Addiction and Illegal Markets

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Abstract

This paper studies dynamic contracts in illegal addictive markets where individuals' tastes for addictive goods develop through prolonged consumption and contract enforcement is limited. Our theoretical analysis uncovers the optimality of a 'free-first-dose' strategy where sellers intensify buyers' addiction by offering consumption credit to newcomers. We show that buyers default a certain portion of the debts for early period consumption but are never imposed any penalty on the equilibrium path. This implies that illegal markets might favor non-violent interactions over violent ones, defying the stereotypical association of illegality with violence. Meanwhile, in illegal gambling markets, a distinct equilibrium phenomenon known as the long-shot bias emerges due to the influence of addiction, illustrating another complex dynamic within these markets. We discuss the implications of the model in the context of illegal sports wagering, narcotics, and religious sects.

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1 Introduction

Individuals' preferences for addictive goods evolve through sustained consumption, while economic transactions are shaped by dynamic incentives and expectations that arise in repeated interactions, particularly when formal contract enforcement is limited or absent. Since addiction is often tied to illegality, the dual dynamics of addiction and informal enforcement are crucial in understanding optimal trading mechanisms in addictive markets. In these markets, sellers must manage the evolving incentives of buyers, who become increasingly addicted through consumption over time, while navigating the absence of formal enforcement mechanisms due to the illegality of the activity or the significant infringement of individual rights that the illegal activity itself entails.

Seminal work by Becker and Murphy (1988) laid the foundation for understanding addiction within an economic framework, offering insights into how rational behavior can coexist with addictive consumption. More recent studies have explored various facets of addiction, including the role of digital technologies (Allcott, Gentzkow and Song (2022)) or self-control (Schilbach (2019)). However, the interaction between addiction and the absence of formal enforcement mechanisms remains underexplored. Our study addresses this gap by focusing on the dual dynamics of addiction and informal enforcement.

While our model can capture market dynamics across various illicit sectors, we focus on illegal gambling markets due to their unique attributes and their representation of broader market dynamics. The global scale of gambling, particularly the illicit segment, is enormous, with estimates suggesting it may surpass $1,700 billion globally (United Nations Office on Drugs and Crime (2021)). This far exceeds other major industries, including tobacco and pharmaceuticals, underscoring its economic importance. Illegal gambling markets are also highly centralized, with gamblers placing bets with a limited number of bookmakers, making them an ideal case for studying price formation and market behavior. Compared to more dispersed markets like narcotics, this centralization provides a clearer lens through which to observe the dynamics of illicit markets. Moreover, the structural features of illegal gambling markets, such as horse race betting, including repeated transactions and reliance on informal enforcement mechanisms, make them a valuable context for examining broader questions about the functioning and organization of illicit economies.

In our model, a bookmaker (seller) operates over a finite number of periods, and gamblers (buyers) exhibit addictive behavior.[1] Each period, the bookmaker selects between two betting formats: a 'cash-bet' system, requiring immediate payment, and a 'book-bet' system, allowing for deferred payments. This framework allows us to analyze the strategic choices made within these markets and their broader implications. Since gambling is illegal, there are no formal institutions to enforce debt repayments from gamblers, and the bookmaker is exposed to default risks under the book-bet, in contrast to the cash-bet.[2]

[1] Our main model focuses on two periods to simplify the argument. We extend the basic model to allow for an infinite time horizon in Section 7.

[2] We omit consideration of liquidity risk, which could arise from a widespread default, potentially hindering the bookmaker's ability to pay winning gamblers.

Since gamblers always default all their debts in the last period, the bookmaker has no choice but to rely on the cash-bet in that period. On the contrary, the bookmaker may choose the book-bet in the other periods, together with a punishment instrument to reduce the default risk. The cornerstone of our model is the potential for gambler addiction, a factor the bookmaker is aware of. In this context, we characterize the market equilibrium, detailing the bookmaker's optimal punishment strategy in the event of default, and the gamblers' betting trajectory.

This model captures several key features of illegal markets. First, transactions are not protected. Second, sellers, such as those in illegal gambling or illicit drug markets, extend consumption credit to buyers. Third, the model details how sellers may enforce non-formal punitive measures to counteract default risk. These measures can range from exclusion from future transactions to the exertion of psychological or physical violence, tactics seldom seen or severely restricted within legal markets.[3]

Our first theoretical finding reveals that sellers utilize a 'free-first-dose' approach. Much like drug dealers who may offer a free sample to cultivate dependency in their clientele, we find that sellers strategically allows the buyers to initially borrow and consume beyond their means. This tactic is designed to cultivate an addiction, thereby ensuring increased future revenues.

This result unveils that in equilibrium, the seller anticipates and accepts a certain degree of default from each buyer. More specifically, we show that the seller can always replicate the equilibrium outcome attained under any punishment policy by the following *bang-bang* punishment policy (see Proposition 2 below): buyers are allowed to default up to a certain upper bound, which we call *maximum forgiveness*, and are never punished unless they default more than it in equilibrium. Furthermore, the seller can be weakly better off by this bang-bang punishment policy upon any other punishment policy. Such "allowed default" plays a strategic role: it cultivates buyers' addiction by inducing large consumption amounts in the initial period, making them willing to consume aggressively later on.

The second main finding probes further into this dynamic, assessing the influence of violence on market equilibrium. Interestingly, while violence might enforce more prompt debt repayments, it could simultaneously weaken the initial 'free-first-dose' incentive, thereby stifling the development of addiction. This paradoxical outcome proposes that illegal markets might favor non-violent interactions over violent ones, defying the stereotypical association of illegality with violence.

Third, we investigate welfare implications about illegal market equilibrium and show that buyers may consume more addictive goods than the social optimum, that is, their consumption becomes "socially excessive". This is because sellers try to extract buyers' surplus by making them addicted to consuming goods in the initial period and then increasing their willingness to pay for a later period of consumption.

[3]In Section 7, the model is formally extended to more general illegal market settings.

Fourth, we show that in the case of illegal gambling markets, equilibrium odds exhibit the feature of so-called *long-shot bias*, which has been well-known as an empirical puzzle in the existing literature, such that buyers (gamblers) place larger bets on long-shots than favorites (Chiappori et al. (2019), Ottaviani and Sørensen (2008)). For instance, horse race bettors tend to place higher wagers on horses with lower odds of winning. In contrast to prior scholarship on the long-shot bias, which often attributes this behavior to gamblers' risk-loving preferences or behavioral factors, we explain this phenomenon through the lens of gamblers' default incentives in illegal gambling markets. Indeed, when gamblers cannot default, the equilibrium odds of horses necessarily offset their winning probabilities so that their expected returns must be equalized across all horses. However, when gamblers can default, the relationship between odds and winning probabilities breaks down, as their expected gains from default increase with bets on less likely outcomes. Consequently, rational gamblers place more bets on these outcomes, generating the long-shot bias.

Finally, we expand the model in two primary directions. The first extension accounts for endogenous commission fees, or 'gambling house' fees, a prominent feature of these markets. This seems particularly relevant in our context since one could argue that negative fees - or bet subsidies - are substitutes for book-bet strategies. We show that this is not the case, as the bookmaker still relies on book-bet strategies when commission fees are endogenous. Rather than being a substitute, bet subsidies and book-bet seem rather complementary. Second, we relax the assumption of the bookmaker's commitment to the announced punishment policy. We also assume an infinite time horizon, with the bookmaker having a long lifespan and gamblers being short-lived. When non-repayment from the book-maker is punished by gamblers, the bookmaker faces a dynamic enforcement constraint. We find that this constraint is more easily satisfied when addiction is strong, as foregoing future bets becomes particularly costly for the bookmaker. This result further corroborates the prediction that addiction facilitates transactions when formal enforcement mechanisms are limited.

The dynamics within illegal markets of addictive goods, as presented in our model, exhibit a strong parallel to the framework of relational contracts (Baker, Gibbons and Murphy 2002). In these contexts, the relational contracts between parties—such as bookmakers and gamblers or drug dealers and users—rely heavily on mutual trust and reputation, with our analysis also emphasizing the role of the addictive nature of the goods in shaping these relationships. For instance, when bookmakers extend credit to gamblers, or when drug dealers employ a 'free-first-dose' strategy, they are initiating a relational contract that exploits the addictive behavior of the other party. This addiction becomes a critical component of the agreement, ensuring repeat engagement and a form of enforced loyalty. Thus, addiction serves not only as a driving force for consistent market participation but also as an informal enforcement mechanism that underpins these relational contracts, sustaining the market's equilibrium in the absence of formal legal structures.

The predictions of our model find particular support in the functioning of different markets. First, the model's theoretical predictions find empirical support in the functioning of the illegal gambling market. Despite the challenges associated with data collection in such opaque markets, Mehmood and Chen (2022) collect and provide some stylized facts from the illegal horse race betting market in Pakistan. The authors calculate that approximately 55% of wagers within this market are placed on credit. Our model is able to square this puzzling manifestation as the 'free-first-dose' strategy in action. According to our model, this approach is not merely about extending credit; it is a calculated bid to cultivate gambling addiction, aiming to ensure a lucrative and enduring customer base. Mehmood and Chen (2022) collected data on debt repayment too, and found that on average, 35% of gamblers do not repay their debt in full. The authors give suggestive evidence that despite the significant rate of default, violence is rare in this market. Only 0.5% of gamblers express fear of violent repercussions for unpaid debts. This empirical evidence underscores the bookmakers' strategic choice to foster addiction through leniency and trust rather than through coercion and fear, a choice that aligns with our model's predictions.

Second, our model's implications also find support in the functioning of illegal drug markets, where the "free-first dose" strategy is a well-documented method to foster dependency among new users. Galenianos and Gavazza (2017) estimate a model using data on the crack cocaine market in the United States. The authors rely on the STRIDE dataset, which contains records of the acquisition of illegal drugs by undercover agents and DEA informants. Hence, the data may contain information about purchases from "new" consumers. Consistent with the prediction of our model, Galenianos and Gavazza (2017) find that drug dealers do not necessarily provide low-quality products to new users by diluting (or "cutting") the products. That way, drug dealers may invest in relationships, and cultivate stronger addictions. Finally, the nuanced role of violence in illegal drug markets has been studied, among others, by Curtis and Wendel (2000) in their analysis of the heroin market of New York City. Violence can either be a tool for enforcement, or something deliberately avoided to maintain customer loyalty. This mirrors our model's predictions about the complex interplay between coercion and addiction sustainability.

Finally, the dynamics explored in our model can also be observed in the interaction of religion and addiction, particularly within the context of religious sects. Initial engagements often come with tangible benefits and a strong sense of community, serving as the "free-first dose" that attracts individuals and fosters attachment (Dawson (1998)). As commitment deepens, a form of "debt repayment" emerges, where members feel obliged to contribute more significantly, financially or otherwise, driven by emotional or spiritual indebtedness.

Related Literature. This paper contributes to the literature in several ways. First, we contribute to the large and multifaceted literature on addiction. In a seminal article, Becker and Murphy (1988) developed a theory of addiction were decision-makers have rational expectations about the consequences of their choices. Since then, several works investigated the rational nature of addiction and explored addictions in various environments.[4] Several papers investigated the interaction between addiction and market structure (Becker, Grossman and Murphy (1994), Chaloupka (1991), Fethke and Jagannathan (1996), Driskill and McCafferty (2001)). We complement these studies most specifically, as we investigate addiction in illegal markets. This aspect seems particularly crucial, as addictive goods frequently fall into the realm of illegality.

Second, this paper also contributes to the literature on relational contracts. This literature addressed issues related to how transactions are self-enforced without formally written contracts (for example, Levin (2003); Malcomson (2012)). However, to our knowledge, there are few theoretical attempts to investigate how individuals' preferences over addictions are interlinked with optimal trading arrangements in the lack of formal enforcement institutions. Because illegal markets of addictive goods are characterized by both addiction preferences and informal contracts, it is an important research venture to understand what informal trading arrangements emerge when individuals consume addicted goods and engage in informal contract agreements that may be sustained via relational contracts. More specifically, both addiction preferences and relational contracts dynamically evolve altogether: individuals accumulate the tastes for addictions by consuming addicted goods over time while relational contracts are sustained by individuals' future concerns. Therefore, it is important to address how these two-way dynamic incentives are intertwined to characterize optimal trading mechanisms in illegal markets without formal enforcement institutions.

Third, this paper contributes to the theoretical literature on illicit drug markets. Galenianos, Pacula and Persico (2012) and Galenianos and Gavazza (2017) propose search theoretic models where sellers of illicit drugs are randomly matched buyers and choose qualities of drugs that are not observable to buyers. The main focuses of these papers are on the moral hazard problem of sellers, and, more specifically, the issue of whether sellers choose low quality for first-time buyers or offer high qualities for loyal buyers. We complement their work by focussing more specifically on buyers' addiction preferences, which dynamically interact with their default incentives, and sellers' optimal selling strategy.

Finally, this paper contributes to the literature on decision-making that leverages gambling data. Most notably, gambling data have been used to recover risk preference (Jullien and Salanié (2000), Feess, Müller and Schumacher (2016), Chiappori et al. (2019)), and behavioral biases affecting risky choices (Snowberg and Wolfers (2010), Losak, Weinbach and Paul (2023)). We complement these studies by focusing on addiction in illegal gambling markets. Moreover, we explain the long-shot bias rooted in rational choice theory. In our model, the long-shot bias emerges because the possibility of defaulting incentivizes gamblers to place larger bets on horses with lower winning probabilities.

[4]See, among others, Hoch and Loewenstein (1991), Orphanides and Zervos (1995), Chaloupka and Warner (2000), Gruber and Koszegi (2001), Courtemanche, Heutel and McAlvanah (2014), Grossman and Chaloupka (2017), Schilbach (2019), and Allcott, Gentzkow and Song (2022).

2 Model

2.1 Illegal Market for Addictive Goods

One risk neutral seller (she) produces and delivers an "addictive good" such as illicit drugs to a unit mass of buyers (he) for two periods in *the illegal* market where formal contracts are not enforced. In the baseline model, we assume that the buyers are identical, but we extend the model to allow heterogeneous buyers in Section ??. The buyers have the addiction preference such that they are addicted to consuming the good in period 1 and accumulating the addiction stock in the same spirit of Becker and Murphy (1988). The buyers are also *liquidity constrained*: each buyer is endowed $w_t > 0$ units of the numèraire good and cannot pay more than w_t in period $t = 1, 2$.[5]

When a buyer consumes $A_t \geq 0$ units of the addictive good and pays P_t the seller in period $t = 1, 2$, he obtains the following payoffs over two periods:

$$v(A_1) - P_1 + u(A_2, S) - P_2 \tag{1}$$

where $v(A_1)$ is the gross utility from consuming A_1 units of the addictive good in period 1 and $u(A_2, S)$ is the gross utility from consuming A_2 units of the addictive good in period 2 respectively. Here, $S = \beta A_1$ denotes the addiction stock that measures how much the buyer is addicted to consuming the good in period 2 when consuming A_1 in period 1, where $\beta > 0$ captures the degree of buyer's addiction preference. We assume that v and u are continuously differentiable and that $u_S > 0$ and $u_{AS} > 0$ where subscripts denote partial derivatives such as $u_S \equiv \partial u / \partial S$, $u_{AS} \equiv \partial^2 u / \partial A_2 \partial S$, and so on. We also assume that there is no time discounting between the two periods to simplify the argument.

The seller can produce and deliver one unit of the addictive good at the unit cost $c_t > 0$ in the period $t = 1, 2$.

2.2 Cash-Based and Credit-Based Selling

Because the market is illegal, there are no formal contracts to enforce. Then the seller may renege on agreed upon delivery of the goods after she receives the payments from the buyers. The buyers may also default on agreed upon payments to the seller after they receive the goods. Which party, the seller or the buyer, is more relevant party to renege on agreed upon transactions depends on the timing of payment and delivery.

We consider two options for the seller on how to sell the addictive good to buyers in each period. One is *the cash-based selling* where buyers pay first and then the seller decides how much to deliver the good, while the other is *the credit-based selling* where the seller delivers the good to buyers first and then buyers decide how much to pay the seller.

First, suppose that the seller chooses the cash-based selling in period t, specifying that the seller delivers A_t units of the goods to each buyer with payment of P_t. At the beginning

[5] The buyers cannot also save/borrow as well.

of period t each buyer pays P_t to the seller, and then the seller decides much to produce the goods to the buyer. At this stage the seller can renege on delivering A_t units of the goods to the buyer (Figure 1).

If the seller incurs no costs of deviating from the delivery of the goods, she would always deliver nothing after receiving payments from the buyers. To avoid this, we need some punishments to discipline the seller to deliver the agreed amounts to the buyers. In the baseline setting, we model these punishments in such a simple way that, if the seller does not deliver agreed amounts A_t to any buyer in period $t = 1, 2$, she will lose *the reputation value* of $V \geq 0$ at the end of period 2. This value V might represent the future gains that the seller would obtain from future buyers if she honored the informal contracts agreed with the current buyer. In Section?? we extend the model to determine the seller's reputation value V endogenously.

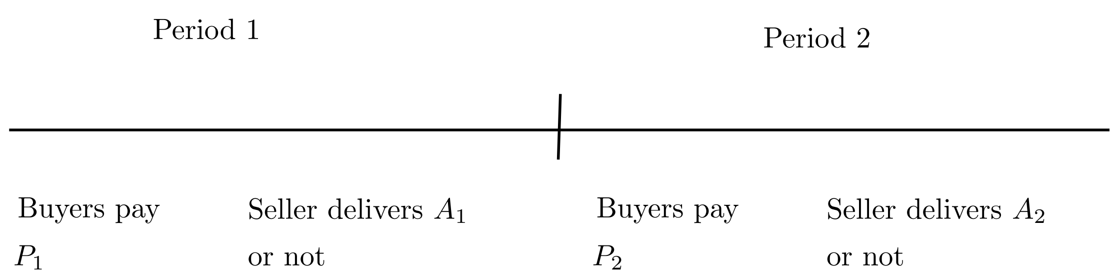

Figure 1: Timing of the game under cash-based selling

Suppose next that the seller chooses the credit-based selling in period 1, specifying that the seller delivers A_1 units of the good to each buyer with payment of P_1. At the beginning of period 1 the seller delivers A_1 units to each buyer. The buyer can then default on the payment P_1, in which case the buyer is punished at the beginning of the next period (period 2) in two ways: (i) exclusion from the market in period 2 and (ii) utility-based penalty such as violence (Figure 2).

First, the seller can use *the exclusion policy* q that specifies the probability $q \in [0, 1]$ to exclude each defaulting buyer from the participation in the transactions of the second period. In illegal markets, it is relevant and effective to prevent those who defaulted on payments in the past from participating in the future gambling. For example, Mehmood and Chen (2022) empirically assess this sort of punishment in illegal horse race betting in Pakistan.[6] If a buyer defaults $D \in [0, P_1]$ so that he pays only $P_1 - D$ to the seller in the end of period 1, he is excluded from the market in period 2 with a probability $q(D) \in [0, 1]$.

Second, the seller can also impose non-monetary sanctions on defaulting buyers. For example, as often discussed about illegal markets, the buyers who defaulted on payments

[6] Mehmood and Chen (2022) consider two types of exclusion policies called local blacklisting and global blacklisting in their empirical study about the illegal horse race betting in Pakistan: under the former defaulting gamblers are excluded from participating in the betting station where they default while under the latter defaulting gamblers are excluded from not only the betting station where they default but also from other betting stations.

may face the violence threatened by sellers and associated crime groups. Additionally, buyers may lose social reputation and suffer from being labeled bad social images when they default promised payments (see Mehmood and Chen (2022) for related penalties imposed on defaulting gamblers in the illegal horse race betting in Pakistan.) We denote by $\xi(D) \in [0, H]$ such an utility-based penalty imposed on the buyer who defaults $D \in [0, P_1]$ in period 1.[7] Here $H \geq 0$ is the exogenous upper bound for available utility-based penalties. For example, the availability of violence as a sanction on defaults depends on legal and police institutions of a society, which we capture by the exogenous upper bound for possible penalties $H \geq 0$. We can allow the case that the utility-based penalty is never available, that is, $H = 0$.

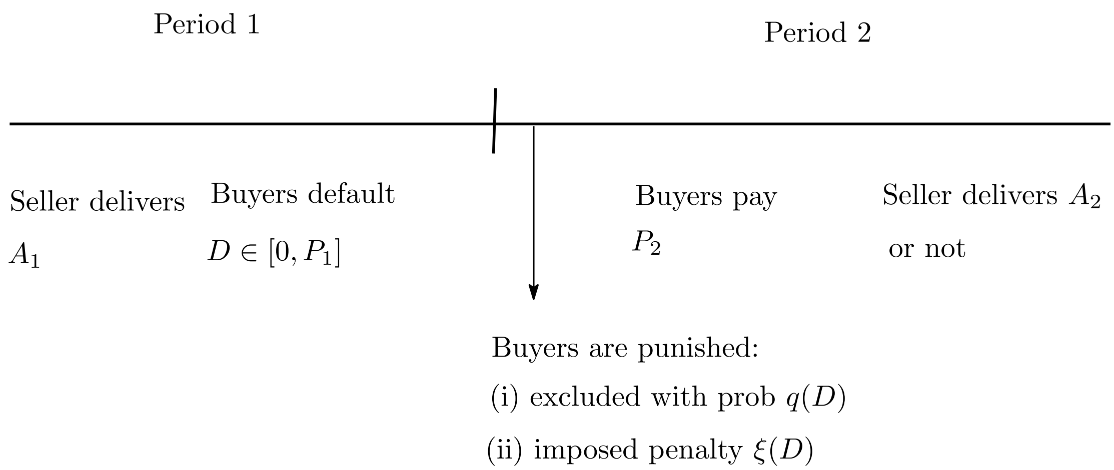

Figure 2: Timing of the game under credit-based selling

Because period 2 is the last period so that there are no future periods to punish the buyer's default, the buyers always default all the payment of P_2 (hence $D = P_1$) if they can buy the goods by the credit-based selling in period 2, yielding no profits to the seller in period 2. Given this, the seller optimally chooses the cash-based selling in period 2 (details are given in the proof of ?? below).

3 Cash-Based Selling

First, we consider the equilibrium when the seller chooses the cash-based selling at the beginning of period 1. Specifically, the seller offers an informal contract $\{A_1, P_1, A_2, P_2\}$ to each buyer at the beginning of period 1, specifying the amount of A_t to deliver after receiving the payment of P_t in period $t = 1, 2$ (see Figure 1).

Because the contract is informal, it must satisfy the set of several constraints to self-enforce. First, if the seller receives P_2 from each buyer but does not deliver A_2 units to any

[7]See Dubey, Geanakoplos and Shubik (2005) for a related approach to default and punishment in market equilibrium framework.

buyer at the end of period 2, she can save the costs of c_2A_2 per buyer but will incur the reputation loss of V. Therefore, for the seller not to make such a deviation, the following *dynamic enforcement* (DE) constraint must be satisfied in period 2:

$$V \geq c_2 A_2 \qquad (\text{DE}_2).$$

Second, suppose that, after the seller receives P_1 from each buyer, she deviates from delivering A_1 units to any buyer at the end of period 1 and trades with no buyers in period 2. By this deviation, the seller can save the production costs of c_1A_1 in period 1 but will lose the profit of $P_2A_2 - c_2A_2$ in period 2 and the reputation value V. Therefore, for such deviation not to be profitable, the following DE constraint must be satisfied in period 1:

$$P_2A_2 - c_2A_2 + V \geq c_1A_1 \qquad (\text{DE}_1)$$

Third, each buyer must not quit in period 2, implying that

$$u(A_2, S) - P_2 \geq 0 \qquad (\text{IR}_2)$$

as well as he must not quit in period 2, implying that

$$v(A_1) - P_1 + u(A_2, S) - P_2 \geq 0 \qquad (\text{IR}_1)$$

We call these constraints *individual rationality* (IR) constraints of the buyers.

Finally, payments must satisfy the buyer's *liquidity* (L) constraint in each period:

$$w_t \geq P_t, \quad t = 1, 2. \qquad (\text{L})$$

The seller chooses the contract $\{A_1, P_1, A_2, P_2\}$ so as to maximize her payoffs

$$\sum_{t=1,2} \{P_t - A_t\} \qquad (2)$$

over two periods subject to all the constraints of DE_1, DE_2, IR_1, IR_2 and L.

The key implication about the optimal cash-based selling is that, when the buyers are more liquidity-constrained in period 2 (w_2 is smaller), the seller finds it more difficult to commit to deliver A_1 and A_2 in both periods: DE_1 and L imply that

$$w_2 + V \geq \sum_{t=1,2} c_t A_t. \qquad (3)$$

This is because the seller cannot extract larger payments P_2 in period 2 from the buyers as they are more liquidity constrained in period 2, which in turn makes the seller's *commitment in period 1* more difficult. Note that the buyer's initial wealth w_1 does not affect the dynamic enforcement constraints of the seller. Rather the buyer's second period wealth

w_2 plays a crucial role to determine how likely the seller can commit to deliver agreed amounts.

4 Credit-Based Selling

4.1 Optimal Contract

Now we consider the optimal credit-based selling for the seller. At the beginning of period 1 the seller offers an informal contract $\{A_1, P_1, A_2, P_2, q, \xi\}$ that specifies the amount to deliver A_t and the corresponding payment P_t in period $t = 1, 2$ as well as the punishment policy $\{q, \xi\}$, which is implemented at the beginning of period 2 depending on how much buyers default in period 1. As we mentioned, $q : [0, P_1] \to [0, 1]$ is the probability to exclude each buyer from trading in period 2 and $\xi : [0, P_1] \to [0, H]$ is the utility-based penalty imposed on the buyer when he defaults $D \in [0, P_1]$ in period 1.

Again we here suppose that, if the seller does not execute the announced punishment policy $\{q, \xi\}$, she will lose the reputation value V at the end of period 2. Given this, we consider the following punishment equilibrium: if the seller deviates from the equilibrium policy $\{q, \xi\}$ in period 2, she optimally follows not to deliver A_2 regardless of how much she receives from the buyers in period 2 while each buyer never pays P_2 to the seller, provided that he anticipates that the seller does not deliver anything at the end of period 2. Therefore, the seller's deviation payoff becomes zero in period 2 when she deviates from the announced punishment policy $\{q, \xi\}$ at the beginning of period 2.

The following constraints must be satisfied for the credit-based selling contract $\{A_1, P_1, A_2, P_2, q, \xi\}$ to be self-enforcing. First, as in the case of cash-based selling, the seller must commit to deliver the agreed amount A_2 at the end of period 2:

$$V \geq c_2 A_2 \tag{DE$_2$}$$

Second, each buyer optimally chooses how much to default $D \in [0, P_1]$ at the end of period 1 so as to maximize his continuation payoff from the end of period 1, given as follows:

$$\max_{0 \leq D \leq P_1} \quad D + (1 - q(D))\{u(A_2, S) - P_2\} - \xi(D) \tag{IC}$$

which we call buyer's *incentive compatibility* (IC) constraint.

Third, there must be the *individual rationality* (IR) constraints in each period, that is, each buyer must accept the contract offered by the seller in period 1:

$$v(A_1) - P_1 + D + (1 - q(D))\{u(A_2, S) - P_2\} - \xi(D) \geq 0 \tag{IR$_1$}$$

and period 2:

$$u(A_2, S) - P_2 \geq 0 \tag{IR$_2$}$$

Finally, the liquidity constraints must be satisfied:

$$w_1 \geq P_1 - D \tag{L$_1$}$$

in period 1 and

$$w_2 \geq P_2 \tag{L$_2$}$$

in period 2 respectively.

The seller chooses the informal contract $\{A_1, P_1, A_2, P_2, q, \xi, D\}$ to maximize her payoffs over two periods:

$$P_1 - D - c_1 A_1 + (1 - q(D))\{P_2 - c_2 A_2\}$$

subject to all the above constraints.

The crucial difference from the cash-based selling is that under the credit-based selling the seller does not face the dynamic enforcement constraint in period 1 (DE$_1$) because it is not the seller but the buyer who reneges the agreed contract in period 1. Therefore, the credit-based selling can make the commitment constraint on the seller's side relaxed whereas it causes the costs associated with buyers' defaults.

