pay the full personal cost and completely refrain from dissenting. It is thus clear that this mechanism is only a variant of our main model in the body of the paper. We prefer our own model because it does not require the additional assumption of increasing social pressure and because we believe the personal cost is likely to apply to the number of dissents at least as much as it applies to the size of dissent.

D.2.9 Risk of impeachment.

**Mechanism:** The risk of impeachment, given dissent, could be increasing in extremeness of the dissenting opinion. **Refutation:** Impeachment is essentially a form of pressure, hence the refutation in mechanism D.2.8 applies.

D.2.10 Extreme dissents get fewer citations

**Mechanism:** Minority opinions that are very extreme get fewer citations hence are not worth the effort of writing. **Refutation:** We can see two reasons why a judge may want citations. The first is the value of simply being cited many times. Such a motive is devoid of ideology hence is silent on Fact 2. The second motive is ideological, where the purpose of the dissent is to improve the total ideological image of the particular case by moving the attention (citations) from a (subjectively) less favorable majority opinion to a more favorable minority opinion. To see why this cannot explain the observations, note first that while it may be true that extreme opinions are cited less, an extremist can always write less extreme dissents thereby increasing her citations (essentially emulating the moderate). Suppose now that a moderate chooses to write a moderate dissent while the extremist does not (a necessary, but not sufficient, condition for the moderate’s dissent rate to be higher than the extremist’s dissent rate). Then it must be that the moderate thinks the ideological utility increases substantially (at least sufficiently to be worth the writing effort) when moving attention from, say, a centrist opinion to a moderate one, but that the extremist does not think there is a large utility difference when doing precisely the same. Following the same intuition of our main model, this means that the judge’s perceived cost of an unfavorable opinion getting cited has to be concave in the ideological distance of the opinion from the judge’s bliss point, which makes it clear that this mechanism is essentially a variant of our main model where the dissent pattern is driven by concave ideological costs.

D.2.11 Unobserved heterogeneity among judges.

**Mechanism:** Extreme judges have some personal characteristics that are different from the others, and these characteristics make them dissent less. **Refutation:** i) As mentioned in the main text our results are driven by ideology relative to Center of Judge Pool and not by ideology per se (as evident by using judge fixed effects in Section 2.3; the attenuation of Fact 3 and disappearance of the S-shaped voting pattern presented in A.4.3 when using raw ideology score; and the alternative score based on party of appointing President where a judge’s ideology score only depends on her own party and the number of judges of the other party). That is, relative position is not a personal attribute, and when considering extremism per se the empirical patterns are weak to non-existent. ii) Our results are robust to using controls for judge personal characteristics (Table A.10). iii) The judge characteristics driving the result have to be positively correlated with ideology for extreme judges but negatively correlated for less extreme judges (since Fact 3 shows non-monotonicity).

D.2.12 Score bias.

**Mechanism:** Our ideology score is constructed by the voting behavior of the appointing President and home state senators (see Section A). This score may be flawed if extreme Presidents appoint non-extreme judges to show they are non–biased. **Refutation:** i) Figure 2 shows an almost linear relationship between ideology of voting and judge’s ideology score that is not demeaned by the Center of Judge Pool. This is an indication that the scoring system we use is indeed a good proxy of judges’ ideology. ii) The results are robust
to the alternative ideology score based on party of appointing President, i.e., a score that is not dependent on how extreme the President is. iii) This mechanism cannot explain non-monotonicity unless one assumes that moderately ideological Presidents for some reason do not at all need to signal that they are not biased.

D.2.13 Results driven by outliers.

**Mechanism:** The result of low dissent rate for extreme judges could be driven by outliers **Refutation:** i) The pattern is robust to using concurrences rather than dissents (see Appendix Figure 9). ii) There is a lower bound of dissent at zero, hence single outliers cannot pull down the average very much. iii) We test robustness for outlier Circuits: Appendix Table A.9 shows that the results are robust to dropping one Circuit at a time.

D.2.14 Random opinion writing.

