

Wealth inequality over time

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Outline

- 1 Question/motivation
- 2 Model
- 3 Calibration
- 4 Transitions
- 5 Additional channels + observations

References:

- Kaymak, B. & Poschke, M. (2016). The evolution of wealth inequality over half a century: The role of taxes, transfers and technology. *Journal of Monetary Economics* 77, 1-25.
- Kaymak, B., Leung, D., Poschke, M. (2022). Accounting for Wealth Concentration in the US.

Recent Trends in Inequality

Table: Share of Top 1%

	1960	2010
Wealth	28%	40%
Earnings	7%	17%
Income	10%	20%

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Recent Trends in Inequality

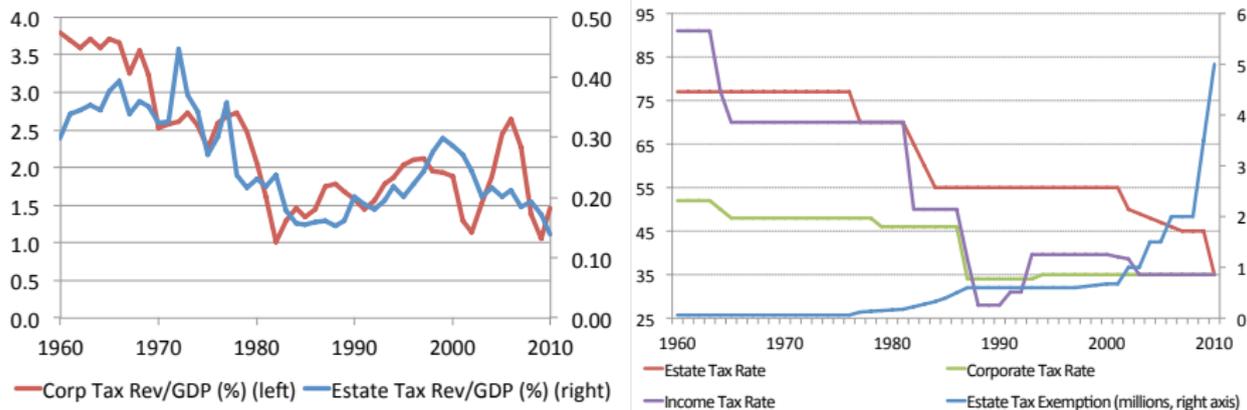
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Q: What caused the upward trend in wealth inequality?

- Higher Wage Inequality
 - Top Income Tax Cuts
 - Larger Government Transfers (Social security)
- Other channels (later):
- Heterogeneous investment returns
 - Non-homothetic bequests

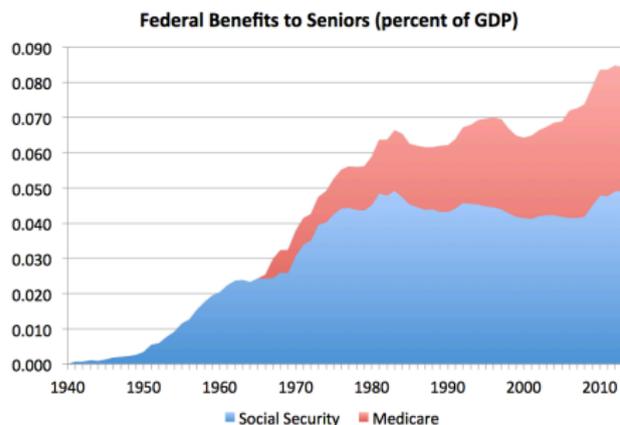
Corporate and estate taxes and top MTR declined, 1960-2010



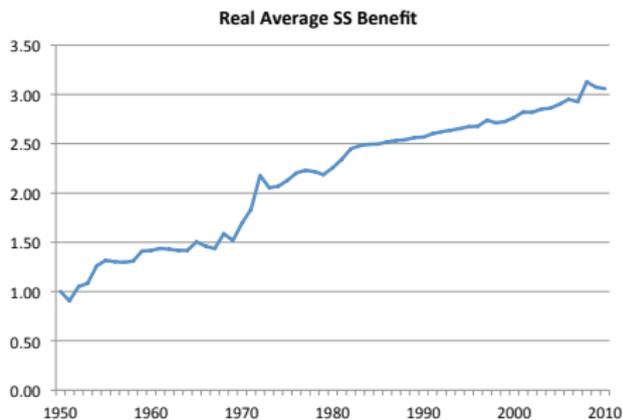
Sources: NIPA, Joulfaian (2013), IRS

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Transfers/GDP and individual benefits increased strongly



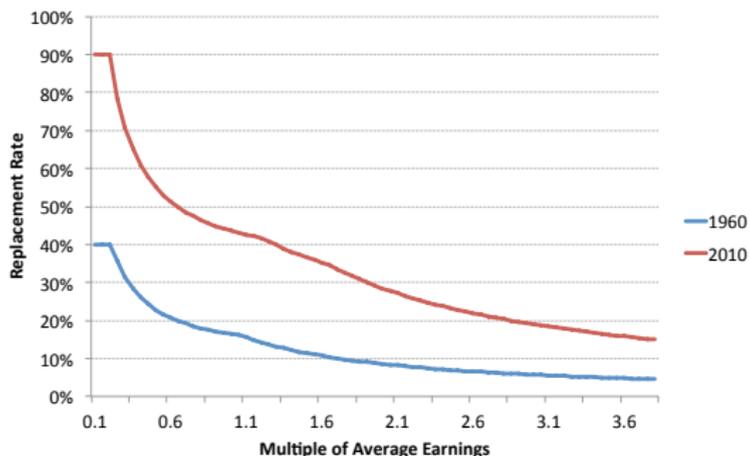
Federal Benefits to Seniors/GDP



Real Average Benefit

Social security benefits

Replacement rates increase, more so for low earnings:



Replacement Rates

Which factors drove higher wealth concentration?

Strategy for answering the question:

- Build a quantitative model of an economy with large earnings and wealth inequality
- Calibrate the model economy to the U.S. economy in 1960
- Simulate the effects of observed
 - changes in transfers (social security)
 - changes in taxes
 - changes in wage inequality
- Evaluate the effect of each change on income and wealth inequality
 - steady states and
 - year-by-year transition

Model

Aiyagari-Bewley-Huggett with...

- Life-Cycle and Intergenerational Income Risk
- Some top earners (à la Castañeda et al., *JPE* 2003)
- ⇒ Matches income and wealth inequality well

Institutions:

- Social security
- Corporate, Estate, Income and Sales Taxation
- Exogenous government expenditures

Households

- value consumption c and dislike working
- are perfectly altruistic towards their children
- have heterogeneous productivity z
- decide how much to consume, work and invest in capital
- take prices w, r , taxes and transfers as given

Households face risks

- workers ($\mathcal{R} = 0$) retire with a constant probability μ_r
- retirees
 - cannot work, but receive a pension
 - die with a constant probability μ_d
 - leave a bequest upon death
- z may change, for workers every period, for retirees upon death
 - ⇒ wage dynamics/imperfect transmission of human capital
 - ⇒ households differ in productivity and wealth ($\Gamma(k, z)$)
- three saving motives
 - life cycle (because of retirement)
 - bequest
 - precautionary (because z can change within and between generations)

Household's Problem

$$V(k, z, \mathcal{R}) = \max_{c, x \geq 0, h \in [0, 1]} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - \theta \frac{h^{1+\epsilon}}{1+\epsilon} + \beta \mathbb{E}[V(k', z', \mathcal{R}') | z] \right\}$$

subject to

$$\begin{aligned} c(1 + \tau_s) + x &= y^d(wzh, rk, \omega(z, \mathcal{R})) + k, \\ k' &= x - E(x, \mathcal{R}, \mathcal{R}') \end{aligned}$$

Tax System and Disposable Income

- Taxation of Corporate Income:

$$\tau_c \max(rk - d_c, 0)$$

- Adjusted Gross Income:

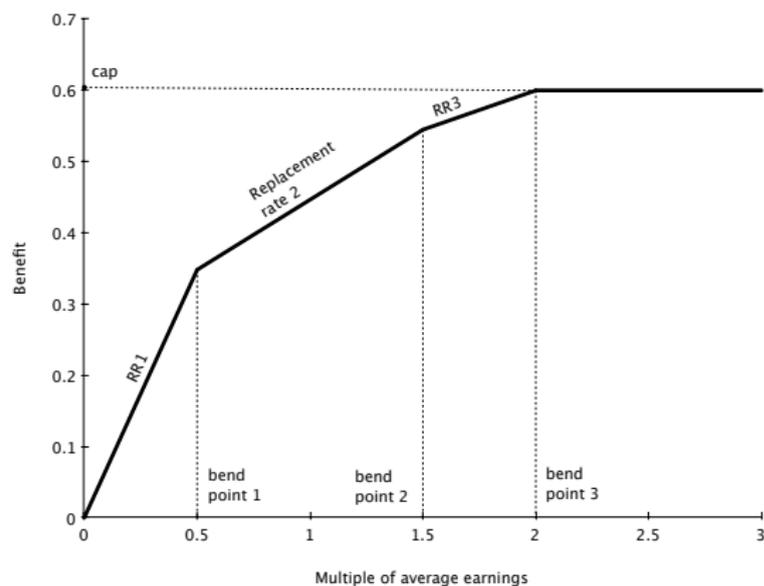
$$y_{agi} = wz h + \min(rk, d_c) + \omega(z, \mathcal{R})$$

- Taxation of Personal Income:

$$y_{agi} - \lambda [\min(y_{agi}, y_b)]^{1-\tau_l} - (1 - \tau_{\max}) \max(y_{agi} - y_b, 0)$$

- $0 \leq \tau_l \leq 1$ measures the degree of progressivity.
 - Permits net transfers (e.g. EITC).
 - τ_{\max} is the top MTR, applicable for $y > y_b$.
- Taxation of Estates: $E(x)$ piecewise linear as in the law.

Social Security and Medicare



Changes over time:

- replacement rates up at all income levels
- cap up from 0.17 to ca. $0.6 \times$ average wages
- increase in income-independent transfers (mimicking Medicare and pension assistance)

For now: compute using earnings of worker with same (k, z)

Demographics and Labor Productivity

$$\Pi = \left[\begin{array}{c|cc} & z_W & z_R \\ \hline z_W & \Pi_{WW} & \Pi_{WR} \\ z_R & \Pi_{RW} & \Pi_{RR} \end{array} \right]$$

Demographics and Labor Productivity

$$\Pi = \left[\begin{array}{c|cc} & z_W & z_R \\ \hline z_W & \Pi_{WW} & \Pi_{WR} \\ z_R & \Pi_{RW} & \Pi_{RR} \end{array} \right]$$

$$\Pi_{WW} = \left(\begin{array}{c|cccccc} & f_L + a_L & f_L + a_H & f_H + a_L & f_H + a_H & z_{awe_l} & z_{awe_h} \\ \hline f_L + a_L & A_{11} & A_{12} & 0 & 0 & \lambda_{in} & 0 \\ f_L + a_H & A_{21} & A_{22} & 0 & 0 & \lambda_{in} & 0 \\ f_H + a_L & 0 & 0 & A_{11} & A_{12} & \lambda_{in} & 0 \\ f_H + a_H & 0 & 0 & A_{21} & A_{22} & \lambda_{in} & 0 \\ z_{awe_l} & \lambda_{out} & \lambda_{out} & \lambda_{out} & \lambda_{out} & \lambda_{ll} & \lambda_{lh} \\ z_{awe_h} & 0 & 0 & 0 & 0 & \lambda_{hl} & \lambda_{hh} \end{array} \right)$$

▶ other Π 's

Closing the Model

- Firms

$$r = F_K(K, N) - \delta$$

$$w = F_N(K, N)$$

- Markets Clear
- Government budget constraint holds at all times:

$$\begin{aligned} \text{Corporate Tax} + \text{Income Tax} + \text{Estate Tax} + \text{Sales Tax} \\ = \text{Transfers} + G \end{aligned}$$

Quantitative Exercise

- Calibrate the model to match the 1960 economy.
- Introduce observed expansion of transfers, tax cuts and path of wage inequality.
- Transition analysis.
- Steady-state decomposition exercise.

Calibration: Preset Parameters

General:

σ	1.1	Risk Aversion
ϵ	1.67	Frisch elasticity of 0.6
α	0.36	Capital Income Share
δ	0.079	$K/Y = 3.0$
μ_r	0.022	Average Career Length of 45 yrs.
μ_d	0.067	Average Retirement Length of 15 yrs.

Productivity Process:

ρ_{ig}	0.30	Solon (1992)
σ_a	0.46×0.38	variance of log earnings in 1960 = 0.5
σ_f	0.46×0.62	share of fixed effects = 0.62

Calibration: Jointly Calibrated Parameters

General:

β	0.958	Interest Rate	0.041
θ	12	mean hours	0.34
d_c/r	$0.44 \times K$	Corporate tax revenue/GDP	0.038

Productivity Process:

z_{awe_l}	top 1% income share
z_{awe_h}	top 0.5% income share
λ_{in}	income Gini (workers)
λ_{lh}	wealth Gini
λ_{ll}	top 1% wealth share
λ_{hh}	top 0.5% wealth share

Tax Parameters

	1960	2010	
τ_l	0.08	0.08	top 1% average tax rate
τ_{\max}	0.91	0.35	tax code
τ_c	0.42	0.236	Marginal Corporate Tax Rate, Gravelle (2004)
$E(\cdot)$			Actual Estate Tax Schedule
γ	0.108	0.108	$(G + \text{Transfers})/Y = 0.17$ (1960)
λ	endogenous		GBC

▶ $E(\cdot)$

Results: Income Process

$z_W \backslash z_W$	6.7	19.2	20.5	58.4	61.4	1222
6.7	0.967	0.009	0	0	0.002	0
19.2	0.006	0.970	0	0	0.002	0
20.5	0	0	0.967	0.009	0.002	0
58.4	0	0	0.006	0.970	0.002	0
61.4	0.034	0.034	0.034	0.034	0.826	0.014
1222	0	0	0	0	0.205	0.773

 $\Pi_{WW} :$

<i>Top 1% earnings dynamics:</i>	model	data
persistence	0.73	ca. 0.75
std. dev. of log earnings growth	0.76	1.1
skewness of log earnings growth	-1.72	-1.26
kurtosis of log earnings growth	14	18


 ▶ Π_{RW}

Calibration Results: Inequality in 1960

	Top Percentile							Gini
	0.5%	1%	5%	10%	20%	40%	60%	
Wealth Share (Data)	0.21	0.28	n/a	0.71	<i>0.81</i>	<i>0.95</i>	<i>1.00</i>	0.80
Wealth Share (Model)	0.22	0.26	0.45	0.62	0.79	0.91	0.98	0.74
Income Share (Data)	0.07	0.10	0.23	0.33	0.49	0.73	0.89	0.34
Income Share (Model)	0.09	0.11	0.18	0.41	0.54	0.75	0.89	0.34
Earnings Share (Data)	0.05	0.07	0.20	0.33				0.34
Earnings Share (Model)	0.08	0.10	0.22	0.35				0.33

Calibration Results: Taxes in 1960

	Corporate Tax			Estate Tax			Income Tax		
	1%	99%	<i>R/Y</i>	1%	99%	<i>R/Y</i>	1%	99%	<i>R/Y</i>
Data	14.4	5.1	3.8	6.0	0.0	0.3	24.0	13.8	10.6
Model	17.2	5.5	5.0	3.6	0.1	0.4	22.2	11.2	10.0

