

Fertility and Social Security*

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Abstract

We examine the effects of changes in government provided old age pensions on fertility choices in the context of two models of fertility, the one by Barro and Becker (1989), and the one inspired by Caldwell and developed by Boldrin and Jones (2002). In the Barro and Becker model parents have children because they perceive their children's lives as a continuation of their own. In the Boldrin and Jones' framework parents procreate because the children care about their old parents' utility, and thus provide them with old age transfers. We find that the direction of the effect on fertility in the Barro and Becker model depends on whether child rearing costs are primarily in goods or in time, but that the size of the effect is always very small. This is inconsistent with empirical results which find a reduction of between 0.7 and 1.2 children born per woman over the relevant range. In the Boldrin and Jones model increases in the size of the public pension system always decrease fertility, regardless of the type of costs incurred to raise the children. In addition, the model accounts for about half of the observed variation. We also find that, in the Boldrin and Jones model, access to capital markets has important quantitative effects on fertility choices, and could, potentially, account for as much of the change in fertility seen in developed countries over the last 70 years as the increase in the size of Social Security systems.

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1 Introduction

In recent years, demographic trends in many advanced countries have been cause for concern among both researchers and policy makers.¹ For example, in 2000, the total fertility rate (TFR) was 1.2 in Italy, 1.3 in Germany and 1.4 across Europe; fertility rates were slightly higher in the USA (at around 2.06 in 2001, up from about 1.8 in the 1980s and early 1990s) and in the Scandinavian countries (where it has varied between 1.8 and 2.1.) From a historical perspective, these figures represent a continuation of a long-term downward trend which has been ongoing for the last two hundred years. Demographers are unsure how and when this trend will reverse, if at all. Given these figures for fertility, projections are that by year 2050 there will be as few as 2.5 persons of working age for every elderly person aged 65 and over. The comparable figure for the year 2000 is around 4. (See Davis, et. al. (1986)) Do these trends in fertility imply that the solvency of government funded pension programs is in danger? The general view is that demographic change, including both the reduction in fertility outlined above accompanied by a continuous growth in life expectancy at retirement, will cause the collapse of public social security systems. Thus, dramatic changes in the design of these systems will be required.

What is less often discussed is causation in the opposite direction. That is, might the generosity of the pension plans themselves be one of the causes of these demographic trends? If it is the case that the generosity of current public pension plans is a contributing cause to the fertility decline, then proposals to reform those plans should take this effect into account. In other words, reforming pension systems because of demographic changes also requires knowledge of the feedback effect that these pension systems have on demographic change.²

This is the view that we explore in this paper. We conduct our analysis on three distinct levels. First, we present data showing a strong and robust negative relationship between the size of government provided social security systems and TFR's in both cross section and in time series. This holds despite controlling for a number of other key determinants of fertility. Second, we analyze two distinct theoretical approaches to fertility and study the effects of changes in the size of the social security system on fertility choice. We find that models based on 'parental altruism' (e.g., Barro and Becker (1989)) have mixed predictions for the fertility effects of changes in social security while those based on 'old age security' motivations (e.g., Boldrin and Jones (2002)) unambiguously predict that fertility is decreasing in the size of the social security system. Third, we calibrate versions of both of these theoretical models to gauge the size of these effects. We find that the fertility effects are small, and often in a direction opposite to that in the data, in models based on

¹The literature on this is too lengthy to cite in detail, but a good example is the volume edited by Davis, Bernstam and Ricardo-Campbell (1986) which deals primarily with this issue.

²The possibility of a feedback from pensions to fertility has long been argued at the informal level, see, e.g., National Academy (1971) for an early example.

parental altruism, but large, and economically important in models based on old age security.³

In the data, we examine first a cross section of countries and find that an increase in the size of the social security system on the order of 10% is associated with a reduction in TFR (Total Fertility Rate—the number of children expected to be born per woman) of between 0.7 and 1.2 children. These findings are highly statistically significant and fairly robust to the inclusion of other possible explanatory variables. Similar estimates are obtained when a panel data set of European countries is used. These results complement and improve upon earlier empirical work on both the statistical determinants of fertility and its relation to the existence and size of government run social security systems. Early work using cross sectional evidence includes National Academy (1971), Friedlander and Silver (1967), and Hohm (1975). Analysis of the relationship between social security and fertility based on individual country time series include Swidler (1983) for the U.S., Cigno and Rosati (1996) for Germany, Italy, the UK and U.S., and Cigno, Casolaro and Rosati (2002) for Germany.

Theoretically, we study the effects of changes of government provided old age pension plans on fertility in two distinct types of models of fertility, both the Barro and Becker (1989) model of fertility and the Caldwell-type model,⁴ as developed in Boldrin and Jones (2002). These two models are grounded in diametrically opposite assumptions about intergenerational altruism and, hence, intergenerational transfers. Both of them have a bearing on late age consumption and the means through which individuals account for its provision. In the Barro and Becker model parents have children because they perceive their children's lives as a continuation of their own. Barro and Becker (1989) suggest, on the basis of a partial equilibrium analysis, that a growing lump-sum social security system should lead to a decrease in fertility and an increase in the capital-output ratio; on the other hand, a social security system financed with a tax on labor income may lead to an increase in fertility when the cost of raising children consists mostly of time. Thus, the sign of the effect depends on parameters even in partial equilibrium. We find that these intuitive predictions are only partially confirmed. First, we find that in the dynastic model, a lump-sum public pension system with a balanced budget is neutral with respect to both fertility and saving. Second, when it is not lump-sum, we find that the fertility effects work through changes in interest rates and are thus dependent on general equilibrium considerations. (This is true of both models that we study.) The sign of these effects depend on whether child rearing costs are primarily in goods or in time, but they are always quantitatively very small. In the dynastic model, when the cost of children consists mostly of time, increasing the social security tax/transfers increases fertility;

³Throughout this paper, we ignore a number of equally important determinants of long run fertility: the decrease in mortality and child mortality in particular, the urbanization process, the increase in the goods cost of children and in the opportunity cost of the time it takes to raise them, the increase in female labor force participation, etcetera.

⁴See, e.g., Caldwell (1978, 1982) for an informal but clear and articulated presentation.

when the cost is mostly in goods it decreases it but slightly, and not for all acceptable combinations of parameter values.

In the Boldrin and Jones' framework parents procreate because the children care about their parents' utility, and thus provide their parents with old age transfers. Thus, this is a formal implementation of what a number of researchers in demography would call the "old age security" motivation for childbearing. When this motive dominates, one would intuitively expect that higher social security benefits lead to lower fertility. In this model, increases in the size of the public pension system always decrease fertility, regardless of the type of costs incurred to raise the children and of the lump-sum nature of the tax. Moreover, the variations in fertility induced by changes in social security in a Caldwell-type model, are large quantitatively, with perhaps as much as 50% of the reduction in fertility seen in developed countries in the past 50 years being accounted for by this source alone. Since government provided pensions are a larger portion of retirement savings for families at the low end of the income distribution our results are also consistent with the empirical finding that fertility has declined more for those individuals.

We also consider the impact on fertility that results from improved access to financial instruments to save for retirement. The same empirical literature that has found evidence of a strong correlation between pensions and fertility, has also reported an equally strong correlation between measures of accessibility to saving for retirement and fertility. We provide a simple parameterization of the degree of capital market accessibility and find that even relatively small reductions in financial market efficiency have strong impacts on fertility; societies where it is harder to save for retirement or where the return on capital is particularly low, *ceteris paribus*, have substantially higher fertility levels.

In sum, these findings give indirect support for a strong role for the 'old age security' motive for fertility. As such, they are generally indicative of a more general hypothesis – Since children are perceived by parents as a component of their optimal retirement portfolio, any social or institutional change that affects the economic value of other components of the retirement portfolio will have a first order impact on fertility choices. The fact that models of children as investments work so well here, and in a fashion which is strongly coherent with data, is supportive of the basic hypothesis.

1.1 Relation with Earlier Work

Empirical analyses of the correlation between fertility indices and different measures of the size or the generosity of the public pension system abound. Hohm (1975) is the first econometrically sound attempt we are aware of; he examines 67 countries, using data from the 1960-1965 periods and concludes that social security programs have a measurable negative effect on fertility of about the same magnitude as the more traditional long-run determinants of fertility, i.e. infant mortality, education,

and per capita income.

Cigno and Rosati (1992) present a co-integration analysis of Italian fertility, saving, and social security taxes. They study the potential impact on fertility of both the availability of public pensions and the increasing ease with which financial instruments can be used to provide for old age income. They conclude that "[...] both social security coverage and the development of financial markets, controlling for the other explanatory variables, affect fertility negatively." (p. 333). Their long-run quantitative findings, covering the period 1930-1984 are particularly interesting in the light of one of the models we use here. The point estimates of the (negative) impact of social security and capital market accessibility on fertility are practically identical (Figure 8, p. 338) to what we find here.

The theoretical effects of pension systems on fertility have been studied extensively. Early work includes Bental (1989), Cigno (1991), and Prinz (1990) in addition to the original discussion in Barro and Becker (1988). More recent examples include Nishimura and Zhang (1992), Cigno and Rosati (1992), Cigno (1995), Rosati (1996), Wigger (1999), Yakita (2001) and Zhang, Zhang and Lee (2001). These papers cover both the B-B model of fertility and the B-J one, but are limited in scope. For example, in both the Nishimura and Zhang and the Cigno papers, models are analyzed which are based on reverse altruism like that in Boldrin and Jones. However, they assume that all generations make choices simultaneously and hence, parental care provided by children does not react to changes in savings behavior. Moreover, they do not consider the problem of shirking in parental care resulting from the public goods problem among siblings that is created when reverse altruism is present.

In the dynastic model of endogenous fertility we find that the introduction of a PAYGO social security system leads either to no change or to an increase in the fertility rate in the steady state. This is contrary to the argument presented in the original Barro and Becker paper and therefore deserves further discussion. As mentioned above, Barro and Becker (1989) argue that a growing social security system should reduce fertility. Their analysis is based on a partial equilibrium model and they argue that a social security system "...has the same substitution effect as an increase in the cost of raising a child [...] therefore [...] holding fixed the marginal utility of wealth [...], and the interest rate, we found that fertility declines in the initial generation while fertility in later generations does not change." That is, there will be a transitional effect of lower fertility when the system is introduced. This is then followed by a return to the original fertility level in steady state. Our analysis show that these conclusions are dependent on the partial equilibrium assumption. In a general equilibrium model, both the interest rate and the marginal utility of wealth adjust in such a way that an increase in fertility occurs in the new BGP. Furthermore, there is no evidence in the data of a return to the previous level of fertility after a transition in those countries with large social security systems, as would be predicted by the model.⁵

⁵Cigno and Rosati (1992) also use a simplified two-period version of the dynastic model claiming

A number of other authors, especially in the demographic and sociological literature, have provided various kinds of evidence of the strong empirical link between availability of independent income sources for late age and fertility rates. This literature is too large to be properly reviewed here, hence only short references to two, in our view particularly significant studies will be made.

Rendal and Bahchieva (1998) use data on poor and disabled elderlies in the USA to estimate the market value of the support they receive from family relatives in the form of time inputs in the household production function and find that, even in recent times and in spite of broad social security and welfare programs, these estimates make children a very valuable economic investment for the poorest fifty per cent of the population.

Direct empirical evidence aimed at distinguishing the altruism hypothesis from the old age support one is scarce. One example is Ortuño-Ortín and Romeu (2003), where an original and interesting econometric analysis of micro data measuring parental health care effort and expenditure is shown to provide substantial backing for the ‘old age support’ hypothesis and against the ‘parental altruism’ alternative.

One of the main contributions of this paper is to estimate the size of the effect of Social Security on fertility decisions by studying calibrated, quantitative, versions of the theoretical models. To our knowledge, no previous papers have undertaken such an endeavor. Rather, their focus has been on attempting to sign the partial derivative of the balanced growth level of fertility as a function of the size of the Social Security system.

In section 2, we present the empirical evidence on the relationship between the size of the social security system and fertility. In sections 3 and 4, we lay out, successively the basics of the models of fertility that we study, the Boldrin and Jones model is discussed in section 3, while the Barro and Becker one is analyzed in section 4. In section 5, we present the results of theoretically estimating the effects of social security system within the confines of calibrated versions of the two models. Section 6 carries out a number of sensitivity analysis exercises and extends the models to account for the differential effects of the “forced saving” and “lump-sum” transfer components of real world social security systems. Conclusions are offered in section 7.

that fertility decreases when a (lump-sum) social security transfer to the first generation is increased. This is also orthogonal to the result we report in Section 4, and it is due to the fact that the authors fail to notice that the (net) social security transfers (T_1 and T_2 respectively) must add up to zero, when properly discounted by the interest factor (r) and the population growth factor (n). When $T_1 + \frac{n}{r}T_2 = 0$, social security drops out of the dynastic budget constraint, and both fertility and saving are invariant to changes in T_i . Their algebra is apparently based on Wildasin (1990) who also seems to ignore the straightforward implication of the intertemporal restriction $T_1 + \frac{n}{r}T_2 = 0$.

2 Background Data

In this section, we present some evidence, both from Cross Section and Time Series, on the relationship between the size of government pension plans and fertility.