**Mechanism:** The opinion is written by a random judge who gets to decide the content, hence extreme judges are not under-represented among the judges who set the court’s opinion. **Refutation:** i) This cannot explain the spider. ii) It is false since we show that the median determines the ideological color of the opinion (Fact 1).

D.2.15 Outlier circuits.

**Mechanism:** The results are driven by a few circuits with many judges since only large circuits would have large enough variation in judges to include extremists **Refutation:** i) This cannot explain the spider. ii) Appendix Table A.9 shows that the results are robust to dropping one Circuit at a time.

D.2.16 Empirical testing of our model against an alternative model that can explain the spider pattern

This section describes an alternative model that may explain the main stylized fact (Fact 3), derives two predictions from this model that differ from the two predictions outlined in Section B.2, and tests the different predictions against each other empirically.

The alternative theory is one where a judge dissent, at a cost of collegial pressure, in the hope that the U.S. Supreme Court (SCOTUS) will use this as a signal to review the case and overturn the (binary) verdict. Majority voting within panels implies also here that the median judge decides the verdict for the panel. In Appendix D.3 we develop a simple model capturing this mechanism (the model is inspired by the model in Beim et al. 2014). The intuition for why this alternative SCOTUS model can produce a spider-shape dissent pattern is as follows. A judge compares, case by case, the cost of dissent ($W$) with how wrongful she thinks a certain verdict is. This means that two prerequisites need to be in place for a judge to dissent: i) she needs to think that the verdict is sufficiently bad to warrant the cost of dissent and ii) she needs to have the Supreme Court on her side as otherwise the verdict will not be overturned anyway. Here, centrists on the one hand usually have the Supreme Court on their side but on the other hand, often being the median, seldom encounter verdicts that are too far from what they think is right. Hence they rarely dissent. Conversely, extremists often dislike the verdict sufficiently to dissent but rarely have the Supreme Court on their side, hence dissent seldom too. Finally, moderately ideological judges may have a larger set of cases where they both sufficiently oppose the verdict and have the Supreme Court on their side. In the appendix we show that this may create a spider pattern of dissent. Two additional predictions (the equivalent of Proposition 5) can be derived from the SCOTUS model.

**Proposition 12** Consider the SCOTUS model. Then: (i) if $W = 0$, $P(t)$ is monotonically decreasing in $|t|$; and (ii) $\argmax_{|t|} P(|t|)$ is increasing in $W$.

58% of the Circuit Court decisions are appealed to the Supreme Court, of which 30% are affirmed.
Proof: See Appendix D.3.

Q.E.D.

Prediction (i) of the SCOTUS model says that, as the collegial cost of dissent (W) approaches zero, the dissent rate becomes a decreasing function of judge’s extremeness. This is intuitive since, when the collegial pressure is low, the only factor that determines whether a judge dissents is whether she has the Supreme Court on her side (because there is no collegial pressure against dissenting). This means that centrist judges will dissent very often. Furthermore, the more extreme a judge is, the less likely it is that her preferences will be aligned with the Supreme Court, which means that the dissent rate falls. This way, the prediction of the SCOTUS model is opposite to the prediction of our main model (with a concave ideological cost) where, as the collegial pressure goes to 0, the dissent rate increases with how extreme the judge is (Proposition 5 part i).

Prediction (ii) of Proposition 12 refers to the consequences of an increase in the cost of dissent. In the SCOTUS model, the judge at the peak of the spider pattern is the one for whom the threshold cutoff for dissent, as determined by the cost of dissent, exactly equals her ideological distance to the Supreme Court, implying that she dissents against any verdict that both she and the Supreme Court view as biased to the “wrong” side. Judges who are more extreme dissent in these cases as well, but in total they are predicted to dissent less because, compared to the judge at the peak, they have less objection to verdicts that manifest extreme ideology on their side of the ideological spectrum, yet do not have the Supreme Court on their side for overturning verdicts they consider to be too moderate. If the cost of dissent increases, judges have to censor themselves more, hence the cutoff for dissent is larger, implying that a judge has to be more ideologically extreme to be at the peak of the spider pattern. Therefore, the prediction of the SCOTUS model is that, as the cost of dissent increases, the spider peak would move outwards. In the main model we had the opposite prediction: an increase in the cost of dissent would have pushed the peak of the spider inwards (Proposition 5 part ii).

