

# Sources and Mechanisms of Cyclical Fluctuations in the Labor Market \*

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## Abstract

I develop a model that accounts for the cyclical movements of hours and employment in the U.S. over the past 60 years. The model pays close attention to evidence about preferences for work and consumption. About a third of cyclical variations in total hours of work per person are in hours per worker and the remainder in the employment rate, workers per person. I show that reasonable volatility in the driving force and a reasonable elasticity of labor supply provide a believable account of the observed cyclical movements in hours per worker. I define and estimate an employment-rate function, analogous to the supply function for hours per worker. My work differs from previous attempts to place cyclical movements of total hours on a labor supply curve by its explicit treatment of unemployment in a framework parallel to the supply of hours of work by the employed.

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# 1 Introduction

I take up the challenge of accounting for volatility in the labor market, in hours per worker and in the employment rate, without contradicting the evidence about the elasticity of labor supply. Many contributions to the literature on aggregate labor-market volatility rest on explicit or implicit assumptions of unreasonably high elasticities of labor supply. The model of the paper describes labor supply in a broad sense, including unemployment. The model integrates labor supply and consumption demand.

Figure 1 shows first differences of log nondurables consumption per person, weekly hours per worker, the employment rate (fraction of the labor force working in a given week, one minus the unemployment rate), and the average product of labor for the United States since 1949. Common movements associated with the business cycle are prominent in all four measures. Consumption, hours, and employment are fairly well correlated with each other, while their correlation with productivity is lower, especially in the last 15 years of the sample.

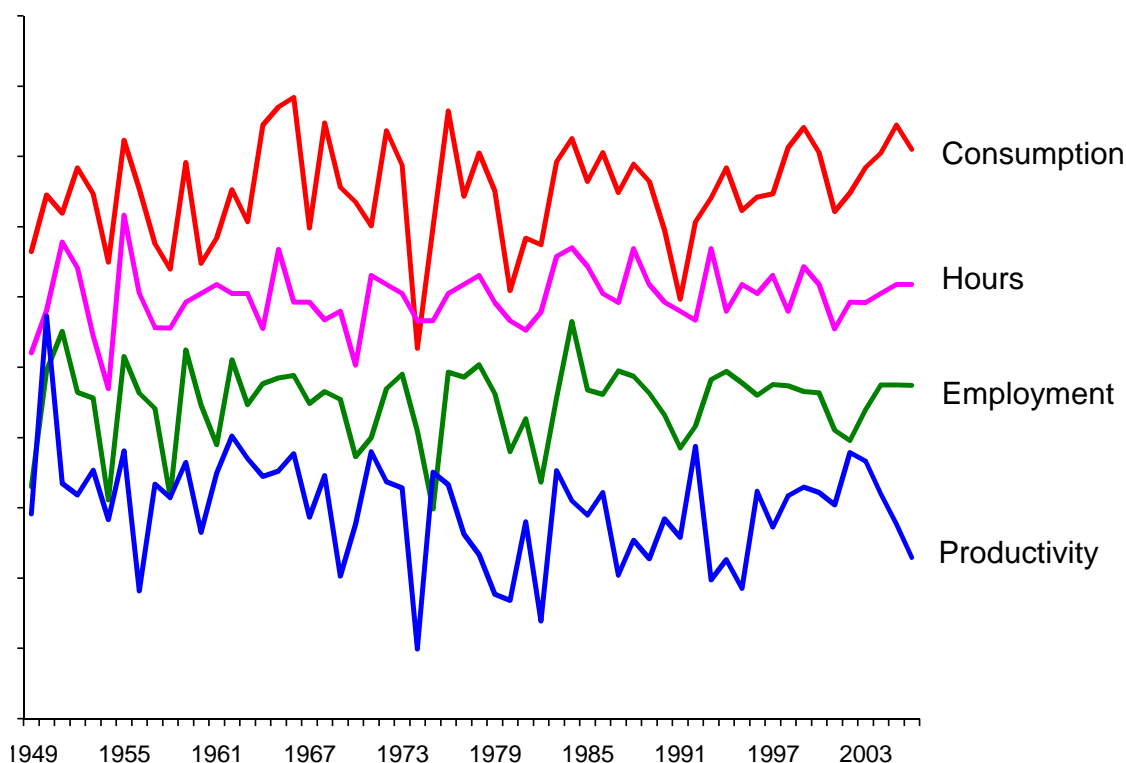


Figure 1: Growth in Consumption, Hours per Worker, Employment Rate, and Productivity  
Note: The tick marks on the vertical axis are one percentage point apart. Constants are added to the series to separate them vertically.

