Optimal Financial Knowledge and Wealth Inequality*

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Abstract

Recent studies show that financial knowledge is strongly positively related to household wealth, but there is also considerable cross-sectional variation in both financial knowledge and wealth levels. To explore these patterns, we develop a calibrated stochastic life cycle model featuring endogenous financial knowledge accumulation. It generates substantial wealth inequality, over and above that of standard life cycle models; this is because higher earners typically have more hump-shaped labor income profiles and lower retirement benefits which, when interacted with precautionary saving motives, boosts their need for private wealth accumulation and thus financial knowledge. In our simulations, endogenous financial knowledge accumulation accounts for around half of overall wealth inequality. The fraction of the population rationally "financially ignorant" depends on the generosity of the retirement system and the level of means-tested benefits. Educational efforts that enhance financial savvy early in the life cycle and provide an expected excess return of one percentage point, would be worth more than 80 percent of median initial wealth for high school dropouts, and close to 60 percent for college graduates.

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1 Introduction[1]

Wealth levels have been shown to vary considerably over the life cycle and across the population of workers on the verge of retirement.[2] There is also much dispersion in observed levels of consumer financial sophistication, and this heterogeneity in financial knowledge is positively associated with differences in retirement wealth.[3] Accordingly, analysts and policymakers interested in retirement system reforms seek to understand what drives these outcomes, particularly in an environment where consumers are increasingly required to save for their own retirement.

In this paper, we argue that financial knowledge itself is an endogenous variable and that individuals can increase their human capital by investing in financial knowledge. Specifically, we devise an explicit multiperiod theoretical model in a world of imperfect insurance and uncertainty that generates inequality in wealth profiles. Such a model is useful to evaluate which types of consumers would benefit most from investment in financial knowledge and the use of sophisticated investment products. The mechanism we propose is that financial knowledge enables individuals to better allocate resources over the life cycle. Accordingly, our model explores two important questions: 1) What forces shape financial knowledge accumulation over the life cycle? and 2) How much wealth inequality might be attributable to resulting differences in financial knowledge?

We build and calibrate a stochastic life cycle utility maximization model featuring uncertainty in income, capital market returns, and medical expenditures, and which also incorporates an endogenous knowledge accumulation process and a sophisticated saving technology. In the model, financial knowledge allows consumers to use more sophisticated financial products which can potentially help them raise the return they earn on financial assets. Individuals who wish to transfer resources over time by saving will benefit most from financial knowledge. Moreover, because of how the U.S. social insurance system works, more

[1]An earlier version of this paper was circulated as Lusardi, Michaud, and Mitchell (2011).

[2]See Moore and Mitchell (2000); Poterba et al. (2012), and Venti and Wise (2000).

[3]See Behrman, Mitchell, Soo, and Bravo (hereafter BMSB 2012); Lusardi and Mitchell (2007a, b, and 2009); Lusardi, Mitchell, and Curto (2010); and Van Rooij, Lusardi, and Alessie (2011a).

educated individuals have the most to gain from investing in financial knowledge. As a result, allowing for endogenous financial knowledge accumulation allows for an amplification of differences in accumulated retirement wealth over the life cycle.

Our contributions to the literature are several. First, we take account of the fact that many consumers appear not to know as much as economists often assume in theoretical models of decision making.[4] Accordingly, specifying how knowledge is acquired in a life cycle setting should be of keen interest to those seeking to explain differences in financial knowledge and behavior. Second, existing economic models of saving often have a difficult time explaining several stylized facts without appealing to exogenous preference differences or heterogeneity in the fixed cost of investing in financial products.[5] For instance, Venti and Wise (2000) show that permanent income differences and chance alone can explain only 30-40 percent of observed differences in retirement wealth, implying that other factors should be taken into account. Yet to date, economists have not explained why a significant fraction of the population reaches retirement with little or no wealth, without assuming that some subset of consumers is extremely impatient or for other behavioral reasons.

Moreover, those seeking to replicate observed heterogeneity in wealth across education and permanent income groups have invoked a range of factors including means-tested programs (Hubbard, Skinner and Zeldes, hereafter HSZ, 1994), and impatience in the form of hyperbolic discounting (Angeletos, Laibson, Repetto, Tobacman, and Weinberg, 2001). Still others assume that consumers use rules of thumb when making saving decisions (Campbell and Mankiw, 1989). Instead, here we draw on the fact that expected returns from financial products can differ across income groups.[6] Such differences in returns can generate a considerable amount of wealth inequality: for example, a dollar invested at a six percent versus a two percent return over 50 years grows to be nearly seven times as large. To simply assume that there is substantial heterogeneity in returns does not help much in explaining wealth heterogeneity. Rather, we contend that a more satisfactory approach is to generate such

[4] For evidence on this point, see Lusardi and Mitchell (2007a, 2011) and Lusardi, Mitchell, and Curto (2010).

[5] See, for instance, Cagetti (2003), and Huang and Caliendo (2009).

[6] See Yitzhaki (1987) and Calvet et al. (2007).

heterogeneity endogenously within the context of an economic model. Accordingly, here we ask how such differences in returns can arise from endogenous accumulation of financial knowledge.

Our model generates wealth inequality above and beyond what traditional models of saving normally deliver. Thus it helps rationalize some of the large differences in wealth found in many empirical studies by relying on the fact that individuals do not start their economic lives with full financial knowledge and that knowledge is acquired endogenously over the life cycle. We also show that some level of financial ignorance may, in fact, be optimal. As demonstrated in many empirical papers, some subsets of individuals display little financial knowledge. Our work can explain why they may rationally fail to invest, since it is expensive to acquire financial knowledge and not everybody benefits from greater financial sophistication. Also, financial knowledge can be an important public policy lever. For example, a policy initiative that reduced the cost of financial knowledge – such as providing financial education in high school or establishing websites that made it easier to acquire information – could have large effects on both wealth accumulation and welfare. We find that a 25-year-old college graduate would be willing to pay more than half of his initial wealth to boost financial knowledge giving him an expected permanent increase of one percent in his annual rate of return. Also, according to our estimates, more than half of wealth inequality can be attributed to financial knowledge giving people access to a sophisticated technology generating higher returns. Policies such as personal accounts under Social Security and increased reliance on individually-managed retirement accounts, for example, could also be accompanied by more financial knowledge accumulation.

The remainder of the paper is structured as follows: Section 2 briefly summarizes prior studies; Section 3 describes empirical evidence on the life cycle path of assets, consumers' use of financial products, and financial knowledge accumulation by education groups. Section 4 presents our model; Section 5 outlines the model calibration; and Section 6 presents simulation results. A conclusion and discussion appear in Section 7.

2 Prior Literature

Our research builds on several related literatures, including work on household life-cycle saving patterns.[7] We depart from conventional intertemporal models, however, in that we allow for the endogenous choice of a saving technology with returns and costs that depend on consumers' levels of financial knowledge. In this way we extend the portfolio choice literature (e.g. Cocco, Gomes, and Maenhout, 2005) in which returns are assumed to be exogenous and consumers decide only how much they will invest in risky assets.

We also draw on prior studies examining whether individuals are well-equipped to make financial decisions. Bernheim (1995, 1998) was among the first to document that many U.S. consumers display low levels of financial literacy. Hilgert, Hogarth, and Beverly (2003) showed that most Americans do not understand basic financial concepts, including key aspects of bonds, stocks, and mutual funds. In a survey of Washington state residents, Moore (2005) found that homeowners did not understand the terms and conditions of consumer loans and mortgages. The National Council for Economic Education's report (NCEE, 2005) detailed widespread knowledge gaps regarding fundamental economic concepts among high school students, as did Mandell (2008). Lusardi and Mitchell's (2008, 2011) modules on planning and financial literacy for the Health and Retirement Study (HRS) confirm that many older (50+) individuals cannot do simple interest-rate computations such as calculating how money would grow at an interest rate of 2 percent, nor do they demonstrate an understanding of inflation and risk diversification. These findings have been confirmed for younger adults as well as other population subgroups (Lusardi, Mitchell, and Curto, 2010; Lusardi and Mitchell, 2009).[8]

Still other authors have suggested that financial knowledge is a choice variable and endogenously determined. For example, Delavande, Rohwedder, and Willis (2008) posited

[7] See, for instance, Cagetti (2003); DeNardi, French, and Jones (2011); Gourinchas and Parker (2002); HSZ (1994); and Scholz, Seshadri, and Khitatrakun (hereafter SSK, 2006).

