Addiction and Illegal Markets*

Daniel Chen† Shingo Ishiguro‡ Sultan Mehmood§ Avner Seror¶

Abstract

This paper studies optimal dynamic contracts in addictive markets where individuals' tastes for addictive goods develop through prolonged consumption and contract enforcement is limited. Our theoretical analysis uncovers the optimality of a 'free-first-dose' strategy where sellers intensify buyers' addiction by offering consumption credit to newcomers. We show that the provision of credit is strategically designed to exceed what buyers can feasibly repay. Such aggressive selling strategies cultivate deeper addictions among buyers, enhancing their propensity for continued consumption. Finally, our analysis uncovers that illegal markets might favor non-violent interactions over violent ones, defying the stereotypical association of illegality with violence. Indeed, enforcing debt repayment with violence can disrupt the addiction development process, which is crucial for maintaining long-term buyers' engagement, and the partial self-enforcement of contracts in illegal markets. We discuss these insights in the context of illegal sports wagering, narcotics, and religious sects.

This draft: March 2024

JEL Classification Numbers: D40, D91, D21.

Keywords: Addiction, Dynamic Contracts, Illegal Markets.

*Daniel L. Chen acknowledges IAST funding from the French National Research Agency (ANR) under the Investments for the Future (Investissements d'Avenir) program, grant ANR-17-EUR-0010. This research received financial support from the research foundation TSE-Partnership and ANITI funding. Shingo Ishiguro acknowledges the financial support from the Japan Society for the Promotion of Science KAKENHI Grant Number 18K01508 and 18H03640, and from the Japan Center for Economic Research. Avner Seror acknowledges funding from the French government under the "France 2030" investment plan managed by the French National Research Agency (reference :ANR-17-EURE-0020) and from Excellence Initiative of Aix-Marseille University - A*MIDEX.

†Toulouse School of Economics (Email: daniel.chen@iast.fr).

‡Graduate School of Economics, Osaka University (Email: ishiuro@econ.osaka-u.ac.jp).

§New Economic School (Email: smehmood@nes.ru)

¶Aix Marseille Univ, CNRS, AMSE, Marseille, France; avner.seror@univ-amu.fr.

1 Introduction

Individuals' tastes for addictive goods develop through prolonged consumption, while economic transactions are shaped by their dynamic incentives and expectations about future interactions when contract enforcement is limited. These dual dynamics are crucial in understanding optimal trading mechanisms in addictive markets, where sellers manage dynamic incentives of buyers who become addicted by consumption over time, and formal enforcement is absent because the activity itself is illegal or requires severe infringement of individual rights. Our study aims to clarify this complex relationship, a topic that remains overlooked in existing literature on dynamic contracts.

While our model can capture market dynamics across a variety of illicit markets, we chose illegal gambling markets as our focal industry. This selection is primarily underscored by the remarkable scale of the global gambling market, which significantly eclipses the market values of other major industries, including tobacco and pharmaceuticals. It is estimated that the illicit segment of the gambling market alone may surpass an astounding $1,700 billion globally (United Nations Office on Drugs and Crime (2021)). Furthermore, illegal gambling serves as an exemplary model for studying other illegal markets, as it embodies many of the features found in other illegal markets, as discussed below.

In the model, a bookmaker (seller) illegally holds horse race betting for a finite number of periods, and gamblers (buyers) become addicted to gambling.[1] In each period, the bookmaker chooses either 'cash-bet' or 'book-bet' as a format for horse race betting. Under the cash-bet format, gamblers are required to immediately pay the amounts they bet whereas under the book-bet format, gamblers defer payments. Since gambling is illegal, there are no formal institutions to enforce debt repayments from gamblers, and the bookmaker is exposed to default risks under the book-bet, in contrast to the cash-bet.[2] Since gamblers always default all their debts in the last period, the bookmaker has no choice but to rely on the cash-bet in that period. On the contrary, the bookmaker may choose the book-bet in the other periods, together with a punishment instrument to reduce the default risk. The cornerstone of our model is the potential for gambler addiction, a factor the bookmaker is aware of. In this context, we characterize the market equilibrium, detailing the bookmaker's optimal punishment strategy in the event of default, and the gamblers' betting trajectory.

This model captures several key features of illegal markets. First, transactions are not protected. Second, sellers, such as those in illegal gambling or illicit drug markets, extend consumption credit to buyers. Third, the model details how sellers may enforce non-formal punitive measures to counteract default risk. These measures can range from

[1] Our main model focuses on two periods to simplify the argument. We extend the basic model to allow for an infinite time horizon in Section 6.

[2] We omit consideration of liquidity risk, which could arise from a widespread default, potentially hindering the bookmaker's ability to pay winning gamblers.

exclusion from future transactions to the exertion of psychological or physical violence, tactics seldom seen or severely restricted within legal markets.[3]

Our first theoretical finding reveals that sellers utilize a 'free-first-dose' approach. Much like drug dealers who may offer a free sample to cultivate dependency in their clientele, we find that sellers strategically allows the buyers to initially borrow and consume beyond their means. This tactic is designed to cultivate an addiction, thereby ensuring increased future revenues.

Our second theoretical result unveils that in equilibrium, the seller anticipates and accepts a certain degree of default from each buyer. More specifically, we show that the seller can always replicate the equilibrium outcome attained under any punishment policy by the following *bang-bang* punishment policy (see Proposition 2 below): buyers are allowed to default up to a certain upper bound, which we call *maximum forgiveness*, and are never punished unless they default more than it. Furthermore, the seller can be weakly better off by this bang-bang punishment policy upon any other punishment policy. Such "allowed default" plays a strategic role: it cultivates buyers' addiction by inducing large consumption amounts in the initial period, making them willing to consume aggressively later on.

The third main finding probes further into this dynamic, assessing the influence of violence on market equilibrium. Interestingly, while violence might enforce more prompt debt repayments, it could simultaneously weaken the initial 'free-first-dose' incentive, thereby stifling the development of addiction. This paradoxical outcome proposes that illegal markets might favor non-violent interactions over violent ones, defying the stereotypical association of illegality with violence.

The dynamics within illegal markets of addictive goods, as highlighted in our model, bear a striking resemblance to the concept of relational contracts. In these markets, the relational contracts formed between participants, such as bookmakers and gamblers or drug dealers and users, are not only built on mutual trust and reputation but are also deeply influenced by the addictive nature of the transactions. For instance, when bookmakers extend credit to gamblers, or when drug dealers employ a 'free-first-dose' strategy, they are initiating a relational contract that exploits the addictive behavior of the other party. This addiction becomes a critical component of the agreement, ensuring repeat engagement and a form of enforced loyalty. Thus, addiction serves not only as a driving force for consistent market participation but also as an informal enforcement mechanism that underpins these relational contracts, sustaining the market's equilibrium in the absence of formal legal structures.

The predictions of our model find particular support in the functioning of different markets. First, the theoretical predictions find significant support in the functioning of illegal gambling, the focal industry of the model. Mehmood and Chen (2022) collected data on illegal horse betting in Pakistan. The authors calculate that approximately 55% of wagers within this market are placed on credit, a clear manifestation of the 'free-first-dose' strategy in action. According to our model, this approach is not merely about extending

[3] In Section 7, the model is formally extended to more general illegal market settings.

credit; it is a calculated bid to cultivate gambling addiction, aiming to ensure a lucrative and enduring customer base. Mehmood and Chen (2022) collected data on debt repayment too, and found that on average, 35% of gamblers do not repay their debt in full. The authors give suggestive evidence that despite the significant rate of default, violence is rare in this market. Only 0.5% of gamblers express fear of violent repercussions for unpaid debts. This empirical evidence underscores the bookmakers' strategic choice to foster addiction through leniency and trust rather than through coercion and fear, a choice that aligns with our model's predictions.

Second, our model's implications also find support in the functioning of illegal drug markets, where the "free-first dose" strategy is a well-documented method to foster dependency among new users. Galenianos and Gavazza (2017) estimate a model using data on the crack cocaine market in the United States. The authors rely on the STRIDE dataset, which contains records of the acquisition of illegal drugs by undercover agents and DEA informants. Hence, the data may contain information about purchases from "new" consumers. Consistent with the prediction of our model, Galenianos and Gavazza (2017) find that drug dealers do not necessarily provide low-quality products to new users by diluting (or "cutting") the products. That way, drug dealers may invest in relationships, and cultivate stronger addictions. Finally, the nuanced role of violence in illegal drug markets has been studied, among others, by Curtis and Wendel (2000) in their analysis of the heroin market of New York City. Violence can either be a tool for enforcement, or something deliberately avoided to maintain customer loyalty. This mirrors our model's predictions about the complex interplay between coercion and addiction sustainability.

Finally, the dynamics explored in our model can also be observed in the interaction of religion and addiction, particularly within the context of religious sects. Initial engagements often come with tangible benefits and a strong sense of community, serving as the "free-first dose" that attracts individuals and fosters attachment (Dawson (1998)). As commitment deepens, a form of "debt repayment" emerges, where members feel obliged to contribute more significantly, financially or otherwise, driven by emotional or spiritual indebtedness.

Related Literature.

This paper contributes to the literature in several ways. First, we contribute to the large and multifaceted literature on addiction. In a seminal article, Becker and Murphy (1988) developed a theory of addiction were decision-makers have rational expectations about the consequences of their choices. Since then, several works investigated the rational nature of addiction and explored addictions in various environments.[4] Several papers investigated the interaction between addiction and market structure (Becker, Grossman and Murphy (1994), Chaloupka (1991), Fethke and Jagannathan (1996), Driskill and McCafferty (2001)). We complement these studies most specifically, as we investigate addiction in illegal markets.

[4]See, among others, Hoch and Loewenstein (1991), Orphanides and Zervos (1995), Chaloupka and Warner (2000), Gruber and Koszegi (2001), Courtemanche, Heutel and McAlvanah (2014), Grossman and Chaloupka (2017), Schilbach (2019), and Allcott, Gentzkow and Song (2022).

This aspect seems particularly crucial, as addictive goods frequently fall into the realm of illegality.

Second, this paper also contributes to the literature on relational contracts. This literature addressed issues related to how transactions are self-enforced without formally written contracts (for example, Levin (2003); Malcomson (2012)). However, to our knowledge, there are only a few theoretical attempts to investigate how individuals' preferences over addictions are interlinked with optimal trading arrangements in the lack of formal enforcement institutions. Because illegal markets of addictive goods are characterized by both addiction preferences and informal contracts, it is an important research venture to understand what informal trading arrangements emerge when individuals consume addicted goods and engage in informal contract agreements that may be sustained via relational contracts. More specifically, both addiction preferences and relational contracts dynamically evolve altogether: individuals accumulate the tastes for addictions by consuming addicted goods over time while relational contracts are sustained by individuals' future concerns. Therefore, it is important to address how these two-way dynamic incentives are intertwined to characterize optimal trading mechanisms in illegal markets without formal enforcement institutions.

Third, this paper contributes to the theoretical literature on illicit drug markets. Galenianos, Pacula and Persico (2012) and Galenianos and Gavazza (2017) propose search theoretic models where sellers of illicit drugs are randomly matched buyers and choose qualities of drugs that are not observable to buyers. The main focuses of these papers are on the moral hazard problem of sellers, and, more specifically, the issue of whether sellers choose low quality for first-time buyers or offer high qualities for loyal buyers. We complement their work by focussing more specifically on buyers' addiction preferences, which dynamically interact with their default incentives, and sellers' optimal selling strategy.

Finally, this paper contributes to the literature on decision-making that leverages gambling data. Most notably, gambling data have been used to recover risk preference (Jullien and Salanié (2000), Feess, Müller and Schumacher (2016), Chiappori et al. (2019)), and behavioral biases affecting risky choices (Snowberg and Wolfers (2010), Losak, Weinbach and Paul (2023)). We complement these studies by focusing on addiction in illegal gambling markets.

2 Model

2.1 Gambling Market for Horse Race Betting

We present the model of an illegal market where a seller sells addictive goods to buyers who form the tastes for addictions over time and their transactions are illegal. Specifically, to fix the idea, we consider the gambling market for horse race betting. We will discuss how our model can be applied to other illegal markets than horse race betting in Section 7 later.

There are two periods, denoted by $t = 1, 2$, and a bookmaker runs a horse race in each period. There are n gamblers and m horses where $n \geq 2$ and $m \geq 2$. We denote by I and J the sets of gamblers and horses. We use the feminine pronoun for the bookmaker and masculine pronoun for each gambler in what follows.

We first drop time index $t = 1, 2$ and describe how the gambling market for horse race betting works. We consider the so called *parimutuel betting* that is common in many wagering markets including not only horse race betting but also other sports betting. In the parimutuel betting gamblers decide how much bet to place before they know the odds of outcomes. In particular we focus on the "win bet" that gamblers bet on which horse to come first, given expectations about the odds of horses. Suppose that gambler i places bet of $a_{i,j} \geq 0$ on horse $j \in J$, where amounts bet are measured in terms of the numèraire good. We denote by $\mathbf{a}_i = (a_{i,1}, ..., a_{i,m})$ a profile of amounts bet by gambler i on all horses. Given a collection of amounts bet by all gamblers $\{\mathbf{a}_i\}_{i \in I}$, the bookmaker receives the commission fee ("house take") as a constant fraction $f \in (0, 1)$ of the aggregate amount bet $\sum_{i \in I, j \in J} a_{i,j}$, that is, the bookmaker receives the commission fee equal to $f \sum_{i \in I, j \in J} a_{i,j}$. We will maintain throughout the paper the assumption that the commission fee f is exogenously given.[5]

In the parimutuel betting the bookmaker does not face a market risk; the winning bets are paid with the losing bets, net of the fee f. The odds of horse j, denoted by h_j, is defined as follows:

$$h_j \equiv \frac{(1-f) \times \text{Amount bet on all horses}}{\text{Amount bet on horse } j} = \frac{(1-f)\sum_{i \in I, j \in J} a_{i,j}}{\sum_{i \in I} a_{i,j}}, \qquad (1)$$

provided the denominator is positive.[6] [7] That is, the odds of horse j is determined by reflecting how much gamblers bet on horse j relative to the aggregate amounts bet on all horses. Specifically, the odds of horse j is higher as gamblers bet less for it. We define $h_j \equiv 0$ when nobody bets on horse j.

We denote by $p_j \in (0, 1)$ the probability that horse j wins the race. The returns of horse racing are determined as follows: when horse j wins the race, gambler i who places bet of $a_{i,j} > 0$ on horse j will receive from the bookmaker the payout equal to the odds of horse j times the amount he bet on horse j, that is, $h_j a_{i,j}$. When gambler i bets $A_i \equiv \sum_{j \in J} a_{i,j}$ in total, his net return equals to $h_j a_{i,j} - A_i$, which is positive (when he wins) or negative (when he loses). When horse j wins the race but nobody bets on that horse, each gambler

[5] The commission/participation fee of the bookmaker is about 5% in the case of the illegal horse racing we observed in Pakistan.

[6] The odds h_j defined here represents the gross return of betting one dollar on winning horse j. Alternatively, its net return $h_j - 1$ is sometimes called odds.

[7] Levitt (2004) shows that bookmakers might also take some market risks when setting odds. Accounting for this would slightly change equation (1), although the logic and the results of this paper would remain unchanged.

i is refunded $(1-f)A_i$ which equals to the amount bet A_i minus the commission fee of the bookmaker fA_i.

2.2 Gamblers' Payoffs with Addiction Preference

We now turn to the two-period model ($t=1,2$). We denote by $a_{i,j,t} \geq 0$ the amount bet by gambler i on horse j in period $t=1,2$. We then define by $A_{i,t} \equiv \sum_{j \in J} a_{i,j,t}$ the amount bet by gambler i in total in period $t=1,2$. To model gamblers' preferences, we follow the "gambling-in-utility" approach (Conlisk (1993)) as follows. Gamblers have the preferences over both gambling activities and private consumption of the numèraire good. Then, gamblers obtain higher payoffs not only when they consume more goods but also when they enjoy more gambling activities by betting larger amounts. More specifically, when gambler i bets $A_{i,1}$ in total and consumes the private good of $c_{i,1}$ in period 1, he obtains the payoff of $v(A_{i,1}) + c_{i,1}$ in period 1. Here, $v(A_{i,1})$ is the gambler i's payoff obtained from enjoying the gambling by betting $A_{i,1}$ in total in period 1. We assume that v is continuously differentiable and strictly concave. Additionally, gambler i obtains the payoff of $u(A_{i,2}, S_i) + c_{i,2}$ in period 2 when he bets $A_{i,2}$ in total and consumes the private good of $c_{i,2}$ in period 2. Here the payoff obtained from gambling in period 2 is given by $u(A_{i,2}, S_i)$ that depends on how much to bet $A_{i,2}$ in total in period 2 as well as how much the gambler is addicted by the first-period gambling, which we capture by the *addiction stock* S_i:

$$S_i = \alpha + \beta A_{i,1} \tag{2}$$

where $\alpha \geq 0$ and $\beta > 0$. Due to $\beta > 0$, the addiction stock S_i positively depends on how much gambler i bet in period 1. Furthermore, $\alpha \geq 0$ represents the fixed benefit from participating in the gambling in period 1: $\alpha > 0$ when $\sum_{j \in J} a_{i,j,1} > 0$ and $\alpha = 0$ otherwise. Therefore, gamblers cultivate more addiction preferences in period 2 by accumulating the addiction stock $S_i = \alpha + \beta A_i$ when they engage in more gambling activities in period 1 (they place larger bet A_i in period 1).

We assume that the second-period utility u is continuously differentiable and strictly concave with respect to $A_{i,2}$ as well as it is increasing in the addiction stock S_i. In what follows we denote by u_A and u_S partial derivatives of u with respect to $A_{i,2}$ and S_i: $u_A \equiv \partial u / \partial A$ and $u_S \equiv \partial u / \partial S$. We assume the boundary conditions on u_A such that $u_A(0, S) > 1$ and $u_A(\infty, S) < f$ for all $S \geq 0$, which will ensure the interior solution.

In summary gambler i obtains total payoffs in two periods as follows

$$v\left(A_{i,1}\right) + c_{i,1} + \left\{u\left(A_{i,2}, S_i\right) + c_{i,2}\right\} \tag{3}$$

where we assume no time discounting between the two periods for simplicity.

2.3 Default and Punishment

The gambling market for horse race betting is illegal such that there are no formal institutions to enforce payments from gamblers to the bookmaker.

There are two forms of betting: cash-bet format and book-bet format. Under the cash-bet, gamblers decide how much to bet and then immediately pay the bookmaker the chosen amount. Under the book-bet, gamblers also decide on their bet amount, but payment is deferred. They are only required to settle their debts after a certain period of time. The amount they owe is the initial bet minus any winnings and potential additional fees. Essentially, the book-bet functions as a loan from the bookmaker. In Pakistan's illegal horse gambling market, book-bets are commonly utilized, with the expectation that gamblers repay their debts within a week. However, since the gambling market is illegal and not formally organized, gamblers may renege on debt paybacks to the bookmaker. Indeed, when gamblers use a book-bet strategy and lose, they may refuse to pay the bookmaker. The bookmaker cannot however legally force gamblers to pay back their debts, because the gambling is illegal.

The bookmaker chooses which cash-bet or book-bet to use in period 1, although she must rely only on the cash-bet in period 2 because period 2 is the last period, so gamblers will never make paybacks in period 2 when the gambling is held as the book-bet in period 2. We will show later that, even though the book-bet causes default risks to the bookmaker, she may prefer the book-bet to the cash-bet.

In order to deter the default by gamblers under the book-bet the bookmaker can resort to a punishment policy in period 1. More specifically, in the beginning of period 1 the bookmaker offers *the punishment policy* q that specifies the probability $q \in [0, 1]$ to exclude each defaulting gambler from the participation in the second-period gambling (see Mehmood and Chen (2022) for an empirical assessment on this sort of punishment.[8]) Suppose that a gambler owes a debt $z \geq 0$ to repay to the bookmaker but defaults the amount of $D \in [0, z]$ in the end of period 1; he pays back only $z - D$ to the bookmaker in the end of period 1. Then the gambler is excluded from the second-period gambling with the probability $q(D, A) \in [0, 1]$ that is contingent on how much to bet A in total and how much to default $D \in [0, z]$ in period 1.

Two remarks are in order.

First, to simplify, we suppose throughout the main analysis that the default incentive is relevant only for gamblers under the book-bet while it is not for the bookmaker. That is, the bookmaker cannot renege on payouts to gamblers once the market odds are determined. We discuss the mechanism by which the bookmaker can endogenously commit to enforce payouts as well as punishment policies in Section 5 and Appendix. In particular we present the dynamic model in which the bookmaker who is a long-lived player interacts with gamblers who are short-lived players over infinite periods. Then we show that there exists an equilibrium in which the bookmaker self-enforces agreed upon payouts and punishment

[8] Mehmood and Chen (2022) empirically investigate different punishment forms to exclude defaulting gamblers in the illegal horse race betting in Pakistan.

policies. In this respect we can view the current two-period setting as a short-cut of such a dynamic equilibrium.[9] As a result, the bookmaker can make promised payouts to gamblers according to the market odds $\{h_j\}_{j \in J}$.