4.2 'Free-First-Dose' Strategy

We now characterize the optimal punishment policy $\{q, \xi\}$ to maximize the seller's payoffs. Define by

$$U_2 \equiv u(A_2, S) - P_2 \tag{4}$$

the buyer's payoff in period 2 when he is allowed to participate in the transaction in period 2. If a buyer defaults $D \in [0, P_1]$ at the end of period 1, he will be excluded with probability $q(D)$ in which he will lose the payoff U_2 in period 2, and he will be imposed the penalty $\xi(D)$ as well. Then define the sum of the expected costs each buyer incurs when he defaults $D \in [0, P_1]$ in period 1 as follows:

$$\psi(D) \equiv q(D)U_2 + \xi(D). \tag{5}$$

Each buyer then chooses $D \in [0, P_1]$ to maximize $D - \psi(D)$. In what follows we call ψ *punishment policy* instead of q and ξ. Note that $0 \leq \xi(D)U_2 + H$ for all $D \in [0, P_1]$.

We define the *bang-bang punishment* policy $\tilde{\psi}$ as follows: each buyer is imposed no penalties, zero, if he does not default more than the *maximum forgiveness*, denoted by $\tilde{D}(P_1) \geq 0$, while he is imposed the largest possible penalty $U_2 + H$ otherwise respectively. More formally, the bang-bang punishment policy $\tilde{\psi}$ is defined as follows:

$$\tilde{\psi}(D) \equiv \begin{cases} 0 & \text{if } D \leq \tilde{D}(P_1) \\ U_2 + H & \text{otherwise} \end{cases} \tag{6}$$

We denote by $C^d = \{P_1, A_1, P_2, A_2, q, \xi\}$ a contract offered by the seller to each buyer under the credit-based selling at the beginning of period 1. Then we say that a contract C^d is optimal when it maximizes the seller's payoff (?) subject to (??).

We show that we can confine our attention to the bang-bang punishment policy without loss of generality.

Proposition 1. *Suppose that the seller adopts the credit-based selling in period 1. Then the bang-bang punishment policy defined as (?) becomes optimal. Furthermore, buyers are induced to default up to the maximum forgiveness $\tilde{D}$ in period 1 but are never punished under the optimal contract.*

Proof. See the Appendix.

The key prediction of Proposition 1 is that, even when buyers default strictly *positive* amounts in equilibrium, they are never punished at all: equilibrium punishment becomes zero. This implies that defaulting buyers are never excluded from participating in the second-period transactions and are imposed no utility-based penalties such as violence even for defaulting on paybacks. Therefore, the seller optimally adopts the "free-first-dose" strategy such that she induces buyers to purchase the addictive goods initially by allowing them to default without punishment and then fosters addictions, resulting in large consumption of those goods in the later period. This tactic is often adopted by sellers of addicted goods such that individuals get goods for free initially and increase the willingness to pay for the goods later after they are addicted. We discuss several case studies that are relevant to this strategy in Section 8.

We provide the intuition behind this result as follows. Consider any arbitrary punishment policy ψ as depicted in Figure 1. Given ψ, each buyer optimally defaults $D^* \in [0, P_1]$ on his payment P_1 at the end of period 1, giving him the expected costs of default $\psi(D^*) = q(D^*)U_2 + \xi(D^*)$ in period 2.

Now, consider the new punishment policy. First, define the maximum forgiveness as

$$\tilde{D} \equiv D^* - \{\psi(D^*) - \underline{\psi}\}$$

where $\underline{\psi} \equiv \inf_{0 \leq D \leq P_1} \psi(D)$ is the lowest possible penalty under the original policy ψ. Then define the new punishment policy $\tilde{\psi}$ as follows:

$$\tilde{\psi}(D) = \equiv \begin{cases} \underline{\psi} & \text{if } D \leq \tilde{D} \\ \underline{U}_2 + H & \text{otherwise} \end{cases}$$

for the maximum forgiveness $\tilde{D}$ as defined above, where each buyer is never punished unless he defaults more than $\tilde{D}$ while he is most harshly punished otherwise (Figure 2). Note that the maximum forgiveness $\tilde{D}$ defined above is not larger than the optimal default D^* chosen by the buyer under the original punishment policy ψ.

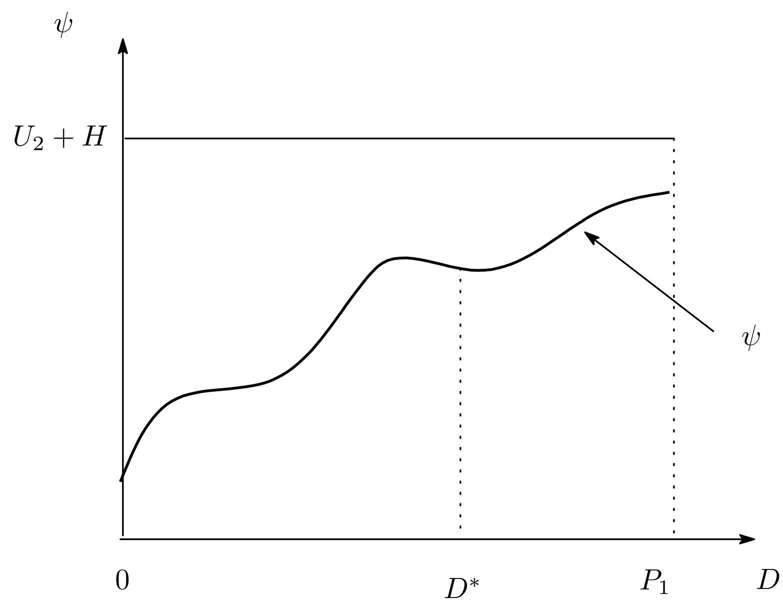

Figure 3: Punishment policy

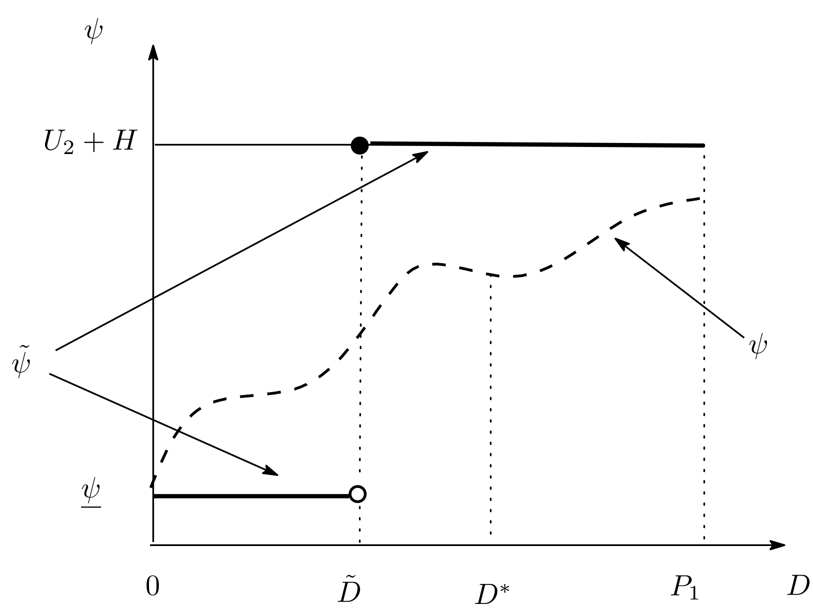

Figure 4: Bang-Bang Punishment policy

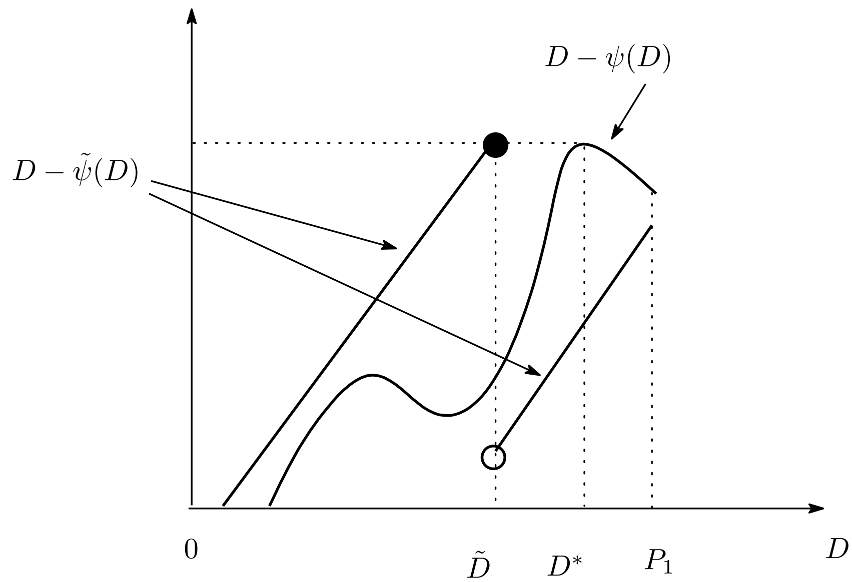

Figure 5: Buyer's Payoffs

Under the original punishment policy ψ each buyer obtains the continuation payoff of $D - \psi(D)$ at the end of period 1, as depicted in Figure 3. Under the newly defined policy $\tilde{\psi}$ the buyer's payoff is changed to $D - \tilde{\psi}(D)$ that is discontinuous at the maximum forgiveness $\tilde{D}$. As depicted in Figure 3, the buyer's payoff under $\tilde{\psi}$ is maximized at $D = \tilde{D}$, and attains the same maximum payoff as what he obtains under the original policy ψ. Therefore, buyers' incentive to default is not changed by the new policy $\tilde{\psi}$. By this new policy the seller can improve her payoffs because the defaulted amounts $\tilde{D}$ can be reduced from D^* to $\tilde{D}$. Furthermore, the seller can increase her payoffs by setting the lowest possible penalty $\underline{\psi}$ to be zero: if the seller reduces the probability q to exclude buyers, more buyers are encourage to purchase the good in period 2, increasing the seller's payoff in period 2. Therefore the bang-bang punishment policy defined as (??) becomes optimal.

5 Optimal Selling Strategy

Given the results obtained above, we investigate which cash-based or credit-based selling is optimal for the seller to choose at the beginning of period 1.

First we show that the buyer's wealth w_2 in the second period plays a crucial role to determine the optimal selling strategy. Specifically, if buyers own sufficiently large wealth in period 2 and the use of utility-based penalties is severely limited, then cash-based selling becomes always optimal.

Proposition 2. *Suppose that w_2 is sufficiently large and that H is sufficiently small. Then, by using cash-based selling, the seller can attain at least the same payoff as what she* *obtains under credit-based selling.*

Proof. We denote by $C^d = \{A_1^d, P_1^d, A_2^d, P_2^d, \psi\}$ the optimal contract under credit-based selling. Then the buyer's IC constraint implies that

$$v(A_1^d) - P_1^d + \tilde{D} + \{u(A_2^d, S^d) - P_2^d\} \geq v(A_1^d) - H,$$

which shows that

$$P_1^d - \tilde{D} \leq H + \{u(A_2^d, S^d) - P_2^d\}.$$

Then the seller's payoff under C^d is bounded above by

$$\begin{aligned}
\pi^d &= P_1^d - \tilde{D} - c_1 A_1^d + P_2^d - c_2 A_2^d \\
&\leq H + \{u(A_2^d, S^d) - P_2^d\} - c_1 A_1^d + P_2^d - c_2 A_2^d \\
&= H + u(A_2^d, S^d) - c_2 A_2^d - c_1 A_1^d.
\end{aligned}$$

Since $\pi^d \geq 0$ must be satisfied in equilibrium, we have

$$H + u(A_2^d, S^d) - c_2 A_2^d \geq c_1 A_1^d$$

6 Model

6.1 Gambling and Preference for Addiction

We present the model of an illegal market where a seller sells addictive goods to buyers who form the tastes for addictions over time and their transactions are illegal. Specifically, to fix the idea, we consider the gambling market for horse race betting. We will discuss how our model can be applied to other illegal markets than horse race betting in Section 7.3 later.

There are two periods, denoted by $t = 1, 2$, and a bookmaker runs a horse race betting in each period. There are n gamblers and m horses where $n \geq 2$ and $m \geq 2$. We denote by I and J the sets of gamblers and horses. We use the feminine pronoun for the bookmaker and masculine pronoun for each gambler in what follows.

To model gamblers' preferences, we follow the "gambling-in-utility" approach (Conlisk (1993)) as follows. Gamblers have the preferences over both gambling activities and private consumption of the numèraire good in each period. Gamblers decide how much amounts to bet on each horse in terms of the numèraire good. We denote by $a_{i,j,t} \geq 0$ the amount bet by gambler i on horse j in period $t = 1, 2$, and by $\mathbf{a}_{i,t} = (a_{i,1,t}, ..., a_{i,m,t})$ a profile of amounts bet by gambler i on horses in period $t = 1, 2$. We also define by $A_{i,t} \equiv \sum_{j \in J} a_{i,j,t}$ the amount bet by gambler i in total in period $t = 1, 2$. When gambler i bets $A_{i,1}$ in total and consumes $c_{i,1}$ units of the private good in period 1, his payoff in period 1 becomes $v(A_{i,1}) + c_{i,1}$. Here $v(A_{i,1})$ is the gambler i's payoff of gambling activities by betting $A_{i,1}$ in total in period 1. We assume that v is continuously differentiable, strictly concave, and

$v(0) = 0$. Additionally, gambler i obtains the payoff of $u(A_{i,2}, S_i) + c_{i,2}$ in period 2 when he bets $A_{i,2}$ in total and consumes $c_{i,2}$ units of the private good in period 2. Here the payoff obtained from gambling activities in period 2 is given by $u(A_{i,2}, S_i)$ that depends on how much to bet $A_{i,2}$ in total in period 2 as well as how much to be addicted to gambling, which we capture by the *addiction stock* S_i (see Becker and Murphy (1988) and Rozen (2010) for related approaches):

$$S_i = \alpha + \beta A_{i,1} \tag{7}$$

where $\alpha \geq 0$ and $\beta > 0$. Due to $\beta > 0$, the addiction stock S_i positively depends on how much gambler i bet in period 1. Furthermore, $\alpha \geq 0$ represents the fixed benefit from participating in the gambling in period 1: $\alpha > 0$ when $\sum_{j \in J} a_{i,j,1} > 0$ and $\alpha = 0$ otherwise. Therefore, gamblers develop more addiction preferences in period 2 by accumulating the addiction stock $S_i = \alpha + \beta A_i$ which increases as they engage in more gambling activities in period 1 (they place larger bet A_i in period 1).

We assume that the second-period utility u is continuously differentiable and strictly concave with respect to $A_{i,2}$ with $u(0, S) = 0$ as well as it is increasing in the addiction stock S_i. In what follows we denote by u_A and u_S partial derivatives of u with respect to $A_{i,2}$ and S_i: $u_A \equiv \partial u / \partial A$ and $u_S \equiv \partial u / \partial S$. We assume the boundary conditions on u_A such that $u_A(0, S) > 1$ and $u_A(\infty, S) < f$ for all $S \geq 0$, which will ensure the interior solution.

In summary gambler i obtains total payoffs over two periods as follows

$$v\left(A_{i,1}\right) + c_{i,1} + \left\{u\left(A_{i,2}, S_i\right) + c_{i,2}\right\} \tag{8}$$

where we assume no time discounting between the two periods for simplicity.

The bookmaker is risk-neutral and maximizes the profits over two periods by running the horse race betting, as we will define below.

6.2 Gambling Market for Horse Race Betting

Gamblers participate in the gambling market for horse race betting in period $t = 1, 2$. In each period we consider the so called *parimutuel betting* that is common in many wagering markets including not only horse race betting but also other sports betting. In the parimutuel betting gamblers decide how much bet to place before they know the odds of outcomes. In particular we focus on the "win bet" that gamblers bet on which horse to come first, given expectations about the odds of horses.

To save notation, we drop time index $t = 1, 2$ from amounts bet by gamblers $\mathbf{a}_{i,t}$ and consider how the gambling market works in each period. To simplify, suppose that all gamblers in I participate in the horse race betting. Given a collection of amounts bet by all gamblers $\{\mathbf{a}_i\}_{i \in I}$ where $\mathbf{a}_i = (a_{i,1}, ..., a_{i,m})$ for $i \in I$, the bookmaker receives the commission fee ("house take") as a constant fraction $f \in (0, 1)$ of the aggregate amount bet $\sum_{i \in I, j \in J} a_{i,j}$, that is, the bookmaker receives the commission fee equal to $f \sum_{i \in I, j \in J} a_{i,j}$. We will maintain throughout the basic model the assumption that the commission fee

$f \in (0, 1)$ is exogenously given.[8] We will discuss the extension of the model to allow endogenous commission fees in Section 7.1.

In the parimutuel betting the bookmaker does not face a market risk; the winning bets are paid with the losing bets, net of the fee f. The odds of horse j, denoted by h_j, is defined as follows:

$$h_j \equiv \frac{(1-f)\sum_{i\in I, j\in J} a_{i,j}}{\sum_{i\in I} a_{i,j}}, \tag{9}$$

provided the denominator is positive.[9] [10] That is, the odds of horse j is determined by reflecting how much gamblers bet on horse j relative to the aggregate amounts bet on all horses. Specifically, the odds of horse j is higher as gamblers bet less for it. We define $h_j \equiv 0$ when nobody bets on horse j.

We denote by $p_j \in (0, 1)$ the probability that horse j wins the race. The returns of horse racing are determined as follows: when horse j wins the race, gambler i who places bet of $a_{i,j} > 0$ on horse j will receive from the bookmaker the payout equal to the odds of horse j times the amount he bet on horse j, that is, $h_j a_{i,j}$. When gambler i bets $A_i \equiv \sum_{j\in J} a_{i,j}$ in total, his net return equals to $h_j a_{i,j} - A_i$, which is positive (when he wins) or negative (when he loses).[11]

6.3 Default and Punishment

The gambling market for horse race betting is illegal so that there are no formal institutions to enforce payments from gamblers to the bookmaker.

There are two forms of betting: cash-bet format and book-bet format. Under the cash-bet, gamblers decide how much to bet and then immediately pay the bookmaker the chosen amount. Under the book-bet, gamblers also decide on their bet amount, but payment is deferred. They are only required to settle their debts after a certain period of time. The amount they owe is the initial bet minus any winnings and potential additional fees. Essentially, the book-bet functions as a loan from the bookmaker. In Pakistan's illegal horse gambling market, book-bets are commonly utilized, with the expectation that gamblers repay their debts within a week. However, since the gambling market is illegal and not formally organized, gamblers may renege on debt paybacks to the bookmaker. Indeed,

[8] The commission/participation fee of the bookmaker is about 5% in the case of the illegal horse racing we observed in Pakistan.

[9] The odds h_j defined here represents the gross return of betting one dollar on winning horse j. Alternatively, we can call its net return $h_j - 1$ odds.

[10] Levitt (2004) shows that bookmakers might also take some market risks when setting odds. Accounting for this would slightly change equation (9), although the logic and the results of this paper would remain unchanged.

[11] There may be the case that horse j wins the race but nobody bets on that horse. Then the practice used in that case may be that each gambler i is refunded $(1-f)A_i$ which equals to the amount bet A_i minus the commission fee of the bookmaker fA_i. However, since we will focus on the equilibrium in which odds are positive for any horse as we will explain more details later, such case never arises.

when gamblers use a book-bet strategy and lose, they may refuse to pay the bookmaker. The bookmaker cannot however legally force gamblers to pay back their debts, because the gambling is illegal.

The bookmaker chooses which cash-bet or book-bet to use in period 1, although she must rely only on the cash-bet in period 2 because period 2 is the last period, so gamblers will never make paybacks in period 2 when the gambling is held as the book-bet in period 2. We will show later that, even though the book-bet causes default risks to the bookmaker, she may prefer the book-bet to the cash-bet.

In order to deter the default by gamblers under the book-bet the bookmaker can resort to a punishment policy announced in period 1. More specifically, we consider two instruments to punish the gamblers who default on debt repayments, both of which are relevant in illegal markets where monetary penalties are difficult to enforce formally due to the lack of legal commitments.

First, the bookmaker can use *the exclusion policy* q that specifies the probability $q \in [0, 1]$ to exclude each defaulting gambler from the participation in the second-period gambling. In illegal gambling markets it is relevant and effective to prevent those who defaulted on payments in the past from participating in the future gambling. For example, Mehmood and Chen (2022) empirically assess this sort of punishment in illegal horse race betting in Pakistan.[12] Suppose that a gambler owes a debt $z \geq 0$ to repay to the bookmaker but defaults an amount $D \in [0, z]$ in the end of period 1; he pays back only $z - D$ to the bookmaker in the end of period 1. Then the gambler is excluded from the second-period gambling with the probability $q(D, z, A) \in [0, 1]$ that is contingent on how much to bet A in total and how much to owe debt z and default $D \in [0, z]$ in period 1.

Second, the bookmaker can also impose non-monetary sanctions on defaulting gamblers. For example, as often discussed about illegal markets, the buyers who defaulted on payments may face the violence threatened by sellers and associated crime groups. Additionally, buyers may lose social reputation and suffer from being labelled bad social images when they default promised payments (see Mehmood and Chen (2022) for related penalties imposed on defaulting gamblers in the illegal horse race betting in Pakistan.) We denote by $\xi(D, z, A) \in [0, H]$ such an utility-based penalty imposed on the gambler who places bet of $A \geq 0$ in total, and owes debt of $z \geq 0$ but defaults $D \in [0, z]$ in period 1.[13] Here $H \geq 0$ is the exogenous upper bound for available utility-based penalties. For example, the availability of violence as a sanction on defaults depends on legal and police institutions of a society, which we capture by the exogenous upper bound for possible penalties $H \geq 0$. We can allow the case that the utility-based penalty is never available, that is, $H = 0$.

[12] Mehmood and Chen (2022) consider two types of exclusion policies called local blacklisting and global blacklisting in their empirical study about the illegal horse race betting in Pakistan: under the former defaulting gamblers are excluded from participating in the betting station where they default while under the latter defaulting gamblers are excluded from not only the betting station where they default but also from other betting stations.

[13] See Dubey, Geanakoplos and Shubik (2005) for a related approach to default and punishment in market equilibrium framework.

In the beginning of period 1 the bookmaker offers and commits to a *punishment policy* $\{q, \xi\}$.

Two remarks are in order.

First, to simplify, we suppose throughout the main analysis that the default incentive is relevant only for gamblers under the book-bet while it is not for the bookmaker. That is, the bookmaker cannot renege on payouts to gamblers once the market odds are determined. We discuss the mechanism by which the bookmaker can endogenously commit to enforce payouts as well as punishment policies in Section 7.2 and the Online Appendix. In particular we present the dynamic model in which the bookmaker who is a long-lived player interacts with gamblers who are short-lived players over infinite periods. Then we show that there exists an equilibrium in which the bookmaker self-enforces agreed upon payouts and punishment policies. In this respect we can view the current two-period setting as a short-cut of such a dynamic equilibrium.[14] As a result, the bookmaker can make promised payouts to gamblers according to the market odds $\{h_j\}_{j \in J}$.

Second, because of some exogenous reasons such as institutional and historical constraints, the bookmaker cannot use other mechanisms than the punishment policy $\{q, \xi\}$ we mentioned above. In other words there are limits on available instruments which the bookmaker can use. For example, the commission fee $f \in (0, 1)$ is exogenous so that the bookmaker cannot control it, and it cannot be negative. Also the bookmaker cannot charge fixed fees to extract surplus from gamblers at the outset of period 1.[15] The main purpose of the paper is not to consider general mechanism design problems but rather focus on the realistic situation in which the bookmaker can use only limited set of instruments. This is the reasonable scenario when informal rules and institutions governing illegal gambling markets have been already established historically so that the current bookmakers cannot drastically change them. We will discuss how our main insights still remain valid even when the bookmaker chooses commission fees in Section 7.1. We will also show that commission fees are endogenously constrained by the commitment problem in illegal gambling markets, thereby justifying our assumption that the bookmaker has only limited instruments (see Section 7.2 and the Online Appendix).

[14] Besides the dynamic mechanism we will discuss later, there are other justifications for the bookmaker to commit to payouts and punishment policies. For instance, the bookmaker may hire agents who are paid constant wages and work on her behalf. These agents are simply required to execute the payout and punishment policy according to ex ante specified rule. Then, since they are paid constant wages regardless of horse racing outcomes, they have no incentives to deviate from the ex ante specified rule. In fact, gambling may be managed by several betting stations but not the bookmaker herself. This might play a role for the bookmaker to commit to payouts and punishment policy.

[15] If the bookmaker can charge both commission fees and fixed fees without any restriction, she would set commission fees equal to zero in both periods together with a certain fixed fee charged in the beginning of period 1 such that all the surplus of gamblers is extracted. However, such scheme encounters the difficulty to enforce in illegal markets where the bookmaker may renege on payouts to gamblers. As we will show in the extended model in Section 7.2, it becomes more difficult for the bookmaker to commit to payouts when the commission fee f is smaller. Therefore, transfer schemes with zero commission fees may not be self-enforcing.

Additionally, the punishment policy $\{q, \xi\}$ is anonymous in the sense that it relies only on the information about how much to bet A in total and how much to default $D \in [0, z]$ but not on other detailed information such as the identities of gamblers.

Let $I_2 \subseteq I$ denote the set of gamblers who participate in the gambling market in period 2. Then the bookmaker obtains aggregate commission fees minus defaulted amounts, that is, her profits over two periods are given as follows

$$\begin{aligned}
\Pi &\equiv \sum_{j \in J} \mathbf{1}_j \left\{ \sum_{i \in I} \{A_{i,1} - h_j a_{i,j,1} - D_{i,j}\} \right\} + \sum_{i \in I_2} f A_{i,2} \\
&= \sum_{i \in I} \left\{ f A_{i,1} - \sum_{j \in J} \mathbf{1}_j D_{i,j} \right\} + f \sum_{i \in I_2} A_{i,2}
\end{aligned}$$

where $\mathbf{1}_j \in \{0, 1\}$ and it takes one when horse j wins the race, and $D_{i,j} \geq 0$ denotes the amount defaulted by gambler i conditional on horse j winning the race in period 1 (define $I_2 = I$ and $D_{i,j} = 0$ for all $j \in J$ and all $i \in I$ when the cash-bet is adopted in period 1).[16] Note that, since the cash-bet is used in period 2, gamblers never default in period 2.

6.4 Gambling Equilibrium

We now define equilibrium in the gambling market for horse race betting in each period, $t = 1, 2$, given a gambling format (cash-bet or book-bet). The gambling market opens for horse race betting after the bookmaker chooses its format, the cash-bet or the book-bet, in each period.[17] In the parimutuel betting gamblers do not know market odds when they decide how much to bet on horses. One reasonable equilibrium concept used in such a market is *rational expectations equilibrium* in which (i) gamblers optimally decide how much to bet on horses $\mathbf{a}_i = (a_{i,1}, ..., a_{i,m})$ by forming expectations about market odds $(h_1, ..., h_m)$, and (ii) market odds $(h_1, ..., h_m)$ are determined according to condition (3) in order to be consistent with the amounts $(\mathbf{a}_1, ..., \mathbf{a}_n)$ bet by gamblers in the market. Therefore, the gamblers' expectations about the odds are self-fulfilled in equilibrium. In this definition each gambler takes his expectations about market odds $(h_1, ..., h_m)$ as given, and hence believes that he cannot influence the determination of market odds. This is the reasonable case when the number of gamblers n is so large that each of them has no market powers, as considered in the standard notion of competitive equilibrium.

We call a profile of amounts bet by gamblers and the odds $\{\mathbf{a}_1, ..., \mathbf{a}_n, h_1, ..., h_m\}$ that satisfy these conditions (i) and (ii) *gambling equilibrium*. In our two-period model we will look for a gambling equilibrium in each period $t = 1, 2$, given the bookmaker's strategy.

[16] Note that the aggregate payments from gamblers to the bookmaker without defaults conditional on horse j winning the race, defined as $\sum_{i \in I}(A_i - h_j a_{i,j})$, equal to the aggregate commission fees $f \sum_{i \in I} A_i$ received by the bookmaker due to the definition of odds (3).

[17] As we noted, only the cash-bet is used in period 2 because it is the last period so that gamblers never repay debts under the book-bet.