[8] Low levels of financial skills are not only a problem in the United States; as illustrated by the Organization for Economic Co-operation and Development (OECD, nd) and the Survey of Health, Aging and Retirement in Europe (SHARE), respondents there also score poorly on several numeracy and financial literacy scales (Christelis, Jappelli, and Padula, 2010). Lusardi and Mitchell (2011) review a range of other studies documenting low financial literacy in many additional countries.

that investment in financial knowledge is akin to human capital investment, but their static model cannot trace life cycle wealth patterns. Jappelli and Padula (2011) discuss investments in financial knowledge, but they used a simple two-period model and did not evaluate whether differences in knowledge levels produce wealth inequality. Both papers built on the seminal work by Ben Porath (1967) and Becker (1975) who modeled the economic decision to invest in human capital by linking education to wages. By contrast, we dynamically model investments in financial knowledge in a rich intertemporal setting with decisionmaking under many sources of uncertainty, which allows us to evaluate the quantitative importance of financial knowledge and to perform several important policy experiments. Moreover, our work relates to several recent empirical findings; for example, our model is consistent with the findings of those who document a positive empirical link between financial knowledge and wealth holdings.[9] Additionally, our analysis is consistent with research showing that highly knowledgeable consumers are more likely to participate in the stock market, which in our model is represented by the use of a sophisticated investment technology.[10]

3 Life Cycle Wealth and Financial Knowledge

3.1 The Evolution of Income and Assets by Education

The simplest life cycle model posits that consumers will save to transfer resources to life stages where the marginal utility of consumption is highest. Given concavity of the utility function, consumers seek to transfer resources from periods of their lives when they earn substantial income, to periods when they earn less. To illustrate typical household income profiles over the life cycle, Figure 1 plots median net household income by education groups, constructed from the Panel Study of Income Dynamics (PSID).[11] Education groups refer to household heads who have completed less than a high school education, high school

[9] See for instance BMSB (2012), Lusardi and Mitchell (2011), and Van Rooij, Lusardi, and Alessie (2011b).

[10] See Christelis, Jappelli, and Padula (2010) and van Rooij, Lusardi, and Alessie (2011a) among others.

[11] These calculations use the PSID CNEF files from 1980 to 1997. All monetary figures in the paper are in 2004 dollars. We first run median regressions with age and cohort effects, and then we predict incomes for the 1935-1945 cohort; the age dummies are smoothed with a lowess filter.

graduates, and those with at least some college. We focus on white males throughout the paper to keep our sample as homogeneous as possible. We also drop individuals with business assets and censor all variables at the 99th percentile.[12]

[Figure 1 here]

Household income for this cohort is hump shaped over the life cycle. It also rises at a faster rate for the college-educated than for the less educated. All groups' incomes slowly decrease from around age 50 onward. After retirement, income falls due to the fact that social security and pension benefit amounts are generally lower than labor earnings. Old-age benefit replacement rates are relatively higher for the less-educated groups due to the progressivity of public safety net programs, so better-educated consumers see their incomes fall relatively more. In retirement, net household income declines somewhat for all groups, probably because of changes in household composition (e.g. loss of a spouse).

Figure 2 traces life cycle paths of median net worth (bank accounts, stocks, IRAs, mutual funds, bonds, net real estate, minus debt) for these same individuals.[13] For the typical household, net assets grow steadily up to the mid-60s and then flatten or decline. Again, there are sharp differences by educational attainment, with the median college educated household having more than \$375,000 in net assets at age 65 (in \$2004). By contrast, at the same age, the median household with less than a high school education has accumulated less than \$125,000, with most of that in the form of housing wealth.

[Figure 2 here]

In the simplest version of the life cycle model, individuals optimally consume only a portion of their lifetime incomes each period, borrowing in some periods and saving in others. A key prediction from this framework is that the life cycle path of assets normalized by lifetime income should be the same across groups; so, as noted by HSZ (1994), the simple life cycle model for higher earners will simply be a scaled-up version of the lower earner's

[12]Hurst, Kennickell, Lusardi, and Torralba (2010) show that including those with business assets skews the interpretation of saving motives compared to the general population, because of the large amount of wealth held in these ventures, as well as the volatility of this income.

[13]For a description of PSID wealth measures and a comparison with the Survey of Consumer Finances, see Juster, Stafford, and Smith (1999).

profile. For this reason, this simple model cannot explain retirement wealth heterogeneity. Another motive for saving is precautionary: when income is uncertain and borrowing is difficult, this raises the possibility that a consumer might have a very high marginal utility of consumption in the future, when income is low. When the utility function is concave and exhibits prudence (Kimball, 1990), such a consumer will want to save more in anticipation of this possibility. While precautionary saving can explain some of the heterogeneity observed in the data, it still falls short of explaining wealth differences among those facing similar uncertain incomes.

Yet another explanation for why the less educated fail to save is offered by HSZ (1994), who point out that the U.S. social insurance system protects families with limited resources against bad states of the world. That is, means-tested and redistributive transfer programs such as the Social Security, Medicaid, and Supplemental Security Income provide an explicit consumption floor in the event that households face poverty in old age. In turn, this consumption floor dampens consumers' precautionary saving motives, particularly when people are rather likely to become eligible for such benefits. While this helps explain why the less educated save little, it does not rationalize wealth inequality in the upper half of the income distribution, where the consumption floor is less likely to be reached.

Other authors resort to differences in preferences to explain differences in wealth accumulation. For example, Cagetti (2003) allows consumers to have different high rates of time preference and low rates of risk aversion, and he concludes that this combination can lead to small precautionary wealth accrual among less-educated and young consumers. Differences in household composition over the life-cycle can also affect consumption by directly changing discount factors or the marginal utility of consumption. Since household size is negatively correlated with education, this could explain some portion of wealth inequality (Attanasio et al., 1999; SSK, 2006). Another potential channel generating wealth inequality is differential anticipated mortality patterns. It is well documented that the more educated live longer (Duggan, Gillingham, and Greenlees, 2007), which could also account for some of the divergence in wealth accumulation across groups.

3.2 Differentials in Sophisticated Financial Products by Education

In view of the income paths illustrated above, it should be apparent that college-educated consumers would optimally do relatively more saving (and borrowing), compared to the less-educated. In turn, this could make the better-educated group more interested than their less-educated peers in a technology that enhanced returns on resources transferred across periods. Table 1 shows the fraction of PSID respondents holding stocks, mutual funds, bonds, and/or individual retirement accounts (IRAs), arrayed by age and education. We denote these products as relatively "sophisticated," as compared to having only a simple bank account (or no saving at all).

[Table 1 here]

From these data, it is evident that college educated households are much more likely to use a sophisticated technology for saving compared to high school dropouts.[14] In particular, more than three-quarters of older (age 55-65) college educated respondents use sophisticated products, compared to fewer than one-third of those with less than a high school education (of the same age). The fact that people investing in more complex financial products earn higher returns relative to a bank account recalls Yitzhaki's (1987) evidence from tax returns and capital gains, where he showed that households earning higher income also held more sophisticated assets paying higher returns. [15]

The ability of the highly educated and better paid to enjoy better returns may result from greater knowledge about financial products. Some authors have suggested that limited numeracy and cognitive ability, or lack of financial sophistication, may explain people's generally low levels of investment and low participation in the stock market (Guiso and Jappelli, 2007; Van Rooij, Lusardi, and Alessie, 2011a). By contrast, in what follows, we endogenize the motivation to take up sophisticated financial products as a way to motivate

[14] See Curcuru, Heaton, Lucas, and Moore (2005) and Campbell (2006).

[15] One could argue that financial knowledge is not needed, if individuals can rely on financial advisers. Below we incorporate the cost of financial advice in the model below. Yet we also note that there are several impediments to using financial advice when consumers lack financial knowledge, as documented by Mullainathan and Schoar (2010) and the Government Accountability Office (U.S. GAO, 2011). In other words, financial literacy is plausibly more of a complement to, than a substitute for, financial advice.

the emergence and persistence of wealth differences over the life cycle.