I take the driving force of the movements shown in the figure to be changes in the marginal product of labor, arising from random changes in total factor productivity growth, in the terms of trade, and in the prices of factors other than labor. I portray the movements of hours per worker in terms of a standard labor supply schedule without extreme wage elasticity.

Understanding the cyclical movements of consumption in this framework is a challenge. With preferences additively separable in work and consumption, it is difficult to construct a model on standard principles that generates a strong hours response—as seen in Figure 1—and a strong pro-cyclical consumption response. The approach I take is to invoke fairly high complementarity between consumption and hours of work. High marginal productivity induces households to substitute purchased consumption goods and services to replace the diminished time at home resulting from longer hours of work.

The second big challenge is to understand the movements of the employment rate in this framework. I do so by making job search an integral part of the model and a distinct use of peoples' time. In this area, the model draws on the Mortensen and Pissarides (1994) theory of equilibrium unemployment. I develop an employment function that is in some ways analogous to an hours supply function. But it does not depend solely on choices made by workers. That is, job search is not just a use of time determined by individual choice in response to a market wage. Rather, it is an equilibrium of interaction among jobseekers and recruiting employers.

In broad summary, the model in this paper considers a worker in a family that maximizes the expected discounted sum of future utility, which depends positively on the members' levels of consumption and negatively on their hours of work. The worker has an hourly marginal product  $w$ . I denote it  $w$  because it functions as the wage in the determination of the worker's hours. The family's marginal utility of goods consumption,  $\lambda$ , set at the same level for all members, describes the long-run or permanent level of well-being in the economy. The marginal product  $w$  captures the deviation of current conditions from normal. When  $w$  is higher than the level corresponding to current consumption, hours will be higher than normal as workers take advantage of the temporarily exceptional benefit of working.

Given the state variables  $\lambda$  and  $w$ , hours of work per worker,  $h$ , is a function  $h(\lambda, w)$  expressing the level that equates the marginal disutility of work to  $\lambda w$ . Hours supply is an increasing function of both  $\lambda$  and  $w$ . A companion function,  $c_e(\lambda, w)$  describes the cor-

responding choice of consumption for employed family members. I view the increases in consumption that occur when  $w$  is unusually high (that is, relative to  $\lambda$ ) as resulting from the positive response of consumption to  $w$  through the consumption-work complementarity. In addition, the function  $c_u(\lambda, w)$  describes the family’s choice of consumption for its unemployed members—with consumption-hours complementarity, it will be lower than consumption of the employed.

I consider a broad class of models where the employment rate is a function  $n(\lambda, w)$  of the same two variables. The class includes the Mortensen and Pissarides (1994) (MP) model, the basic statement of the theory of unemployment widely in use today. The employment rate is an increasing function of both  $\lambda$  and  $w$ . Other members of the class of models differ from the MP model by the principle governing the compensation paid to newly hired workers. Some other members yield much higher responses of unemployment to the two driving forces than is present in the MP model, but unemployment remains a function of the two driving forces alone. Higher responses of unemployment are the result of more limited response of compensation to driving forces. I note that the efficiency-wage model is a member of the broader class—the efficiency-wage principle stabilizes compensation at the point needed to prevent shirking.

In this paper, I do not consider the small procyclical movements of participation in the labor force—Hall (forthcoming) documents these movements. The function

$$h(\lambda, w)n(\lambda, w). \tag{1}$$

governs the total volatility of hours of work, apart from participation. When the marginal product  $w$  rises temporarily above the level corresponding to  $\lambda$ , employment and hours rise, creating a cyclical bulge in total hours per person. Recessions are times when the opposite occurs.

I treat the state variables  $\lambda$  and  $w$  as unobserved latent variables. I take each of the four indicators—consumption, hours, the employment rate, and productivity—as a function of the two latent variables plus an idiosyncratic residual. The model falls short of identification. I use information from extensive research on some of the coefficients to help identify the remaining coefficients. I also use inequalities derived from the model to limit the ranges of the coefficients. I embody the information in a prior distribution and compute the posterior distribution of the parameters from the prior and the sample evidence shown in Figure 1.

The posterior distribution shows that the empirical employment function is much more

sensitive to  $\lambda$  and  $w$  than is its counterpart in the MP model. Although I treat this as an empirical finding not associated with any specific theory of compensation determination, it implies that compensation paid to newly hired workers is stickier than it would be with Nash bargaining—unemployment rises in recessions because the marginal product of labor falls relative to the compensation paid to newly hired workers, so employers cut their job-creation efforts.