Transition analysis

Transition analysis

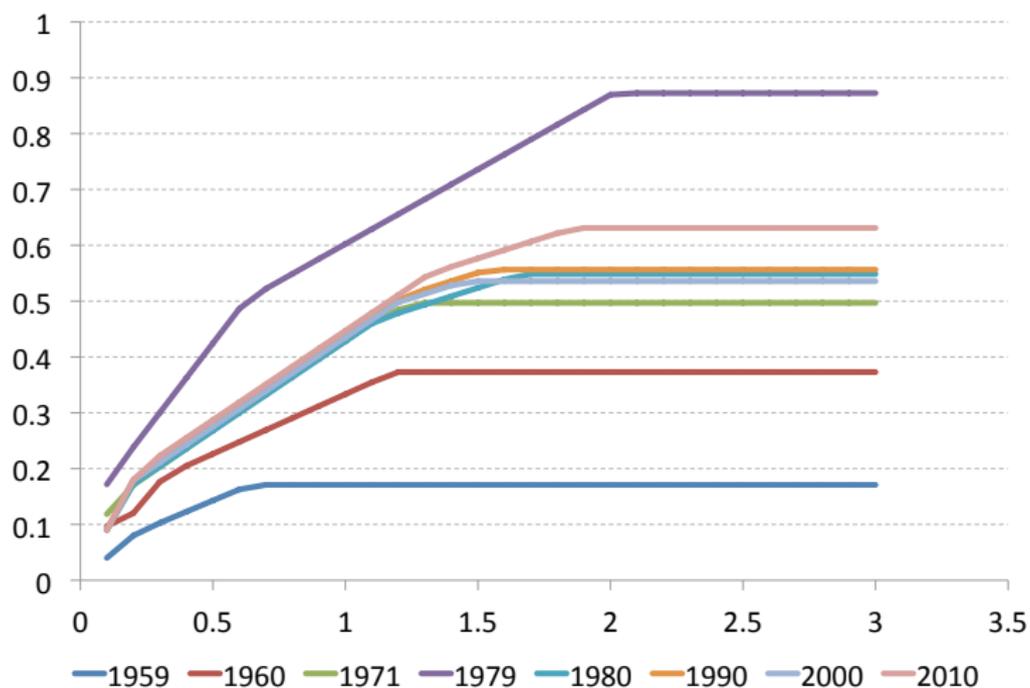
Series of data inputs for the transition:

- Social Security and Medicare
- Tax Cuts (corporate and estate taxes and top MTR)
- Earnings Inequality

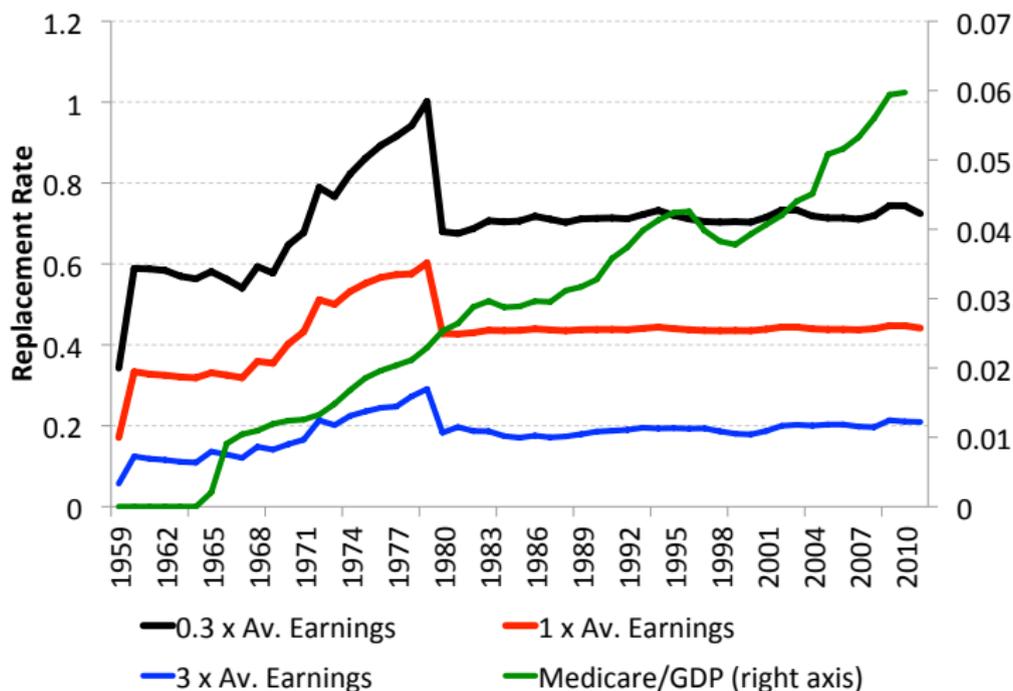
Notes:

- everything constant after 2010.
- expectations: perfect foresight.

Transition inputs: Some social security PIA formulas



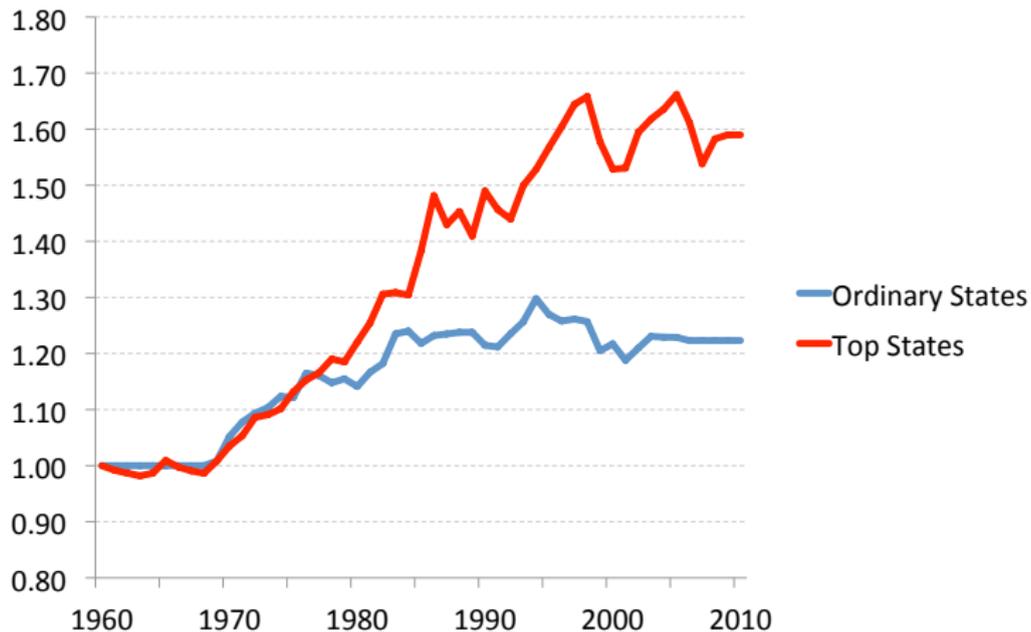
Transition inputs: Social security replacement rates



Transition inputs: Wages

$$z_{it} = \exp(\mu_z - \kappa_t + \zeta_t \bar{z}_i + v_{5/6t})$$

Technical Change and Wage Inequality



Transition analysis: approach

- The environment changes
- ⇒ behavior and equilibrium variables will change over the transition
- ⇒ cannot consider a *stationary equilibrium*, but need to solve for an *equilibrium path*:
Need to find **sequence** of equilibrium objects $\{r_t, \lambda_t, K_t\}_{t=1}^T$, not just single SS value.

Approach, in short:

- Guess a sequence.
- Solve problems.
- Check market clearing.
Similar to approach for stationary equilibrium, but with a higher-dimensional equilibrium object.

Transition analysis: algorithm (1/3)

- 1 Solve final steady state.
- 2 Fix
 - a length of the transition, T , and
 - criteria for convergence for r , λ and K : ε_r , ε_λ and ε_K .
- 3 Guess a sequence $\{r_t, \lambda_t, K_t\}_{t=1}^T$.
 Computed implied values:
 - $r_t \Rightarrow K_t/N_t \Rightarrow w_t$ from firm's FOC
 - $K_t, K_t/N_t \Rightarrow N_t \Rightarrow Y_t \Rightarrow$ transfer amounts (which in our model are indexed to GDP)

Transition analysis: algorithm (2/3)

- ④ Solve the **household problem for each transition year t** using the sequences of price and environment inputs and the final steady state, *backwards* (starting with year $T + 1$, then $T \dots$):

$$V_t(k, z, \mathcal{R}) = \max_{c, x \geq 0, h \in [0, 1]} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - \theta \frac{h^{1+\epsilon}}{1+\epsilon} + \beta \mathbb{E}[V_{t+1}(k', z', \mathcal{R}') | z] \right\}$$

s.t. constraints

where V_t is the year t value function, and $V_{T+1} = V_{SS2}$.

From this, obtain the **policy functions** $c_t(\cdot)$, $k'_t(\cdot)$ and $h_t(\cdot)$.