3.3 Financial Knowledge and Wealth Accumulation

To understand how financial knowledge can alter the invariance of wealth to income in the standard models, we first build an illustrative two-period model where the individual receives labor income y only in the first period. Denoting wealth in period 2 as w, we seek to understand wealth accumulation in period 2 as a function of lifetime income. The consumer can choose how much to consume, c, in the first period, and how much to invest in raising R, the return factor on saving, s. Thus, $w = Rs$ and $c = y - \pi R - w/R$ where π is the monetary cost of raising R. Assuming the consumer has a discount factor β, he maximizes:

$$\max_{w,R} u(y - \pi R - w/R) + \beta u(w)$$

From the first order conditions to this problem and assuming power utility, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, we obtain the following condition for optimal wealth:

$$w^{1-\frac{1}{2\sigma}}(y - 2\sqrt{\pi w}) = \left(\frac{\sqrt{\pi}}{\beta}\right)^{1/\sigma}$$

While the right-hand side is constant with wealth, the left hand side is not. The left-hand side is decreasing in w for reasonable values of σ and π . A rise in income increases the left-hand side for a given wealth level. If the wealth ratio is to increase to equal to right hand side, wealth must rise by more than income. We use simulations to show that this is indeed always the case for reasonable parameter values. We consider a two-period model to assess the behavior of a young person planning to save for retirement. If we posit that $\sigma = 1.6$, $\beta = 0.96$, $\pi = 2.5$, all reasonable parameter values as will be discussed below, Figure 3 shows how the wealth ratio varies with income, for income levels between 20 and 100.

[Figure 3 here]

The slope of this line is positive, and the intuition behind the result is clear. There is a complementarity between an agent's need to save and his willingness to invest in raising R. For high values of y, the reward to investing in R rises because saving needs are relatively important. In this two-period model, the lifetime income and the income trajectory are the same. One can already see, therefore, that it is not higher income *per se* that raises the incentive to invest in financial knowledge, but the need to smooth marginal utility over both periods. That need is greater when there is a large gap between first and second period consumption. This implies that heterogeneity in retiree benefit replacement rates can affect the incentives to invest in financial knowledge, and in turn, this can lead to additional differences in wealth accumulation. The same can be said of differences in demographic factors that shift the marginal utility of consumption over the life-cycle, as well as differences in expected mortality.

A richer setting with uncertainty and borrowing constraints offers additional motivations to save. If consumers are liquidity constrained, they may be unwilling to invest in financial knowledge. Faced with uncertainty, a consumer may want to save more and also invest more in financial knowledge for precautionary reasons. Furthermore, the sensitivity of saving to the interest rate can be smaller than in the certainty case (Cagetti, 2003), which may also affect incentives to invest in knowledge. Accordingly, it is useful to investigate the effect of financial knowledge on wealth inequality in a richer model of saving. To this we turn next.

4 The Model

We extend the two-period model in several directions to allow cross-sectional variation in both financial knowledge and wealth levels. Our model of consumption over the life cycle allows uncertainty over asset returns, household income, and out-of-pocket medical expenditures. The individual is assumed to choose his consumption stream by maximizing expected discounted utility, where utility flows are discounted by β. The consumer also faces stochastic mortality risk, and decisions are made from time $t = 0$ (assumed to be age 25) to age T (or as long as the consumer is still alive; and $T = 100$). Adding to the heterogeneity created by the stochastic components, we also focus on three different education groups (less than high school, high school, and college). Across these, we allow for heterogeneity in income, mortality, demographics, out-of-pocket medical expenditure levels, and risk. Importantly, we do not allow for differences in preferences, and we assume consumers start their life cycles with no financial knowledge, so as to make clearer how our model works regarding investment in financial knowledge.

The utility function is assumed to be strictly concave in consumption and defined as $n_t u(c_t/n_t)$, where n_t is an equivalence scale capturing (known) changes in demographics (SSK, 2006). The marginal utility of consumption is $u'(c_t/n_t)$ and thus rises with n_t. Since the path of n_t is hump shaped over the life cycle, this contributes to generate a hump shaped consumption profile with age (Attanasio et al., 1999).

The consumer may elect to invest a portion of his resources in two different investment technologies. The first is a basic technology (for example, a checking account) which yields a certain (low) return $\overline{r}$ ($\overline{R} = 1 + \overline{r}$). The second is more sophisticated and enables the consumer to receive higher returns, but it comes at a cost. Specifically, the consumer must pay a direct cost (fee) to use the technology, c_d, as well as invest time in acquiring the knowledge. Additionally there is an opportunity cost of time spent obtaining knowledge with a price of $\pi_i(i_t)$; we assume that this cost function is convex, reflecting decreasing returns in the production of knowledge. A convex cost has the advantage of avoiding bang-bang solutions where consumers invest massively in one period; hence it encourages the smoothing of investment over time (Delavande et al., 2008).[16]

The rate of return of the sophisticated technology is stochastic, and the expectation of the return depends on the agent's level of financial knowledge at the end of t, $\widetilde{R}(f_{t+1})$. The stochastic return function is given by

$$\widetilde{R}(f_{t+1}) = \overline{R} + r(f_{t+1}) + \sigma_\varepsilon \varepsilon_{t+1}$$

[16] Alternatively one might allow for a direct disutility of investing in financial knowledge and try to estimate it from the data. Here, because we are concerned mainly with the model's properties rather than its precise fit to the data, we abstract from the direct disutility channel.

where ε_{t+1} is a N(0,1) iid shock and σ_ε is the standard deviation of returns on the sophisticated technology. The function $r(f_{t+1})$ is increasing in f_{t+1} and can be interpreted as an excess return function. Since the variance is assumed fixed, this also implies that agents with higher financial literacy obtain a higher Sharpe ratio for their investments. We denote by $\kappa_t = 1$ an indicator that the consumer invests in the sophisticated technology in period t, and $\kappa_t = 0$ if not.[17]

Financial knowledge evolves according to

$$f_{t+1} = \delta f_t + i_t$$

where δ is a depreciation factor and i_t is gross investment. Depreciation exists both because consumer financial knowledge may decay, and also because some knowledge becomes obsolete as new financial products are developed.

The consumer may also receive a government transfer tr_t and is guaranteed a minimum consumption floor of c_{min} by the government (as in HSZ, 1994). This consumption floor may lower the expected variance of future consumption, which diminishes the precautionary motive for saving. Transfers are defined as $tr_t = \max(c_{min} - x_t, 0)$ where cash on hand is:

$$x_t = a_t + y_t - oop_t$$

where y_t is net household income and oop_t represents out-of-pocket medical expenditures. Both of these variables are stochastic over and above a deterministic trend. The sophisticated technology cannot be purchased if $x_t - c_d < c_{min}$ (that is, the government will not pay for costs of obtaining the technology). End-of-period assets are given by

$$a_{t+1} = \widetilde{R}_\kappa(f_{t+1})(x_t + tr_t - c_t - \pi(i_t) - c_d\kappa_t)$$

[17]We model only the extensive margin of investment in the sophisticated technology. An extension of the model would consist of allowing for a decision of "how much" to invest in the technology. In preliminary work, we found that differences in participation across education groups is larger than at the intensive margin in the PSID.

where $\widetilde{R}_\kappa(f_{t+1}) = (1-\kappa_t)\overline{R} + \kappa_t \widetilde{R}(f_t)$. We impose a borrowing constraint on the model such that assets a_{t+1} have to be non-negative.

We posit that an employed individual's net household income equation (in logs) is given by a deterministic component which depends on education, age, and an AR(1) stochastic process:

$$\log y_{e,t} = g_{y,e}(t) + \mu_{y,t} + \nu_{y,t} = g_{y,e}(t) + \eta_{y,t}$$
$$\mu_{y,t} = \rho_{y,e}\mu_{y,t-1} + \varepsilon_{y,t}$$
$$\varepsilon_{y,t} \sim N(0, \sigma^2_{y,\varepsilon}),\ \nu_{y,t} \sim N(0, \sigma^2_{y,v})$$

where e represents his education group, and $g_{y,e}(t)$ is an age polynomial (quadratic). The error term $\eta_{y,t}$ is the sum of an persistent component $\mu_{y,t}$ and an idiosyncratic component $\nu_{y,t}$.

Retirement is exogenous at age 65. After retirement, the household receives retirement income which is a function of his pre-retirement income.