The model provides an internally consistent account of cyclical movements in the labor market. It attains the goal of explaining the large observed cyclical volatility of labor input without invoking an unrealistically high elasticity of labor supply. The main way that it attains the goal is to explain the movements of unemployment as responses to the two driving forces. Because most of the decline in labor input that occurs in a recession takes the form of rising unemployment rather than reduced hours of those at work, the shift in emphasis from the elasticity of labor supply to the elasticity of unemployment is appropriate.

This paper makes progress on the issues in Hall (1997). That paper had a similar factor structure to this one, but did not include unemployment. It found unexplained movements in total labor input that it labeled shifts in preferences. This paper interprets the same movements as the result of changes in equilibrium unemployment and succeeds in matching the observed data without invoking any shifts in preferences.

## 2 Insurance

The analysis in this paper makes the assumption that workers are insured against the personal risk of the labor market and that the insurance is actuarially fair. The insurance makes payments based on outcomes outside the control of the worker that keep all workers' marginal utility of consumption the same. This assumption—dating at least back to Merz (1995)—results in enormous analytical simplification. In particular, it makes the Frisch system of consumption demand and labor supply the ideal analytical framework. Absent the assumption, the model is an approximation based on aggregating employed and unemployed individuals, each with a personal state variable, wealth.

I do not believe that, in the U.S. economy, consumption during unemployment behaves literally according to the model with full insurance against unemployment risk. But families and friends may provide partial insurance. I view the fully insured case as a good and convenient approximation to the more complicated reality, where workers use savings

and partial insurance to keep consumption close to the levels that would maintain roughly constant marginal utility. See Hall (2006) for evidence supporting the view that the fully insured case is a good approximation for the response of workers to unemployment. I make no claim that workers are insured against idiosyncratic permanent changes in their earnings capacities, only that the transitory effects of unemployment can usefully be analyzed under the assumption of insurance.

### 3 Dynamic Labor-Market Equilibrium

I now consider an economy with many identical families, each with a large number of members. All workers face the same pay schedule and all members of all families have the same preferences. The family insures its members against personal (but not aggregate) risks and satisfies the Borch-Arrow condition for optimal insurance of equal marginal utility across individuals. In each family, a fraction  $n_t$  of workers are employed and the remaining  $1 - n_t$  are searching. These fractions are outside the control of the family—they are features of the labor market. In my calibration, a family never allocates any of its members to pure leisure—it achieves higher family welfare by assigning all non-working members job search and it never terminates the work of an employed member. Thus, as I noted earlier, I neglect the small variations in labor-force participation that occur in the actual U.S. economy. To generate realistically small movements of participation in the model I would need to introduce heterogeneity in preferences or earning powers.

This section develops a model that generalizes the canonical model of Mortensen and Pissarides (1994). I adopt the undirected search and matching functions of their model, but replace the Nash bargain with a more general characterization of the determination of a newly hired worker’s compensation. I also follow other authors in generalizing preferences and incorporating choice over hours of work. I will refer to the result as the *extended MP model*.

#### 3.1 Search and matching

Employers post vacancies. Each period, the probability that a worker will become available to fill the vacancy is  $q$ . In tighter labor markets, vacancies are harder to fill and  $q$  is lower. The MP model characterizes the tightness of the labor market in terms of the vacancy/unemployment ratio  $\theta$ . The job-finding rate is an increasing and concave function  $\phi(\theta)$

and the vacancy-filling rate is the decreasing function  $\phi(\theta)/\theta$ . The model assumes a constant exogenous rate of job destruction,  $s$ . Employment follows a two-state Markoff process with stochastic equilibrium

$$n = \frac{\phi(\theta)}{s + \phi(\theta)}. \quad (2)$$

Because the job-finding rate  $\phi(\theta)$  is high—more than 25 percent per month—the dynamics of unemployment are rapid. Essentially nothing is lost by thinking about unemployment as if it were at its stochastic equilibrium and treating it as a jump variable. I will adopt this convention in the rest of the paper. I invert equation (2) to find  $\theta(n)$  and take the job-filling probability to be the decreasing function

$$q(n) = \phi(\theta(n))/\theta(n). \quad (3)$$

In a tighter labor market with higher employment rate  $n$ , the job-filling rate  $q(n)$  is lower.