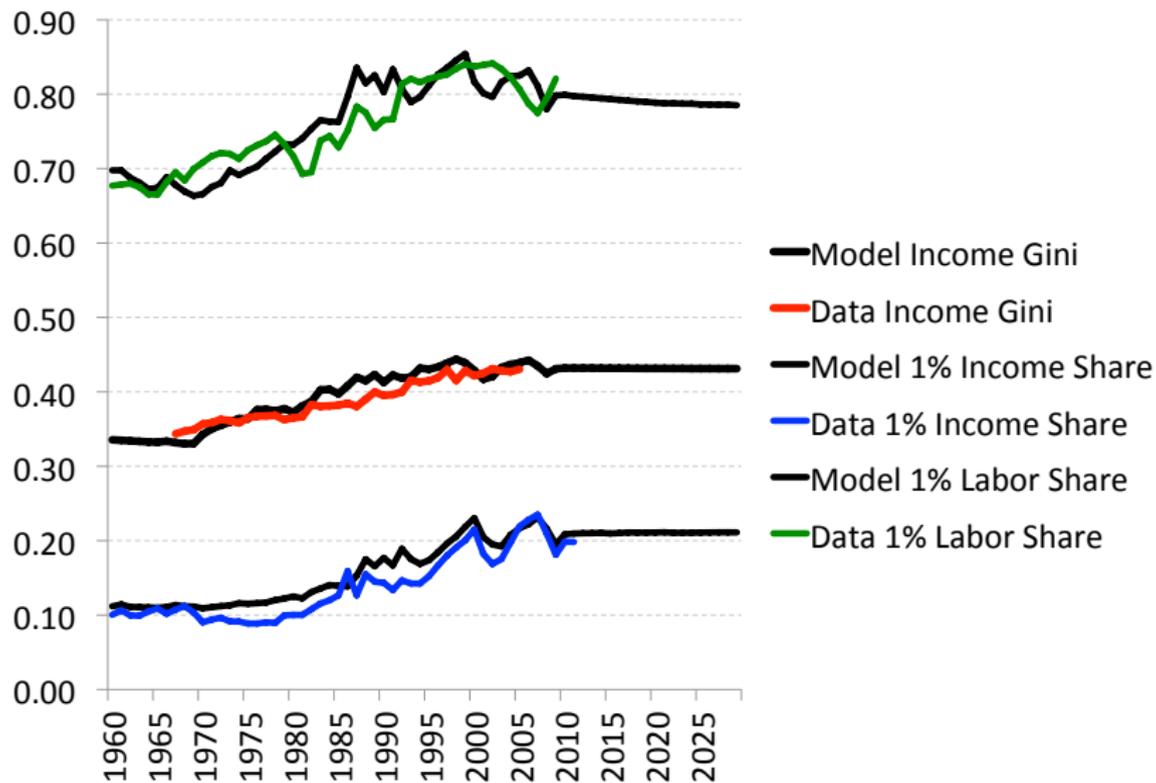
Note: value function and policy functions indexed by t .

Transition analysis: algorithm (3/3)

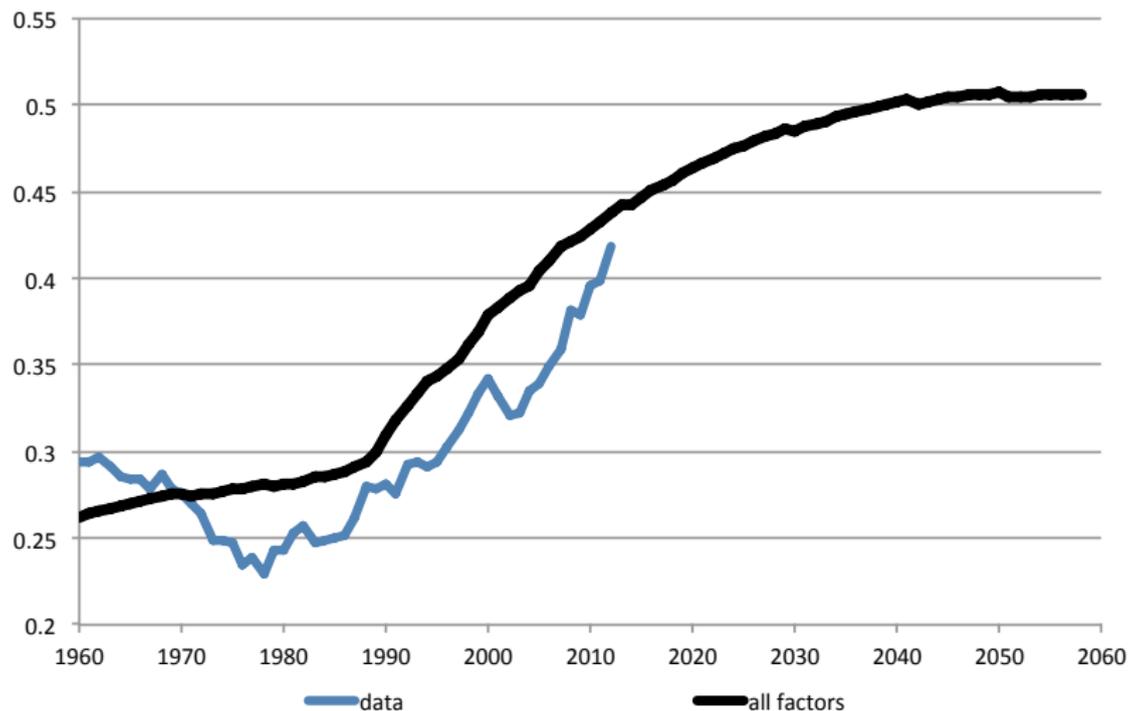
- 5 Compute the **distribution of assets** $\Gamma_{t+1}(\cdot)$ for each t , using $\Gamma_t(\cdot)$ and the policy functions $k'_t(\cdot)$, starting from Γ_{SS1} .
- 6 Compute aggregate asset supply K_t^S and aggregate labor supply N_t^S for each t by integrating over the policy functions using $\Gamma_t(\cdot)$.
- 7 Check **market clearing**:
 - 1 Compute implied \tilde{r}_t for each t using K_t^S and N_t^S in the firm's FOC:

$$\tilde{r}_t = \alpha(K_t^S/N_t^S)^{\alpha-1} - \delta$$
 - 2 Compute implied $\tilde{\lambda}_t$ that clears the government budget constraint, given policy functions and $\Gamma_t(\cdot)$.
 - 3 Compute largest deviation $dx = \max(\tilde{x}_t - x_t)$ for $x = r, \lambda, K$.
- 8 If $dx < \varepsilon_x$ for $x = r, \lambda, K$: done.
 Otherwise, return to step 3 and update sequences $\{r_t, \lambda_t, K_t\}_{t=1}^T$:
 $r_t^{\text{new}} = .6r_t + .4\tilde{r}_t$, $\lambda_t^{\text{new}} = .6\lambda_t + .4\tilde{\lambda}_t$, $K_t^{\text{new}} = K_t^S$.

Model fit: The evolution of top incomes



Model fit: The evolution of the top 1% wealth share



Transition analysis: decompositions

Benchmark:

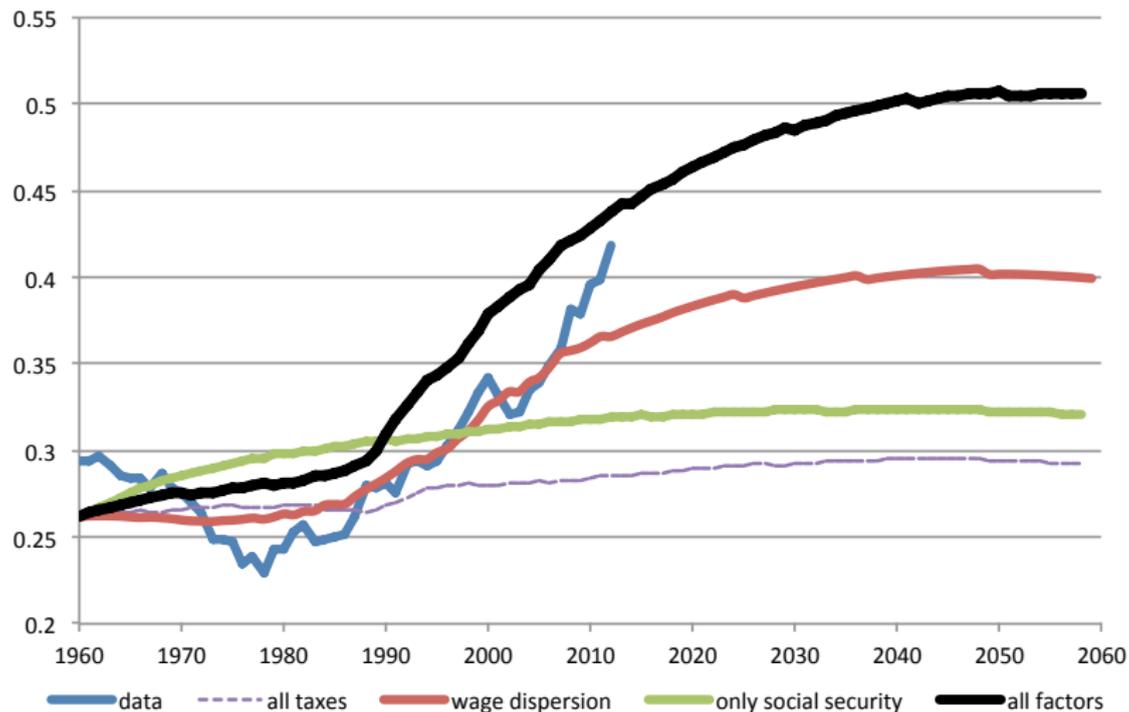
- Social security, taxes, wages all change.

Decomposition:

- Some inputs change.
- Others remain as in the 1960 steady state.

Compute equilibrium transition path for this configuration of inputs.

The evolution of the top 1% wealth share: Decomposition

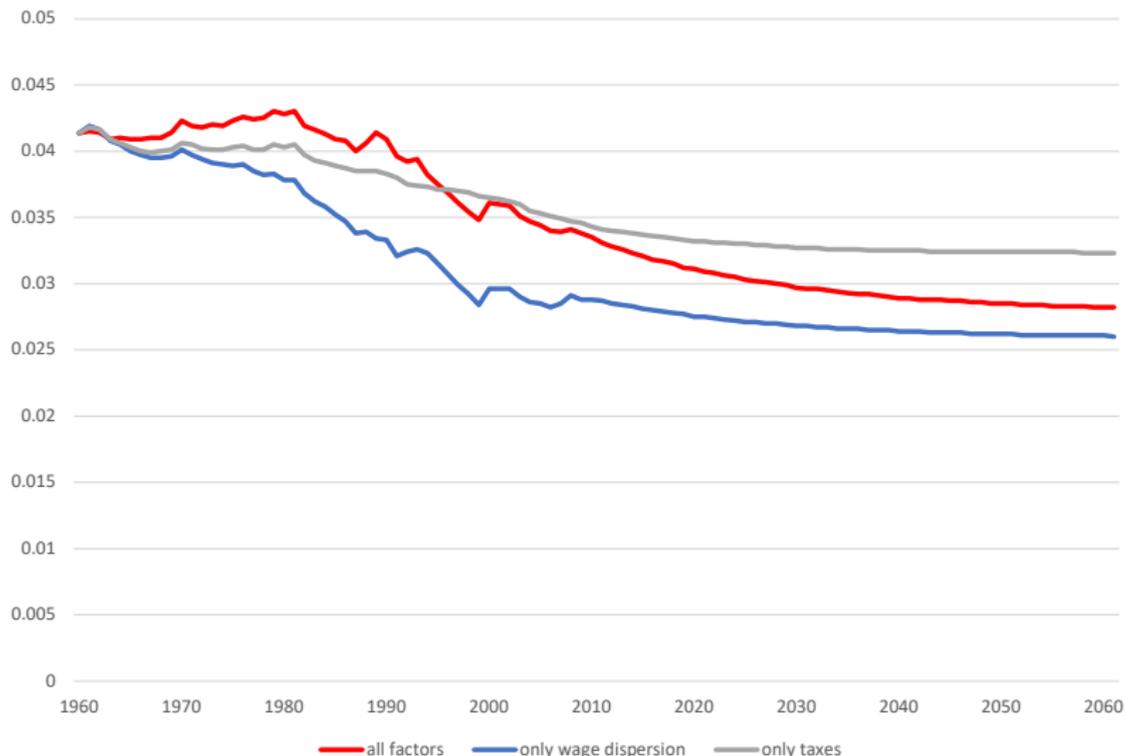


Remarks on the transition

- Model matches the increase in the data from 1980 to 2010 almost exactly.
Overstates increase since 1960.
- Increase in wage inequality stops in 2003 in the model.
Convergence of the top wealth share not completed then.
- Model top wealth share continues to rise for another 50 years,
and 10 percentage points.
- Speed of the transition:

Completing	1/2	of the change in the	36	years.
	3/4	top 1% wealth share takes	56	
	9/10		76	
	99/100		106	

Higher inequality reduces r by almost 1.5 percentage points



Additional channels + observations

What determines wealth concentration, revisited

- A recent, mostly theoretical literature stresses the importance of *heterogeneous investment returns* for wealth concentration (see in particular Benhabib, Bisin and Zhu 2011).
- Life cycle was very stylized in JME paper.

Richer approach: Model as above, plus:

- Life cycle
- Heterogeneous investment returns
- Non-homothetic warm glow bequest motive

Consumption-Savings Problem

Workers ($j < J_R - 1$)

$$V_j^W(k, z, \kappa) = \max_{c, k' \geq 0, h \in [0, 1]} \left\{ \frac{c^{1-\sigma_c}}{1-\sigma_c} - \theta \frac{h^{1+\sigma_l}}{1+\sigma_l} + \beta s_j \mathbb{E}[V_{j+1}^W(k', z', \kappa') | z, \kappa] \right. \\ \left. + (1 - s_j) \phi(k') \right\}$$

subject to

$$(1 + \tau_s)c + k' = y^d(z \varepsilon_j h \omega, r \kappa k) + k + Tr, \\ \phi(k) = \phi_1[(k + \phi_2)^{1-\sigma_c} - 1]$$

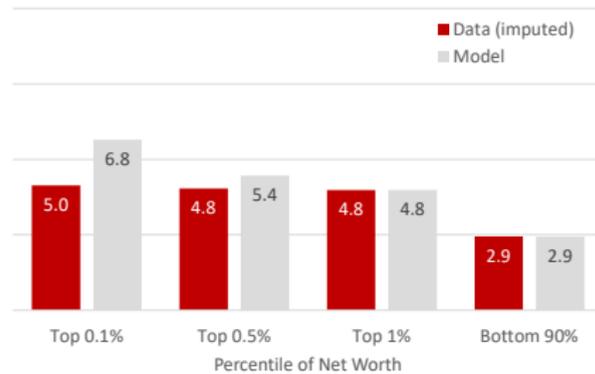
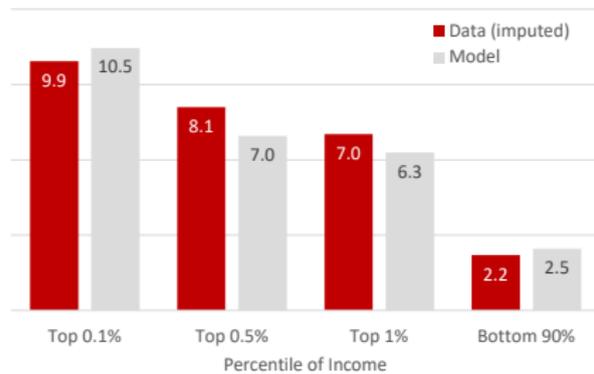
Retirees ($j \geq J_R$)

receive social security benefits b instead of labor earnings $z w \varepsilon_j h$

Calibration

- Larger model, more parameters – need more calibration targets.
- New here: match the joint distribution of income, earnings and net worth.
- Key moments:
 - Labor income share of top 1% income earners: 59%.
(55% for top 1% of wealth.)
 - Relative saving returns of top 1%: 3.2 times those of bottom 90%.