A similar stochastic AR(1) process is assumed for out-of-pocket medical expenditures. Log out-of-pocket expenditures follow the process:

$$\log oop_{e,t} = g_{o,e}(t) + \mu_{o,t} + \nu_{o,t} = g_{o,e}(t) + \eta_{o,t}$$
$$\mu_{o,t} = \rho_{o,e}\mu_{o,t-1} + \varepsilon_{o,t}$$
$$\varepsilon_{o,t} \sim N(0, \sigma^2_{o,\varepsilon}),\ \nu_{o,t} \sim N(0, \sigma^2_{o,v})$$

Because these expenditures are generally low prior to retirement (and to save on computation time), we allow only for medical expenditure risk after retirement (as in HSZ, 1994).

Finally, we allow for mortality risk at all ages, denoting $p_{e,t}$ as the one-year survival probability. These are allowed to differ across education groups.

If we denote the non-deterministic components of income and out-of-pocket expenditures as η_y and η_o, respectively, then the state-space in period t is defined as $s_t = (\eta_{y,t}, \eta_{o,t}, e, f_t, a_t)$. The consumer's decisions are given by (c_t, i_t, κ_t). Hence, there are two continuous control variables (consumption and investment) and a discrete one (participation). There are five state variables. We represent the problem as a series of Bellman equations such that, at each age, the value function has the following form:[18]

$$
\begin{aligned}
V_d(s_t) &= \max_{c_t, i_t, \kappa_t} n_{e,t} u(c_t/n_{e,t}) + \beta p_{e,t} \int_{\varepsilon} \int_{\eta_y} \int_{\eta_o} V(s_{t+1}) dF_e(\eta_o) dF_e(\eta_y) dF(\varepsilon) \\
a_{t+1} &= \widetilde{R}_{\kappa}(f_{t+1})(a_t + y_{e,t} + tr_t - c_t - \pi(i_t) - c_d I(\kappa_t > 0)),\ a_{t+1} \geq 0 \\
f_{t+1} &= \delta f_t + i_t \\
\widetilde{R}_{\kappa}(f_{t+1}) &= (1 - \kappa_t)\overline{R} + \kappa_t \widetilde{R}(f_t)
\end{aligned}
$$

We index variables by e where education differences are assumed to be present.[19]

The model is solved by backward recursion after discretizing the continuous state-variables. At each point in the state space, we use a grid-search method to search for the optimal solution of consumption, financial knowledge investment, and investment in the sophisticated technology. We solve for optimal decisions for a grid of 40 net asset points and 25 financial knowledge points. We use bi-linear interpolation to find the value function when net assets or the financial knowledge stock at $t + 1$ falls off the grid; the value function behaves smoothly and is concave except at low levels of net assets, where liquidity constraints and the consumption floor bind. Accordingly, the grid for assets in the state-space is defined as equally spaced points on $a^{0.5}$, which leads to more points at lower levels of net assets. We use the method proposed by Tauchen (1986) to discretize the processes for income and out-of-pocket median expenditures (with 9 points each). Finally, we use three points for

[18] This formulation abstracts from bequest motives. While an extension to include bequests could be relevant if they were luxury goods, others (DeNardi, French, and Jones, 2011) show that they have a minimal effect on wealth decumulation among the elderly. Moreover, incorporating bequests would only increase wealth inequality, without changing the qualitative nature of our results.

[19] There are three risks over which the value function is integrated: rate of return, out-of-pocket expenditures and income. These risks are assumed independent.

rate of return shocks. In total, optimal decisions are computed more than 14 million times and it takes approximately six minutes to solve for all three education groups distributing the computation load on two servers with 32 cores.[20]

5 Calibration

Our goal is to show how endogenous financial knowledge affects wealth holding, and to understand the determinants of financial knowledge accumulation. Since we lack information on individual returns over the life cycle by education groups, we do not try to estimate all relevant parameters of the model. Instead, we proceed with a calibration using plausible values from the literature for preferences and constraints. An extensive sensitivity analysis is reported in Section 6.

To implement the model, we assume that $u(c_t/n_t)$ has a CRRA form with relative risk aversion σ. The value of 3 for this parameter used by HSZ (1994) is reasonable in their context, since their main mechanism for creating dispersion in saving patterns is the differential impact of the precautionary saving motive due to a consumption floor. Accordingly, in their setting, the precautionary saving motive governed by the coefficient of relative prudence, $1+\sigma$, needs to be large. By contrast, our model has an additional channel for creating wealth dispersion, so there is no need for such a strong precautionary saving motive. We use a value of $\sigma = 1.6$, which is close to that estimated by Attanasio et al. (1999) using consumption data.

The portfolio choice literature typically assumes risk aversion parameters in excess of 4 (e.g. Campbell and Viceira, 2002; Cocco, Gomes, and Manheout, 2005). There are two reasons why we do not need such high degree of risk aversion. First, we assume that excess returns are costly because they need to be built-up and depreciate over time. Second, the participation cost lowers incentives to use the sophisticated technology (Vissing-Jorgensen,

[20] The fact that even low literacy individuals act as though they can solve the complex model above may seem incompatible with their lack of sophistication. But in our setup, an approximation to these people's optimal decision rules can be quite simple as noted by Deaton (1992) in his discussion of complex precautionary saving models.

2002). Both help produce a reasonable fit for participation patterns in the sophisticated technology. To fit the share of wealth invested in the technology would probably require higher risk aversion.

Following SSK (2006), we define an equivalence scale which takes account of consumption differences in household size by education group and changes in demographics over the life cycle. Let $z(j,k) = (j + 0.7k)^{0.75}$ where j is the number of adults in household and k is the number of children (under 18 years old). We then define $n_{e,t} = z(j_{e,t}, k_{e,t})/z(2,1)$ where $j_{e,t}$ and $k_{e,t}$ are the average number of adults and children in the household by age and education group. We use PSID data to estimate the time series of average equivalence scales by education group. The age profile of those scales is hump shaped and more amplified for less educated households[21]We use a discount factor of 0.96 as in SSK (2006) and Campbell and Viceira (2002).[22]The annual minimum consumption floor is set at $10,000 per couple with one child.[23]

Computing post-retirement income as a function of pre-retirement income is notoriously difficult because retirement is, in actuality, somewhat endogenous. Here we estimate fixed-effect regressions of net household income on age and a retirement dummy, estimated separately by education level. This produces replacement rates of 0.81 for dropouts, 0.72 for high-school graduates, and 0.68 for college graduates. These are higher than those rates based only on Social Security benefits, since older households have other sources of retirement income (e.g. spousal earnings, employer pension benefits, annuities, etc). On the other

[21]We compute the average number per household of adults and children (under 18 years old) by the head's education and age. We then compute the equivalence scale according to the formula above.

[22]This is consistent with De Nardi, French, and Jones (2011) who estimate that value to be 0.97 and Cagetti (2003) who estimates a value of 0.948 for those with less than high school education and 0.989 for the college educated.

[23]We arrive at this value using data from the Office of the Assistant Secretary for Planning and Evaluation (ASPE, 2008), where the maximum monthly benefit payable to a couple with one child under the Temporary Assistance for Needy Families (TANF) program was $495 (in $2006). The average monthly benefit of recipients on food stamps (for a 3-person household) was $283. Hence, prior to age 65, the sum of TANF and food stamp benefits totaled $778/month for a 3-person household or $9,336/year (omitting the lifetime TANF receipt limit). The Social Security Administration (http://www.ssa.gov/pressoffice/factsheets/colafacts2004.htm) reports that the 2004 maximum monthly federal payment for SSI for a single household was $552 and $829 for couples; including food stamps yields an annual total of $7,620 for singles and $12,180 for couples. Accordingly we use a value of $10,000/year, comparable to the $12,000 used by HSZ (1994; in 2004 dollars).

hand, these levels are close to total retirement income estimates in the literature (c.f. Aon Consulting, 2008). Following retirement, we let income decline at the rate found in PSID data, controlling for educational groups and cohort effects. This decline is mostly due to changes in household composition.