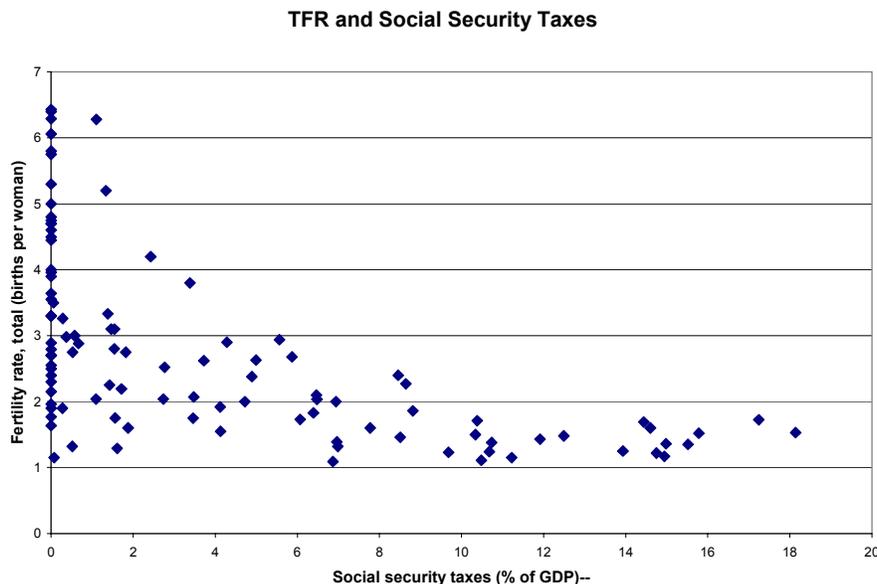


Figure 1: Cross-country correlation, SS tax and TFR

Although one must be careful about causal interpretations, the data in cross section show a strong negative relationship between the Total Fertility Rate (TFR) in a country and the size of its Social Security and pension system. Data from 1997 at the country level for 104 countries is shown in Figure 2.1. This plots TFR for the country in 1997 versus Social Security expenditures as a fraction of GDP in 1997, denoted SST, for these countries. Since this second variable is a measure of the average tax rate for the Social Security system as a whole, we identify it with the Social Security Tax (SST) in what follows. Although the relationship is far from perfect, as can be seen, there is a strong negative relationship between these two variables. Most notably, there are only four countries for which SST is at least 6% and TFR is above 2 (children per woman).⁶

In contrast to this, in those countries where TFR is above 3, none has an SST above 4%. This is suggestive of the overall relationship between these two variables. Regression results from this data set confirm and quantify the visual impression. These are given in the table below. For the most part, this table is self-explanatory.

⁶The source for this data is the "World Development Indicators", 2002, published by the World Bank.

In each regression, the dependent variable is TFR, SST is the Social Security tax rate estimated as total expenditures on the Social Security System as a fraction of GDP (in 1997), GDP is per capita GDP in 1995 (in USD 1,000), IMR is the Infant Mortality Rate, estimated as the number of deaths per 1,000 live births (in 1997), finally, 65% is the fraction of the population that is 65 years or older (in 1997).

Regression	I	II	III	IV	V
Data Set	Cross Section	Cross Section	Cross Section	Cross Section	Panel
Variable					
Constant	3.396 (23.38)	3.496 (22.98)	1.865 (11.74)	2.183 (11.25)	3.496 (33.86)
SST	-16.149 (-7.29)	-12.967 (-5.23)	-6.708 (-4.17)	-12.493 (-4.83)	-12.233 (-14.25)
GDP		-0.028 (-2.51)	0.008 (1.15)	-0.021 (-1.61)	
IMR			0.036 (12.73)	0.032 (10.59)	
65%				6.72($E - 7$) (2.79)	
n	104	102	102	100	123
R ²	.34	.38	.76	.78	.63

Table 1: Fertility and Social Security, Cross Section and Time Series

As can be seen, the coefficient on SST is negative and highly statistically significant in all regressions performed. It is also economically significant. Most LDC's have either no social security system or a very small one. In contrast, SST is between 7% and 16% for most developed countries. Thus, the relevant range for calculations is in changes in SST from 0% (0.00) to 10% (0.10). Column 1 implies that, everything else the same, an increase in SST of this size (i.e., from 0% to 10%) is associated with a reduction in the number of children per woman of about 1.6.

In Regressions II through IV in the table, we include some other variables in the regression that might either give alternative explanations for the results in column 1 or allow for a sharper estimation of the conditional correlation between SST and TFR. Regression II includes per capita GDP as a second variate. Although the significance of SST does fall somewhat, and the coefficient on GDP is negative and significant, the coefficient on SST only changes slightly, and still has a t-Value that is much higher than the one on GDP. Other variates of interest are IMR, and the fraction of the population 65 and above. In Regression III, where IMR is added, the coefficient on SST is negative and highly significant, if somewhat smaller. For

purposes of comparison, this regression suggests that the reduction in TFR resulting from a 10% increase in SST is 0.67, still a significant change in fertility. Note that in this regression the coefficient on GDP is positive and insignificant, suggesting that the relevance of GDP as a determinant of fertility is mostly due to the changes in IMR it strongly correlates with. Regression IV is also relevant, because it quantifies more precisely the impact of SST on TFR. The former, obviously, may be higher just because the share of elderly in the population is particularly high. Once we control for this effect, GDP remains insignificant, while IMR and SST are still highly significant and the coefficient of the latter almost doubles, going back to the point estimates of Regression I and II especially. According to Regression IV an increase of SST from 0.0% to 10% would bring about a reduction of roughly 1.3 children.

In sum, then, an increase in the SST of 10% is associated with a reduction in TFR of between 0.7 and 1.6 children. As already mentioned, these estimates are both statistically and economically meaningful. Most developed countries have SST's in the range of 10 to 15%, while most developing countries have SST's below 5%; hence, an increase in SST of 10% means going from the SST system of an LDC to that of a developed country. The regressions suggest that this is associated with a reduction in the number of children of between 0.7 and 1.6 per woman, with 1.3 the most likely point estimate.

These findings are subject to the same cautions which always accompany cross sectional regression studies, but they are highly suggestive that SST does indeed have an effect on fertility decisions, that this effect is to reduce the number of children that people have and that this effect is fairly large in size.

We find similar results when we look at time series data. Here, we look at a panel data set of TFR's and SST's in 8 developed countries over the period from 1960 to the present.⁷ The 8 countries are: Austria, Belgium, Denmark, Finland, France, Ireland, Norway and Spain. This data is shown in Figure 2.2. The column labeled Regression V shows the results of a simple regression for this panel data set. (Uncorrected for autocorrelation and/or heteroscedasticity.)

As can be seen, the results from this panel regression are quite similar to what we saw above in the cross section— viz., an increase in SST from 5% to 15% is associated with a fall in TFR of roughly 1.2-1.3 children per woman. Indeed, the proximity of the point estimate for SST in equations IV and V is quite remarkable.

In sum then, the data suggests a strong, negative correlation between the size of the Social Security System and the number of children born per woman. This effect is robust to the inclusion of other variates and is also robust to the data set examined. Although the estimated effects vary to some degree over the different data sets, it

⁷The data on Social Security for Austria, Belgium, Denmark, Finland, France, Ireland and Norway is from MZES (Mannheimer Zentrum für Europäische Sozialforschung) and EURODATA in cooperation with ILO (International Labour Organization) "The Cost of Social Security: 1949-1993". For Spain, the data comes from private communication from Sergi Jimenez-Martin.

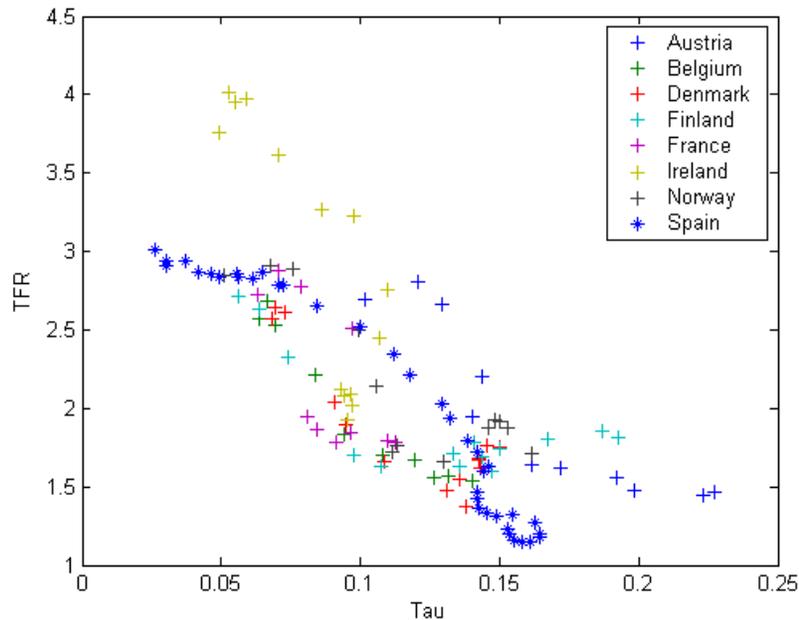


Figure 2: SS tax and TFR in 8 European Countries

suggests that an increase in the size of the social security system on the order of 10% is associated with a reduction in TFR of between 0.7 and 1.6 children per woman.

These results are of considerable interest but also must be interpreted with care. In many countries, the social security system not only provides old-age insurance (i.e., an annuity) provided by the government, but also has an element of forced savings. That is, the benefits paid out to an individual are also dependent, to varying degrees in different countries, on the contributions made over the working lifetime of the payee. Because of this, the exact relationship between SST in these regressions and the social security tax rate in subsequent sections is imperfect. That is, in the models, we will assume that SST is financed through a labor income tax and is paid out lump sum. Thus, from the point of view of testing the model predictions, we would ideally like to have data on that part of SST that most closely mirrors this. Data limitations prevent us from this however. Thus, the effective change in the SST that is relevant for the models is probably smaller than what we find here.

3 Social Security in the Caldwell Model of Fertility

In this section, we lay out the basic model of children as a parental investment in old age care. In doing this, we follow the development in Boldrin and Jones (2002) quite closely. That is, we assume that there is an altruistic effect going from children to parents, that parents know that this is present, and that they use it explicitly in choosing family size. Thus, the utility of children is increasing in the consumption of their parents, when the latter are in the third and last period of their lives. In our calibration exercise an effort is made to impose a certain degree of discipline on our modeling choice; we use available micro evidence to calibrate the size of the intergenerational transfers in relation to wage and capital income. In modeling the pension system we will make the simplifying assumption that Social Security payments go only to the old and are lump sum. In many real world Social Security Systems, the size of the payment received when old is linked (sometimes only weakly) to the contributions the worker makes when in the labor force. They typically have a redistributive component in addition to an annuity structure. We will abstract from these considerations for simplicity. It is likely that, since social security systems are a larger fraction of overall wealth for those agents in the lower part of the income distribution, and those individuals also have slightly more children, inclusion of this source of heterogeneity would only increase the size of the effects that we are capturing here; a quantitative evaluation of the impact of these features of actual social security systems will be presented in Section 6.

Our baseline characterization of the Social Security system is therefore one in which pensions are lump-sum, while financing is provided via a payroll tax. Accordingly, let T_t^o denote the transfer received by the old in period t , and let τ_t denote the labor income tax rate on the middle aged in period t .

As is standard in fertility models, we will write the cost of children in terms of both goods and labor time components (a_t and $b_t w_t$, respectively). We assume that labor is inelastically supplied, but that it can be used either for market work or for childrearing. Thus, total labor income, after taxes is given by $(1 - \tau_t)w_t(1 - b_t n_t)$, where n_t denotes the number of young people born at time t . Capital, which in our formulation encompasses all kinds of durable assets, is owned by the old; a fraction of its total value is assumed to be automatically transferred to the middle-aged at the end of the period. We will also assume that the pension system is of the “pay as you go” kind, so that, in equilibrium, $T_t^o = n_{t-1} \tau_t w_t (1 - b_t n_t)$. Notice that we use superscripts, y , m , and o to denote, respectively, young, middle-age and old people. Thus, the problem of an agent i , born in period $t - 1$, $i = 1, \dots, n_{t-1}$, is to:

$$\text{Max } U_{t-1} = u(c_t^m) + \zeta u(c_t^o) + \beta u(c_{t+1}^o),$$

subject to the constraints:

$$\begin{aligned}
d_t^i + s_t + c_t^m + a_t n_t &\leq (1 - \tau_t) w_t (1 - b_t n_t) \\
c_t^o &\leq d_t^i + \sum_{\substack{j=1 \\ j \neq i}}^{j=n_{t-1}} d_t^j + (1 - \xi) R_t x_t + T_t^o \\
c_{t+1}^o &\leq \sum_{j=1}^{j=n_t} d_{t+1}^j + (1 - \xi) R_{t+1} x_{t+1} + T_{t+1}^o \\
x_{t+1} &\leq \xi R_t x_t / n_{t-1} + s_t.
\end{aligned}$$

Here, c_t^m is the consumption of the typical middle aged person in period t , c_t^o is the consumption of an old person, s_t is the amount of savings, n_t is the number of children, d_t^i is the level of support the agent gives to his/her parents, x_t is the amount of the capital stock each old person controls in period t , w_t is the wage rate, R_t is the gross return on capital in the period, T_t^o is the lump sum transfer received when old, and τ_t is the Social Security tax rate on labor income. Note we assume that the decision maker, i , takes d_t^j , $j \neq i, j = 1, \dots, n_{t-1}$, x_t , n_{t-1} , R_t , R_{t-1} , and the taxes, T_t^o , T_{t+1}^o and τ_t as given. Among other things, this implies that, when choosing a donation level, the representative middle age agent does not cooperate with his own siblings to maximize total utility. Instead, he takes their donations to the parents as given, and maximizes his own utility by choosing a best response level of donations. In Boldrin&Jones (2002) we call this behavior “non-cooperative” and contrast it with a “cooperative” behavior in which members of the same generation choose donations in such a way that the sum of their utilities is maximized. Also, note that we have assumed that middle aged individuals work, but that the elderly do not; it is not our intention to model here the impact that the presence of a Social Security system may or may not have on the life-cycle labor supply of individuals. Notice that we can rewrite the middle aged budget constraint as:

$$d_t^i + s_t + c_t^m + \theta_t(\tau) n_t \leq (1 - \tau_t) w_t,$$

where $\theta_t(\tau) = a_t + (1 - \tau_t) b_t w_t$. Since θ_t is exogenous to the individual decision maker, using this shorthand will simplify the presentation. In addition to introducing a social security tax and transfers, we also have deviated from the original Boldrin&Jones paper in that we have included a change in the law of motion of wealth per old person:

$$x_{t+1} = \xi R_t x_t / n_{t-1} + s_t.$$

The parameter ξ affords us a simple way of modeling differences, both across countries at a given time, and across time in a given country, in both the inheritance mechanisms and the access to financial institutions. This will allow us to study the idea that increased access to financial markets increases the rate of return on private

savings to physical capital, which also lessens the value of within-family support in old age, thereby causing fertility to fall. This allows us to capture capital depreciation while providing some freedom in our handling of the effective life-time rate of return on wealth accumulation. To do this we proceed as follows. Let $0 < \delta < 1$ be the depreciation rate per period. Write $R_t = (1 - \delta) + F_k(K, AL)$, where F is the aggregate production function, K is capital, L is aggregate labor supply and A is the level of TFP; subscripts denote, here and in what follows, partial derivatives. We will let ξ range in the interval $[0, 1]$. When $\xi = 0$ capital markets are fully operational, there no involuntary or legally imposed bequests, and the old people are able to consume the total return from their middle age savings. On the contrary, when $\xi = 1$, old people have no control whatsoever on their savings, which are entirely and directly passed to the offsprings, whom in turn will be unable to get anything out of them, and so on. In this extreme case, no saving will take place and children's donations are the only viable road to consumption in old age. As usual, reality will lay somewhere in between these two extremes, as discussed in the calibration section.