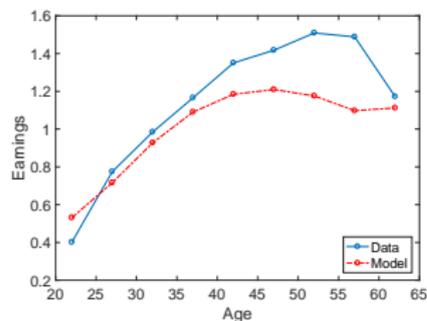
Rates of return



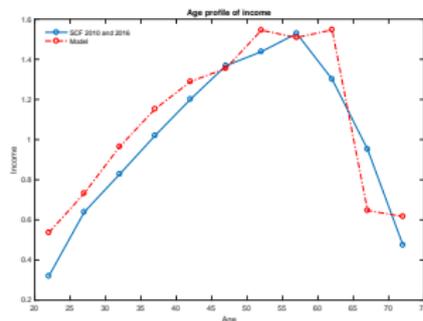
Data: group rates of return from SCF, implied by group's earning share, wealth share, and labor income share.

Life-Cycle Patterns: Averages

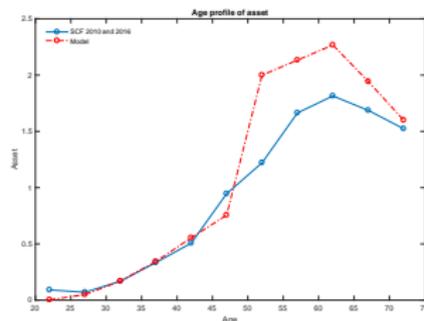
DATA vs MODEL



(a) earnings

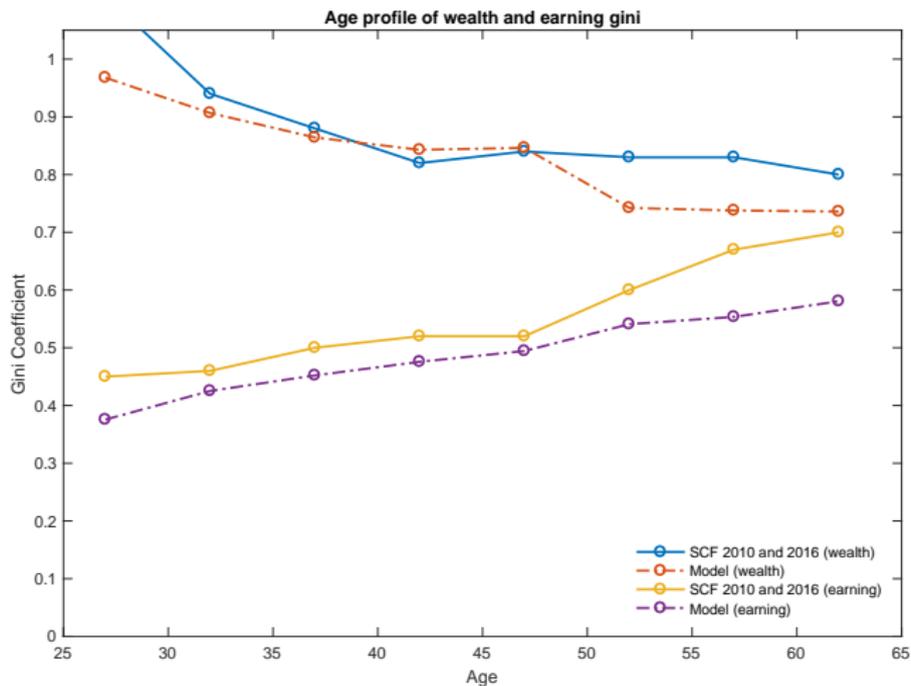


(b) income



(c) wealth

Life-Cycle Patterns: Dispersion


[more](#)

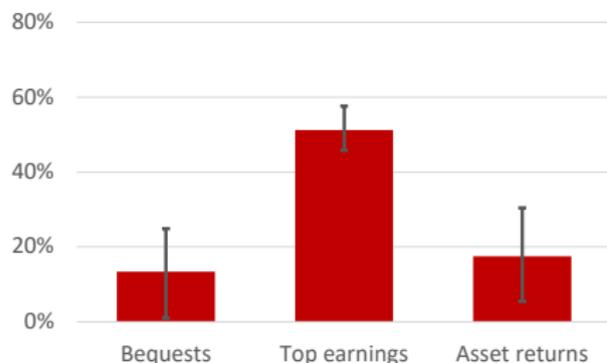
Decomposition: determinants of wealth concentration

Compare benchmark economy to counterfactual stationary equilibrium with

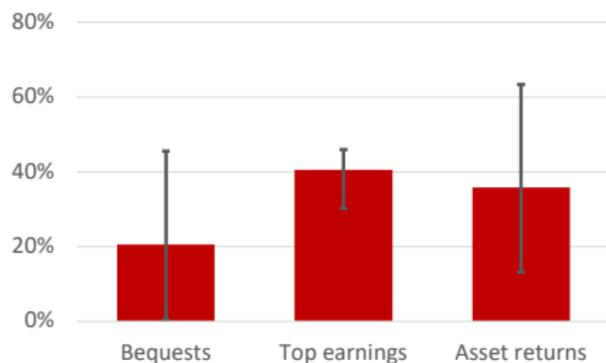
- equal bequests and/or
- no top earners ($z_8 = z_7 = z_6$)
- common asset returns.

We compute the *marginal effect* of each channel, in each possible configuration. (4 marginal effects per channel.)

Accounting for Wealth Concentration



(d) Top 1% Wealth Share



(e) Top 0.1% Wealth Share

Note.— Percent contribution to top wealth shares. The whiskers represent the range of values obtained by permuting the order of decomposition. The column height represents the average value.

Some interesting issues

- 1 Benchmark results versus those with a single mechanism: The role of LIS
- 2 Entrepreneurs
- 3 Why do heterogeneous returns have little impact?
- 4 The timing of bequests

An economy without superearners

Counterfactual:

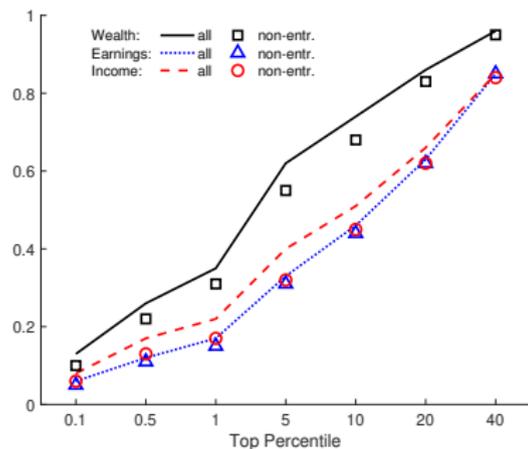
eliminate superearners and match top 0.1% wealth share with rate of return differences alone

Results:

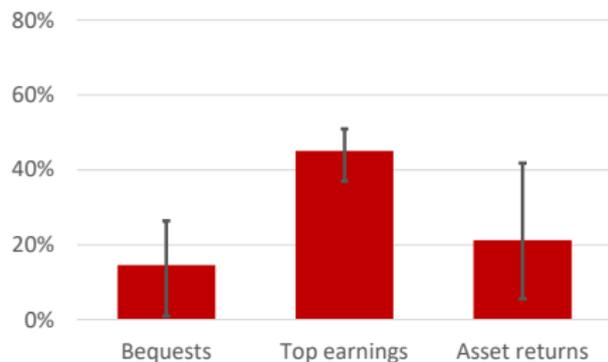
	Top 1% earnings	labor income share of top 1% by	
		income	wealth
data	0.17	0.59	0.50
benchmark	0.18	0.61	0.48
simulation	0.04	0.31	0.07

Entrepreneurs

Calibrate model for *non-entrepreneurs*:



(f) Marginal distributions



(g) Top 1% wealth share decomposition

Why do heterogeneous returns have little impact?

