Addiction and Illegal Markets

Shingo Ishiguro (Osaka) Sultan Mehmood (NES)
Avner Seror (Aix-Marseille) Daniel Chen (TSE)

2025

What we do

We develop a theory to understand *illegal* markets where

- Addiction preference: Buyers are addicted to consuming goods over time.
- No formal laws: Buyers default on payments, and instruments to punish defaults are limited.

Our focus: illegal gambling market

- Horse race betting, sports betting, etc.
- Huge market: USD 1.7 trillion wagered globally (United Nations Office on Drugs and Crime (2021))

Evidence: Illegal Gambling in Pakistan

Mehmood and Chen (2024), “Contract Enforcement in a Stateless Economy”:

- Illegal horse race betting in Pakistan

Key facts:

1. Book-bet: 55% of initial gamblers are allowed to place bets by “credit” but not “cash” (they can defer payments).
2. Default: 35% of gamblers do not repay debts in full.
3. Few violence: only 0.5% of gamblers reported any apprehension of encountering violence in case of non-payment situations.

Questions

- Why do bookmakers use “book-bet” rather than “cash-bet” even though the former results in gamblers’ defaults?
- Why do bookmakers allow gamblers to default?
- Why is violence so rare to punish defaulting gamblers?

Relevant not only in illegal gambling but also other illegal markets (e.g., illicit drug markets).

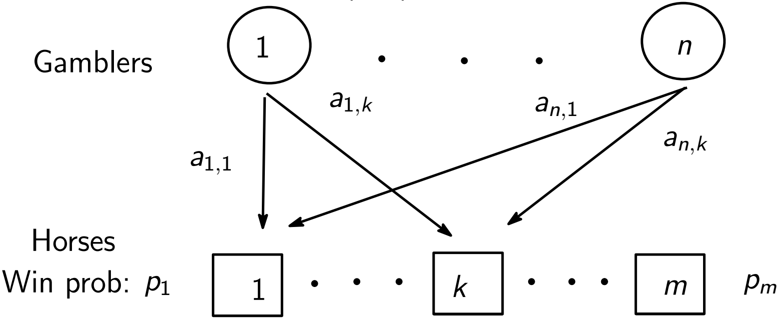
Main Insights: What we show

In dynamic illegal markets with addiction preferences,

1. 'Free-first-dose' strategy becomes optimal for sellers.
   - Sellers initially offer to new buyers goods for free or cheap prices, cultivate their addiction preferences, and then exploit their willingness to pay later on.
2. It becomes optimal for sellers to let new buyers owe indebtedness initially (book-bet rather than cash-bet).
3. Buyers default but are never punished (no violence).
4. Equilibrium odds in gambling market have "long-shot bias," known as an empirical puzzle.
5. Buyers' addictions are socially excessive.

Model: Gambling Market of Horse Racing

(i) Gamblers bet on which horse to come first. (ii) Bookmaker collects "house-take" $f \in (0,1)$ per \$1 bet.



Odds: $h_k = (1-f)\sum_j \sum_i a_{i,j} / \sum_i a_{i,k}$

Payout: $h_j a_{i,j}$ conditional on horse j wining the race.

Addiction Preference: Gamblers' Payoffs

Time: $t = 1, 2$ (can be extended)

- In period 1

$$\underbrace{v(A_{i,1})}_{\text{utility from gambling}} + \underbrace{c_{i,1}}_{\text{gambling returns}}$$

- In period 2

$$\underbrace{u(A_{i,2}, S_i)}_{\text{utility from gambling}} + \underbrace{c_{i,2}}_{\text{gambling returns}}$$

- $A_{i,t} \equiv \sum_{j \in J} a_{i,j,t}$: amounts bet in total in period t.
- Addiction stock in period 2: $S_i = \beta A_{i,1}$ where $\beta > 0$ (Rational Addiction (Becker and Murphy, 1988)).

