Scientific Disclosure and the Faces of Competition

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We construct a theoretical model that allows us to examine disclosure and the different faces of competition across fields. Our modeling choices are motivated by interviews with scientists in fields such as molecular biology, computer science, and mathematics, which exhibit quite different disclosure patterns. We consider a researcher who has made a discovery of partial value in solving a problem in her field, who decides whether to keep it secret until completion of the project or to disclose it to her colleagues. We first consider disclosure in an environment that risks unwanted competition despite the fact that it gives her credit for discovery and may attract collaborators with skills and/or resources to augment the project, in either cost reduction or quality improvement. In our model, collaboration also has the potential to remove a competitor. We then examine the contrasting situation where the researcher wants to attract others to work independently on her line of research. In this version of the model, the disclosing scientist gains a reputational benefit if she convinces other researchers to pursue her line of investigation—potentially creating a hot new area of research. Despite some similarities, these two environments present somewhat different faces of competition.

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Introduction

A growing body of research suggests that the ideal of communalism among academic scientists is just that—an ideal (Blumenthal et al. 1996; Campbell et al. 2002; Cohen and Walsh 2008; Murray 2010).[2] In many cases, commercial incentives associated with commercial potential or industrial support for the research itself may limit dissemination (Cohen and Walsh 2008; Gans and Murray 2010; and Gans et al. 2011). But, more fundamentally, the scientific reward system of giving priority to the first to discover a result creates competitive incentives which limit disclosure of preliminary ideas during the research process (Dasgupta and David 1994; Murray and Mahony 2007; Haeussler et al. 2012). This type of withholding is well documented in life sciences, and our informal interviews suggest it occurs across many fields. In considering disclosure of early results, scientists often said the incentive was not merely early credit, but the potential to attract collaborators with additional skills or resources, which they weighed against the risk of increasing competition by giving others (particularly those with a large lab or resource base) projects they could adopt and complete on their own. But our interviews also suggest a different environment in fields such as mathematics and computer science, where examples abound of researchers reporting unsolved problems in current research and inviting entry and competition (von Hippel 2005; Hong and Walsh 2009; Franzoni and Sauermann 2012; http://polymathprojects.org/tag/tim-gowers/).

We construct a theoretical model that allows us to examine disclosure and competitive incentives among scientific researchers in these different environments. We consider a game in which a researcher who has made a discovery of partial value in solving a problem in her field decides whether to keep it secret until completion of the project or to disclose it to her colleagues. We first consider disclosure that risks unwanted competition despite the fact that it gives credit for discovery and may attract collaborators with skills and/or resources to augment the project, in either cost reduction or quality improvement. In our model, collaboration also has the potential to remove a competitor. We then examine the contrasting situation where the researcher wants to attract others to work independently on her line of research. In this version of the model, the disclosing scientist gains a reputational benefit if she convinces other researchers to pursue her line

[2] Although some will claim that mathematics is not a science, if one views science as the production of knowledge that can be falsified by evidence, mathematics clearly qualifies. Thus we use the terms scientific inquiry in its most general terms.

of investigation—potentially creating a hot new area of research. We refer to the first scenario as the competition/collaboration case and the latter as the research leader/hot area case.

In both cases the focal researcher's expected payoff has three components, a deterministic benefit associated with disclosure, a probabilistic component which is realized only if she is the first to complete the project, and an opportunity cost. The deterministic portion of the benefit turns out to be important for disclosure in both cases. In both, she moves first, but her choice variables differ to fit the situation. In the competition/collaboration case, the focal researcher decides to disclose and to whom, where we allow for two types of colleagues, those who can be trusted not to compete once they know the result and general colleagues who might compete. In the research leader/hot area case, where she views competition more positively, we simplify by abstracting from trusted colleagues and explicit collaboration, but we allow her to choose the stage at which she discloses her result.

Throughout, the other researchers make their decisions, taking into account their opportunity cost. Thus, in both cases, the opportunity cost, which we specify as including the base value of the focal researcher's project relative to the value of an alternative, is an important factor. If we interpret the opportunity cost component of the expected payoffs in terms of project novelty, we obtain the result that, all else equal, less novel projects are more likely to be generally disclosed in equilibrium in the competition/collaboration case. This is in contrast to the research leader/hot area case where disclosure occurs earlier as the novelty of the project rises.

Not surprisingly, in both cases, the quality of potential competitors relative to the focal researcher is important. Our preliminary results are quite rich, and we only highlight a few here. In the competition/collaboration case, where we introduce a distinction between trusted and general colleagues, we find that an increase in the quality of one's *trusted* colleagues, all else equal, increases the likelihood of disclosing to the *general* audience. An important factor behind this result is the fact that the focal researcher can improve the quality of her project if she attracts a higher quality trusted colleague to work with her, which acts as a deterrent to potential general competitors.

A comparison of the marginal effects of increasing the quality of the focal researcher's general colleagues highlights the different *faces of competition.* In the competition/collaboration case, the marginal impact of an increase in the quality of general colleagues is negative, but it is the greatest in magnitude when the focal researcher is more capable than her general colleagues.

This result occurs because of the fact that collaboration rarely occurs in equilibrium in this case, so that the dominant factor in the focal researcher's expected payoff is the reduction in her probability of being first to solve the complete problem.

In the research leader/hot area case, the marginal impact of an increase in the quality of general colleagues on entry and the stage of disclosure are both positive. With higher quality general colleagues, the focal researcher's probability of being first to solve her problem declines, but if she can attract others to this area, she benefits from having introduced the problem. Her role, if one of her colleagues wins, is analogous to that of the conceptual-helpfulness star scientist in Oettl (2012). The higher quality colleagues are more likely to be first to complete the problem. The focal researcher discloses at a later stage, however, because this increases the deterministic portion of her expected payoff. This *face* of competition, though quite intuitive in the context of the research leader/hot area case, is not present in the traditional view of competition.