Testing predictions for $W \to 0$

When testing these opposite predictions (part i of propositions 5 and 12), it is important to note that in the main model it is the Distance to Center of Judge Pool that measures the extremeness, while in the SCOTUS model it is the Distance to SCOTUS that measures a judge’s extremeness. To get a measure of the Distance to SCOTUS, we use the Martin Quinn Supreme Court scores to put the judicial scores of the Circuit Court and the Supreme Court on the same metric (Martin and Quinn 2002).\(^{59}\)

To test the predictions, we use retired status of a judge as a proxy for a very low cost of dissent. The motivation for this is as follows. Firstly, judges who have retired take a reduced caseload, hence have more time to write dissents. Secondly, they arguably have lower collegial pressure from colleagues or are less sensitive to such pressure. We first verify that judges who have retired have a discontinuous drop in caseload from about 100 per year to 30 per year (Figure 12) and that caseload continues to decline gradually thereafter. Next, we show that retired judges dissent more, discontinuously at the year of retirement (Figure 13), and verify that the increase in dissents is not due to age. In fact, older judges are less likely to dissent, which also explains the decline in dissent before retirement. Figure 13 visualizes the following regression.\(^{60}\)

$$\text{Dissent Rate}_{it} = a + b \times 1(\text{Years after Retirement} \geq 0)_{it} + c \times \text{Years after Retirement}_{it} + d \times \text{Years after Appointment}_{it} + \nu_{it}$$

for judge $i$ and year $t$.

\(^{59}\)A histogram of Score Relative to Supreme Court is presented in the right panel of Appendix Figure 4.

\(^{60}\)We have verified that age and experience vary smoothly around the retirement decision. .
Appendix Figure 12.— Caseload and Years from Retirement

Notes: Each dot represents the average caseload of judges with the same number of years relative to retirement. Data on cases comes from OpenJurist (1950-2007).

Appendix Figure 13.— Dissent or Concurrence and Years from Retirement vs. Age

Notes: Each dot represents the average sum of dissent rate and concurrence rate for judges with the same number of years relative to retirement (left panel) or the same age (right panel) in a Circuit-year. The average is a weighted average to account for the number of times the judge actually appeared on cases in that Circuit-year. Data on cases comes from OpenJurist (1950-2007).
To test the opposing prediction (i) of Propositions 5 and 12 we run the regression in equation (3), limiting the sample to retired judges only.

As can be seen in Table A.11, for retired judges, the rate at which judges dissent or concur is positively correlated with Distance to Center of Judge Pool (columns 1 and 5), which supports our main model. The rate at which judges dissent or concur is also positively correlated with Distance to SCOTUS (columns 3 and 7), which goes against the prediction of the SCOTUS model. Note also that the spider pattern disappears (columns 2, 4, 6 and 8), as predicted in Proposition 5. In total, these results provide support for the main model and go against the prediction of the SCOTUS model.

**APPENDIX TABLE A.11**

**Dissent or Concurrence and Ideology Score among Retired Judges**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
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<td>Distance to Center of Judge Pool</td>
<td>0.0323***</td>
<td>0.000569</td>
<td>0.0337***</td>
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<td></td>
<td>(0.00810)</td>
<td>(0.0244)</td>
<td>(0.00814)</td>
<td>(0.0250)</td>
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<td></td>
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<td></td>
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<td>Distance to Center of Judge Pool(^2)</td>
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<td></td>
<td>0.0365</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0418)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance to Supreme Court</td>
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<td>0.0488*</td>
<td>0.0253***</td>
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<tr>
<td></td>
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<td>Distance to Supreme Court(^2)</td>
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<td>Y</td>
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<tr>
<td>Control for Age and Experience</td>
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<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>Control for Age and Experience</td>
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<td>Y</td>
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<td>Y</td>
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</tr>
<tr>
<td>Control for Age and Experience</td>
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<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
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<tr>
<td>Control for Age and Experience</td>
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<td>Y</td>
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<td>3353</td>
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</tr>
<tr>
<td>R-sq</td>
<td>0.090</td>
<td>0.091</td>
<td>0.081</td>
<td>0.082</td>
<td>0.094</td>
<td>0.094</td>
<td>0.087</td>
<td>0.088</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Absolute values of the distance to the center of the judge pool or the Supreme Court are the main independent variables. The dependent variable is the judge’s sum of dissent rate and concurrence rate in a Circuit-year. Fixed effects include year and circuit. Observations are weighted by the number of votes cast by the judge in the Circuit-year. Columns 1-4 use the same set of controls as in Table A.3.