Equilibrium in Gambling Market

"Parimutuel" Betting:

- In each period $t = 1, 2$ gamblers decide how much to place bets on horses *before* knowing odds.
- Gambling (rational expectations) equilibrium in period $t = 1, 2$: (i) Gamblers optimally place bets $\{a_{i,j}\}_{j \in J}$ given expectations about odds $\{h_j\}_{j \in J}$, and (ii) odds $\{h_j\}_{j \in J}$ are determined to be consistent with gamblers' bets.
- Odds of horse j:

$$h_j \equiv \frac{(1 - f) \sum_{i \in I} \sum_{\ell \in J} a_{i,\ell}}{\sum_{i \in I} a_{i,j}}$$

Gambling Formats

In period 1 bookmaker chooses either one of

- Cash-bet: gamblers must pay immediately when they place bets.
- Book-bet: gamblers can defer payments to the next period.
  - Default can be punished in period 2.

In period 2 only Cash-Bet is available (period 2 is the last period).

Bookmaker's Payoffs

Bookmaker's payoffs over two periods:

$$\sum_{i \in I} \left\{ f \sum_{j \in J} a_{i,j,1} - D_i \right\} + f \sum_{i \in I_2} \sum_{j \in J} a_{i,j,2}.$$

- D_i: amount defaulted by gambler i in period 1.
- Bookmaker collects commission fees minus defaulted amounts.
- $I_2 \subseteq I$: set of gamblers participating in the gambling in period 2.

Book-bet: Timing

Bookmaker chooses book-bet in period 1.

Period 1 (Book-bet):

1. Bookmaker offers a punishment policy (given below).
2. Gamblers place bets and odds are determined in gambling equilibrium.

Period 2 (Cash-bet):

1. Gamblers decide how much to default.
2. Punishment policy is implemented.
3. Gamblers place bets and pay under cash-bet.
4. Odds are determined in gambling equilibrium.

Punishment Policy under Book-Bet

Punishments $\{q, \xi\}$:

1. Exclusion from gambling in period 2 with prob q.

$$q(D, z, A) \in [0, 1]$$

   contingent on how much to owe debt $z \geq 0$, how much to default $D \in [0, z]$ and how much to bet $A \geq 0$ in total in period 1.

2. Utility-based penalty (e.g., violence) ξ:

$$\xi(D, z, A) \in [0, H]$$

   for an exogenous upper bound $H \geq 0$.

Remarks on Punishment Policy

- Limited instruments of punishment: (i) exclusion and (ii) utility-based penalty (violence):
  - Relevant in illegal markets
  - Exclusion: Loss of future gain from gambling (linked to addiction stock S_i)
  - Violence: direct penalty but may be limited by exogenous reasons (e.g. crime prevention policies).
- Bookmaker can commit to punishment policy $\{q, \xi\}$ and payouts to gamblers (can be extended).
  $\Rightarrow$ interpreted as a short-cut of dynamic equilibrium.

(jump to illegal market)

Gambling Equilibrium under Cash-Bet in Period 2

- Gambler i places bets $\{\hat{a}_{i,1}, ..., \hat{a}_{i,m}\}$ to maximize

$$\underbrace{u(A_i, S_i)}_{\text{gambling utility}} + \underbrace{\sum_{j \in J} p_j \hat{h}_j a_{i,j} - A_i}_{\text{expected returns}}$$

where $A_i = \sum_{j \in J} a_{i,j}$, given equilibrium odds $\{\hat{h}_j\}_{j \in J}$.

- Equilibrium odds: $\hat{h}_j = (1 - f) \sum_i \sum_\ell \hat{a}_{i,\ell} / \sum_i \hat{a}_{i,j}$.
- Equilibrium payoff:

$$\hat{U}(S_i) \equiv \max_{\{a_{i,j}\}} u(A_i, S_i) + \sum_{j \in J} p_j \hat{h}_j a_{i,j} - A_i$$

Equilibrium under Book-bet in Period 1

In period 1

- Gambler i places bet $\{a_{i,j}\}_{j \in J}$ with $A_i \equiv \sum_{j \in J} a_{i,j}$.
- Gambler i's net return when horse j wins the race:

$$h_j a_{i,j} - A_i.$$

- Gambler i's debt:

$$z_{i,j} \equiv \max\{0, A_i - h_j a_{i,j}\}$$

- Gambler i's default $D_{i,j}$ s.t. $0 \leq D_{i,j} \leq z_{i,j}$.