After substituting in the constraints and using symmetry for donations of future children, this problem can be reformulated as one of solving:

$$\max_{s_t, n_t, d_t} V(s_t, n_t, d_t),$$

where the concave maximizer is defined as

$$\begin{aligned} V(s, n, d) = & \\ = & u[(1 - \tau_t)w_t - d - s - \theta_t n] + \zeta u \left[d + \sum_{j \neq i, j=1}^{j=n_t-1} d_t^j + (1 - \xi)R_t x_t + T_t^o \right] + \\ & + \beta u [nd_{t+1} + (1 - \xi)R_{t+1}[\xi R_t x_t / n_{t-1} + s] + T_{t+1}^o]. \end{aligned}$$

This gives rise to First Order Conditions:

$$\begin{aligned} 0 &= \partial V / \partial d, \text{ or, } u'(c_t^m) = \zeta u'(c_t^o) \\ 0 &= \partial V / \partial s, \text{ or, } u'(c_t^m) = \beta u'(c_{t+1}^o) \frac{\partial c_{t+1}^o}{\partial s} \\ 0 &= \partial V / \partial n, \text{ or, } \theta_t u'(c_t^m) = \beta u'(c_{t+1}^o) \frac{\partial c_{t+1}^o}{\partial n} \end{aligned}$$

A fundamental Rate of Return condition follows immediately from the last two equations; this is:

$$\text{R of R) } \quad \frac{\partial c_{t+1}^o}{\partial s} = \frac{\partial c_{t+1}^o}{\partial n} / \theta_t.$$

Assuming now that $u(c) = c^{1-\sigma} / (1 - \sigma)$, the three first order conditions can be

written in a form which allows for further algebraic manipulation, i.e.

$$c_t^o = \zeta^{1/\sigma} c_t^m \quad (1)$$

$$c_{t+1}^o = \beta^{1/\sigma} c_t^m \left[\frac{\partial c_{t+1}^o}{\partial s_t} \right]^{1/\sigma}, \quad (2)$$

$$\theta_t^{1/\sigma} c_{t+1}^o = \beta^{1/\sigma} c_t^m \left[\frac{\partial c_{t+1}^o}{\partial n_t} \right]^{1/\sigma} \quad (3)$$

Substituting in the budget constraints and imposing symmetry in the choice of donations (i.e., that $d_t = d_t^j$) equation (3.1) gives:

$$n_{t-1}d_t + (1 - \xi)R_t x_t + T_t^o = \zeta^{1/\sigma} [(1 - \tau_t)w_t - d_t - s_t - \theta_t n_t].$$

Solving this for d_t gives:

$$d_t = \frac{1}{\zeta^{1/\sigma} + n_{t-1}} \left[\zeta^{1/\sigma} ((1 - \tau_t)w_t - s_t - \theta_t n_t) - (1 - \xi)R_t x_t - T_t^o \right].$$

Using this in the budget constraint for the old, we see that

$$c_t^o = \frac{\zeta^{1/\sigma}}{\zeta^{1/\sigma} + n_{t-1}} [n_{t-1} ((1 - \tau_t)w_t - s_t - \theta_t n_t) + (1 - \xi)R_t x_t + T_t^o].$$

Thus, after some algebra, we obtain the two rates of return:

$$\frac{\partial c_{t+1}^o}{\partial s_t} = \frac{\zeta^{1/\sigma} (1 - \xi) R_{t+1}}{\zeta^{1/\sigma} + n_t},$$

$$\begin{aligned} \frac{\partial c_{t+1}^o}{\partial n_t} &= \\ &= \frac{\zeta^{1/\sigma}}{(\zeta^{1/\sigma} + n_t)^2} \left[\zeta^{1/\sigma} ((1 - \tau_{t+1})w_{t+1} - s_{t+1} - \theta_{t+1} n_{t+1}) \right] \\ &\quad - \frac{\zeta^{1/\sigma}}{(\zeta^{1/\sigma} + n_t)^2} [(1 - \xi)R_{t+1} x_{t+1} + T_{t+1}^o]. \end{aligned}$$

In principle, the last two equations can be substituted into the first order conditions above for solution if that is desirable computationally. Alternatively, we can view them as definitions of the ‘free variables’ $\frac{\partial c_{t+1}^o}{\partial s_t}$, and, $\frac{\partial c_{t+1}^o}{\partial n_t}$, with their respective equations included as the extra equations that they must satisfy. What remains is to determine the three prices w_t , R_t , and θ_t from the other endogenous variables. We write feasibility in per old person terms:

$$n_{t-1}c_t^m + c_t^o + n_{t-1}a_t n_t + n_{t-1}s_t \leq Y_t = F(x_t, A_t n_{t-1}(1 - b_t n_t)),$$

where $K_t = x_t$ is the amount of capital per old person, and $L_t = A_t n_{t-1}(1 - b_t n_t)$ is the amount of labor supplied per old person; F is assumed to be CRS. We are assuming that only middle age people supply labor. From this, it follows that

$$\begin{aligned} w_t &= F_\ell(x_t, A_t n_{t-1}(1 - b_t n_t)), \\ R_t &= F_k(x_t, A_t n_{t-1}(1 - b_t n_t)), \text{ and,} \\ \theta_t &= a_t + (1 - \tau_t)b_t w_t. \end{aligned}$$

Thus, given the initial conditions n_{-1} , n_0 , x_0 , the sequence of exogenous variables a_t , b_t , A_t , τ_t , and T_t^o , and the model's parameters, the full system of equations determining the equilibrium sequences is thereby determined.

3.1 Exogenous Growth and BGP's

We assume that there is exogenous labor augmenting technological change, $A_t = \gamma_A^t A_0$. As it is well known, for there to be balanced growth it must also be that $a_t = \gamma_A^t a_0$, $b_t = b$, and $\tau_t = \tau$. Accordingly we define the detrended variables in the standard way. That is, $\hat{c}_t^o = c_t^o / \gamma_A^t$, $\hat{c}_t^m = c_t^m / \gamma_A^t$, $\hat{d}_t = d_t / \gamma_A^t$, $\hat{s}_t = s_t / \gamma_A^t$, $\hat{x}_t = x_t / \gamma_A^t$, and $\hat{T}_t^o = T_t^o / \gamma_A^t$. Finally, we denote $n_t / n_{t-1} = \gamma_{nt}$. Under our assumptions, if \hat{x}_t , \hat{s}_t , and γ_{nt} converge to constants then, so do \hat{w}_t , R_t , and $\hat{\theta}_t$ and, consequently, the equilibrium quantities. The Balanced Growth Equations that these must satisfy are given by:

$$\hat{c}^o = \zeta^{1/\sigma} \hat{c}^m \quad (4)$$

$$\hat{c}^o = \frac{\beta^{1/\sigma}}{\gamma_A} \hat{c}^m \left[\frac{\partial c^o}{\partial s} \right]^{1/\sigma} \quad (5)$$

$$\hat{c}^o = \left[\frac{\beta}{\hat{\theta} \gamma_A^{(\sigma-1)}} \right]^{1/\sigma} \hat{c}^m \left[\frac{\partial c^o}{\partial n} \right]^{1/\sigma} \quad (6)$$

$$\frac{\partial c^o}{\partial s} = \frac{\zeta^{1/\sigma} (1 - \xi) R}{\zeta^{1/\sigma} + \gamma_n} \quad (7)$$

$$\frac{\partial c^o}{\partial n} = \frac{\zeta^{1/\sigma}}{(\zeta^{1/\sigma} + \gamma_n)^2} \left[\zeta^{1/\sigma} \left((1 - \tau) \hat{w} - \hat{s} - \hat{\theta} \gamma_n \right) - (1 - \xi) R \hat{x} - \hat{T}^o \right] \quad (8)$$

$$\hat{c}^m = (1 - \tau) \hat{w} (1 - b \gamma_n) - \hat{a} \gamma_n - \hat{d} - \hat{s} \quad (9)$$

$$\hat{c}^o = \gamma_n \hat{d} + (1 - \xi) R \hat{x} + \hat{T}^o \quad (10)$$

$$\hat{x} = \frac{\xi R \hat{x}}{\gamma_A \gamma_n} + \frac{\hat{s}}{\gamma_A} \quad (11)$$

$$\hat{w} = F_\ell(\hat{x}, A_0\gamma_n(1 - b\gamma_n)), \quad (12)$$

$$R = (1 - \delta) + F_k(\hat{x}, A_0\gamma_n(1 - b\gamma_n)), \quad (13)$$

$$\hat{\theta} = \hat{a} + (1 - \tau)b\hat{w}, \quad (14)$$

$$\hat{T}^o = \gamma_n\tau\hat{w}(1 - b\gamma_n). \quad (15)$$

We can use (4) in (5), simplify consumption, and substitute $\frac{\partial c^o}{\partial s}$ using (7) to obtain the following equation for γ_n :

$$\gamma_n = \zeta^{1/\sigma} \left(\frac{\beta(1 - \xi)R}{\gamma_A^\sigma \zeta} - 1 \right)$$

From the above equation it is clear that steady state fertility only depends on the preference parameters ζ , β , and σ , the exogenous rate of growth of technological progress γ_A , the equilibrium interest rate, R , and the degree of capital market imperfection ξ . This implies that the other parameters, such as the costs of having children or the size of the social security system, impact steady state fertility only indirectly, through general equilibrium effects embedded in the interest rate. Therefore, in small closed economies, or in economies with a linear technology and fixed prices, there would be no such effects. Most notably, fertility would be invariant to both the size of the social security system and the costs of having children. The Barro and Becker model of fertility, as we will show, displays a similar feature. In both models, the effects of social security on fertility come from general equilibrium effects.

Increasing ξ corresponds to forcing the old to pass on more of their savings to their children and thus represents reducing access to capital markets. This has a direct effect on the growth rate of population as can be seen. Surprisingly, holding R constant and increasing ξ causes γ_n to fall, the opposite of what one would expect. There is also an indirect effect of a change in ξ on R . A careful examination of the RofR condition shows that the indirect effect goes in the opposite direction. In fact, due to the general equilibrium equalization of the rate of return on saving with the rate of return on fertility, an increase in ξ leads to lower investment in physical capital and, hence, a higher value of R in equilibrium. Because of these offsetting effects, the overall impact of more efficient capital markets on the value of $(1 - \xi)R$ and, hence, on the growth rate of population depends on parameters. In section 6, below, we find that the overall effect is negative as would be expected.

4 Social Security in the B&B Model of Fertility

In this section, we develop the equations determining the fertility effects of a social security system in the Barro and Becker model. There is a basic problem with trying to study Social Security in a Barro&Becker model. This is that they assume that

people only live two periods, youth and adulthood, and hence there is no time when the middle aged can be taxed to finance consumption of the old. Because of this, we will adapt the model to allow for three period lives. As above, we assume that individuals work when they are middle aged, but do not when they are old. As in this case we want to consider also the impact of a pure lump sum pension system, let T_t^m denote the lump sum tax on the middle age in period t . That is, we will write the problem of the dynasty as choosing N_t^y , N_t^m , N_t^o , c_t^m , c_t^o and k_t to solve:

$$\max U_0 = \sum_{t=0}^{\infty} \beta^t [g(N_t^m)u(c_t^m) + \zeta g(N_t^o)u(c_t^o)],$$

subject to:

$$N_t^o c_t^o + N_t^m c_t^m + N_t^y (a_t + k_{t+1}) \leq (1 - \tau_t)(N_t^m - bN_t^y)w_t + N_t^m R_t k_t + N_t^m T_t^m + N_t^o T_t^o.$$

As above, we let

$$\theta_t(\tau_t) = a_t + (1 - \tau_t)bw_t = a_t + b(\tau_t)w_t,$$

and use the simplification that

$$N_t^o = N_{t-1}^m = N_{t-2}^y.$$

Then, we can rewrite this problem as:

$$\max U_0 = \sum_{t=0}^{\infty} \beta^t [g(N_t^m)u(c_t^m) + \zeta g(N_{t-1}^m)u(c_t^o)],$$

subject to:

$$N_{t-1}^m c_t^o + N_t^m c_t^m + N_{t+1}^m (\theta_t + k_{t+1}) \leq N_t^m [(1 - \tau_t)w_t + R_t k_t + T_t^m] + N_{t-1}^m T_t^o.$$

Note that we have assumed that although the tax variables, τ_t , T_t^m , and T_t^o are taken as given, the dynasty head understands that changing N_t^m (for example), changes both his tax obligation and his transfer receipts. An alternative formulation would have the head taking total transfers in each period as given. Presumably, since we want to model the Social Security system as transferring money from workers to retirees, we have that $T_t^m < 0$, and $T_t^o > 0$. Note that a version of Ricardian Equivalence follows immediately in this version of the model if $\tau_t = 0$ for all t , and $N_t^m T_t^m = -N_{t-1}^m T_t^o$ for all t , that is, if the system is PAYGO with lump sum transfers only. That is, changes in these lump sum transfers do not affect the equilibrium. Notice also that in this model we cannot take into consideration the impact that financial markets imperfections may or may not have over the rate of population growth; because all capital accumulation decisions and the associated return are

handled within the infinitely lived dynasty, capital markets external to the family cannot possibly matter.