As in the MP model, employers incur a cost  $\gamma$  at the beginning of a period to maintain a vacancy for the period, with probability  $q(n)$  of filling the job at the end of the period.

### 3.2 The employment contract

Employers pay workers  $w_t$  for each hour of work in period  $t$ . Employers collect an amount  $y_t$  from a new worker. Both workers and employers are price-takers with respect to  $w_t$ , so the employment contract embodies efficient two-part pricing. I discuss the determination of  $y_t$  shortly; it is a key feature of the model. For simplicity I develop the model as if  $y_t$  were collected at the beginning of the period, but the results would be identical if it were spread over the period of employment and  $y_t$  were the present value as of the beginning of the period of the amount deducted from  $w_t h_t$  by the employer.

### 3.3 Production and the firm's decisions

The economy has a single kind of output, with production function

$$F(H_t, K_t, \eta_t). \quad (4)$$

Here  $H_t = n_t h_t$  is total hours of work,  $K_t$  is the capital stock, and  $\eta$  is a vector of random disturbances.

Firms make three decisions: (1) the number of vacancies to try to fill each period, (2) the hours to demand from the existing work force, and (3) the demand for capital.

(1) Under the standard employment contract, firms exactly break even from employing a new worker during the worker’s tenure. They decide whether to recruit workers based upon the immediate payoff,

$$q(n_t)y_t - \gamma. \tag{5}$$

They invest  $\gamma$  in holding a vacancy open for the period and have a probability  $q(n_t)$  of gaining the payoff  $y_t$ . Firms are large enough to absorb the fully diversifiable risk associated with the probability of successful recruiting. Firms would create infinitely many vacancies if the payoff were positive and zero if it were negative. Equilibrium requires that the payoff to recruiting be zero:

$$q(n)y = \gamma. \tag{6}$$

The employment rate that solves this zero-profit condition is a function  $n(y)$ , which I call the *employment function*.

(2) The number of employees at a firm is a state variable. The first-order condition,

$$\frac{\partial F(nh, K)}{\partial H} = w_t, \tag{7}$$

describes the firm’s demand for their hours.

(3) A capital services market allocates the available capital efficiently among firms in proportion to their employment levels. The first-order condition,

$$\frac{\partial F(nh, K)}{\partial K} = r_t, \tag{8}$$

describes the firm’s demand for capital.

### 3.4 The family’s decisions

As in most research on choices over time, I assume that preferences are time-separable, though I am mindful of Browning, Deaton and Irish’s (1985) admonition that “the fact that additivity is an almost universal assumption in work on intertemporal choice does not suggest that it is innocuous.” In particular, additivity fails in the case of habit.

The family orders levels of hours of employed members,  $h_t$ , consumption of employed members,  $c_{e,t}$ , and consumption of unemployed members,  $c_{u,t}$ , within a period by the utility function,

$$n_t U(c_{e,t}, h_t) + (1 - n_t) U(c_{u,t}, 0) \tag{9}$$



The family orders future uncertain paths by expected utility with discount factor  $\delta$ . The family solves the dynamic program,

$$V(W_t, \eta_t) = \max_{h_t, c_{e,t}, c_{u,t}} \{n_t U(c_{e,t}, h_t) + (1 - n_t) U(c_{u,t}, 0) + \mathbb{E} \delta V((1 + r_t)[W_t - n_t c_{e,t} - (1 - n_t) c_{u,t}] - \phi(n_t)(1 - n_t)y_t + w_t n_t h_t, \eta_{t+1})\} \quad (10)$$

Here  $V(W_t, \eta_t)$  is the family's expected utility as of the beginning of period  $t$  and  $W_t$  is wealth. The expectation is over the conditional distribution of  $\eta_{t+1}$ . The amount  $\phi(n_t)(1 - n_t)$  is the flow of new hires of family members, each of which costs the family  $y_t$ .

The family utility function may serve as a reduced form for a more complicated model of family activities that includes home production.

### 3.5 Equilibrium

Let  $\eta_{(t)}$  be the history of the random driving forces up to time  $t$ . An equilibrium in this economy is a wage function  $w(\eta_{(t)})$ , a return function  $r(\eta_{(t)})$ , and an employment rate function  $n_t(\eta_{(t)})$  such that the supply of hours  $h(\eta_{(t)})$  and the supply of savings,  $W(\eta_{(t)})$ , from the family's maximizing program in equation (10) equal the firm's demands from equations (7) and (8), and the recruiting profit in equation (5) is zero, for every  $\eta_{(t)}$  in its support.