Second, the main purpose of the paper is not to consider complicated mechanisms, which specify general transfer rules contingent on amounts bet. Instead we focus on the realistic situation in which the bookmaker can use only limited set of instruments, and consider more practical implications. Specifically, the bookmaker can design only the exclusion probability q and we impose the reasonable restriction on q such that $q(0, A) = 0$ for any $A \geq 0$: the bookmaker cannot exclude non-defaulting gamblers. This may be because, if the bookmaker punishes non-defaulting gamblers, she may lose the reputation in the gambling market and, as a result, may lose large future profits. Additionally, the punishment probability $q(D, A)$ relies only on the information about how much to bet A in total and how much to default $D \in [0, z]$ but not on other detailed information such as the identities of gamblers and how much they bet on *each* horse $a_{i,j}$. We will discuss the role of non-monetary penalty such as violence in Section 4.

2.4 Gambling Equilibrium

We now define equilibrium in the gambling market for horse race betting in each period, $t = 1, 2$. The gambling market opens for horse race betting after the bookmaker chooses its format, the cash-bet or the book-bet, in each period. In the parimutuel betting gamblers do not know market odds when they decide how much to bet on horses. Then one reasonable equilibrium concept used in such a market is so called *rational expectations equilibrium* in which (i) gamblers optimally decide how much to bet on horses $\mathbf{a}_i = (a_{i,1}, ..., a_{i,m})$ by forming expectations about market odds $(h_1, ..., h_m)$ and taking them as given, and (ii) market odds $(h_1, ..., h_m)$ are determined according to condition (1) in order to be consistent with the amounts $(\mathbf{a}_1, ..., \mathbf{a}_n)$ bet by gamblers in the market. Therefore, the gamblers' expectations about the odds are self-fulfilled in equilibrium. In this definition each gambler takes his expectations about market odds $(h_1, ..., h_m)$ as given, and hence believes that he cannot influence the determination of market odds. This is the reasonable case when the number of gamblers n is so large that each of them has no market powers, as considered in the standard notion of competitive equilibrium.

We call a profile of amounts bet by gamblers and the odds $\{\mathbf{a}_1, ..., \mathbf{a}_n, h_1, ..., h_m\}$ that satisfy these conditions (i) and (ii) *gambling equilibrium*.[10] In our two-period model we will look for a gambling equilibrium in each period $t = 1, 2$, given the bookmaker's strategy. In

[9] Besides the dynamic mechanism we will discuss later, there are other justifications for the bookmaker to commit to payouts and punishment policies. For instance, the bookmaker may hire agents who are paid constant wages and work on her behalf. These agents are simply required to execute the payout and punishment policy according to ex ante specified rule. Then, since they are paid constant wages regardless of horse racing outcomes, they have no incentives to deviate from the ex ante specified rule. In fact, gambling may be managed by several betting stations but not the bookmaker herself. This might play a role for the bookmaker to commit to payouts and punishment policy.

[10] See Chiappori et al. (2019) and Ottaviani and Sørensen (2010) for related approaches.

particular, to avoid complication, we will focus on the gambling equilibrium in which every horse attracts some positive amounts bet, so the odds of every horse is positive, $h_j > 0$ for all $j \in J$. In such equilibrium there are no horses which win the race but nobody bet on: no gamblers are refunded $(1-f)A_i$.[11]

2.5 Timing of the Game

We consider the following timing of the games under the cash-bet and the book-bet used in period 1 (see Figure 1 for the cash-bet and Figure 2 for the book-bet).

When the bookmaker uses the cash-bet in both periods, the following game is repeated twice:

1. Gamblers decide how much to bet $\mathbf{a}_i = (a_{i,1}, ..., a_{i,m})$, by forming expectations about the odds $\{h_j\}_{j \in J}$ and taking them as exogenously given. Then gamblers immediately pay the bookmaker the amounts they bet.
2. Given amounts $\{\mathbf{a}_i\}_{i \in I}$ bet by gamblers, the equilibrium odds $\{h_j\}_{j \in J}$ are determined by condition (1).
3. The wining horse is determined, and the bookmaker makes payouts to gamblers.

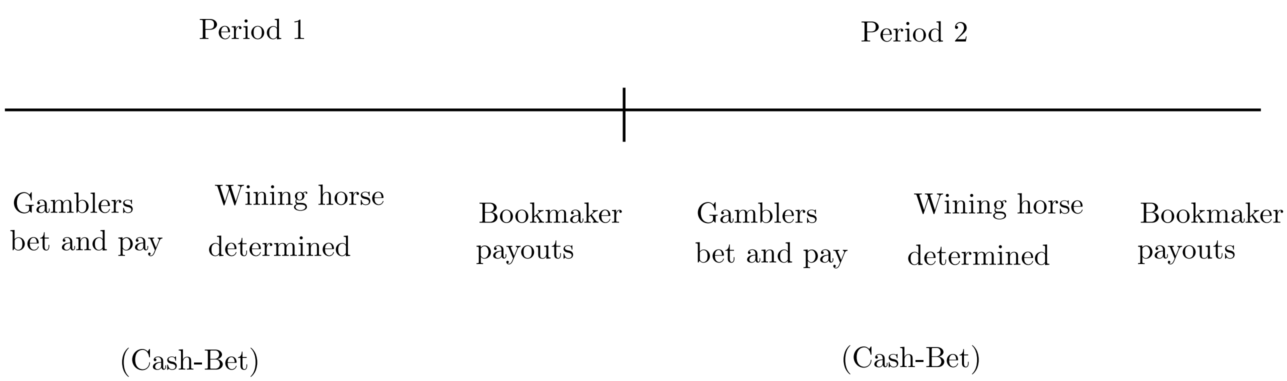

Figure 1: Timing of the game under cash-bet

Next we consider that the bookmaker uses the book-bet in period 1 while using the cash-bet in period 2. As we mentioned, gamblers may default on paybacks under the book-bet because they defer payments to the end of period 1. Suppose that horse j wins the race and gambler i receives net returns of $R_{i,j}$ from the bookmaker according to his

[11] This restriction can be justified when each gambler places a small but positive bet $\varepsilon > 0$ on each horse by a "mistake". Alternatively, there are some irrational gamblers who always bet a positive but small amount $\varepsilon > 0$ on each horse. Then our equilibrium might be viewed as the limit as such irrational/mistaken bet ε goes to zero.

amount bet $a_{i,j}$ on horse j and the odds of horse j, h_j. If $R_{i,j} < 0$, gambler i has the debt equal to $-R_{i,j}$ but can default any amount $D_{i,j} \in [0, -R_{i,j}]$, in which case he will pay back only $-R_{i,j} - D_{i,j}$ to the bookmaker. We denote by $z_{i,j} \equiv \max\{0, -R_{i,j}\}$ the debt gambler i owes to the bookmaker when horse j wins the race.

The game under the book-bet proceeds as follows.

In period 1:

1. The bookmaker offers a punishment policy q.

2. Gamblers decide how much to bet $\mathbf{a}_i = (a_{i,1}, ..., a_{i,m})$, by forming expectations about the odds $\{h_j\}_{j \in J}$ and taking them as exogenously given.

3. Given amounts $\{\mathbf{a}_i\}_{i \in I}$ bet by gamblers, the equilibrium odds $\{h_j\}_{j \in J}$ are determined by condition (1).

4. The wining horse is determined.

5. Gamblers decide how much to default on paybacks when their net gambling returns $R_{i,j}$ are negative. When gambler i does not default, he can certainly participate in the gambling in period 2. When gambler i defaults an amount of $D_{i,j} \in [0, z_{i,j}]$, he will be excluded from the second-period gambling with probability $q(D_{i,j}, A_i) \in [0, 1]$.

In period 2:

1. Those who were not excluded in the end of period 1 decide how much to bet under the cash-bet, by forming expectations about the odds.

2. The odds are determined by condition (1), the winning horse is determined, and the bookmaker makes payouts to gamblers.

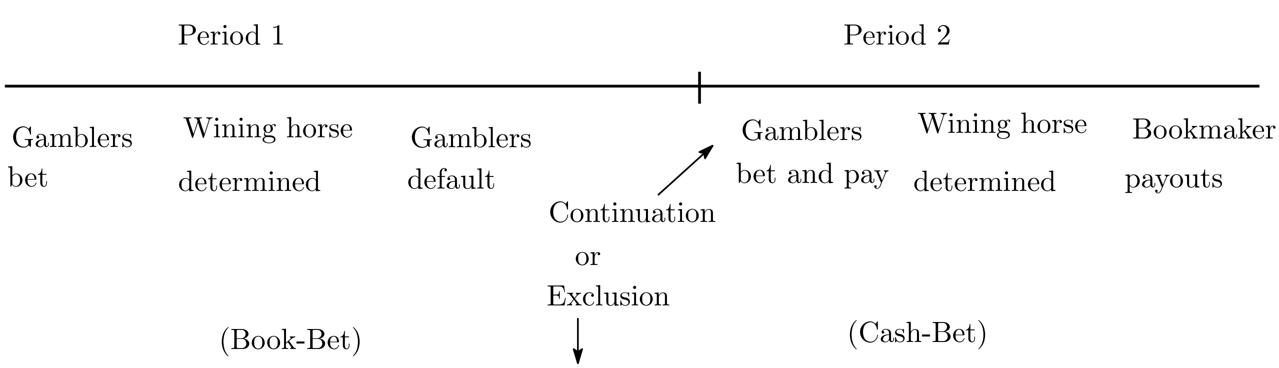

Figure 2: Timing of the game under book-bet

We proceed to provide equilibrium characterization as follows. First, we show the equilibrium outcome in period 2, in which the bookmaker holds the cash-bet. Given this, we then show that the optimal punishment policy for the bookmaker under the book-bet in period 1 becomes the bang-bang form and has the feature of "free-first-dose" strategy such that gamblers are allowed to default but are never punished in equilibrium.

3 Gambling Equilibrium in Period 2

We begin with gambling equilibrium in period 2, given the addiction stock of gamblers $(S_1, ..., S_n)$ that have been already determined by the amounts they bet in period 1, $(A_{1,1}, ..., A_{n,1})$, where $S_i = \alpha + \beta A_{i,1}$.

Suppose that there are $I_2 \subseteq I$ gamblers who are allowed to participate in the gambling in period 2. Although it may be the case that $I_2 \neq I$ in general, we will simplify the following argument by setting $I_2 = I$; all gamblers participate in period 2. We consider the equilibrium in period 2, given the addiction stocks of participating gamblers $(S_1, ..., S_n)$. As mentioned, gambling is always held as the cash-bet in period 2 regardless of which the cash-bet or the book-bet was used in period 1. Under the cash-bet gamblers cannot default. To save notation, we drop time index $t = 2$ from subscripts of all the variables in this section.

Gambler i's payoff in period 2 is given by

$$u(A_i, S_i) + c_{i,j} \tag{4}$$

where $c_{i,j}$ is his private consumption of the numèraire good when horse j wins the race, and it equals to the net return $R_{i,j}$ he earns, given by[12]

$$c_{i,j} = R_{i,j} \equiv h_j a_{i,j} - A_i.$$

Here, when gambler i bets $\mathbf{a}_i$ and horse j wins the race, gambler i earns the net return $R_{i,j}$ that equals to the payout $h_j a_{i,j}$ received from the bookmaker minus the total amount he bet A_i. Then, gambler i places bet of $\mathbf{a}_i$ so as to maximize his expected payoff

$$u(A_i, S_i) + \sum_{j \in J} p_j \{h_j a_{i,j} - A_i\} \tag{5}$$

given expectations about the odds $\{h_j\}_{j \in J}$.

We denote by $\hat{a}_{i,j} > 0$ the equilibrium amount bet by gambler i on horse j, and by $\hat{A}_i \equiv \sum_{j \in J} \hat{a}_{i,j}$ the aggregate amount bet by gambler i. Let also denote by $\hat{A} \equiv \sum_{i \in I} \hat{A}_i$

[12] We here allow $c_{i,j}$ to be negative for simplicity. We can however ensure non-negative consumption $c_{i,j} \geq 0$ by assuming that gamblers are exogenously endowed a large income $w > 0$ in each period.

the aggregate amount bet by all gamblers and by $\hat{a}_j \equiv \sum_{i\in I} \hat{a}_{i,j}$ the aggregate amount bet by all gamblers on horse $j \in J$.

Then, according to (1), equilibrium odds $\{\hat{h}_j\}_{j\in J}$ are determined as follows[13]

$$\hat{h}_j = \frac{(1-f)\hat{A}}{\hat{a}_j}, \quad \text{for } j \in J. \tag{6}$$

We then obtain the following proposition (we relegate all proofs to the Appendix.)

Proposition 1. *Given a collection of addiction stocks of gamblers $(S_1, ..., S_n)$, gambling equilibrium in period 2 is characterized as follows:*

- *Gambler i who has addiction stock S_i bets the amount $\hat{A}_i = \hat{A}(S_i)$ in total satisfying*

$$u_A(\hat{A}_i, S_i) - f = 0. \tag{7}$$

- *Gambler i who has addiction stock S_i obtains the equilibrium payoff given as follows:*

$$\hat{U}(S_i) \equiv u(\hat{A}(S_i), S_i) - f\hat{A}(S_i). \tag{8}$$

Proposition 1 states that gambler i's equilibrium aggregate bet $\hat{A}(S_i)$ is determined by the first-order optimality condition (7), which depends on his addiction stock S_i. When the gambler's marginal utility of betting u_A increases as he is addicted more (that is, $\partial u^2/\partial A\partial S \equiv u_{AS} > 0$), he will bet more aggressively as he is addicted more; $\hat{A}$ is increasing in S_i. This result implies that the bookmaker has the incentive to let gamblers bet more in period 1, and hence make them addicted more, inducing larger amounts bet in period 2. Also, as we can see from (8), gambler's equilibrium payoff of $\hat{U}(S_i)$ increases with his addiction stock S_i; $\hat{U}'(S_i) = u_S(\hat{A}(S_i), S_i) > 0$. This has the important implication about the optimal strategy of the bookmaker in period 1. When the bookmaker induces gamblers to bet more in period 1, she must compensate them by allowing them to default more. Such compensation is costly for the bookmaker but can be lower when gamblers are addicted more by the first-period gambling because they are then more willing to participate in the second-period gambling.

By Proposition 1, the bookmaker obtains the payoff of $f\sum_{i\in I}\hat{A}(S_i)$ in period 2, which equals to the commission fee times the aggregate amount bet by participating gamblers.[14]

[13] Note that we are considering the equilibrium in which $\hat{h}_j > 0$ for all $j \in J$, so $\hat{a}_j > 0$ for all $j \in J$.

[14] When $I_2 \neq I$, the bookmaker's payoff in period 2 is modified to $f\sum_{i\in I_2}\hat{A}(S_i)$.

4 “Free-First-Dose” Strategy

4.1 Bang-Bang Punishment Policy

We move to period 1 and characterize the optimal punishment policy q chosen by the bookmaker in period 1. Then we show that, when the book-bet is used in period 1, the bookmaker optimally adopts the ‘free-first-dose’ strategy in period 1 by reducing initial constraints weighing on gamblers to foster addictions. In order to save notations, we drop time index $t = 1$ from the variables chosen in period 1. The equilibrium outcome in the second-period game is summarized by the aggregate bet $\hat{A}(S_i)$ of gambler i and his equilibrium payoff $\hat{U}(S_i)$ as shown in Proposition 1.

Under the book-bet, gambler i does not immediately pay the amount bet $A_i = \sum_j a_{i,j}$ to the bookmaker, but it is deferred in the end of period 1. After horse j wins the race, gambler i has the debt of $z_{i,j} \equiv \max\{0, -R_{i,j}\}$ where $R_{i,j} \equiv h_j a_{i,j} - A_i$ is gambler i's net return. Given his debt $z_{i,j}$, gambler i can default any amount of $D_{i,j}$ such that

$$0 \leq D_{i,j} \leq z_{i,j}. \tag{9}$$

In the beginning of period 1 the bookmaker offers the punishment policy, q, defined as

$$q : \mathbb{R}_+ \times \mathbb{R}_+ \to [0, 1] \tag{10}$$

that specifies the probability $q(D, A) \in [0, 1]$ to exclude each defaulting gambler from the participation in the gambling in period 2, contingent on the amount bet in total A and the amount defaulted $D \in [0, z]$. As we mentioned, we make the exogenous restriction on q such that $q(0, A) = 0$ for any $A \geq 0$: the bookmaker cannot exclude gamblers who did not default in period 1. Also, as we discussed, the bookmaker commits herself to use the punishment policy q initially announced in the beginning of period 1.

Given a punishment policy q offered by the bookmaker, we consider the resulting equilibrium in period 1 as follows: gamblers decide how much bet to place by forming expectations about the odds $\{h_j\}_{j \in J}$. Then the horse race outcome is realized and the equilibrium odds $\{h_j\}_{j \in J}$ are determined such that the gamblers' expectations are self-fulfilled according to (1). Following this, gamblers decide how much to default, and only those who are not excluded will participate in the gambling market in period 2; they then obtain the second-period equilibrium payoff $\hat{U}(S_i)$ as shown in Proposition 1.

When gambler i defaults $D_{i,j} \in [0, z_{i,j}]$, he will be excluded from the gambling in period 2, with probability $q(D_{i,j}, A_i) \in [0, 1]$, and obtain the following *ex post* payoff after horse j wins the race in period 1:

$$\max\{R_{i,j}, 0\} - z_{i,j} + D_{i,j} + (1 - q(D_{i,j}, A_i))\hat{U}(S_i). \tag{11}$$

Gambler i decides how much to default $\{D_{i,j}\}_{j \in J}$ such that $0 \leq D_{i,j} \leq z_{i,j}$ for each winning horse $j \in J$, given the punishment policy q and realized odds $\{h_j\}_{j \in J}$. The *ex ante* payoff of gambler i in period 1 is then given by

$$v(A_i) + \sum_{j \in J} p_j \{h_j a_{i,j} - A_i + D_{i,j} + (1 - q(D_{i,j}, A_i)\hat{U}(S_i))\}. \tag{12}$$

Now we define the "bang-bang" punishment policy: each gambler is never punished unless he defaults more than a cutoff value $\tilde{D}(z, A)$ while he will be most harshly punished otherwise. That is, it is defined as follows:

$$\tilde{q}(D, A) = \begin{cases} 0 & \text{if } D \leq \tilde{D}(z, A) \\ 1 & \text{otherwise} \end{cases} \tag{13}$$

We call such cut-off value $\tilde{D}(z, A)$ the *maximum forgiveness*.

Suppose that a punishment policy q implements the gambling equilibrium outcome $\{\mathbf{a}^*, \mathbf{h}^*\}$ under the book-bet in period 1 such that gamblers bet $\mathbf{a}^* \equiv \{\mathbf{a}_i^*\}_{i \in I}$ and the associated odds $\mathbf{h}^* \equiv \{h_j^*\}_{j \in J}$ are determined in order to satisfy condition (1) in period 1: $h_j^* = (1 - f) \sum_i A_i^* / \sum_i a_{i,j}^*$ for each $j \in J$. Following this, gambler i places the equilibrium bet $\hat{A}(S_i^*)$ in total in period 2 where $S_i^* \equiv \alpha + \beta A_i^*$ is the addiction stock of gambler i in the equilibrium (Proposition 1). Let denote by $\hat{\mathbf{A}} \equiv (\hat{A}_1(S_1^*), ..., \hat{A}_n(S_n^*))$ the profile of these equilibrium bets in period 2. The entire equilibrium outcome in the two-period game is therefore given by the collection $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{A}}\}$.

Then we show that the bookmaker can replace any punishment policy q by the bang-bang punishment policy (13) such that she can implement the same equilibrium outcome $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{A}}\}$ as that attained under the original policy q, and can be weakly better off by the new policy $\tilde{q}$.

Proposition 2. *Suppose that the bookmaker adopts the book-bet in period 1 and that a punishment policy q implements an equilibrium outcome $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{A}}\}$. Then, the bookmaker can always implement the same equilibrium outcome $\{\mathbf{a}^*, \mathbf{h}^*, \hat{\mathbf{A}}\}$ by the bang-bang punishment policy, defined as $\tilde{q}$ above (13), such that gamblers are induced to default the amount equal to the maximum forgiveness and are never punished in equilibrium. Furthermore, the bookmaker can be weakly better off by such a bang-bang punishment policy.*

The key predictions of Proposition 2 are (i) that gamblers default a strictly *positive* amount and (ii) that they are however not punished in equilibrium. Therefore, the bookmaker optimally adopts the "free-first-dose" strategy such that she induces gamblers to place large bets initially by allowing them to default without punishment and then fosters addictions, resulting in large bets in the later period. Such "allowed default" plays a strategic role to compensate individuals for being addicted to gambling over time. This tactic is often adopted by sellers of addicted goods such that individuals get goods for free initially and increase the willingness to pay for the goods later after they are addicted.

Proposition 2 can help explain seemingly paradoxical evidence about illegal markets that sellers do not often penalize buyers even when they default on payments. For example, Mehmood and Chen (2022) conducted an empirical analysis on illegal horse race betting in Pakistan, and showed that on average 35% of gamblers in their sample do not repay their debts in full. Mehmood and Chen (2022) also found that only 0.5% of surveyed gamblers reported any apprehension of encountering violence in case of non-repayment. Violence plays the similar punishment role to the exclusion of defaulting gamblers from the future gambling activities because both of them impose certain penalties on defaulting gamblers (see Section 4.2 for the formal analysis on violence). These facts look surprising at a first glance: a significant fraction of gamblers defaults on debt repayments but few of them are penalized. Proposition 2 provides a theoretical insight to explain these paradoxical observations. Furthermore, despite the high frequency of gamblers' defaults, race clubs (bookmakers) allow gamblers to place bets by credit rather than cash. Mehmood and Chen (2022) reported that 55% of wagers in their sample are placed on credit, as opposed to cash-in-advance bets. We will provide a rationale for this phenomenon later (Proposition 3 given below).