In particular, to avoid complication, we will focus on the gambling equilibrium in which every horse attracts positive amounts bet, so the odds of all horses are positive, $h_j > 0$ for all $j \in J$.[18] [19]

6.5 Timing of the Game

We consider the following timing of the games under the cash-bet and the book-bet used in period 1 (see Figure 1 for the cash-bet and Figure 2 for the book-bet). In either case of adopting the cash-bet or the book-bet in period 1, we model the market outcome in each period $t = 1, 2$ as the gambling equilibrium defined above.

When the bookmaker uses the cash-bet in both periods, the following game is repeated twice:

1. Gamblers decide how much to bet $\mathbf{a}_i = (a_{i,1}, ..., a_{i,m})$, given expectations about the odds $\{h_j\}_{j \in J}$. Then gamblers immediately pay the bookmaker the amounts they bet.

2. Given amounts $\{\mathbf{a}_i\}_{i \in I}$ bet by gamblers, the equilibrium odds $\{h_j\}_{j \in J}$ are determined by condition (3).

3. The wining horse is determined, and the bookmaker makes payouts to gamblers.

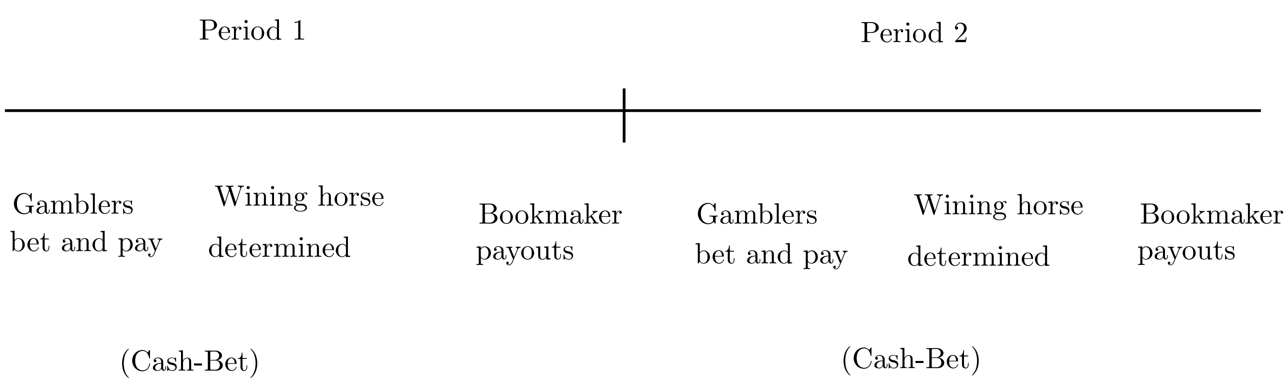

Figure 6: Timing of the game under cash-bet

Next we consider that the bookmaker uses the book-bet in period 1 while using the cash-bet in period 2. As we mentioned, gamblers may default on paybacks under the book-bet because they defer payments to the end of period 1 and no formal contracts exist to

[18] In such equilibrium there are no horses which win the race but nobody bet on: no gamblers are refunded $(1 - f)A_i$.

[19] This restriction can be justified when each gambler places a small but positive bet $\varepsilon > 0$ on each horse by a "mistake". Alternatively, there are some irrational gamblers who always bet a positive but small amount $\varepsilon > 0$ on each horse. Then our equilibrium might be viewed as the limit as such irrational/mistaken bet ε goes to zero.

enforce their payments. When gambler i bets $\mathbf{a}_i$ and horse j wins the race, he receives the payout $h_j a_{i,j}$, which equals to horse j's odds h_j multiplied by his bet $a_{i,j}$ on horse j, from the bookmaker and pays the amount bet A_i in total, thereby obtaining the net return of $h_j a_{i,j} - A_i$ which is positive or negative. Then gambler i owes the debt equal to $z_{i,j} \equiv \max\{A_i - h_j a_{i,j}, 0\}$ and can default any amount $D_{i,j} \in [0, z_{i,j}]$, in which case he will pay back only $z_{i,j} - D_{i,j}$ to the bookmaker.

The game under the book-bet proceeds as follows.

In period 1:

1. The bookmaker offers a punishment policy $\{q, \xi\}$.
2. Gamblers decide how much to bet $\mathbf{a}_i = (a_{i,1}, ..., a_{i,m})$, given expectations about the odds $\{h_j\}_{j \in J}$.
3. Given amounts $\{\mathbf{a}_i\}_{i \in I}$ bet by gamblers, the equilibrium odds $\{h_j\}_{j \in J}$ are determined by condition (3).
4. The wining horse is determined.
5. Gamblers decide how much to default on paybacks. When gambler i defaults an amount of $D_{i,j} \in [0, z_{i,j}]$, he will be excluded from the second-period gambling with probability $q(D_{i,j}, z_{i,j}, A_i) \in [0, 1]$ as well as he will be imposed an utility-based penalty $\xi(D_{i,j}, z_{i,j}, A_i) \in [0, H]$.

In period 2:

1. Those who were not excluded in the end of period 1 decide how much to bet under the cash-bet, given expectations about the odds.
2. The equilibrium odds are determined by condition (3), the winning horse is determined, and the bookmaker makes payouts to gamblers.

We proceed to show the equilibrium of the two-period game described above as follows. First, we provide the equilibrium outcome in period 2, in which the bookmaker holds the cash-bet. Given this, we then show that the optimal punishment policy for the bookmaker under the book-bet in period 1 becomes the bang-bang form and has the feature of "free-first-dose" strategy such that gamblers are allowed to default but are never punished in equilibrium.

7 Gambling Equilibrium in Period 2

We begin with gambling equilibrium in period 2, given the addiction stock of gamblers $(S_1, ..., S_n)$ that has been already determined by the amounts they bet in period 1, $(A_{1,1}, ..., A_{n,1})$, where $S_i = \alpha + \beta A_{i,1}$.

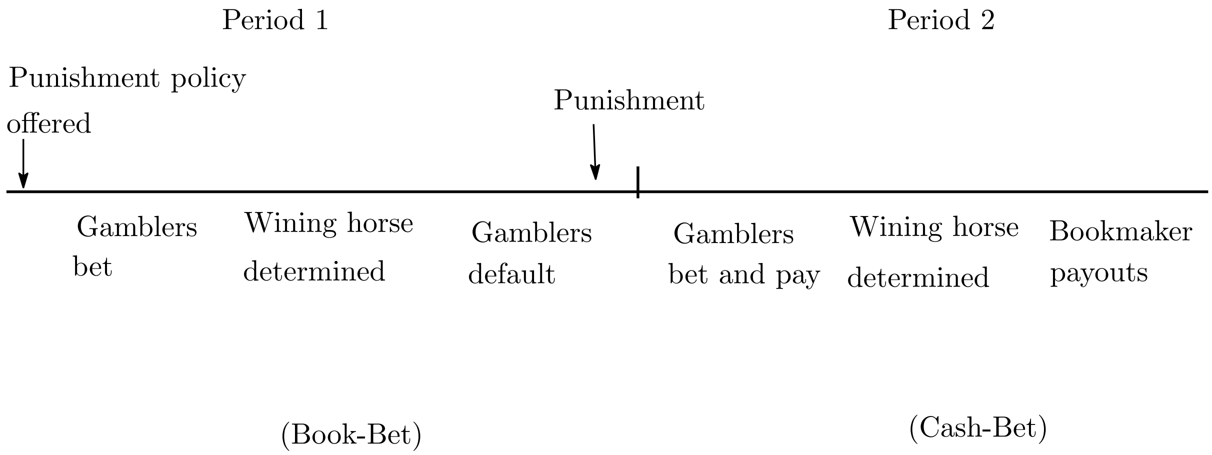

Figure 7: Timing of the game under book-bet

As mentioned, the cash-bet is used for the horse race betting in period 2 regardless of the gambling format in period 1. Suppose that gamblers in $I_2 \subseteq I$ participate in the gambling in period 2. When the book-bet is used in period 1, some of them may be excluded so that $I_2 \neq I$ may be the case, according to a exclusion probability q specified in the punishment policy, while we have $I_2 = I$ when the cash-bet is used in period 1. In either case we simplify the following argument by setting $I_2 = I$; all gamblers participate in period 2. To save notation, we also drop time index $t = 2$ from subscripts of all the variables in this section.

Gambler i's payoff in period 2 is given by

$$u\left(A_i, S_i\right) + c_{i,j} \tag{10}$$

where $c_{i,j}$ is his private consumption of the numèraire good when horse j wins the race, and it equals to the net return $R_{i,j}$ he earns, given by $c_{i,j} = R_{i,j} \equiv h_j a_{i,j} - A_i$.[20] Then, gambler i places bet of $\mathbf{a}_i$ to maximize his expected payoff

$$u\left(A_i, S_i\right) + \sum_{j \in J} p_j \left\{h_j a_{i,j} - A_i\right\} \tag{11}$$

given expectations about the odds $\{h_j\}_{j \in J}$.

We denote by $\hat{a}_{i,j} \geq 0$ the equilibrium amount bet by gambler i on horse j, and by $\hat{A}_i \equiv \sum_{j \in J} \hat{a}_{i,j}$ the aggregate amount bet by gambler i. Let also denote by $\hat{A} \equiv \sum_{i \in I} \hat{A}_i$ the aggregate amount bet by all gamblers on all horses and by $\hat{a}_j \equiv \sum_{i \in I} \hat{a}_{i,j}$ the aggregate amount bet by all gamblers on horse $j \in J$.

[20] We here allow $c_{i,j}$ to be negative for simplicity. We can however ensure non-negative consumption $c_{i,j} \geq 0$ by assuming that gamblers are exogenously endowed a large income $w > 0$ in each period.

Then, according to (3), equilibrium odds $\{\hat{h}_j\}_{j\in J}$ are determined as follows[21]

$$\hat{h}_j = \frac{(1-f)\hat{A}}{\hat{a}_j}, \quad \text{for } j \in J. \tag{12}$$

We then obtain the following proposition (we relegate all proofs to the Appendix.)

Proposition 1. *Given a collection of addiction stocks of gamblers $(S_1, ..., S_n)$, gambling equilibrium in period 2 is characterized as follows:*

- *Gambler i who has addiction stock S_i bets the amount $\hat{A}_i = \hat{A}(S_i)$ in total satisfying*

$$u_A(\hat{A}_i, S_i) - f = 0. \tag{13}$$

- *Gambler i who has addiction stock S_i obtains the equilibrium payoff given as follows:*

$$\hat{U}(S_i) \equiv u(\hat{A}(S_i), S_i) - f\hat{A}(S_i). \tag{14}$$

Proposition 1 states that gambler i's equilibrium aggregate bet $\hat{A}(S_i)$ is determined by the first-order optimality condition (7), which depends on his addiction stock S_i. When the gambler's marginal utility of betting u_A increases as he is addicted more (that is, $u_{AS} \equiv \partial u^2/\partial A\partial S > 0$), he will bet more aggressively as he is addicted more; $\hat{A}$ is increasing in S_i. This result implies that the bookmaker has the incentive to let gamblers bet more in period 1, and hence make them addicted more, inducing larger amounts bet in period 2. Also, as we can see from (8), gambler's equilibrium payoff of $\hat{U}(S_i)$ increases with his addiction stock S_i; $\hat{U}'(S_i) = u_S(\hat{A}(S_i), S_i) > 0$. Note also that $\hat{U}(S) > 0$ holds due to $u_A(0, S) > f$

By Proposition 1, the bookmaker obtains the payoff of $f\sum_{i\in I}\hat{A}(S_i)$ in period 2, which equals to the commission fee times the aggregate amount bet by participating gamblers.[22]

8 "Free-First-Dose" Strategy

8.1 Bang-Bang Punishment Policy

We move to period 1 and characterize the optimal punishment policy $\{q, \xi\}$ chosen by the bookmaker in the beginning of period 1, provided that the book-bet is chosen in period 1. Then we show that the bookmaker optimally adopts the 'free-first-dose' strategy in period 1 by reducing initial constraints weighing on gamblers to foster addictions. In order to save notations, we drop time index $t = 1$ from the variables chosen in period 1. The equilibrium

[21] Note that we are considering the equilibrium in which $\hat{h}_j > 0$ for all $j \in J$, so $\hat{a}_j > 0$ for all $j \in J$.

[22] When $I_2 \neq I$, the bookmaker's payoff in period 2 is modified to $f\sum_{i\in I_2}\hat{A}(S_i)$.

outcome in the second-period game is summarized by the aggregate amount bet $\hat{A}(S_i)$ and equilibrium payoff $\hat{U}(S_i)$ for gambler i, as shown by (7) and (8) in Proposition 1.

Under the book-bet, gambler i does not immediately pay the amount bet $A_i = \sum_j a_{i,j}$ to the bookmaker, but it is deferred in the end of period 1. When horse j wins the race, gambler i has the debt of $z_{i,j} \equiv \max\{0, A_i - h_j a_{i,j}\}$. Given his debt $z_{i,j}$, gambler i can default any amount of $D_{i,j}$ such that

$$0 \leq D_{i,j} \leq z_{i,j}. \tag{15}$$

When gambler i places bet A in total and then defaults $D \in [0, z]$ in period 1, he will then lose the second-period equilibrium payoff $\hat{U}(S)$ with probability $q(D, z, A) \in [0, 1]$ and be imposed an utility-based penalty $\xi(D, z, A)$ in the end of period 1. In total gambler i incurs the expected loss of

$$\psi(D, z, A) \equiv q(D, z, A)\hat{U}(S) + \xi(D, z, A) \tag{16}$$

by defaulting an amount $D \in [0, z]$ in period 1. Note that

$$0 \leq \psi(D, z, A) \leq \hat{U}(S) + H, \quad \text{for any } A \geq 0 \text{ and any } D \in [0, z].$$

Gamblers care only about the total expected loss of default $\psi(D, z, A)$ but not separate values of q and ξ. We will thus call ψ a *punishment policy* in the following.

Given a punishment policy ψ offered by the bookmaker, we consider the resulting equilibrium in period 1 as follows: gamblers decide how much bet to place by forming expectations about the odds $\{h_j\}_{j \in J}$. Then the horse race outcome is realized and the equilibrium odds $\{h_j\}_{j \in J}$ are determined such that the gamblers' expectations are self-fulfilled according to (3). Following this, gamblers decide how much to default, and only those who are not excluded will participate in the gambling market in period 2; they then obtain the second-period equilibrium payoff $\hat{U}(S_i)$ as shown by (8) in Proposition 1.

Suppose that horse j wins the race and gambler i defaults $D_{i,j} \in [0, z_{i,j}]$ in the end of period 1. Then gambler i consumes $c_{i,j} = h_j a_{i,j} - A_i + D_{i,j}$ of the numèraire good in the end of period 1, followed by incurring the expected loss of default $\psi(D_{i,j}, z_{i,j}, A_i)$. Thus gambler i obtains the following *continuation* payoff in the end of period 1:

$$h_j a_{i,j} - A_i + D_{i,j} - \psi(D_{i,j}, z_{i,j}, A_i) + \hat{U}(S_i). \tag{17}$$

Gambler i decides how much to default $D_{i,j} \in [0, z_{i,j}]$ for each winning horse $j \in J$, given the punishment policy ψ and realized odds $\{h_j\}_{j \in J}$. The *ex ante* expected payoff of gambler i in period 1 is given by

$$v(A_i) + \sum_{j \in J} p_j \{h_j a_{i,j} - A_i + D_{i,j} - \psi(D_{i,j}, z_{i,j}, A_i))\} + \hat{U}(S_i). \tag{18}$$

Now we define the "bang-bang" punishment policy $\tilde{\psi}$: each gambler is imposed the lowest penalty $\underline{\psi}(z, A)$ unless he defaults more than a cutoff value $\tilde{D}(z, A)$ while he is imposed the maximum penalty $\hat{U}(S) + H$ otherwise. That is, it is defined as follows:

$$\tilde{\psi}(D, z, A) \equiv \begin{cases} \underline{\psi}(z, A) & \text{if } D \leq \tilde{D}(z, A) \\ \\ \hat{U}(S) + H & \text{otherwise} \end{cases} \tag{19}$$

We call the above cut-off value $\tilde{D}(z, A)$ the *maximum forgiveness*.

Suppose that a punishment policy ψ implements the gambling equilibrium outcome $\{\mathbf{a}^*, \mathbf{h}^*\}$ under the book-bet in period 1 such that gamblers bet $\mathbf{a}^* \equiv \{\mathbf{a}_i^*\}_{i \in I}$ and the associated odds $\mathbf{h}^* \equiv \{h_j^*\}_{j \in J}$ are determined in order to satisfy condition (3) in period 1: $h_j^* = (1-f) \sum_i A_i^* / \sum_i a_{i,j}^*$ for each $j \in J$. Following this, gambler i places the equilibrium bet $\hat{A}(S_i^*)$ in total in period 2 where $S_i^* \equiv \alpha + \beta A_i^*$ is the addiction stock of gambler i in the equilibrium (Proposition 1). Let denote by $\hat{\mathbf{A}} \equiv (\hat{A}_1(S_1^*), ..., \hat{A}_n(S_n^*))$ the profile of these equilibrium bets in period 2. The entire equilibrium outcome in the two-period game is therefore given by the collection $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{A}}\}$. Let denote by $z_{i,j}^* \equiv \max\{0, A_i^* - h_j^* a_{i,j}^*\}$ the equilibrium debt gambler i owes when horse j wins the race.

Then we show that the bookmaker can replace any punishment policy ψ by the bang-bang punishment policy $\tilde{\psi}$ defined as (13) such that she can implement the same equilibrium outcome $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{A}}\}$ as that attained under the original policy ψ, and can be weakly better off by the new policy $\tilde{\psi}$.

Proposition 2. *Suppose that the bookmaker adopts the book-bet in period 1 and that a punishment policy ψ implements an equilibrium outcome $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{A}}\}$. Then there always exists a bang-bang punishment policy $\tilde{\psi}$, defined as (13), such that the bookmaker can implement the same equilibrium outcome $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{A}}\}$, and can be weakly better off from the original punishment policy ψ.*

Proposition 2 shows that we can confine our attention only to the bang-bang punishment policy, defined as (13), without loss of generality.

To see the intuition behind Proposition 2, consider any punishment policy ψ (as given in Figure 3) and suppose that there exists an equilibrium $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{A}}\}$ implemented by ψ. Thus in any subgame given any $A_i \geq 0$ and any $z_{i,j} \geq 0$, gambler i optimally chooses the default level $D_{i,j} \in [0, z_{i,j}]$, denoted by $D^*(z_{i,j}, A_i) \in [0, z_{i,j}]$.[23] Gambler i is then imposed a penalty $\psi(D^*(z_{i,j}, A_i), z_{i,j}, A_i)$, and chooses the optimal default $D = D^*(z_{i,j}, A_i)$ to maximize his continuation payoff $D - \psi(D, z_{i,j}, A_i)$ subject to $0 \leq D \leq z_{i,j}$.

[23] Because we are supposing the existence of the equilibrium $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{A}}\}$, the optimal default choice must exist in any subgame, ensuring that $D^*(z_{i,j}, A_i)$ exists for any $A_i \geq 0$ and any $z_{i,j} \geq 0$.

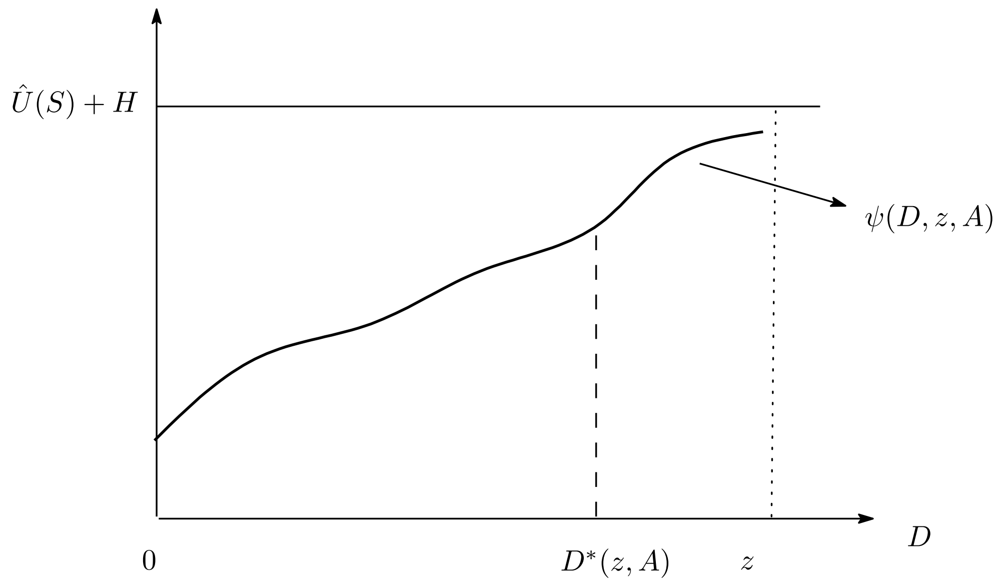

Figure 8: Punishment policy

Then we define the bang-bang punishment policy $\tilde{\psi}$ as follows: First, we define the lowest possible penalty as

$$\underline{\psi}(z,A) \equiv \inf_{0 \leq D \leq z} \psi(D,z,A).$$

Next we set the maximum forgiveness equal to

$$\tilde{D}(z_{i,j},A_i) \equiv D^*(z_{i,j},A_i) - \{\psi(D,z,A) - \underline{\psi}(z,A)\} \tag{20}$$

where we can show that $\tilde{D}(z,A) \geq 0$ (Lemma A1 in the Appendix) and $\tilde{D}(z,A) \leq z$ for all $A \geq 0$ and all $z \geq 0$. In particular $\tilde{D}(z,A) \leq D^*(z,A)$ holds. Then gambler i is punished according to the bang-bang punishment policy $\tilde{\psi}$ defined as (13) for the maximum forgiveness $\tilde{D}(z_{i,j},A_i)$ defined as (14) (see Figure 4).

This newly defined policy can induce gamblers to choose the same amounts bet as those in the equilibrium under the original punishment policy ψ and induce them to default the amount equal to $\tilde{D}(z_{i,j},A_i)$ for each wining horse $j \in J$. First, under the newly defined punishment policy, gambler i never defaults less than the maximum forgiveness, $D < \tilde{D}(z_{i,j},A_i)$. This is because gambler i is never punished as long as $D < \tilde{D}(z_{i,j},A_i)$, but then he can slightly increase the amount to default without being imposed any punishment. Second, gambler i never defaults more than the maximum forgiveness, $D > \tilde{D}(z_{i,j},A_i)$. If this is the case, gambler i obtains the ex post payoff equal to $D - \{\hat{U}(S)+H\} + \hat{U}(S) = D - H$ because he will be then most harshly punished. However, due to $\hat{U}(S_i) \geq 0$ and the

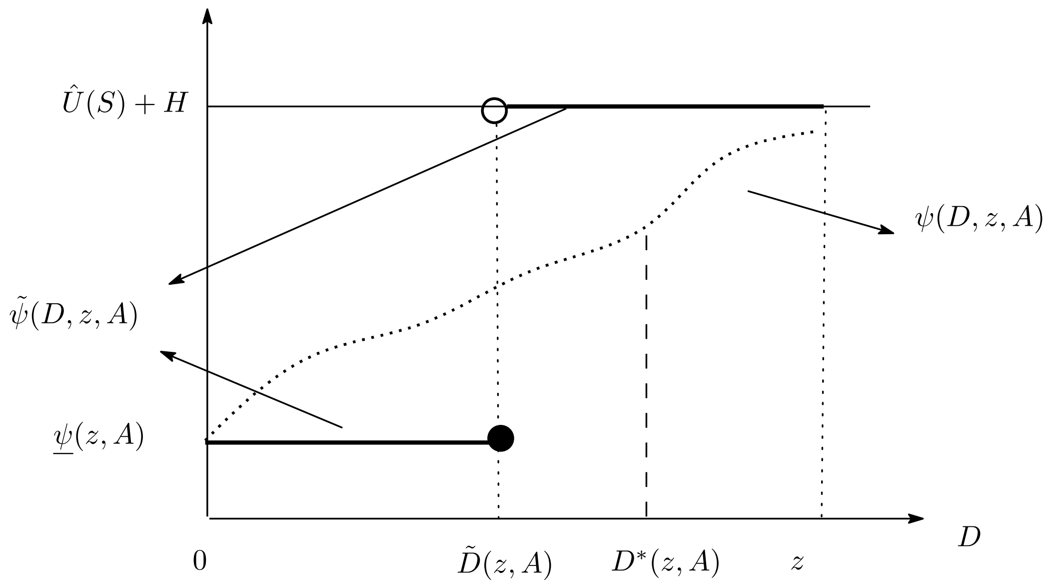

Figure 9: Bang-bang punishment policy

definition of $\tilde{D}(z_{i,j}, A_i)$, we can verify that

$$\begin{aligned} D - H &\leq D - \psi(D, z_{i,j}, A_i) + \hat{U}(S_i) \\ &\leq \max_{D \in [0, z_{i,j}]} D - \psi(D, z_{i,j}, A_i) + \hat{U}(S_i) \\ &= D^*(z_{i,j}, A_i) - \psi(D^*(z_{i,j}, A_i), z_{i,j}, A_i) + \hat{U}(S_i) \\ &= \tilde{D}(z_{i,j}, A_i) - \underline{\psi}(z_{i,j}, A_i) + \hat{U}(S_i) \end{aligned}$$

so that gambler i cannot be better off by deviating from the default of $\tilde{D}(z_{i,j}, A_i)$, in which case he is least harshly punished and obtains the ex post payoff of $\tilde{D}(z_{i,j}, A_i) - \underline{\psi}(z_{i,j}, A_i) + \hat{U}(S_i)$. Therefore, gambler i's optimal choice is to default $D_{i,j} = \tilde{D}(z_{i,j}, A_i)$ when horse j wins the race.

Given the above result, gambler i places bets $\mathbf{a}_i$ to maximize the following expected payoff in period 1:

$$v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + \tilde{D}(z_{i,j}, A_i) - \underline{\psi}(z_{i,j}, A_i) + \hat{U}(S_i)\}. \tag{21}$$

Due to the definition of $\tilde{D}(\cdot, \cdot)$ (see (14)), the above payoff (15) is equivalent to

$$v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + D^*(z_{i,j}, A_i) - \psi(D_j^*(z_{i,j}, A_i), z_{i,j}, A_i)) + \hat{U}(S_i)\},$$

which is same as what gambler i obtains under the original punishment policy ψ and the same odds $\mathbf{h}^*$ as those in the original equilibrium. Therefore, in period 1 gamblers bet the same amounts $\mathbf{a}^*$ as those in the original equilibrium with the same equilibrium odds $\mathbf{h}^*$. This also leads to the same equilibrium amounts bet $\hat{\mathbf{A}}$ as those in the original equilibrium in period 2. Therefore, without loss of generality we can confine our attention to the bang-bang punishment policy $\tilde{\psi}$ defined as (13).

Additionally, we associate the lowest possible probability to exclude defaulting gamblers, defined as $\underline{q}(z^*_{i,j}, A^*_i) \equiv \inf_{0 \le D \le z^*_{i,j}} q(D, z^*_{i,j}, A^*_i)$, with the lowest equilibrium penalty $\underline{\psi}(z^*_{i,j}, A^*_i)$. This can benefit the bookmaker because more gamblers can participate in the second-period gambling than those under the original punishment policy ψ.