To keep as close as possible with the standard Barro-Becker formulation we will assume that $g(N) = N^\eta$ and $u(c) = c^{1-\sigma}/(1-\sigma)$.⁸ Retaining the notation from the previous section and assuming the same production function we get (after some algebra) that on a Balanced Growth Path, the system must satisfy:

$$\hat{c}^o = \zeta^{1/\sigma} [\gamma_n]^{(1-\eta)/\sigma} \hat{c}^m, \quad (16)$$

$$\gamma_n^{1-\eta} \gamma_A^\sigma = \beta R, \quad (17)$$

$$\begin{aligned} \hat{a} + b(1-\tau)\hat{w} &= \frac{\gamma_A}{R} \left[(1-\tau)\hat{w} + \hat{T}^m + \frac{(\eta+\sigma-1)}{(1-\sigma)} \hat{c}^m \right] \\ &+ \frac{\gamma_A^2}{R^2} \left[\hat{T}^o + \frac{(\eta+\sigma-1)}{(1-\sigma)} \hat{c}^o \right] \end{aligned} \quad (18)$$

$$\hat{c}^o + \gamma_n \hat{c}^m + \gamma_n^2 (\hat{a} + b(1-\tau)\hat{w} + \gamma_A \hat{k}) = \gamma_n [(1-\tau)\hat{w} + R\hat{k} + \hat{T}^m] + \hat{T}^o \quad (19)$$

$$R = (1-\delta) + F_k(\hat{k}, 1 - b\gamma_n), \quad (20)$$

$$\hat{w} = F_\ell(\hat{k}, 1 - b\gamma_n), \quad (21)$$

$$\hat{T}^o = \gamma_n (\tau \hat{w} (1 - b\gamma_n) - \hat{T}^m). \quad (22)$$

Notice that equation (4.2) shows, once again, that steady state fertility only depends on the preference parameters η , β , σ , the growth rate of exogenous technological progress γ_A and the equilibrium interest rate. Therefore, as in the Boldrin and Jones' framework, all other parameters, including the size of the social security system, and the costs of having children, only impact fertility indirectly through the interest rate. These effects are thus absent in presence of fixed prices. In sum then, neither of the two models delivers an explicit and unambiguous prediction about the direction of the effect of the introduction of a PAYGO social security system on fertility and the growth rate of population. Thus, any effect can only be identified through a more thorough, quantitative exercise. This is what we turn to next.

5 Quantitative Results

In this section, we present quantitative comparative statics results for calibrated versions of the two models. We start by calibrating the model economies to match some key facts of the US economy, and then perform comparative statics by changing the payroll tax over the interval from zero to 25%, a number consistent with the total employee and employer Social Security contributions in some countries in Europe. We have also done extensive sensitivity analysis with respect to all of the parameter

⁸Monotonicity and concavity place some restrictions on the allowable values of η and σ . These are $\sigma < 1$, $1 < \eta + \sigma < 2$, and $0 < \eta < 1$. See the appendix for details.

values. We have found that our key conclusions are the same for a very wide range of all of the parameter values. Throughout, we assume that a period is 20 years; this choice distorts some of the model's predictions as it implies that, over the life cycle, working and retirement years are equal, whereas they stand in a ration of 2 to 1 in reality.

We will begin by considering the Caldwell model in the case where financial markets are frictionless, $\xi = 0$. The impact of $\xi > 0$ on fertility will be considered in the Sensitivity Analysis section, below.

5.1 Utility

Recall from Section 3 that for the Boldrin-Jones model, the period utility function is given by:

$$u(c_t^m, c_t^o, c_{t+1}^o) = \frac{(c_t^m)^{1-\sigma}}{1-\sigma} + \zeta \frac{(c_t^o)^{1-\sigma}}{1-\sigma} + \beta \frac{(c_{t+1}^o)^{1-\sigma}}{1-\sigma},$$

while for the Barro-Becker model it is given by:

$$u(N_t^m, c_t^m, N_t^o, c_t^o) = (N_t^m)^\eta \frac{(c_t^m)^{1-\sigma}}{1-\sigma} + \zeta (N_t^o)^\eta \frac{(c_t^o)^{1-\sigma}}{1-\sigma}.$$

The two utility functions are not strictly equivalent. This is because the two models are built on very different, almost opposite, assumptions about the nature and direction of intra-familial relationships, filial altruism in one case, parental altruism in the other. One of our goals is to evaluate quantitatively how these differential assumptions about the endogenous source of fertility impact on the model's ability to explain the observed correlation between TFR and size of the Social Security system. In calibrating the model we will try, to the extent possible, to treat the two models symmetrically, hence we will try to use the same elasticity of substitution in consumption parameter σ , the same discount factor β , and to attribute the same degree of importance ζ to the consumption of the elderlies. When we deviate from this rule it is due, as shown below, to the inability of one of the two models to match crucial and clearly measurable features of the US economy.

5.2 Production

We assume that the production function is CRS with constant depreciation, and is given by:

$$(1 - \delta)K + F(K, L) = (1 - \delta)K + AK^\alpha L^{1-\alpha}$$

Inputs and output markets are competitive.

5.3 Parameter values

Setting $\xi = 0$, there are a total of ten parameters in the two models we are considering, nine of which are in common while the tenth, $\eta \in (0, 1)$, is specific to the Barro-Becker model. A number of these parameters are used in macroeconomic models of growth and the business cycle, hence, in calibrating them we follow the existing literature for as many as we can. Accordingly, we normalize A to 1, we set annual depreciation to 8%, and we fix the share of income that goes to capital to either 0.22 or 0.33.⁹ The time horizon of our exercise is, roughly, 1930-2000 and we are interested in capturing long run movements in fertility. The period 1930-1940 coincides with the Great Depression, during which Total Factor Productivity was abnormally low. Estimating the average growth rate of productivity from 1935 to 2000 gives an abnormally high number, which would not be representative of the century-long actual trend. Hence, we have set the parameter γ_A equal to 1.2% on a yearly basis following Dennison's calculations for the 20th century.

Additionally, we have made the choice to set the relative weights on the flow utility from current consumption of the old (ζ) to be one for both models. While it makes our life easier this choice implies, obviously, that on a per capita basis consumption of old parents is equal to that of middle age children. This contradicts the evidence from the empirical life time consumption literature that suggests a drop in all measures of per capita consumption after retirement; a reasonable estimate of the ratio between average consumption while working and while retired yields a value of about 0.80. For this ratio to be obtained by c^o/c^m we need to set $\zeta = 0.83$, which is significantly different than our choice of $\zeta = 1.0$. The impact of this choice is also considered in the Sensitivity Analysis section below.

Given these choices, we still need to determine the values of the four common parameters β , σ , a , and b , and for the Barro-Becker model we must also choose a value of η . To make our results as clear as possible, for each model we consider two extreme cases: one in which all of the costs of raising children are in terms of goods ($b = 0$), the other in which they are completely in terms of time ($a = 0$). This implies calibrating three parameters at a time for the Caldwell model, and four for

⁹The choice of $\alpha = .22$, come from the macroeconomic home production literature, e.g., McGrattan, Rogerson and Wright (1997), while the choice of $\alpha = .33$ is typical of the aggregate business cycle literature. The difference between these two values comes from the recognition that much of the measured capital stock (residential real estate and durable goods) but a relatively smaller share of measured output (imputed service from residential housing) is properly assigned to home production. Depending on which classification one adopts, the measured capital/output ratio varies substantially. Our model does not include home production and, as such, we find the value of $\alpha = .33$ more coherent; nevertheless we have performed simulations with both values to check for the robustness of the basic results. An interesting extension of this work would follow the home production literature more closely using both parental time and home capital goods to produce, jointly, child care and other home goods. This is beyond the scope of the current paper, but see Schoonbroodt (2004) for an interesting step in that direction.

the Barro-Becker model. Both models make either explicit or implicit predictions about a large number of potentially measurable variables that could be used to help in the calibration: the real rate of return on safe investments, donations as a share of income or consumption, the total fertility rate and the growth rate of the population, the amount of time and/or resources devoted to rearing children, the composition of the population by age group, etcetera. As we need to pin down only three parameters we will need three independent observations.

To do this, we need to choose the historical period to which the calibrated predictions should refer. The Social Security Administration was created in 1935 and, until after the end of the Second World War, it covered only a limited number of people.¹⁰ It would be incorrect to believe, though, that no PAYGO pension scheme existed before 1935; a number of such plans had emerged since the early 1900s. These were primarily limited to state and federal employees and army veterans, but this had a non-negligible impact at the aggregate level. Further, the years between 1900 and the 1930s were witness to very rapid growth of the urban population; in 1920 for the first time in the USA's history, more people were living in cities than on farms; this process continued all the way to the WWII period and was in some sense "completed" only at the end of it. As argued in Boldrin and Jones (2002), the "non-cooperative" solution to the donation game, which we have used here, is better suited than the cooperative one to model the behavior of a urban society since, in this case, extended-family ties and direct within-family transmission of land are limited to a minority of the population. Finally, two other major factors, the Great Depression and the onset of World War II combine to make the 1930-1945 period a unique one in U.S. history.

These historical events had a huge impact on realized fertility which are visibly apparent in Figure 3. Measured TFR, which had been steadily decreasing since 1800, took a sharp swing downward around 1920 and was particularly low during the 1930-1940 decade. It then rose back to much higher levels (about 50% higher, in fact) during the 'baby boom' between 1940 and 1960, after which it decreased again to the current low levels.¹¹ These considerations make it difficult to use any of the dates over this period as a benchmark for calibration, viz., the parameter values necessary to match the low levels of fertility in 1935 are significantly different from those that work for the boom of 1950.¹² Because of this, we adopt the opposite approach. We

¹⁰Beneficiaries were about fifty thousands in 1937 and had grown to two hundred thousand by 1940; in 1950 the number of beneficiaries reached 3.5 million; see <http://www.ssa.gov/history/briefhistory3.html> for more details.

¹¹This pattern is even more striking in the time series of 'completed fertility' by cohort, the total number of children per capita that women of a given cohort have over their life time. Using that measure, women born between 1880 and 1915 averaged about 2.2 births over their lifetimes. This climbed to a peak of about 3.1 for women born around 1935 and then slowly fell, reaching 2.0 for the 1950 birth cohort. Since this statistic matches up better with the concept of life time fertility choices for a given individual, this is even more telling.

¹²Also, the model we analyze here is purposely stylized and thus misses some important factors: mortality rates, female labor force participation and human capital accumulation being the three

choose parameter values that match figures for the current U.S. economy (i.e., TFR, interest rates, K/Y ratios and childbearing costs) and ‘go backwards’ in time. That is, we ask: Given a parameterization of the models that matches the relevant facts for 2000 with the Social Security system in place in 2000, what does the model imply about what fertility, etc., should have been like when $\tau = 0$?

This approach does not completely avoid the difficulties of matching the data during the 1930 to 1950 period, since we still have to compare the model predictions with the data from that period. However, it is also clear from Figure 3 that there is a natural long run trend of decreasing fertility which mimics most of the last two centuries and around which the 1920-1960 fertility bust-boom swings almost perfectly. All of this suggests taking the fitted values for that period as the standard of comparison for the model. For this, we need to obtain a reasonable estimate of how large the change in fertility was between 1935 and 2000. If one draws a trend line through the TFR data of Figure 3¹³, the predicted values for 1935 is 2.83 children per woman. This is the value that we will use as being representative of pre-Social Security fertility in the U.S. At the other end of the horizon, TFR was at 1.75 in 1980, at 2.03 in 1990, and it is around 2.06 currently. Thus, we will take a TFR of 2.00 to be the current “steady-state” level. The total variation in TFR we are interested in explaining is, therefore, around 0.83 children per woman. Since our model features monoparental families, this corresponds to a variation of about 0.41 between the two BGP values of γ_n .

Finally, the parameters describing the Social Security system must be chosen for the model. The exact form of the US Social Security system is much more complex than what we allow for here. It is not exactly PAYGO, payments received depend, to some extent on what was paid in, etc. Figure 4 shows the time paths of both receipts and expenditures of the Social Security system from 1937 to date. These figures include both Social Security and Medicare, but omit Social Security Disability Insurance since this is not restricted to the elderly. As can be seen these are approximately 7% of GDP over the last 20 years. Since labor’s share in income is 67% in the model, this corresponds to an average labor income tax rate of approximately 10%, and this is the value we used in the calibration.