The return on the safe asset is set to $\overline{r}$ =2 percent (following Campbell and Viceira, 2002). As there is no natural scale for f_t, we allow it to take a value between 0 and 100. To calibrate what returns can be obtained from this knowledge, we simply rescale on a range of excess returns. For simplicity, we assume a linear production function $r(f_{t+1}) = .01 \times r_{max} f_t$ and fix the maximum excess return, r_{max}, at 4 percent; this roughly matches the equity premium used in other studies of portfolio choice.[24] The standard deviation of the excess return is set to 0.16, again consistent with prior studies.[25]

To compute the deterministic part of net household income, we draw on data from CNEF values in the PSID, pooling all available waves (1980-2005). The NBER's Taxsim is used to compute net household income. We account for cohort effects when computing income profiles, setting the cohort effect for our calibration baseline to the 1935-1945 birth group. For comparability with prior studies, we use the AR(1) error structure estimated by HSZ (1994) for net household income prior to age 65. We use data from the Health and Retirement Study to compute the profile of household out-of-pocket medical expenditures, allowing for cohort effects; we predict the profiles for those born in 1940, again using the error structure estimated by HSZ (1994). Both income and out-of-pocket expenditures prove to be highly persistent, and differences in persistence and variance across education groups are relatively small. Following the literature (HSZ, 1994; SSK, 2006), we set the variance of the transitory error component to zero in the simulations since most of it likely reflects measurement error.

Estimating the price of acquiring financial knowledge from available data is difficult because little information is available on inputs to the production process – time and expenditures on financial services – let alone data on investments in, as opposed to the stock of,

[24] See Campbell and Viceira (2002) and Cocco, Gomes, and Maenhout (2005).

[25] Campbell and Viceira (2002) and Cocco, Gomes, and Maenhout (2005).

financial knowledge. One older study by Lewellen et al. (1977) reported on the distribution of expenditures on formal advice and time spent researching stocks for a cross-section of stockholders in 1972. More than 50 percent of males spent fewer than five hours per year doing their own research, 20 percent spent 5-10 hours, 14 percent devoted 10-20 hours, and 16 percent more than 20 hours. Hence the distribution is relatively skewed, with a median of fewer than five hours but a mean between 10 and 20 hours per year. Using an average wage rate of $25 as the opportunity cost of time, this yields an expenditure worth between $250 and $500 annually. That study also reported the distribution of use of formal services (newspapers, magazines, financial advisers) by income levels. For someone with an average income, 30 percent spent under $80, 28 percent around $145, and 34 percent over $270 (in 2004 dollars). We use the midpoint to arrive at a rough estimate of the average time and monetary expenditures of between $395 and $645. According to Turner and Muir (2011), the average cost of a one-hour financial advice consultation is currently about $250. Veritat.com offers financial planning at $25 a month for singles and $40 for families ($35 for retirees), on top of an initial planning fee of $250. Accordingly, the cost ranges from $550 for singles and $730 for families. Less-expensive alternatives include financial advice software such as ESPlanner, where a one-year license costs $40 (the upgraded ESPlanner costs $149; see esplanner.com/product_catalog). For our analysis, we seek to match an average annual expenditure of $500 on financial literacy. We use the function $\pi(i_t) = 100 i_t^{1.75}$ which matches such average expenditures in the simulations and yields a smooth financial literacy investment age profile. For the participation cost of the sophisticated technology, we use the median estimate of $750 (in $2004) from Vissing-Jorgensen (2002).

We also require an estimate of the depreciation factor for financial knowledge,δ, but there is little information on the size of this parameter. One study has reported that undergraduates' economic learning depreciated at 4-10 percent annually (Kipps, Kohen, and Paden, 1984). Wage and labor supply information have also been used to measure human capital depreciation; for instance, Heckman (1976) estimated annual depreciation rates of 3-7 percent. For the sensitivity analysis, we start with a value of 6 percent and vary the parameter. We could permit the depreciation rate to rise with age, to reflect the possibility of cognitive decline. Nevertheless, this is not needed to produce a hump-shaped financial knowledge profile. Furthermore, it is unclear whether consumers can predict cognitive decline, particularly when it comes to memory, and self-reported memory does not change with age in the HRS.[26]

We also allow for mortality risk differences across education groups, estimated using Gompertz hazard regressions in HRS data for persons over age 50, allowing for proportional education effects.[27] We assume the same proportionality by education prior to age 50, but we use age/mortality profiles taken from population life tables.

Upon finding the consumer's optimal consumption, financial knowledge investment, and technology participation at each point in the state-space and at each age, we then use our decision rules to simulate 5,000 individuals moving through their life cycles. These consumers are given the initial conditions for education, earnings, and assets derived from the PSID for persons age 25-30. We initialize financial knowledge at the lowest level (0), because we lack baseline information on financial knowledge. This also makes clear how endogenous accumulation of financial knowledge affects wealth outcomes, and abstracts from differences in initial conditions. We draw income shocks, out-of-pocket medical expenditure surprises, and rate of return shocks, and we then simulate the life cycle paths of all consumers. Bilinear interpolation is used when simulated state variables fall between gridpoints.

6 Simulations

To discuss the simulation results, we focus on outcomes around the time of retirement since this is when heterogeneity in net assets is most evident. Table 2 reports statistics for each education group at the time of retirement, where we see that wealth patterns are quite unequal across education groups. By retirement, the median high school dropout

[26]Objectively measured memory scores do fall after about age 65, but self-assessed perceptions are more likely to affect individual behavior.

[27]These regressions are available upon request. Life expectancy at age 25 is five years higher for the college educated compared to high school dropouts.

accumulates less than half as much wealth compared to high school graduates ($61,500 versus $180,300), and college graduates accumulate two times as much in retirement assets ($370,200). In fact, compared to the outcomes reported in Figure 2, the model somewhat over-predicts wealth inequality, due mainly to very low predicted assets among dropouts. The ratio of median wealth to income (average lifetime income for each group) is 1.91 for dropouts and 7.8 for college graduates. Accordingly, the difference is more than 308 percent between these two groups, for a ratio of 4.08. In other words, our model generates a positive relationship between accumulated wealth (adjusted for income) and income. We proxy retirement shortfalls by the fraction of consumers that reaches retirement age with assets below their current incomes: among high school droputs, it reaches 40 percent, but it declines to 17 percent for college graduates.

[Table 2 here]

At retirement, the fraction of consumers investing in the sophisticated technology also varies by educational group, with only 35 percent of the dropouts, 54 percent of the high school graduates, and 69 percent of the college graduates doing so. This pattern nicely matches the participation patterns for participation in sophisticated financial products shown in Table 1 for the PSID. There we found that 32 percent of dropouts, 53 percent of high school graduates, and 76 percent of college graduates participated in what we termed sophisticated saving technologies.

Finally, we compute the fraction of consumers with low financial knowledge at the time of retirement. Given the production function, a threshold of 25 units would imply that such households can expect an excess return of only one percentage point or less. In our model, such a low level of financial knowledge turns out to be optimal for many, given the constraints and shocks the individuals face. These "optimally ignorant" individuals include 67 percent of the dropouts, 47 percent of the high school graduates, and 33 percent of college graduates. Since financial knowledge strongly influences participation in the sophisticated technology, it is perhaps not surprising that almost all of those with a financial knowledge level of over 25 do use the technology. In this way, financial knowledge can be seen as a type of entry cost, allowing users to deploy the technology effectively. This entry cost varies by education groups, since incentives to invest in financial knowledge also differ.

Figure 4 illustrates the life cycle path of average financial knowledge across education groups, which proves to be hump shaped. Financial knowledge peaks around the age of 65 and declines thereafter. In the accumulation phase, better-educated consumers invest more because they have more to gain from higher returns that help them smooth lifetime marginal utilities. At some point, the opportunity cost of investing becomes too large in terms of foregone consumption and depreciation, and also because the marginal benefit decreases due to the shorter horizon over which they will enjoy the investments. Raising the depreciation rate of knowledge with age would only make this decline more marked.

[Figure 4 here]

6.1 Quantitative Importance of Endogenous Financial Knowledge

Our model embodies several differences across education groups which can generate differential wealth accumulation patterns. First, the consumption floor acts as a tax on saving for those most likely to experience a substantial negative income shock, since subsistence benefits are means tested (HSZ, 1994). Second, differences in replacement rates, demographics, and mortality patterns can create different incentives to save. Finally, there is the mechanism we propose: financial knowledge, which creates a positive relationship between normalized wealth and income. To understand the relative contribution of each mechanism, we next carry out a decomposition exercise. In our baseline simulation, at retirement, the ratio of median wealth-to-income for college graduates to dropouts is 4.03. Next, we eliminate the possibility of accumulating knowledge, along with all differences across education groups other than income and medical expenditure differences; here we fix all constraints to those of high school graduates, and we eliminate the consumption floor. Figure 5 shows that, in the setup with uncertainty only in income and medical expenditures, the wealth-to-income ratio of college graduates is roughly the same as that of dropouts, at 0.97. Accordingly, all groups accumulate wealth in the same proportion to income, which is the basic prediction of the life cycle model.