These results contribute to an emerging theoretical literature on information sharing in research which has largely focused on firms (Von Hippel, 1987; Anton and Yao, 2002; 2004; Lerner and Tirole, 2002, Gächter *et al.,* 2010; Baker and Mezzetti, 2005; Stein, 2008; Hellmann and Perotti, 2010; Gans *et al*., 2011; and Gill, 2008). Stein (2008) is sufficiently general to characterize communication among academics, but it has the limitation of assuming strong complementarity of players so that they must communicate for an idea to move forward. Theoretical studies of disclosure in an academic setting have largely focused on issues related to publication and secrecy (Mukherjee and Stern 2009; Gans and Murray 2010; Gans et al. 2011). The studies which examine prepublication disclosure have largely focused on projects of interest to industry and differences in industrial and academic incentives (Jensen et al. 2003, Aghion et al. 2009; Lacetera 2009). The study closest to ours is Haeussler et al. (2012) which focuses on the role of potential misappropriation in discouraging early presentation of ideas. Additionally, researchers in their model do not consider potential for collaboration, nor do they consider presentation only to a selective group of researchers outside the lab.

In this paper, misappropriation plays no role. The negative face of competition in both the competition/collaboration and research leader/hot area cases is simply that disclosing one's initial results decreases the discloser's probability of getting a publication when competitors enter. This negative effect is weighed against the benefits of attracting a collaborator in the competition/collaboration case and against the benefit from creating a hot new area in the research leader case.

Finally, it is important to recognize that this is not a general model of collaboration, but rather a model in which collaboration potential is one of the benefits of early disclosure. Thus we abstract from the types of issues considered in the extensive work on the economics and management of teams and collaboration in organizations.[3] The consideration of collaboration most closely related to our paper is Bikard and Murray (2011) analysis of academic collaboration with reference to quality of output versus coordination costs, which is factored into our theoretical specification.

2 The Researcher's Disclosure Problem

Throughout, we model the disclosure problem of Researcher 1 (our focal researcher) who is working to solve a defined problem (the focal problem) in a given field. The problem, if solved, will generate value iV where V is some base value for solving the problem and i is a measure of the researcher's quality in this field; without loss of generality we assume that $i \in (0,1)$. A natural interpretation of V is the reputation effect of a publication; not all publications in the same journal or set of journals are equal thus we model the value of the final product - a journal publication - multiplied by a measure of researcher 1's quality.[4] Quality, $i,$ represents the lab's resources or expertise in the given field and can reflect specialized human or physical capital. The higher the i, the more capable her lab is in solving problems in this field. Thus, i can also approximately indicate the probability that Researcher 1 can solve a problem (e.g., the focal problem) in this field.

While other labs may be working on the same overarching problem, none is working on precisely the same approach or specification. Examples of this phenomenon abound. For example, in the 1980's researchers attempting to show the existence of high-temperature superconductivity approached the problem using different materials, and the same is true today for scientists trying

[3] A survey of these works is far beyond our scope. Much of the work in economics relates to team composition and is reviewed in Conti et al. (2012). In the innovation literature there is much on open source which is clearly related to our research leader scenario (von Hippel (2005) is a reference of note). Bikard and Murray (2011) provide an interesting review of work on creativity and collaboration.

[4] One could also think of V in terms of monetary value, as in the case of patents. In fact much of the literature on academic researchers sharing information is related to secrecy surrounding patented discoveries (Gans et al. 2011, Haeussler et al. 2012).

to show room-temperature superconductivity. Similarly, research into the causes and/or treatments of various diseases focuses on a variety of approaches.[5] In such settings, it is reasonable to assume that each lab works on a unique and specific problem. If solved, each specific problem adds value to the field. Thus researchers in the field may compete indirectly, each taking a different approach, or they may compete head-to-head if they work on the same specification. While initially Researcher 1 is the only researcher working on her approach, by disclosing her progress she could attract others to work on her specific problem --either in direct competition or collaboration.

3. Disclosure, Collaboration, and Competition

We consider first a situation in which the focal researcher has made a discovery during her problem solving, and this initial discovery has a value of rV, with r<1, and this initial discovery is independent of her quality i. That is, she has preliminary findings, but the findings do not merit a publication. We treat r as independent of the researcher's quality to allow for any researcher to make a random discovery that merits the attention from others. Quality is quite important in the value of the final scientific output that builds on the discovery, so that high quality researchers can turn the initial discovery into a result of higher value than lower quality researchers, should the high quality researchers become aware of it. Thus if researcher 1 is the only researcher to continue working on the problem, she will receive benefit $[\mathrm{rV} + (1-\mathrm{r})iV]$ which is also the quality of the completed project. The opportunity cost of continuing to work independently is the value of an alternative project K times her quality i. Note that $V > K$, otherwise the focal researcher would not have undertaken her current line of work (the focal project) rather than K. Suppose the time she needs to spend on completing the project is equivalent to (1-r), then the opportunity cost to complete the remaining project is $(1-r)Ki$. Thus the net benefit of completing the project before disclosing is

$$[\mathrm{r} + (1-\mathrm{r})i]V - (1-r)Ki.$$

Given her discovery r, the researcher may want to disclose it to other researchers outside of her lab to obtain credit or to attract collaborators to complete the project. We model the other re-

[5]There is a stream of research on the benefits of openness in science because of complementary approaches. See for example, Stein (2008), Boudreau (2011), Hellmann and Perrotti (2011), and Jeppesen and Lakhani (2010).

searchers in her field as either trusted or general colleagues and their qualities as t and g, respectively, and t and g are both in $(0,1)$.[6] A trusted colleague is someone she knows well and can trust not to compete with her and not to pass private information to others. If she shares r with him, and he becomes interested in working on her problem, he might offer to help Researcher 1; otherwise, he maintains his own research direction. Thus, she bears no risk by sharing with a trusted colleague and she might gain a collaborator who could improve the quality of the final solution, depending on the trusted colleague's lab capabilities.

Sharing with a general colleague, on the other hand, has both benefits and risks. The risk is that it exposes Researcher 1 to possible competition on project V. If interested, a general colleague can replicate Researcher 1's work and work to solve the problem on his own. Suppose the replicability (or the level of spillover) is s (0<s<1). Then the higher the replicability, the easier it if for the general colleague to replicate what the focal researcher has done to date hence the less there is to do by the general colleague. Thus the general colleague's opportunity cost is (1-rs) times the value of his alternative project Kg. The main risk of competition is that someone else may solve the (1-r) problem and get the credit. This is more likely to happen when competitors have greater quality, when there are more such competitors, and when Researcher 1 has lower quality. Thus we assume the probability of winning as $\frac{i}{ng+i}$ if Researcher 1 works alone and competes with the n symmetric general colleagues, where n (n ≥ 1) is the number of general colleagues.