**Testing predictions about the most dissenting judge**

Our second test examines the effect of an increase in the cost of dissent on who the most dissenting judge is (part ii of propositions 5 and 12). To test this we follow Berdejó and Chen (2014), who find that dissents decrease during wartime (Figure 14). For the purpose of our test, our assumption is that this decrease is due to wars tending to increase social cohesion (increase \( W \)). We test the main (concave ideological cost) model’s prediction by examining how the dissent rate is affected by the interaction between a judge’s Distance to Center of Judge Pool and wartime.

\[
DissentRate_{cit} = a + b \times Distance_{cit} + c \times Distance^2_{cit} + d \times Distance_{cit} \times wartime_t + e \times Distance^2_{cit} \times wartime_t + f \times wartime_t + \nu_{cit}
\]

(48)

for judge \( i \) in Circuit \( c \) and year \( t \).
Appendix Figure 14.— The Effect of Wartime on Dissents

Notes: Each dot represents the proportion of dissents over many votes on cases with the same publication year. Figure reproduced from Berdejó and Chen (2014).

We test the SCOTUS model’s prediction by running the same regression but using Distance to SCOTUS as a measure of extremeness. The peak of the spider is determined by the first-order condition of the regression equation. Therefore, to test for a shift in the peak of the dissent rate, we test for a significant difference between $\frac{-b}{2c}$ and $\frac{-(b+d)}{2(c+e)}$. If wartime shifts the peak inwards ($\frac{-b}{2c} > \frac{-(b+d)}{2(c+e)}$), this would corroborate the main model and weaken the SCOTUS model and vice versa. The regressions, the ratios, and the test statistics for the equality of the coefficient ratios are reported in Table A.12. Using Distance to Center of Judge Pool, Column 2 reports that during war there is a significant inward shift of the peak of the spider, which is consistent with the main model. Using Distance to Supreme Court, Column 4 rejects the significant outward shift that is predicted by the SCOTUS model (in fact the coefficient even has the wrong sign).61 Judged together, the two tests seem to refute the SCOTUS model while supporting the main model presented in this paper.

61 Upon request we can provide a table showing the results are the same if we add controls for judge age and experience.
APPENDIX TABLE A.12

Dissent or Concurrence and Ideology Score among Judges: Tests for changes in who dissents the most due to increase in dissent costs during wartime

<table>
<thead>
<tr>
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<th>(1)</th>
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<th>(4)</th>
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<tbody>
<tr>
<td></td>
<td>Dissent or Concur</td>
<td>Dissent or Concur</td>
<td>Dissent or Concur</td>
<td>Dissent or Concur</td>
</tr>
<tr>
<td>Distance(t)</td>
<td>0.0806***</td>
<td>0.0962***</td>
<td>(0.0120)</td>
<td>(0.0103)</td>
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<tr>
<td>Distance(2)</td>
<td>-0.0819***</td>
<td>-0.137***</td>
<td>(0.0182)</td>
<td>(0.0160)</td>
</tr>
<tr>
<td>Distance * War</td>
<td>-0.0861***</td>
<td>-0.127***</td>
<td>(0.0255)</td>
<td>(0.0255)</td>
</tr>
<tr>
<td>Distance(2) * War</td>
<td>0.127***</td>
<td>0.172***</td>
<td>(0.0450)</td>
<td>(0.0379)</td>
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<tr>
<td>Test for Difference in</td>
<td>-0.862**</td>
<td>0.182</td>
<td>(0.414)</td>
<td>(0.284)</td>
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<tr>
<td>Ratio of Coefficients</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(t\): Distance to:

- Center of Judge Pool
- Supreme Court

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>Y</th>
<th>Y</th>
<th>Y</th>
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<tbody>
<tr>
<td>Circuit Fixed Effects</td>
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<tr>
<td>Year Fixed Effects</td>
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<tr>
<td>N</td>
<td>8760</td>
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<td>8760</td>
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<tr>
<td>R-sq</td>
<td>0.128</td>
<td>0.125</td>
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</table>

Notes: Results of regression 48. Robust standard errors clustered at the circuit-year level in parentheses (* \(p < 0.10\); ** \(p < 0.05\); *** \(p < 0.01\)). Data on cases comes from OpenJurist (1950-2007). Absolute value of the distance to the center of the judge pool (columns 1 and 2) and absolute value of the distance to supreme court (columns 3 and 4) are the main independent variables of interest. The dependent variable is the sum of a judge’s rates of dissent and concurrence in a Circuit-year. Observations are weighted by the number of votes cast by the judge in the Circuit-year.

D.3 The SCOTUS model

In this section we present a hierarchical model that is able to produce the spider-shaped pattern of dissent rate as a function of ideological distance to SCOTUS and we derive the predictions that are presented in Proposition 12 and are tested empirically in Section D.2.16. The hierarchical model presented here is developed along the lines of the model of Beim et al. (2014).

Every period, three judges are randomly and independently drawn from a uniform distribution of types \(t \sim U(-1, 1)\) to sit together on a panel. The panel produces a binary verdict ("conservative" or "liberal") for a case with characteristics \(x\), where \(x \in [-1, 1]\). A judge of type \(t\) prefers a conservative verdict over a liberal one iff \(x < t\). The panel determines the verdict by a majority voting, implying that the verdict is conservative if and only if the median judge, denoted \(t_m\), is such that \(x < t_m\).

A panel member may also dissent. Upon noticing a dissent, the Supreme Court may decide to review the case. The bliss point of the Supreme Court is normalized to 0. Thus, the Supreme Court rules conservatively on a reviewed case iff \(x < 0\). The cost of dissenting is denoted \(W\) (and represents writing costs of the minority opinion or collegial pressure). When a judge \(t\) is able to reverse the binary verdict in case \(x\) her utility gain is \(|t - x|\). Hence, a judge will never dissent if \(|t - x| \leq W\). We can calculate the type-dependent probability of dissent \(P(t)\) while considering only a judge with \(t > 0\) (by symmetry the same applies to judges \(t < 0\)).

Under this framework, judge \(t > 0\) may dissent in two scenarios:

1. \(t_m > 0\) and \(x \in [t, t_m]\), so that both \(t\) and the Supreme Court prefer a liberal verdict while the panel
produces a conservative verdict.

2. \( t_m < 0 \) and \( x \in [t_m, 0] \), so that both \( t \) and the Supreme Court prefer a conservative verdict while the panel produces a liberal verdict.

Under scenario 1, the judge indeed dissents if \( x - t > W \), i.e., if \( x \in [t + W, t_m] \). Under scenario 2, the judge indeed dissents if \( t - x > W \), i.e., if \( x \in [t_m, \min \{0, t - W\}] \). We will now show that the model produces a spider-shaped pattern of dissent rate for \( W < 1/2 \). In this case, we have \( W < 1 - W \).

Figure 15 is helpful in distinguishing between three regions of \( t \).

**APPENDIX Figure 15.— Regions in SCOTUS model**

<table>
<thead>
<tr>
<th>Region</th>
<th>[0, W]</th>
<th>(W, 1-W)</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region I</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region II</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region III</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A judge \( t \) in region I dissents if either \( x \in [t + W, t_m] \) or \( x \in [t_m, t - W] \).
A judge \( t \) in region II dissents if either \( x \in [t + W, t_m] \) or \( x \in [t_m, 0] \).
A judge \( t \) in region III dissents if \( x \in [t_m, 0] \).