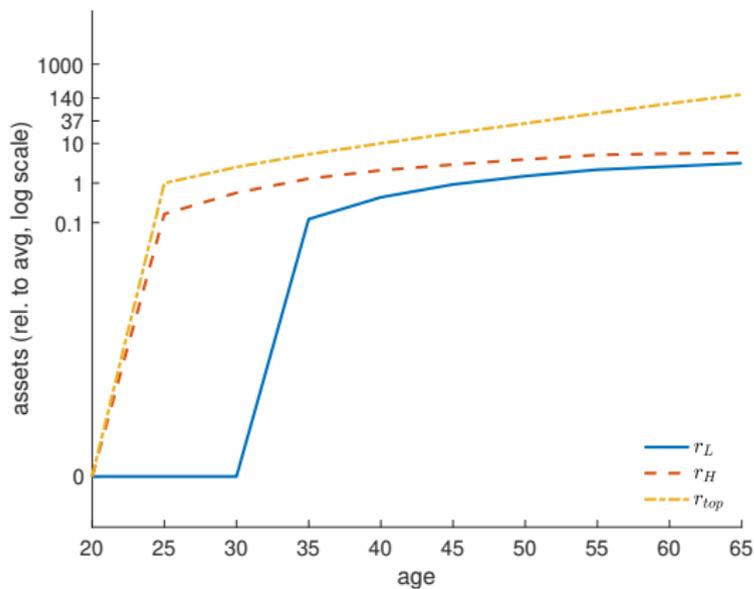


Figure: Path of assets if z always z_6 , return fixed

Why do heterogeneous returns have little impact?

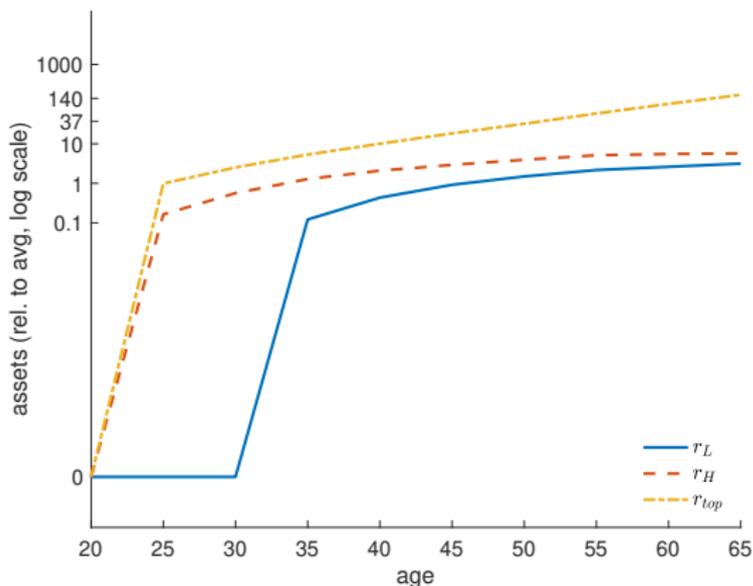


Figure: Path of assets if z always z_6 , return fixed

Answer: because life is too short.

Reaching the top 0.1% takes 35 years at the top return of 25.3%.

The timing of bequests

Receive bequests at age 20 instead of age 50:

- Top 1% wealth share rises 3ppts.
- Somewhat greater role of return heterogeneity.

Summary

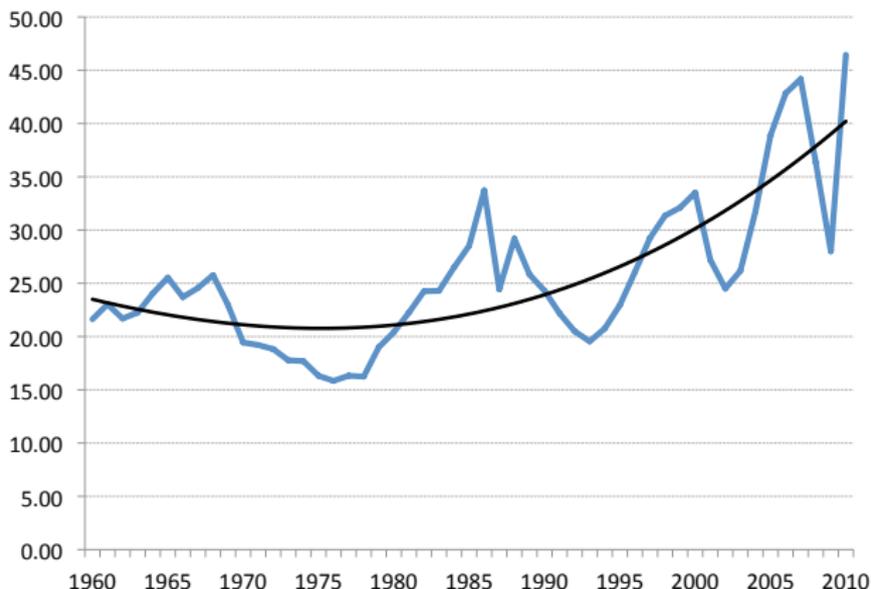
Summary

- We saw different ways of modeling wealth concentration,
 - how to calibrate models with such channels,
 - how to compute deterministic transitions in heterogeneous agent models.
-
- My substantive takeaway: top income earners have a lot of labor income
- ⇒ top earners play a large role for wealth concentration.
- A note on measurement/interpretation: Top earners here include those with high wages and salaries (CEOs, finance...) but also high-earning entrepreneurs.

Appendix

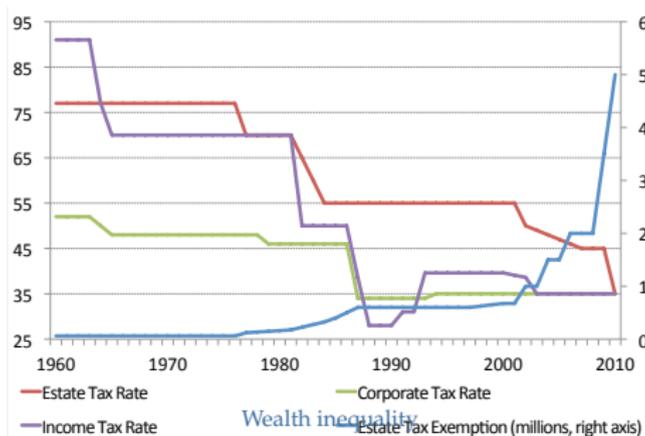
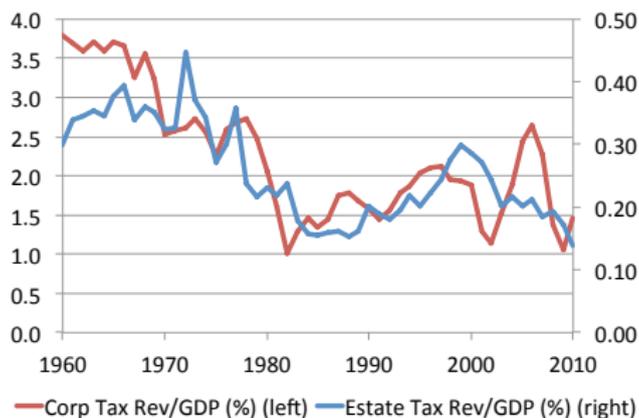
Wealth Share of Top Income Groups: 1%

$$s_{w,1\%} = \frac{s_{y,1\%} f_{K,1\%}}{rK / (Y - \delta K)}$$



Source: Author's calculations based on Piketty and Saez (AER, 2006)

Corporate and estate taxes and top MTR declined, 1960-2010



Demographics

$$\Pi = \left[\begin{array}{c|cc} & z_W & z_R \\ \hline z_W & \Pi_{WW} & \Pi_{WR} \\ z_R & \Pi_{RW} & \Pi_{RR} \end{array} \right]$$

Π_{WR} : constant retirement probability μ_r

Π_{RR} : constant survival probability $1 - \mu_d$

▶ back

Income Process: Intergenerational

$$\Pi = \left[\begin{array}{c|cc} & z_W & z_R \\ \hline z_W & \Pi_{WW} & \Pi_{WR} \\ z_R & \Pi_{RW} & \Pi_{RR} \end{array} \right]$$

$$\Pi_{RW} = \left(\begin{array}{c|cccccc} & f_L + a_L & f_L + a_H & f_H + a_L & f_H + a_H & z_{awe_1} & z_{awe_h} \\ \hline f_L + a_L & F_{11} & 0 & F_{12} & 0 & \phi_{in} & 0 \\ f_L + a_H & F_{11} & 0 & F_{12} & 0 & \phi_{in} & 0 \\ f_H + a_L & F_{21} & 0 & F_{22} & 0 & \phi_{in} & 0 \\ f_H + a_H & F_{21} & 0 & F_{22} & 0 & \phi_{in} & 0 \\ z_{awe_1} & \phi_{out_1} & 0 & \phi_{out_2} & 0 & \phi_{ff} & 0 \\ z_{awe_h} & \phi_{out_1} & 0 & \phi_{out_2} & 0 & \phi_{ff} & 0 \end{array} \right)$$