The basic idea of Proposition 2 is as follows: whenever the punishment probability $q(D, A)$ is strictly positive and less than 1 for some $D > 0$, the bookmaker can improve her payoff by replacing it by the bang-bang punishment policy such that gamblers are induced to default less but face more severe punishment.

To see this more precisely, consider any punishment policy q (as given in Figure 3) and then suppose the following: (i) gambler i optimally chooses to default $D^*(z_{i,j}, A_i) \in [0, z_{i,j}]$ when horse j wins the race, and (ii) gambler i is then excluded from the gambling in period 2 with probability $q(D^*_j(z_{i,j}, A_i), A_i) \in [0, 1]$. Here, after horse j won the race, gambler i optimally defaults the amount of $D^*(z_{i,j}, A_i)$ so as to maximize his ex post payoff by choosing D such that

$$\max\ D + (1 - q(D, A_i))\hat{U}(S_i)$$

subject to $0 \leq D \leq z_{i,j}$, where $S_i = \alpha + \beta A_i$.

Then we define the bang-bang punishment policy $\tilde{q}$ as follows: First, we set the maximum forgiveness equal to

$$\tilde{D}(z_{i,j}, A_i) \equiv D^*(z_{i,j}, A_i) - q(D^*(z_{i,j}, A_i), A_i)\hat{U}(S_i). \tag{14}$$

Second, gambler i is punished according to the bang-bang punishment policy $\tilde{q}$ defined as (13) for the maximum forgiveness $\tilde{D}(z_{i,j}, A_i)$ defined as (14) (see Figure 4).

This newly defined policy can induce gamblers to choose the same amounts bet as those in the equilibrium under the original punishment policy q and induce them to default the amount equal to $\tilde{D}(z_{i,j}, A_i)$ for each wining horse $j \in J$. Note first that under the newly defined punishment policy, gambler i never defaults less than the maximum forgiveness, $D < \tilde{D}(z_{i,j}, A_i)$. This is because gambler i is never punished as long as $D < \tilde{D}(z_{i,j}, A_i)$, but then he can slightly increase the defaulted amount without being imposed any punishment. Second, we show that gambler i never defaults more than the maximum forgiveness, $D >$

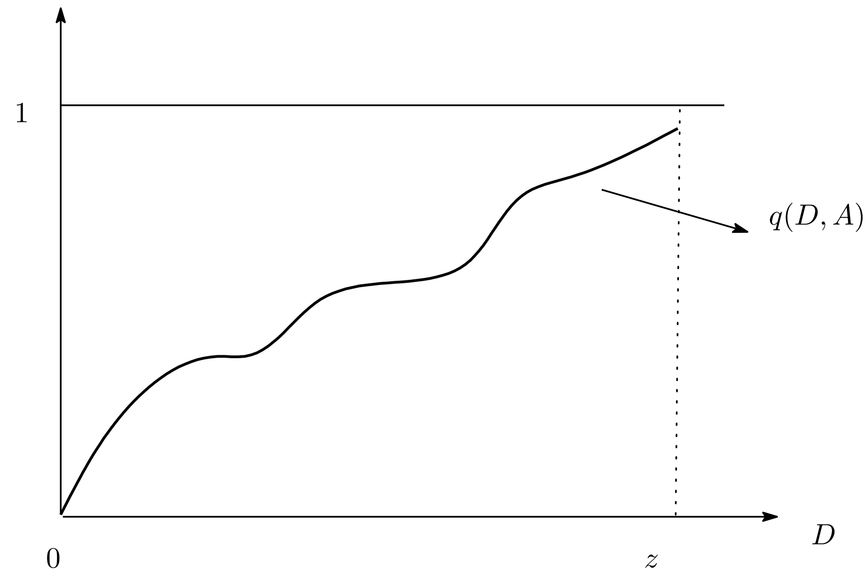

Figure 3: Punishment policy

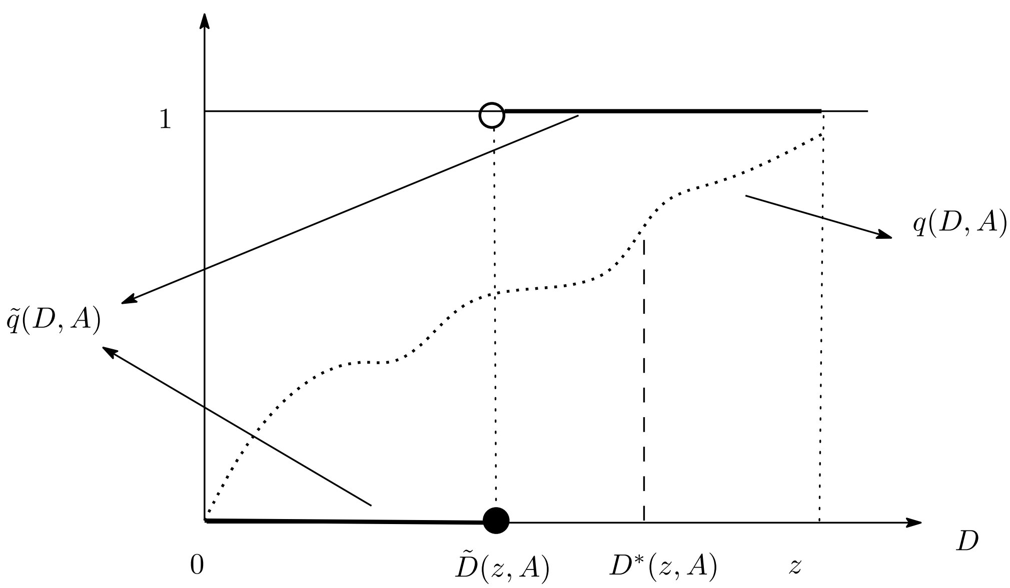

Figure 4: Bang-bang punishment policy

$\tilde{D}(z_{i,j}, A_i)$. If this is the case, gambler i obtains the ex post payoff equal to $D + 0 \times \hat{U}(S_i) = D$ when horse j wins the race in period 1 because he will be certainly excluded from the gambling in period 2. However, due to $\hat{U}(S_i) \geq 0$ and the definition of $\tilde{D}(z_{i,j}, A_i)$, we can verify that

$$
\begin{aligned}
D &\leq D + (1 - q(D, A_i))\hat{U}(S_i) \\
&\leq \max_{D \in [0, z_{i,j}]} D + (1 - q(D, A_i))\hat{U}(S_i) \\
&= D^*(z_{i,j}, A_i) + (1 - q(D^*(z_{i,j}, A_i), A_i)\hat{U}(S_i) \\
&= \tilde{D}(z_{i,j}, A_i) + \hat{U}(S_i)
\end{aligned}
$$

so that gambler i cannot be better off from defaulting the amount of $\tilde{D}(z_{i,j}, A_i)$, in which case he is never punished and obtains the ex post payoff of $\tilde{D}(z_{i,j}, A_i) + \hat{U}(S_i)$. Therefore, gambler i's optimal choice is to default $D_{i,j} = \tilde{D}(z_{i,j}, A_i)$ when horse j wins the race.

Given the above result, gambler i places bets $\mathbf{a}_i$ so as to maximize the following expected payoff in period 1:

$$
v(A_i) + \sum_{j \in J} p_j \{h_j a_{i,j} - A_i + \tilde{D}(z_{i,j}, A_i) + \hat{U}(S_i)\}. \tag{15}
$$

Due to the definition of $\tilde{D}(\cdot, \cdot)$ (see (14)), the above payoff (15) is equivalent to

$$
v(A_i) + \sum_{j \in J} p_j \{h_j a_{i,j} - A_i + D^*(z_{i,j}, A_i) + (1 - q(D_j^*(z_{i,j}, A_i), A_i))\hat{U}(S_i)\},
$$

which is same as what gambler i would obtain under the original punishment policy q and the same odds $\{h_j\}_{j \in J}$ as those in the original equilibrium. Therefore, in period 1 gamblers bet the same amounts $\mathbf{a}^*$ as those in the original equilibrium with the same equilibrium odds $\mathbf{h}^*$. This also leads to the same equilibrium amounts bet $\hat{\mathbf{A}}$ as those in the original equilibrium in period 2.

The bookmaker's payoff under the original punishment policy q is given by

$$
\Pi(q) \equiv \sum_{i \in I} \sum_{j \in J} p_j \{A_i^* - h_j^* a_{i,j}^* - D(z_{i,j}^*, A_i^*) + (1 - q(D(z_{i,j}^*, A_i^*), A_i^*)) f \hat{A}(S_i^*)\}
$$

where $z_{i,j}^* \equiv \max\{A_i^* - h_j^* a_{i,j}^*, 0\}$. On the contrary, the bookmaker obtains the following payoff by the newly defined policy $\tilde{q}$:

$$
\Pi(\tilde{q}) \equiv \sum_{i \in I} \sum_{j \in J} p_j \{A_i^* - h_j^* a_{i,j}^* - \tilde{D}(z_{i,j}^*, A_i^*)\} + \sum_{i \in I} f \hat{A}(S_i^*).
$$

The bang-bang punishment policy $\tilde{q}$ can bring two gains to the bookmaker. First, gamblers reduce the defaulted amount from $D^*(z_{i,j}, A_i)$ to $\tilde{D}(z_{i,j}, A_i)$. This can benefit the bookmaker. Second, all gamblers can certainly participate in the gambling in period 2 because $\tilde{q}(\tilde{D}(z_{i,j}, A_i), A_i) = 0$ holds in equilibrium. Therefore, the market size in period 2 does not shrink under the bang-bang punishment policy. These two positive effects make the bookmaker never worse off by the bang-bang punishment policy. In general the bookmaker can be strictly better off by the bang-bang punishment policy whenever the original punishment policy q excludes gamblers with some positive probability in equilibrium.

4.2 Role of Violence

We have so far restricted the available punishment instrument for the bookmaker only to the probability $\tilde{q}$ to exclude defaulting gamblers in period 2. One might wonder if more direct threats such as violence are often used to enforce payments in illegal markets. However, violence may not be necessarily involved but rather may be uncommon in illegal markets (see Mahar (2022) for a related evidence). In fact, according to Mehmood and Chen (2022), in the illegal horse race betting in Pakistan only 0.5% of gamblers reported threats of violence in the case of non-payments (see Section 7 for more detailed discussion).

In this subsection we extend the model to allow the bookmaker to use non-monetary penalty such as violence as an additional instrument to punish defaulting gamblers. We then show that the bookmaker still optimally sticks to the bang-bang punishment policy such that gamblers default but are never punished by violence in equilibrium, implying that violence is available but not observed.

We consider the scenario in which the bookmaker can punish defaulting gamblers under the book-bet by using not only exclusion but also violence in the end of period 1. More specifically, the bookmaker can impose a non-monetary penalty $\psi(D, A) \in [0, H]$ in terms of utility on a defaulting gambler, by making it contingent on how much to bet in total A and how much to default D. Here, $H > 0$ is the exogenous upper bound for penalty imposed on gamblers.

Consider the book-bet in period 1 and suppose that the bookmaker offers a punishment policy $\{q, \psi\}$ in the beginning of period 1, where q is the exclusion probability as considered in the main model, and ψ specifies a non-monetary penalty:

$$\psi : \mathbb{R}_+ \times \mathbb{R}_+ \to [0, H]$$

contingent on the defaulted amount $D \geq 0$ and the aggregate bet $A \geq 0$. As assumed for the punishment probability q, we impose the same reasonable restriction on ψ such that $\psi(0, A) = 0$: when gamblers do not default, they will be imposed no penalties.

Given this policy, suppose that gambler i bets $\mathbf{a}_i$ and then defaults $D_{i,j} \geq 0$ when horse j wins the race in period 1, where $0 \leq D_{i,j} \leq z_{i,j} \equiv \max\{A_i - h_j a_{i,j}, 0\}$. Then, gambler i is imposed a penalty $\psi(D_{i,j}, A_i)$ and is excluded from gambling in period 2 with probability $q(D_{i,j}, A_i) \in [0, 1]$. Therefore, gambler i obtains the following expected payoff in period 1

$$v(A_i) + \sum_{j \in J} p_j \{h_j a_{i,j} - A_i + D_{i,j} - \psi(D_{i,j}, A_i) + (1 - q(D_{i,j}, A_i)\hat{U}(S_i)\}.$$

Then, gambler i optimally defaults $D_{i,j}$ to maximize his payoff $D_{i,j} - \psi(D_{i,j}, A_i) + (1 - q(D_{i,j}, A_i))\hat{U}(S_i)$ subject to $0 \leq D_{i,j} \leq z_{i,j}$. We denote by $D^*_{i,j}(z_{i,j}, A_i)$ the optimal defaulted amount of gambler i.

Now we define the maximum forgiveness as

$$\tilde{D}(z_{i,j}, A_i) \equiv D^*(z_{i,j}, A_i) - \psi(D^*(z_{i,j}, A_i), A_i) - q(D^*(z_{i,j}, A_i), A_i)\hat{U}(S_i).$$

and the bang-bang punishment policy as

$$\tilde{q}(D, A) = \begin{cases} 0 & \text{if } D \leq \tilde{D}(z, A) \\ 1 & \text{otherwise} \end{cases}$$

and

$$\tilde{\psi}(D, A) = \begin{cases} 0 & \text{if } D \leq \tilde{D}(z, A) \\ H & \text{otherwise} \end{cases}$$

This is the extension of the bang-bang punishment policy considered in the basic model. Gamblers are never punished unless they default more than the maximum forgiveness $\tilde{D}(z, A)$ while they are most harshly punished otherwise in the sense that they are certainly excluded from gambling in period 2 and are imposed the maximum penalty H.

Then we can use the same argument as in Proposition 2 and show that the optimal punishment policy $\{q, \psi\}$ can be restricted to the bang-bang policy without loss of generality. Since the formal proof of this result is the simple modification of Proposition 2, we omit it. The intuition is the same as that of Proposition 2: First, the bang-bang punishment policy $\{\tilde{q}, \tilde{\psi}\}$ defined above can induce gamblers to bet the same amounts as those they choose under the original policy $\{q, \psi\}$. This is because, if gamblers deviate from the equilibrium bet, they are punished under the new policy in the essentially same way as they are under the original policy. Second, since the bang-bang policy reduces the defaulted amount and induces gamblers to participate certainly in the gambling in period 2, the bookmaker can be better off by the bang-bang policy.

One important implication of this result is that gamblers default a positive amount but are never punished by violence in equilibrium. This theoretical insight is consistent with the evidence about illegal horse race betting in Pakistan as we will discuss in Section 7 later.

5 Equilibrium Characterization

In this section we investigate how gamblers' addictions affect the optimal policy chosen by the bookmaker and the resulting patterns of equilibrium amounts bet over time.

5.1 Optimal Punishment Policy under the Book-bet

We begin with the optimal punishment policy q chosen by the bookmaker, provided that she uses the book-bet in period 1. Thanks to Proposition 2, without loss of generality we can focus only on the bang-bang punishment policy $\tilde{q}$ defined as (13), when the bookmaker uses the book-bet in period 1. In particular, since all gamblers are identical in terms of their payoff functions, we pay our attentions to the symmetric equilibrium in which all gamblers place bets and default according to the same strategy: $a^*_{i,j,1} = a^*_j$ and $A^*_{i,1} = A^*$ for all $i \in I$ in period 1 and $A_{i,2} = \hat{A}(S^*)$ for all $i \in I$ in period 2 where $S_i = S^* \equiv \alpha + \beta A^*$ for all $i \in I$. We denote by $\mathbf{a}^* = (a^*_1, ..., a^*_m)$ an equilibrium profile of amounts bet by each gambler in period 1, where a^*_j is the amount bet by each gambler on horse j, and $A^* = \sum_{j \in J} a^*_j$ is the aggregate amount bet by each gambler.

We now consider the optimal bang-bang punishment policy that maximizes the bookmaker's payoff $\Pi(\tilde{q})$ given by

$$\Pi(\tilde{q}) \equiv n \left\{ A^* - \sum_{j \in J} p_j \{h^*_j a^*_j + \tilde{D}(z^*_j, A^*)\} \right\} + nf\hat{A}(S^*) \tag{16}$$

in the symmetric equilibrium where

$$z^*_j \equiv \max\{A^* - h^*_j a^*_j, 0\}$$

is the equilibrium debt that should be paid back from each gambler to the bookmaker conditional on horse j wining the race. The first term of (16) is the bookmaker's payoffs in period 1: she receives the aggregate amount bet A^* from each gambler and gives him the expected payout equal to $\sum_j p_j h^*_j a^*_j$ and loses the defaulted amount $\tilde{D}(z^*_j, A^*)$ conditional on horse $j \in J$ wining the race. The second term of (16) is the bookmaker's payoffs from holding the cash-bet in period 2, where each gambler bets $A_2 = \hat{A}(S^*)$ in total in period 2 according to Proposition 1.

We say that an aggregate amount bet A^* by each gambler is *implementable* when there exists an equilibrium in which each gambler chooses A^* in total in period 1, defaults the amount equal to the maximum forgiveness $\tilde{D}(z^*_j, A^*)$, and is never punished, in association with the corresponding equilibrium odds $\{h^*_j\}_{j \in J}$ that satisfies (1) and is consistent with equilibrium amount bet $\mathbf{a}^*$ in period 1.

For an aggregate amount bet A^* to be implementable, it must satisfy a certain set of constraints by taking into account the default incentive of gamblers. To see this, we define the payoff of the gambler who never defaults no matter what horse racing outcomes are, given the equilibrium odds $\{h^*_j\}_{j \in J}$ in period 1, as follows

$$U^* \equiv \max_{\{a_{ij}\}_{j \in J}} v(A_i) + \sum_{j \in J} p_j h^*_j a_{i,j} - A_i + \hat{U}(\alpha + \beta A_i) \tag{17}$$

where $A_i = \sum_{j \in J} a_{i,j}$ is the aggregate amount bet by gambler i in period 1.

We then obtain the following lemma.

Lemma 1. *$v(A^*) + \hat{U}(S^*) \geq U^*$ must be satisfied for any implementable A^*.*

The intuition of this result is as follows: first, each gambler can obtain at least the payoff of U^*, defined as (17), in any equilibrium because he can always pay back all debts in period 1 and then participate in the gambling in period 2, yielding the second-period payoff of $\hat{U}(S_i)$. Therefore, U^* is a lower bound for gambler's equilibrium payoff. Second, gambler's equilibrium payoff is bounded above by $v(A^*) + \hat{U}(S^*)$. This is because each gambler cannot do better than defaulting all debts in period 1 and obtaining the equilibrium payoff of $\hat{U}(S^*)$ in period 2 without any punishment, yielding the total payoffs of $v(A^*) + \hat{U}(S^*)$ without any debt repayment. Therefore, $v(A^*) + \hat{U}(S^*)$ is an upper bound for gambler's equilibrium payoff. Then equilibrium payoff of each gambler must lie between these two payoffs, $v(A^*) + \hat{U}(S^*)$ and U^*, to implement an equilibrium aggregate bet A^* from him. This results in Lemma 1.

We call the constraint stated in Lemma 1 *feasibility* (F) constraint. When the bookmaker chooses the book-bet in period 1, she optimally offers the bang-bang punishment policy $\tilde{q}$ in period 1 so as to maximize her payoff $\Pi(\tilde{q})$, given by (16), subject to the feasibility (F) constraint. We denote by Π_b the bookmaker's optimal value in this problem:

$$\Pi_b \equiv \max_{\tilde{q}} \Pi(\tilde{q}) \quad \text{s.t. F constraint.}$$

5.2 Bookmaker's Payoff under the Cash-Bet

To compare the book-bet with the cash-bet, we consider the case that the bookmaker chooses the cash-bet in period 1 as well as in period 2. Under the cash-bet gamblers cannot default in period 1 and can then surely participate in the gambling in period 2. As a result, gambler i places bet $\mathbf{a}_i$ in period 1 to maximize the following expected payoff

$$v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i\} + \hat{U}(S_i) \tag{18}$$

given expectations about equilibrium odds $\{h_j^*\}_{j \in J}$.

In the symmetric equilibrium the odds are determined by $h_j^* = (1 - f)/p_j$ for each $j \in J$.[15] Given this, gamblers bet the aggregate amount A in period 1 to maximize the expected payoff given by (18), which we can re-write by $v(A) - fA + \hat{U}(S)$. We denote by $\tilde{A}$ the equilibrium aggregate amount bet by each gambler in period 1 and by $\hat{A}(\tilde{S})$ the associated amount bet by each gambler in period 2 where $\tilde{S} \equiv \alpha + \beta \tilde{A}$. Then the bookmaker obtains total payoffs in two periods as follows

$$\Pi_c \equiv nf\tilde{A} + nf\hat{A}(\tilde{S})$$

[15] Again we here focus on the equilibrium in which the odds are positive for any horse.

provided that she chooses the cash-bet in period 1.

5.3 Book-Bet or Cash-Bet?

The bookmaker is exposed to default risks when she uses the book-bet in period 1 in contrast to the cash-bet. However, according to Mehmood and Chen (2022), in the illegal horse race betting in Pakistan more than 50% of gamblers are allowed to place bets by credit rather than cash-in-advance bets. If we focus only on default costs, this observation is puzzling. Why does the bookmaker extend credit to gamblers even when they default? In the following we show the condition under which the bookmaker can gain from using the book-bet in period 1 by strategically inducing gamblers' defaults and making them addicted to gambling, resulting in large amounts bet in a later period.