Calculating the type-dependent probability of dissent \( P(t) \), we get (each line represents one region in the graph)

\[
P(t) = \begin{cases} 
\frac{1}{3} \left[ \frac{1}{2} \left[ 1 - (t + W) \right] \right]^3 + \frac{1}{3} \left[ \frac{1}{2} \left[ (t - W) - (-1) \right] \right]^3 & \text{if } t \in [0, W] \\
\frac{1}{3} \left[ \frac{1}{2} \left[ 1 - (t + W) \right] \right]^3 + \frac{1}{3} \left( \frac{3}{2} \right)^3 & \text{if } t \in [W, 1 - W] \\
\frac{1}{3} \left( \frac{3}{2} \right)^3 & \text{if } t \in [1 - W, 1] 
\end{cases}
\]

To understand the calculations of the expression of \( P(t) \), note first that the event \( x \in [t_m, 0] \) is independent of \( t \) and it occurs iff \( \min t < t_m < x < 0 \). As \( \min t, t_m \) and \( x \) are all drawn from a uniform distribution over \([-1, 1] \), the probability that all three of them are negative is \( \left( \frac{1}{2} \right)^3 \), and the probability that \( x \) is the largest among the three is \( 1/3 \), yielding the expression \( \frac{1}{3} \left( \frac{3}{2} \right)^3 \). Next, the event \( x \in [t_m, t - W] \) is an adjustment of this calculation for the event \( x \in [t_m, t - W] \). In particular, we now need \( \min t < t_m < x < t - W \), so \( x \) needs to be the largest of the three uniformly-distributed variables, which all need to be in the region \([-1, t - W] \), and this event corresponds to probability \( \frac{1}{3} \left[ \frac{1}{2} \left[ (t - W) - (-1) \right] \right]^3 \) (i.e., the probability of being negative, 1/2, is replaced with the probability of being smaller than \( t - W \), which is \( \frac{1}{3} \left[ (t - W) - (-1) \right] \)). Finally, the event \( x \in [t + W, t_m] \) occurs iff \( t + W < x < t_m < \max t \). So \( x \) needs to be the smallest of three uniformly-distributed variables, which all need to be in the region \([t + W, 1] \), and this event has probability \( \frac{1}{3} \left[ \frac{1}{2} \left[ 1 - (t + W) \right] \right]^3 \).

Differentiating with respect to \( t \) yields

\[
\frac{dP(t)}{dt} = \begin{cases} 
-\frac{1}{8} \left[ 1 - (t + W) \right]^2 + \frac{1}{8} \left[ (t - W) + 1 \right]^2 & \text{if } t \in [0, W] \\
-\frac{1}{8} \left[ 1 - (t + W) \right]^2 & \text{if } t \in [W, 1 - W] \\
0 & \text{if } t \in [1 - W, 1] 
\end{cases}
\]

It is immediate to see that \( dP(t)/dt \) is negative in region II. To get the sign of \( dP(t)/dt \) in region I, note that \( t > 0 \) implies that \( t + W \) is closer to the right edge of the type distribution (1) than \( t - W \) is to the left edge of the type distribution (1), implying that

\[
[1 - (t + W)]^2 < [(t - W) + 1]^2
\]

implying that \( dP(t)/dt > 0 \) in region I. Overall, we get that \( P(t) \) increases in region I and then decreases in region II and stays flat in region III, implying a spider-shaped pattern of dissent rate.
D.3.1 Proof of Proposition 12

(i) When $W = 0$, regions I and III disappear and we are left only with region II where $dP(t)/dt$ is negative. Symmetry implies that for any type $t$, $P(t)$ is decreasing in $|t|$.

(ii) The value of $t$ for which $P(t)$ is maximal is the border between regions I and II, i.e. $t = W$. It is thus immediate that $\arg \max_t P(t)$ increases in $W$. 