[Figure 5 here]

Next we reintroduce the consumption floor, which is predicted to lower precautionary savings of dropouts by more than that of college graduates, as well as increase the gap in wealth-to-income ratios. In fact, as illustrated, this ratio rises to 1.07, indicating that in this model, the consumption floor plays a relatively inconsequential role. As a result, our findings differ from HSZ (1994), because our precautionary saving motive is much smaller due to lower risk aversion. We then reintroduce differences in old-age income replacement rates, recalling that college graduates have much lower replacement rates than do dropouts. This change will alter both wealth accumulation and lifetime income patterns; the net effect, of course, depends on the substitutability of retirement wealth and private wealth. Our simulation shows that this does have a sizable effect on the ratio: indeed it rises from 1.07 to 1.5. Introducing differences in demographics contributes another increase of roughly 0.5 in the ratio. What this means is that differences in household composition are as important as differences in replacement rates, in the model. Taking account of mortality differences again increases the ratio, now to 2.5; college-educated households now need to finance consumption over a longer horizon while high school dropouts face a shorter horizon. In other words, this is the amount of inequality generated with a life cycle model that lacks endogenous financial knowledge.

But the outcomes change markedly when we introduce the possibility of consumer investments in financial knowledge, which permits them to access the sophisticated technology and earn higher expected returns. Now the wealth-to-income ratio across education groups rises from 2.5 to 4.08, an increase in the ratio of 1.58. Clearly, of all the explanations examined here for heterogeneity in wealth outcomes, financial knowledge accounts for more than half the cross-group wealth inequality.

To more fully appreciate the effect of interest compounding, we undertake a simple counterfactual exercise. For each of the three schooling groups, we take average simulated consumption, investment, and medical expenditures by age. Then we compute the average return factor of each education group by age using its accumulated financial knowledge, and compare this to the average wealth path assuming all groups only received average returns earned by high school dropouts. We find that wealth would have been 39 percent lower for college graduates at the time of retirement if they had experienced the returns paid to dropouts; for high school graduates, the decline would have been just over 30 percent compared to the paths using their actual average rates of return. Since rates of return differ by roughly one percent between education groups, these differences compounded over many years generate substantial differences in wealth. And our model generates these wealth differences endogenously, drawing only on differences in marginal utilities of consumption over the life cycle.

6.2 Policy Simulations

In the real world, several institutional factors can help shape the process of financial knowledge accumulation. For instance, means-tested benefits protect consumers against bad states of nature; when consumers seek financial knowledge to create a buffer stock of saving, having such programs may provide disincentives to invest in financial knowledge. Similarly, Social Security benefits may crowd out household saving and also discourage the accumulation of financial knowledge. Finally, the educational system can be influential in boosting initial levels of financial knowledge, as demonstrated by Bernheim (1998) and Lusardi and Mitchell (2011).

To explore the relative importance of each type of policy, we next conduct three policy simulations. First, we reduce expected retirement benefits by 20 percent, which might mimic what Social Security can pay future retirees unless tax revenues are increased (Cogan and Mitchell 2003). Second, we examine the impact of a reduction in means-tested benefits by half, which could mean either that generosity is decreased or that eligibility is restricted. Finally, our last scenario is aimed at understanding what would happen if all consumers starting their life cycle had a financial knowledge level of 25. Our simulations assumes that they would start the life cycle with a possible excess rate of return of one percent. The results are reported in Table 3.

[Table 3 here]

Lowering the generosity of retirement income payments raises median assets for all three education groups. This is not surprising, because retirement income crowds out private wealth accumulation in the life cycle model. Median assets more than double for dropouts and they rise by roughly $100,000 for college graduates. We can also compute the change in the present value of retirement income by education group. Expressing the change in median wealth as a fraction of the change in the expected present value of retirement income yields an estimate of the displacement or crowd-out effect of retirement income. The simplest life cycle model would predict a complete offset, once adjustment is made for the fact that wealth is measured at the time of retirement and thus the reduction in lifetime income has only been partially offset by that age. The unadjusted displacement effects in our simulations range from -0.79 to -1.03. Perhaps even more interestingly, the fraction of respondents optimally ignorant is much lower given reduced retirement benefits. In other words, since all consumers must now save for retirement, investment in financial knowledge increases overall. Of course, this comes at a cost: the present value of investment expenditures rises by about $8,000 for dropouts, $4,000 for high school graduates, and $6,000 for college graduates. Lowering retirement income generosity thus decreases wealth inequality, instead of increasing it.

The next scenario halves means-tested benefits from $10,000 to $5,000 per year. As anticipated, this boosts incentives to save for precautionary reasons. But because the precautionary saving motive here is less important than in other studies, the policy change has little differential effect on wealth accumulation and/or financial knowledge. Both rise following the cut in benefits, but the increase is relatively similar across groups. Accordingly, in this model, means-tested benefits do not appear to be an important factor shaping saving and investment in financial knowledge.

Our final policy scenario considers the possibility that consumers could start on their life cycle paths already having a positive amount of financial knowledge. This could happen, for instance, if financial education were included in high school curricula. To explore how results change, we chose a level of 25; this implies consumers can earn an initial excess return of one percent. As indicated in the last panel of Table 3, this increases retirement outcomes only slightly. Since financial knowledge is endogenous in the model, people who do not need the knowledge will let it depreciate to their target optimal levels. Although wealth is slightly larger at retirement, and financial ignorance less prevalent, these effects are small in comparison to the initial change in financial knowledge.

The fact that outcomes do not change at retirement does not, however, mean that financial education is not beneficial in terms of welfare. In fact, the last row of Table 3 reports the change in initial wealth at age 25 that would be equivalent to the change in expected utility from the boost in financial knowledge. It would take an additional \$6,781 dollars to make a dropout equally well off, \$6,859 for a high school graduate, and \$15,410 for a college graduate. Relative to median initial wealth, it would take an increase of 82 percent of initial wealth from dropouts, for them to have the same expected utility as in the baseline; the value is 56 percent for college graduates. These large wealth equivalent measures demonstrate that consumers do value financial knowledge, even when they make no new investments thereafter. A large part of this value is due to the fact that investment costs are reduced when people are endowed with a positive level of initial financial knowledge.

In sum, our last simulation confirms that a policy which exogenously raised financial knowledge early in life might not have measurable long-term effects: our consumers have both optimal financial knowledge and optimal target wealth levels in mind when solving their life cycle problems. Hence it is possible that an educational program could enhance saving in the short run, with little enduring impact in terms of additional future wealth. Nonetheless, the training will still bring important welfare benefits, since additional short-term saving increases lifetime consumption and thus utility.

6.3 Sensitivity Analysis

We have not estimated all parameters of the model due to lack of data on individual rates of return, so it is important to assess how our results might differ when key parameters are varied. To this end, Table 4 reports how altering various key parameters of the simulation model alters the asset ratio of college educated individuals relative to dropouts, the fraction using the sophisticated technology, and the fraction optimally ignorant at retirement. Lowering the risk aversion level to 1.1 increases dispersion in wealth and other outcomes. This is because when risk aversion is low, consumers save mostly for retirement; since the intertemporal elasticity of substitution is large, they are more willing to invest in financial knowledge. Such complementarity increases dispersion in wealth. But the opposite occurs with higher risk aversion. In the latter case, the precautionary motive is more important and so all groups accumulate more wealth and financial knowledge, in turn reducing wealth inequality.

[Table 4 here]

Heterogeneity in outcomes does not differ much when the discount factor is changed, since it is common across groups. A higher depreciation rate for financial knowledge appears to lower heterogeneity in assets, since college graduates reduce their investments in financial knowledge. Changing the average cost of investing in financial knowledge does not change heterogeneity by much. By contrast, changing the convexity of the cost function is consequential. The heterogeneity results are also relatively insensitive to the fixed cost of investing in the technology. Finally, changing the maximum attainable excess return does not have a large impact on wealth heterogeneity. This is because very few respondents optimally attain maximum financial knowledge in our model, so they do not attain maximum returns.