Equilibrium under Book-Bet in Period 1

Given a punishment policy $\{q, \xi\}$ and book-bet in period 1,

- Gambler i places bets $\{a_{i,j}\}_{j \in J}$ and decides how much to default $\{D_{i,j}\}_{j \in J}$ so as to maximize

$$v(A_i) + \sum_{j \in J} p_j \{h_j a_{i,j} - A_i + D_{i,j} - \xi(A_i, z_{i,j}, D_{i,j})\}$$
$$+ \sum_{j \in J} p_j (1 - q(A_i, z_{i,j}, D_{i,j})) \hat{U}(S_i)$$

  where $0 \leq D_{i,j} \leq z_{i,j}$, and
- $z_{i,j} \equiv \max\{0, A_i - h_j a_{i,j}\}$: debt owed conditional on horse j wining the race.
- $A_i \equiv \sum_{j \in J} a_{i,j}$: amounts bet in total by gambler i.

(jump to equi under book-bet)

Bang-Bang Punishment Policy

- Expected loss of default:

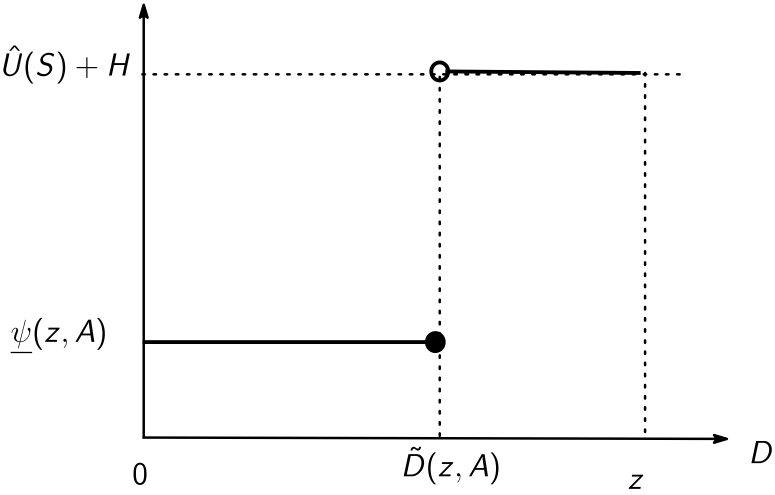
$$\psi(A, D, z) \equiv \underbrace{q(A, D, z)\hat{U}(S)}_{\text{loss of 2nd period payoff}} + \underbrace{\xi(A, D, z)}_{\text{utility-based penalty}}$$

- Punishment policy: ψ instead of q and ξ.
- Bang-bang punishment policy:

$$\tilde{\psi}(A, D, z) \equiv \begin{cases} \underline{\psi}(z, A) & \text{if } D \leq \tilde{D}(z, A) \\ \hat{U}(S) + H & \text{otherwise} \end{cases}$$

- $\tilde{D}(z, A)$: maximum forgiveness.

Bang-Bang Punishment Policy



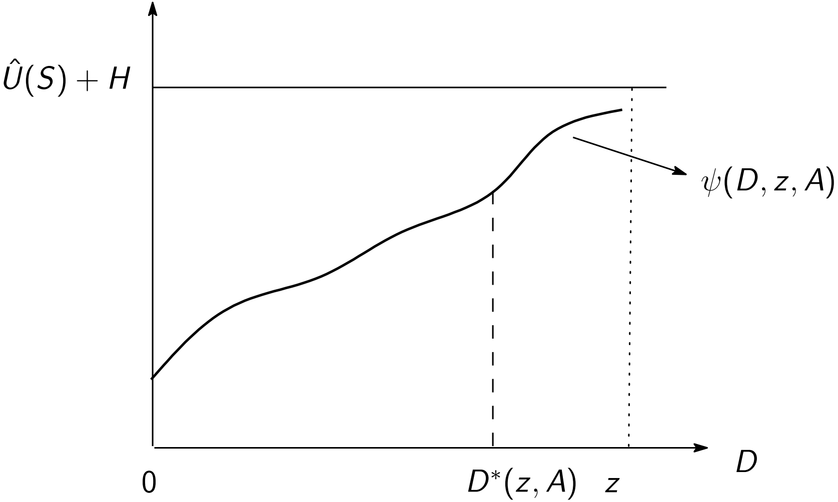
Optimality of Bang-Bang Punishment Policy