However, one benefit of general sharing is letting the entire community of scientists know that she discovered r, with a reputational value to her of $(1+u)rV$ associated with recognition from the scientific community. We make the reasonable assumption that $0 < u < 1$. Another benefit is that she may find a collaborator among the general colleagues. Collaboration could reduce research cost since the collaborator would bring his lab resources to bear on the remainder of the problem. Collaborating with a higher quality researcher will also improve the quality of the final output. Additionally, collaboration would reduce the number of potential competitors to $n-1$. If there were only one general colleague, collaborating would eliminate competition, but if $n > 1$, collaboration risks competition from $n-1$ general colleagues if they decide to compete with Researcher 1. If this situation is highly risky, Researcher 1 may

[6] We assume all general colleagues are of the same quality. An interesting extension would be to allow general colleagues to differ in their quality.

want to keep her results secret or share only with a trusted colleague. From the general colleague's point of view, collaboration would save him the replication cost.

Given the above premise about collaboration, we assume the value of the collaborative output is a convex combination of the quality of researcher 1 and the highest quality of the researchers in collaborative work. The value is also determined by the fraction of the research yet to be completed. Specifically, the value of the collaborative effort is $V(1-r)max(i,j) \times p$. $max(i,j)$ is the maximum of i and j where $j = t$ if the collaborator is a trusted colleague and $j = g$ if the colleague is a general colleague. That is, the remainder of the work reflects the quality of the "best" researcher in the collaboration. For example, if Researcher 1 collaborates with a trusted colleague whose quality is $t > i$, then the collaboration will generate a value of $V(1-r)tp$. On the other hand if the trusted colleague has value $t \leq i$, then the final solution will have same value that Researcher 1 would have obtained if she had completed the project alone.[7] p is the probability of winning the competition if there is competition. It is assumed to increase with the pair's maximum quality and decrease with competitors' quality: $p = \frac{\max(i,t)}{\max(i,t)+ng}$ if Researcher 1 works with her trusted colleague and competes against the general colleagues; or $p = \frac{\max(i,g)}{\max(i,g)+(n-1)g}$ if Researcher 1 works with a general colleague and competes against the other general colleagues.

If such collaboration takes place, Researcher 1 receives a portion of value (credit) that results from the collaboration. Let this portion to be a, where $0 < a < 1$. Thus the colleague (trusted or general) receives $(1-a)$ of the credit. For simplicity, we assume $a = 0.5$ since we do not model the process that leads to a value of a. Our purpose is instead to model situations in which collaboration will occur given the likely value of parameters. The assumption of $a = 0.5$ is as if that the researchers split the credit evenly for their publication once their work is published jointly.[8]

We have modeled collaboration as increasing in the quality of the final output but only if the collaborator has higher quality than Researcher 1. However, we assume that collaboration, regardless of the quality of the collaborator, reduces the opportunity cost of the project by re-

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[7] In our simulations of the model we also considered the average quality of the collaborators. Results are very similar.

[8] An interesting extension to our model would be to make a depend on a function of the relative qualities of the collaborators. This is beyond the scope of the current work.

ducing the time to completion from $(1-r)$ to $(1-r)(1-ij)$, $j=t$ or g. Following Bikard and Murray (2011) we also assume that collaboration can have a negative effect due to time involved in coordinating across several researchers. Coordination costs are measured as the extra time needed to complete the project. An important aspect of this coordination is that the higher quality person needs to educate the lower quality person so that the collaborative outcome will reach the higher quality level. The coordination time, which increases the greater the differences in researcher qualities, is modeled as $(1-r)(i-j)^2$. Thus the cost of solving the remaining problem collaboratively is the time to completion times the value of the alternative project Ki or $(1-r)[1-ij+(i-j)^2]Ki$.

The Appendix lists all the choices and the payoff functions. From the game tree, it should be clear that Researcher 1 decides among sharing with no one, sharing only with trusted colleagues, or generally sharing. If she shares with no one, the game ends. If she shares only with a trusted colleague, then the trusted colleague may offer to collaborate, in which case Researcher 1 accepts or rejects. If she shares generally, then she may be approached by trusted and/or general colleagues with offers of collaboration, which she can accept or reject. If collaborating with either the general or trusted colleague has the same payoff, we assume she will collaborate with the trusted colleague. While we can solve for the subgame perfect equilibria, clearly the comparative statics, such as how the likelihood of general sharing or the likelihood of collaboration (and with whom) vary with parameters such as the spillover, s, the progress of the project, r, or researcher qualities is not simple.

4. The Research Leader and a Hot New Area

Now consider disclosure in a different environment, one loosely characterizing our interviews with computer scientists and mathematicians. The focal researcher wants to invite other people to work on the same topic in order to increase her visibility. If she can induce others to enter and conduct research on her topic she can be seen as a "research leader." Although having more people working on the same topic means more competition, it has the positive effect of raising her stature in the field. In this model, we do not consider inviting trusted versus general colleagues because the researcher's current focus is to invite entry to a topic rather than to deter competition. For the same reason, we do not consider inviting collaboration.

Given these assumptions, the game tree becomes much simpler. Researcher 1 moves first and decides at what stage (r_1) to disclose a topic before she turns it to the public to invite entry. Given the focal researcher's decision, the other people decide whether to enter this topic area. Thus, Researcher 1's decision will influence the number of other people, e, who would join the same topic. There are n > 0 potential entrants each with quality g < 1. The focal researcher has quality i.

If the she does not disclose until the project is complete ($r_1 = 1$ so that e = 0) or if disclosure when $r_1 < 1$ does not induce entry, then her payoff is Vi – Ki where, as before, K is the value of the alternative project and V > K (otherwise, she would not be engaged in solving the focal project). If she discloses when $r_1 < 1$ and if early disclosure induces entry she earns recognition as a research leader expressed as $(1+ue)r_1Vi$ which increases with the number of people who enter the area e, her own quality i, and the value of the topic V. As in the competition version of the model, the researcher receives this benefit whether or not she successfully completes the solution. In contrast, however, in this scenario she obtains this credit only if she induces other researchers to enter and compete. In addition, the discovery is not random with respect to the focal researcher's ability i since the focal researcher in this model is selecting the stage of discovery at which to disclose.