7 Discussion and Conclusions

In this paper we have developed an augmented stochastic life cycle model that endogenizes the decision to acquire financial knowledge. Our goals were to explore the forces that shape financial knowledge accumulation over the lifetime and to examine how much wealth inequality might be attributable to differences in financial knowledge. The formulation acknowledges that financial knowledge does offer higher expected returns, but it is costly to acquire and depreciates with time. The profile of optimal financial knowledge was shown to be hump shaped over the life cycle, and it also differs by education group because of differences in life cycle income paths. We also demonstrate that allowing for endogenous financial knowledge creates large differences in wealth holdings and that social insurance influences the incentives to acquire financial knowledge. Thus our model can rationalize and account for at least some of the observed differences in wealth holdings across education groups, while other authors have had to rely on social insurance and preference parameters to produce similar dispersion.

In generating wealth inequality above and beyond what traditional models of saving have delivered, we can also rationalize some of the large differences in wealth found in most empirical works on saving, by relying on an important fact: individuals do not start their economic lives with full financial knowledge and that knowledge is acquired endogenously over the life cycle. We also show that some level of financial ignorance may actually be optimal. Some may rationally fail to invest as it is expensive to acquire financial knowledge and not everybody benefits from greater financial sophistication. And financial knowledge can be an important public policy lever. For example, we predict that moving to personal accounts under Social Security along with increased reliance on individually managed 401(k) accounts, would be accompanied by more financial knowledge as well as wealth inequality. And an increase in labor income risk, such as that which characterizes the current macroeconomy, is likely to be accompanied by an increase not only in precautionary saving but also an increase in financial knowledge.

In a world where individuals are increasingly asked to take on responsibility to save and provide for their own retirements, and where consumers are confronted with complex financial markets, it is important to start incorporating financial knowledge into our models of saving. People display different levels of financial knowledge early in life, and this simple feature has important implications for how much people save. By incorporating more realistic features in our theoretical models, we will be better equipped to match the data, make predictions for the future, and generate better recommendations for public policy.

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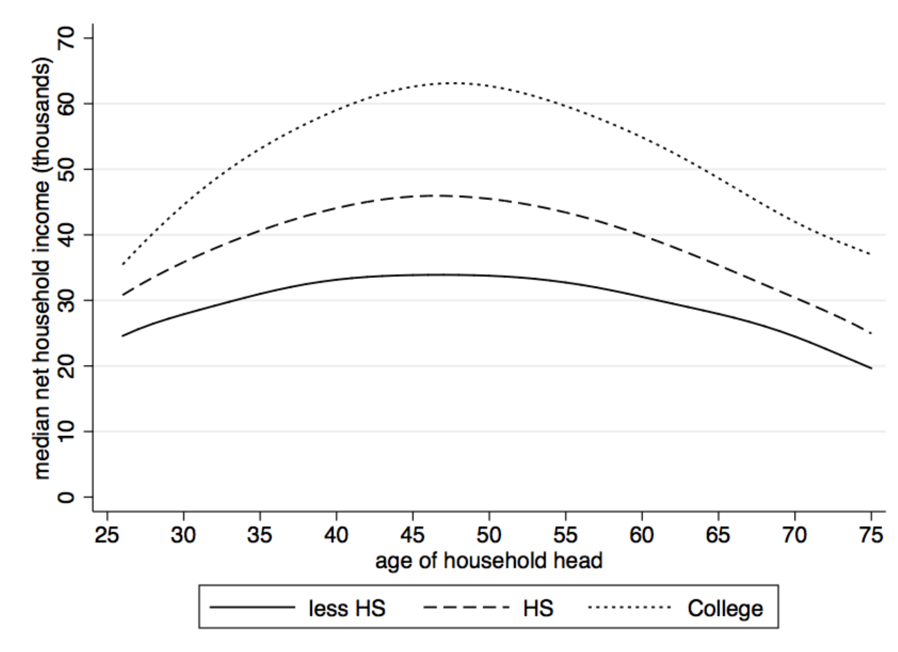

Figure 1: **Life Cycle Net Household Income Profiles by Educational Attainment**. This figure shows median average net household income computed from PSID data, waves 1980-1999 (see text). The figure adjusts for cohort effects based on median regressions with age controls; predictions are for those born 1935-1945. Age profiles are smoothed using a lowess filter. ($2004).

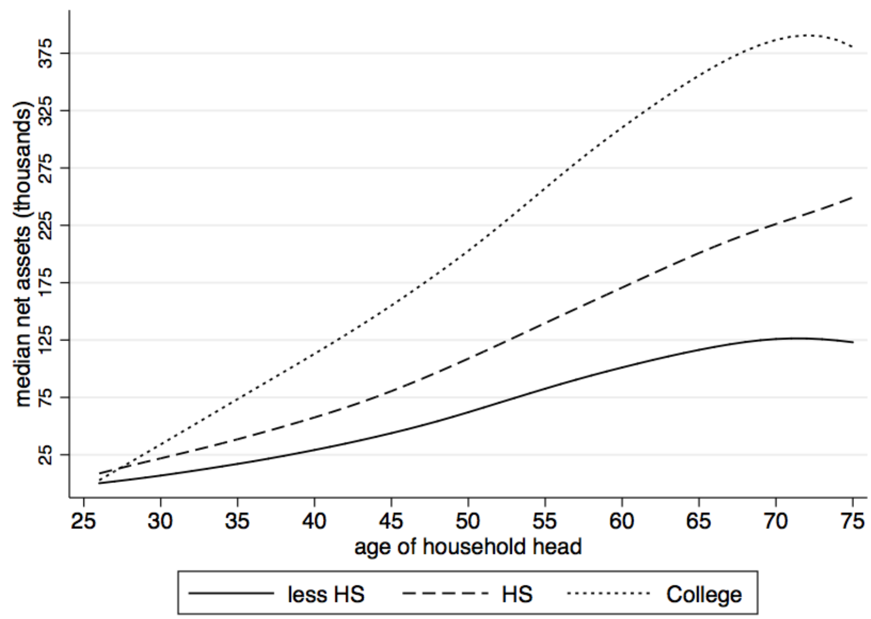

Figure 2: **Life Cycle Net Assets by Educational Attainment**. This figure shows median asset profiles by education group using PSID data (see text). The lines are predicted from median regressions where a correction is made for cohort effects (following French, 2005); assets include the sum of assets minus all debt. We predict for all persons born 1935-1945 and smooth the age profile using a lowess filter. ($2004).

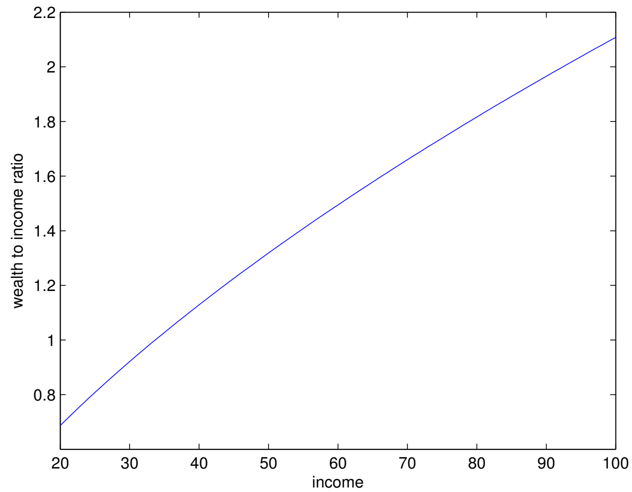

Figure 3: **Relationship of the Wealth-to-Income Ratio and Income in a Two-period Model**. This figure shows how the wealth to income ratio increases with income in a two-period model with first period consumption y, and cost of investing $\pi = 2.5$, also $\sigma = 1.6$, and $\beta = 0.96$.