The bookmaker hence obtains the following payoff by the bang-bang punishment $\tilde{\psi}$ modified in the above way:

$$\Pi(\tilde{\psi}) \equiv \sum_{i \in I} \left\{ fA^*_i - \sum_{j \in J} p_j \{ \tilde{D}(z^*_{i,j}, A^*_i) + (1 - \underline{q}(z^*_{i,j}, A^*_i)) f \hat{A}(S^*_i) \} \right\}. \tag{22}$$

The bookmaker's payoff under the original punishment policy ψ is given by

$$\Pi(\psi) \equiv \sum_{i \in I} \left\{ fA^*_i - \sum_{j \in J} p_j \{ D^*(z^*_{i,j}, A^*_i) + (1 - q(D(z^*_{i,j}, A^*_i), A^*_i)) f \hat{A}(S^*_i) \} \right\}. \tag{23}$$

We then obtain $\Pi(\tilde{\psi}) \ge \Pi(\psi)$ due to $\tilde{D}(z^*_{i,j}, A^*_i) \le D^*(z^*_{i,j}, A^*_i)$ and $\underline{q}(z^*_{i,j}, A^*_i) \le q(D^*(z^*_{i,j}), z^*_{i,j}, A^*_i)$, thereby showing that the bookmaker can be weakly better off by the bang-bang punishment policy $\tilde{\psi}$. The bang-bang punishment policy $\tilde{\psi}$ can bring two gains to the bookmaker. First, gamblers reduce the amount to default from $D^*(z_{i,j}, A_i)$ to $\tilde{D}(z_{i,j}, A_i)$. Second, more gamblers can participate in the gambling in period 2 so that the market size in period 2 does not shrink under the bang-bang punishment policy. These two positive effects make the bookmaker never worse off by the bang-bang punishment policy.

By using Proposition 2, we show that the bookmaker can maximize her payoff by allowing gamblers to default without any punishment.

Proposition 3. *Suppose that the book-bet is used in period 1 and a bang-bang punishment policy $\tilde{\psi}$ maximizes the bookmaker's payoff $\Pi(\tilde{\psi})$ with its associated equilibrium $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{A}}\}$. Then we obtain the following:*

(i) *Gambler i defaults the amount equal to the maximum forgiveness $\tilde{D}(z^*_{i,j}, A^*_i) \ge 0$ and is never punished, that is, $\underline{\psi}(z^*_{i,j}, A^*_i) = 0$, whenever the defaulted amount $\tilde{D}(z^*_{i,j}, A^*_i)$ is strictly positive on the equilibrium path.*

(ii) *Without loss of generality we can set the lowest expected penalty among gamblers, defined as $\min_{i \in I} \sum_{j \in J} p_j \underline{\psi}(z^*_{i,j}, A^*_i)$, to be to zero.*

The key prediction of Proposition 3 (i) is that, whenever gamblers default strictly *positive* amounts in equilibrium, they are never punished at all: equilibrium punishment becomes zero $\underline{\psi}(z^*_{i,j}, A^*_i) = 0$ on the path of play. This implies that defaulting gamblers are never excluded from participating in the second-period gambling and are imposed no utility-based penalties such as violence even for defaulting on paybacks. Therefore, the bookmaker optimally adopts the "free-first-dose" strategy such that she induces gamblers to place bets initially by allowing them to default without punishment and then fosters addictions, resulting in large bets in the later period. This tactic is often adopted by sellers of addicted goods such that individuals get goods for free initially and increase the willingness to pay for the goods later after they are addicted. We discuss several case studies that are relevant to this strategy in Section 8.

Proposition 3 (ii) shows that we can confine our attention only to the bang-bang punishment policy $\tilde{\psi}$ such that some gambler, say $i \in I$, is imposed no penalties on the equilibrium path regardless of whether he defaults a strictly positive amount. This is because $\min_{i \in I} \sum_{j \in J} p_j \underline{\psi}(z^*_{i,j}, A^*_i) = 0$ and $\underline{\psi}(z^*_{i,j}, A^*_i) \geq 0$ together imply that $\underline{\psi}(z^*_{i,j}, A^*_i) = 0$ is satisfied for any $j \in J$ for such gambler i.

8.2 Optimal Punishment Policy under the Book-bet

We now turn to derive the optimal punishment policy ψ chosen by the bookmaker, provided that she uses the book-bet in period 1. Thanks to Proposition 2, without loss of generality we can focus only on the bang-bang punishment policy $\tilde{\psi}$ defined as (13). In particular, since all gamblers are identical in terms of their payoff functions, we pay our attentions to the symmetric equilibrium in which all gamblers place bets and default according to the same strategy: $a^*_{i,j,1} = a^*_j$ and $A^*_{i,1} = A^*$ for all $i \in I$ in period 1 and $A_{i,2} = \hat{A}(S^*)$ for all $i \in I$ in period 2 where $S_i = S^* \equiv \alpha + \beta A^*$ for all $i \in I$. We denote by $\mathbf{a}^* = (a^*_1, ..., a^*_m)$ an equilibrium profile of amounts bet by each gambler in period 1, where a^*_j is the amount bet by each gambler on horse j, and $A^* = \sum_{j \in J} a^*_j$ is the amount bet by each gambler in total. Thus we obtain the equilibrium odds as $h^*_j = (1 - f)A^*/a^*_j$ for $j \in J$.

We now consider the bang-bang punishment policy $\tilde{\psi}$ that maximizes the bookmaker's payoff $\Pi(\tilde{\psi})$, given by (16), which we can simplify to

$$\Pi(\tilde{\psi}) \equiv n\left\{fA^* - \tilde{D}(z^*, A^*)\right\} + nf\hat{A}(S^*) \tag{24}$$

in the symmetric equilibrium where

$$z^*_j \equiv \max\{A^* - h^*_j a^*_j, 0\} = z^* \equiv fA^*$$

is the equilibrium debt that each gambler owes conditional on horse j wining the race. We call a punishment policy $\tilde{\psi}$ *optimal* when it induces the symmetric equilibrium outcome $\{\mathbf{a}^*, \mathbf{h}^*, \hat{A}(S)\}$ maximizing the bookmaker's payoff $\Pi(\tilde{\psi})$ given by (18).

We say that an aggregate amount bet A^* by each gambler is *implementable* when there exists an equilibrium in which each gambler chooses A^* in total in period 1, defaults the amount equal to the maximum forgiveness $\tilde{D}(z_j^*, A^*)$, and is imposed the lowest penalty $\underline{\psi}(z^*, A^*)$, in association with the corresponding equilibrium odds $\mathbf{h}^* = \{h_j^*\}_{j\in J}$ that satisfies (3) and is consistent with equilibrium amount bet $\mathbf{a}^*$ in period 1. Thanks to Proposition 3 (ii) we can set the lowest penalty $\underline{\psi}(z^*, A^*)$ to be zero without loss of generality in the symmetric equilibrium maximizing the bookmaker's payoff $\Pi(\tilde{\psi})$, given by (18).

For an aggregate amount bet A^* to be implementable, it must satisfy a certain set of constraints by taking into account the default incentive of gamblers.

First, the equilibrium payoff of each gambler is given by

$$\begin{aligned} V^* &\equiv v(A^*) - fA^* + \tilde{D}(z^*, A^*) - \underline{\psi}(z^*, A^*) + \hat{U}(S^*) \\ &= v(A^*) - fA^* + \tilde{D}(z^*, A^*) + \hat{U}(S^*) \end{aligned}$$

where the lowest penalty $\underline{\psi}(z^*, A^*)$ is set to be zero as mentioned above. For each gambler not to deviate from the equilibrium bet A^*, the following *incentive compatibility* (IC) constraint must be satisfied:

$$V^* \geq \max_{\mathbf{a}_i, \{D_{i,j}\}_{j\in J}} v(A_i) + \sum_{j\in J} p_j\{h_j^* a_{i,j} - A_i + D_{i,j} - \tilde{\psi}(D_{i,j}, z_{i,j}, A_i)\} + \hat{U}(S_i). \quad (25)$$

That is, the gambler's equilibrium payoff V^* must be at least as large as the deviation payoff obtained by placing different bets $\mathbf{a}_i \neq \mathbf{a}^*$ from the equilibrium one and making optimal defaults subsequently, yielding the payoff equal to the right hand side of (19).

Second, determining the amount a_j^* bet by each gambler on horse $j \in J$ is equivalent to determining its odds h_j^* given A^*, due to the definition of equilibrium odds $h_j^* = (1 - f)A^*/a_j^*$. Then, since $A^* = \sum_j a_j^*$ holds, we have the following identity:

$$\sum_{j\in J}(1/h_j^*) = 1/(1-f). \quad (26)$$

Third, each gambler cannot default more than his debt $z \geq 0$, that is, the maximum forgiveness $\tilde{D}(z, A)$ must satisfy

$$0 \leq \tilde{D}(z, A) \leq z \quad (27)$$

for all $z \geq 0$ and all $A \geq 0$.

Finally, available penalties are constrained by the upper bound:

$$0 \leq \tilde{\psi}(D, z, A) \leq \hat{U}(S) + H \quad (28)$$

for all $D \in [0, z]$, all $z \geq 0$ and all $A \geq 0$.

We consider the *best* equilibrium that maximizes the bookmaker's payoffs per gambler over two periods $\Pi(\tilde{\psi})/n = fA^* - \tilde{D}(z^*, A^*) + f\hat{A}(S^*)$ subject to all the constraints obtained above, that is, it is determined by the solution to the following problem:[24]

Problem B:

$$\max_{\mathbf{h}^*, A^*, \tilde{\psi}, \tilde{D}} \; fA^* - \tilde{D}(z^*, A^*) + f\hat{A}(S^*)$$

subject to (19)-(22).

We denote by Π^b the maximum value of Problem B, which gives the bookmaker the largest payoff attained under the book-bet.

9 Equilibrium Characterization under the Book-Bet

In this section we characterize the best equilibrium for the bookmaker under the book-bet, which solves Problem B given above. The fundamental difficulty for this purpose is that the IC constraints given by (19) are so complicated that we need to handle many possible deviations of gamblers. First, we address this issue and show the condition under which we can confine our attention only to a single IC constraint without loss of generality, thereby reducing the number of possible deviations by gamblers. Second, given this result, we show that the best equilibrium for the bookmaker under the book-bet exhibits the following interesting features: (i) equilibrium odds have so called *long-shot bias* such that gamblers place more bets on the long-shots which are less likely to win the race, and (ii) equilibrium amounts bet are "socially excessive" in the sense that these are larger than the social optimal bets that maximize the social welfare of the economy.

9.1 Equilibrium Odds: Long-Shot Bias

First, we identify the lower bound for payoffs of each gambler in any symmetric equilibrium under the book-bet. Suppose that gambler i places bet of $\mathbf{a}_i$ in period 1 and then defaults all the debt $z_{i,j} \equiv \max\{A_i - h_j^* a_{i,j}, 0\}$, that is, $D_{i,j} = z_{i,j}$ for all $j \in J$, followed by being punished most harshly $\tilde{\psi}(D_{i,j}, z_{i,j}, A_i) = \hat{U}(S_i) + H$ no matter what horse race outcome is realized in period 1. This deviation gives gambler i the payoff of

$$\begin{aligned} \overline{V} &\equiv \max_{\mathbf{a}_i} \; v(A_i) + \sum_{j \in J} p_j \max\{h_j^* a_{i,j} - A_i, 0\} - \{\hat{U}(S_i) + H\} + \hat{U}(S_i) \\ &= \max_{\mathbf{a}_i} \; v(A_i) + \sum_{j \in J} p_j \max\{h_j^* a_{i,j} - A_i, 0\} - H. \end{aligned}$$

[24] One remark is here that the odds $\mathbf{h}^*$ in period 1 are determined in the gambling market equilibrium as defined in Section 2.4. Therefore the bookmaker cannot directly choose the market odds themselves. By the best equilibrium, we mean the gambling market equilibrium such that the odds $\mathbf{h}^*$ are consistent with equilibrium condition (3) and maximize the bookmaker's payoffs.

This is a lower bound for gambler i's payoff. Furthermore, in order to achieve $\overline{V}$, gambler i's optimal deviation is to bet all the amount of A_i to the horse that maximizes the expected net return $p_j \max\{h_j^* - 1, 0\}$, provided that he defaults all debts when horse j wins the race but $h_j^* < 1$ or loses the race, in which either case he pays nothing. Therefore, gambler i's deviation payoff $\overline{V}$ can be re-written as follows

$$\overline{V} = \max_{A} v(A) + \max_{j \in J} \max\{p_j(h_j^* - 1), 0\}A - H. \tag{29}$$

Then, for $\mathbf{a}^*$ to be implementable, the following IC must be satisfied:

$$V^* \equiv v(A^*) - fA^* + \tilde{D}(z^*, A^*) + \hat{U}(S^*) \geq \overline{V} \tag{IC*}$$

Note that incentive constraint (19) implies IC* because each gambler can always default all the debt whatever horse race outcome is in period 1, followed by being most harshly punished $\tilde{\psi}(D_{i,j}, z_{i,j}, A_i) = \hat{U}(S_i) + H$. Therefore, IC* is a necessary condition for (19) to hold. We can also show its converse under Assumption 1 given below and greatly simplify the characterization of optimal punishment policy.

Without loss of generality, suppose that $p_1 \leq \cdots \leq p_m$. We make the assumption that horses are not so heterogeneous with respect to their winning probabilities.

Assumption 1. $(1 - p_1)(p_m - p_1) \leq p_1$.

We then show the following result.

Proposition 4. *Suppose that Assumption 1 is satisfied. Then the best equilibrium for the bookmaker under the book-bet adopted in period 1 is characterized as follows: (i) the amount bet A^* per gambler in total in period 1 is given by a solution to the following problem:*

Problem B*:

$$\max_{A^* \geq 0, \tilde{D}^*} \quad \Phi(A^*) \equiv fA^* - \tilde{D}^* + f\hat{A}(S^*)$$

subject to IC and $0 \leq \tilde{D}^* \leq fA^*$, where $\tilde{D}^* \equiv \tilde{D}(z^*, A^*)$.*

(ii) Furthermore, equilibrium odds h_j^* are given by $h_j^* = k^*/p_j + 1$ for $j \in J$, where $k^* \in (p_1, \infty)$ is a unique solution to

$$\sum_{j \in J} \frac{p_j}{p_j + k^*} = \frac{1}{1 - f}. \tag{30}$$

We provide an intuitive explanation for Proposition 4 below.

First, the original IC constraints (19) can be replaced by only IC* under Assumption 1, thereby reducing the original optimization problem, Problem B, to the relaxed one, Problem B* in Proposition 4. To see this, suppose that the bookmaker wants to implement a profile of amounts bet $\mathbf{a}^* = (a_j^*)_{j \in J}$ with its total amount $A^* \equiv \sum_j a_j^*$ from each gambler in period 1 in a symmetric equilibrium with market odds $\mathbf{h}^* = (h_j^*)_{j \in J}$. Then, if gambler i deviates to bet $A_i \neq A^*$ in period 1, the bookmaker can easily detect this deviation by imposing the most severe penalty $\underline{\psi}(z, A) = \hat{U}(S) + H$ for any deviation bet $A \neq A^*$. Next gambler i is also never better off by any deviation such that he places the equilibrium bet $A_i = A^*$ in total but changes its allocation among horses. Since horses are not so heterogeneous with respect to their winning probabilities under Assumption 1, gambler i can gain little from changing allocations of amounts bet $\mathbf{a}_i$ while keeping $A_i = A^*$ in total. Thus the gambler can be better off by allocating all the amount A^* to a single horse and default all the debts for all other horses, yielding at most the deviation payoff $\overline{V}$ to him. However, this is not profitable due to IC* again.

Next we see how equilibrium odds are determined. In the best equilibrium in which the bookmaker's payoffs are maximized the gambler's deviation payoff $\overline{V}$ should be minimized. We can achieve this by setting equilibrium odds as $h_j^* = k^*/p_j + 1$ defined in Proposition 4. To see this, note that, when a gambler bets one dollar on horse j, he will receive the payout $h_j^* - 1$ whenever it is positive and horse j wins the race while he can default paybacks in all other cases. Therefore, the expected net return of betting one dollar on horse j is given by $\max\{p_j(h_j^* - 1), 0\}$. Then, if $\max_j p_j(h_j^* - 1) = p_s(h_s^* - 1) > p_\ell(h_\ell^* - 1)$ for some horses ℓ and s, we can reduce the deviation payoff $\overline{V}$ by slightly increasing h_ℓ^* and decreasing $h_s^* > 0$ while keeping the feasibility constraint (20) unchanged. Therefore, to minimize $\overline{V}$, the expected returns taking account into the default incentive should be equalized across horses, implying that $p_j(h_j^* - 1) = k$ for all $j \in J$ for some constant k. Then, by using (20) and (24), we can determine the value of such k, given by k^* in Proposition 4.

The equilibrium odds determined in the above way $h_j^* = k^*/p_j + 1$ exhibit the feature of so called *long-shot bias*: gamblers place more bets on the horses which are less likely to win the race. This bias has been observed in many gambling markets and has been known as an empirical puzzle in the literature because such bias is not consistent with the standard theory of expected utility maximization (Chiappori et al. (2019),Ottaviani and Sørensen (2008)). To understand why the long-shot bias exists in our equilibrium, note that equilibrium odds $h_j^* = k^*/p_j + 1$ imply that

$$p_1 h_1^* \leq p_1 h_2^* \leq \cdots \leq p_m h_m^* \tag{31}$$

because p_j is increasing in j. This means that the expected gross returns $p_j h_j^*$ to bet one dollar are higher for the horses having higher wining probabilities. Put differently, equilibrium odds h_j^* of horses do not fully offset their winning probabilities p_j in equilibrium. Since equilibrium odds $h_j^* = k^*/p_j + 1$ become higher for the horses less likely to win the race (with lower j and hence lower p_j), the expected returns to bet one dollar $p_j h_j^*$ are negatively associated with higher odds h_j^*. This shows that the long-shot bias emerges in our equilibrium.

The key behind this result is that, as shown above, the expected net return to bet one dollar on horse j is given by $p_j(h_j^* - 1)$ and these returns are equalized across horses, $p_j(h_j^* - 1) = p_\ell(h_\ell^* - 1)$ for any $j \neq \ell$, in the gambling equilibrium in which gamblers default on payments to the bookmaker. Suppose that a gambler bets one dollar on horse j but loses the race, which happens with probability $1 - p_j$. In that case, since the gambler can default the amount of one dollar bet, his expected gain of default is measured by $(1-p_j) \times \$1$ which is lower for the horses which are more likely to win the race (with larger p_j). Thus it becomes less attractive for gamblers to bet on the horses having higher winning probabilities when they can default than when they cannot default. Thus gamblers bet more on the horses less likely to win the race, thereby resulting in the long-shot bias. This is in contrast to the case that gamblers *cannot* default at all, in which case the expected gross returns to bet one dollar $p_j h_j$ are equalized across horses, that is, $p_j h_j = p_\ell h_\ell$ for any $j \neq \ell$, so the long-shot bias never arises.

The existing studies have tried to explain the long-shot bias in the view point of gamblers' risk-loving or behavioral preferences (Chiappori et al. (2019)). In contrast we provide a new rationale for the long-shot bias in the perspective of strategic defaults without these preferences of gamblers. Our result shows that the long-shot bias emerges as an equilibrium feature of illegal gambling markets where individuals can renege on debts due to the lack of formal contracts. Therefore Proposition 4 has the important implication that the long-shot bias might be more prevalent in illegal gambling markets than legal gambling markets. This theoretical finding is worth addressing further to match actual data about illegal horse race and sports betting, which will be left for future research.

9.2 Socially Excessive Addiction

We now investigate welfare implications about the gambling market equilibrium obtained in Proposition 4. To this end, we define the social welfare of the economy as the sum of all players' payoffs over two periods. Since transfers made between the bookmaker and gamblers are cancelled out in total, the social welfare is equivalent to the sum of gamblers' gross payoffs over two periods as follows:

$$W(A_1, A_2) \equiv n\{v(A_1) + u(A_2, S)\}$$

where $S = \alpha + \beta A_1$, and $A_t \geq 0$ is the total amount bet per gambler in period $t = 1, 2$. We then define by (A_1^{fb}, A_2^{fb}) *the first-best* amounts bet to maximize the social welfare $W(A_1, A_2)$.

We then consider the optimal punishment policy ψ with the equilibrium odds $\mathbf{h}^*$, and the associated equilibrium amounts bet $(A^b, \hat{A}(S^b))$ in periods 1 and 2 where A^b is the optimal solution to Problem B* given in Proposition 4 and $S^b \equiv \alpha + \beta A^b$ is the addition stock corresponding to A^b.

Regarding the addiction preference of gamblers, we make the following assumption.

Assumption 2. *$\hat{U}(S) > A$ for any $A \geq 0$.*

Assumption 2 says that gambler's addiction preference, captured by the parameter values $\alpha > 0$ and $\beta > 0$, is so strong that his second-period payoff $\hat{U}(S)$ is sufficiently large to cover any amount bet $A \geq 0$ in the first period.[25]

We then show the following result.

Proposition 5. *Suppose that Assumption 1-2 are satisfied, and that the commission fee $f \in (0, 1)$ is sufficiently small. Then the equilibrium amounts bet A^b and $\hat{A}(S^b)$, which solve Problem B* of Proposition 4, are socially excessive in the sense that these are larger than the first-best amounts bet A_1^{fb} and A_2^{fb} in both periods, that is, $A^b > A_1^{fb}$ and $\hat{A}(S^b) > A_2^{fb}$.*

The intuition behind this result is as follows.

We define the *constrained* social welfare as the sum of all gamblers' gross payoffs over two periods, $v(A) + u(\hat{A}(S), S)$, where the amount bet in period 2 is constrained by the gambling market equilibrium condition such that $\hat{A}(S)$ satisfies the equilibrium condition (7): $u_A(\hat{A}(S), S) = f$. We also define the *constrained* efficient amounts bet $(A^{**}, \hat{A}(S^{**}))$ that maximize the constrained social welfare $v(A) + u(\hat{A}(S), S)$ over $A \geq 0$, where $S^{**} \equiv \alpha + \beta A^{**}$ denotes the corresponding addiction stock in period 2. Then, instead of comparing the equilibrium amounts bet $(A^b, \hat{A}(S^b))$ that solve Problem B* with the first-best amounts bet (A_1^{fb}, A_2^{fb}), we will make a comparison between the equilibrium amounts bet $(A^b, \hat{A}(S^b))$ and the constrained efficient amounts bet $(A^{**}, \hat{A}(S^{**}))$. This alternative comparison makes sense when the commission fee f is sufficiently small because then the constrained efficient amounts bet can be sufficiently close to the first-best amounts bet.

Now we consider which A^b or A^{**} is larger. The key for this exercise is that the bookmaker must ensure that gambler's equilibrium payoff $v(A) - fA + \tilde{D}(z, A) + \hat{U}(S)$ cannot be lower than his deviation payoff $\overline{V}$, corresponding to IC*. To this end, the bookmaker may need to compensate gamblers by allowing them to default some positive amount, that is, $\tilde{D}(z, A) > 0$ in equilibrium. If this is the case, IC* becomes binding so that the optimal default should satisfy $\tilde{D}(z, A) = \overline{V} - \{v(A) - fA + \hat{U}(S)\} > 0$, and thus the bookmaker's payoff coincides with the constrained welfare

$$fA - \tilde{D}(z, A) + f\hat{A}(S) = v(A) + u(\hat{A}(S), S) - \overline{V}$$

[25] To see this, assume that the marginal utility of gambling u_A is increasing in the addiction stock S and its effect is so large that $u_{SA}^2 \geq u_{SS}u_{AA}$. Then, by the definition of $\hat{U}(S)$, we have $\hat{U}'(S) = u_S(\hat{A}(S), S) > 0$, and thus $\partial\hat{U}/\partial A = \beta\hat{U}'(S) > 0$ as well as $\partial\hat{U}^2/\partial^2 A = \beta^2\{u_{SA}\hat{A}'(S) + u_{SS}\} \geq 0$ because $\hat{A}'(S) = -u_{AS}/u_{AA}$. When α and β are so large that $\beta u_S(\hat{A}(\alpha), \alpha) > 1$ and $\hat{U}(\alpha) \geq 0$ are satisfied, we obtain $\hat{U}(S) > A$ for all $A \geq 0$.

up to the constant of $\overline{V}$. Then the bookmaker's choice of the amount bet A is same as the constrained efficient one A^{**}.

However, when gamblers' addiction preference is so strong that Assumption 2 is satisfied, IC* is slack at the constrained efficient level A^{**} even when they are not allowed to default any amount, that is, $\tilde{D}^* = 0$. In other words gamblers are willing to place larger bets than the efficient level A^{**} even when they are not compensated by any positive amount allowed to default. In such a situation the optimal policy chosen by the bookmaker should set the maximum forgiveness to the lowest possible level, zero, $\tilde{D}^* = 0$, and increase the amount bet A in period 1 as much as possible until the gambler's equilibrium payoff $v(A) - fA + \hat{U}(S)$ is reduced down to the deviation payoff $\overline{V}$. This can raise the bookmaker's payoff $fA + f\hat{A}(S)$. In that way the bookmaker extracts the surplus from gamblers. This makes the equilibrium amount bet A^b in period 1 larger than the constrained efficient one A^{**}. Since the first-best amounts bet (A_1^{fb}, A_2^{fb}) are close to the constrained efficient one $(A^{**}, \hat{A}(S^{**}))$ when the commission fee $f \in (0, 1)$ is sufficiently small, by continuity the equilibrium amounts bet $(A^b, \hat{A}(S^b))$ can be larger than the first-best amounts bet (A_1^{fb}, A_2^{fb}). Thus the equilibrium addiction, measured by the addiction stock of $S^b \equiv \alpha + \beta A^b$, becomes larger than the socially efficient level $S^{fb} \equiv \alpha + \beta A^{fb}$, and hence individuals are too much addicted to gambling in terms of social efficiency.

The above result suggests that some policy interventions to restrict the use of credit-based gambling (book-bet) are effective to improve the efficiency by making individuals less addicted to gambling. If gamblers cannot bet by credit but use only cash-bet, they bet A to maximize the payoff of $v(A) - fA + \hat{U}(S)$ in period 1, resulting in a lower amount bet than the equilibrium bet A^b under the book-bet.[26] This might support the related policies, which were recently announced in several countries such as Australia and UK, to ban the credit card use for gambling and reduce addiction-related problems.

10 Optimal Gambling Format: Cash-bet or Book-bet?

We next address the issue about which book-bet or cash-bet becomes optimal for the bookmaker to use in period 1. We then show that gamblers' addiction preferences play the crucial role to determine the optimal gambling format in period 1. More specifically, we show that the bookmaker prefers the book-bet to the cash-bet when gamblers are strongly addicted to gambling. On the contrary, when gamblers are not addicted to gambling at all, the cash-bet may be optimal for the bookmaker. These results can help understand why sellers often provide credits to newcomers for purchasing addictive goods despite the default risk in illegal markets.

[26] The equilibrium bet A^b is defined as the largest value of A satisfying $v(A) - fA + \hat{U}(S) = \overline{V}$ so that A^b is larger than A maximizing $v(A) - fA + \hat{U}(S)$.

10.1 Equilibrium under the Cash-Bet in Period 1

Suppose that the bookmaker chooses the cash-bet in period 1 as well as in period 2. Under the cash-bet gamblers cannot default in period 1 and then certainly participate in the gambling in period 2. As a result, gambler i places bet $\mathbf{a}_i$ in period 1 to maximize the following expected payoff

$$v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i\} + \hat{U}(S_i) \tag{32}$$

given expectations about equilibrium odds $\{h_j^*\}_{j \in J}$ in period 1.

In the symmetric equilibrium the market odds are determined by $h_j^* = (1-f)/p_j$ for each $j \in J$.[27]. Given this, each gambler places the bet of total amount A in period 1 to maximize the expected payoff given by (26), which we can re-write by $v(A) - fA + \hat{U}(S)$. We denote by $\tilde{A}$ the equilibrium amount bet by each gambler in total in period 1, that is, $\tilde{A}$ maximizes $v(A) - fA + \hat{U}(S)$. Also we denote by $\hat{A}(\tilde{S})$ the associated amount bet by each gambler in total in period 2 where $\tilde{S} \equiv \alpha + \beta \tilde{A}$.

Then the bookmaker obtains the following payoffs over two periods

$$\Pi_c \equiv n\{f\tilde{A} + f\hat{A}(\tilde{S})\}$$

by using the cash-bet in period 1.

10.2 Book-Bet or Cash-Bet?

We now compare the bookmaker's payoff Π^c under the cash-bet with that under the book-bet Π^b which attains the maximum value in Problem B given in Section 4. We do not impose Assumption 1 in this section.

The bookmaker is exposed to default risks when she uses the book-bet in period 1 in contrast to the cash-bet. However, according to Mehmood and Chen (2022), in the illegal horse race betting in Pakistan more than 50% of gamblers are allowed to place bets by credit rather than cash-in-advance bets. If we focus only on default costs, this observation is puzzling. Why does the bookmaker extend credit to gamblers even when they default? In the following we show the condition under which the bookmaker can gain from using the book-bet in period 1 by strategically inducing gamblers' defaults and making them addicted to gambling, resulting in large amounts bet in period 2.