Equilibrium outcome given a punishment policy ψ:

- $\mathbf{a}^* = \{a^*_{i,j}\}_{i \in I, j \in J}$: equilibrium bets in period 1.
- $\mathbf{h}^* = \{h^*_j\}_{j \in J}$: equilibrium odds in period 1.
- $\hat{\mathbf{a}}(\mathbf{S}^*) = \{\hat{a}_{i,j}(S^*_i)\}_{j \in J, i \in I}$: equilibrium bets in period 2.

Proposition 1. *Suppose that a punishment policy ψ implements an equilibrium outcome $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{a}}(\mathbf{S}^*)\}$. Then the same outcome is implemented by a bang-bang punishment policy $\tilde{\psi}$. Furthermore, the bookmaker can be weakly better off by $\tilde{\psi}$.*

Intuition



Intuition

- Gambler's equilibrium default $D^*(z, A)$ under ψ:

$$\max_D \; D - \psi(D, z, A)$$

subject to $0 \le D \le z$ where $z \ge 0$ is the debt owed.

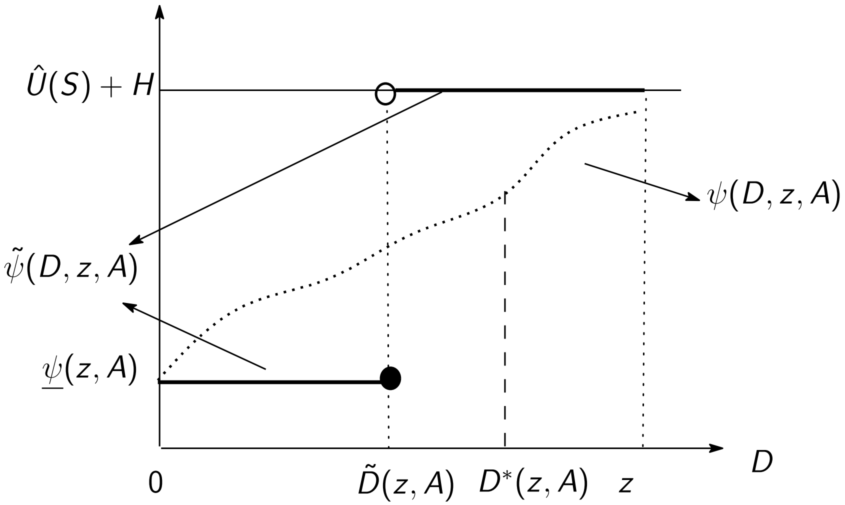
- Define the lowest penalty as