Finally, we need to have an estimate of the cost of raising a child. We focus on the case in which this cost is entirely in time, i.e., $a = 0$, and $b > 0$. For this, we set b to be 3% of the available family time, which corresponds to roughly 6% of the mother’s time per child. When total fertility is about 2.0 children per woman this number is consistent with the estimates on time-use data reported by Juster and Stafford (1991), with the one estimated by Merlo (1999) using data fitted to an international

most prominent ones See Boldrin and Jones (2002) and Boldrin, Jones and De Nardi (2004) for the first, Schoonbroodt (2004) for the second, and Boldrin, Jones and De Nardi (2004) for the third.

¹³This data is taken from Haines (1994).

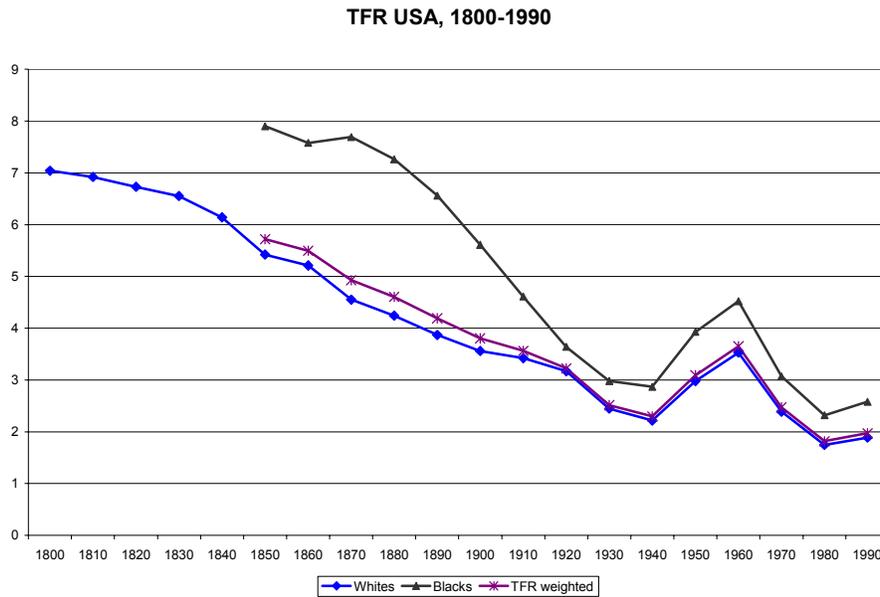


Figure 3: TFR in the USA: 1800-1990

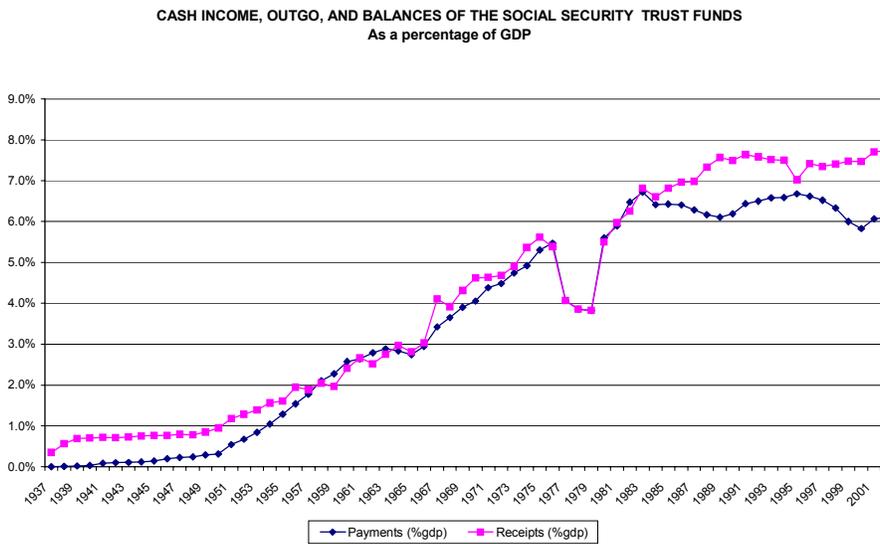


Figure 4: Social Security Receipts and Expenditures/GDP: 1937-2004

cross section, and also with the estimates reported by Moe (1998) which are based on Peruvian micro data. This number ($b = 3\%$) seems surprisingly low. This has been discussed to some extent in the time use literature and the consensus seems to be that it is due to the fact that these studies neglect the possibility of joint production in the home. That is, these studies report that the fraction of mothers time in the home which the *primary* activity is child care is 6%, but they do not include options for multiple activities – cooking and child care simultaneously for example. In our context, it should be added, using $b = 3\%$ actually overestimates the time cost of a child; this is because, in our model, the fraction b is applied to the total time available for work during the whole working life (20 years, in our calibration), while the 6% of mother’s time per child reported in the quoted studies refers only to the infancy-childhood years, which are generally substantially less than 20. From this point of view, then, a value of b between 2% and 2.5% may be more appropriate; again, we refer to the Sensitivity Analysis section for this case.

Given this discussion, we will adopt the following three target values for our calibration:

- (a) capital output ratio: 2.4 (annual basis),
- (b) the total fertility rate: 2.0 children per woman, and,
- (c) the amount of time allocated to rearing children: 3% of family time per child.

These should all hold for the year 2000, when $\tau = 10\%$. As we will see below, both of the models have some difficulties matching these three targets perfectly.

For the **Boldrin and Jones’ version of the Caldwell model** we need to pick a combination of β , σ , and a (or b) consistent with the real world values of (a)-(c). Notice next that, when $\zeta = 1.0$, the elasticity of intertemporal substitution in consumption plays a very secondary role. Our choice of $\sigma = 0.95$ and $\beta = .99$ (yearly) yields a TFR of 1.82 and an annual capital output ration of about 2.4 when $\tau = 0.10$. These two choices together imply an interest rate of about 2.9% per year, perhaps a bit on the low side, when $\alpha = 0.33$.¹⁴ For the case in which the time cost (b) is zero, we keep all other parameters the same and we set the good cost of raising children (a) so that the resulting good cost of raising children as a fraction of per-capita output turns out to be 4.5%. This is a value for which we have a hard time finding real-world estimated counterparts, so we picked it only because it was consistent with observed TFR, capital/output ratios and interest rates at $\tau = .10$ and when all other calibrated parameters remained the same as above.

For the **Barro and Becker model** we have, instead, four parameters: β, σ, η , and either a or b to choose. There are, as discussed in the theoretical section, restrictions on these parameters: $\sigma < 1$, and $\sigma + \eta > 1$. Given these restrictions, we were not able to set the time cost of having children anywhere near what one would call “realistic” values while, at the same time, matching a TFR of 2.0 for any value of τ in the

¹⁴When, instead, $\alpha = 0.22$ is used we obtain a K/Y ratio of around 1.4 which is also not dissimilar from the one observed in the data when adjustments are made for residential structures and consumer durables. This also allows a considerable reduction in ζ still holding γ_n at about 1.

relevant range. To match targets similar to those described above, we must set the time cost of having a child to 33% of the available time and then adjust the other three parameters, β , σ , and η , accordingly. Despite this, the parameter η needs to be set as low as possible to keep fertility at a reasonable level in the Barro-Becker model. When we set b to zero and follow the same procedure as with the Boldrin and Jones model, the level of child care costs that this requires is $a = 27\%$ of per capita income.

The parameter values used in the baseline calibration are summarized in Table 2.

Parameter	Caldwell model	B and B model	Source
γ_A	1.012	1.012	Dennison
A	1.0	1.0	Normalization
α	0.33 or 0.22	0.33 or 0.22	RBC or MRW
δ	8%	8%	RBC
ζ	1.0	1.0	Arbitrary
β	0.99	0.96	Targets (a)-(c)
σ	0.95	0.95	Targets (a)-(c)
η	n/a	0.12	Targets (a)-(c)
(a, b)	(0, 3%) or (4.5%, 0)	(0, 33%) or (27%, 0)	Time use data

Table 2: Model Parameters

Thus, the two frameworks require very different costs (both in terms of time and goods) to match the key facts about population growth and capital accumulation. This is due to the type and intensity of the external effects present in the two frameworks. In the Barro-Becker setup, children are such a powerful generator of utility for their parents that it takes a very high cost of child rearing to prevent families from having a very large number of children. In the Boldrin and Jones' framework the benefit from having children is relatively smaller, hence, the costs of raising children are more in line with the available empirical estimates.

In all of these experiments we have assumed that there are perfect capital markets. For the Boldrin and Jones' framework, we have also experimented with some degree of capital market imperfection, meaning that the old parents cannot consume all of their gross return on capital, but are forced to bequeath a fraction of it to their children. These results are reported in the Sensitivity Analysis section.

5.4 Quantitative Effects

There are two kinds of comparisons that we are interested in corresponding to the two different types of data sources discussed in section 2. These are: differences across time for a given country, and differences across countries at a given time.

We report first the main findings for the Boldrin and Jones model, with perfect capital markets and only time costs of having children. The figures graph different

BGP values for a given variable as a function of the Social Security tax rate. Figures 5-9 plot, in order, the values of γ_n , K/Y , c^m/y and c^o/y ¹⁵, s/y and d and nd corresponding to the values of τ on the horizontal axis.

As we can see, in a Caldwell-type of framework when the Social Security tax moves from zero to about 10%, the number of children decreases from about 1.15 to about 0.91 (0.9 if there are only good costs to raising children), the capital-output ratio increases from about 2.2 to 2.4, and there is a sizeable decline in consumption of about 3.0% for both middle aged and old. Finally, donations (both total and per-child) and savings also decrease. The drop in output caused by the the introduction of social security is large, roughly a 10% deviation from the undistorted balanced growth path level. This drop is larger than that for savings, generating an increase in the capital-output ratios. The drop in fertility is also large as it is equivalent to 0.48 children per woman.

Comparing this to US time series data, we see that the drop predicted by the model is equal to 60% of the observed total drop in TFR between 1935 and 2000. See Table 3.

variable	Data, 2000	Model, 2000	Data, 1935	Model 1935
τ	10%	10%	0%	0%
TFR	2.0*	1.8	2.8	2.3
c_o/Y		0.43		0.40
c_m/Y		0.43		0.40
K/Y	2.4	2.4		2.2
d/Y		0.04		0.05
nd/Y		0.07		0.12

Table 3: Model and Data, US: 1935 to 2000

As another benchmark, we constructed two subgroups of countries, one with ‘large’ Social Security systems, one with small.¹⁶

¹⁵Recall they are the same in this parameterization.

¹⁶The countries for the ‘high SST’ group are: Austria, Belgium, France, Germany, Italy, Netherlands, and Sweden. These are all the European countries for which SST/GDP exceeds 14%. The ‘low SST’ group includes: Argentina, Chile, Colombia, Iceland, Ireland, Korea, Panama, and Venezuela. This is an ad hoc group of countries from the 1997 cross section discussed in section 2. They share three properties: (i) low SST/GDP (all under 4%), (ii) low IMR’s (between 5 and 20 per 1000), and (iii) low share of population older than 64 (between 5% and 11%). In our cross country regressions these are the statistically significant variables.

Variable	τ	TFR
Low SST, Data 1997	3.6%	2.34
Low SST, Model	3.6%	2.10
US, 2000	10%	2.06
US, Model	10%	1.82
High SST, Data 1997	23.67%	1.47
High SST, Model	23.67%	1.37

Table 4: *Model and Data for 3 Groups of Countries*

From this we can see that the changes predicted by the model are roughly in line with what is seen in the data. Indeed, the size of fertility difference predicted from the model when moving from the low SST group to the high SST group is 0.73 children per woman, while that in the data is 0.87 children per woman. With respect to the cross section regressions presented earlier on, notice that the low SST group has, roughly, the same IMR rate as the high SST group but much lower values for the 65% variable: the range is 4.6-11.5, averaging at 7.9%, versus a range of 13.5-17.5 averaging at 15.8% for the high SST group. Regression IV shows that the quantitative impact of 65% on the predicted TFR is positive but very small, which provides is consistent with our simulated prediction. We should, however, compare our results also to what we found in our econometric estimates; there, once we control for infant mortality and the fraction of the population over 65, a 20% increase in the social security tax is associated with a drop in TFR of between 1.3 and 2.5 children per woman. Thus, our model accounts for between 30% and 55% of the observed differences in fertility in the overall cross-section.

The channels through which a change in the social security system affects fertility in this model are complex. From the budget constraint of a middle age agent, we see that disposable income is reduced, while from the budget constraint for the same agent when old, disposable income is increased, *ceteris paribus*. This, in and of itself, reduces the need for savings as a method for smoothing consumption—*ceteris paribus*, income is smoother with a higher social security tax rate. In the model, there are two methods that an individual agent has for moving wealth across time, direct savings and fertility. Thus, quite naturally, the use of both of these falls across BGPs when τ is increased. There are several other effects also present. In particular, an increase in the social security tax and the associated transfers may increase the expected wealth of a middle age agent when he is old, but has offsetting effects on the wealth of the potential children. This in turn, reduces donations causing total consumption of the old people to fall. In fact, and this is the main channel through which fertility is affected by the social security system, the fact that transfers from the social security system increase has the direct effect of lowering support payments from children to their parents (cf., the expression for d in section 3). This reduces the importance of children as an investment good, lowering fertility. Finally, note that in the case where $b > 0$, there is an implicit substitution effect induced by an increase in the social security system. This is that the cost of a child is b units of time. The

opportunity cost of this time is the after tax wage $-(1-\tau)bw$ – which, *ceteris paribus*, is decreasing in τ . The substitution effect resulting from this relative price change, goes in the direction of increasing fertility. Simulations show that this substitution effect is dominated, in the Caldwell-type model, by the rate of return effect, which makes children a less attractive investment once pensions are introduced.