Later: $\phi_{in} = \phi_{ff} = 0$, $\phi_{out_1} = F_{21}$, $\phi_{out_2} = F_{22}$

Results: Income Process

$$\Pi = \left[\begin{array}{c|cc} & z_W & z_R \\ \hline z_W & \Pi_{WW} & \Pi_{WR} \\ z_R & \Pi_{RW} & \Pi_{RR} \end{array} \right]$$

$z_R \backslash z_W$	6.7	19.2	20.5	58.4	61.4	1222
0.0	0.043	0	0.023	0	0	0
0.0	0.043	0	0.023	0	0	0
0.0	0.023	0	0.043	0	0	0
0.0	0.023	0	0.043	0	0	0
0.0	0.023	0	0.043	0	0	0
0.0	0.023	0	0.043	0	0	0

▶ back

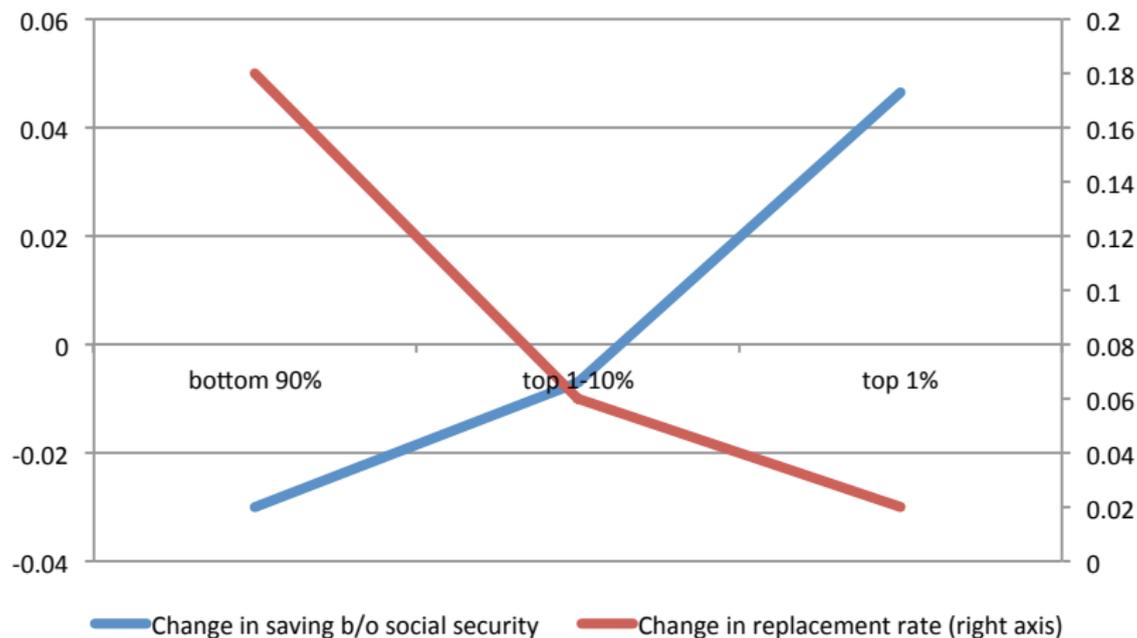
Calibration Results: Taxes in 1960

	Corporate Tax			Estate Tax			Income Tax		
	1%	99%	<i>R/Y</i>	1%	99%	<i>R/Y</i>	1%	99%	<i>R/Y</i>
Data	14.4	5.1	3.8	6.0	0.0	0.3	24.0	13.8	10.6
Model	14.4	4.6	4.6	3.1	0.1	0.3	22.6	10.6	9.6

► inequality

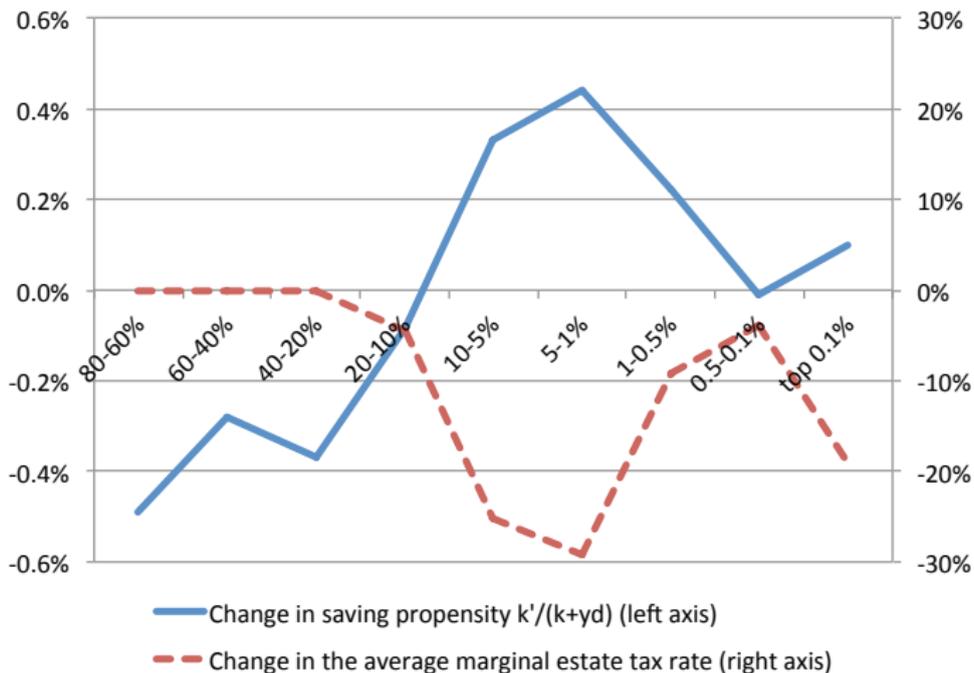
Expanding social security and saving behavior

More generous SS crowds out saving:



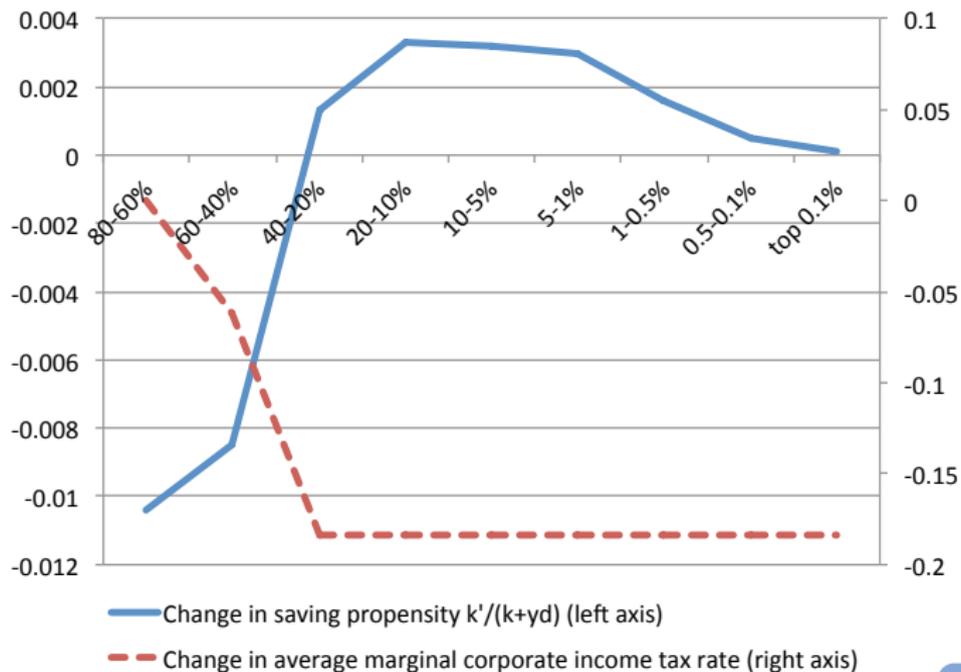
Estate taxes and saving behavior

The top 10% face the largest change in estate taxes, and react most:



Corporate taxes and saving behavior

Lower corporate taxes raise gross saving rates for $k > d_c/r$ (ca. median):



Aggregates