$$\underline{\psi}(z, A) \equiv \inf_{0 \le D \le z} \psi(D, z, A).$$

- Define maximum forgiveness as

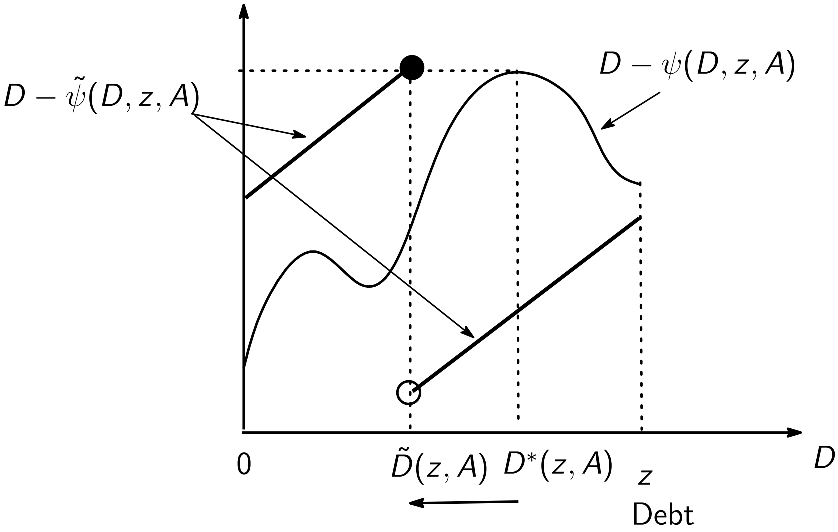
$$\tilde{D}(z, A) \equiv D^*(z, A) - \{\psi(D^*(z, A), z, A) - \underline{\psi}(z, A)\}$$

Intuition



$\hat{U}(S) + H$
$\psi(D, z, A)$
$\tilde{\psi}(D, z, A)$
$\underline{\psi}(z, A)$
D
0
$\tilde{D}(z, A)$
$D^*(z, A)$
z

Intuition: Gambler's Ex Post Payoffs



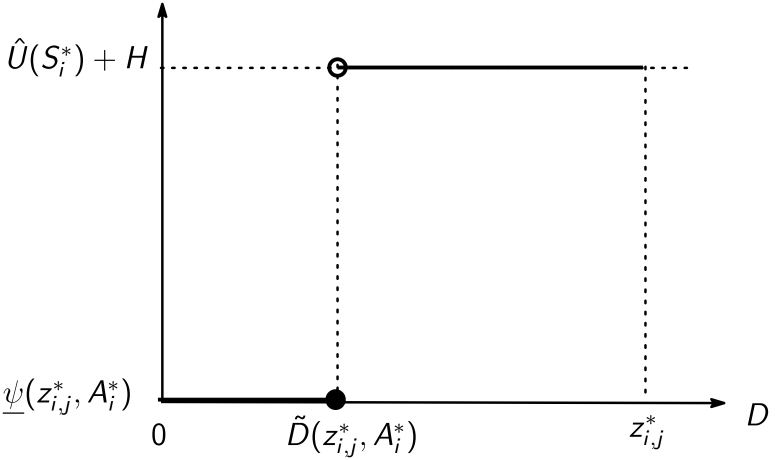
Intuition

Under the newly defined punishment policy $\tilde{\psi}$

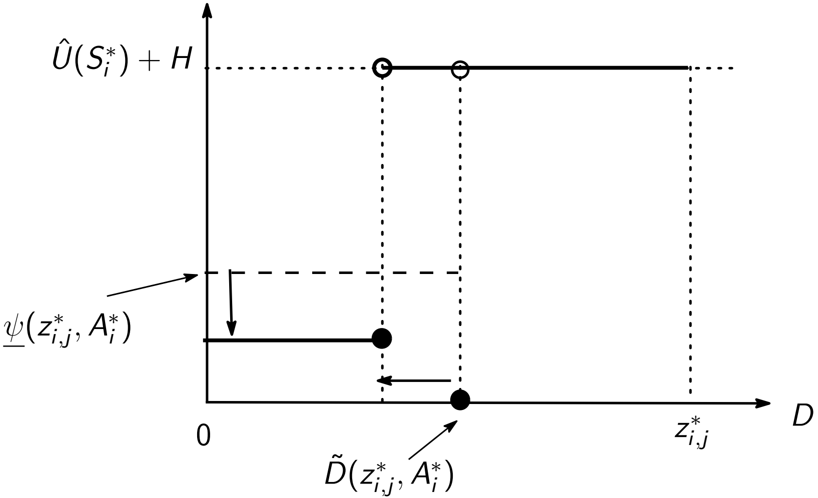
- Gamblers default $\tilde{D}(z, A)$ and obtain the same expected payoffs as those under original ψ.
- Gamblers choose the same amounts bet $\{\mathbf{a}^*, \hat{\mathbf{a}}(\mathbf{S}^*)\}$ as those in equilibrium under original ψ.
- Gamblers default less: $\tilde{D}(z, A) \leq D^*(z, A)$.
  $\Rightarrow$ Bookmaker can be better off.