Which book-bet or cash-bet the bookmaker prefers in period 1 depends on how strongly gamblers are addicted to gambling in the first period. We measure the degree of gamblers' addictions by the parameter values α and β that determine the addiction stock $S_i = \alpha + \beta A_i$ in period 2. When α and β are larger, gamblers are addicted more by engagements in the first period gambling.

We then show that, when gamblers' addictions are so strong that α and β are sufficiently large, the bookmaker prefers the book-bet to the cash-bet in period 1: the bookmaker obtains a higher payoff Π_b under the book-bet than that Π_c under the cash-bet. On the contrary, in the absence of addiction preferences in the sense that $\alpha = \beta = 0$, the bookmaker may prefer the cash-bet to the book-bet under a certain condition. Therefore, gamblers' addictions cause a significant effect on the bookmaker's choice of the gambling format.

To derive these results, we make the following assumption, which states that the second-period equilibrium payoff of each gambler $\hat{U}(S)$ is large relative to his aggregate bet A in period 1.

Assumption 1. *$\hat{U}(\alpha + \beta A) \geq A$ for all $A \geq 0$.*

Assumption 1 ensures that each gambler never defaults more than the maximum forgiveness: otherwise he will be excluded from participating in the second-period gambling and then lose the second-period payoff $\hat{U}(S_i)$. When the second-period payoff is so large that it can cover any aggregate bet A_i in period 1, each gambler avoids the exclusion by defaulting less than the maximum forgiveness.[16]

When gamblers are never addicted ($\alpha = \beta = 0$, so $S_i = 0$), we define by $u(A_{i,2}) \equiv u(A_{i,2}, 0)$ the second-period utility of gambler i who places bet of $A_{i,2}$ in total. Since the

[16] Assumption 1 is satisfied when the addiction effects, captured by the magnitudes of u_S and u_{AS}, are sufficiently large in period 2. To see this, note that, by the definition of $\hat{U}(S)$, we have $\hat{U}'(S) = u_S(\hat{A}(S), S) > 0$. Suppose that $u_{AS} + u_{SS} > 0$ and $u(\hat{A}(\alpha), \alpha) \geq 0$. Then, we can verify that $\hat{U}(S)$ is increasing, and convex with respect to A, as well as $\hat{U}(\alpha) \geq 0$ at $A = 0$. Then, we obtain that $\hat{U}(S) \geq A$ for all $A \geq 0$ because $\hat{U}'(\alpha) = u_S(\hat{A}(\alpha), \alpha) > 0$ at $A = 0$ and $\hat{U}'' \geq 0$.

differences in payoff functions between the two periods do not matter when there is no addiction, we assume that the payoff functions of gambling are identical in both periods: $v(A) \equiv u(A)$ for all $A \geq 0$.

We define by $\tilde{A}^*$ the amount bet to maximize $u(A) - fA$ over $A \geq 0$. Note that each gambler obtains the payoff of $\hat{U} \equiv \max_{A\geq 0} u(A) - fA$ in period 2 when $S = 0$. Assume also that $\overline{u} \equiv \max_{A\geq 0} u(A) < +\infty$.

We then show the following result.

Proposition 3. *(i) Suppose that gamblers are strongly addicted ($\alpha > 0$ and $\beta > 0$) such that Assumption 1 is satisfied. Then the book-bet becomes optimal for the bookmaker in period 1 ($\Pi_b \geq \Pi_c$). (ii) Suppose that gamblers are not addicted at all ($\alpha = \beta = 0$). Then, if $u(\tilde{A}^*) < 2f\tilde{A}^*$, it becomes optimal for the bookmaker to hold the cash-bet in period 1 ($\Pi_c > \Pi_b$).*

When gamblers are so strongly addicted that Assumption 1 is satisfied, the bookmaker can gain from allowing them to default without punishment initially and making them addicted in the later period. This induces gamblers to place large bets in the second period, thereby benefiting the bookmaker more than the default cost in the first period. This gives the first part of Proposition 3. This result helps understand why the bookmaker adopts the book-bet even when she faces default risks, in consistent with the evidence about illegal horse race betting in Pakistan (Mehmood and Chen (2022)): 55% of wagers at the race club are placed on credits instead of cash-in-advance bets, suggesting that credit-based betting becomes a significant component of the race club's economic system.

Regarding the second part of Proposition 3, the bookmaker faces the trade-off between the book-bet and the cash-bet as follows: first, under the book-bet gamblers can always default all debt repayments in period 1 and then quit the gambling in period 2. This "take-money-run-away" strategy can give each gambler the payoff of $\overline{u} \equiv \max_A v(A)$. Therefore, the bookmaker must leave at least this rent $\overline{u}$ to each gambler when she uses the book-bet in period 1.[17] This rent never arises under the cash-bet. Second, if gamblers have the addiction preference, there is the benefit of using the book-bet relative to the cash-bet such that they are addicted to gambling when they are allowed to default in period 1 and then subsequently bet aggressively in period 2. However, such gains disappear when gamblers are not addicted at all. Then only the benefit of the book-bet is that the allowed default can be used as a tool to compensate gamblers for inducing large amounts bet. However, such benefit is outweighed by the first two negative effects when gamblers place a relatively large bet $\tilde{A}^*$ under the cash-bet as assumed in Proposition 3 ($u(\tilde{A}^*) < 2f\tilde{A}^*$). Then, the bookmaker prefers the cash-bet to the book-bet.

Remark. Proposition 3 says nothing about the intermediate range of α and β. When gamblers' addiction preferences are moderate in that α and β are positive but not so large,

[17] This is never the case when Assumption 1 is satisfied because gamblers always avoid default and participate in the gambling in period 2 whatever amounts they bet in period 1.

we cannot give a definite answer to the question about which cash-bet or book-bet is optimal for the bookmaker. Also, if gamblers have no addiction preferences ($\alpha = \beta = 0$) but $u(\tilde{A}^*) \geq 2f\tilde{A}^*$ holds in contrary to the condition in Proposition 3, then the bookmaker may choose the book-bet even without gamblers' addictions. This is because the book-bet can be used to compensate gamblers for placing a large amount bet by allowing them to default, and such benefit exists even in the absence of addiction preferences.

5.4 Aggregate Amounts Bet with and without Addictions

Next we discuss how addictions affect the dynamic patterns of aggregate bets placed by gamblers. More specifically, we compare the equilibrium aggregate amount bet under addiction ($\alpha > 0$ and $\beta > 0$) with that under no addiction ($\alpha = \beta = 0$). We then find the following equilibrium features: (i) Gamblers' aggregate bets increase over time when they are sufficiently addicted to gambling, whereas their bets never increase over time when they are not addicted at all. (ii) Whenever the cash-bet becomes optimal without addiction preference, gamblers bet more aggressively in both periods when they are addicted than when they are not addicted at all.

These results are important to understand how the bookmaker manages the individuals' addictions over time. When individuals have the preference of addictions over gambling, the bookmaker strategically cultivates such a preference by using the 'free-first-dose' tactic to allow them to default without punishments in early periods. In the course of transactions individuals then develop the taste for gambling further, and bet more in later periods. Without addiction preferences, the bookmaker has no such strategic incentives so that individuals never increase their bets over time. In what follows we explain more detailed intuitions behind this result by using figures, although we relegate more formal analysis to the Appendix.

In Figures 5 and 6 we depict the equilibrium aggregate amounts bet per gambler with and without addictions over time. The straight lines in these figures display the equilibrium aggregate amounts bet per gambler over two periods when gamblers have the addiction preference such that $\alpha > 0$ and $\beta > 0$. They place more bet in period 2 than in period 1. This is because the book-bet allows gamblers to default and then develops the taste for gambling such that they are willing to bet more in period 2.

The dashed lines in the figures display the equilibrium aggregate amounts bet per gambler when gamblers have no addiction preferences ($\alpha = \beta = 0$). In contrast to the case of addictive gamblers, their aggregate bets weakly decrease over time: when gamblers do not develop the taste for gambling, their bets drop down in period 2. More detailed intuition is as follows. Suppose contrary to this claim that the bookmaker's optimal policy is to implement a higher amount bet in period 2 than in period 1, $A_2 > A_1$. Then, we can verify that the bookmaker can improve her payoff by deviating from the optimal policy due to the following reasons. First, if the bookmaker chooses the cash-bet in both periods, she can always implement the aggregate amount bet $\tilde{A}^*$ per gambler in each period where $\tilde{A}^*$ maximizes the payoff of each gambler $v(A) - fA$. Second, since the bookmaker must rely

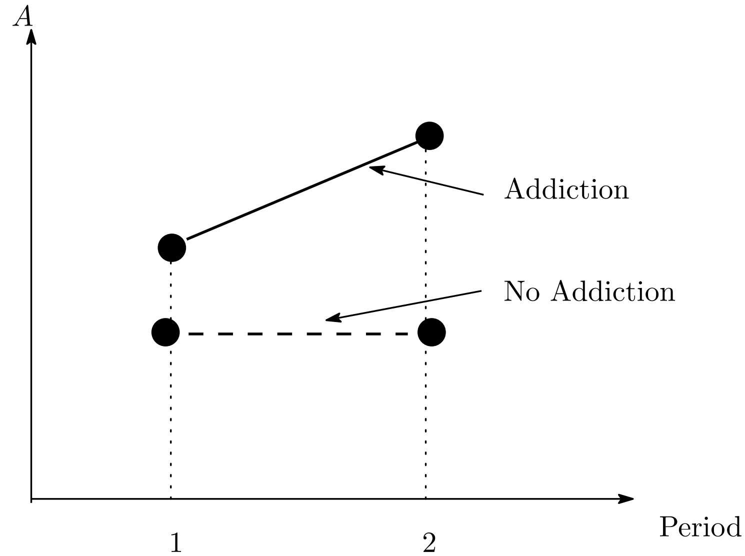

Figure 5: Aggregate amounts bet with and without addictions

only on the cash-bet in period 2, the aggregate amount bet in period 2 becomes $A_2 = \tilde{A}^*$. By using these two facts and our supposition that $A_2 > A_1$, gamblers must bet a lower amount than $\tilde{A}^*$ in period 1 ($A_2 = \tilde{A}^* > A_1$). However, if this is the case, the bookmaker can increase gambler's bet up to $\tilde{A}^*$ in period 1 by adopting the cash-bet in period 1. Since this can improve the bookmaker's payoff, our original supposition that $A_2 > A_1$ is not true. Therefore, it must be that gamblers place larger bet in period 1 than in period 2 in the equilibrium that maximizes the bookmaker's payoff, when gamblers have no addiction preferences.

In particular, when the cash-bet becomes optimal for the bookmaker to choose in period 1, the equilibrium aggregate amount bet becomes constant at $A_1 = A_2 = \tilde{A}^*$, when gamblers are never addicted ($\alpha = \beta = 0$). As shown in Proposition 3, this is the case when $u(\tilde{A}^*) < 2f\tilde{A}^*$ is satisfied (see Figure 5). On the contrary, when the opposite inequality $u(\tilde{A}^*) \geq 2f\tilde{A}^*$ holds, the aggregate amounts bet without addictions may not be constant over time because the book-bet may be optimal even without addiction preference. However, as we already discussed, even in such case the aggregate amounts bet never increase over time (Figure 6).

5.5 Addiction and Default

The addiction preferences of individuals affect their incentive to default on payback. As shown in Propositions 2 and 3, the optimal policy of the bookmaker becomes the free-first-dose strategy such that individuals develop the taste for addictions by placing large bets with defaults in period 1. Because the bookmaker's profits decrease with the defaulted amount, the gambling market works less effectively if individuals default more. Therefore,

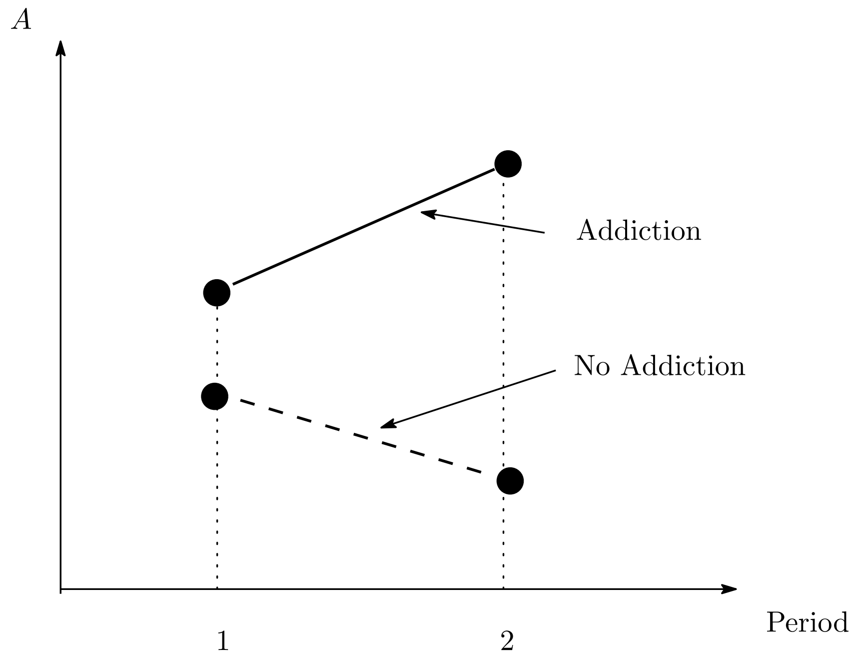

Figure 6: Aggregate amounts bet with and without addictions

it is important to understand how individuals' addiction preferences interact with their default incentives.

As we mentioned, β is the key parameter of the model capturing the speed of addiction about how likely the amount bet in period 1, $A_{i,1}$, increases the addiction stock, $S_i = \alpha + \beta A_{i,1}$. We consider the optimal punishment policy and the associated equilibrium obtained in Proposition 3: (i) the bookmaker chooses the optimal punishment probability $\tilde{q}$ to maximize her payoff $\Pi(\tilde{q})$ subject to the feasibility (F) constraint, together with the maximum forgiveness $\tilde{D}(z, A)$; (ii) each gambler places the equilibrium bet $\mathbf{a}^*$ and then defaults the amount equal to the maximum forgiveness $\tilde{D}(z^*, A^*)$ in period 1, where $z^* \equiv A^* - h_j^* a_j^* = fA^*$ is the debt each gambler owes to the bookmaker, and A^* is the aggregate amount bet by each gambler in period 1. Then each gambler follows betting $\hat{A}(S^*)$ in total in period 2 where $S^* \equiv \alpha + \beta A^*$.

We then define the *default ratio* conditional on each horse wining the race, as follows:

$$DR \equiv \tilde{D}(z^*, A^*)/z^*. \tag{19}$$

DR measures how much gamblers default relative to the debt z^* required to pay back to the bookmaker.

Suppose that the speed of addictions β is large. Then gamblers place larger bets when they are addicted more to gambling in period 1. This increases the amount bet A^* in equilibrium, so gamblers owe larger debts $z^* = fA^*$. Accordingly, gamblers default more when they bet more in period 1. However, when gamblers are addicted more as β is larger, they obtain a larger second-period payoff $\hat{U}(S^*)$, and are more eager to participate in the gambling in period 2. Note that the defaulted amount $\tilde{D}(z^*, A^*)$ is used to compensate gamblers for betting large amounts in period 1. Then the bookmaker does not need to compensate gamblers more when they gain more from the second-period gambling. This can reduce the defaulted amounts relative to the debts owed in period 1. Therefore, the default ratio DR may decrease with the speed of addiction β. Although the general result is difficult to obtain without further specifications of the model, we provide the example in the Appendix to show that DR is decreasing in some β.

6 Endogenous Commitment by the Bookmaker

We have so far assumed that the bookmaker can commit to the punishment policy announced in the beginning of period 1 as well as promised payouts in both period 1 and period 2. As we discussed, this assumption was made to simplify the analysis by paying our main focus to default incentives of only gamblers but not the bookmaker.

In this section we provide the extension of the basic model such that the bookmaker endogenously self-enforces promised payouts and punishment policies. Then we view the two-period model presented so far as a short-cut of the dynamic equilibrium in which the bookmaker's payouts and punishment policy are self-enforcing. Since the full analysis of the dynamic model is complicated, we relegate its details to the Appendix and discuss only its core idea in this section.

We consider the following dynamic extension of the two-period model. Time is discrete and is extended over infinity $t = 1, 2, ...$ The bookmaker is a long-lived player who lives forever over time while gamblers are short-lived players who live only for one period. For example, the bookmaker is an organization which runs the horse race betting over time, so it is the going-concern over infinite periods.

To maintain the consistency with the two-period model, we suppose that each period t is divided into two sub-periods, which we call "sub-period 1" and "sub-period 2", as follows. Sub-period 1 corresponds to the game played as "period 1" in the basic model while sub-period 2 corresponds to the game played as "period 2" in the basic model respectively. More specifically, in the beginning of period t, new gamblers are born and enter the economy. Then these gamblers and the bookmaker play the "two-period game" in sub-period 1 and sub-period 2 within period t in the same way as they do in the basic model. In the end of period t these gamblers leave the economy and are replaced by new gamblers.

The information structure of this dynamic game is as follows. For each gambler i born in period t, there is a corresponding gambler born in next period $t+1$, who can observe the events happened between the bookmaker and gambler i in period t. We call such gambler *successor* of gambler i, denoted by $\phi(i)$. This information structure can be used to construct the punishment equilibrium in which the bookmaker will lose her payoffs from future gambling if she deviates against gambler i in the current period, for example, by reneging on payouts to him: when such deviation happens, gambler i and all his successors will punish the bookmaker by terminating the relationships with her. This punishment is possible because the information about the deviation made by the bookmaker against i in period t will be transmitted over all i's successors directly or indirectly in the future.

We now discuss the equilibrium in the above dynamic game in which the bookmaker implements an aggregate amount bet A from each gambler in sub-period 1 and the corresponding amount bet $\hat{A}(S)$ from each gambler in sub-period 2 within each period. In such an equilibrium the bookmaker obtains the payoff of $\Pi(\tilde{q})$ in each period, defined as (16) in the basic model. The important difference from the basic model is that, in addition to the feasibility (F) constraint $v(A) + \hat{U}(S) \geq U^*$ as we imposed in the basic model, we also need to take into account the following constraint, which we call *dynamic enforcement* (DE) constraint:

$$\frac{\delta \Pi(\tilde{q})}{1-\delta} \geq (1-f)\hat{A}(S) \qquad \text{(DE)}$$

where $S = \alpha + \beta A$ is the addiction stock per gambler and $\delta \in [0, 1)$ is the discount factor of the bookmaker across two successive periods.[18]

To understand the DE constraint, we consider the equilibrium in which each gambler places the bet of A in total under the book-bet in sub-period 1 and $\hat{A}(S)$ in total under the cash-bet in sub-period 2 within each period. Also, each gambler places the equilibrium bet of $\hat{a}_j$ on horse j under the cash-bet in sub-period 2, resulting in the equilibrium odds $\hat{h}_j \equiv (1-f)/p_j$ in sub-period 2. Then the bookmaker is required to give the payout of $\hat{h}_j \hat{a}_j = (1-f)\hat{A}(S)$ to each gambler i conditional on horse j wining the race in sub-period 2. Since the payout is not formally enforced, the bookmaker must be given the incentive not to renege on the equilibrium payout of $\hat{h}_j \hat{a}_j = (1-f)\hat{A}(S)$. If the bookmaker deviates from this payout against gambler i today, she can save that amount today while she will be punished by gambler i's successors who will terminate the relationships with her in the future, thereby losing the future discounted value of payoffs $\delta\Pi(\tilde{q})/(1-\delta)$. The DE constraint ensures that the bookmaker has no incentives to make such deviation against any gambler in each period.[19]

In the Appendix we show that there exists an equilibrium in the above dynamic game such that (i) the bookmaker follows promised payouts and punishment policy $\{\tilde{q}, \tilde{D}\}$ as those considered in the basic model, and (ii) she implements the aggregate amount bet A from each gambler in order to maximize her payoff $\Pi(\tilde{q})$ subject to the feasibility (F)

[18] We assume no time discounting between two sub-periods within each period.

[19] There are also two other possible deviations by the bookmaker: she may renege on the net payout $h_j a_{i,j} - A_i$ under the book-bet in sub-period 1 and may not exclude those who defaulted in sub-period 1. However, in the Appendix we will show that these deviations do not constrain the bookmaker's payoffs. The reasons for this are as follows. First, gamblers owe debts but do not receive positive net payouts under the book-bet in equilibrium, so there are no rooms for the bookmaker to renege on net payouts under the book-bet. Second, if the bookmaker does not implement the announced punishment policy against some gamblers in sub-period 1, she will be immediately punished by these gamblers and their successors who will quit the bookmaker.

constraint and the DE constraint given above.

One important implication obtained from this extension is that addiction preferences might help self-enforce relational contract agreements between the bookmaker and gamblers over time. The parameter of the model capturing the addiction preference is given by $\beta > 0$ that determines how likely/quickly gamblers are addicted to gambling over time. Here, the increase in β has two effects: first, as β increases, gamblers accumulate larger addiction stock $S_i = \alpha + \beta A_i$, so they place larger bets $\hat{A}(S_i)$ under the cash-bet in sub-period 2 within each period. This increases the payouts that the bookmaker must commit to give gamblers, raising the right hand side of the DE constraint and hence making her commitment more difficult. Second, as β increases, the bookmaker can exploit larger profits from future gamblers who will accumulate larger addiction stock and then bet more. This raises the left hand side of the DE constraint, making the bookmaker's commitment easier. In the Appendix we show that the latter effect dominates the former effect when the bookmaker is sufficiently patient, so the bookmaker can more easily self-enforce promised payouts when gamblers' addictions become stronger. This result suggests that individuals' addictions can *complement* relational contract agreements, implying that informal markets where goods/services are traded without formal contracts can work more effectively even without well-functioning institutions when individuals are addicted more by consuming goods/services. This implication is useful to understand how and why illegal markets such as illegal drugs and illegal wagering markets work efficiently even without formal enforcement.