With competition, her likelihood of winning is $\frac{i}{eg+i}$. The additional payoff from winning the competition is not only an additional $(1-r_1)Vi$ but also an additional payoff reflecting the fact that she has completed a project that has attracted others. The added payoff is given by $be(1-r_1)Vi,$ which we call the "hot area" effect because a large value of e reflects a "hot" or "popular" research topic. The presence of the factor b allows for an examination of cases where the hot area effect is absent (b = 0). As such, Researcher 1's payoff is

$$\pi_1 = (1+ue)r_1Vi + \frac{i}{eg+i}[be(1-r_1)Vi + (1-r_1)Vi] - Ki \tag{1}$$

$$= Vi\left[(1+ue)r_1 + \frac{i}{eg+i}(be+1)(1-r_1) - k\right].$$

Where k $\equiv \frac{K}{V}$. Note that we assume the focal researcher continues to work on the project if $r_1 < 1$ so that she pays opportunity cost on the entire project even if she does not win.

If $r_1 < 1$, other researchers decide whether to enter the topic area. As long as entry is more worthwhile than working on an alternative topic, entry will take place. If a researcher enters, the probability of winning the completion is $\frac{g}{eg+i}$. If an entrant wins the competition, then the hot field effect is b times the number of entrants e times the value that the entrant provides, or $be(1- r_1)Vg$. Thus the expected net payoff for an entrant is

$$\pi_e = \frac{g}{eg+i}[be(1-r_1)Vg + (1-r_1)Vg] - (1-sr_1)Kg.$$

$$= Vg\left[\frac{g}{eg+i}(be+1)(1-r_1) - (1-sr_1)k\right].$$

As before, the replicability (or the level of spillover) is s (0<s<1).

The first order derivative of π_e with respect to e is:

$$\frac{\partial \pi_e}{\partial e} = Vg^2(1-r_1)\frac{bi-g}{(eg+i)^2}.$$

Thus,

1) when $b > g/i$, $\frac{\partial \pi_e}{\partial e} > 0$. Entry will take place until e=n. Thus,

$$\pi_1 = Vi[(1+un)r_1 + \frac{i}{ng+i}(bn+1)(1-r_1) - k]$$

and

$$\frac{\partial \pi_1}{\partial r_1} = iV\left(1+un - \frac{i(bn+1)}{ng+i}\right).$$

When $u > \frac{ib-g}{ng+i}$, the equilibrium r_1 that maximizes π_1 is 1. Intuitively, that is when the credit of being a sole leader in a niche weighs more than competing in a hot field. Otherwise, the equilibrium r_1=0.

2) When $b \leq g/i$, $\frac{\partial \pi_e}{\partial e} \leq 0$. As more people enter, the entry payoff of the next person who faces the entry decision reduces. Thus, entry will continue until the π_e=0. That is when

$$e^* = \frac{(1-sr_1)\frac{i}{g}k - (1-r_1)}{(1-r_1)b - (1-sr_1)k} = \frac{(\frac{i}{g}k - 1) - r_1(sk\frac{i}{g} - 1)}{(b-k) - r_1(b-sk)}.$$

Thus,

$$\pi_1 = iV[(1+ue^*)r_1 + \frac{i}{e^*g+i}(be^*+1)(1-r_1)] - Ki.$$

Note that

$$\frac{\partial e^*}{\partial r_1} = \frac{\left(\frac{Ki}{Vg} - 1\right)\left(b - s\frac{K}{V}\right) - \left(s\frac{Ki}{Vg} - 1\right)\left(b - \frac{K}{V}\right)}{\left(\left(b - \frac{K}{V}\right) - r_1\left(b - s\frac{K}{V}\right)\right)^2}$$

And

$$\frac{\partial \pi_1}{\partial r_1} = Vi\left(1 + ue^* + ur_1\frac{\partial e^*}{\partial r_1} + i\frac{(1 - r_1)(bi - g)\frac{\partial e^*}{\partial r_1} - (be^* + 1)(e^*g + i)}{(e^*g + i)^2}\right).$$

5. Simulated Results for the Competition/Collaboration Case

5.1 Simulation Set-Up

The competition/collaboration case is simulated using multiple values of each parameter. A total of 4,100,625 cases are examined and 2.28% are cases of mixed strategies which are dropped from further consideration. Attention is directed to the decision to disclose (that is, to generally share, GS, or target share only with trusted colleagues, TS) and the decision to collaborate (whether the focal researcher discloses or not). The decisions are analyzed using tabulations of results as well as logit models. GS = 1 if disclosure occurs (0, otherwise) and COLLAB = 1 if there is collaboration (0, otherwise), and GS or COLLAB are the dependent variables in the logit models. Notice that the focal researcher will always share with her trusted colleagues since TS weakly dominates NS. We use the following notation to indicate the intersection of GS and TS with COLLAB:

	Generally share	Collaborate with general colleague	Collaborate with trusted colleague
GS_G	Yes	Yes	No
GS_T	Yes	No	Yes
GS_N	Yes	No	No
TS_T	No	No	Yes
TS_N	No	No	No

The logit regressors include the levels of each of the parameters as well as all possible interactions. Model parameters are i, g, t, s, r, n, k, a and u. k $\equiv$ K/V, and, as discussed earlier, k < 1 which imposes the condition that the research direction of the focal researcher is more important than the alternative research project (otherwise, she would not have undertaken this line of research). The case of small k is one in which the discovery is important and large k can be thought of as an incremental discovery. Recall that the opportunity cost of working on the focal project is the quality of the researcher (i, t, or g) times the value of the alternative project K which is now replaced by k. The number of general colleagues, n, is set to 1, 2 and 5. These values were picked based on Walsh et al. (2011) survey results on numbers of competitors for research labs across a variety of fields. When n = 1, competition will not occur if the focal researcher generally shares and then collaborates with the general colleague. When n = 5 the probability of entry is very low; if the focal researcher generally shares entry occurs in only 0.6% of the cases when n = 5. As noted earlier, we assume a = 0.5. Attention is restricted to u = 0.1; clearly, increases in its value would increase the likelihood of GS. All other parameters vary over (0, 1).