### 3.6 State variables

I let  $\lambda_t$  be the marginal utility of wealth (and also marginal utility of consumption):

$$\lambda_t = \frac{\partial V}{\partial W_t} = \delta(1 + r_t) \mathbb{E} \frac{\partial V}{\partial W_{t+1}} \quad (11)$$

I take  $\lambda_t$  and the hourly wage  $w_t$  as the state variables of the economy relevant to labor-market equilibrium. Both state variables are complicated functions of the underlying driving forces  $\eta$ . In particular,  $\lambda_t$  embodies the entire forward-looking optimization of the household based on its perceptions of future earnings.

### 3.7 Hours, consumption, and employment

The family's first-order conditions for hours and the consumption levels of employed and unemployed members are are:

$$U_h(c_{e,t}, h_t) = -\lambda_t w_t \quad (12)$$

$$U_c(c_{e,t}, h_t) = \lambda_t \tag{13}$$

$$U_c(c_{u,t}, 0) = \lambda_t \tag{14}$$

These conditions define three functions,  $c_e(\lambda_t, w_t)$ ,  $h(\lambda_t, w_t)$ , and  $c_u(\lambda_t)$  giving the consumption and hours of the employed and the consumption of the unemployed. With consumption-hours complementarity,  $c_u < c_e$ .

### 3.8 The compensation bargain

I am agnostic about the principles underlying the bargain—the only restriction is that the bargained payment is a function  $y(\lambda, w)$  of the two state variables. One could interpret this assumption as a Markoff property, the exclusion of any other endogenous state variable arising from the bargaining game between worker and employer. This exclusion has substance, as it rules out a state variable that might capture the inertia of compensation. In the setup of this paper, compensation can be sticky in the sense of being unresponsive to the state of the labor market, but it cannot be sticky in the sense of being under the influence of a slow-moving state variable other than  $\lambda$  and  $w$ . A bargaining theory that implies an endogenous state variable that imparts inertia to compensation would be an exciting addition to the post-MP literature, but it has yet to be developed.

I note that the Nash wage bargain is a member of the class of models where  $y$  is a function of the two state variables alone. The reservation payment for the employer, having encountered a worker, is zero—the employer is indifferent to hiring at that point and comes out definitely ahead if the worker makes any positive payment. The family’s upper limit on the payment is the amount of the increase in its value function from shifting a member from unemployment to employment. From equation (10), that amount is

$$U(c_e, h) - U(c_u, 0) + \lambda(-c_e + c_u + w_t h_t) \tag{15}$$

in utility terms. This is the change in utility when a member moves from unemployment to employment (a negative amount) plus the budgetary effect of the increase in consumption spending (a negative consideration) plus the added earnings. In terms of purchasing power, the reservation payment is

$$R(\lambda, w) = \frac{U(c_e, h) - U(c_u, 0)}{\lambda} - c_e + c_u + w_t h_t. \tag{16}$$

All of the terms in this expression are functions of  $\lambda$  or  $w$  or both. Let the Nash bargaining weight of the job-seeker be  $\nu$ . The Nash-bargain upfront payment is  $y(\lambda, w) = (1-\nu)R(\lambda, w)$ .

The employment function  $n(y(\lambda_t, w_t))$  can now be written  $n(\lambda_t, w_t)$ , so it joins consumption and hours as functions of the two state variables, a property I will exploit shortly in the empirical analysis.

### 3.9 Volatility

Volatility in the labor market occurs because of movements in the wage  $w(\eta_t)$ , arising from the shifts in technology that  $\eta_t$  induces. These could be changes in productivity or in other factors that appear in the technology as a reduced form, such as changes in the terms of trade. The volatility of hours operates in the standard way—an increase in the wage raises  $h(\lambda, w)$  through the direct effect of  $w$  but the resulting decline in  $\lambda$ , arising from the favorable effect of a higher wage on wealth, lowers hours. Most volatility in the U.S. economy comes from variations in the employment rate  $n(\lambda, w)$ . Here again a higher wage raises employment while the resulting higher wealth and lower value of  $\lambda$  lowers employment, but, according to the evidence in this paper, employment is more sensitive to both variables than is the supply of hours.