7 Applications to Other Illegal Markets

In this section we show that our results can include a broad range of applications to understand how illegal markets work beyond the specific setting of illegal horse race betting. More specifically, we discuss how our model can be applied to any illegal markets, such as illegal drugs, where sellers sell addictive goods to buyers who form addiction preferences over time, and their transactions are illegal and are hence not formally enforceable. Therefore our theory is not restricted to the specific environment of horse race betting.

For this purpose, we make the following interpretations of the basic model of horse race betting: first, the bookmaker and gamblers are interpreted as a "seller" and "buyers" respectively, where the seller sells an addictive good to buyers. Second, the amount bet $A_{i,t}$ by gambler i in period $t = 1, 2$ is interpreted as consumption level of the addictive good by buyer i in period $t = 1, 2$. Buyers decide how much to consume the addictive good in each period, and are addicted to consuming the good, which we capture by the addiction stock $S_i = \alpha + \beta A_{i,1}$ again. Third, the seller has two formats to sell the addictive good: one is to require buyers to immediately pay when they purchase the good. The other is to allow buyers to defer payments later. The former corresponds to the cash-bet format while the latter corresponds to the book-bet format respectively as we considered in the model of horse race betting. Fourth, we interpret the commission fee of the bookmaker $f > 0$ as the price of the addictive good, which are assumed to be exogenous.[20]

Given the above interpretations of the model, we define buyer i's payoff in period 2 as $u(A_{i,2}, S_i) - fA_{i,2}$ where he consumes $A_{i,2}$ of the addictive good at the price of f, and pays the seller $fA_{i,2}$ in terms of the numèraire good. Then buyer i chooses the consumption level of the addictive good $A_{i,2}$ to satisfy the first-order optimality condition as follows

$$u_A(A_{i,2}, S_i) - f = 0$$

given his addiction stock S_i. This implies that the optimal consumption level of the addictive good is equivalent to the optimal amount bet obtained in the basic model, $\hat{A}(S_i)$. Then buyer i obtains the equilibrium payoff of $\hat{U}(S_i)$ in period 2 as defined in the basic model.

In period 1 buyer i decides how much to consume the addictive good $A_{i,1}$ at the price of f and decides how much to default $D_i \in [0, fA_{i,1}]$ where he is supposed to pay $fA_{i,1}$ to the seller. The seller can use a punishment policy q such that each buyer will be excluded from purchasing the good in period 2 with probability $q(D, A) \in [0, 1]$. The exclusion probability q is contingent on how much to default $D \in [0, fA]$ and how much to owe the debt fA. Given a punishment policy $q(D_i, A_{i,1}) \in [0, 1]$, buyer i's payoff in period 1 is given by

$$v(A_{i,1}) - fA_{i,1} + D_i + (1 - q(D_i, A_{i,1}))\hat{U}(S_i).$$

Here, buyer i obtains the utility $v(A_{i,1})$ by consuming $A_{i,1}$ of the addictive good and pays the seller $fA_{i,1} - D_i$ when he defaults $D_i \in [0, fA_{i,1}]$, in which case he will be excluded from purchasing the good in the next period with probability $q(D_i, A_{i,1}) \in [0, 1]$ and lose the second-period payoff $\hat{U}(S_i)$.

Then we can verify that the seller can replace any punishment policy q by the bang-bang punishment policy $\tilde{q}$ such that buyer i is never punished unless he defaults more than the maximum forgiveness $\tilde{D}(A)$ while he is most harshly punished otherwise. To see this, we denote by $D^*(A)$ the optimal default given the consumption of A in period 1; $D^*(A)$ maximizes the buyer's ex post payoff $-fA + D + (1 - q(D, A))\hat{U}(S)$ over $D \in [0, fA]$. Then we define the maximum forgiveness as

$$\tilde{D}(A) \equiv D^*(A) - q(D^*(A), A)\hat{U}(S)$$

for $A \geq 0$. We can then obtain the bang-bang punishment policy as follows:

$$\tilde{q}(D, A) = \begin{cases} 0 & \text{if } D \leq \tilde{D}(A) \\ 1 & \text{otherwise} \end{cases}$$

[20] For example, each seller of illegal drugs is so small relative to its market size that she has no market powers to control these prices. Or the prices of illegal goods may be fixed by customs/traditions prevailing in black markets.

By the same step as in Proposition 2, we can verify that the seller can be weakly better off by the bang-bang punishment policy upon any other punishment policy. Given this result, the propositions we have so far obtained still remain valid in a broad range of illegal markets where sellers sell addictive goods to buyers who form addiction preferences over time, and transactions of the goods are illegal and are hence not formally enforceable.

8 Case studies

In this section, we discuss several case studies. We first discuss illegal horse betting, a case that closely aligns with the formal model. We then turn to illegal drugs and religious sects.

8.1 Illegal Horse Betting in Pakistan

Gathering data from markets prone to addiction and weak contract enforcement poses significant challenges due to the clandestine nature of these transactions, which are deliberately concealed to evade legal penalties. The first case study we present focuses on the underground horse race betting market in Pakistan, where Mehmood and Chen (2022) collected data, which we discuss below in light of the main predictions of the model.

The horse races take place every Sunday from noon to 6 pm, with races scheduled every 30 minutes. Gambling takes place at betting stations inside the premises of the race club. The betting stations at the race club. The entry at the club requires a ticket of PKR 500 (USD 2.25), with anyone who has a ticket allowed entry into the club and, by default, the ability to bet at any of the 12 betting stations that issue identical odds. Every station charges a constant 5% participation fee, and gamblers are allowed to bet on credit. Below, we discuss our three main theoretical results in light of available data.

Free-first dose effect. In their examination of the horse race betting market, Mehmood and Chen (2022) leave an intriguing puzzle unaddressed: the policy allowing patrons to place credit bets of up to PKR 5000, or roughly USD 20, upon entry to the race club. The economic rationale for such an institutional arrangement is not immediately apparent, raising questions about its profitability. Approximately 55% of wagers at the race club are placed on credit, as opposed to cash-in-advance bets, suggesting that credit betting constitutes a significant component of the race club's economic ecosystem. Our model offers a simple explanation. Offering bets on credit cultivates gamblers' addiction, ensuring a steady stream of future revenues.

Debt repayment. Mehmood and Chen (2022) find that on average, 35% of gamblers do not repay their debt in full. Our model explains this surprising feature as well. Indeed, we find (Proposition 2) that bookmakers allow a certain degree of default on the equilibrium path. That way, gamblers can place larger bets, so their addiction becomes stronger.

Violence. Mehmood and Chen (2022) collected data about perceived violence from betters. Their findings reveal that such occurrences are exceedingly rare. To be specific, a mere 0.5% of surveyed gamblers reported any apprehension of encountering violence in case of non-payment situations. This empirical observation is in line with anecdotal evidence, as exemplified by recent ethnographic research. For instance, during interviews, Paa'h Sadiq, a prominent bookie and key informant, expressed astonishment at the mere suggestion of violence in his line of work. He aptly countered, "Do I look like Amresh Puri [famous Indian actor, notably known for villainous roles]? You guys see too many gangster films. Gambling debts are debts of honor. If I resort to violence, I lose honor and the [very] right to collect debts." (Mahar, 2022, p.5) These testimonies underscore the exceptional rarity of violence associated with unpaid gambling debts within this betting market. These evidence are also in line with the predictions of the model. We find that while violence might enforce more prompt debt repayments, it could simultaneously weaken the ability of book-makers to cultivate addictions.

8.2 Illegal Drugs

Illegal drug markets are among the most elusive yet economically significant sectors, marked by their adaptability and resilience. While empirical data on these markets are scarce due to the illicit and secretive nature of transactions, qualitative evidence provides insights into their operational dynamics and the strategies employed by participants.

Free-first dose effect. The strategy of offering the first dose for free or at a significantly reduced price to potential new users is a well-known method in drug markets, aimed at fostering dependency. Another illustration of the free-first dose effect is through drug dealers' incentive to dilute ("cut") the products they sell. This dilution is unobservable to buyers until after they consume, and thus creates a moral hazard issue (Galenianos and Gavazza (2017), Galenianos, Pacula and Persico (2012)). Refraining from cutting on drugs can be seen as a particular example of the free-first dose effect. Indeed, on the one hand, offering high-quality products to new consumers is costly for drug dealers. On the other hand, offering high-quality products cultivates stronger addictions. Galenianos and Gavazza (2017) estimate a model using data on the crack cocaine market in the United States. Their estimation reveals that although they are short-lived, relationships between buyers and sellers are valuable to sellers, as regular buyers consume more frequently and account for the vast majority of crack cocaine purchases. Galenianos and Gavazza (2017) rely on the STRIDE dataset, which contains records of acquisitions of illegal drugs by undercover agents and DEA informants. Hence, the data may contain more specifically information about purchases from "new" consumers. Consistent with our model, Galenianos and Gavazza (2017) find that drug dealers do not necessarily cut on quality in the STIDE dataset, suggesting that creating relations and cultivating addictions might be valuable.

Debt repayment. Credit plays an important part in drug distribution, acting as a pivotal mechanism for sustaining and expanding consumer bases in environments where immediate payment may not be an option. This reliance on credit not only facilitates transactions but also embeds a level of trust and dependency between dealers and users (Jacobs and Wright (2006)). This aspect of illegal drug markets is consistent with our model, where offering addictive goods on credit strengthens addiction and future transactions.

Violence. The role of violence in drug markets is multifaceted, serving both as a tool for enforcement and as a potential deterrent to the stability of these markets. The strategic use of violence, or the deliberate avoidance thereof, is a critical consideration for drug dealers who must balance the immediate benefits of enforcing payment and loyalty against the long-term consequences of scaring away customers or attracting law enforcement attention. In their analysis of the heroin market of New York city, Curtis and Wendel (2000) illustrate how violence is strategically used or avoided to maintain market stability and customer loyalty. The authors suggest that while violence can be an effective means of debt collection, it can also undermine the very foundation of trust and repeat business upon which these markets rely.

8.3 Religious Sects

Religious sects, particularly those with more exclusive or intense commitment requirements, can sometimes exhibit dynamics similar to the patterns of addiction and enforcement seen in the contexts of gambling and drug markets. The initial engagement with these groups is often marked by a welcoming atmosphere and various forms of support, which can be seen as analogous to the "free-first dose" effect. Over time, the deepening of commitment can introduce elements of indebtedness to maintain consumption and addiction.

Free-first dose effect. Many religious sects initially offer new members a sense of belonging, community, and sometimes tangible benefits such as meals, social support, or even housing. This welcoming approach serves to attract individuals seeking community or spiritual fulfillment, providing an initial "dose" of the benefits of membership with little to no upfront cost. This phase can be critical for building attachment to the group and its beliefs (Dawson (1998)).

Debt repayment. As members become more integrated into the sect, they may feel an increasing obligation to contribute financially, dedicate time, or engage in proselytizing activities. This sense of obligation can be akin to "debt repayment," where the perceived debt is not just financial but also emotional or spiritual. Members might believe that their salvation, enlightenment, or the well-being of their community depends on their contributions. In some cases, the failure to meet these expectations can lead to feelings of guilt or indebtedness, further binding members to the group as they strive to "repay" their perceived debt.

Violence. While physical violence is rare within the vast majority of religious sects, subtler forms of coercion, such as psychological pressures or social ostracism, may be employed by more controlling groups to ensure adherence to their norms. The complexity and diversity of these coercive practices, especially when balanced against the need for voluntary and genuine commitment, present challenges that our model might not fully capture.

In the economic literature on religion, Iannaccone (1992) argued that costly practices within religious cults allow for the screening of free-riders and increase the overall benefits for group members. Our model complements Iannaccone (1992), suggesting that addiction might be a channel through which costly practices in religious groups are self-enforced. Additionally, in a world of increasing religious pluralism, cultivating an addiction to a specific cult might also be a way to decrease group members' outside options. Hence, our work also connects to the literature on religious competition, suggesting an explanation for the persistence of multiple sects.[21]

9 Conclusion

In this study, we introduce a dynamic model to analyze addiction dynamics in markets lacking formal enforcement mechanisms. Our model delineates how sellers employ a 'free-first-dose' strategy to foster addiction, thereby establishing a reliable consumer base. Key to our findings is the strategic tolerance of non-repayment by sellers, which, paradoxically, deepens addiction and customer loyalty. Contrary to prevalent views, our results suggest that early-stage violence and coercion may detrimentally affect addiction cultivation, offering new insights into the operation of illegal markets.

Understanding of the economic and psychological forces at play suggests novel policies for effective regulation of non-legal markets. Targeting the 'free-first-dose' strategy could significantly disrupt the cycle of addiction, curtailing the pathway to long-term dependency. Surprisingly, protecting sellers against defaults may alter market dynamics.

References

Allcott, Hunt, Matthew Gentzkow and Lena Song. 2022. "Digital Addiction." American Economic Review 112(7):2424–63.

Becker, Gary S. and Kevin M. Murphy. 1988. "A Theory of Rational Addiction." Journal of Political Economy 96(4):675–700.

Becker, Gary S., Michael Grossman and Kevin M. Murphy. 1994. "An Empirical Analysis of Cigarette Addiction." The American Economic Review 84(3):396–418.

Chaloupka, Frank. 1991. "Rational Addictive Behavior and Cigarette Smoking." Journal of Political Economy 99(4):722–42.

Chaloupka, Frank J. and Kenneth E. Warner. 2000. "The Economics of Smoking." Handbook of Health Economics 1:1539–1627.

Chiappori, Pierre-André, Bernard Salanié, Francçis Salanié and Amit Gandhi. 2019. "From Aggregate Betting Data to Individual Risk Preferences." Econometrica 87(1):1–36.

Conlisk, John. 1993. "The Utility of Gambling." Journal of Risk and Uncertainty 6:255–275.

[21] On the literature on religious markets, see, among others, McBride (2008) and McBride (2010).

Courtemanche, Charles, Garth Heutel and Patrick McAlvanah. 2014. "Impatience, Incentives and Obesity." The Economic Journal 125(582):1–31.

Curtis, Ric and Travis Wendel. 2000. Toward the development of a typology of illegal drug markets. Criminal Justice Press pp. 121–152.

Dawson, Lorne L. 1998. Comprehending Cults: The Sociology of New Religious Movements. Oxford University Press.

Driskill, Robert and Stephen McCafferty. 2001. "Monopoly and Oligopoly Provision of Addictive Goods." International Economic Review 42(1):43–72.

Feess, Eberhard, Helge Müller and Christoph Schumacher. 2016. "Estimating risk preferences of bettors with different bet sizes." European Journal of Operational Research 249(3):1102–1112.

Fethke, Gary and Raj Jagannathan. 1996. "Habit persistence, heterogeneous tastes, and imperfect competition." Journal of Economic Dynamics and Control 20(6):1193–1207.

Galenianos, Manolis and Alessandro Gavazza. 2017. "A Structural Model of the Retail Market for Illicit Drugs." American Economic Review 107(3):858–896.

Galenianos, Manolis, Rosalie Liccardo Pacula and Nicola Persico. 2012. "A Search-Theoretic Model of the Retail Market for Illicit Drugs." Review of Economic Studies 79(3):1239–1269.

Grossman, Michael and Frank J. Chaloupka. 2017. The Demand for Cocaine by Young Adults: A Rational Addiction Approach. New York Chichester, West Sussex: Columbia University Press pp. 620–665.

Gruber, Jonathan and Botond Koszegi. 2001. "Is Addiction "Rational"? Theory and Evidence." Quarterly Journal of Economics 116(4):1261–1303.

Hoch, Stephen J. and George Loewenstein. 1991. "Time-Inconsistent Preferences and Consumer Self-Control." Journal of Consumer Research 17(4):492–507.

Iannaccone, Laurence R. 1992. "Sacrifice and stigma: Reducing free-riding in cults, communes, and other collectives." Journal of political economy 100(2):271–291.

Jacobs, B.A. and R. Wright. 2006. Street Justice: Retaliation in the Criminal Underworld. Cambridge Studies in Criminology Cambridge University Press.

Jullien, Bruno and Bernard Salanié. 2000. "Estimating Preferences under Risk: The Case of Racetrack Bettors." Journal of Political Economy 102(3):503–530.

Levin, Jonathan. 2003. "Relational Incentive Contracts." American Economic Review 93(3):835–857.

Levitt, Steven D. 2004. "Why are Gambling Markets Organised so Differently from Financial Markets?" The Economic Journal 114(495):223–246.

Losak, Jeremy M., Andrew P. Weinbach and Rodney J. Paul. 2023. "Behavioral Biases in Daily Fantasy Baseball: The Case of the Hot Hand." Journal of Sports Economics 24(3):374–401.

Malcomson, James. 2012. Chapter 25 - Relational Incentive Contracts. In Handbook of Organizational Economics, ed. John Roberts Roberts Gibbons. Vol. 1 of Handbook of Organizational Economics North-Holland pp. 1014–1065.

McBride, Michael. 2008. "Religious Pluralism and Religious Participation: A Game Theoretic Analysis." American Journal of Sociology 114(1):77–106.

McBride, Michael. 2010. "Religious Market Competition in a Richer World." Economica 77(305):148–171.

Mehmood, Sultan and Daniel L Chen. 2022. "Contract Enforcement in a Stateless Economy." Center for Research on Economic Development Working Paper Series 162021:12.

Orphanides, Athanasios and David Zervos. 1995. "Rational Addiction with Learning and Regret." Journal of Political Economy 103(4):739–758.

Ottaviani, Marco and Peter Norman Sørensen. 2010. "Noise, Information, and the Favorite-Longshot Bias in Parimutuel Predictions." American Economic Journal: Microeconomics 2(1):58–85.

Schilbach, Frank et al. 2019. "Alcohol and Self-Control: A Field Experiment in India." American Economic Review 109(4):1290–1322.

Snowberg, Erik and Justin Wolfers. 2010. "Explaining the Favorite–Long Shot Bias: Is it Risk-Love or Misperceptions?" Journal of Political Economy 118(4):723–746.

United Nations Office on Drugs and Crime. 2021. "2021 Report on Safeguarding Sport from Corruption." https://www.unodc.org/res/safeguardingsport/grcs/section9_html/SPORTS_CORRUPTION_2021_S9.pdf. Accessed: September 20, 2022.

10 Appendix A: Proofs

10.1 Proof of Proposition 1

In period 2 gambler i places a bet $\mathbf{a}_i$ to maximize the following expected payoff

$$u(A_i, S_i) + \sum_{j \in J} p_j \{h_j a_{i,j} - A_i\},$$

given expectations about the market odds $\{h_j\}_{j\in J}$. Then the equilibrium amount bet $\hat{a}_{i,j}$ is determined by the following first-order optimality condition:

$$u_A\left(\hat{A}_i, S_i\right) + p_j h_j - 1 = 0, \quad j \in J. \tag{A1}$$

Since (A1) holds for any $j \in J$ in equilibrium, we obtain

$$p_j h_j = p_\ell h_\ell, \quad \text{for any } j, \ell \in J.$$

By using the definitions of $\hat{a}_j \equiv \sum_{i\in I} \hat{a}_{i,j}$ and $\hat{A} \equiv \sum_{i\in I} \hat{A}_i$, we have

$$p_j(1-f)\hat{A}/\hat{a}_j = p_\ell(1-f)\hat{A}/\hat{a}_\ell,$$

so that

$$\hat{a}_j/\hat{a}_\ell = p_j/p_\ell$$

for any $j, \ell \in J$. Therefore, gamblers bet more on the horses which are more likely to win the race. Then we verify that

$$\begin{aligned}\hat{A} &= \hat{a}_j + \sum_{\ell\neq j} \hat{a}_\ell \\ &= \hat{a}_j \left(1 + \sum_{\ell\neq j} (\hat{a}_\ell/\hat{a}_j)\right) \\ &= \hat{a}_j \left(1 + \sum_{\ell\neq j} (p_\ell/p_j)\right)\end{aligned}$$

so that gamblers place the bet equal to

$$\hat{a}_j = \frac{\hat{A}}{1 + \sum_{\ell\neq j}(p_\ell/p_j)}$$

on horse $j \in J$ in total in equilibrium. Note that the amount bet by each gambler on each horse $\hat{a}_{i,j}$ is indeterminate. Only the aggregate amount $\hat{a}_j$ bet by all gamblers on horse $j \in \hat{J}$ is determined.

Since $h_j = (1-f)\hat{A}/\hat{a}_j$ holds for $j \in J$ in equilibrium, we substitute $\hat{a}_j$ into h_j in order to obtain the following:

$$p_j h_j = (1-f), \quad \text{for } j \in J.$$

Therefore, the equilibrium amount bet $\hat{A}_i$, given by (A1) above, is determined by the following condition:

$$u_A(\hat{A}_i, S_i) - f = 0, \quad \text{for } i \in I$$

yielding the desired result (7) in the main text. This gives the equilibrium amount $\hat{A}_i = \hat{A}(S_i)$ bet by gambler i in total, which depends on his addiction stock S_i.