"Free-First-Dose" Strategy

Optimal $\tilde{\psi}$ maximizing bookmaker's payoff:



"Free-First-Dose" Strategy



Symmetric Equilibrium

Symmetric equilibrium induced by a bang-bang punishment policy $\tilde{\psi}$:

- Period 1: $a^*_{i,j} = a^*_j$ and $A^*_i = A^*$ for all $i \in I$.
- Period 2: $a_{i,j} = \hat{a}_j(S^*)$ and $\hat{A}(S^*) \equiv \sum_j \hat{a}_j(S^*)$ for all $i \in I$ where $S^* = \beta A^*$.
- Equilibrium odds in period 1: $h^*_j \equiv (1-f)A^*/a^*_j$ for $j \in J$.

Best (symmetric) equilibrium: $\tilde{\psi}$ and induced $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{a}}^*(S^*)\}$ maximize the bookmaker's payoff per gambler:

$$\Pi(\tilde{\psi}) \equiv \{fA^* - \tilde{D}^*\} + f\hat{A}(S^*)$$

where gamblers default $\tilde{D}^* \geq 0$ without penalties $\xi = q = 0$ under $\tilde{\psi}$.

Equilibrium Feature #1: Long-Shot Bias

Equilibrium odds have

- Long-shot bias: *gamblers bet more on the horses less likely to win the race.*

- Observed in the data about horse race betting
- Empirical puzzle (Chiappori et al, 2019; Ottaviani and Sørensen, 2008).

Long-Shot Bias

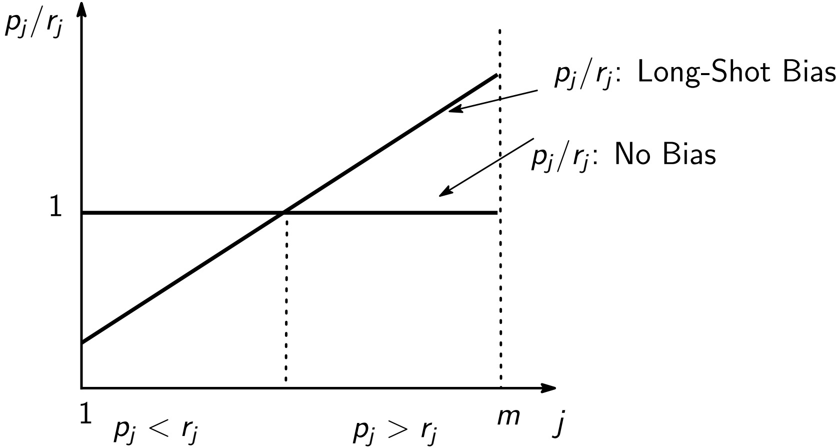
- Support rate of horse j (how much j attracts bets):

$$r_j \equiv \frac{\sum_{i \in I} a_{i,j}}{\sum_{i \in I} \sum_{\ell \in J} a_{i,\ell}}$$

- Odds: $h_j = (1 - f)/r_j$ inversely related to r_j.
- No Bias: $r_j = p_j$ for all $j \in J$.
  $\Rightarrow$ Support rates of horses completely offset their winning probabilities!

Long-Shot Bias

- Wining prob of horses: $p_1 < \cdots < p_m$.



Why Long-Shot Bias in our Equilibrium?

- Expected return to bet \$1 on horse j:

$$\underbrace{p_j(h_j^* - 1)}_{\text{win}} + \underbrace{(1 - p_j) \times 0}_{\text{lose and default}} = p_j(h_j^* - 1)$$

- Default gain: gamblers have more chances to default by betting more on horses with lower p_j.
- No long-shot bias exists when gamblers cannot default.
- Implication: Long-shot bias is more likely in illegal markets where gamblers often default.

Equilibrium Feature #2: Optimality of Book-Bet

- Which book-bet or cash-bet the bookmaker chooses in period 1?