The marginal effect of each parameter in the logit regression is the partial of the probability of general sharing or collaboration with respect to the parameter. Since the partial involves the levels of the other parameters, the marginal effect of a parameter is computed as the average partial where the partials vary across all values of the other parameters.

Four situations are of interest. The first involves only those cases in which the focal researcher (Researcher 1) is more capable than either her trusted colleague or the general colleague(s) (alternatively, we can think of this case as one in which the focal researcher has more resources relevant for solving the remainder of the problem). ISMARTER is the condition that i > t and i > g. The second situation is one in which the focal researcher is the least capable researcher. IDUMBER is the condition that i < t and i < g. The final two are cases where the focal researcher capability lies between that of the general and the trusted colleague: IBET_TIG is the condition that if t < i < g and IBET_GIT is the condition if g < i <t.

In Table 1 are the results for the fractions of sharing and collaborative behavior for each of the four cases of interest. Several results stand out. First, regardless of the relative values of i, g, and t, general sharing (GS_G = 1, GS_T = 1, or GS_N = 1) occurs more often than sharing only with a trusted colleague (TS_T = 1 or TS_N = 1). GS is most likely when g < i < t (IBET_GIT) and least likely when t < i < g (IBET_TIG).

Second, the majority of the equilibrium outcomes involve general sharing without collaboration. This is particularly true for ISMARTER, where collaboration occurs in only 4.42% of the cases. In this case collaboration does not improve the quality of a completed project, though it can still decrease the cost of completing the project. In the other cases, collaboration is much more likely to occur (IDUMBER, IBET_GIT and IBET_TIG), the focal researcher has more incentive to collaborate because collaboration not only reduces the cost of completing the project, but it also leads to an improved solution if the collaborator is more capable. Of course, the trusted and/or general colleague must also receive a net benefit in order for collaboration to be possible. Incentives are aligned roughly 34% of the time the when the focal researcher is less capable than both her trusted and general colleagues. When she collaborates with the trusted colleague it is almost always after general sharing.

Table 1

	IDUMBER	ISMARTER	IBET_TIG	IBET_GIT
GS_G	13.06%	2.21%	23.35%	0.06%
GS_T	20.79%	2.21%	0.00%	27.95%
GS_N	49.22%	82.36%	55.18%	61.76%
TS_T	3.20%	0.00%	0.00%	0.38%
TS_N	13.72%	13.23%	21.47%	9.85%
Total cases	1,139,446	1,233,225	552,825	552,825

The interesting question is how relative abilities and other parameters, such as the opportunity cost colleagues to collaborate, and spillovers affect the outcomes. Section 5.2 discusses the averages of marginal effects on disclosure and 5.3 discusses the effects on collaboration.

5.2 The Disclosure Decision

Notice that the focal researcher will always share with her trusted colleagues since TS weakly dominates NS. In considering whether to also share with general colleagues, she considers three issues. First, by disclosing generally she earns credit in addition to any expected return from solving the entire problem. Second, if she generally shares she may attract a collaborator from among the general colleagues. In the absence of general sharing her only option for collaboration is a trusted colleague. A more capable collaborator leads to an improvement in the final product and lower cost if the collaborators solve the problem (that is, if a competitor does not first solve the problem). On the other hand collaboration imposes a coordination cost. Finally, there is the potential for competition from a general colleague if the focal researcher generally shares. Because all three effects are relevant, the sign of the marginal effects can be positive or negative, in general.

In Table 2 are the average marginal effects for the logit model of general sharing regressed on the levels and interactions of the parameters.

Table 2. Average Marginal Effects for General Sharing

Parameter	IDUMBER	ISMARTER	IBET_TIG	IBET_GIT
i	-0.075	0.117	0.055	-0.039
g	-0.144	-0.299	-0.056	-0.356
t	0.152	0.002	0.012	0.144
s	-0.098	-0.083	-0.131	-0.056
r	0.515	0.254	0.439	0.271
k	0.580	1.532	0.824	0.695
n	0.047	0.047	0.075	0.016
R-square	0.670	0.920	0.884	0.798
# Cases	1139466	1233225	552825	552825

Note: *All marginal effects are statistically different from zero (p value = 0.000).*

Effect of i

An increase in own quality, i, reduces the chance that a general colleague could successfully compete with the focal researcher. This makes general sharing less risky from the point of view of inviting competition and occurs regardless of the relation of i to t and g. However, the impact of increases in i on coordination cost or the benefit from collaborating with a researcher with other skills or resources (i.e., different g or t) depends on the position of i in the community. For example, when $i < g$, the coordination cost of collaborating with a general colleague decreases with an increase in i, while the gain in the quality of the project from such collaboration decreases. Similarly, the impact of i on sharing for collaborative purposes is ambiguous for $i > g$, but the direction of the coordination cost and quality gains are the reverse. Thus the effect of an increase in i on general sharing for collaborative purposes can be positive or negative.

The negative effect dominates for IDUMBER and IBET_GIT. For IBET_GIT, the competitive threat is small because the focal researcher already has a higher quality than the general colleagues. In fact, if the focal researcher generally shares entry occurs in only 2.52% of the cases for IBET_GIT. Note also that the focal researcher has fewer resources than the trusted colleague in IDUMBER and IBET_GIT, so that collaboration with the trusted colleague will increase the quality of the competed project. In fact, as shown in Table 1, these are the cases where collaboration with t after generally sharing is very likely.

The positive effect dominates for ISMARTER and IBET_TIG. For ISMARTER, entry occurs only 1.52% of the time when the focal researcher generally shares, thus the downside of generally sharing (competition from a general colleague) is already very low. The chance of competition is much higher for IBET_TIG. In this case, if the focal researcher discloses entry occurs 7.99% of the time; however, since the general colleagues have more resources there is a chance of collaboration that will increase the quality of the project.

Effect of g

The average marginal effects of changes in g in Table 2 are always negative, but the magnitudes vary substantially. Larger values of g, all else equal, increase the likelihood of a competitor solving the problem before the focal researcher. For IDUMBER and IBET_TIG increasing values of g lead to improvement in the final solution in the case of collaboration, but it also increases the coordination costs of collaboration. On the other hand, for ISMARTER and IBET_GIT, increases in g do not lead to an improved product, but, since $g < i$, increases in g for a given value of i are associated with lower coordination costs. The smallest (in absolute value) marginal effects are for IDUMBER and IBET_TIG.