Which book-bet or cash-bet the bookmaker prefers in period 1 depends on how strongly gamblers are addicted to gambling in the first period. We measure the degree of gamblers' addictions by the parameter values α and β that determine the addiction stock $S_i = \alpha + \beta A_i$ in period 2. When α and β are larger, gamblers are addicted more to gambling. We then show that, when gamblers' addictions are so strong that α and β are sufficiently large to satisfy Assumption 2, the bookmaker prefers the book-bet to the cash-bet in period 1:

[27] Again we here focus on the equilibrium in which the odds are positive for any horse.

the bookmaker obtains a higher payoff Π_b under the book-bet than the payoff Π_c under the cash-bet. On the contrary, in the absence of addiction preferences in the sense that $\alpha = \beta = 0$, the bookmaker may prefer the cash-bet to the book-bet under a certain condition. Therefore, gamblers' addictions cause a significant effect on the bookmaker's choice of the gambling format.

When gamblers are never addicted ($\alpha = \beta = 0$, so $S_i = 0$), we define by $u(A_{i,2}) \equiv u(A_{i,2}, 0)$ the second-period utility of gambler i who places bet of $A_{i,2}$ in total in period 2. Since the differences in payoff functions between the two periods do not matter when there is no addiction, we then assume that the payoff functions of gambling are identical in both periods: $v(A) \equiv u(A)$ for all $A \geq 0$. We define by $\tilde{A}^*$ the amount bet to maximize $u(A) - fA$ over $A \geq 0$. Note that each gambler obtains the payoff of $\hat{U} \equiv \max_{A \geq 0} u(A) - fA$ in period 2 when $S = 0$. We also assume that $\overline{u} \equiv \max_{A \geq 0} u(A) < +\infty$.

We then show the following result.

Proposition 6. *(i) Suppose that gamblers are strongly addicted to gambling ($\alpha > 0$ and $\beta > 0$) such that Assumption 2 is satisfied. Then the book-bet becomes optimal for the bookmaker in period 1 ($\Pi_b \geq \Pi_c$). (ii) Suppose that gamblers are not addicted at all ($\alpha = \beta = 0$). Then, if $u(\tilde{A}^*) + H < 2f\tilde{A}^*$, it becomes optimal for the bookmaker to hold the cash-bet in period 1 ($\Pi_c > \Pi_b$).*

When gamblers are so strongly addicted that Assumption 2 is satisfied, the bookmaker can gain from allowing them to default without punishment initially and making them addicted. This induces gamblers to place large bets in the second period, thereby benefiting the bookmaker more than the default cost in the first period. This gives the first part of Proposition 6.

Regarding the second part of Proposition 6, the bookmaker faces the trade-off between the book-bet and the cash-bet as follows: first, under the book-bet gamblers can always default all debt repayments in period 1 and then quit the gambling in period 2 after he is imposed the largest penalty $\hat{U} + H$. This "take-money-run-away" strategy can give each gambler the payoff of $\max_A v(A) - H = \overline{u} - H$. Therefore, the bookmaker must leave at least this rent $\overline{u} - H$ to each gambler when she uses the book-bet in period 1.[28] This rent never arises under the cash-bet. Second, if gamblers have the addiction preference, there is the benefit for the bookmaker to use the book-bet relative to the cash-bet such that gamblers are addicted to gambling when they are allowed to default in period 1 and then subsequently bet aggressively in period 2. However, such gains disappear when gamblers are not addicted at all. Then only the benefit of using the book-bet is that the allowed default can be used as a tool to compensate gamblers for inducing large amounts bet in period 1. However, such benefit is outweighed by the first two negative effects when gamblers place a relatively large bet $\tilde{A}^*$ under the cash-bet as assumed in Proposition 6 ($u(\tilde{A}^*) + H < 2f\tilde{A}^*$). Then, the bookmaker prefers the cash-bet to the book-bet.

[28] This is never the case when Assumption 2 is satisfied because then gamblers always avoid default and participate in the gambling in period 2 whatever amounts they bet in period 1.

11 Extensions

In this section we discuss three extensions of the basic model (we relegate more detailed analysis to the Online Appendix.)

11.1 Endogenous Commission Fees

One might wonder if the bookmaker can more directly subsidize gamblers to place large bets in period 1 without allowing them to default. More specifically, the bookmaker may cultivate gamblers' addiction preferences by offering a negative commission fee in period 1. To address this issue, we extend the basic model by allowing the bookmaker to choose a commission fee in each period.

Our main insight still remains valid: when gamblers have strong addictive preferences as made in Assumption 2, the bookmaker prefers the book-bet to the cash-bet even when she chooses commission fees which can be negative so that gamblers are directly subsidized to bet. Therefore, subsidizing gamblers by negative commission fees are not sufficient to maximize the bookmaker's payoffs but adding the 'free-first-dose' strategy still becomes effective to raise her payoffs. The intuition behind this result is that the bookmaker can always mimic the optimal commission fees chosen under the cash-bet while using the book-bet and a bang-bang punishment policy. More specifically, the bookmaker can use the book-bet with a bang-bang punishment policy by setting the maximum forgiveness $\tilde{D}(z, A)$ to be zero so that gamblers are never allowed to default at all in period 1 together with offering the same commission fees as those under the cash-bet. By doing this, the bookmaker can achieve at least the same payoff as that under the cash-bet (we provide more formal analysis in the Online Appendix.)

11.2 Endogenous Commitment by the Bookmaker

We have so far assumed that the bookmaker can commit to the punishment policy announced in the beginning of period 1 as well as promised payouts in both period 1 and period 2. As we discussed, this assumption is made to simplify the analysis by paying our main focus to default incentives of only gamblers but not the bookmaker.

In this section we discuss how to extend the basic model in such a way that the bookmaker endogenously self-enforces promised payouts and punishment policies. Then we view the two-period model presented so far as a short-cut of the dynamic equilibrium in which the bookmaker's payouts and punishment policy are self-enforcing. Since the full analysis of the dynamic model is complicated, we relegate its details to the Online Appendix and discuss only its basic idea in this section.

In the dynamic extension of the model the bookmaker lives forever and encounters a sequence of shot-lived gamblers who lives for two periods. The bookmaker holds the cash-bet or the book-bet when gamblers are young while she must offer only the cash-bet when they are old, as considered in the basic model. In contrast to the two-period model the bookmaker can default on payouts and renege on announced punishment policies. However, if the bookmaker makes such defection, she will be punished by the future gamblers who are informed of the past history of the game from the previous generations: the future gamblers will punish the bookmaker by quitting the gambling because they expect the bookmaker to renege on payouts. In that way promised payouts and punishment policies are self-enforced as a dynamic equilibrium (more detailed analysis is given in the Online Appendix).

We obtain two important implications about the above dynamic equilibrium as follows.

First, when the commission fee f is smaller, the payouts $(1-f)\hat{A}(S)$ which the bookmaker promises to pay old gamblers under the cash-bet is larger so that the bookmaker finds it more difficult to commit to payouts. Thus the self-enforcing condition on the side of the bookmaker severely constrains the use of non-positive commission fees. This result provides a reason about why transfer schemes which combine zero commission fees with fixed fees do not work effectively when the bookmaker's commitment to payouts is limited. Therefore, we can justify the assumption made in the basic model that the commission fee is bounded away from zero, $f > 0$, on the ground of limited commitment by the bookmaker.

Second, addiction preferences help self-enforce relational contract agreements between the bookmaker and gamblers over time. The parameter of the model capturing addiction preferences is given by $\beta > 0$ that determines how likely/quickly gamblers are addicted to gambling over time. Here, the increase in β has two effects: first, as β increases, gamblers accumulate larger addiction stock $S_i = \alpha + \beta A_i$, so they place larger bets $\hat{A}(S_i)$ under the cash-bet when old within each period. This increases the payouts that the bookmaker must commit to give old gamblers, making her commitment more difficult. Second, as β increases, the bookmaker can exploit larger profits from future gamblers who will accumulate larger addiction stock and then bet more. This makes the bookmaker's commitment easier. We then show that the latter effect dominates the former one when the bookmaker is sufficiently patient, so the bookmaker can more easily self-enforce promised payouts when gamblers' addictions become stronger.

This result suggests that individuals' addictions can *complement* relational contract agreements, implying that informal markets where goods/services are traded without formal contracts can work more effectively even without well-functioning institutions when individuals are addicted more to consuming goods/services. This implication is useful to understand how and why illegal markets such as illicit drugs and illegal wagering markets work efficiently even without formal enforcement.

11.3 Applications to Other Illegal Markets

In this section we discuss how our results can include a broad range of applications to understand how illegal markets work beyond the specific setting of illegal horse race betting. More specifically, we show that our model can be applied to any illegal markets, such as illegal drugs, where sellers sell addictive goods to buyers who form addiction preferences over time, and their transactions are illegal and are hence not formally enforceable.

For this purpose, we make the following interpretation of the basic model of horse race betting: first, the bookmaker and gamblers are interpreted as a "seller" and "buyers" respectively, where the seller sells an addictive good to buyers. Second, the amount bet $A_{i,t}$ by gambler i in period $t = 1, 2$ is interpreted as the consumption level of the addictive good by buyer i in period $t = 1, 2$. Buyers decide how much to consume the addictive good in each period, and are addicted to consuming the good in period 2, which we capture by the addiction stock $S_i = \alpha + \beta A_{i,1}$ again. Third, the seller has two formats to sell the addictive good: one is the cash-in-advance purchase, meaning that buyers must immediately pay the seller when they purchase the good. The other is the credit-based purchase, meaning that buyers can defer payments. The former corresponds to the cash-bet format while the latter corresponds to the book-bet format respectively as we considered in the model of horse race betting. Fourth, we interpret the commission fee of the bookmaker $f > 0$ as the price of the addictive good, which buyers pay the seller for consuming one unit of the addictive good. Here the price f is exogenous as made in the basic model or it can be chosen by the seller as considered in the extended model in Section 7.1.

Given the above re-interpretation of the basic model, we can show that most of the results we have so far obtained still remain valid in a broad range of illegal markets where sellers sell addictive goods to buyers who form addiction preferences over time, and transactions of the goods are illegal and are hence not formally enforceable (see the Online Appendix for more formal analysis).

12 Case studies

In this section, we discuss several case studies. We first discuss illegal horse betting, a case that closely aligns with the formal model. We then turn to illegal drugs and religious sects.

12.1 Illegal Horse Betting in Pakistan

Gathering data from markets prone to addiction and weak contract enforcement poses significant challenges due to the clandestine nature of these transactions, which are deliberately concealed to evade legal penalties. The first case study we present focuses on the underground horse race betting market in Pakistan, where Mehmood and Chen (2022) collected data, which we discuss below in light of the main predictions of the model.

The horse races take place every Sunday from noon to 6 pm, with races scheduled every 30 minutes. Gambling takes place at betting stations inside the premises of the race club. The entry at the club requires a ticket of PKR 500 (USD 2.25), with anyone who has a ticket allowed entry into the club and, by default, the ability to bet at any of the 12 betting stations that issue identical odds. Every station charges a constant 5% participation fee, and gamblers are allowed to bet on credit. Below, we discuss our three main theoretical results in light of available data.

Free-first dose effect. In their examination of the horse race betting market, Mehmood and Chen (2022) leave an intriguing puzzle unaddressed: the policy allowing patrons to place credit bets of up to PKR 5000, or roughly USD 20, upon entry to the race club. The economic rationale for such an institutional arrangement is not immediately apparent, raising questions about its profitability. Approximately 55% of wagers at the race club are placed on credit, as opposed to cash-in-advance bets, suggesting that credit betting constitutes a significant component of the race club's economic ecosystem. Our model offers a simple explanation. Offering bets on credit cultivates gamblers' addiction, ensuring a steady stream of future revenues.

Debt repayment. Mehmood and Chen (2022) find that on average, 35% of gamblers do not repay their debt in full. Our model explains this surprising feature as well. Indeed, we find (Proposition 2) that bookmakers allow a certain degree of default on the equilibrium path. That way, gamblers can place larger bets, so their addiction becomes stronger.

Violence. Mehmood and Chen (2022) collected data about perceived violence from betters. Their findings reveal that such occurrences are exceedingly rare. To be specific, a mere 0.5% of surveyed gamblers reported any apprehension of encountering violence in case of non-payment situations. This empirical observation is in line with anecdotal evidence, as exemplified by recent ethnographic research. For instance, during interviews, Paa'h Sadiq, a prominent bookie and key informant, expressed astonishment at the mere suggestion of violence in his line of work. He aptly countered, "Do I look like Amresh Puri [famous Indian actor, notably known for villainous roles]? You guys see too many gangster films. Gambling debts are debts of honor. If I resort to violence, I lose honor and the [very] right to collect debts." (Mahar, 2022, p.5) These testimonies underscore the exceptional rarity of violence associated with unpaid gambling debts within this betting market. These evidence are also in line with the predictions of the model. We find that while violence might enforce more prompt debt repayments, it could simultaneously weaken the ability of book-makers to cultivate addictions.

12.2 Illegal Drugs

Illegal drug markets are among the most elusive yet economically significant sectors, marked by their adaptability and resilience. While empirical data on these markets are scarce due to the illicit and secretive nature of transactions, qualitative evidence provides insights into their operational dynamics and the strategies employed by participants.

Free-first dose effect. The strategy of offering the first dose for free or at a significantly reduced price to potential new users is a well-known method in drug markets, aimed at fostering dependency. Another illustration of the free-first dose effect is through drug dealers' incentive to dilute ("cut") the products they sell. This dilution is unobservable to buyers until after they consume, and thus creates a moral hazard issue (Galenianos and Gavazza (2017), Galenianos, Pacula and Persico (2012)). Refraining from cutting on drugs can be seen as a particular example of the free-first-dose effect. Indeed, on the one hand, offering high-quality products to new consumers is costly for drug dealers. On the other hand, offering high-quality products cultivates stronger addictions. Galenianos and Gavazza (2017) estimate a model using data on the crack cocaine market in the United States. Their estimation reveals that although they are short-lived, relationships between buyers and sellers are valuable to sellers, as regular buyers consume more frequently and account for the vast majority of crack cocaine purchases. Galenianos and Gavazza (2017) rely on the STRIDE dataset, which contains records of acquisitions of illegal drugs by undercover agents and DEA informants. Hence, the data may contain more specifically information about purchases from "new" consumers. Consistent with our model, Galenianos and Gavazza (2017) find that drug dealers do not necessarily cut on quality in the STIDE dataset, suggesting that creating relations and cultivating addictions might be valuable.

Debt repayment. Credit plays an important part in drug distribution, acting as a pivotal mechanism for sustaining and expanding consumer bases in environments where immediate payment may not be an option. This reliance on credit not only facilitates transactions but also embeds a level of trust and dependency between dealers and users (Jacobs and Wright (2006)). This aspect of illegal drug markets is consistent with our model, where offering addictive goods on credit strengthens addiction and future transactions.

Violence. The role of violence in drug markets is multifaceted, serving both as a tool for enforcement and as a potential deterrent to the stability of these markets. The strategic use of violence, or the deliberate avoidance thereof, is a critical consideration for drug dealers who must balance the immediate benefits of enforcing payment and loyalty against the long-term consequences of scaring away customers or attracting law enforcement attention. In their analysis of the heroin market of New York city, Curtis and Wendel (2000) illustrate how violence is strategically used or avoided to maintain market stability and customer loyalty. The authors suggest that while violence can be an effective means of debt collection, it can also undermine the very foundation of trust and repeat business upon which these markets rely.

12.3 Religious Sects

Religious sects, particularly those with more exclusive or intense commitment requirements, can sometimes exhibit dynamics similar to the patterns of addiction and enforcement seen in the contexts of gambling and drug markets. The initial engagement with these groups is often marked by a welcoming atmosphere and various forms of support, which can be seen as analogous to the "free-first dose" effect. Over time, the deepening of commitment can introduce elements of indebtedness to maintain consumption and addiction.

Free-first dose effect. Many religious sects initially offer new members a sense of belonging, community, and sometimes tangible benefits such as meals, social support, or even housing. This welcoming approach serves to attract individuals seeking community or spiritual fulfillment, providing an initial "dose" of the benefits of membership with little to no upfront cost. This phase can be critical for building attachment to the group and its beliefs (Dawson (1998)).

Debt repayment. As members become more integrated into the sect, they may feel an increasing obligation to contribute financially, dedicate time, or engage in proselytizing activities. This sense of obligation can be akin to "debt repayment," where the perceived debt is not just financial but also emotional or spiritual. Members might believe that their salvation, enlightenment, or the well-being of their community depends on their contributions. In some cases, the failure to meet these expectations can lead to feelings of guilt or indebtedness, further binding members to the group as they strive to "repay" their perceived debt.

Violence. While physical violence is rare within the vast majority of religious sects, subtler forms of coercion, such as psychological pressures or social ostracism, may be employed by more controlling groups to ensure adherence to their norms. The complexity and diversity of these coercive practices, especially when balanced against the need for voluntary and genuine commitment, present challenges that our model might not fully capture.

In the economic literature on religion, Iannaccone (1992) argued that costly practices within religious cults allow for the screening of free-riders and increase the overall benefits for group members. Our model complements Iannaccone (1992), suggesting that addiction might be a channel through which costly practices in religious groups are self-enforced. Additionally, in a world of increasing religious pluralism, cultivating an addiction to a specific cult might also be a way to decrease group members' outside options. Hence, our work also connects to the literature on religious competition, suggesting an explanation for the persistence of multiple sects.[29]

13 Conclusion

In this study, we introduce a dynamic model to analyze addiction dynamics in markets lacking formal enforcement mechanisms. Our model delineates how sellers employ a "free-first-dose' strategy to foster addiction, thereby establishing a reliable consumer base. Key to our findings is the strategic tolerance of non-repayment by sellers, which, paradoxically, deepens addiction and customer loyalty. Additionally, contrary to prevalent views, our results suggest that early-stage violence and coercion may detrimentally affect addiction cultivation, offering new insights into the operation of illegal markets.

We also explore the welfare implications of illegal market equilibrium, and find that consumption of addictive goods is socially excessive. This finding underscores the potential need for regulation aimed at curbing the consumption of illegal addictive goods. Additionally, in the context of illegal gambling markets, we show that addiction distorts market odds. Specifically, the long-shot bias arises because gamblers place relatively larger bets on horses with lower winning probabilities, given that they are allowed to partially default.

While the economic significance of illegal markets is undeniable, important questions remain about the types of regulation that can be effectively implemented. Disrupting

[29] On the literature on religious markets, see, among others, McBride (2008) and McBride (2010).

early-stage consumption appears to be a particularly promising approach to curbing addiction and excessive consumption. For example, in the context of drug markets, partial regulation—such as promoting controlled consumption of substitutes—could prove effective. Similarly, in gambling markets, regulation might focus on limiting practices akin to book-betting or subsidized bets, which are still prevalent in online platforms. In the realm of online social media, a comparable strategy might involve restricting features designed to enhance user engagement for new users, such as notifications or algorithm-driven content suggestions. Future research should explore the efficacy of these regulatory strategies across different markets and assess their adaptability to various market settings, helping to identify which interventions are most effective at reducing the harmful impacts of illegal addictive markets.

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14 Appendix A: Proofs

We first prove that the seller never chooses the credit-based selling in period 2 no matter what selling strategy she chooses in period 1.

Lemma A1. *The credit-based selling becomes never optimal for the seller in period 2 whichever selling strategy she chooses in period 1.*

Proof. Suppose contrary to this claim that the seller adopts the credit-based selling in period 2, where $P_2 \geq 0$. Then any buyer always defaults on all the payment of P_2 (that is, $D = P_2$) at the end of period 2. Therefore, the seller earns $-c_2 A_2$ in period 2. Also each buyer obtains the equilibrium payoff of $\max\{0, u(A_2, S)\}$ in period 2. If $u(A_2, S) \leq 0$, the seller can reduce A_2 to some A_2'' such that $u(A_2'', S) > 0$, which ensures that each buyer can accept A_2'' as well as DE_1 and DE_2 are relaxed ($-c_2 A_2'' + V > c_1 A_1$ and $V > c_2 A_2''$) but the seller can increase her payoff from $-c_2 A_2$ to $-c_2 A_2''$ in period 2. Hence we have $u(A_2, S) > 0$ in the equilibrium.

Now consider the cash-based selling in period 2 as follows: $P_2'' \in (0, w_2)$ and $u(A_2, S) \geq P_2''$. Such P_2'' exists. Then the seller can increase her payoff in period 2 from $-c_2 A_2$ to $P_2'' - c_2 A_2$, showing that the credit-based selling cannot be optimal. Q.E.D.

14.1 Proof of Proposition 1

Suppose that a punishment policy ψ implements an equilibrium outcome $\{A_1^*, A_2^*\}$ and gives the seller the equilibrium payoff of

$$\pi^* \equiv P_1^* - c_1 A_1^* - D^* + (1 - q(D^*))\{P_2^* - c_2 A_2^*\} - \xi(D^*)$$

where each buyer defaults $D^* \in [\underline{D}, P_1^*]$ in period 1, so as to maximize

$$D - q(D)U_2 - \xi(D)$$

subject to $\underline{D} \leq D \leq P_1^*$, where $\underline{D} \equiv \max\{P_1 - w_1, 0\}$.

Define

$$\underline{\psi} \equiv \inf_{D \in [\underline{D}, P_1^*]} \psi(D)$$

Then we show the following claim.

Lemma A1. *$D^* - \psi(D^*) \geq -\underline{\psi}$ holds.*

Proof. Due to the optimal default choice of $D^* \in [0, P_1^*]$, it must be that

$$D^* - \psi(D^*) \geq D - \psi(D)$$

for all $D \in [\underline{D}, P_1^*]$. Also, due to the definition of $\underline{\psi} = \inf_{\underline{D} \leq D \leq P_1^*} \psi(D)$, for any $\varepsilon > 0$, some $D'' \in [\underline{D}, P_1^*]$ exists such that

$$\underline{\psi} + \varepsilon > \psi(D'').$$

Therefore, we obtain

$$\begin{aligned} D^* - \psi(D^*) &\geq D'' - \psi(D'') \\ &> D'' - \underline{\psi} - \varepsilon \\ &\geq -\underline{\psi} - \varepsilon \end{aligned}$$

for all small $\varepsilon > 0$, so that

$$D^* - \psi(D^*) > -\underline{\psi} - \varepsilon$$

for all small $\varepsilon > 0$. By taking $\varepsilon \to 0$, we obtain the desired result. Q.E.D.

Now define the bang-bang punishment policy as follows: first, define the maximum forgiveness as follows

$$\tilde{D} \equiv \max\{\underline{D}, D^* - \{\psi(D^*) - \underline{\psi}\}\}.$$

Such $\tilde{D}$ satisfies $\tilde{D} \geq 0$ due to Lemma A1 as well as $\tilde{D} \leq D^* \leq P_1^*$, so $\tilde{D} \leq P_1^*$. Also $\tilde{D} \geq \underline{D} \geq P_1 - w_1$, so $w_1 \geq P_1 - \tilde{D}$. Therefore we have $P_1 - w_1 \leq \tilde{D} \leq P_1^*$. Second, define the new punishment policy as follows

$$\tilde{\psi}(D) = \begin{cases} \underline{\psi} & \text{if } D \leq \tilde{D} \\ U_2 + H & D > \tilde{D} \end{cases}$$

Given this new policy, each buyer faces the following expected payoff at the end of period 1:

$$D - \tilde{\psi}(D) + U_2.$$

Since it becomes never optimal for the buyer to choose $D < \tilde{D}$ under $\tilde{\psi}$, we then show that

$$\begin{aligned}
\tilde{D} - \tilde{\psi}(\tilde{D}) &= \tilde{D} - \underline{\psi} \\
&\geq D^* - \{\psi(D^*) - \underline{\psi}\} - \psi(D^*) \\
&= D^* - \psi(D^*) \\
&\geq D - \psi(D) \quad \text{for any } D \in [\underline{D}, P_1^*] \\
&\geq D - (U_2 + H) \\
&= D - \tilde{\psi}(D)
\end{aligned}$$

for any $D \in [\underline{D}, P_1^*]$. Therefore, each buyer optimally chooses to default $\tilde{D}$ given the newly defined policy $\tilde{\psi}$. Furthermore, each buyer's payoff under the new policy is given by

$$\begin{aligned}
& v(A_1^*) - P_1^* + \tilde{D} + U_2 - \tilde{\psi}(\tilde{D}) \\
\geq \;& v(A_1^*) - P_1^* + D^* - \psi(D^*) + U_2,
\end{aligned}$$

so that he can be weakly better off from the original punishment policy ψ. This implies that the buyers accept the new policy $\tilde{\psi}$ at the beginning of period 1 as well as they accept $\{A_2^*, P_2^*\}$ in period 2.

We then decompose $\tilde{\psi} \equiv \inf_{\underline{D} \leq D \leq P_1^*} q(D)U_2 + \xi(D)$ into $\tilde{q} \in [0,1]$ and $\tilde{\xi} \in [0, H]$ as follows: define $\underline{q} \equiv \inf_{\underline{D} \leq D \leq P_1^*} q(D)$ and set $\tilde{q}(\tilde{D}) = \underline{q}$, where $\underline{q} \in [0,1]$. Also define

$$\tilde{\xi}(\tilde{D}) = \underline{\psi} - \underline{q}U_2.$$

Here, we have

$$\begin{aligned}
\underline{\psi} &= \inf_{\underline{D} \leq D \leq P_1^*} \{q(D)U_2 + \xi(D)\} \\
&\leq \inf_{\underline{D} \leq D \leq P_1^*} q(D)U_2 + H \\
&= \underline{q}U_2 + H
\end{aligned}$$

implying that $\tilde{\xi}(\tilde{D}) = \underline{\psi} - \underline{q}U_2 \leq H$. Also, we have

$$\begin{aligned}
\underline{\psi} &= \inf_{\underline{D} \leq D \leq P_1^*} \{q(D)U_2 + \xi(D)\} \\
&\geq \inf_{\underline{D} \leq D \leq P_1^*} q(D)U_2 + \inf_{\underline{D} \leq D \leq P_1^*} \xi(D) \\
&\geq \underline{q}U_2
\end{aligned}$$

implying that $\tilde{\xi}(\tilde{D}) \geq 0$. Therefore, $\tilde{\xi}(\tilde{D}) \in [0, H]$.

We show the following claim.

Lemma A2. *$P_2 \geq c_2 A_2$ at the optimal contract.*

Proof. Suppose that $P_2 < c_2 A_2$: seller's payoff becomes negative in period 2 under the optimal contract. Since $u(A_2, S) > c_2 A_2$ for a small enough $A_2 > 0$, we can find some small $A_2'' \in (0, A_2)$ and $P_2'' \in (c_2 A_2'', w_2)$ such that $u(A_2'', S) \geq P_2'' \geq c_2 A_2''$. Then DE_2 is satisfied: $V \geq c_2 A_2 \geq c_2 A_2''$, while satisfying IR_2 and LL_2. However, then the seller can improve her payoff in period 2 from $P_2 - c_2 A_2 < 0$ to $P_2'' - c_2 A_2'' \geq 0$. Q.E.D.

Suppose that the seller offers the informal contract $\{A_1^*, P_1^*, A_2^*, P_2^*, \tilde{\psi}\}$ to buyers in period 1, where $\tilde{\psi}$ is defined as above. Then the buyers accept this contract as shown above. Also, the seller can be weakly better off: first, if $\tilde{D} = D^* - \{\psi(D^*) - \underline{\psi}\}$, then

$$
\begin{aligned}
& P_1^* - c_1 A_1^* - \tilde{D} + (1 - \underline{q})\{P_2^* A_2^* - c_2 A_2^*\} \\
\geq\ & P_1^* - c_1 A_1^* - D^* + (1 - \underline{q})\{P_2^* A_2^* - c_2 A_2^*\} \\
\geq\ & P_1^* - c_1 A_1^* - D^* + (1 - q(D^*))\{P_2^* A_2^* - c_2 A_2^*\} \\
=\ & \pi^*
\end{aligned}
$$

because $\tilde{D} \leq D^*$ and $q(D^*) \geq \underline{q}$. Second, if $\tilde{D} = \underline{D} = P_1^* - w_1 \geq 0$, then we have

$$
\begin{aligned}
& P_1^* - c_1 A_1^* - \tilde{D} + (1 - \underline{q})\{P_2^* - c_2 A_2^*\} \\
=\ & w_1 - c_1 A_1^* + (1 - \underline{q})\{P_2^* - c_2 A_2^*\} \\
\geq\ & P_1^* - c_1 A_1^* - D^* + (1 - q(D^*))\{P_2^* - c_2 A_2^*\}
\end{aligned}
$$

because $w_1 \geq P_1^*$, $D^* \geq 0$ and $q(D^*) \geq \underline{q}$.