Consider the game with cash-bet in both periods (without defaults):

Period 1 (Cash-bet):

1. Gamblers place bets and pay.
2. Odds are determined.

Period 2 (Cash-bet):

1. Gamblers place bets and pay.
2. Odds are determined.

Cash-Bet or Book-Bet?

Proposition 4. *Suppose that gamblers have so strong addiction preference (with large β). Then the bookmaker prefers book-bet to cash-bet: she optimally allows gamblers to bet by credit (not cash) and default in period 1.*

Intuition: Consider a bang-bang punishment $\tilde{\psi}$ where

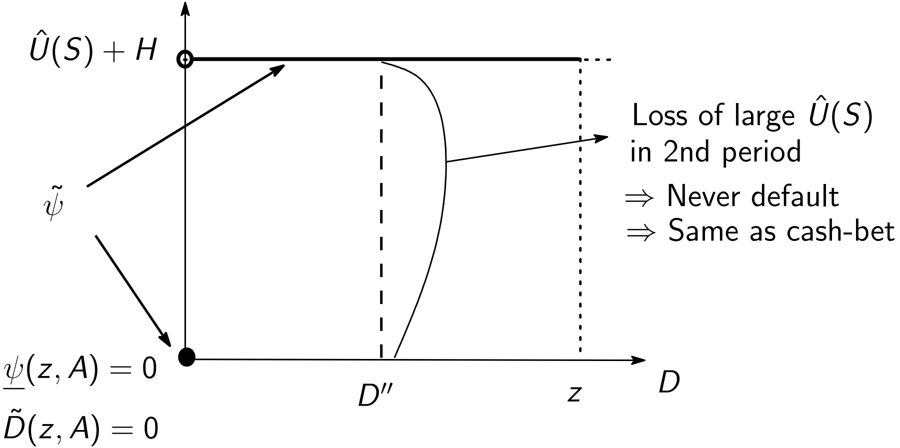
- Maximum forgiveness: $\tilde{D}(z, A) = 0$ for all $z \geq 0$ and all $A \geq 0$.
- Gamblers are most harshly punished for any default $D > 0$.

Then

- Gamblers never default (otherwise they lose large addicted gains in $t = 2$).
- $\tilde{\psi}$ can replicate the same outcome as that under cash-bet.

Optimality of Book-bet

$\tilde{\psi}$ can replicate the same outcome as that under cash-bet.



Equilibrium Feature #3: Excessive Addiction

- Social welfare: $W = n\{v(A_1) + u(A_2, S)\}$ where $S = \beta A$.

Proposition 5. *Suppose that gamblers' addiction preference is sufficiently strong, and that the commission fee $f \in (0, 1)$ is sufficiently small. Then the equilibrium amounts bet in both periods are larger than those maximizing the social welfare.*

Intuition: Under large β and small f the bookmaker

- optimally adopts book-bet in period 1
- excessively cultivates gamblers' addiction by allowing them to bet larger while allowing defaults in period 1.
- induces them to bet aggressively in period 2.

Applications to Other Markets

We apply our theory to understand the 'free-first-dose' strategy in several environments:

- Illicit drug dealers
  - offer new users "genuine" drugs without diluting 'purity' and cultivate their addiction preferences
  - offer high prices later as they are addicted to using drugs.
- Religious sects
  - initially offer new members a sense of belonging, community, and tangible benefits such as meals, social support, or even housing
  - increase their feelings of obligations to contribute to the groups, resulting in financial and spiritual "debts."

Endogenous Commitment

- Time: $t = 1, 2, ...$
- Bookmaker infinitely lives and faces a sequence of short-lived gamblers over time.
- Dynamic enforcement (DE): bookmaker self-enforces punishment policy $\tilde{\psi}$ and payout $(1-f)\sum_j \hat{a}_{j\in J}(S^*)$ under cash-bet if

$$\underbrace{\frac{\delta}{1-\delta}\Pi(\tilde{\psi})}_{\text{Discounted Future Payoff}} \geq \underbrace{(1-f)\sum_j \hat{a}_{j\in J}(S^*)}_{\text{Payout under Cash-Bet}} .$$

- Complementarity between addiction and relational contracts: higher β makes DE easier to satisfy.
  $\Rightarrow$ Answer to why Illegal markets work even without formal contracts when people have addiction preferences.