The largest marginal effects are in cases where the focal researcher is more capable than a general colleague. While this may seem counterintuitive, the combined results in Tables 1 and 2 show that for $g < i$, collaboration rarely occurs in equilibrium, so that the dominant effect of general sharing is the increase in the probability that the focal researcher loses. When $g > i$, collaboration is more likely to occur in equilibrium, which means that cost saving and/or quality improvement play a significant role.

Effect of t

The marginal effects of t are all positive, but for ISMARTER and IBET_TIG they are very small (effectively zero). For IDUMBER and IBET_GIT increases in t, holding i constant, increase the quality of the final project if the trusted colleague becomes a collaborator thus making it less likely that a general colleague could successfully compete. Although the increase in t also increases coordination costs, it is clear from Table 1 that collaboration with trusted colleagues is common for IDUMBER and IBET_GIT, which suggests the quality effect dominates.

Effect of r

An important reason for disclosing is to obtain credit. The results are intuitive. As r increases, the fraction of the final solution that has been discovered by the focal researcher increases. This is an incentive to generally share and thus to obtain credit for what has been discovered. In addition, greater r means that more must be replicated by a competitor. The average value of r when sharing occurs is 0.529 and it is 0.318 in the case of no disclosure (results are similar across the four conditions on relative capabilities). In Table 2 the average marginal effect of r is always positive, but it is largest when $i < g$ (IDUMBER and IBET_TIG) and these are the cases when the focal researcher is most at risk of losing to a competitor. For these cases, earning credit for intermediate results is more attractive.

Effect of s

The effect of s is quite intuitive—a decrease in replication cost reduces general sharing. Thus any policy attempts to reduce replication costs has the counterproductive effect of reducing the amount of sharing, all else equal.

Effect of K/V

The marginal effect of $k = K/V$ is always positive. This is not surprising since larger values of k represent higher opportunity cost, all else equal, and the threat of entry is less.

Threat of Entry

General sharing always has the positive benefit of giving credit to the focal researcher for her discovery, but it can have the negative effect of inducing entry. If the threat of entry is sufficiently low, the focal researcher will chose to generally share. In fact, if the focal researcher discloses, entry occurs in only 1.52% of the cases for ISMARTER *versus* 11.46% of the cases for IDUMBER. When $g < i < t$ (IBET_GIT) entry occurs in only 2.52% of the cases, while in the case of $t < i < g$ (IBET_TIG) entry occurs in 7.99% of the cases. The differences in entry rates are not surprising. The focal researcher faces competition only from a general colleague and a general colleague has less incentive to enter when the focal researcher is more capable

(ISMARTER and IBET_GIT). In the case of IBET_GIT, a general colleague potentially competes with quality that is the max of i and t; that is, a team with trusted colleagues is tougher competition. We also see this in the fact that the marginal effect of t is positive and largest when t > i (IDUMBER and IBET_GIT). As t increases and t > i collaboration between i and t leads to tougher competition; this does not hold for t < i.

This low rate of entry raises the question of why general sharing does not induce greater levels of competition. Entry is least likely when k and/or n are high, when g is low relative to i, and when s is low. When k is high the discovery is not path breaking (at least in comparison with the alternative project) thus the opportunity cost of entering and competing with the focal researcher is high (that is, the potential gains from entry are low). In Table 3 are the rates of entry conditional on general sharing by the focal researcher and for the events k ≥ 0.7 and k ≤ 0.3.

Table 3. Rates of Entry and k = K/V

	k ≤ 0.3	k ≥ 0.7
IDUMBER	19.99%	0.63%
ISMARTER	3.16%	0.14%
IBET_GIT	7.53%	0.0%
IBET_TIG	20.02%	3.75%

For each of our conditions on relative capabilities, entry is much more likely for path breaking discoveries. Note that IDUMBER and IBET_TIG are more at risk of entry than are ISMARTER and IBET_GIT. This follows from the fact that an entrant is competing with a more capable researcher for the first two conditions and an entrant in these cases is more likely to solve the problem first.

The threat of entry also increases with g and s. An increases in g, holding i constant, means that the general colleagues are more capable in competing with the focal researcher and thus a greater competitive threat. The focal researcher's incentive to generally share is lower for higher values of g. The marginal effect of an increase in g is always negative and it is largest in absolute value when i is greater than g (columns ISMARTER and IBET_GIT of Table 2).

High values of s are associated with discoveries that are easier to replicate, hence the opportunity costs of entry are lower for small versus large values of s. This leads to less disclosure.

To provide additional context for the number of competitors, we examine entry by competitors when n varies over 1, 2 and 5. When n = 1 and disclosure takes place, the likelihood of entry varies from a high of 25.77% for IDUMBER to a low of 3.75% for ISMARTER. For n = 2 the respective percentages are 8.86% and 0.97%. Finally, when n increases to 5, the likelihood of entry falls to 1.65% for IDUMBER and 0.04% for ISMARTER. The entry percentages for IBET_GIT and IBET_TIG lie between the ISMARTER and IDUMBER percentages.

5.3 The Decisions to Collaborate

Here we consider the case of collaboration between the focal researcher and either the trusted or general colleague (COLLAB = 1). The average marginal effects for are in Table 4; all are significantly different from zero with the exception of n for IDUMBER and IBET_TIG. Collaboration patterns are very different depending on relative capabilities.

Table 4. Average Marginal Effects for Collaboration

Parameter	IDUMBER	ISMARTER	IBET_TIG	IBET_GIT
i	-0.550	0.168	-0.249	-0.677
g	0.089	0.244	1.198	0.105
t	0.382	0.159	0.174	1.417
s	-0.065	0.003	-0.024	0.007
r	0.185	0.015	0.189	0.037
k	-0.566	0.257	0.230	0.108
n	NS	-0.003	NS	-0.002
R-square	0.42	0.82	0.41	0.625
# Cases	1139466	1233225	552825	552825

Note: *NS = average marginal effect not significantly different from zero.*
All other marginal effects are statistically different from zero (p value = 0.000).