Thus we can pay our attention to the punishment policy $\tilde{\psi}$ that specifies two penalties $\underline{\psi}$ and $U_2 + H$ depending on whether $D \leq \tilde{D}$ or not to hold.

Now, if $\underline{\psi} > 0$, we can set the new punishment policy $\tilde{\psi}$ such that $\underline{\psi} = 0$, and show that such a newly defined policy can improve the seller's and buyers' payoffs at the same time: for the buyer's payoff, we have

$$v(A_1^*) - P_1^* + \tilde{D} + U_2 > v(A_1^*) - P_1^* + \tilde{D} - \underline{\psi} + U_2$$

for any $\underline{\psi} > 0$. For the seller's payoff, we have

$$P_1^* A_1^* - c_1 A_1^* - \tilde{D} + (1 - \underline{q})\{P_2^* A_2^* - c_2 A_2^*\} \geq P_1^* A_1^* - c_1 A_1^* - \tilde{D} + \{P_2^* A_2^* - c_2 A_2^*\}$$

for any $\underline{q} \in [0, 1]$ due to Lemma A2. Therefore the newly defined punishment policy that set $\underline{\psi} = 0$ can improve the seller's payoff without violating all other constraints. Q.E.D.

14.2 Proof of Proposition 1

In period 2 gambler i places bet $\mathbf{a}_i$ to maximize his expected payoff

$$u(A_i, S_i) + \sum_{j \in J} p_j \{h_j a_{i,j} - A_i\},$$

given expectations about market odds $\{h_j\}_{j \in J}$. In the equilibrium in which $\hat{a}_j \equiv \sum_i \hat{a}_{i,j} > 0$ for all $j \in J$ we must have $p_j h_j = p_\ell h_\ell = d$ for any $j, \ell \in J$ for some constant $d \geq 0$ because otherwise no gamblers place any bet on the horse ℓ such that $p_j h_j^* > p_\ell h_\ell^*$. Then we obtain $1 = \sum_{j \in J} p_j = d \sum_{j \in J} (1/h_j) = d/(1-f)$ due to the definition of h_j, so we have $d = 1 - f$ and hence $p_j h_j = 1 - f$. Then the equilibrium bet $\hat{a}_{i,j} > 0$ placed by gambler i on horse j is determined by the following first-order optimality condition:

$$u_A \left(\hat{A}_i, S_i \right) - f = 0, \quad j \in J, \tag{A1}$$

yielding the desired result (7) in the main text. This gives the equilibrium amount $\hat{A}_i = \hat{A}(S_i)$ bet by gambler i in total, which depends on his addiction stock S_i. Note that the amount bet by each gambler on each horse $\hat{a}_{i,j}$ is indeterminate. Only the aggregate amount $\hat{a}_j$ bet by all gamblers on horse $j \in J$ is determined.

As a result, we can write the equilibrium second-payoff of gambler i who has the addiction stock S_i as follows

$$\begin{aligned} \hat{U}(S_i) &\equiv u(\hat{A}(S_i), S_i) + \sum_{j \in J} p_j h_j \hat{a}_{i,j} - \hat{A}(S_i) \\ &= u(\hat{A}(S_i), S_i) - f \hat{A}(S_i) \end{aligned}$$

because $\hat{A}(S_i) \equiv \sum_{j \in J} \hat{a}_{i,j}$, and $p_j h_j = 1 - f$ for all $j \in J$, yielding the desired result (8) in the main text. Q.E.D.

14.3 Proof of Proposition 2

Consider any punishment policy ψ implementing an equilibrium $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{A}}\}$ in the two-period game. That is, following the punishment policy ψ offered by the bookmaker, there exists the resulting equilibrium in which gambler $i \in I$ places equilibrium bet $\mathbf{a}_i^* \equiv (a_{i,1}^*, ..., a_{i,m}^*)$ in period 1. The associated equilibrium odds in period 1 are given by $\mathbf{h}^* \equiv \{h_j^*\}_{j \in J}$ such that $h_j^* = (1-f) \sum_{i \in I} A_i^* / \sum_{i \in I} a_{i,j}^*$.[30] Let $A_i^* \equiv \sum_{j \in J} a_{i,j}^*$ denote the aggregate amount bet by gambler i in the equilibrium.

Since we are here supposing that there exists an equilibrium $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{A}}\}$ implemented by a punishment policy ψ, in any subgame given any $A_i \geq 0$ and any $z_{i,j} \geq 0$, gambler i must optimally choose the default level $D_{i,j} \in [0, z_{i,j}]$, denoted by $D^*(z_{i,j}, A_i) \in [0, z_{i,j}]$.

[30] Note that we focus on the equilibrium in which every horse has positive odds, that is, $\sum_{i \in I} a_{i,j}^* > 0$.

That is, $D^*(z, A) \in [0, z]$ is chosen to maximize $D - \psi(D, z, A)$ over $0 \leq D \leq z$, given $z \geq 0$ and $S = \alpha + \beta A$.[31] Gambler i places bet $\mathbf{a}_i$ to maximize the following expected payoff in period 1:

$$v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + D^*(z_{i,j}, A_i) - \psi(D^*(z_{i,j}, A_i), z_{i,j}, A_i)\} + \hat{U}(S_i) \tag{A2}$$

given the equilibrium odds $\mathbf{h}^* = \{h_j^*\}_{j \in J}$. These gamblers' choices lead to the equilibrium amounts bet $\mathbf{a}^*$ in period 1 and the associated amounts bet $\hat{\mathbf{A}}$ in period 2 respectively.

Define

$$\underline{\psi}(z, A) \equiv \inf_{D \in [0,z]} \psi(D, z, A)$$

for $z \geq 0$ and $A \geq 0$. Then we show the following claim.

Lemma A1. *$D^*(z, A) - \psi(D^*(z, A), z, A) \geq -\underline{\psi}(z, A)$ holds for all $z \geq 0$ and all $A \geq 0$.*

Proof. See the Online Appendix.

Define

$$\tilde{D}(z, A) \equiv D^*(z, A) - \{\psi(D^*(z, A), z, A) - \underline{\psi}(z, A)\}$$

where $\tilde{D}(z, A) \geq 0$ due to Lemma A1, and $\tilde{D}(z, A) \leq z$ because $D^*(z, A) \leq z$. Then we define the following new punishment policy:

$$\tilde{\psi}(D, A) \equiv \begin{cases} \underline{\psi}(z, A) & \text{if } D \leq \tilde{D}(z, A) \\ \hat{U}(S) + H & \text{otherwise} \end{cases}$$

Note that the new policy $\tilde{\psi}$ has the bang-bang form. We show that the new policy defined above can induce gamblers to place the same bets as those under the original policy ψ while reducing defaulted amounts.

As shown in the main text, gamblers are never better off not only by defaulting less than the maximum forgiveness $\tilde{D}(z, A)$ but also by defaulting more than it. Thus gambler i's optimal default is to choose $D_{i,j} = \tilde{D}(z_{i,j}, A_i)$ when horse j wins the race, he bets A_i in total and owes the debt of $z_{i,j}$ in the end of period 1.

[31] Note that the functional form of $D^*(\cdot, \cdot)$ is the same for all gamblers because all of them face the same functional forms of $\psi(\cdot, \cdot)$ and $\hat{U}(\cdot)$. However, the realized amount of default $D^*(z_{i,j}, A_i)$ can differ across gamblers because they may face the different race outcomes $z_{i,j}$ and place different bets $\mathbf{a}_i$.

Then, for any profile of amounts bet $\mathbf{a}_i$, we can show that

$$
\begin{aligned}
& v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + D_{i,j} - \tilde{\psi}(D_{i,j}, z_{i,j}, A_i)\} + \hat{U}(S_i) \\
\leq \ & v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + \tilde{D}(z_{i,j}, A_i) - \underline{\psi}(z_{i,j}, A_i)\} + \hat{U}(S_i) \\
= \ & v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + D^*(z_{i,j}, A_i) - \psi(D^*(z_{i,j}, A_i), z_{i,j}, A_i)\} + \hat{U}(S_i) \\
\leq \ & \max_{\mathbf{a}_i} \ v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + D^*(z_{i,j}, A_i) - \psi(D^*(z_{i,j}, A_i), z_{i,j}, A_i)\} + \hat{U}(S_i) \\
= \ & v(A_i^*) + \sum_{j \in J} p_j \{h_j^* a_{i,j}^* - A_i^* + D^*(z_{i,j}^*, A_i^*) - \psi(D^*(z_{i,j}^*, A_i), z_{i,j}^*, A_i^*)\} + \hat{U}(S_i^*) \\
= \ & v(A_i^*) + \sum_{j \in J} p_j \{h_j^* a_{i,j}^* - A_i^* + \tilde{D}(z_{i,j}^*, A_i^*) - \underline{\psi}(z_{i,j}^*, A_i^*)\} + \hat{U}(S_i^*)
\end{aligned}
$$

where the first inequality follows from the optimal default choice of $\tilde{D}(z_{i,j}, A_i)$, and the first and last equalities from the definition of $\tilde{D}(z, A)$ respectively. Therefore, gambler i never deviates from the amount bet $\mathbf{a}_i^*$ which he chooses in the original equilibrium with the equilibrium odds $\mathbf{h}^*$. Thus the bang-bang punishment policy $\tilde{\psi}$ implements the same amounts bet as those in the equilibrium under the original punishment policy ψ.

We next decompose the lowest equilibrium penalty $\underline{\psi}(z_{i,j}^*, A_i^*)$ under the above bang-bang punishment policy $\tilde{\psi}$ into q and ξ as follows: define $\underline{q}(z, A) \equiv \inf_{0 \leq D \leq z} q(D, z, A)$ and

$$\underline{\xi}(z, A) \equiv \underline{\psi}(z, A) - \underline{q}(z, A)\hat{U}(S).$$

Since $\underline{\psi}(z, A) = \inf_{0 \leq D \leq z}\{q(D, z, A)\hat{U}(S) + \xi(D, z, A)\} \geq \inf_{0 \leq D \leq z} q(D, z, A)\hat{U}(S) = \underline{q}(z, A)\hat{U}(S)$, we have $\underline{\xi}(z, A) \geq 0$. Also, since $\underline{\psi}(z, A) \leq \inf_{0 \leq D \leq z} q(D, z, A)\hat{U}(S) + H = \underline{q}(z, A)\hat{U}(S) + H$, we have $\underline{\xi}(z, A) \leq H$. Thus $\underline{\xi}(z, A) \in [0, H]$ holds. Then we set $\underline{\psi}(z_{i,j^*}, A_i^*) = \underline{q}(z_{i,j}^*, A_i^*)\hat{U}(S_i^*) + \underline{\xi}(z_{i,j}^*, A_i^*)$ for $i \in I$ and $j \in J$. Given this modification, the corresponding payoff of the bookmaker who uses the newly defined policy $\tilde{\psi}$ becomes $\Pi(\tilde{\psi})$ given as (16) in the main text. This is not less than her payoff $\Pi(\psi)$, given as (17) in the main text, under the original punishment policy ψ because $\underline{q}(z, A) \leq q(D, z, A) \in [0, 1]$ and $\tilde{D}(z, A) \leq D^*(z, A)$. Q.E.D.

14.4 Proof of Proposition 3

We here give a sketch of the proof and relegate its details to the Online Appendix.

(i) Suppose contrary to the claim that $\tilde{D}(z_{i,j}^*, A_i^*) > 0$ implies that $\underline{\psi}(z_{i,j}^*, A_i^*) > 0$ for some $i \in I$ and $j \in J$ under the punishment policy $\tilde{\psi}$ that maximizes the bookmaker's

payoff. Then consider the new punishment policy by slightly reducing both $\tilde{D}(z^*_{i,j}, A^*_i)$ and $\underline{\psi}(z^*_{i,j}, A^*_i)$ by the same amount, say $\varepsilon > 0$, for such $i \in I$ and $j \in J$. This new punishment policy gives gamblers the same expected payoffs as those under the original punishment policy because the net gain of defaulting $\tilde{D}(z^*_{i,j}, A^*_i) - \varepsilon - \{\underline{\psi}(z^*_{i,j}, A^*_i) - \varepsilon\}$ is unchanged while weakly improving the bookmaker's payoff because she can then reduce the probability to exclude gamblers in period 2 whenever $q(\tilde{D}(z^*_{i,j}, A^*_i), z^*_{i,j}, A^*_i) > 0$.
(ii) Suppose contrary to the claim that $\min_{i \in I} \sum_{j \in J} p_j \underline{\psi}(z^*_{i,j}, A^*_i) > 0$. Then we have $\sum_j p_j \underline{\psi}(z^*_{i,j}, A^*_i) > 0$ for all $i \in I$. We define by $j(i) \in J$ the horse for which $\underline{\psi}(z^*_{i,j(i)}, A_i) > 0$ for each $i \in I$. Then, by using the similar logic to the above proof of (i), we can slightly reduce the lowest expected penalty $\underline{\psi}(z^*_{i,j(i)}, A_i) > 0$ for all $i \in I$ and all $j(i) \in J$, by which the bookmaker and gamblers are better off. Q.E.D.

14.5 Proof of Proposition 4

We set the candidate for the optimal solution to Problem B as follows: (i) $\underline{\psi}(z^*, A^*) = 0$ and

$$\tilde{D}(z^*, A^*) = \tilde{D}^* \equiv \max\{0, \{\overline{V} - \{v(A^*) - fA^* + \hat{U}(S^*)\}\},$$

(ii) $\underline{\psi}(D, z, A) = \hat{U}(S) + H$ and $\tilde{D}(z, A) = 0$ when $z \neq z^*$ or $A \neq A^*$, and (iii)

$$h^*_j = 1 + k^*/p_j, \quad j \in J$$

where $k^* \in (-p_1, \infty)$ is the unique solution to

$$\sum_{j \in J} \left(\frac{p_j}{p_j + k^*} \right) = 1/(1 - f). \tag{A3}$$

We can then show that, by using the above candidate for the optimal policy, IC* becomes sufficient to detect all possible deviations of gamblers under Assumption 1. The idea of the proof is as follows: first, the equilibrium odds $h^*_j = 1 + k^*/p_j$ minimize the gambler's deviation payoff $\overline{V}$ by equalizing the expected returns $p_j(h^*_j - 1)$ across all horses when taking into account gamblers' default incentives. Second, given these equilibrium odds, we show that gamblers have no incentives to deviate from the amounts bet $\mathbf{a}^*$ solving Problem B. Since the formal proof for this is complicated, we provide it to the Online Appendix. Q.E.D.

14.6 Proof of Proposition 5

Under Assumption 2, gambler's payoff $v(A) - fA + \hat{U}(S)$ can be larger than his deviation payoff $\overline{V}$ at the constrained efficient level $A = A^{**}$ even when he is not allowed to default at all, that is $\tilde{D}(z^{**}, A^{**}) = 0$ with $z^{**} \equiv fA^{**}$. Then the bookmaker can increase her payoff $fA + f\hat{A}(S)$ as much as possible by increasing the amount bet A until the gambler's equilibrium payoff $v(A) - fA + \hat{U}(S)$ is reduced down to the deviation payoff $\overline{V}$. Therefore the bookmaker chooses A to satisfy $v(A) - fA + \hat{U}(S) = \overline{V}$, implying that such A is larger than the constrained efficient one A^{**}. When f is sufficiently small, we have $A^{**} \simeq A_1^{fb}$ and $\hat{A}(S^{**}) \simeq A_2^{fb}$ so that the equilibrium amounts bet can be larger than the first-best choices in both periods. We provide the formal proof in the Online Appendix. Q.E.D.

14.7 Proof of Proposition 6

We here give a sketch about the idea of proof (more details are given in the Online Appendix).

(i) Consider the extreme form of the bang-bang punishment policy $\tilde{\psi}$ by setting the maximum forgiveness as $\tilde{D}(z, A) = 0$ for all $z \geq 0$ and all $A \geq 0$. Thus gamblers are imposed the maximum penalty $\hat{U}(S) + H$ if they default any positive amount. Then under Assumption 2 gamblers optimally choose not to default at all because, if they default, they lose the large second-period payoffs. By using this punishment policy, the bookmaker can mimic the same outcome as that attained under the cash-bet, thereby making her never worse off by offering the book-bet.

(ii) Under the book-bet each gambler can always default all debts, follow being imposed the maximum penalty $H + \hat{U}$ in period 1, and quit in period 2; this strategy gives him at least the payoff of $\max_A u(A) - H$. To deter such deviation, the bookmaker must compensate at least such payoff by allowing gamblers to default some positive amounts in period 1. However, this is costly for the bookmaker when there are no gains from making gamblers addicted to gambling in period 1. Then the cash-bet dominates the book-bet in period 1. Q.E.D.

15 Online Appendix A: Additional Proofs

In this online appendix we provide additional proofs for the results presented in the main paper.

15.1 Proof of Lemma A1

Due to the optimal default choice of $D^*(z, A)$, it must be that

$$D^*(z, A) - \psi(D^*(z, A), z, A) \geq D - \psi(D, z, A)$$

for all $D \in [0, z]$. Also, due to the definition of $\underline{\psi}(z, A) = \inf_{0 \leq D \leq z} \psi(D, z, A)$, for any $\varepsilon > 0$, some $D'' \in [0, z]$ exists such that

$$\underline{\psi}(z, A) + \varepsilon > \psi(D'', z, A).$$

Therefore, we obtain

$$\begin{aligned} D^*(z, A) - \psi(D^*(z, A), z, A) &\geq D'' - \psi(D'', z, A) \\ &> D'' - \underline{\psi}(z, A) - \varepsilon \\ &\geq -\underline{\psi}(z, A) - \varepsilon \end{aligned}$$

for all small $\varepsilon > 0$, so that

$$D^*(z, A) - \psi(D^*(z, A), z, A) > -\underline{\psi}(z, A) - \varepsilon$$

for all small $\varepsilon > 0$. By letting $\varepsilon \to 0$, we obtain the desired result. Q.E.D.

15.2 Proof of Proposition 3

(i) Suppose contrary to the claim that $\tilde{D}(z^*_{i,j}, A^*_i) > 0$ implies that $\underline{\psi}(z^*_{i,j}, A^*_i) > 0$ for some $i \in I$ and $j \in J$ under the punishment policy $\tilde{\psi}$ that maximizes the bookmaker's payoff. Let $\tilde{I}$ and $\tilde{J}$ denote the set of such $i \in I$ and $j \in J$. Then define the new punishment policy $\tilde{\psi}''$ as follows: define $\tilde{D}''(z^*_{i,j}, A^*_i) \equiv \tilde{D}(z^*_{i,j}, A^*_i) - \varepsilon$ and $\underline{\psi}''(z^*_{i,j}, A^*_i) \equiv \underline{\psi}(z^*_{i,j}, A^*_i) - \varepsilon$ for $(z^*_{i,j}, A^*_i)$ such that $i \in \tilde{I}$ and $j \in \tilde{J}$, for a small $\varepsilon > 0$, while $\tilde{D}''(z, A) = \tilde{D}(z, A)$ and $\underline{\psi}''(z, A) = \underline{\psi}(z, A)$ for all other values of z and A. Then the punishment policy $\tilde{\psi}''$ newly defined in this way induces gamblers to place the same equilibrium bets as those under the original policy $\tilde{\psi}$ because

$$\begin{aligned}
& v(A_i^*) + \sum_{j \in J} p_j \{h_j^* a_{i,j}^* - A_i^* + \tilde{D}''(z_{i,j}^*, A_i^*) - \underline{\psi}''(z_{i,j}^*, A_i^*)\} + \hat{U}(S_i^*) \\
= \; & v(A_i^*) + \sum_{j \in J} p_j \{h_j^* a_{i,j}^* - A_i^* + \tilde{D}(z_{i,j}^*, A_i^*) - \underline{\psi}(z_{i,j}^*, A_i^*)\} + \hat{U}(S_i^*) \\
\geq \; & \max_{\mathbf{a}_i, 0 \leq D_{i,j} \leq z_{i,j}} v(A_i) + \sum_j p_j \{h_j^* a_{i,j} - A_i + D_{i,j} - \tilde{\psi}(D_{i,j}, z_{i,j}, A_i)\} + \hat{U}(S_i) \\
= \; & \max_{\mathbf{a}_i, 0 \leq D_{i,j} \leq z_{i,j}} v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + D_{i,j} - \tilde{\psi}''(D_{i,j}, z_{i,j}, A_i)\} + \hat{U}(S_i).
\end{aligned}$$

Here note that $-z_{i,j}^* + \tilde{D}(z_{i,j}^*, A_i^*) - \tilde{\psi}(\tilde{D}(z_{i,j}^*, A_i^*) \geq -\hat{U}(S_i^*) - H$ is equivalent to $-z_{i,j}^* + \tilde{D}''(z_{i,j}^*, A_i^*) - \tilde{\psi}''(\tilde{D}''(z_{i,j}^*, A_i^*)) \geq -\hat{U}(S_i^*) - H$ for any $(z, A) = (z_{i,j}^*, A_i^*)$ for $(i, j) \in \tilde{I} \times \tilde{J}$, showing that gamblers' optimal default is not changed for all such (z, A). Also, since $\psi(D, z, A) = \tilde{\psi}''(D, z, A)$ holds for any $(z, A) \neq (z, A) = (z_{i,j}^*, A_i^*)$ for $(i, j) \in \tilde{I} \times \tilde{J}$, gamblers' default choice is changed for all other (z, A) as well.

We next define the exclusion probability q'' and the utility-based penalty ξ'' under $\tilde{\psi}''$. Define $q_{i,j}^* \equiv q(\tilde{D}(z_{i,j}^*, A_i^*), z_{i,j}^*, A_i^*)$ and $\xi_{i,j}^* \equiv \xi(\tilde{D}(z_{i,j}^*, A_i^*), z_{i,j}^*, A_i^*)$ as the exclusion probability and the utility-based penalty used in the equilibrium under the original punishment policy $\tilde{\psi}$. Then we define $\underline{q}''(z_{i,j}^*, A_i^*)$ and $\underline{\xi}''(z_{i,j}^*, A_i^*)$ for $(z_{i,j}^*, A_i^*)$ such that $(i, j) \in \tilde{I} \times \tilde{J}$ as follows: if $q_{i,j}^* > 0$, then define $\underline{q}''(z_{i,j}^*, A_i^*) \equiv q_{i,j}^* - \rho_{i,j}$ for a small $\rho_{i,j} > 0$ such that $\rho_{i,j} \hat{U}(S_i^*) = \varepsilon$, and $\underline{\xi}''(z_{i,j}^*, A_i^*) = \xi_{i,j}^*$. If $q_{i,j}^* = 0$, then set $\underline{q}''(z_{i,j}^*, A_i^*) = 0$ while $\underline{\xi}''(z_{i,j}^*, A_i^*) \equiv \xi_{i,j}^* - \varepsilon$. Also define $q''(D, z, A) = q(D, z, A)$ and $\xi''(D, z, A) = \xi(D, z, A)$ for all other values of (D, z, A).

By the new punishment policy $\tilde{\psi}''$ defined above, the bookmaker's payoff is changed to

$$\sum_i \left\{ f A_i^* - \sum_{j \in J} p_j \{\tilde{D}''(z_{i,j}^*, A_i^*) + (1 - \underline{q}''(z_{i,j}^*, A_i^*)) \hat{A}(S_i^*)\} \right\}$$

which can be greater than the payoff under the original punishment policy $\tilde{\psi}$.

(ii) Suppose contrary to the claim that $\min_{i \in I} \sum_{j \in J} p_j \underline{\psi}(z_{i,j}^*, A_i^*) > 0$. Then we have $\sum_j p_j \underline{\psi}(z_{i,j}^*, A_i^*) > 0$ for all $i \in I$. We define by $j(i) \in J$ the horse for which $\underline{\psi}(z_{i,j(i)}^*, A_i) > 0$ for each $i \in I$.

For a small $\rho > 0$, define $\varepsilon_{i,j(i)} \equiv \rho / p_{j(i)}$ for each $i \in I$. Then define the new punishment policy $\tilde{\psi}''$ as follows: set $\underline{\psi}''(z_{i,j(i)}^*, A_i^*) \equiv \underline{\psi}(z_{i,j(i)}^*, A_i^*) - \rho$ and $\tilde{\psi}''(D, z, A) = \tilde{\psi}(D, z, A)$ for all other values of (D, z, A) than $(z_{i,j(i)}^*, A_i^*)_{i \in I}$ while keeping the maximum forgiveness $\tilde{D}(z, A)$ unchanged. Then we show that the newly defined punishment policy $\tilde{\psi}''$ can still implement the same equilibrium outcome while making the bookmaker weakly better off.

First, if gambler i chooses $A_i \neq A_i^*$, he obtains the payoff of

$$v(A_i) + \sum_j p_j\{h_j^* a_{i,j} - A_i + D_{i,j} - \tilde{\psi}''(D_{i,j}, z_{i,j}, A_i)\} + \hat{U}(S_i)$$

which is equal to

$$V_i'' \equiv v(A_i) + \sum_j p_j\{h_j^* a_{i,j} - A_i + D_{i,j} - \tilde{\psi}(D_{i,j}, z_{i,j}, A_i)\} + \hat{U}(S_i)$$

because $\tilde{\psi}''(D, z, A) = \tilde{\psi}(D, z, A)$ for all $(z, A) \neq (z_{i,j(i)}^*, A_i^*)$. However, if gambler i follows the equilibrium bet $\mathbf{a}_i^*$, he obtains the payoff of

$$\begin{aligned}
& v(A_i^*) + \sum_j p_j\{h_j^* a_{i,j}^* - A_i^* + \tilde{D}(z_{i,j}^*, A_i^*) - \underline{\psi}''(z_{i,j}^*, A_i^*)\} + \hat{U}(S_i^*) \\
&= \ v(A_i^*) + \sum_j p_j\{h_j^* a_{i,j}^* - A_i^* + \tilde{D}(z_{i,j}^*, A_i^*) - \underline{\psi}(z_{i,j}^*, A_i^*)\} + \hat{U}(S_i^*) + \rho \\
&\equiv V_i^* + \rho \\
&> \ V_i^*.
\end{aligned}$$

Since the original punishment policy $\tilde{\psi}$ implements $\mathbf{a}_i^*$, it must be that $V_i^* \geq V_i''$, implying that gambler i never deviates from $\mathbf{a}_i^*$.