In the Caldwell-type framework, the quantitative effects of changes in the size of the social security system are similar for the two alternative cost structures (time costs and goods costs). This is because in this framework the key mechanism governing fertility is how fertility translates into transfers to parents. The introduction of a social security system reduces per-child donations, and hence fertility. The difference between the two is in the last effect mentioned above – distortionary effect of taxation on the child-rearing vs. market activities. If the costs of children is solely in terms of goods, in this framework with inelastic labor supply, there is no offsetting substitution effect when τ is increased. Thus, the effects on fertility are larger, if only slightly, in this case.

Leaving aside the obvious effects on output and wealth that the introduction of a distortionary tax causes, the economic impact of Social Security in the Barro-Becker framework is quite different from the one just described. Figures 9-12 plot the values taken in the Barro-Becker model by γ_n , K/Y , Y , and per-child bequests as τ varies. Indeed, whether fertility increases or decreases in the Barro-Becker model when Social Security is introduced depends on whether households need mainly goods or time to raise children. If mainly time is needed to raise children, fertility increases as the size of the Social Security program increases, which is the case reported in Figure 9. Notice that the increase is basically insignificant, in the case reported, for example, TFR increases of about 0.05 children as the Social Security tax goes from zero to ten per cent. Conversely, if only goods are needed to raise children, the introduction of Social Security decreases the number of children per woman by about 0.014. These effects are orders of magnitude smaller than those seen in the data. The effects on capital accumulation and consumption (of both middle-aged and old) go in the opposite direction, but are also quantitatively small. The donations effect is missing in this framework, and the different impact on fertility depending on type of cost needed to raise children is due to distortionary taxation: time spend raising children is not taxed, while time spent working is taxed.

The basic reason for the small effects on fertility in the Barro-Becker model is that in a balanced PAYGO system, the effects of transfers are netted out by the dynasty planner. Thus, as noted above, if the system was funded entirely through lump sum taxation, the effects would be literally zero. If childrearing requires time, however, there is a change due to, as discussed above, a change in the effective relative prices of the two uses for time, work and childcare. This effect is typically small, however, and present only if the cost of children is in terms of time, hence the resulting effect on fertility is also small. An additional effect is also present. This is that, even when the costs are entirely in terms of goods, the dynasty views per capita transfers as

fixed, whereas in equilibrium, through the government budget constraint, these are determined by average fertility across dynasties. This connection is not recognized by the individual dynasty however, and is the sole reason why, with only goods cost of children, the effect of changes in social security are not exactly zero.

As an additional dimension along which the two models' predictions should be compared, we note that the Caldwell-type model predicts an increase in the capital-output ratio, while the Barro and Becker model predicts a decrease of the capital-output ratio as social security increases. In the data, the U.S. capital-output ratio has either remained constant or increased since early in the XX century; also, the capital-output ratio is substantially higher among the European countries, relative to the U.S., and the European countries have, with the sole exception of the UK, a substantially higher SST than the U.S. . This lends further empirical support to Caldwell-type models of fertility as an alternative to dynastic models.

6 Sensitivity Analysis

6.1 The Role of Financial Markets Imperfections.

In our version of the Caldwell model the parameter $\xi \in (0, 1)$ measures the extent to which financial market imperfections prevent middle age individuals from using private saving as a means of financing late age consumption. In the baseline model we assumed $\xi = 0$, so that financial markets are functioning perfectly. As reported in the introduction, a number of empirical studies have found evidence that different measures of the ability to save for retirement are strongly correlated with fertility decisions. In fact, a study by Cigno and Rosati using Italian and German micro data have estimated that the impact of financial market accessibility on fertility is comparable to that of public pensions: the easier it is to save for retirement, the lower is fertility. In the Caldwell model the intuition for this result is simple: in the equation for the equilibrium donations (see Section 3) the terms $(1 - \xi)R_t x_t$ and T_t^o are interchangeable – a variation in ξ has the same effect as a change in the public pension transfer. The more imperfect capital markets are, the less valuable physical capital is for financing consumption in late age and, therefore, the more valuable children are in this regard. One would expect, then, that when $\xi > 0$ fertility would be higher than in the baseline case; the question is: how much higher?

The answer, reflected in Figure 13, is: a lot higher. Figure 13 plots the mapping from the pair $(\tau, \xi) \in [0.0, 0.25] \times [0.0, 0.20]$ into the equilibrium values of γ_n , while Figure 14 has K/Y on the vertical axis and the same two parameters on the horizontal plane. As the reader can verify, even small changes in the efficiency of financial markets make children a very valuable form of investment. This in turn pushes fertility to levels similar to those observed in the earlier part of the 20th century. In quantitative terms, we find that, even in the presence of a social security system of

roughly the same magnitude as the current one, a reduction in the rate of return on capital of about 20% ($\xi = 0.2$) would increase fertility 30%, or 0.66 more children per woman in our setting. Equally important, the same degree of financial market inefficiency leads to a substantial decrease in aggregate savings resulting in a K/Y ratio which is almost 50% lower than in the baseline case. These are large effects by historical standards.

Financial instruments through which one can reliably save for retirement are limited both historically in the developed countries and currently in the developing countries. It is difficult to know if changing ξ from 0 to 0.2 corresponds to an interesting quantitative exercise without further data work. But, the fact that the effects that we find are so large makes this an interesting possibility to explore further.

6.2 Parameters of Preferences and Technology

There are three preference parameters common to both models, β , σ , and ζ , and one specific to the Barro-Becker model, η ; further, in the Barro-Becker model the restriction $\sigma + \eta > 1$ must hold. The long and the short of the sensitivity analysis results is: varying preference parameters within reasonable intervals does not change the qualitative predictions of the two models, nor the magnitude of $\Delta\gamma_n/\Delta\tau$ as a percentage of the initial value of γ_n . It is still and uniformly true that increasing τ from about 0% to 10% decreases TFR by between 20% and 25% in a Caldwell-type model, while it increases fertility slightly less than 10% in the Barro-Becker model.

What varies substantially, and sometimes dramatically, with the preference parameters are the levels of both fertility and the capital-output ratio, and this sensitivity in levels is common to both models.

As illustrated earlier on, at the baseline parameter values the implied TFR is slightly below the current value of 2.06 for the US for the Caldwell model (at 1.82 it is closer to the US level of 1980 than 2000), and the values for b (resp. a) needed for the Barro and Becker model are much larger than the estimated 3% of time. This seems to point to a lack of richness of the models overall. Clearly, however, a model with features of both would do much better¹⁷. Since the aim of this paper is partially to compare the two models, this was not attempted.

Our findings for changes in the parameters governing technology are similar to those for preferences: small changes in either α , γ_A , a , b , or δ bring about changes in fertility and in the capital-output ratio that are sometimes substantial. However, they leave the comparative static results basically unaltered when it comes to fertility. Indeed, in the Caldwell model, reducing the time cost of children from the $b = 3\%$ value adopted in the baseline case to values slightly higher than $b = 2\%$ suffices to make the predicted level of fertility to match current averages in the U.S., i.e. about

¹⁷Other possibilities are to lower α to .22 as discussed above, or to make children 'less expensive' in the Caldwell model by including joint production in the home of both childcare and other home goods.

2.06 per woman. This choice may be justified by the fact that in our model the effective time cost of having children is artificially increased by the assumption that, with only three periods, the length of working life is equal to that of the retirement period. As explained above, this is a gross distortion of the real world, where the number of years spent working is roughly twice the number of years spent in retirement. Because of this fact, one may argue that $b = 2\%$ is a preferable baseline calibration for the Caldwell-type setting; should this choice be made, our model can easily match current U.S. fertility levels when $\tau = 10\%$ and the remaining parameters are as in Table 2, without affecting any of the comparative statics results.

One experiment that is of particular interest is the effects of changes in the growth rate of productivity. Our value of 1.012 is fairly low and is based on Dennison's work, which makes substantial adjustments for the observed changes in labor quality. We also performed our baseline experiments on the effects of changes in τ on γ_n for values of γ_A up to and including 1.02. These gave rise to very similar results: when the size of the SST increases from zero to ten percent, fertility drops of almost half a child per woman.

Another alteration that is particularly relevant concerns changing α . In the household production literature (which treats the stock of housing and durables as inputs into the production of home goods, and removes the housing service component from GNP) an estimate of $\alpha = .22$ has been found in McGrattan, Rogerson and Wright (1997). Recalibrating the Caldwell model to this target does not change the overall effects of changes in Social Security on fertility, but it does greatly enhance our ability to hit the targets set out in the previous section. In particular, with $\alpha = .22$, $\gamma_n = 1$, is attainable even with $\zeta = .65$ (see Schoonbroodt (2004)). When this alternative calibration is adopted for the dynastic model, increasing the social security tax rate still increases fertility, and still only very marginally. In the Caldwell-type model, increasing the social security tax rate reduces fertility of more or less the same percentage as in the base line model.

7 Conclusion

A number of authors have suggested that the welfare state, and the public pension system in particular, might be an important factor behind the drop in fertility to the bare (or even below) replacement levels most western countries are experiencing. Controlling for infant mortality, income level, and female labor force participation, almost all regression exercises, including ours, point to a strong negative correlation between the size of the Social Security system and the Total Fertility Rate, both across countries and over time.

In this paper we test the ability of two models of endogenous fertility to replicate this correlation when they are calibrated to match other very elementary facts of the US economy. The results are mixed. We find that in models based on parental altruism changes in the size of Social Security systems like those we have seen over

the last 100 years generate only small (and typically positive) effects on fertility. In contrast, models based on the ‘old age security’ motive for fertility are more in accord with the patterns seen in the data. Although imperfect, even simple, calibrated models of this type account between 40% and 60% of the observed differences in fertility over time in the U.S. or between the U.S. and other developed countries. Since the introduction of government funded pension systems has a much larger effect on incentives at the lower end of the income distribution, this finding is also consistent with the observation that the reduction in fertility over this period has been much larger for poorer households.

In addition to this, we study the effects of improved access to savings instruments on fertility. We find that even small improvements (on the order of 20%) have the potential to account for about 50% of the observed changes in fertility over time. This channel is one which requires more exploration, but, apparently it is quite promising.

Taken together then, we find that these two effects account for between 50% and 100% of the drop in fertility in the U.S. from 3.00 children per woman to 2.0 over the period from 1920 to now.

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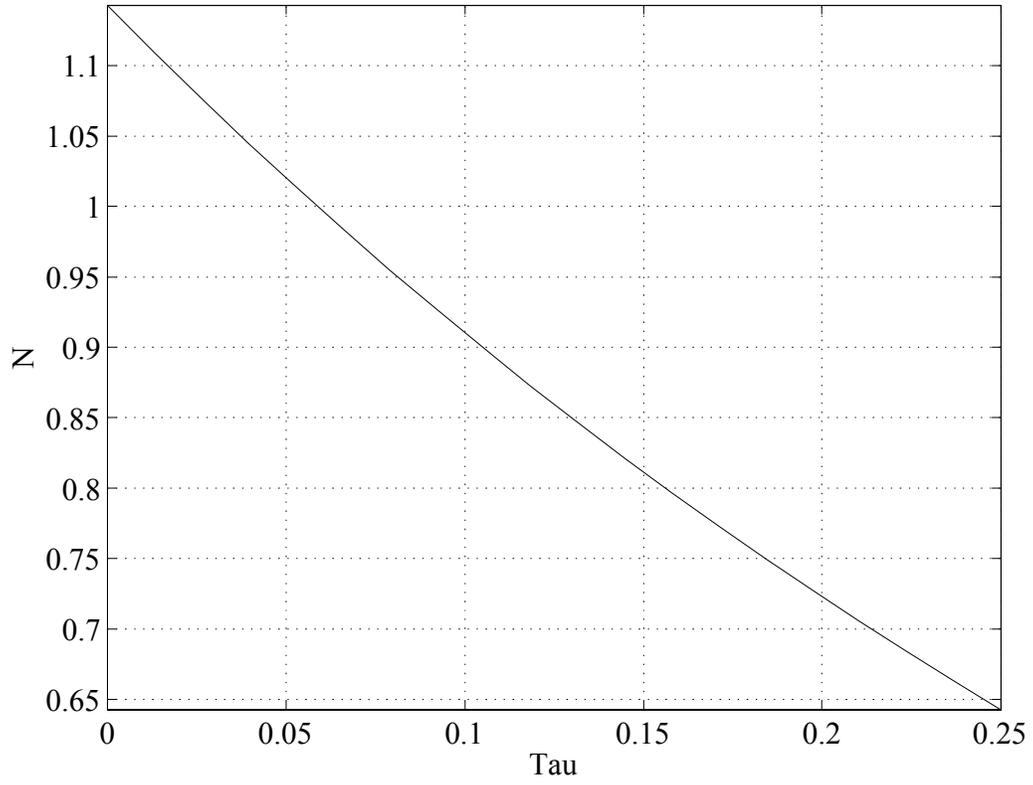