As a result, we can write the equilibrium second-payoff of gambler i who has the addiction stock S_i as follows

$$\begin{aligned}\hat{U}(S_i) &\equiv u(\hat{A}(S_i), S_i) + \sum_{j\in J} p_j h_j \hat{a}_{i,j} - \hat{A}(S_i) \\ &= u(\hat{A}(S_i), S_i) + (1-f)\sum_{j\in J} \hat{a}_{i,j} - \hat{A}(S_i) \\ &= u(\hat{A}(S_i), S_i) + (1-f)\hat{A}(S_i) - \hat{A}(S_i) \\ &= u(\hat{A}(S_i), S_i) - f\hat{A}(S_i)\end{aligned}$$

yielding the desired result (8) in the main text.

10.2 Proof of Proposition 2

First we show the following lemma.

Lemma A1. *$D^*(z_{i,j}, A_i) \geq q(D^*(z_{i,j}, A_i), A_i)\hat{U}(S_i)$ for any $A_i \geq 0$.*

Proof. Each gambler can always decide not to default, ensuring the full participation in the gambling in period 2. In that case gambler i can obtain the equilibrium payoff of $\hat{U}(S_i)$ in period 2. Then, since gambler i optimally decides how much to default given his debt $z_{i,j} \geq 0$,[22] we must have

$$D^*(z_{i,j}, A_i) + (1 - q(D^*(z_{i,j}, A_i), A_i))\hat{U}(S_i) \geq \hat{U}(S_i)$$

yielding the desired result $D^*(z_{i,j}, A_i) \geq q(D^*(z_{i,j}, A_i), A_i)\hat{U}(S_i)$. Q.E.D.

We now move to the proof of Proposition 2.

Consider any punishment policy q and the resulting equilibrium amount bet $\mathbf{a}_i^* \equiv (a_{i,1}^*, ..., a_{i,m}^*)$ of gambler $i \in I$ in period 1. The associated equilibrium odds in period 1 are given by $\{h_j^*\}_{j\in J}$ such that $h_j^* = (1-f)\sum_{i\in I} A_i^* / \sum_{i\in I} a_{i,j}^*$.[23] Let $A_i^* \equiv \sum_{j\in J} a_{i,j}^*$ denote the aggregate amount bet by gambler i in the equilibrium. We also denote by $D^*(z, A) \in [0, z]$ the defaulted amount that solves

$$\max_{0\leq D\leq z} \; D + (1 - q(D, A))\hat{U}(S)$$

[22] When $z_{i,j} = 0$, it must be that $D = 0$, and hence $q(D, A_i) = q(0, A_i) = 0$. Then the statement of Lemma A1 holds trivially.

[23] Note that we are considering the equilibrium in which every horse has positive odds, that is, $\sum_{i\in I} a_{i,j}^* > 0$.

given z, and $S = \alpha + \beta A$. Note that the functional form of $D^*(\cdot,\cdot)$ is the same for all gamblers because all of them face the same functional forms of $q(\cdot,\cdot)$ and $\hat{U}(\cdot)$. However, the realized amount of default $D^*(z_{i,j}, A_i)$ can differ across gamblers because they may face the different race outcomes $z_{i,j}$ and place different bets $\mathbf{a}_i$.

Gambler i places bet $\mathbf{a}_i$ to maximize the following expected payoff in period 1:

$$v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + D^*(z_{i,j}, A_i) + (1 - q(D^*(z_{i,j}, A_i), A_i))\hat{U}(S_i) \qquad \text{(A2)}$$

given the equilibrium odds $\{h_j^*\}_{j \in J}$.

Define

$$\tilde{D}(z, A) \equiv D^*(z, A) - q(D^*(z, A), A)\hat{U}(S_i),$$

where $\tilde{D}(z, A) \geq 0$ due to Lemma A1, and $\tilde{D}(z, A) \leq z$ because $D^*(z, A) \leq z$, and $\hat{U}(S_i) \geq 0$. Note that, when $z = 0$, we have $D^*(z, A) = 0$ and $q(D^*(z, A), A) = q(0, A) = 0$, so that $\tilde{D}(0, A) = 0$. Then we define the following new punishment policy:

$$\tilde{q}(D, A) \equiv \begin{cases} 0 & \text{if } D \leq \tilde{D}(z, A) \\ 1 & \text{otherwise} \end{cases}$$

Note that the new policy $\tilde{q}$ has the bang-bang form.

We show that the new policy can induce gamblers to place the same bets as those under the original policy q while reducing defaulted amounts.

First, gamblers never choose $D < \tilde{D}(z, A)$: if so, they can slightly increase D while keeping $D \leq \tilde{D}(z, A)$ so that they can still certainly participate in the gambling in period 2. This deviation can strictly improve their payoffs. Therefore, we consider only the case that $D \geq \tilde{D}(z, A)$ for any $j \in J$. Suppose that gambler i chooses $\mathbf{a}_i = (a_{i,1}, ..., a_{i,m})$ and D_{ij} where $D_{ij} \geq \tilde{D}(z_{i,j}, A_i)$ for each $j \in J$. Let

$$J_i^* \equiv \{j \in J \mid D_{ij} = D^*(z_{i,j}, A_i)\}.$$

Here, $\tilde{q}(D_{ij}, A_i) = 1$ for any $j \notin J_i^*$ while $\tilde{q}(D_{ij}, A_i) = 0$ for any $j \in J_i^*$. Let denote $z_{i,j} \equiv \max\{A_i - h_j a_{i,j} 0\}$ and $z_{i,j}^* \equiv \max\{A_i^* - h_j a_{i,j}^*, 0\}$. Let also denote $S_i^* \equiv \alpha + \beta A_i^*$.

Then we can show that

$$
\begin{aligned}
& v(A_i) + \sum_{j \in J_i^*} p_j \{h_j^* a_{i,j} - A_i + \tilde{D}(z_{i,j}, A_i) + \hat{U}(S_i)\} \\
& \quad + \sum_{j \notin J_i^*} p_j \{h_j^* a_{i,j} - A_i + D_{i,j}\} \\
&= v(A_i) + \sum_{j \in J_i^*} p_j \{h_j^* a_{i,j} - A_i + D^*(z_{i,j}, A_i) + (1 - q(D^*(z_{i,j}, A_i), A_i))\hat{U}(S_i)\} \\
& \quad + \sum_{j \notin J_i^*} p_j \{h_j^* a_{i,j} - A_i + D_{ij}\} \\
&\le v(A_i) + \sum_{j \in J_i^*} p_j \{h_j^* a_{i,j} - A_i + D^*(z_{i,j}, A_i) + (1 - q(D^*(z_{i,j}, A_i), A_i))\hat{U}(S_i)\} \\
& \quad + \sum_{j \notin J_i^*} p_j \{h_j^* a_{i,j} - A_i + D_{ij} + (1 - q(D_{ij}, A_i))\hat{U}(S_i)\} \\
&\le v(A_i) + \sum_{j \in J_i^*} p_j \{h_j^* a_{i,j} - A_i + D^*(z_{i,j}, A_i) + (1 - q(D^*(z_{i,j}, A_i), A_i))\hat{U}(S_i)\} \\
& \quad + \sum_{j \notin J_i^*} p_j \{h_j^* a_{i,j} - A_i + D^*(z_{i,j}, A_i) + (1 - q(D^*(z_{i,j}, A_i), A_i))\hat{U}(S_i)\} \\
&= v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + D^*(z_{i,j}, A_i) + (1 - q(D^*(z_{i,j}, A_i), A_i))\hat{U}(S_i)\} \\
&\le \max_{\mathbf{a}_i} \ v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + D^*(z_{i,j}, A_i) + (1 - q(D^*(z_{i,j}, A_i), A_i))\hat{U}(S_i)\} \\
&= v(A_i^*) + \sum_{j \in J} p_j \{h_j^* a_{i,j}^* - A_i^* + D^*(z_{i,j}^*, A_i^*) + (1 - q(D^*(z_{i,j}^*, A_i^*), A_i^*))\hat{U}(S_i^*)\} \\
&= v(A_i^*) + \sum_{j \in J} p_j \{h_j^* a_{i,j}^* - A_i^* + \tilde{D}(z_{i,j}^*, A_i^*) + \hat{U}(S_i^*)\}
\end{aligned}
$$

where the first equality follows from the definition of $\tilde{D}(\cdot, \cdot)$, the first inequality from $\hat{U}(S_i) \geq 0$, the second inequality from the fact that $D^*(z, A)$ maximizes $D + (1 - q(D, A))\hat{U}(S)$ subject to $0 \leq D \leq z$, and the third equality from the definition of $\mathbf{a}_i^*$ that maximizes the expected payoff (A2) of gambler i given q and $\{h_j^*\}_{j \in J}$.

Therefore, given the same odds $\{h_j^*\}_{j \in J}$ as those in the original equilibrium, each gambler i bets the same amount $\mathbf{a}_i^*$ as that under the original punishment policy, and defaults the amount equal to the maximum forgiveness $\tilde{D}(z_{i,j}^*, A_i^*)$, ensuring the full participation in the second-period gambling. Thus, under the new punishment policy, these betting and default behaviors of gamblers constitute an equilibrium with the same odds $\{h_j^*\}$ as those in the original equilibrium.

The bookmaker's payoff under the original punishment policy q is given by

$$\Pi(q) \equiv \sum_{i\in I}\sum_{j\in J} p_j\{A_i^* - h_j^* a_{i,j}^* - D^*(z_{i,j}^*, A_i^*)\} + \sum_{i\in I}\sum_{j\in J} p_j(1 - q(D^*(z_{i,j}^*, A_i^*), A_i^*))f\hat{A}(S_i^*)$$

where $\hat{A}(S_i^*)$ is the aggregate amount bet by gambler i in period 2 in the original equilibrium. However, the bookmaker can be weakly better off by the new punishment policy $\tilde{q}$:

$$\begin{aligned}
\Pi(\tilde{q}) &\equiv \sum_{i\in I}\sum_{j\in J} p_j\{A_i^* - h_j^* a_{i,j}^* - \tilde{D}(z_{i,j}^*, A_i^*)\} + f\sum_{i\in I}\hat{A}(S_i^*) \\
&= \sum_{i\in I}\sum_{j\in J} p_j\{A_i^* - h_j^* a_{i,j}^* - D^*(z_{i,j}^*, A_i^*) + q(D^*(z_{i,j}^*, A_i^*), A_i^*)\hat{U}(S_i^*)\} \\
&\quad + f\sum_{i\in I}\hat{A}(S_i^*) \\
&\geq \sum_{i\in I}\sum_{j\in J} p_j\{A_i^* - h_j^* a_{i,j}^* - D^*(z_{i,j}^*, A_i^*)\} \\
&\quad + f\sum_{i\in I}\sum_{j\in J} p_j(1 - q(D^*(z_{i,j}^*, A_i^*), A_i^*))\hat{A}(S_i^*) \\
&= \Pi(q)
\end{aligned}$$

because $\hat{U}(S_i^*) \geq 0$, $\hat{A}(S_i^*) \geq 0$, and $q(D, A) \leq 1$. Q.E.D.

10.3 Proof of Lemma 1

Note that $h_j^* a_j^* - A^* = -fA^*$ holds for each wining horse $j \in J$ because $h_j^* = (1-f)A^*/a_j^*$ in equilibrium. Then $z_j^* = fA^*$ for any $j \in J$. Since each gambler can always decide not to default and can protect himself from being excluded in period 2, he can secure at least U^* so that

$$\begin{aligned}
U^* &\leq v(A^*) + \sum_{j\in J} p_j\{h_j^* a_j^* - A^* + \tilde{D}(z_j^*, A^*) + \hat{U}(S^*)\} \\
&\leq v(A^*) + \sum_{j\in J} p_j\{h_j^* a_j^* - A^* + z_j^* + \hat{U}(S^*)\} \\
&= v(A^*) + \hat{U}(S^*)
\end{aligned}$$

because of the definition about $h_j^* = (1-f)A^*/a_j^*$ for each $j \in J$ and the fact that $\tilde{D}(z_j^*, A^*) \leq z_j^*$. Q.E.D.

10.4 Proof of Proposition 3

(i) Suppose that Assumption 1 is satisfied. Then we derive the optimal bang-bang punishment policy chosen by the bookmaker under the book-bet in period 1. Also we show that the bookmaker prefers the book-bet to the cash-bet in period 1.

First, by using Lemma 1, we derive the upper bound payoff of the bookmaker as follows. Since each gambler can obtain at least U^* in equilibrium, it must be that

$$v(A^*) + \sum_{j \in J} p_j \{h_j^* a_j^* - A^* + \tilde{D}(z^*, A^*)\} + \hat{U}(S^*) \geq U^*. \tag{A3}$$

By using this, the bookmaker's payoff $\Pi(\tilde{q})$ is bounded above by

$$\begin{aligned} \Pi(\tilde{q}) &\equiv n\left\{A^* - \sum_j p_j \{h_j^* a_j^* + \tilde{D}(z^*, A^*)\}\right\} + nf\hat{A}(S^*) \\ &\leq n\Phi(A^*) \\ &\equiv n\{v(A^*) + \hat{U}(S^*) - U^*\} + nf\hat{A}(S^*) \end{aligned}$$

where

$$\Phi(A) \equiv v(A) + \hat{U}(S) - U^* + f\hat{A}(S) \tag{A4}$$

is the upper bound payoff of the bookmaker per gambler when implementing an aggregate amount bet A from each gambler in period 1.

By taking into account Lemma 1, we then consider the aggregate amount bet A^* per gambler in period 1, which solves the following problem.

Problem P:

$$\Phi^* \equiv \max_{A \geq 0} \ \Phi(A)$$

subject to $v(A) + \hat{U}(\alpha + \beta A) \geq U^*$.

The value Φ^* defined in Problem P becomes the maximum payoff for the bookmaker among all implementable amounts bet A.

First, we show that it becomes optimal for the bookmaker to hold the book-bet rather than the cash-bet in period 1, under Assumption 1. Holding the cash-bet in period 1 is equivalent to holding the book-bet in period 1 with the maximum forgiveness equal to $\tilde{D}(z, A) = 0$ for all $A \geq 0$ and all $z \geq 0$. To see this, note that under such book-bet gambler i never defaults: if gambler i defaults $D_{i,j} > 0$ when horse j wins the race, he will obtain the ex post payoff of $h_j a_{i,j} - A_i + D_{i,j}$ whereas, if he does not default, he will obtain $h_j a_{i,j} - A_i + \hat{U}(S_i)$. Since $0 < D_{i,j} \leq z_{i,j} \equiv \max\{A_i - h_j a_{i,j} 0\}$ and $-A_i + \hat{U}(S_i) > 0$ for all $A_i \geq 0$ under Assumption 1, the former payoff is smaller than the latter, $h_j a_{i,j} - A_i + D_{i,j} \leq 0 < h_j a_{i,j} - A_i + \hat{U}(S_i)$. Therefore, gambler i never defaults ex post. Given this result, gambler i bets $\mathbf{a}_i$ to maximize his expected payoff

$$v(A_i) + \sum_{j\in J} p_j\{h_j a_{i,j} - A_i\} + \hat{U}(S_i).$$

This is equivalent to the case that the bookmaker holds the cash-bet in period 1 as well as in period 2. Then the bookmaker can always choose the punishment policy under the book-bet that implements the same payoff as attained under the cash-bet. The bookmaker may be better off further by optimally choosing the punishment policy $\{\tilde{q}, \tilde{D}\}$. Therefore under Assumption 1 the bookmaker weakly prefers the book-bet to the cash-bet in period 1.

Next we look for a symmetric equilibrium in which the bookmaker can attain her upper bound payoff Φ^*. To this end, we define the equilibrium odds as

$$h_j^* = (1-f)/p_j$$

for each $j \in J$. We denote by $\mathbf{a}^* = \{a_j^*\}_{j\in J}$ the candidate for an equilibrium amount bet by each gambler where we define $a_j^* = p_j A^* > 0$ for each $j \in J$ and $h_j^* = (1-f)/p_j$ for $A^* \equiv \sum_{j\in J} a_j^*$. Note that $U^* = \max_{A\geq 0} v(A) - fA + \hat{U}(\alpha + \beta A)$.

Suppose that the aggregate amount bet A^* by each gambler in period 1 attains Φ^*. Let $A_2^* \equiv \hat{A}(S^*)$ denote the aggregate amount bet by each gambler in period 2 corresponding to $S^* \equiv \alpha + \beta A^*$.

Define the maximum forgiveness as follows: given $\mathbf{a}_i = (a_{i,1}, ..., a_{i,m})$ and $z_{i,j} \equiv \max\{A_i - h_j a_{i,j}, 0\}$,

$$\tilde{D}(z_{i,j}, A_i) \equiv \begin{cases} \min\{z_{i,j}, d\} & \text{if } A_i = A^* \\ 0 & \text{if } A_i \neq A^* \end{cases} \tag{A5}$$

where $d \equiv U^* - \{v(A^*) - fA^* + \hat{U}(S^*)\}$.

Then we consider the following bang-bang punishment policy:

$$\tilde{q}(D, A) \equiv \begin{cases} 0 & \text{if } D \leq \tilde{D}(z, A) \\ 1 & \text{otherwise} \end{cases}$$

First, suppose that gambler i follows to choose the equilibrium bet $a_{i,j} = a_j^* \equiv p_j A^*$ for each $j \in J$. Then we show that $\tilde{D}(z_j^*, A^*) \geq 0$ because $U^* = \max_A v(A) - fA + \hat{U}(S) \geq v(A^*) - fA^* + \hat{U}(S^*)$, and hence $d \geq 0$ holds. Also, when $a_{i,j} = a_j^* \equiv p_j A^*$, we have $\tilde{D}(z_j^*, A^*) \leq fA^*$ due to the fact that A^* satisfies the constraint in Problem P: $v(A^*) + \hat{U}(S^*) \geq U^*$. Therefore, if gambler i places the equilibrium bet $\mathbf{a}^* = (a_1^*, ..., a_m^*)$ and defaults $\{\tilde{D}(z_j^*, A^*)\}_{j\in J}$, he obtains the following expected payoff given the equilibrium odds $h_j^* = (1-f)/p_j$ for each $j \in J$:

$$\begin{aligned} & v(A^*) + \sum_{j \in J} p_j \{h_j^* a_j^* - A^* + \tilde{D}(z_j^*, A^*) + \hat{U}(\alpha + \beta A^*)\} \\ = \ & U^* \end{aligned}$$

that is same as the payoff gambler i would obtain if he pays back all the debt ($D_{i,j} = 0$ for all $j \in J$).

Now we check unilateral deviations by each gambler i. First, suppose that gambler i chooses $\mathbf{a}_i = (a_{i,j})_{j \in J}$ and $\{D_{i,j}\}_{j \in J}$ such that $A_i \neq A^*$. Then, the second part in the definition about the maximum forgiveness $\tilde{D}(z_{i,j}, A_i) = 0$ is applied. Gambler i then obtains

$$v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + D_{i,j} + (1 - q(D_{i,j}, A_i))\hat{U}(S_i)\}.$$

We can verify that gambler i will choose $D_j = \tilde{D}(z_{i,j}, A_i) = 0$ for all $j \in J$. To see this, suppose that $D_{i,j} > \tilde{D}(z_{i,j}, A_i) = 0$ for some $j \in J$. This implies that $D_{i,j} > 0$ and hence $z_{i,j} \equiv A_i - h_j^* a_{i,j} > 0$. Also $\tilde{q}(D_{i,j}, A_i) = 1$ must hold for $D_{i,j} > 0$. Then gambler i's returns are at most $h_j^* a_{i,j} - A_i + D_{i,j} \leq 0$ for such j whereas, if he chooses $D_{i,j} = \tilde{D}(z_{i,j}, A_i) = 0$, he will obtain $h_j^* a_{i,j} - A_i + \hat{U}(S_i) \geq 0$ due to Assumption 1. Therefore, gambler i will optimally choose $D_{i,j} = \tilde{D}(z_{i,j}, A_i) = 0$. Given this result, we show that

$$\begin{aligned} & v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + D_{i,j} + (1 - q(D_j, A_i))\hat{U}(S_i)\} \\ \leq \ & v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + \tilde{D}(z_{i,j}, A_i) + \hat{U}(S_i)\} \\ = \ & v(A_i) - fA_i + \hat{U}(S_i) \\ \leq \ & U^* \end{aligned}$$

because $h_j^* = (1-f)/p_j$ for all $j \in J$, and $\tilde{D}(z_{i,j}, A_i) = 0$ holds for all $z_{i,j}$ whenever $A_i \neq A^*$. Then each gambler cannot improve his payoff more than U^*, which he would obtain by choosing $\mathbf{a}^*$ and $\{\tilde{D}(z_j^*, A^*)\}_{j \in J}$.