Very little collaboration takes place for the condition ISMARTER. Collaboration takes place in only 4.42% of the cases, and there is little difference between the likelihood of collaborating with a trusted colleague and with a general colleague (see Table 1). Since the focal researcher is more capable (has more resources), collaboration does not improve the quality of a completed project, though it can still decrease the cost of completing the project.

Compared with ISMARTER, when i is less than g and/or t, the focal researcher has more incentive to collaborate because collaboration offers the advantage not only of reducing the cost of completing the project, but it also leads to an improved solution if the collaborator is more capable. Of course, the trusted and/or general colleague must also receive a net benefit in order for collaboration to be possible. Incentives are aligned frequently since collaboration frequently occurs and occurs most often for the cases of IDUMBER when the focal researcher is less capable than both the trusted and general colleagues. When the focal researcher collaborates with the trusted colleague it is almost always after general sharing.

Effect of i

In Table 4 the average marginal effect of a change in i is positive only for ISMARTER. When the focal researcher is the most capable researcher, there is no possibility of improving the project, though the time to completion can be shortened. For the other cases, increases in i, relative to t and/or g, reduce the benefit of an improved project and this dominates a reduction in costs of completion.

Effects of g and t

The marginal effects of g and t are always positive. In the case of g, the largest effect is for IBET_TIG. In this case the only improvement in the quality of the project comes from increases in g. A similar argument holds for t, which has its largest marginal effect for IBET_GIT so that the only improvement in the quality of the project comes from increases in t

Effects of r and s

The marginal effects of these two are always small but they are the largest in magnitude when $i < g$ (IDUMBER and IBET_TIG). As s rises, and/or r falls, the costs of entry for a general colleague falls which makes collaboration less appealing for him.

Effect of k

The average marginal effect of k in Table 4 is negative for IDUMBER and positive for all other cases. Consider the case of ISMARTER first. When $k \leq 0.3$ collaboration occurs in only 0.46% of the cases, whereas collaboration is the result in 10.59% of the cases when $k \geq 0.7$. Thus, when the focal project is important (so that the opportunity cost to the focal researcher of working on the project is low), collaboration rarely takes place. On the other hand, when the focal project is less important relative to the alternative project, collaboration is more likely.

In the case of IDUMBER collaboration is more likely for the cases where the focal project is more important (collaboration occurs 61.11% of the time where $k \leq 0.3$, versus 9.55% of the time where $k \geq 0.7$). The different effects of k depending on the relative capability of i versus g and t and results from the fact that the trusted and general colleague cannot improve on the project in the case of ISMARTER, they can only reduce costs. When k is small (and the project is important), the potential reduction in costs is small relative to the case of k large; thus the cost-saving benefit – the main reason for ISMARTER to collaborate -- is smaller when k is smaller. For IDUMBER, on the other hand, there are both cost savings and project improvement. Further, the trusted and general colleagues are much more willing to work with the less capable focal researcher when that researcher has a very good research result.

Thus collaboration with a more capable researcher takes place rarely, and when it occurs it is for the less important projects. For the less capable researcher, collaboration is more likely and it is most likely with more important projects.

6 Simulated Results for the Research Leader Case

SIMULATION RESULTS INCOMPLETE

In the research leader section some analytic results were presented. For example, in cases where $b > g/i$, if the focal researcher discloses before completing the project (that is, where $r_1 < 1$) entry takes place until $e = n$. The case of $b \leq g/i$ is more complicated and simulation will be used to characterize equilibrium.

Preliminary results are quite interesting. Restricting attention to interior solutions (that is, when ($0 < e^* < n$), we find, for example, that equilibrium entry rises as project novelty rises (that is, as $k = K/V$ falls). This is accompanied by falling equilibrium r_1.

Somewhat counterintuitive in the context of traditional views of competition, we find that, all else equal, a researcher who faces increasingly capable competitors chooses to disclose at a stage that induces higher entry. That is, an increase in g/i leads to higher equilibrium e^*, and hence to higher equilibrium r_1. To understand this result, it is useful to decompose the focal researcher's expected benefit from disclosing into its certain and probabilistic components (see equation 1). By increasing r_1 she increases the certain relative to the uncertain component. The end result is our finding of higher g/i associated with higher e^*.

If there is not research leader effect (u = 0) and b ≤ g/i, there is no value of b for which the focal researcher will encourage entry by disclosing.

6. Concluding Remarks

There is growing body of evidence that patterns of disclosure and competition among scientists are quite complex and markedly different across fields (Haeussler 2012; Stephan 1996, 2012). In this paper, we constructed a framework that allows us to begin to understand differences among fields such as molecular biology where disclosure is often postponed until publication and/or patenting and computer science and mathematics where incomplete results are posted for the entire community quite frequently. Interesting, computer scientists claim their field is quite competitive, as do biologists; so clearly they view competition somewhat differently.

While we have only begun to explore the differences in the competition/collaboration and research leader/hot area cases, the preliminary results provide interesting insights. With regard to the effect of changes in the quality of general colleagues, the direction of marginal effects is opposite in the two cases. In the competition/collaboration case the marginal effects of an increase in the quality of one's potential competitors reduces disclosure. One of the insights from considering the effect of trusted colleagues in this setting is that, by contrast, an increase in the quality of trusted colleagues increases general sharing. The reason comes from the fact that higher quality trusted colleagues decrease entry by competitors in response to disclosure. Further, in the research leader/hot area case, entry is increased by an increase in the quality of colleagues, and, as well, the focal researcher postpones disclosure (but does disclose before publication).

With regard to policy, we find that policies aimed at reducing replication costs have the unintended consequence of reducing disclosure in the competition/collaboration case. Results of such policies in the research leader/hot area case are in progress. Although beyond the scope of this paper, interesting other issues to explore include consideration of misappropriation in the two contexts. Note that we also have not explored collaboration in the research leader/hot area case. Finally, it is well beyond this scope but endogenizing credit to each collaborator is a fruitful direction to explore.