Figure 5: *Fertility and the SS tax, Caldwell Model*

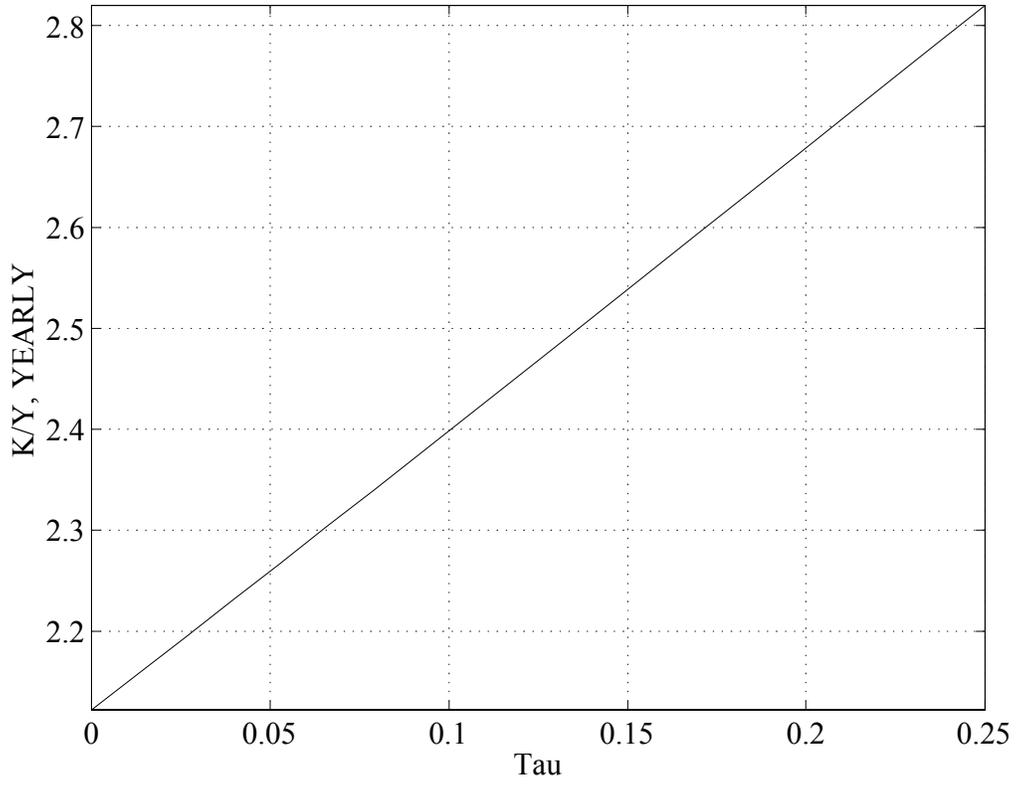


Figure 6: *Capital Output Ratio and the SS tax, Caldwell Model*

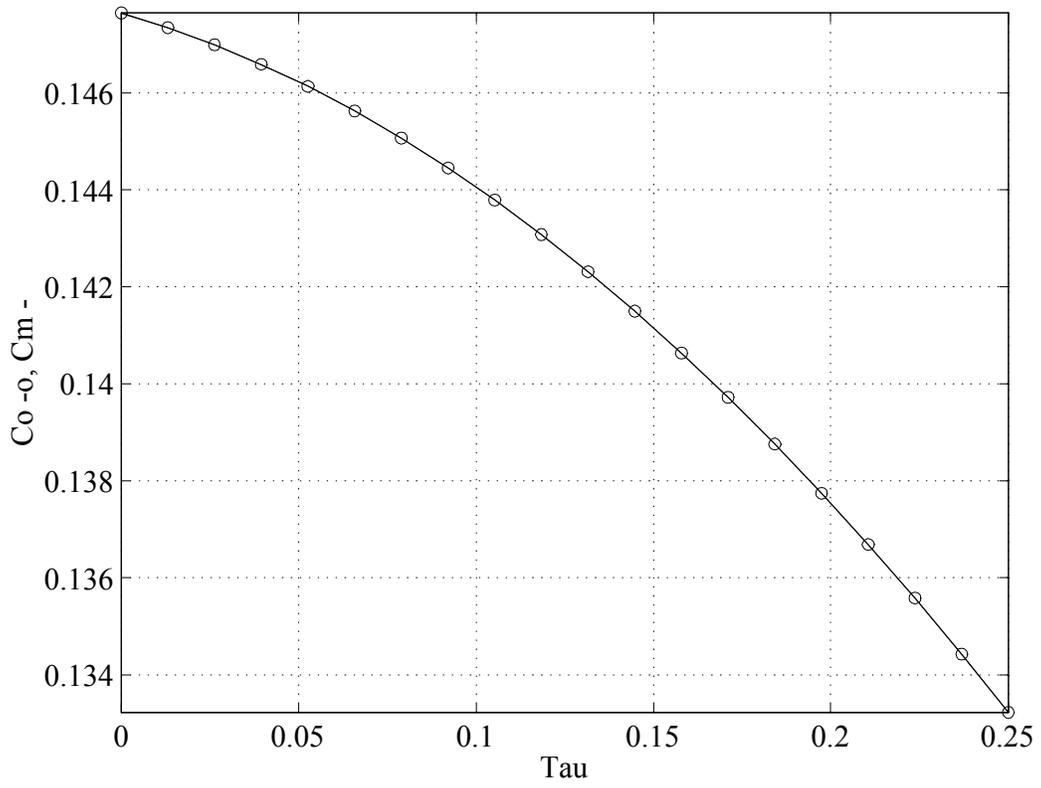


Figure 7: *Consumption of O's and M's and the SS tax, Caldwell Model*

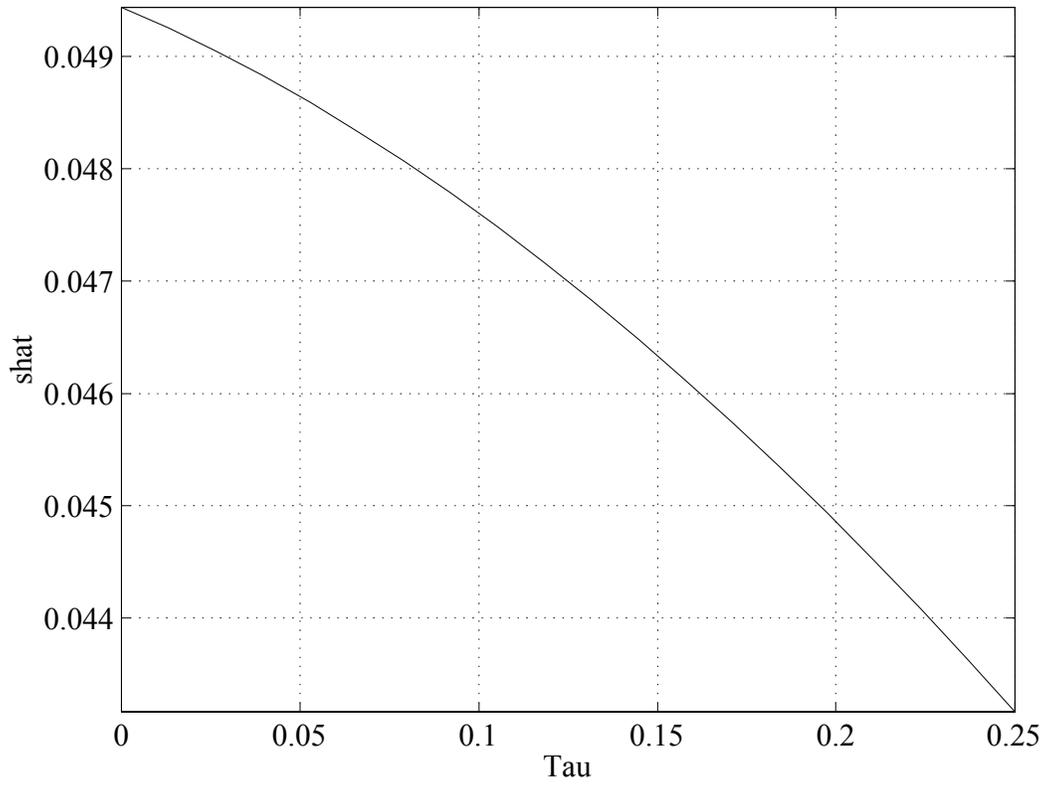


Figure 8: *Savings and the SS tax, Caldwell Model*

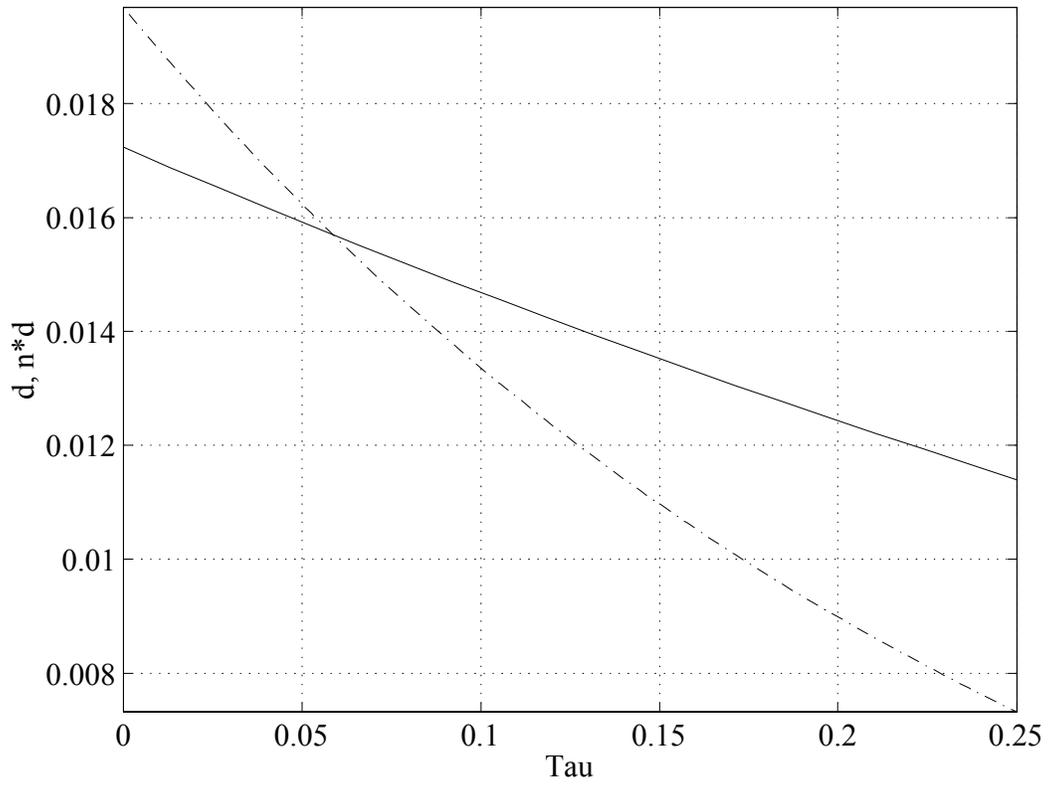


Figure 9: *Old Age Support and the SS tax, Caldwell Model*

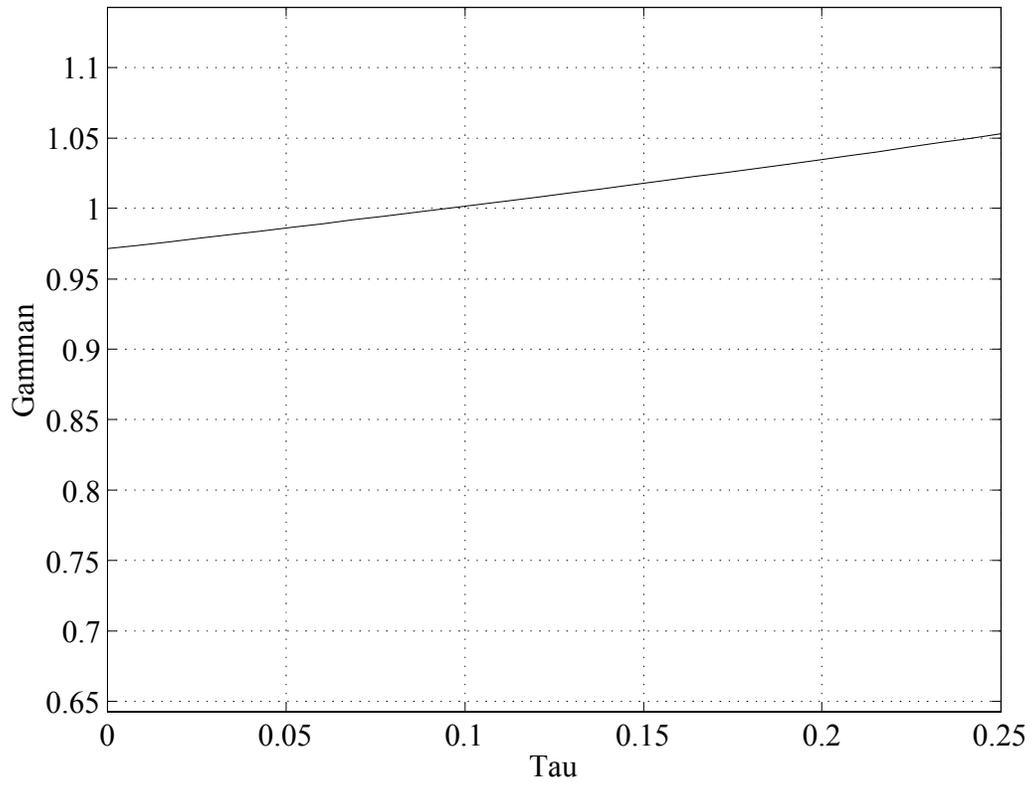