Next suppose that gambler i chooses $A_i = A_s^*$ for some $s \neq i$ but $\mathbf{a}_i \neq \mathbf{a}_i^*$. Define the set of horses J'' such that $a_{i,j} = a_{s,j}^*$ holds. Then we have $z_{i,j} = A_s^* - h_j^* a_{i,j} = z_{s,j}^*$ for $j \in J''$, so that, if gambler i defaults $D_{i,j} \leq \tilde{D}(z_{s,j}^*, A_s^*)$, $\underline{\psi}''(z_{s,j}^*, A_s^*)$ is applied as the penalty. By J^* define the subset of J'' as follows: $J^* \subset J''$ such that $D_{i,j} \leq \tilde{D}(z_{s,j}^*, A_s^*)$. Then, if $j(s) \in J^*$, $\underline{\psi}''(z_{s,j(s)}^*, A_s^*) = \underline{\psi}(z_{s,j(s)}^*, A_s^*) - \rho/p_{j(s)}$ is applied. By using these facts, we verify that gambler i obtains at most the following deviation payoff:

$$\begin{aligned}
& v(A_s^*) + \sum_{j \in J^*} p_j\{h_j^* a_{s,j}^* - A_s^* + \tilde{D}(z_{s,j}^*, A_s^*) - \underline{\psi}''(z_{s,j}^*, A_s^*)\} \\
& + \sum_{j \in J'' \setminus J^*} p_j\{h_j^* a_{s,j}^* - A_s^* + D_{i,j} - \tilde{\psi}''(D_{i,j}, z_{s,j}^*, A_s^*)\} \\
& + \sum_{j \notin J''} p_j\{h_j^* a_{i,j} - A_s^* + D_{i,j} - \tilde{\psi}''(D_{i,j}, z_{i,j}, A_s^*)\} + \hat{U}(S_s^*).
\end{aligned}$$

Since $\tilde{\psi}''$ coincides with $\tilde{\psi}$ except only for $(z^*_{s,j(s)}, A^*_s)$, we can rewrite the above payoff by

$$\begin{aligned} & v(A^*_s) + \sum_{j \in J^*} p_j \{h^*_j a^*_{s,j} - A^*_s + \tilde{D}(z^*_{s,j}, A^*_s) - \underline{\psi}(z^*_{s,j}, A^*_s)\} + \mathbf{1}_{j(s)}\rho \\ & + \sum_{j \in J'' \setminus J^*} p_j \{h^*_j a^*_{s,j} - A^*_s + D_{i,j} - \tilde{\psi}(D_{i,j}, z^*_{s,j}, A^*_s)\} \\ & + \sum_{j \notin J''} p_j \{h^*_j a_{i,j} - A^*_s + D_{i,j} - \tilde{\psi}(D_{i,j}, z_{i,j}, A^*_s)\} + \hat{U}(S^*_s) \\ & = v(A^*_s) + \sum_{j \in J} p_j \{h_j a_{i,j} - A^*_s + D_{i,j} - \tilde{\psi}(D_{i,j}, z_{i,j}, A^*_s)\} + \hat{U}(S^*_s) + \mathbf{1}_{j(s)}\rho \end{aligned}$$

where $\mathbf{1}_{j(s)}$ takes 1 only when $j(s) \in J^*$ and zero otherwise. Since $\tilde{\psi}$ implements $\mathbf{a}^*_i$, it must be that

$$V^*_i \geq v(A^*_s) + \sum_{j \in J} p_j \{h_j a_{i,j} - A^*_s + D_{i,j} - \tilde{\psi}(D_{i,j}, z_{i,j}, A^*_s)\} + \hat{U}(S^*_s)$$

for any $\mathbf{a}_i \neq \mathbf{a}^*_i$ with $A_i = A^*_s$ for any $s \in I$. Thus, the above deviation payoff cannot be larger than $V^*_i + \mathbf{1}_{j(s)}\rho$, which is less than the equilibrium payoff $V^*_i + \rho$ obtained under the new punishment policy $\tilde{\psi}''$. Therefore gambler i never makes the above deviation.

Finally, we set the exclusion probability q'' under $\tilde{\psi}''$ as follows: if $q^*_{i,j(i)} \equiv q(\tilde{D}(z^*_{i,j(i)}, A^*_i), z^*_{i,j(i)}, A^*_i) > 0$, then define $\underline{q}''(z^*_{i,j(i)}, A^*_i) \equiv q^*_{i,j(i)} - \varepsilon_{i,j(i)} \geq 0$ where $\varepsilon_{i,j(i)}\hat{U}(S^*_i) = \rho$. Otherwise define $\underline{q}''(z^*_{i,j(i)}, A^*_i) = 0$. Also define $q''(D, z, A) = q(D, z, A)$ for all $(z, A) \notin \{(z^*_{i,j(i)}, A^*_i)\}_{i \in I}$. Given these definitions, we set the utility-based penalty ξ'' as follows: if $q^*_{i,j(i)} > 0$, then we set $\xi''(z^*_{i,j(i)}, A^*_i) = \xi(z^*_{i,j(i)}, A^*_i)$ while otherwise $\xi''(z^*_{i,j(i)}, A^*_i) \equiv \xi(z^*_{i,j(i)}, A^*_i) - \rho$ (because $\underline{\psi}(z^*_{i,j(i)}, A^*_i) > 0$). Also define $\xi''(D, z, A) = \xi(D, z, A)$ for all $(z, A) \notin \{(z^*_{i,j(i)}, A^*_i)\}_{i \in I}$. Then the bookmaker is never worse off by the newly defined punishment policy $\tilde{\psi}''$. Q.E.D.

15.3 Proof of Proposition 4

We set the candidate for the optimal solution to Problem B as follows:

$$\underline{\psi}(z^*, A^*) = 0,$$

$$\tilde{D}(z^*, A^*) = \tilde{D}^* \equiv \max\{0, \{\overline{V} - \{v(A^*) - fA^* + \hat{U}(S^*)\}\},$$

$$\tilde{D}(z, A) = 0, \quad \underline{\psi}(D, z, A) = \hat{U}(S) + H \;\text{ if } z \neq z^* \text{ or } A \neq A^*,$$

and

$$h^*_j = 1 + k^*/p_j, \quad j \in J$$

where $k^* \in (-p_1, \infty)$ is the unique solution to

$$\sum_{j \in J} \left(\frac{p_j}{p_j + k^*} \right) = 1/(1-f). \tag{A1}$$

We now show that, by using the candidate for the optimal policy defined above, IC* becomes sufficient for (19) in the main text to hold under Assumption 1.

We first show that the odds defined above minimize the deviation payoff of gamblers $\overline{V}$. To this end, fix arbitrary odds $\{h_j^*\}_{j \in J}$. Then we show that some $\ell \in J$ exists such that $a_{i,\ell} = A_i$ and $a_{i,j} = 0$ for any $j \neq \ell$, and we can then re-write $\overline{V}$ as follows

$$\overline{V} = \max_{\mathbf{a}_i} v(A_i) + \max\{0, p_\ell(h_\ell^* - 1)A_i\} - H$$

where $\ell \in J$ maximizes $p_j(h_j^* - 1)$ over J. If this is not the case, some ℓ and j (with $\ell \neq j$) exist such that gambler i places bets of $a_{i,\ell} > 0$ and $a_{i,j} > 0$ on ℓ and j as well as $p_\ell(h_\ell^* a_{i,\ell} - A_i) \geq 0$ and $p_j(h_j^* a_{i,j} - A_i) \geq 0$ to achieve $\overline{V}$. Then the gambler can be however weakly better off by betting all A_i to either ℓ or j depending on whether or not to have $p_\ell h_\ell^* \geq p_j h_j^*$: when $p_\ell h_\ell^* \geq p_j h_j^*$, the gambler should choose $a_{i,\ell} = A_i$ and $a_{i,j} = 0$ to obtain a weakly higher payoff $p_\ell h_\ell^* A_i - p_\ell A_i \geq p_\ell(h_\ell^* a_{i,\ell} - A_i) + p_j(h_j^* a_{i,j} - A_i)$. Therefore, without loss of generality we can suppose that the gambler puts all A_i to horse ℓ that maximizes $p_j(h_j^* - 1)$ over $j \in J$.

We now consider the odds $\{h_j^*\}_{j \in J}$ to minimize the gambler's deviation payoff $\overline{V}$. Suppose that some ℓ exists such that

$$\max_{j \in J} p_j(h_j^* - 1) > p_\ell(h_\ell^* - 1).$$

Then we can slightly reduce $h_\ell^* > 0$ for any ℓ for which $p_\ell(h_\ell^* - 1) = \max_{j \in J} p_j(h_j^* - 1)$ and slightly increase $h_j^* > 0$ for any $j \neq \ell$ while keeping $\sum_{j \in J}(1/h_j^*) = 1/(1-f)$ unchanged. This can strictly reduce $\overline{V}$ or make it unchanged. Therefore, without loss of generality we set

$$p_j(h_j^* - 1) = k, \quad \text{for any } j \in J$$

for some constant k. Then $h_j^* = 1 + k/p_j$ for $j \in J$ so that $1/(1-f) = \sum_{j \in J}(1/h_j^*) = \sum_{j \in J}(p_j/(p_j + k))$, which uniquely determines $k^* \in (-p_1, \infty)$.

We now show that IC* is not only necessary but also sufficient for (19) to hold under $\tilde{\psi}$ defined above. First, suppose that $A_i \neq A^*$. Then gambler i faces $\tilde{\psi}(D, z_{i,j}, A_i) = \hat{U}(S_i) + H$ for any $D \in [0, z_{i,j}]$ and any $j \in J$, so he optimally defaults all the debt; $D_{i,j} = z_{i,j}$. Thus gambler i obtains at most

$$\begin{aligned}&\max_{\mathbf{a}} v(A) + \max_{j\in J} p_j \max\{h_j^* - 1, 0\}A - H \\ &= \max_{A} \; v(A) + \max\{0, k^*\}A - H \\ &= \overline{V}\end{aligned}$$

which is however not larger than the equilibrium payoff V^* due to IC*, that is, $V^* = v(A^*) - fA^* + \tilde{D}^* + \hat{U}(S^*) \geq \overline{V}$. Second, suppose that $A_i = A^*$ but $z_{i,j} \neq z^*$ for some $j \in J$. Let $\tilde{J} \subseteq J$ denote the set of horses such that $z_{i,j} = z^*$ and hence $a_{i,j} = a_j^*$. Then gambler i faces $\tilde{\psi}(D_{i,j}, z_{i,j}, A^*) = \hat{U}(S^*) + H$ for any $D_{i,j} \in [0, z_{i,j}]$ and any $j \notin \tilde{J}$, and obtains at most

$$\max_{\mathbf{a}_i} v(A^*) + \sum_{j\in\tilde{J}} p_j\{h_j^* a_j^* - A^* + \tilde{D}^*(z^*, A^*)\} + \sum_{j\notin\tilde{J}} p_j\{\max\{h_j^* a_{i,j} - A^*, 0\} - \hat{U}(S^*) - H\} + \hat{U}(S^*)$$

subject to $\sum_{j\notin\tilde{J}} a_{i,j} = A^* - \sum_{j\in\tilde{J}} a_j^*$. Letting $\tilde{P} \equiv \sum_{j\in\tilde{J}} p_j$ and noting that $h_j^* a_j^* = (1-f)A^*$, this payoff is bounded above by

$$v(A^*) + \tilde{P}\{-fA^* + \tilde{D}^*\} + \max\left\{0, \max_{j\notin\tilde{J}} p_j h_j^* \left(A^* - \sum_{j\in\tilde{J}} a_j^*\right) - p_j A^*\right\} - (1-\tilde{P})(\hat{U}(S^*) + H) + \hat{U}(S^*)$$

which equals to

$$\begin{aligned}&v(A^*) + \tilde{P}\{-fA^* + \tilde{D}^*\} \\ &+ \max\left\{0, \max_{j\notin\tilde{J}}\left\{p_j h_j^*\left(1 - (1-f)\sum_{j\in\tilde{J}}\left(\frac{p_j}{p_j + k^*}\right)\right) - p_j\right\}\right\} A^* - (1-\tilde{P})(\hat{U}(S^*) + H) + \hat{U}(S^*)\end{aligned}$$

because $a_j^* = (1-f)A^*/h_j^*$ and $h_j^* = 1 + k^*/p_j$. Due to IC* such that $V^* \geq \overline{V}$, this payoff is not larger than the equilibrium payoff V^* if

$$\frac{1}{1-\tilde{P}} \max\left\{0, \left\{p_j h_j^*\left(1 - (1-f)\sum_{j\in\tilde{J}}\left(\frac{p_j}{p_j + k^*}\right)\right) - p_j\right\}\right\} \leq \max\{k^*, 0\} \qquad \text{(A2)}$$

for all $j \notin \tilde{J}$. When the maximum of the left hand side of (A2) is zero for all $j \notin \tilde{J}$, this inequality is trivially satisfied. Therefore, suppose that the above maximum is not zero for some $j \notin \tilde{J}$, implying that $0 < p_j h_j^*(A^* - \sum_{j\in\tilde{J}} a_j^*) - p_j A^*$. Then we have

$$\begin{aligned} 0 &< p_j h_j^* \left(A^* - \sum_{j\in\tilde{J}} a_j^* \right) - p_j A^* \\ &\leq p_j (h_j^* - 1) A^* \\ &= k^* \end{aligned}$$

so that $k^* > 0$. Therefore, by using $p_j h_j^* = k^* + p_j$, we can rewrite (A2) by

$$(p_\ell + k^*)(1-f) \sum_{j\in\tilde{J}} (p_j/\tilde{P}) \left(\frac{1}{p_j + k^*} \right) \geq k^* \tag{A3}$$

for any $\ell \notin \tilde{J}$ such that $p_\ell h_\ell^*(A^* - \sum_{j\in\tilde{J}} a_j^*) - p_\ell A^* > 0$. Since $p_j \leq p_m$ for all $j \neq m$, we obtain that

$$\sum_{j\in\tilde{J}} (p_j/\tilde{P}) \frac{1}{p_j + k^*} \geq \frac{1}{p_m + k^*}.$$

Since $p_1 \leq p_\ell$, the left hand side of (A3) is bounded below from

$$(1-f)\frac{p_1 + k^*}{p_m + k^*}.$$

Then the desired inequality (A3) is satisfied if

$$(1-f)\frac{p_1 + k^*}{p_m + k^*} \geq k^*. \tag{A4}$$

Also, since $\sum_j (p_j/p_j + k^*) = 1/(1-f)$ and $p_1 \leq p_j$ for all $j \neq 1$, we have $k^* \leq \hat{k} \equiv 1 - f - p_1$. We now define the following function

$$g(k) \equiv (1-f)\frac{p_1 + k}{p_m + k} - k$$

for $k \geq 0$, where $g(0) > 0$ and $g(\infty) = -\infty$ as well as $g'' < 0$. Then, inequality (A4) holds if $g(\hat{k}) \geq 0$ so that

$$(1-f)\frac{p_1 + \hat{k}}{p_m + \hat{k}} \geq \hat{k}.$$

This can be written by $(1-f)^2 \geq (1-f-p_1)(p_m - p_1 + 1 - f)$ so that $p_1(p_m - p_1) \geq (1-f)(p_m - 2p_1)$. When $p_m \leq 2p_1$, this inequality is satisfied. When $p_m \geq 2p_1$, this inequality holds if $p_1(p_m - p_1) \geq p_m - 2p_1$ (when $f = 0$), which is satisfied due to Assumption 1: $p_1 \geq (1-p_1)(p_m - p_1)$.

Therefore IC* is sufficient for A^* to be implementable under Assumption 1. Then we can re-write Problem B by Problem B* given in the main text. This completes the proof of the first half of Proposition 4. Furthermore, as we have shown above, the equilibrium odds which solve Problem B are given by $h_j^* = k^*/p_j + 1$, completing the proof of the second half of Proposition 4. Q.E.D.

15.4 Proof of Proposition 5

Define $S^{fb} \equiv \alpha + \beta A_1^{fb}$ and $S^{**} \equiv \alpha + \beta A^{**}$. Then, under Assumption 2 we can verify that, when f is sufficiently small,

$$v(A^{fb}) - fA^{fb} + \hat{U}(S^{fb}) > \overline{V} \tag{A5}$$

and

$$v(A^{**}) - fA^{**} + \hat{U}(S^{**}) > \overline{V}. \tag{A6}$$

To see this, note that for the value of k^* satisfying (A1), we have $k^* \leq 1 - f - p_1 \leq 1 - f$ as shown in the proof of Proposition 4. Then under Assumption 2 we obtain $-fA + \hat{U}(S) > (1-f)A \geq \max\{k^*, 0\}A$ for all $A \geq 0$, and hence

$$\begin{aligned} \max_{A \geq 0} v(A) - fA + \hat{U}(S) &> \max_{A \geq 0} v(A) + \max\{k^*, 0\}A - H \\ &= \overline{V}. \end{aligned}$$

When $f \in (0,1)$ is sufficiently small, the payoff in the left hand side is close to

$$v(A^{**}) - fA^{**} + \hat{U}(S^{**}) \simeq v(A^{**}) + u(\hat{A}(S^{**}), S^{**}) = \max_{A} v(A) + u(\hat{A}(S), S)$$

thereby resulting in (A6). Also, when f is small, A^{**} and A_1^{fb} are close to each other as well, leading to (A5). We fix such small f in what follows.

Claim 1: $\overline{V} \leq v(A^b) - fA^b + \hat{U}(S^b)$ so that $\tilde{D}^* = 0$ holds at the optimum of Problem B*.

Proof. Suppose that

$$\overline{V} > v(A^b) - fA^b + \hat{U}(S^b)$$

so that $\tilde{D}^* = \overline{V} - \{v(A^b) - fA^b + \hat{U}(S^b)\}$. Then the bookmaker's payoff is given by

$$fA^b - \tilde{D}^* + f\hat{A}(S^b) = v(A^b) + u(\hat{A}(S^b), S^b) - \overline{V}.$$

If $A^b \neq A^{**}$, then the bookmaker can increase her payoff by changing A^b slightly without violating $\overline{V} > v(A^b) - fA^b + \hat{U}(S^b)$. Therefore we have $A^b = A^{**}$ but then

$$v(A^{**}) - fA^{**} + \hat{U}(S^{**}) < \overline{V}$$

contradicting to (A6). Therefore, it must be that

$$\overline{V} \leq v(A^b) - A^b + u(\hat{A}(S^b), S^b)$$

so that $\tilde{D}^* = 0$ holds at the optimum. Q.E.D.

Claim 2: $A^b = A_{\max} \equiv \max\{A \mid \overline{V} \leq v(A) - fA + \hat{U}(S)\}$.

Proof. Due to Claim 1, at the optimum of Problem B* the bookmaker's payoff is given by $fA^b + f\hat{A}(S^b)$ where $\overline{V} \leq v(A^b) - fA^b + \hat{U}(S^b)$. Since $fA + \hat{A}(S)$ is increasing in A, it must be that the bookmaker chooses the largest A among those such that $\overline{V} \leq v(A) - fA + \hat{U}(S)$. Q.E.D.

Claim 3: $A^b > A_1^{fb}$ and $\hat{A}(S^b) > A_2^{fb}$.

Proof. Suppose that $A_1^{fb} \geq A^b = A_{\max}$. First, if $A_1^{fb} = A_{\max}$, then

$$v(A_1^{fb}) - fA_1^{fb} + \hat{U}(S^{fb}) \geq \overline{V}.$$

However, we verify from (A5) that a slight increase from A_1^{fb} to A'' results in $v(A'') - fA'' + \hat{U}(S'') \geq \overline{V}$ so that we obtain $A_{\max} \geq A'' > A_1^{fb}$, contradicting to the supposition that $A_1^{fb} = A_{\max}$. Thus suppose that $A_1^{fb} > A_{\max}$. Then we have

$$v(A_1^{fb}) - fA_1^{fb} + \hat{U}(S^{fb}) < \overline{V} \tag{A7}$$

because otherwise inequality (A7) is reversed so that our supposition of $A_1^{fb} > A_{\max}$ contradicts to the definition of $A_{\max}$. However, then (A7) contradicts to (A5), and hence we obtain $A^b = A_{\max} > A_1^{fb}$.

Given the result that $A^b > A^{fb}$, we obtain $\hat{A}(S^b) \simeq A^*(S^b) > A^*(S^{fb}) = A_2^{fb}$ when $f > 0$ is sufficiently small where $A^*(S)$ is defined as A_2 maximizing $u(A_2, S)$, and thus $A_2^{fb} = A^*(S^{fb})$. Q.E.D.

15.5 Proof of Proposition 6

(i) We show that it becomes optimal for the bookmaker to hold the book-bet rather than the cash-bet in period 1, under Assumption 2. Holding the cash-bet in period 1 is equivalent to holding the book-bet in period 1 with the maximum forgiveness equal to $\tilde{D}(z, A) = 0$ for all $A \geq 0$ and all $z \geq 0$ as well as the following penalty: $\tilde{\psi}(D, z, A) = 0$ when $D = 0$ and $\tilde{\psi}(D, z, A) = \hat{U}(S) + H$ when $D > 0$ for any $z \geq 0$ and any $A \geq 0$.

To see this, note that under such book-bet gambler i never defaults: if gambler i defaults $D_{i,j} > 0$ when horse j wins the race, he will obtain the ex post payoff of $h_j a_{i,j} - A_i + D_{i,j} - H$ whereas, if he does not default, he will obtain $h_j a_{i,j} - A_i + \hat{U}(S_i)$. Since $0 < D_{i,j} \leq z_{i,j} \equiv \max\{A_i - h_j a_{i,j} 0\}$ and $-A_i + \hat{U}(S_i) > 0$ for all $A_i \geq 0$ under Assumption 2, the former payoff is smaller than the latter, $h_j a_{i,j} - A_i + D_{i,j} - H \leq 0 < h_j a_{i,j} - A_i + \hat{U}(S_i)$. Therefore, gambler i never defaults ex post. Given this result, gambler i bets $\mathbf{a}_i$ to maximize his expected payoff

$$v(A_i) + \sum_{j \in J} p_j \{h_j a_{i,j} - A_i\} + \hat{U}(S_i).$$

This is equivalent to the case that the bookmaker holds the cash-bet in period 1 as well as in period 2. Then the bookmaker can always choose the punishment policy under the book-bet that implements the same payoff as that attained under the cash-bet. The bookmaker may be better off further by optimally choosing the punishment policy $\tilde{\psi}$. Therefore under Assumption 2 the bookmaker weakly prefers the book-bet to the cash-bet.

(ii) Define $\tilde{A}^*$ as the amount bet A maximizing $u(A) - fA = v(A) - fA$. Suppose that gamblers are not addicted to gambling at all: $\alpha = \beta = 0$. Then, we show that the bookmaker prefers the cash-bet to the book-bet in period 1, provided that $u(\tilde{A}^*) + H < 2f\tilde{A}^*$.

Recall that $\overline{u} \equiv \max_{A \geq 0} u(A) < +\infty$. Suppose that $\mathbf{a}^* \equiv (a_j^*)_{j \in J}$ and $A^* \equiv \sum_{j \in J} a_j^*$ are implemented in an equilibrium under a bang-bang punishment policy $\tilde{\psi}$. The gambler's equilibrium payoff in period 2 is given by $\hat{U} \equiv \max_A u(A) - fA$ when $S \equiv 0$. Then gambler i's equilibrium payoff over two periods is given by

$$\begin{aligned} & u(A^*) + \sum_{j \in J} p_j \{h_j^* a_j^* - A^* + \tilde{D}(z_j^*, A^*) - \underline{\psi}(z_j^*, A^*) + \hat{U}\} \\ = \ & u(A^*) - fA^* + \sum_{j} p_j \{\tilde{D}(z_j^*, A^*) - \underline{\psi}(z_j^*, A^*)\} + \hat{U} \end{aligned}$$

where the equilibrium odds are given by $h_j^* = (1-f)A^*/a_j^*$.

Gambler i can always choose A_i to maximize $u(A)$, and default all the amounts required to pay back, $D_{i,j} = \max\{A_i - h_j^* a_{i,j}, 0\}$, followed by being imposed the largest penalty $\tilde{\psi}(D, z, A) = \hat{U} + H$, and then quitting the gambling in period 2. By doing this choice, gambler i can secure at least the payoff of $\overline{u} - H$. Therefore, for such a deviation to be unprofitable, it must be that

$$u(A^*) - fA^* + \sum_{j} p_j \{\tilde{D}(z_j^*, A^*) - \underline{\psi}(z_j^*, A^*)\} + \hat{U} \geq \overline{u} - H$$

so that

$$\sum_{j} p_j \tilde{D}(z_j^*, A^*) \geq \overline{u} - \{u(A^*) - fA^* + \hat{U} + H\}.$$

Since each gambler bets $A_2 = \tilde{A}^*$ maximizing $u(A) - fA$ in period 2, the expected payoff of the bookmaker from using the book-bet in period 1 is given by

$$\begin{aligned}\Pi_b &\equiv n\left\{fA^* - \sum_j p_j \tilde{D}(z_j^*, A^*)\right\} + nf\tilde{A}^* \\ &\leq n\{u(A^*) + \hat{U} - \overline{u} + H\} + nf\tilde{A}^*\end{aligned}$$

because $\underline{\psi}(z^*, A^*) = 0$ implies $q(\tilde{D}(z^*, A^*), z^*, A^*) = 0$: no gamblers are excluded in the equilibrium. When the bookmaker uses the cash-bet in period 1 as well as in period 2, she obtains the payoff of $\Pi_c \equiv 2nf\tilde{A}^*$. If $\Pi_b \geq \Pi_c$, it must be then that

$$u(A^*) - \overline{u} + H + \hat{U} \geq f\tilde{A}^*.$$

Since $\overline{u} \geq u(A^*)$, this inequality implies that $\hat{U} + H \geq f\tilde{A}^*$ and hence $u(\tilde{A}^*) + H \geq 2f\tilde{A}^*$ due to the definition of $\hat{U} = u(\tilde{A}^*) - f\tilde{A}^*$, contradicting to the assumption that $2f\tilde{A}^* > u(\tilde{A}^*) + H$. Therefore, $\Pi_c > \Pi_b$ must be satisfied. Q.E.D.

16 Online Appendix B: Extensions

In this online appendix we provide the formal analysis for the three extensions of the basic model discussed in Section 7 of the main text.

16.1 Endogenous Commission Fees

In this subsection we extend the basic model to allow the bookmaker to choose commission fees as discussed in Section 7.1 of of the main text.

The bookmaker can choose a commission fee of $f_t \in (-\infty, \overline{f}]$ in period $t = 1, 2$ where $\overline{f} < 1$ ensuring that market odds become positive: $h_{j,t} = (1 - f_t)A_t/a_{j,t} \geq 0$ for any $j \in J$ and $t = 1, 2$. Here f_t can be negative so that the bookmaker can directly subsidize gamblers to place bets. We suppose that the bookmaker offers and commits to commission fees (f_1, f_2) in the beginning of period 1.

We first consider that the bookmaker adopts the cash-bet in period 1 and chooses the commission fees (f_1, f_2) to maximize her payoffs over two periods, given by $n\{f_1A_1 + f_2A_2\}$ subject to the incentive constraint such that each gambler optimally places the bet of $A_t \geq 0$ in total in period $t = 1, 2$ to maximize his payoffs as follows:

$$(A_1, A_2) \in \arg \max_{A_1'' \geq 0, A_2'' \geq 0} \; v(A_1'') - f_1 A_1'' + u(A_2'', S'') - f_2 A_2''$$

where $S'' = \alpha + \beta A_1''$.

We denote by (f_1^c, f_2^c) the optimal commission fees chosen by the bookmaker under the cash-bet. We also denote by (A_1^c, A_2^c) the corresponding amounts bet in two periods, which satisfy the above incentive constraint of gamblers. Here the second-period amount bet A_2^c satisfies the optimality condition as $u_A(A_2^c, S^c) = f_2^c$, where $S^c \equiv \alpha + \beta A_1^c$. We then define by

$$\hat{U}(S; f_2^c) \equiv \max_{A \geq 0} u(A, S) - f_2^c A$$

the second-period equilibrium payoff of each gambler, given an addiction stock S and the optimal commission fee f_2^c in period 2.

Next we consider the book-bet. We make the similar assumption to Assumption 2: gamblers' addiction preference is sufficiently strong such that they obtain large addiction gains from the first-period gambling, given the commission fee f_2^c optimally chosen under the cash-bet.

Assumption 3. $\hat{U}(S; f_2^c) > A$ *for all* $A \geq 0$.

We then show that under Assumption 3 the bookmaker still prefers the book-bet to the cash-bet even when commission fees (f_1, f_2) are endogenous. The idea is similar to Proposition 6. Suppose that in the beginning of period 1 the bookmaker offers the commission fees (f_1^c, f_2^c), which are the optimal choice under the cash-bet, and the bang-bang punishment policy $(\tilde{\psi}, \tilde{D})$ such that the maximum forgiveness is set to be zero, $\tilde{D}(z, A) = 0$ for all $A \geq 0$ and all $z \geq 0$. Then, under Assumption 3 gamblers never default for any amount bet $A \geq 0$ in period 1: if gambler i defaults $D_{i,j} > 0$, then he is most harshly punished and loses the second-period payoff $\hat{U}(S_i; f_2^c)$. Then his payoff is at most $h_j^* a_{i,j} - A_i + D_{i,j} - H \leq -H$ but, if he does not default, he obtains a larger payoff $h_j^* a_{i,j} - A_i + \hat{U}(S_i; f_2^c) > 0$ under Assumption 3. Therefore, gambler i never defaults and faces the same payoff as that under the cash-bet with the optimal commission fees (f_1^c, f_2^c):

$$v(A_i) - f_1^c A_i + \hat{U}(S_i; f_2^c).$$

Then gamblers place the same bets (A_1^c, A_2^c) as those chosen under the cash-bet with the commission fees (f_1^c, f_2^c). The bookmaker thus obtains at least the same payoff $n\{f_1^c A_1^c + f_2^c A_2^c\}$ as that attained under the cash-bet, and may be better off further by choosing the commission fees and punishment policy optimally.