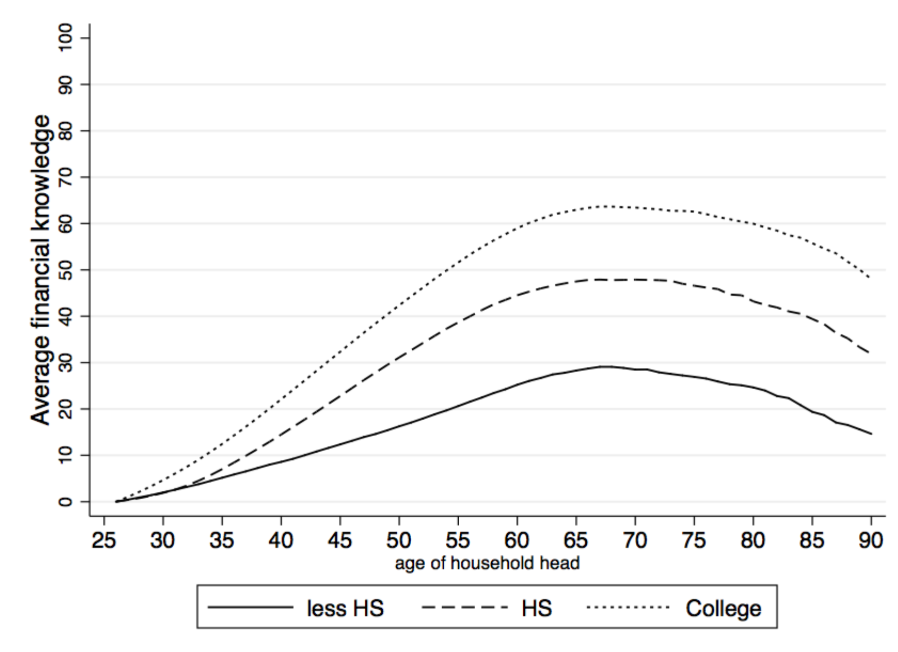

Figure 4: **Simulated Life-cycle Profile of Financial Knowledge in the Baseline Scenario**. We plot the average financial knowledge score (between 0 and 100) by age and education level.

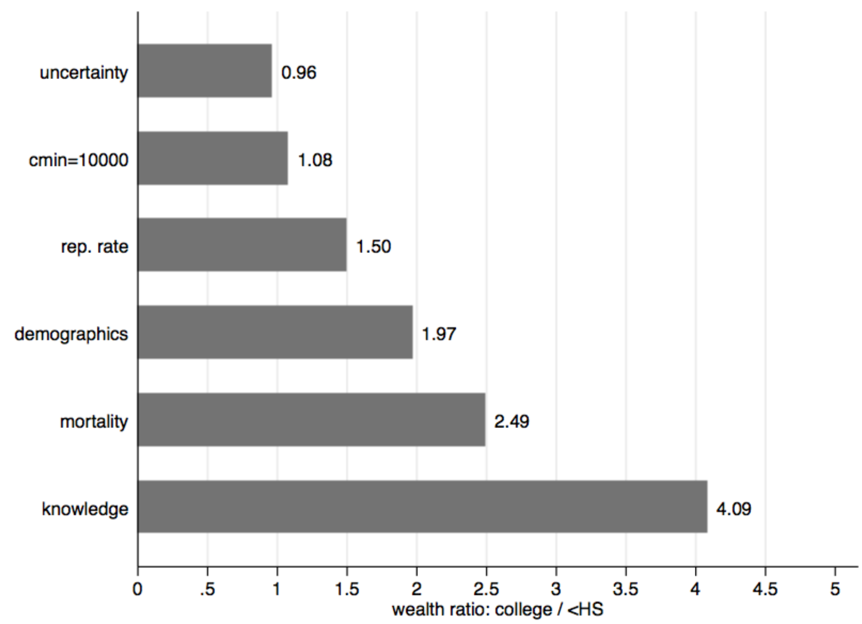

Figure 5: **Decomposition of Wealth Inequality across Education Groups at Retirement**. We compute the ratio of median assets to average lifetime income of each group. We then express the ratio for college educated consumers as a multiple of the ratio for those who did not finish high school. We start with a model that features only differences in uncertain lifetime income and medical expenditures. All other differences across education groups are suppressed. All education groups use the values for those who finished high school. We then progressively add mechanisms that can generate dispersion in asset ratios. In the second row, we put in a consumption floor. In the third row, we add differences across education groups in replacement rates. In the fourth row, we add differences in demographics over the life-cycle. In the fifth row, mortality differences. In the sixth row, we add financial knowledge accumulation.

age group	<High School	High School	College+	Total
25-35	21.8	24.8	51.5	38.6
35-45	24.6	39.8	58.3	48.7
45-55	24.1	42.3	65.5	53.4
55-65	32.1	53.3	75.6	59.5
Total	25.9	38.5	61.1	49.1

Table 1: **Life Cycle Participation (percent) in Sophisticated Financial Products (Stocks and IRAs) by Educational Attainment in the PSID**. Predicted from regressions with controls for age categories and cohort dummies. Cohort born 1935-1945.

at retirement	<HS	HS	College+	Ratio College/<HS
median net assets ($ 000)	61.5	180.3	370.2	6.05
median asset to income ratio	1.91	4.7	7.8	4.08
fraction ($a_t < y_t$)	0.39	0.22	0.17	0.44
fraction ($\kappa_t = 1$)	0.35	0.54	0.69	1.95
fraction with $f_t \leq 25$	0.67	0.47	0.33	0.48

Table 2: **Simulated Outcomes at Retirement (age 65).** These are statistics from the baseline simulations at the time of retirement. The last column is the ratio of college to less than high school. ($2004). Assets ($a_t$), income ($y_t$), use of technology ($\kappa_t$) and financial knoweldge (f_t).

	↓20% in Retirement Income			
At retirement	<HS	HS	College	College / l<HS
median assets ($000)	115.9	258.4	468.9	4.04
fraction ($a_t < y_t$)	0.27	0.12	0.07	0.26
fraction ($\kappa_t > 0$)	0.45	0.63	0.79	1.75
fraction with $f_t \leq 25$	0.57	0.38	0.23	0.39
	↓Means-Tested Benefits to $5000			
At retirement	<HS	HS	College	College / <HS
median assets ($000)	68.1	198.7	384.9	5.65
fraction ($a_t < y_t$)	0.36	0.20	0.16	0.42
fraction ($\kappa_t > 0$)	0.37	0.57	0.71	1.89
fraction with $f_t \leq 25$	0.64	0.44	0.29	0.47
	↑Financial Knowledge at Age 25			
At retirement	<HS	HS	College	College/ <HS
median assets ($000)	65.1	195.4	391.9	6.01
fraction ($a_t < y_t$)	0.38	0.21	0.165	0.42
fraction ($\kappa_t > 0$)	0.37	0.57	0.71	1.90
fraction with $f_t \leq 25$	0.64	0.43	0.30	0.47
Wealth Equiv.($)	6.8	6.9	15.4	2.26

Table 3: **Simulation Results of Policy Experiments**. We report outcomes of the simulation at the time of retirement in three scenarios. The first scenario lowers retirement income by 20 percent. The second lowers means-tested benefits from $10,000 to $5,000. Finally, the last scenario provides a boost of 25 unit of financial knowledge at age 25 for all consumers in the simulation. For this last case, we compute the initial wealth equivalent at age 25 that would make the average consumer at baseline as well-off in terms of utility, compared to the scenario where he inherits 25 units of financial literacy. ($2004). See Table 2 for variable definitions.

As of Age 65, ratio college/<HS	Median assets	fraction ($\kappa > 0$)	fraction ($f \leq 25$)
Baseline	4.08	1.95	0.48
$\sigma = 1.1$	6.25	2.33	0.56
$\sigma = 3$	1.94	1.56	0.36
$\delta = 0.03$	4.08	1.79	0.36
$\delta = 0.09$	3.58	2.14	0.56
$\pi(i) = 75i^{1.75}$	4.01	1.80	0.43
$\pi(i) = 125i^{1.75}$	3.81	2.05	0.53
$\pi(i) = 100i^{1.25}$	2.75	1.58	0.36
$\pi(i) = 100i^{2}$	3.42	2.13	0.53
$c_d = 1000$	4.23	2.10	0.49
$c_d = 500$	4.02	1.82	0.36
$\beta = 0.94$	3.34	2.09	0.75
$\beta = 0.98$	2.69	1.61	0.21
$r_{max} = 0.06$	3.74	1.65	0.33

Table 4: **Sensitivity Analysis of the Simulations**. Each row reports the ratio of college to less than high school statistics at the time of retirement, given a change in the single parameter indicated. r_{max} denotes the excess return when the consumer has perfect knowledge, relative risk aversion (σ), depreciation rate of FL (δ), cost of investment i ($\pi(i)$), participation cost (c_d), discount factor (β) .