Appendix

Bookmaker's Commitment

Even in the two-period model the bookmaker can commit to punishment policy and payouts by

- hiring "agents" who are paid "fixed" wages (independent of race outcomes)
- letting them run betting stations and implement punishment policy and payouts

(jump to Remarks)

Equilibrium under Book-Bet

Period 1 (Book-bet):

1. Bookmaker offers and commits to punishment policy $\{q, \xi\}$.
2. Gamblers place bets and odds are determined.
3. Gamblers decide how much to default.

Period 2 (Cash-bet):

- Gambler i bets $\hat{A}(S_i)$ in total:

$$\hat{U}(S_i) \equiv \max_{A_i \geq 0} \; u(A, S_i) - fA_i.$$

- Gambler i's equilibrium payoff: $\hat{U}(S_i)$.

(jump to Equilibrium under Book-Bet)

Best Equilibrium: Constraints

- Gambler's equilibrium payoff:

$$V^* \equiv v(A^*) - fA^* + \tilde{D}(z^*, A^*) + \hat{U}(S^*)$$

- Incentive Compatibility (IC):

$$\begin{aligned} V^* \geq \max_{\mathbf{a}_i, \{D_{i,j}\}_{j \in J}} & v(A_i) \\ & + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + D_{i,j} - \tilde{\psi}(D_{i,j}, z_{i,j}, A_i)\} \\ & + \hat{U}(S_i). \end{aligned}$$

subject to $0 \leq D_{i,j} \leq z_{i,j} \equiv \max\{0, A_i - h_j^* a_{i,j}\}$, and $S_i = \alpha + \beta A_i$.

Best Equilibrium: Constraints

- Equilibrium odds (EO): $\{h_j^*\}_{j \in J}$ satisfy

$$\sum_{j \in J} (1/h_j^*) = 1/(1-f).$$

(jump to definition of equi odds)

- Feasibility (F):

$$0 \leq \tilde{D}(z, A) \leq z$$

and

$$0 \leq \tilde{\psi}(D, z, A) \leq \hat{U}(S) + H$$

for all $D \in [0, z]$, all $z \geq 0$ and all $A \geq 0$.

Best Equilibrium

Best equilibrium $\{\mathbf{a}^*, \mathbf{h}^*, \hat{A}(S^*)\}$:

- (symmetric) equilibrium bets in period 1: $\mathbf{a}^* = \{a_j^*\}_{j \in J}$.
- equilibrium odds in period 1: $h_j^* = (1 - f)A^*/a_j^*$ where $A^* = \sum_j a_j^*$.
- equilibrium amounts bet in total in period 2: $\hat{A}(S^*)$ where $S^* = \alpha + \beta A^*$.
- Maximize the bookmaker's payoff

$$\Pi(\tilde{\psi}) \equiv fA^* - \tilde{D}(z^*, A^*) + f\hat{A}(S^*)$$

subject to IC, EO and F.

Equilibrium under Cash-Bet in $t = 1, 2$

Under cash-bet in period 1 (no defaults are allowed),

- Gamblers place bets $\{\tilde{a}_j\}_{j \in J}$ *without* defaults in period 1:

$$\max_{a_{i,j}} \; v(A_i) + \sum_{j \in J} p_j h_j a_{i,j} - A_i + \hat{U}(S_i)$$

  where $A_i = \sum_{j \in J} a_{i,j}$ and $S_i = \alpha + \beta A_i$.

- Bookmaker's payoff per gambler:

$$\Pi_c \equiv f \sum_{j \in J} \tilde{a}_j + f \sum_{j \in J} \hat{a}_j(\tilde{S})$$

  where $\tilde{S} \equiv \alpha + \beta \sum_j \tilde{a}_j$.