The main message of this extension is that, when gamblers' addictive preferences are so strong that Assumption 3 holds, the bookmaker can be better off by the book-bet upon the cash-bet even if negative commission fee (subsidy) is allowed and is endogenously chosen. In other words subsidizing gamblers via negative commission fees is not sufficient but adding the book-bet with allowed default yields more gains to the bookmaker. The bookmaker can cultivate more addictive preferences of gamblers by allowing them to default in an early period rather than giving them only subsidies via negative commission fees.

16.2 Endogenous Commitment by the Bookmaker

In this appendix we extend the two-period model presented in the main text in such a way that the bookmaker endogenously self-enforces promised payouts and punishment policies.

16.2.1 Dynamic Game with A Sequence of Short-Lived Gamblers

Time is discrete and is extended over infinity $t = 1, 2, ...$ The bookmaker is a long-lived player who lives forever over time while gamblers are short-lived players who live only for one period (in sub-period 1 and sub-period 2 defined below).

To maintain the consistency with the two-period model, we suppose that each period t is divided into two sub-periods, which we call "sub-period 1" and "sub-period 2", as follows. Sub-period 1 corresponds to the game played as "period 1" in the basic model while sub-period 2 corresponds to the game played as "period 2" in the basic model respectively. More specifically, in sub-period 1 of period t, (i) new gamblers are born and enter the market; (ii) the bookmaker chooses the cash-bet or the book-bet, and announces the punishment policy ψ; (iii) gamblers decide how much to bet; (iv) after the horse race outcome is realized, gamblers decide how much to default; (v) the bookmaker decides whether or not to pay the payouts that are determined by the horse race outcome as well as she decides whether or not to implement the announced punishment policy. In sub-period 2 of period t, (i) gamblers place bets under the cash-bet;[32] (ii) the bookmaker decides whether or not to renege on payouts; (iii) gamblers leave the economy and are replaced by newly born gamblers.

The bookmaker discounts her payoffs across successive two periods at $\delta \in [0, 1)$ while there are no time discounting between sub-period 1 and sub-period 2 within each period.

We introduce the "quitting option" between the bookmaker and each gambler i in the beginning of sub-period 1 and sub-period 2 of each period: the bookmaker decides whether or not to exercise the quitting option against gambler i (exclude him from the gambling) while gambler i decides whether or not to quit the gambling. The bookmaker and gamblers simultaneously make the quitting decision. Then, the relationship between the bookmaker and gambler i ends once at least one of them decides to quit. In that case the bookmaker obtains no revenues from gambler i while gambler i obtains the reservation payoff normalized to zero.

16.2.2 Information Structure

We consider the following information structure: each gambler i born in period t observes what punishment policy the bookmaker has offered and how much she has paid to him within the entire period of t. In addition, for each gambler i in period t, there exists a corresponding gambler, denoted by $\phi(i)$, born in next period $t+1$ who can observe whether or not the bookmaker and gambler i terminated their relationship in period t as well as

[32] The bookmaker has no choices but the cash-bet in sub-period 2: gamblers never repay debts if the book-bet is used in sub-period 2 because they leave the economy in sub-period 2.

how much payouts the bookmaker made to gambler i in sub-period 2 of period t. We then define $\phi(i), \phi(\phi(i)), \phi(\phi(\phi(i))), \ldots$ recursively.

We call the sequence of gamblers $\phi(i), \phi(\phi(i)), \ldots$ *successors* of gambler i. The role of gambler i's successors is to punish the bookmaker when she deviated from the promised payouts or announced punishment policy against gambler i in each period: if the bookmaker deviates against gambler i in some period t, gambler i or/and his immediate successor $\phi(i)$ will punish the bookmaker by exercising the quitting option. Here successor $\phi(i)$ can know directly or indirectly the bookmaker's deviation against gambler i in period t: $\phi(i)$ observes whether or not gambler i terminated the relationship with the bookmaker in sub-period 1 and whether or not the bookmaker did not give the promised payouts to gambler i under the cash-bet in sub-period 2 within period t. The next successor $\phi(\phi(i))$ can observe that the relationship between the bookmaker and gambler $\phi(i)$ was terminated in period $t+1$. Then $\phi(\phi(i))$ will exercise the quitting option in period $t+2$, and so on. In this way all i's successors will punish the bookmaker once she has made the deviation against i in period t.

Let denote by $\phi^{-1}(i)$ the gambler whose immediate successor is i.

16.2.3 Self-Enforcing Equilibrium

Throughout the following, we will maintain Assumption 1-2 made in the main text, ensuring (i) that the book-bet becomes optimal for the bookmaker in sub-period 1 within each period and (ii) that the bookmaker's per period payoff is reduced to

$$\Phi(A) \equiv fA - \tilde{D}^* + f\hat{A}(S)$$

where $\tilde{D}^*$ satisfies IC* constraint:

$$v(A^*) - fA^* + \tilde{D}^* + \hat{U}(S^*) \geq \overline{V}. \qquad (\text{IC}^*)$$

and $0 \leq \tilde{D}^* \leq z^* \equiv fA^*$.

We consider how the bookmaker implements an aggregate amount bet A^* per gambler in sub-period 1 within each period and obtains the corresponding payoff $\Phi(A^*)$ per gambler in each period even *without* formal commitment to promised payouts and punishment policy. To this end, we focus on the bang-bang punishment policy $\{\tilde{\psi}, \tilde{D}\}$ as used in the proof of Proposition 4 as follows.

(*) Punishment policy $\tilde{\psi}$:

$$\underline{\psi}(z^*, A^*) = 0,$$

$$\underline{\psi}(z, A) = \hat{U}(S) + H, \quad \text{for any } (z, A) \neq (z^*, A^*)$$

and the maximum forgiveness $\tilde{D}(z, A) \in [0, z]$ satisfies

$$\tilde{D}(z^*, A^*) = \tilde{D}^* \equiv \max\{0, \overline{V} - \{v(A^*) - fA^* + \tilde{+}\hat{U}(S^*)\}\},$$

$$\tilde{D}(z, A) = 0, \quad \text{for any } (z, A) \neq (z^*, A^*)$$

together with the equilibrium odds:

$$h_j^* \equiv k^*/p_j + 1, \quad j \in J.$$

Note that $q(\tilde{D}^*, z^*, A^*) = \xi(\tilde{D}^*, z^*, A^*) = 0$ and that $q(D, z, A) = 1$ and $\xi(D, z, A) = \hat{U}(S) + H$ for any $(z, A) \neq (z^*, A^*)$ according to the bang-bang punishment policy $\tilde{\psi}$ defined above.

In addition to IC* constraint, we must take into account the *dynamic enforcement* (DE) constraint given as follows

$$\frac{\delta\Phi(A^*)}{1-\delta} \geq (1-f)\hat{A}(S^*) \tag{DE}$$

where $S = \alpha + \beta A^*$. DE ensures that the bookmaker self-enforces the promised payouts $(1-f)\hat{A}(S^*)$ to pay each old gambler under the cash-bet in sub-period 2 within each period on the equilibrium path.

Now consider the equilibrium in which in each period the bookmaker implements the aggregate amount bet A^* in sub-period 1 where each gambler places bet $a_j^* \equiv p_j A^*$ on each horse $j \in J$, and implements the aggregate amount bet $\hat{A}(S^*)$ in sub-period 2 from each gambler where $S^* \equiv \alpha + \beta A^*$. Recall that we set $h_j^* \equiv k^*/p_j + 1$ for the equilibrium odds of horse $j \in J$ in sub-period 1. We also define by $\hat{h}_j \equiv (1-f)/p_j$ for each $j \in J$ the equilibrium odds in sub-period 2 and $\hat{a}_j \equiv p_j \hat{A}(S^*)$ the associated equilibrium amount bet by each gambler on horse j under the cash-bet in sub-period 2 within each period. On the equilibrium path the bookmaker obtains $\Phi(A^*)$ per gambler within each period.

There are three possible deviations by the bookmaker: first, the bookmaker may renege on payouts for some gamblers under the book-bet in sub-period 1. Second, the bookmaker may not implement the announced punishment policy against some gamblers in the end of sub-period 1. Third, the bookmaker may renege on payouts for some gamblers under the cash-bet in sub-period 2.

In the first and second deviations we consider the punishment equilibrium as follows: when the bookmaker did not implement the announced punishment policy $\tilde{\psi}$ against gambler i or she did not give the payout of $R_{i,j} \equiv h_j a_{i,j} - A_i$ to gambler i in sub-period 1 of period t (given $R_{i,j} > 0$), she and gambler i exercise the quitting option simultaneously in the beginning of sub-period 2 of period t. In the beginning of period $t+1$ the bookmaker and i's successor $\phi(i)$ will exercise the quitting option simultaneously. Repeating this, all the successors of gambler i, $\phi(i), \phi(\phi(i)), ...$, and the bookmaker will terminate their relationships in the future. Therefore, the bookmaker will obtain no payoffs in any future period.

In the third deviation we consider the punishment equilibrium as follows: when the bookmaker did not give gambler i the promised payout $h_j a_{i,j}$ under the cash-bet in sub-period 2 of period t, all i's successors and the bookmaker will terminate their relationships in all the future periods. As a result, the bookmaker will lose all future payoffs from period $t+1$.

Define the following condition for $S_i = \alpha + \beta A_i$:

$$\frac{\delta}{1-\delta}\Phi(A^*) \geq (1-f)\hat{A}(S_i). \tag{DE-1}$$

By $D^*(z, A) \in [0, z]$ we also define the optimal default choice of each gambler as follows: $D^*(z^*, A^*) = \tilde{D}^*$ and $D^*(z, A) = z$ for any $(z, A) \neq (z^*, A^*)$.

Then we consider the following strategy profile of the bookmaker and gambler i.

The bookmaker's strategy in period t:

- Sub-period 1:
  - The bookmaker exercises the quitting option against gambler i if she deviated against gambler $\phi^{-1}(i)$ from the equilibrium payouts and/or punishment policy $\tilde{\psi}$ in the previous period. If the bookmaker did not deviate, she offers the book-bet and the punishment policy $\{\tilde{\psi}, \tilde{D}\}$.
  - Suppose that the bookmaker offered $\{\tilde{\psi}, \tilde{D}\}$ and that DE-1 holds for addiction stock S_i. Then the bookmaker pays $R_{i,j} \equiv \max\{h_j a_{i,j} - A_i, 0\}$ to gambler i when horse j wins the race if

    $$\begin{aligned} & z_{i,j} - D^*(z_{i,j}, A_i) + (1 - q(D^*(z_{i,j}, A_i), z_{i,j}, A_i)) f \hat{A}(S_i) + \frac{\delta}{1-\delta}\Phi(A^*) \\ \geq \; & R_{i,j}, \end{aligned}$$

    given the winning odds $h_j > 0$ where $z_{i,j} \equiv \max\{A_i - h_j a_{i,j}, 0\}$.[33] Otherwise, she will not pay gambler i. Suppose that the bookmaker offered $\{\tilde{\psi}, \tilde{D}\}$ but DE-1 fails to hold given S_i. Then the bookmaker will not pay gambler i.
  - The bookmaker excludes the gamblers who defaulted according to the announced punishment policy $\tilde{\psi}$ regardless of how much she has paid to them.

- Sub-period 2:
  - If DE-1 holds, the bookmaker does not exercise the quitting option against gambler i and she quits otherwise.
  - When the bookmaker holds the cash-bet and the horse race outcome is realized, she will pay $h_j a_{i,j}$ to gambler i if

    $$\frac{\delta}{1-\delta}\Phi(A^*) \geq h_j a_{i,j} \tag{DE-2}$$

[33] We are here focusing on the situation in which market odds $h_j > 0$ are positive for all $j \in J$ again, as we discussed in footnote 16 of the main text: for example, each gambler places a small but positive bet $\varepsilon > 0$ on any horse by a mistake or other irrational reasons.

given the winning odds $h_j > 0$.[34] Otherwise, she gives no payouts to gambler i.

Gambler i's strategy in period t:

- Sub-period 1: Suppose that the either one of the following occurs: (i) the bookmaker deviated against $\phi^{-1}(i)$ in the previous period; (ii) gambler $\phi^{-1}(i)$ quit in the previous period; (iii) the bookmaker did not offer the book-bet and/or the equilibrium punishment policy $\{\tilde{\psi}, \tilde{D}\}$ in the current period. Then gambler i quits. Otherwise, he will place bet $\{a_j^*\}_{j\in J}$. When gambler i places the bet of $\{a_j^*\}_{j\in J}$ and the bookmaker pays the promised payout $h_j^* a_j^*$ for wining horse j, he will default $\tilde{D}(z_j^*, A^*)$. When gambler i places the bet of $a_{i,j} \neq a_j^*$, he will default all the debt of $z_{i,j}$.

- Sub-period 2: if DE-1 fails or the bookmaker deviates from the promised payout $R_{i,j} \equiv \max\{h_j a_{i,j} - A_i, 0\}$ in sub-period 1, gambler i quits. Otherwise, he places bet $\{\hat{a}_j\}_{j\in J}$ such that $A_i = \hat{A}(S_i)$ and $\hat{a}_j \equiv p_j \hat{A}(S_i)$ for $j \in J$.

When all the players follow the prescribed strategies, the bookmaker obtains the payoff of $\Phi(A^*)$ per gambler every period while each gambler born in period t bets the aggregate amount of A^* and defaults $\tilde{D}(z^*, A^*)$ in sub-period 1, followed by betting $\hat{A}(S^*)$ in total in sub-period 2. Then the equilibrium payoff of each gambler born in each period is given by

$$V^* \equiv v(A^*) - fA^* + \tilde{D}(z^*, A^*) + \hat{U}(S^*).$$

Given the equilibrium amount bet $\hat{A}(S^*)$ by each gambler in sub-period 2, for the bookmaker to honor the equilibrium payouts $(1-f)\hat{A}(S^*)$ for each gambler, DE-1 must be satisfied at $A_i = \hat{A}(S^*)$, yielding DE constraint. The best equilibrium which the bookmaker can implement is obtained to solve the following problem.

Problem E:

$$\max_{A^* \geq 0, 0 \leq \tilde{D}^* \leq z^*} \quad \Phi(A^*)$$

subject to IC* constraint:

$$v(A^*) - fA^* + \tilde{D}^* + \hat{U}(S^*) \geq \overline{V} \tag{IC*}$$

and DE constraint:

$$\frac{\delta \Phi(A^*)}{1-\delta} \geq (1-f)\hat{A}(\alpha + \beta A^*). \tag{DE}$$

[34] Again we focus on the equilibrium in which $h_j > 0$ for all $j \in J$.

We then show the following result.

Proposition B1. *Suppose that Assumptions 1-2 of the main text are satisfied. Then there exists an equilibrium in which the bookmaker obtains the payoff of* $\Phi(A^*)$ *per gambler attaining the maximum in the Problem E every period.*

Proof. Consider the strategies of the bookmaker and gamblers described above.

Step 1. First, we consider sub-period 2 of period t, provided that DE-1 is satisfied so that the bookmaker and gambler i born in sub-period 1 of period t have not quit.

Suppose then that the bookmaker offered the cash-bet and then horse j won the race in sub-period 2. If the bookmaker pays $h_j a_{i,j}$ to gambler i, she expects to obtain the continuation value $\delta\Phi(A^*)/(1-\delta)$ from subsequent gamblers in future periods. If the bookmaker reneges on the payout of $h_j a_{i,j}$ against gambler i, all the successors of gambler i will quit the gambling, thereby yielding the payoff of zero to the bookmaker in the future. Therefore, it becomes optimal for the bookmaker to pay $h_j a_{i,j}$ to gambler i if (DE-2) is satisfied in sub-period 2 of period t.

Given this, we consider the incentive of gamblers in sub-period 2. Recall the equilibrium odds of $\hat{h}_j = (1-f)/p_j$ for $j \in J$, and take any profile of amounts bet by gambler i, $\mathbf{a}_i$ in sub-period 2 of each period. Then define the set of horses $\tilde{J}_i$ such that DE-2 holds at $h_j = \hat{h}_j \equiv (1-f)/p_j$ for each $j \in J$. When gambler i places bet $\mathbf{a}_i$ in sub-period 2, he will obtain the following expected payoff

$$u(A_i, S_i) + \sum_{j \in \tilde{J}_i} p_j \{\hat{h}_j a_{i,j} - A_i\} + \sum_{j \notin \tilde{J}_i} (-A_i)$$

because the bookmaker will renege on payouts for gambler i when horse $j \notin \tilde{J}_i$ wins the race. Then we verify that

$$\begin{aligned} & u(A_i, S_i) + \sum_{j \in \tilde{J}_i} p_j \{\hat{h}_j a_{i,j} - A_i\} + \sum_{j \notin \tilde{J}_i} (-A_i) \\ \leq\ & u(A_i, S_i) + \sum_{j \in J} p_j \{\hat{h}_j a_{i,j} - A_i\} \\ \leq\ & \hat{U}(S_i). \end{aligned}$$

On the contrary, if gambler i places the equilibrium bet of $\hat{a}_j \equiv p_j \hat{A}(S_i)$ for each $j \in J$, he can ensure that

$$\frac{\delta\Phi(A^*)}{1-\delta} \geq \hat{h}_j \hat{a}_j = (1-f)\hat{A}(S_i)$$

due to DE-1. Therefore, the bookmaker never reneges on payouts, implying that gambler i obtains the equilibrium payoff of $\hat{U}(S_i)$ in sub-period 2. Then gambler i never deviates from betting $\hat{\mathbf{a}}$ where $\hat{a}_j \equiv p_j \hat{A}(S_i)$ for each $j \in J$, given DE-1.

Suppose next that DE-1 fails. Then in the beginning of sub-period 2 the bookmaker and gambler i will terminate the relationship. Furthermore, i's successors will terminate the relationship as well in all the future periods, giving the bookmaker the continuation value of zero.

Step 2. Second, consider the incentive of the bookmaker in the end of sub-period 1, provided that gambler i defaulted $D_{i,j}$. If the bookmaker follows the equilibrium strategy, she will exclude gambler i who defaulted $D_{i,j}$ according to the exclusion probability q specified in the punishment policy $\tilde{\psi}$. This in turn implies that the bookmaker will obtain the following continuation payoff from the end of sub-period 1:

$$(1 - q(D_{i,j}, z_{i,j}, A_i))f\hat{A}(S_i) + \frac{\delta\Phi(A^*)}{1-\delta} \geq 0 \tag{B1}$$

from gambler i if DE-1 holds and zero otherwise respectively. If the bookmaker deviates not to punish the defaulting gambler i, all i's successors will quit. Therefore, the bookmaker obtains the continuation value of zero, which is not however profitable by condition (B1).

Third, consider the incentive of the bookmaker in the stage in which she makes payouts to gamblers when horse j wins the race in sub-period 1 of period t. Recall that $z_{i,j} \equiv \max\{A_i - h_j a_{i,j}, 0\}$ denotes the net payback from gambler i to the bookmaker when horse j wins the race.

Suppose first that DE-1 holds. Recall that $D^*(z_{i,j}, A_i) \in [0, z_{i,}]$ is the optimal default by gambler i, given the punishment policy $\tilde{\psi}$ specified as (*), when he places the bet of A_i in total and owes the debt of $z_{i,j}$ in sub-period 1. Note that it becomes optimal for gambler i to choose $D^*(z^*, A^*) = \tilde{D}^*$ and $D^*(z, A) = z$ for any $(z, A) \neq (z^*, A^*)$, given the punishment policy $\tilde{\psi}$ specified as (*).

Then, if the bookmaker follows the promised payout of $R_{i,j} \equiv \max\{h_j a_{i,j} - A_i, 0\}$ to gambler i, she will obtain the following continuation value of her payoffs:

$$-R_{i,j} + z_{i,j} - D^*(z_{i,j}, A_i) + (1 - q(\tilde{D}(z_{i,j}, A_i), z_{i,j}, A_i))f\hat{A}(S_i) + \frac{\delta\Phi(A^*)}{1-\delta} \tag{B2}$$

from gambler i and his successors. Here, gambler i will optimally defaults the amount $D^*(z_{i,j}, A_i)$ and subsequently places the bet of $\hat{A}(S_i)$ in sub-period 2, provided that the bookmaker will optimally punish him according to the equilibrium punishment policy $\tilde{\psi}$ as well as she holds the cash-bet and never reneges on the equilibrium payout $\hat{h}_j\hat{a}_j$ in sub-period 2 due to the supposition that DE-1 is satisfied. On the contrary, if the bookmaker deviates from $R_{i,j}$, then she will obtain the continuation value of zero because gambler i will quit in sub-period 2 and all the successors of gambler i will quit as well. Note that, if gambler i will quit in sub-period 2, he always defaults on all the debt of $z_{i,j}$ in sub-period 1 and will never place bet in sub-period 2. Therefore, the bookmaker follows the promised payout $R_{i,j}$ when the above value (B2) is non-negative.

Suppose next that DE-1 fails in some period t. Then, the bookmaker and gambler i will simultaneously quit in the beginning of sub-period 2, followed by the termination of relationships between the bookmaker and i's successors in all the future periods. Therefore, the bookmaker and gambler i obtain the payoffs of zero from sub-period 2 of period t. Given this continuation outcome, gambler i defaults all the debt $z_{i,j}$ in sub-period 1 of period t. Anticipating this, the bookmaker reneges on payouts to gambler i in sub-period 1 of period t.

We now define by J_i^* the set of horses for which DE-1 is satisfied and (B2) becomes non-negative at $h_j^* \equiv k^*/p_j + 1$ for each $j \in J$, given a profile of amounts bet $\mathbf{a}_i$ by gambler i. Note that $h_j^* a_{i,j} > A_i$ must hold for any $j \notin J_i^*$, so $z_{i,j} = 0$ and hence $D_{i,j} = 0$ for any $j \notin J_i^*$.[35] Then, if gambler i places bet $\mathbf{a}_i$ in sub-period 1, he expects to obtain

$$v(A_i) + \sum_{j \in J_i^*} p_j\{h_j^* a_{i,j} - A_i + D^*(z_{i,j}, A_i) - \tilde{\psi}(z_{i,j}, A_i)\} + \sum_{j \notin J_i^*} p_j\{-\tilde{\psi}(0, z_{i,j}, A_i)\} + \hat{U}(S_i)$$

where the third bracket term captures the fact that for $j \notin J_i^*$ the bookmaker reneges on payouts followed by the termination of the relationship between herself and gambler i in sub-period 2. Since $h_j^* a_{i,j} > A_i$ for any $j \notin J_i^*$, we verify that for any $\mathbf{a}_i$ and any $D_{i,j} \in [0, z_{i,j}]$,

$$\begin{aligned}
& v(A_i) + \sum_{j \in J_i^*} p_j\{h_j^* a_{i,j} - A_i + D_{i,j} - \tilde{\psi}(D_{i,j}, z_{i,j}, A_i)\} + \sum_{j \notin J_i^*} p_j\{-\tilde{\psi}(0, z_{i,j}, A_i)\} + \hat{U}(S_i) \\
& \leq v(A_i) + \sum_{j \in J} p_j\{h_j^* a_{i,j} - A_i + D_{i,j} - \tilde{\psi}(D_{i,j}, z_{i,j}, A_i)\} \\
& \quad + \sum_{j \notin J_i^*} p_j\{h_j^* a_{i,j} - A_i - \tilde{\psi}(0, z_{i,j}, A_i)\} + \hat{U}(S_i) \\
& = v(A_i) + \sum_{j \in J} p_j\{h_j^* a_{i,j} - A_i + D_{i,j} - \tilde{\psi}(D_{i,j}, z_{i,j}, A_i)\} + \hat{U}(S_i) \\
& \leq v(A^*) + \sum_{j \in J} p_j\{h_j^* a_j^* - A^* + \tilde{D}(z^*, A^*) + \hat{U}(S^*)\} \\
& = v(A^*) - fA^* + \tilde{D}^* + \hat{U}(S^*) \\
& = V^*
\end{aligned}$$

where the last inequality follows from the same step as the proof of Proposition 4 (IC* is sufficient to detect gambler's deviation from $\mathbf{a}^*$ under Assumption 1). If gambler i places the bet of $\mathbf{a}^*$, then DE-1 holds and (B2) can be non-negative at $h_j = h_j^*$,[36] implying that gambler i obtains the equilibrium payoff of V^*. Thus, each gambler never deviates from choosing A^* in sub-period 1. Therefore, the bookmaker attains the payoff of $\Phi(A^*)$ in each

[35] When (B2) is negative, $R_{i,j} > 0$ must hold because $z_{i,j} \geq D^*(z_{i,j}, A_i)$, $\Phi(A^*) \geq 0$, and $\hat{A}(S_i) \geq 0$.
[36] This is because $R_{i,j} = h_j^* a_j^* - A^* = -fA^* < 0$.

period where A^* solves Problem E. Q.E.D.

We now discuss how addiction preferences help self-enforce relational contract agreements between the bookmaker and gamblers over time. We differentiate both sides of DE constraint with respect to the addiction preference parameter β to obtain

$$\frac{\delta}{1-\delta}\frac{\partial\Phi(A)}{\partial\beta} - (1-f)\frac{\partial\hat{A}(S)}{\partial\beta} \tag{B3}$$

for a given amount bet A per gambler. Assuming that $u_{AS} > 0$, we obtain $\partial\Phi(A)/\partial\beta > 0$.[37] Then the first term of (B3) dominates the second one when the discount factor $\delta \in [0, 1)$ is close to 1, in which case larger addiction preference β can weaken DE constraint to implement an amount bet A from each gambler. This result suggests that individuals' addictions can *complement* relational contract agreements, implying that informal markets where goods/services are traded without formal contracts can work more effectively even without well-functioning institutions when individuals are addicted more to consuming goods/services.

16.3 Applications to Other Illegal Markets

Consider the re-interpretation of the model as discussed in the main text. Then individual i chooses how much to consume the good $A_i \geq 0$ in period 2 to maximize his second-period payoff $u(A_i, S_i) - fA_i$. This gives him the second-period payoff $\hat{U}(S_i)$, which is same as (8) obtained in Proposition 1. In period 1 individual i decides how much to consume $A_i \geq 0$ and how much to default $D_i \in [0, fA_i]$ to maximize his payoffs over two periods

$$v(A_i) - fA_i + D_i - \psi(D_i, fA_i) + \hat{U}(S_i)$$

where $\psi(D_i, fA_i)$ is the penalty imposed when he is required to pay fA_i but defaults $D_i \in [0, fA_i]$. This is essentially same as the payoff defined in the case of illegal horse race betting. Then, by using a similar logic to Proposition 2, we can show that the optimal punishment policy for the seller can be the bang-bang form. Furthermore, since each individual can always default all the payment $D = fA$, followed by being most harshly punished (being imposed the maximum penalty H), he can obtain at least

$$\tilde{V} \equiv \max_A v(A) - H.$$

We can then replace $\overline{V}$ by $\tilde{V}$ in Problem B* to obtain the optimal punishment policy for the seller. Thus we can show a similar characterization result to Proposition 4 except

[37] Note that $\Phi(A) = v(A) + u(\hat{A}(S), S) - \overline{V}$ when $\tilde{D}^* > 0$ holds at the optimum of Problem B* and $\Phi(A) = fA + f\hat{A}(S)$ when $\tilde{D}^* = 0$ holds at the optimum respectively. In either case Φ is increasing in β because $\hat{A}$ and $u(A, S)$ are increasing in β given $u_{AS} > 0$.

for the equilibrium odds which are specific to the horse race betting. Thus most of the propositions we have so far obtained still remain valid in a broad range of illegal markets where sellers sell addictive goods to buyers who form addiction preferences over time, and transactions of the goods are illegal and are hence not formally enforceable.