Second, consider the deviation such that $\mathbf{a}_i \neq \mathbf{a}^*$ while keeping $A_i = A^*$. Then gambler i will obtain the following expected payoff:

$$v(A^*) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A^* + D_{i,j} + (1 - q(D_{i,j}, A^*))\hat{U}(S^*)\}$$

where $0 \leq D_{i,j} \leq z_{i,j} \equiv \max\{A^* - h_j^* a_{i,j}, 0\}$. Note also that we have $\tilde{D}(z_{i,j}, A^*) = \min\{z_{i,j}, d\} \leq d$. Again, since gambler i never defaults more than $\tilde{D}(z_{i,j}, A^*)$ due to Assumption 1, he will choose $D_{i,j} = \tilde{D}(z_{i,j}, A^*)$ for any $j \in J$. This implies that gambler

i's deviation payoff can be written by

$$v(A^*) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A^* + \tilde{D}(z_{i,j}, A^*) + \hat{U}(S^*)\}$$

because $\tilde{q}(\tilde{D}(z_{i,j}, A^*), A^*) = 0$. We can then re-write this payoff further as follows

$$\begin{aligned}
& v(A^*) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A^* + \tilde{D}(z_{i,j}, A^*) + \hat{U}(S^*)\} \\
\leq \ & v(A^*) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A^* + \hat{U}(S^*)\} + d \\
= \ & v(A^*) + p_1 h_1^* a_{i,1} + \sum_{j \neq 1} p_j h_j^* a_{i,j} - A^* + \hat{U}(S^*) + d \\
= \ & v(A^*) + (1-f) \left\{ A^* - \sum_{j \neq 1} a_{i,j} \right\} + (1-f) \sum_{j \neq 1} a_{i,j} - A^* + \hat{U}(S^*) + d \\
= \ & v(A^*) + \hat{U}(S^*) - \{v(A^*) + \hat{U}(S^*) - U^*\} \\
= \ & U^*
\end{aligned}$$

where the first inequality follows from $\tilde{D}(z_{i,j}, A^*) \leq d$, and the second equality from $a_{i,1} = A^* - \sum_{j \neq 1} a_{i,j}$ respectively. Therefore, each gambler i cannot improve his payoff by the deviation.

In either case each gambler i cannot gain by deviating from the equilibrium bet $\mathbf{a}^*$. Therefore, it becomes an equilibrium in which gamblers choose $\mathbf{a}^*$ and default $\tilde{D}(z_j^*, A^*)$ for each $j \in J$, giving each of them the equilibrium payoff $v(A^*) + \sum_j p_j \{h_j^* a_j^* - A^* + \tilde{D}(z_j^*, A^*) + \hat{U}(S^*)\}$ equal to U^*. Then, the bookmaker obtains the payoff

$$\begin{aligned}
n \left\{ A^* - \sum_j p_j (h_j^* a_j^* + \tilde{D}(z_j^*, A^*)) \right\} + nf\hat{A}(S^*) \ &= \ n\{v(A^*) + \hat{U}(S^*) - U^*\} + nf\hat{A}(S^*) \\
&= \ n\Phi^*
\end{aligned}$$

equal to her upper bound payoff $n\Phi^*$.

(ii) Suppose that gamblers are not addicted at all; $\alpha = \beta = 0$. Then, we show that the bookmaker prefers the cash-bet to the book-bet in period 1, provided that $u(\tilde{A}^*) < 2f\tilde{A}^*$.

Define $\overline{u} \equiv \max_{A \geq 0} u(A)$. Suppose that $\mathbf{a}^* \equiv (a_j^*)_{j \in J}$ and $A^* \equiv \sum_{j \in J} a_j^*$ are implemented in an euilibrium under a bang-bang punishment policy $\tilde{q}$. Then gambler i's equilibrium payoff is given by

$$\begin{aligned} & u(A^*) + \sum_{j \in J} p_j \{h_j^* a_j^* - A^* + \tilde{D}(z_j^*, A^*) + \hat{U}\} \\ & = \ u(A^*) - fA^* + \sum_j p_j \tilde{D}(z_j^*, A^*) + \hat{U} \end{aligned}$$

where the equilibrium odds are given by $h_j^* = (1-f)A^*/a_j^*$.

Gambler i can always choose A_i to maximize $u(A)$, and default all the amounts required to pay back, $D_{i,j} = \max\{A_i - h_j^* a_{i,j}, 0\}$, followed by quitting the gambling in period 2. Then gambler i can secure at least the payoff of $\overline{u}$. Therefore, for such a deviation to be unprofitable, it must be that

$$u(A^*) - fA^* + \sum_j p_j \tilde{D}(z_j^*, A^*) + \hat{U} \geq \overline{u}$$

so that

$$\sum_j p_j \tilde{D}(z_j^*, A^*) \geq \overline{u} - \{u(A^*) - fA^* + \hat{U}\}.$$

Since each gambler bets $A_2 = \tilde{A}^*$ maximizing $u(A) - fA$ in period 2, the expected payoff of the bookmaker when the book-bet is used in period 1 is given by

$$\begin{aligned} \Pi_b & \equiv & n\left\{A^* - \sum_j p_j \{h_j^* a_j^* + \tilde{D}(z_j^*, A^*)\}\right\} + nfA_2 \\ & = & n\left\{fA^* - \sum_j p_j \tilde{D}(z_j^*, A^*)\right\} + nf\tilde{A}^* \\ & \leq & n\{u(A^*) + \hat{U} - \overline{u}\} + nf\tilde{A}^* \end{aligned}$$

where the equality follows from the fact that $\sum_j p_j h_j^* a_j^* = (1-f)A^*$. When the bookmaker uses the cash-bet in period 1 as well as in period 2, she obtains the payoff of $\Pi_c \equiv 2nf\tilde{A}^*$. If $\Pi_b \geq \Pi_c$, it must be then that

$$u(A^*) - \overline{u} + \hat{U} \geq f\tilde{A}^*.$$

Since $\overline{u} \geq u(A^*)$, this inequality implies that $\hat{U} \geq f\tilde{A}^*$ and hence $u(\tilde{A}^*) \geq 2f\tilde{A}^*$, contradicting to the assumption that $2f\tilde{A}^* > u(\tilde{A}^*)$. Therefore, $\Pi_c > \Pi_b$ must be satisfied. Q.E.D.

11 Appendix B: Aggregate Amounts Bet with and without Addictions

We provide the formal analysis about the equilibrium patterns of aggregate amounts bet with and without addictions, which we discussed in Section 5.3 of the main text. We use the notation A_t to denote the aggregate amount bet by each gambler in period $t = 1, 2$.

First, we consider the case that gamblers are addicted to gambling so that $\alpha > 0$ and $\beta > 0$. Then we define by $A_1 = A^*$ the equilibrium aggregate amount bet by each gambler in period 1 and by $A_2 = \hat{A}(S^*)$ the aggregate amount bet by each gambler in period 2 respectively, where $S^* \equiv \alpha + \beta A^*$. For the comparison, we define by $A_1 = \tilde{A}$ the equilibrium aggregate amount bet by each gambler in period 1 when the cash-bet is used in period 1. Correspondingly, the aggregate amount bet by each gambler in period 2 is given by $A_2 = \hat{A}(\tilde{S})$ where $\tilde{S} \equiv \alpha + \beta \tilde{A}$.

We make the specification on the second-period utility $u(A, S)$ such that it is linearly homogeneous. Then we define by η the optimal amount bet per addiction stock in period 2 that attains $\max_{A \geq 0} u(A, 1) - fA$. Under linear homogeneity of u, η is given by the first-order condition as $u_A(\eta, 1) = f$, so that the second-period aggregate bet $A_2 = \hat{A}(S)$ is determined by $\hat{A}(S) = \eta S$. We also derive the second-period payoff of each gambler as $\hat{U}(S) = \tilde{u} S$ where $\tilde{u} \equiv u(\eta, 1) - \eta f$. Note that both $\hat{A}(S)$ and $\hat{U}(S)$ are proportional to the addiction stock S. We also assume that $u_{AS} > 0$.

We then show the following result on the dynamic patterns of gamblers' bets over time in the presence of addiction preference ($\alpha > 0$ and $\beta > 0$).

Proposition B1. *Consider the gamblers who are addicted to gambling such that $\alpha > 0$ and $\beta > 0$. Suppose also that their second-period utility function $u(A, S)$ is linearly homogeneous with $u_{AS} \geq 0$, and that the speed of addiction β is so large that $u_A(1, \beta) > f$. Then, (i) equilibrium aggregate amounts bet increase over time; $A_1 < A_2$, and (ii) equilibrium aggregate amounts bet under the book-bet are larger than those under the cash-bet in both periods, if Assumption 1 holds.*

Proof. (i) Suppose contrary to the claim that $A_1 \geq A_2$ in some equilibrium. Then, due to the optimality condition about the equilibrium bet $\hat{A}(S_i)$ in period 2, the linear homogeneity of u, and $u_{AS} > 0$, we have

$$\begin{aligned} f &= u_A(A_2, \alpha + \beta A_1) \\ &\geq u_A(A_2, \beta A_2) \\ &= u_A(1, \beta) \end{aligned}$$

contradicting to $u_A(1, \beta) > f$.

(ii) Suppose contrary to the claim that the equilibrium aggregate bet is larger under the cash-bet than under the book-bet in period 1: $A^* < \tilde{A}$. Then, the bookmaker can set $\tilde{D}(z, A) = 0$ for all $z \geq 0$ and all $A \geq 0$, and $\tilde{q}(D, A) = 0$ for any $D \in (0, z]$ and all $A \geq 0$. Under Assumption 1 gamblers never default no matter what debts they owe, so they always choose $D = 0$ given the above alternative policy. Then gamblers bet to maximize $v(A) + \sum_j p_j \{h_j^* - A\} + \hat{U}(S)$ in period 1. Since $h_j^* = (1-f)/p_j$ holds for all $j \in J$ in the symmetric equilibrium, each gambler bets the aggregate amount $\tilde{A}$ that maximizes $v(A) - fA + \hat{U}(S)$ in period 1. Following this, each gambler places bet $A_2 = \hat{A}(\alpha + \beta\tilde{A})$ in period 2, which is larger than $\hat{A}(\alpha + \beta A^*)$ because $\hat{A}$ is increasing.

By the above alternative policy, the bookmaker can improve her payoff per gambler:

$$fA^* - \tilde{D}(z^*, A^*) + f\hat{A}(S^*) < f\tilde{A} + f\hat{A}(\alpha + \beta\tilde{A})$$

a contradiction. Therefore, we have $A^* \geq \tilde{A}$ in period 1, and hence $\hat{A}(S^*) \geq \hat{A}(\alpha + \beta\tilde{A})$. Q.E.D.

Next we consider that the individuals are not addicted to gambling at all; $\alpha = \beta = 0$. Assume that $u(A, 0) = v(A)$. We then show the following result.

Proposition B2. *Suppose that individuals are not addicted to gambling at all* ($\alpha = \beta = 0$)*. Then, equilibrium aggregate amounts bet never increase over time;* $A_1 \geq A_2$*, in contrast to the case of addictive gamblers.*

Proof. Let $\alpha = \beta = 0$.

First, suppose that using the cash-bet becomes optimal for the bookmaker in period 1. Then each gambler bets $\tilde{A}^*$ in total in both periods where $\tilde{A}^*$ maximizes $u(A, 0) - fA = v(A) - fA$. Thus, the equilibrium amounts bet become constant over time.

Second, suppose that using the book-bet becomes optimal for the bookmaker in period 1, associating with the punishment policy $\{\tilde{q}, \tilde{D}\}$ in period 1. Suppose contrary to the claim that $A_1 = A^* < A_2$ holds. Note that gamblers bet $A_2 = \tilde{A}^* \equiv \hat{A}(0)$ in period 2 where $\tilde{A}^*$ maximizes $u(A, 0) - fA$. Thus $A^* < A_2 = \tilde{A}^*$. Then the bookmaker's payoff is given by $fA^* - \tilde{D}(z^*, A^*) + f\tilde{A}^*$ when she uses the book-bet and the associated punishment policy $\{\tilde{q}, \tilde{D}\}$.

Alternatively, consider that the bookmaker offers the cash-bet in period 1 instead of using the book-bet with the punishment policy $\{\tilde{q}, \tilde{D}\}$. Then, the bookmaker can obtain a higher payoff per gambler:

$$f\tilde{A}^* + f\tilde{A}^* \quad > \quad fA^* - \tilde{D}(z^*, A^*) + f\tilde{A}^*$$

contradicting to the optimality of using the book-bet with the punishment policy $\{\tilde{q}, \tilde{D}\}$. Therefore, we have $A_1 = A^* \geq A_2 = \tilde{A}^*$. Q.E.D.

Finally, we show that, if the cash-bet is optimal in period 1 with non-addictive gamblers ($\alpha = \beta = 0$), the bookmaker optimally induces gamblers to bet more in both periods when they are addicted than when they are not addicted.

Proposition B3. *Suppose that the cash-bet becomes optimal in period 1 when gamblers are not addicted at all ($\alpha = \beta = 0$). Then, the bookmaker's optimal policy is to implement higher aggregate bets in both periods when gamblers are so addicted that Assumption 1 is satisfied than when they are not addicted at all ($\alpha = \beta = 0$).*

Proof. We denote by $A_1 = A^*$ and $A_2 = \hat{A}(S^*)$ the equilibrium aggregate amounts bet by each gambler in period 1 and period 2 respectively, when gamblers are addicted ($\alpha > 0$ and $\beta > 0$). Note also that, whenever the bookmaker uses the cash-bet in period 1 (as well as in period 2) given $\alpha = \beta = 0$, the equilibrium aggregate amount bet by each gambler becomes a constant over time and is given by $A_1 = A_2 = \tilde{A}^*$.

Consider the case that $\alpha > 0$ and $\beta > 0$, and suppose contrary to the claim that $\tilde{A}^* > A^*$ in period 1. Here recall that $\tilde{A}^*$ maximizes $v(A) - fA$. Then we consider the alternative policy that the bookmaker sets $\tilde{D}(z, A) = 0$ for all $z \geq 0$ and all $A \geq 0$, not allowing gamblers to default more than zero. Under Assumption 1 gamblers never default and bet $\tilde{A}$ in total to maximize $v(A) - fA + \hat{U}(S)$ in period 1. By the revealed preference argument, we obtain

$$v(\tilde{A}) - f\tilde{A} + \hat{U}(\tilde{S}) \geq v(\tilde{A}^*) - f\tilde{A}^* + \hat{U}(\alpha + \beta\tilde{A}^*)$$

where $\tilde{S} \equiv \alpha + \beta\tilde{A}$, and

$$v(\tilde{A}^*) - f\tilde{A}^* \geq v(\tilde{A}) - f\tilde{A}.$$

Combining these two inequalities yields $\hat{U}(\alpha + \beta\tilde{A}) \geq \hat{U}(\alpha + \beta\tilde{A}^*)$. Since $\hat{U}$ is increasing, it must be that $\tilde{A} \geq \tilde{A}^*$.

Therefore, by the above alternative policy, the bookmaker can obtain the payoff of $f\tilde{A} + f\hat{A}(\tilde{S})$ per gambler that is larger than the optimal one, $fA^* - \tilde{D}(z^*, A^*) + f\hat{A}(S^*)$, because

$$\begin{aligned} fA^* - \tilde{D}(z^*, A^*) + f\hat{A}(S^*) &< f\tilde{A}^* + f\hat{A}(\alpha + \beta\tilde{A}^*) \\ &\leq f\tilde{A} + f\hat{A}(\tilde{S}) \end{aligned}$$

where we used $\tilde{A} \geq \tilde{A}^* > A^*$ and the fact that $\hat{A}$ is increasing. This is a contradiction, hence $A^* \geq \tilde{A}^*$ holds in period 1: gamblers bet more in period 1 when they are addicted to gambling ($\alpha > 0$ and $\beta > 0$) than when they are not addicted at all ($\alpha = \beta = 0$). We next show that $\hat{A}(S^*) \geq \tilde{A}^*$: by the revealed preference argument, we obtain

$$u(\hat{A}(S^*), S^*) - f\hat{A}(S^*) \geq u(\tilde{A}^*, S^*) - \tilde{A}^*$$

and

$$u(\tilde{A}^*, 0) - f\tilde{A}^* \geq u(\hat{A}(S^*), 0) - f\hat{A}(S^*).$$

These two inequalities yield

$$u(\hat{A}(S^*), S^*) - u(\hat{A}(S^*), 0) \geq u(\tilde{A}^*, S^*) - u(\tilde{A}^*, 0).$$

Since $u(A, S) - u(A, 0)$ is increasing in A due to $u_{AS} > 0$, we have $\hat{A}(S^*) \geq \tilde{A}^*$. Therefore, gamblers bet more in period 2 when they are addicted than when they are not addicted. Q.E.D.

12 Appendix C: Addiction and Default

In this appendix we provide the example to show how the default ratio DR defined in Section 5.4 of the main text is affected by the speed of addiction β. To this end, we make the following specification: (i) the second-period utility $u(A, S)$ is given by $u(A, S) = \gamma h(A, S)$ where $\gamma > 0$ and h is linearly homogeneous, and (ii) the first-period utility $v(A)$ is quadratic and given by $v(A) = A - (1/2)A^2$. Recall that η maximizes $u(x, 1) - fx$ over $x \geq 0$, and the equilibrium aggregate bet per gambler in period 2 is given by $\hat{A}(S) = \eta S$. Also, the second-period equilibrium payoff of gambler i becomes $\hat{U}(S_i) = \tilde{u}S_i$ where $\tilde{u} \equiv u(\eta, 1) - f\eta$. To ensure Assumption 1, we consider the range of β as $\beta \in [1/\tilde{u}, \infty)$ such that $-A + \hat{U}(S) \geq 0$ holds when $\beta > 1/\tilde{u}$. Also, note that $\tilde{A}$ maximizes $v(A) - fA + \hat{U}(S)$ and hence satisfies the corresponding first order condition as $v'(\tilde{A}) - f + \hat{U}'(\alpha + \beta\tilde{A}) = 0$, so that $\tilde{A} = 1 + \beta\tilde{u} - f$.

We consider the optimal punishment policy derived in the proof of Proposition 3: $\{\tilde{q}, \tilde{D}\}$ where $\tilde{q}$ is defined as (13) in the main text, and $\tilde{D}$ is defined as (A5) in the Appendix. In the equilibrium each gambler owes the debt $z_j^* = A^* - h_j^* a_j^* = fA^*$ and defaults $\tilde{D}(z_j^*, A^*) = d$ conditional on horse j wining the race, where $d \equiv U^* - \{v(A^*) - fA^* + \hat{U}(S^*)\}$.

We can then re-write the default ratio DR_j as follows

$$DR_j = DR \equiv \frac{U^* - \{v(A^*) - fA^* + \hat{U}(S^*)\}}{fA^*}$$

where A^* maximizes $v(A) + u(\hat{A}(S), S)$ subject to the feasibility (F) constraint that $v(A) + \hat{U}(S) \geq U^*$. In the above specification A^* is given as follows: $v'(A^*) = -\beta u$ if the feasibility constraint is slack, and $v'(A^*) \geq -\beta u$ otherwise respectively. In the latter case we have $DR = 1$; gamblers default on all debts.

Whenever $DR < 1$ so that A^* is given by $v'(A^*) = -\beta u$, we can verify that

$$\begin{aligned}
\partial DR/\partial\beta &= \{\partial U^*/\partial\beta - \{v'(A^*) - f + \hat{U}'(S^*)\}(\partial A^*/\partial\beta) - \partial\hat{U}(S^*)/\partial\beta\}A^* \\
&\quad - \{U^* - \{v(A^*) - fA^* + \hat{U}(S^*)\}\}(\partial A^*/\partial\beta) \\
&= \{-\tilde{u}(A^* - \tilde{A}) + f(1 + \beta\eta)(\partial A^*/\partial\beta)\}A^* - fA^* DR(\partial A^*/\partial\beta) \\
&< 0
\end{aligned}$$

provided that

$$1 + \beta\eta - (1/f)\tilde{u}(A^* - \tilde{A})/(\partial A^*/\partial\beta) < DR.$$

This inequality is equivalent to

$$K(\beta) \equiv (1 + \beta\eta)\,(1 - \tilde{u}/u) < DR.$$

Here, $\tilde{u} \equiv u(\eta, 1) - f\eta = u(\eta, 1) - \eta u_A(\eta, 1)$. Since $u(\eta, 1) = \gamma h(\eta, 1)$ and η is given by η^* such that $h_A(\eta^*, 1) = 0$ when $\gamma \to \infty$, we have $\tilde{u} = \gamma\{h(\eta, 1) - \eta h_A(\eta, 1)\} \to +\infty$ as $\gamma \to \infty$ as well as $\tilde{u}/u = \{h(\eta, 1) - \eta h_A(\eta, 1)\}/\{h(\eta, 1)\} \to h(\eta^*, 1)/h(\eta^*, 1) = 1$ as $\gamma \to \infty$. Therefore, when γ is sufficiently large, we verify that $K(1/\tilde{u}) \to 0$ as $\gamma \to \infty$ and hence $K(\beta) < DR$ at $\beta = 1/\tilde{u}$, where $A^* = 1 + u/\tilde{u} \to 1$ and $\tilde{A} = 2 - f$ at $\beta = 1/\tilde{u}$. Furthermore, we show that $DR < 1$ holds at $\beta = 1/\tilde{u}$ when γ is sufficiently large: $U^* - \{v(A^*) - fA^* + \hat{U}(S^*)\} \to v(2 - f) - f(1 - f) - v(2) = v(2 - f) - f(1 - f)$ at $\beta = 1/\tilde{u}$ when $\gamma \to \infty$. Then, $v(2 - f) - f(1 - f) < fA^* = 2f$ holds at $\beta = 1/\tilde{u}$ given $(2 - f) - (1/2)(2 - f)^2 < f(3 - f)$ which is satisfied for all $f \in (0, 1)$. Therefore, DR is decreasing in $\beta \in (1/\tilde{u}, \overline{\beta})$ for some $\overline{\beta}$, provided that γ is sufficiently large.

13 Appendix D: Endogenous Commitment by the Bookmaker

In this appendix we extend the two-period model presented in the main text in such a way that the bookmaker endogenously self-enforces promised payouts and punishment policies.

13.1 Dynamic Game with A Sequence of Short-Lived Gamblers

We consider the dynamic extension of the basic model as follows. Time is discrete and extended over infinity $t = 1, 2, ...$ The bookmaker is a long-lived player who lives forever over time while gamblers are short-lived players who live only for one period (in sub-period 1 and sub-period 2 defined below). For example, the bookmaker is an organization which runs the horse race betting over time, so it is the going-concern over infinite periods.