Figure 10: *Fertility and the SS tax, B&B Model*

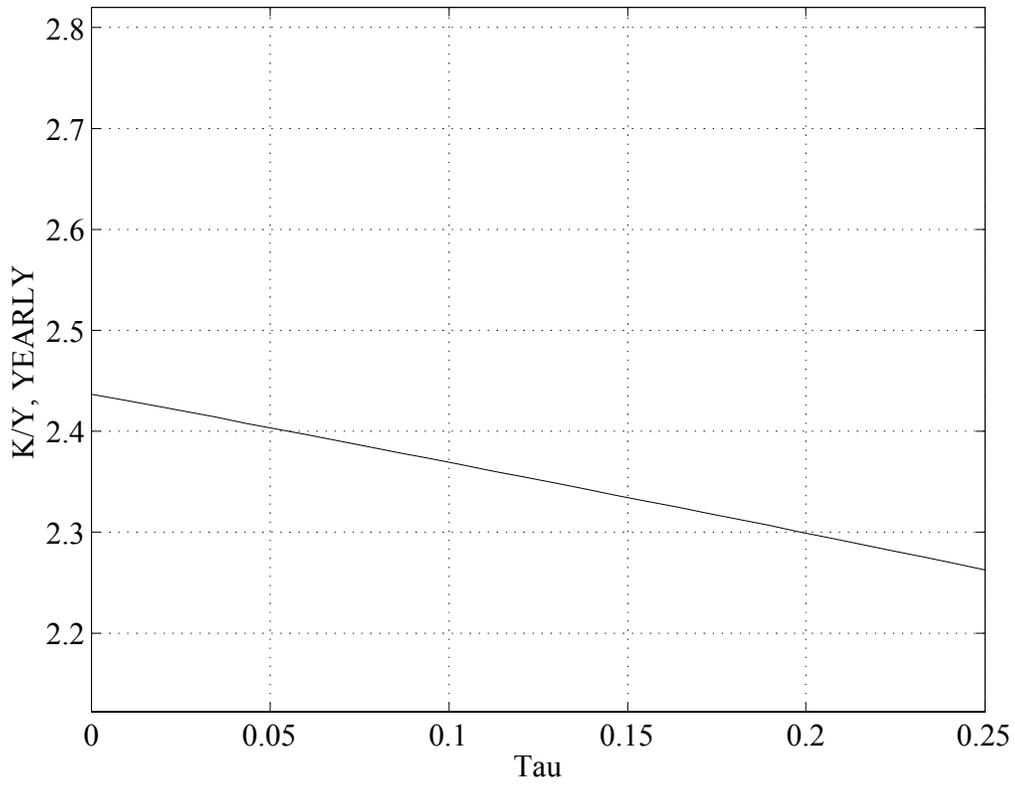


Figure 11: *Capital Output Ratio and the SS tax, B&B Model*

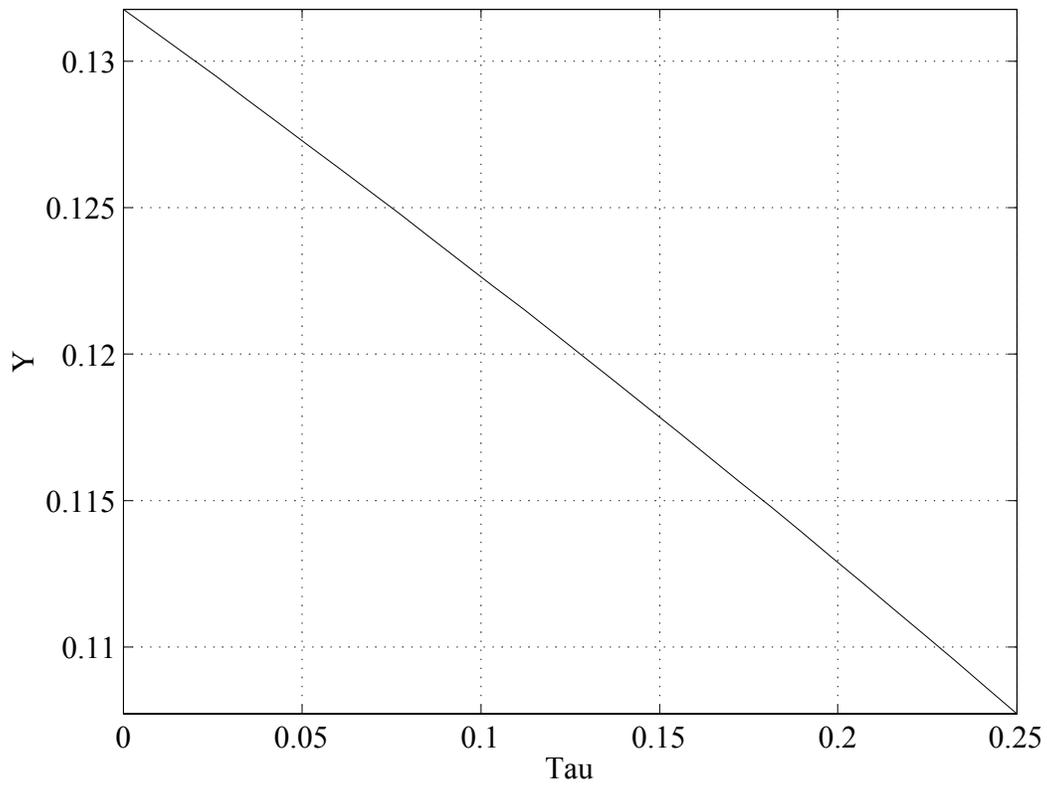


Figure 12: *Output and the SS tax, B&B Model*

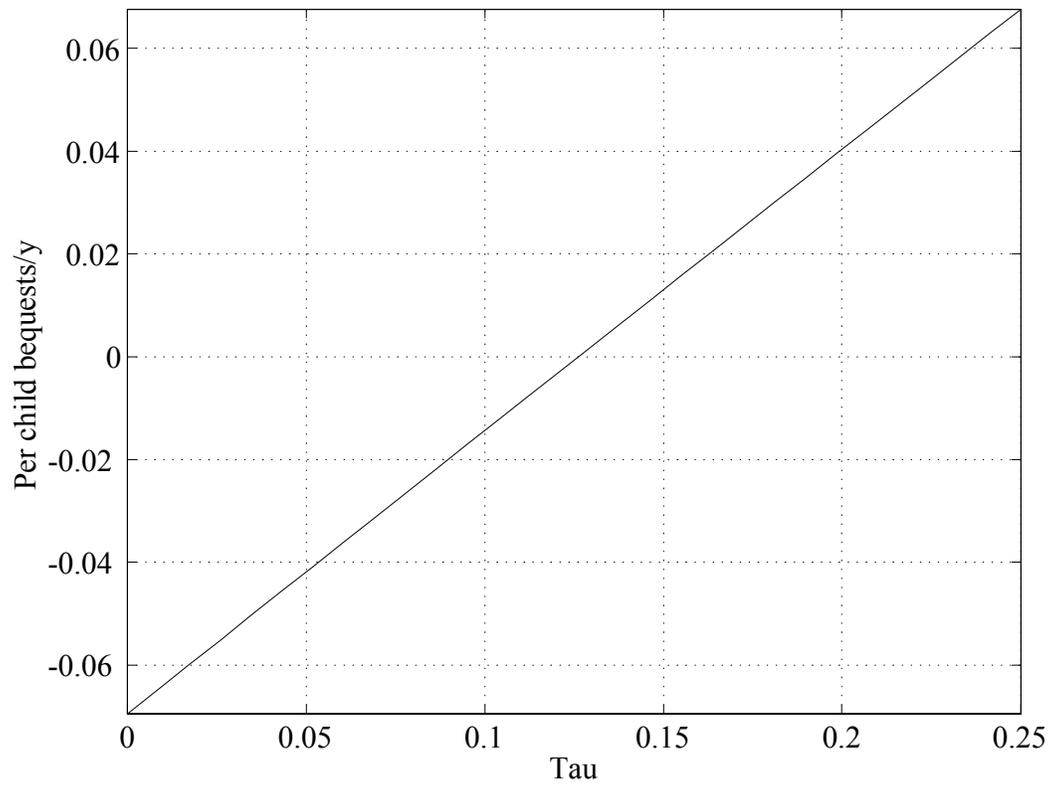


Figure 13: *Bequests and the SS tax in the B&B Model*

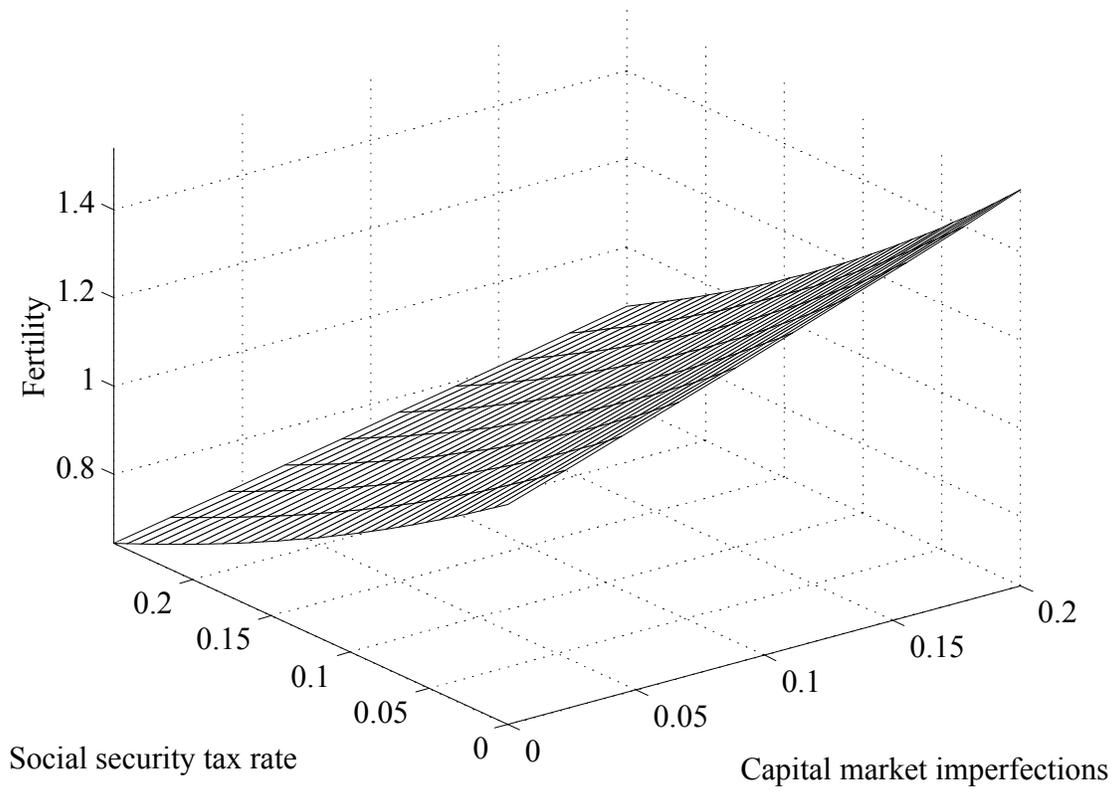


Figure 14: *Fertility, SS tax and ξ , Caldwell Model*

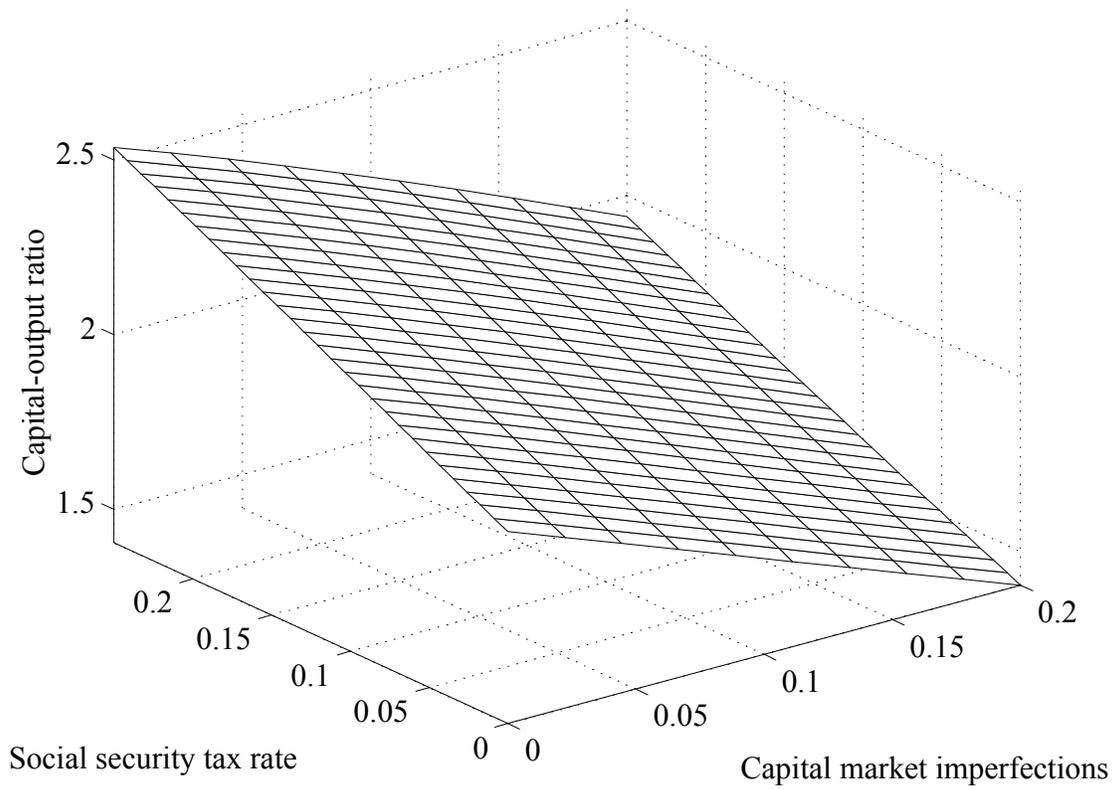


Figure 15: K/Y , SS tax and ξ , *Caldwell Model*