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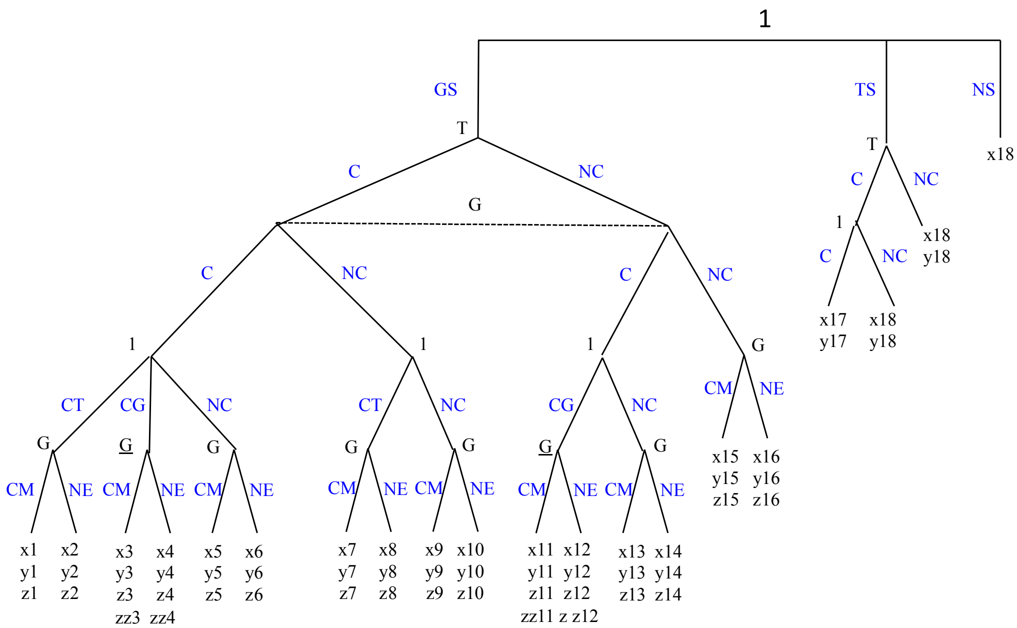
APPENDIX

The game for the competition model:



Payoff functions:

$x1 = (1+u)rV + a(1-r)m_{it}V\frac{m_{it}}{ng+m_{it}} - (1-r)f_{it}Ki$, where $m_{it} = \max(i,t)$ and $f_{it} = 1 - it + (i-t)^2$.

$y1 = (1-a)(1-r)m_{it}V\frac{m_{it}}{ng+m_{it}} - (1-r)f_{it}Kt.$

$z1 = (1-r)gV\frac{g}{ng+m_{it}} - (1-rs)Kg.$

$x2 = (1+u)rV + a(1-r)m_{it}V - (1-r)f_{it}Ki.$

$y2 = (1-a)(1-r)m_{it}V - (1-r)f_{it}Kt.$

$z2 = 0.$

$x3 = (1+u)rV + a(1-r)m_{ig}V\frac{m_{ig}}{(n-1)g+m_{ig}} - (1-r)f_{ig}Ki$, where $m_{ig} = \max(i,g)$ and $f_{ig} = 1 - ig + (i-g)^2$.

$y3 = 0.$

$z3 = (1-a)(1-r)m_{ig}V\frac{m_{ig}}{(n-1)g+m_{ig}} - (1-r)f_{ig}Kg.$

$zz3 = (1-r)gV\frac{g}{(n-1)g+m_{ig}} - (1-rs)Kg.$

$x4 = (1+u)rV + a(1-r)m_{ig}V - (1-r)f_{ig}Ki.$

$y4 = 0.$

$z4 = (1-a)(1-r)m_{ig}V - (1-r)f_{ig}Kg.$

$zz4 = 0.$

$x5 = (1+u)rV + (1-r)iV\frac{i}{ng+i} - (1-r)Ki.$
$y5 = 0.$
$z5 = (1-r)gV\frac{g}{ng+i} - (1-rs)Kg.$

$x6 = (1+u)rV + (1-r)iV - (1-r)Ki.$
$y6 = 0.$
$z6 = 0.$

$x7 = (1+u)rV + a(1-r)m_{it}V\frac{m_{it}}{ng+m_{it}} - (1-r)f_{it}Ki.$
$y7 = (1-a)(1-r)m_{it}V\frac{m_{it}}{ng+m_{it}} - (1-r)f_{it}Kt.$
$z7 = (1-r)gV\frac{g}{ng+m_{it}} - (1-rs)Kg.$

$x8 = (1+u)rV + a(1-r)m_{it}V - (1-r)f_{it}Ki.$
$y8 = (1-a)(1-r)m_{it}V - (1-r)f_{it}Kt.$
$z8 = 0.$

$x9 = (1+u)rV + (1-r)iV\frac{i}{ng+i} - (1-r)Ki.$
$y9 = 0.$
$z9 = (1-r)gV\frac{g}{ng+i} - (1-rs)Kg.$

$x10 = (1+u)rV + (1-r)iV - (1-r)Ki.$
$y10 = 0.$
$z10 = 0.$

$x11 = (1+u)rV + a(1-r)m_{ig}V\frac{m_{ig}}{(n-1)g+m_{ig}} - (1-r)f_{ig}Ki.$
$y11 = 0.$
$z11 = (1-a)(1-r)m_{ig}V\frac{m_{ig}}{(n-1)g+m_{ig}} - (1-r)f_{ig}Kg.$
$zz11 = (1-r)gV\frac{g}{(n-1)g+m_{ig}} - (1-rs)Kg.$

$x12 = (1+u)rV + a(1-r)m_{ig}V - (1-r)f_{ig}Ki.$
$y12 = 0.$
$z12 = (1-a)(1-r)m_{ig}V - (1-r)f_{ig}Kg.$
$zz12 = 0.$

$x13 = x15 = (1+u)rV + (1-r)iV\frac{i}{ng+i} - (1-r)Ki.$
$y13 = y15 = 0.$

$z13 = z15 = (1 - r)gV \frac{g}{ng+i} - (1 - rs)Kg.$

$x14 = x16 = (1 + u)rV + (1 - r)iV - (1 - r)Ki.$
$y14 = y16 = 0.$
$z14 = z16 = 0.$

$x17 = a[r + (1 - r)m_{it}]V - (1 - r)f_{it}Ki.$
$y17 = (1 - a)[r + (1 - r)m_{it}]V - (1 - r)f_{it}Kt.$

$x18 = [r + (1 - r)i]V - (1 - r)Ki.$
$y18 = 0.$