To maintain the consistency with the two-period model, we suppose that each period t is divided into two sub-periods, which we call "sub-period 1" and "sub-period 2", as follows. Sub-period 1 corresponds to the game played as "period 1" in the basic model while sub-period 2 corresponds to the game played as "period 2" in the basic model respectively. More specifically, in sub-period 1 of period t, (i) new gamblers are born and enter the market; (ii) the bookmaker chooses the cash-bet or the book-bet, and announces the punishment policy q; (iii) gamblers decide how much to bet; (iv) after the horse race outcome is realized, gamblers decide how much to default; (v) the bookmaker decides whether or not to pay the payouts that are determined by the horse race outcome as well as she decides whether or not to implement the announced punishment policy. In sub-period 2 of period t, (i) gamblers place bets under the cash-bet;[24] (ii) the bookmaker decides whether or not to renege on payouts; (iii) gamblers leave the economy and are replaced by newly born gamblers.

The bookmaker discounts her payoffs across successive two periods at $\delta \in [0, 1)$ while there are no time discounting between sub-period 1 and sub-period 2 within each period.

We introduce the "quitting option" between the bookmaker and each gambler i in the beginning of sub-period 1 and sub-period 2 of each period: the bookmaker decides whether or not to exercise the quitting option against gambler i (exclude him from the gambling) while gambler i decides whether or not to quit the gambling. The bookmaker and gamblers simultaneously make the quitting decision. Then, the relationship between the bookmaker and gambler i ends once at least one of them decides to quit. In that case the bookmaker obtains no revenues from gambler i while gambler i obtains the reservation payoff normalized to zero.

13.2 Information Structure

We consider the following information structure: each gambler i born in period t observes what punishment policy the bookmaker has offered and how much she has paid to him within the entire period of t. In addition, for each gambler i in period t, there exists a corresponding gambler, denoted by $\phi(i)$, born in next period $t+1$ who can observe whether or not the bookmaker and gambler i terminated their relationship in period t as well as how much payouts the bookmaker made to gambler i in sub-period 2 of period t. We then define $\phi(i), \phi(\phi(i)), \phi(\phi\phi(i)), \ldots$ recursively.

We call the sequence of gamblers $\phi(i), \phi(\phi(i)), \ldots$ *successors* of gambler i. The role of gambler$i$'s successors is to punish the bookmaker when she deviated from the promised payouts or announced punishment policy against gambler i in each period: if the bookmaker deviates against gambler i in some period t, gambler i or/and his immediate successor $\phi(i)$ will punish the bookmaker by exercising the quitting option. Here successor $\phi(i)$ can know directly or indirectly the bookmaker's deviation against gambler i in period t: $\phi(i)$ observes whether or not gambler i terminated the relationship with the bookmaker in sub-period 1 or whether or not the bookmaker did not give the promised payouts to gambler i under the cash-bet in sub-period 2 within period t. The next successor $\phi(\phi(i))$ can observe that the relationship between the bookmaker and gambler $\phi(i)$ was terminated in period $t+1$. Then $\phi(\phi(i))$ will exercise the quitting option in period $t+2$, and so on. In this way all i's successors will punish the bookmaker once she has made the deviation against i in period t.

Let denote by $\phi^{-1}(i)$ the gambler whose immediate successor is i.

[24] The bookmaker has no choices but the cash-bet in sub-period 2: gamblers never repay debts if the book-bet is used in sub-period 2 because they leave the economy in sub-period 2.

13.3 Self-Enforcing Equilibrium

Throughout the following, we will maintain Assumption 1 made in the main text, ensuring that the book-bet becomes optimal for the bookmaker in sub-period 1 within each period. Note also that the bookmaker's payoff $\Pi(\tilde{q})$ is bounded above by $\Phi(A)$ defined as (A4) above, as shown in the proof of Proposition 3.

We consider how the bookmaker implements an aggregate amount bet A per gambler in sub-period 1 within each period and obtains the corresponding upper bound payoff $\Phi(A)$ per gambler in each period t *without* commitment to promised payouts and punishment policy. To this end, we focus on the bang-bang punishment policy $\{\tilde{q}, \tilde{D}\}$ as used in the proof of Proposition 3: $\tilde{D}$ defined as (19) and $\tilde{q}$ defined as (13) in the main text. Then, in addition to the feasibility constraint $v(A) + \hat{U}(S) \geq U^*$ as before, we also need to take into account the *dynamic enforcement* (DE) constraint as discussed in the main text. We replace $\Pi(\tilde{q})$ by its upper bound payoff $\Phi(A)$ in the DE constraint given in the main text. Then we can re-write the DE constraint by

$$\frac{\delta\Phi(A)}{1-\delta} \geq (1-f)\hat{A}(S) \tag{DE}$$

where $S = \alpha + \beta A$.

Now consider the equilibrium in which in each period the bookmaker implements the aggregate amount bet $A_1 = A^*$ in sub-period 1 where each gambler places bet $a_j^* \equiv p_j A^*$ on each horse $j \in J$, and implements the aggregate amount bet $A_2 = \hat{A}(S^*)$ in sub-period 2 from each gambler where $S^* \equiv \alpha + \beta A^*$. We set $h_j^* \equiv (1-f)/p_j$ for the equilibrium odds of horse $j \in J$ in sub-period 1. We also denote by $\hat{h}_j \equiv (1-f)/p_j$ for each $j \in J$ the equilibrium odds in sub-period 2 and $\hat{a}_j \equiv p_j \hat{A}(S^*)$ the associated equilibrium amount bet by each gambler on horse j under the cash-bet in sub-period 2 within each period.

On the equilibrium path the bookmaker obtains the following payoff

$$\Phi(A^*) \equiv \{v(A^*) + \hat{U}(S^*) - U^*\} + f\hat{A}(S^*)$$

per gambler within each period.

There are three possible deviations by the bookmaker: first, the bookmaker may renege on payouts for some gamblers under the book-bet in sub-period 1. Second, the bookmaker may not implement the announced punishment policy against some gamblers in the end of sub-period 1. Third, the bookmaker may renege on payouts for some gamblers under the cash-bet in sub-period 2.

In the first and second deviations we consider the punishment equilibrium as follows: when the bookmaker did not implement the announced punishment policy $\tilde{q}$ against gambler i or she did not give the payout of $R_{i,j} \equiv h_j a_{i,j} - A_i$ to gambler i in sub-period 1 of period t (given $R_{i,j} > 0$), she and gambler i exercise the quitting option simultaneously in the beginning of sub-period 2 within period t. In the beginning of period $t+1$ the bookmaker and i's successor $\phi(i)$ will exercise the quitting option simultaneously. Repeating this, all the successors of gambler i, $\phi(i), \phi(\phi(i)), \ldots$, and the bookmaker will terminate their relationships in the future. Therefore, the bookmaker will obtain no payoffs in any future period.

In the third deviation we consider the punishment equilibrium as follows: when the bookmaker did not give gambler i the promised payout $h_j a_{i,j}$ under the cash-bet in sub-period 2 of period t, all i's successors and the bookmaker will terminate their relationships in all the future periods. As a result, the bookmaker will lose all future payoffs from period $t+1$.

Define the following condition for $S_i = \alpha + \beta A_i$:

$$\frac{\delta}{1-\delta}\Phi(A^*) \geq (1-f)\hat{A}(S_i) \tag{DE-1}$$

Then we consider the following strategy profile of the bookmaker and gambler i.

The bookmaker's strategy in period t:

- Sub-period 1:
  - The bookmaker exercises the quitting option against gambler i if she has deviated against gambler $\phi^{-1}(i)$ from the equilibrium payouts and/or punishment policy in the previous period. If the bookmaker has not deviated, she offers the book-bet and the punishment policy $\{\tilde{q}, \tilde{D}\}$.
  - Suppose that the bookmaker offered $\{\tilde{q}, \tilde{D}\}$ and that DE-1 holds. Then the bookmaker pays $\max\{h_j a_{i,j} - A_i, 0\}$ to gambler i after horse j wins the race if

    $$\max\{A_i - h_j a_{i,j}, 0\} - \tilde{D}(z_{i,j}, A_i) + f\hat{A}(S_i) + \frac{\delta}{1-\delta}\Phi(A^*) \geq \max\{h_j a_{i,j} - A_i, 0\},$$

    given the winning odds $h_j > 0$.[25] Otherwise, she will not pay gambler i. Suppose that the bookmaker offered $\{\tilde{q}, \tilde{D}\}$ but DE-1 fails to hold. Then the bookmaker will not pay gambler i.
  - The bookmaker excludes the gamblers who defaulted according to the announced punishment policy $\tilde{q}$ regardless of how much she has paid to them.

- Sub-period 2:
  - If DE-1 holds, the bookmaker does not exercise the quitting option against gambler i and she quits otherwise.

[25] We are here focusing on the situation in which market odds $h_j > 0$ are positive for all $j \in J$ again, as we discussed in footnote 9 of the main text: for example, each gambler places a small but positive bet $\varepsilon > 0$ on any horse by a mistake or other irrational reasons.

– After the bookmaker holds the cash-bet and the horse race outcome is realized, she will pay $h_j a_{i,j}$ to gambler i if

$$\frac{\delta}{1-\delta}\Phi(A^*) \geq h_j a_{i,j} \tag{DE-2}$$

given the winning odds $h_j > 0$.[26] Otherwise, she will give no payouts to gambler i.

Gambler i's strategy in period t:

- Sub-period 1: Suppose that the either one of the following occurs: the bookmaker has deviated against $\phi^{-1}(i)$ in the previous period; gambler $\phi^{-1}(i)$ has quit in the previous period; the bookmaker did not offer the book-bet and/or the equilibrium punishment policy $\{\tilde{q}, \tilde{D}\}$ in the current period. Then gambler i quits. Otherwise, he will place bet $\{a_j^*\}_{j\in J}$. After the horse race outcome is realized, gambler i will default $\tilde{D}(z_j^*, A^*)$.
- Sub-period 2: if DE-1 fails or the bookmaker deviates from the promised payout $R_{i,j} \equiv \max\{h_j a_{i,j} - A_i, 0\}$ in sub-period 1, gambler i will quit. Otherwise, he will place bet $\{\hat{a}_j\}_{j\in J}$ such that $A_i = \hat{A}(S_i)$ and $\hat{a}_j \equiv p_j \hat{A}(S_i)$ for $j \in J$.

When all the players follow the prescribed strategies, the bookmaker obtains the payoff of $\Phi(A^*)$ every period while each gambler born in period t bets the aggregate amount of A^* and defaults $\tilde{D}(z^*, A^*)$ in sub-period 1, followed by betting $\hat{A}(S^*)$ in total in sub-period 2. Then the equilibrium payoff of each gambler born in each period is given by

$$v(A^*) - fA^* + \tilde{D}(z^*, A^*) + \hat{U}(S^*) = U^*.$$

Given the equilibrium amount bet $\hat{A}(S^*)$ by each gambler in sub-period 2, for the bookmaker to honor the equilibrium payouts $(1-f)\hat{A}(S^*)$ for each gambler, DE-1 must be satisfied at $A_i = \hat{A}(S^*)$, that is, it yields the DE constraint:

$$\frac{\delta}{1-\delta}\Phi(A^*) \geq (1-f)\hat{A}(S^*) \tag{DE}$$

The best equilibrium which the bookmaker can implement is obtained to solve the following problem.

Problem E:

$$\max_{A\geq 0} \Phi(A)$$

[26] Again we focus on the case that $h_j > 0$ for all $j \in J$.

subject to the feasibility (F) constraint $v(A) + \hat{U}(S) \geq U^*$ and the DE constraint:

$$\frac{\delta \Phi(A)}{1-\delta} \geq (1-f)\hat{A}(\alpha + \beta A). \tag{DE}$$

We then show the following result.

Proposition D1. *Let denote by A^* the solution of A to Problem E. Then there exists an equilibrium in which the bookmaker attains the payoff of $\Phi(A^*)$ every period.*

Proof. Consider the strategies of the bookmaker and gamblers described above.

Step 1. First, we consider sub-period 2 of period t, provided that DE-1 is satisfied so that the bookmaker and gamblers born in sub-period 1 of period t have not quit.

Suppose then that the bookmaker offered the cash-bet and then horse j won the race in sub-period 2. If the bookmaker pays $h_j a_{i,j}$ to gambler i, she expects to obtain the continuation value $\delta\Phi(A^*)/(1-\delta)$ from subsequent gamblers in future periods. If the bookmaker reneges on the payout of $h_j a_{i,j}$ against some gambler i, all the successors of gambler i will quit the gambling, thereby yielding the payoff of zero to the bookmaker in the future. Therefore, it becomes optimal for the bookmaker to pay $h_j a_{i,j}$ to gambler i if (DE-2) is satisfied in sub-period 2 of period t.

Given this, we consider the incentive of gamblers in sub-period 2. Define the equilibrium odds $\hat{h}_j = (1-f)/p_j$ for $j \in J$, and take any profile of amounts bet by gambler i, $\mathbf{a}_i$ in sub-period 2 of each period. Then define the set of horses $\tilde{J}_i$ such that DE-2 holds at $h_j = \hat{h}_j \equiv (1-f)/p_j$ for each $j \in J$. Then gambler i places bet $\mathbf{a}_i$ in sub-period 2, he will obtain the following expected payoff

$$u(A_i, S_i) + \sum_{j \in \tilde{J}_i} p_j\{\hat{h}_j a_{i,j} - A_i\} + \sum_{j \notin \tilde{J}_i}(-A_i)$$

because the bookmaker will renege on payouts for gambler i when horse $j \notin \tilde{J}_i$ wins the race. Then we verify that

$$\begin{aligned} & u(A_i, S_i) + \sum_{j \in \tilde{J}_i} p_j\{\hat{h}_j a_{i,j} - A_i\} + \sum_{j \notin \tilde{J}_i}(-A_i) \\ \leq\ & u(A_i, S_i) + \sum_{j \in J} p_j\{\hat{h}_j a_{i,j} - A_i\} \\ \leq\ & \hat{U}(S_i). \end{aligned}$$

On the contrary, if gambler i places the equilibrium bet of $\hat{a}_j \equiv p_j \hat{A}(S_i)$ for each $j \in J$, he can ensure that

$$\frac{\delta \Phi(A^*)}{1-\delta} \geq \hat{h}_j \hat{a}_j = (1-f)\hat{A}(S_i)$$

that is satisfied due to DE-1. Therefore, the bookmaker never reneges on payouts, implying that gambler i obtains the equilibrium payoff of $\hat{U}(S_i)$ in sub-period 2. Then gambler i never deviates from betting $\hat{\mathbf{a}}$ where $\hat{a}_j \equiv p_j \hat{A}(S_i)$ for each $j \in J$, given DE-1.

Suppose next that DE-1 fails. Then in the beginning of sub-period 2 the bookmaker and gambler i will terminate the relationship. Furthermore, i's successors will terminate the relationship as well in all the future periods, giving the bookmaker the continuation value of zero.

Step 2. Second, consider the incentive of the bookmaker in the end of sub-period 1, provided that gambler i defaulted $D_{i,j}$. If the bookmaker follows the equilibrium strategy, she will exclude gambler i who defaulted on $D_{i,j}$ according to $\tilde{q}$. This in turn implies that the bookmaker will obtain the following continuation payoff from the end of sub-period 1:

$$(1-\tilde{q}(D_{i,j}, A_i)) f \hat{A}(S_i) + \frac{\delta \Phi(A^*)}{1-\delta} \geq 0 \tag{D1}$$

from gambler i if DE-1 holds and zero otherwise respectively. If the bookmaker deviates not to punish the defaulting gambler i, all i's successors will quit. Therefore, the bookmaker obtains the continuation value of zero, which is not however profitable by condition (D1).

Third, consider the incentive of the bookmaker in the stage in which she makes payouts to gamblers after horse j won the race in sub-period 1 of each period. Let $z_{i,j} \equiv \max\{A_i - h_j a_{i,j}, 0\}$ denote the net payback from gambler i to the bookmaker when horse j wins the race.

Suppose first that DE-1 holds. Then, if the bookmaker follows the promised payout of $R_{i,j} \equiv \max\{h_j a_{i,j} - A_i, 0\}$ to gambler i, she will obtain the following continuation value of her payoffs:

$$-\max\{R_{i,j}, 0\} + z_{i,j} - \tilde{D}(z_{i,j}, A_i) + (1-\tilde{q}(\tilde{D}(z_{i,j}, A_i), A_i)) f \hat{A}(S_i) + \frac{\delta \Phi(A^*)}{1-\delta} \tag{D2}$$

from gambler i and his successors. Here, gambler i will optimally follows the equilibrium default $\tilde{D}(z_{i,j}, A_i)$ and subsequently places the bet of $\hat{A}(S_i)$ in sub-period 2, provided that the bookmaker will optimally punish him according to the equilibrium punishment policy $\tilde{q}$ as well as she holds the cash-bet and never reneges on the equilibrium payout $\hat{h}_j \hat{a}_j$ in sub-period 2. Note that $\tilde{q}(\tilde{D}(z_{i,j}, A_i), A_i) = 0$ holds due to the definition of $\tilde{q}$ and $\tilde{D}$. On the contrary, if the bookmaker deviates from $R_{i,j}$, then she will obtain the continuation value of zero because gambler i will quit in sub-period 2 and all the successors of gambler i will quit as well. Note that, if gambler i will quit in sub-period 2, he always defaults on all the debt of $z_{i,j}$ in sub-period 1 and will never place bet in sub-period 2. Therefore, the bookmaker follows the promised payout $R_{i,j}$ when the above value (D2) is non-negative.

Suppose next that DE-1 fails in some period t. Then, the bookmaker and gambler i will simultaneously quit in the beginning of sub-period 2, followed by the termination of relationships between the bookmaker and i's successors in all the future periods. Therefore, the bookmaker and gambler i obtain the payoffs of zero from sub-period 2 of period t. Given this continuation outcome, gambler i defaults all the debt $z_{i,j}$ in sub-period 1 of period t. Anticipating this, the bookmaker reneges on payouts to gambler i in sub-period 1 of period t.

We now define by J_i^* the set of horses for which DE-1 is satisfied as well as (D2) becomes non-negative at $h_j = h_j^* \equiv (1-f)/p_j$ for each $j \in J$, given a profile of amounts bet $\mathbf{a}_i$ by gambler i. Then, if gambler i places bet $\mathbf{a}_i$ in sub-period 1, he expects to obtain

$$v(A_i) + \sum_{j \in J_i^*} p_j \{h_j^* a_{i,j} - A_i + \tilde{D}(z_{i,j}, A_i) + \hat{U}(S_i)\} + \sum_{j \notin J_i^*} p_j \times 0$$

where the last term captures the fact that for $j \notin J_i^*$ the bookmaker reneges on payouts followed by the termination of the relationship between her and gambler i in sub-period 2 today. Since $h_j^* a_{i,j} - A_i + \tilde{D}(z_{i,j}, A_i) + \hat{U}(S_i) \geq 0$ for all $j \in J$ under Assumption 1, we verify that

$$\begin{aligned}
& v(A_i) + \sum_{j \in J_i^*} p_j \{h_j^* a_{i,j} - A_i + \tilde{D}(z_{i,j}, A_i) + \hat{U}(S_i)\} \\
& \leq v(A_i) + \sum_{j \in J} p_j \{h_j^* a_{i,j} - A_i + \tilde{D}(z_{i,j}, A_i) + \hat{U}(S_i)\} \\
& \leq v(A^*) + \sum_{j \in J} p_j \{h_j^* a_j^* - A^* + \tilde{D}(z^*, A^*) + \hat{U}(S^*)\} \\
& = U^*
\end{aligned}$$

by using the same step as the proof of Proposition 3. If gambler i places the bet of $\mathbf{a}^*$, then DE-1 holds and (D2) can be non-negative at $h_j = h_j^*$, implying that gambler i obtains the payoff of U^*. Thus, each gambler never deviates from choosing A^* in sub-period 1. Therefore, the bookmaker attains the payoff of $\Phi(A^*)$ in each period where A^* solves Problem E.

13.4 Complementarity between Addictions and Relational Contracts

We now discuss how addictions can ease the dynamic enforcement constraint:

$$\frac{\delta \Phi(A)}{1-\delta} \geq (1-f)\hat{A}(\alpha + \beta A). \tag{DE}$$

To see this, we assume that the second-period utility function $u(A, S)$ is linearly homogeneous. Then we obtain $\hat{U}(S) = \tilde{u}S$ and $\hat{A}(S) = \eta S$ where $\tilde{u} \equiv u(\eta, 1) - f\eta$ and η maximizes $u(x, 1) - fx$ over $x \geq 0$. Given these results, we can see how the difference in the left hand and right hand sides of DE changes with respect to β as follows:

$$\frac{\partial}{\partial \beta} \left\{ \frac{\delta \Phi(A)}{1-\delta} - (1-f)\hat{A}(S) \right\} = A \left\{ \frac{\delta}{1-\delta} \tilde{u} - (1-f)\eta \right\}$$

which is positive when the bookmaker's discount factor $\delta \in (0, 1)$ is so large that

$$\delta/(1-\delta) > (1-f)\eta/\tilde{u}. \tag{D3}$$

Therefore, when (D3) is satisfied, the DE constraint becomes less stringent as the speed of addiction β is larger. This shows that, as individuals are addicted more quickly over time by gambling (hence β is larger), it becomes easier for the bookmaker to self-enforce the relational contract regarding the promised payouts, given (D3).