The response of the employment rate to changes in the driving forces depends directly on the payment  $y(\lambda, w)$  that a newly hired worker makes to the employer—see equation (6). The higher this payment, the tighter is the labor market, because employers recruit new workers more aggressively when the payoff is higher. If the payment were fixed, the employment rate would also be fixed. In fact, when the driving forces raise the wage  $w$ , the employment rate rises, according to the evidence later in this paper. So an increase in the wage induces an increase in the upfront payment,  $y$ . Because the payment is a deduction from the worker’s total compensation, the positive response of the payment to  $w$  means that compensation does not rise in proportion to the wage—it is sticky in that sense. If, as seems likely, the upfront payment is amortized over the duration of a job, then the elasticity of the compensation that workers receive with respect to the underlying wage  $w$  is less than one. A higher  $w$  delivers more value from the employment relation to the employer and induces greater recruiting effort and thus a tighter labor market with a higher employment rate  $n$ .

In this framework, I interpret Shimer (2005) as showing that the value of the upfront payment  $y$  resulting from a Nash bargain with roughly equal bargaining weights has low

sensitivity to  $w$  and results in low volatility of the employment rate. At the other extreme, if compensation to the worker—the present value of  $wh$  over the job less the upfront payment  $y$ —were unresponsive to  $w$ ,  $y$  would move in proportion to  $w$ . In this situation of completely sticky compensation, recruiting effort would rise sharply with  $w$  and the volatility of the employment rate would be high and procyclical. The finding of this paper, that the employment rate is quite sensitive to  $w$ , implies that newly hired workers let employers keep some important part of an increase in  $w$  because the worker makes a higher upfront payment  $y$ . In general, the finding of sensitivity of  $n(\lambda, w)$  to  $w$  implies some stickiness of compensation.

### 3.10 Models within the framework of this paper

Hall and Milgrom (forthcoming) develop an alternating-offer bargaining model and calibration in which compensation is sufficiently insensitive to labor-market conditions that productivity changes cause realistic changes in unemployment. Hagedorn and Manovskii (forthcoming) generate similar responses with Nash bargaining by assuming low bargaining power for the worker and high elasticity of labor supply.

The efficiency-wage model of unemployment volatility, as developed by Alexopoulos (2004) also fits within the framework developed above. Her model omits explicit treatment of the search and matching process, but the substance is the same. Under the efficiency-wage principle, employers set compensation at the level needed to prevent short-run opportunism among workers—their share of the employment surplus needs to be large enough to keep them working effectively. When productivity rises, the benefits go mostly to employers, who respond by recruiting harder and tightening the labor market.

### 3.11 The role of $\lambda$

The marginal utility of consumption,  $\lambda$ , enters the extended MP model by determining the value of time at home in relation to the value of work. When  $\lambda$  is high, job-seekers are more interested in finding work because they value time away from work less. Workers have lower reservation levels of compensation as a result, and the compensation bargain is more favorable to the employer. Thus employment is an increasing function of  $\lambda$ . See Hall and Milgrom (forthcoming) for a discussion of the relation between the MP and related models with full preferences (variable marginal rates of substitution between consumption and hours) and the linear preferences that most of the MP literature assumes. In the model

with full preferences,  $\lambda$  plays the role of the fixed leisure premium  $z$  that Mortensen and Pissarides and most of their followers assumed.

## 4 Unemployment Theories

What theories of employment and unemployment fit the paradigm of the extended MP model, where the employment rate is a function of  $\lambda$  and  $w$ ? I distinguish three broad classes of theories.

First, the *pure equilibrium model* of employment launched by Rogerson (1988) places workers at their points of indifference between work and non-work, so compensation just offsets the disamenity of the loss of time at home. Labor supply is perfectly elastic at that level of compensation. The employed are those who wind up in jobs at the labor demand prevailing at that compensation.

Second, *search-and-matching models*—surveyed recently by Rogerson, Shimer and Wright (2005)—divide the labor market into many sub-markets, each in equilibrium. Unemployment arises because some workers are in markets where their marginal products do not cover the disamenity of work. The canonical Mortensen and Pissarides (1994) model is a leading example: Workers are either in autarky, unmatched with any employer, in which case they have zero marginal product by assumption, or they are matched and are employed at a marginal product above their indifference point. Job-seekers enjoy a capital gain upon finding a job. Although most search-and-matching models assume fixity of hours, that assumption is not essential and is straightforward to relax—Andolfatto (1996) was a pioneer on this point. A key assumption of the MP model is that the firm’s demand for labor is perfectly elastic. This assumption only makes sense if the labor market is at the point where the total supply of hours equals the total demand for hours at the marginal product  $w$ .

Third, *allocational sticky-wage models* invoke a state variable, the sticky wage, that controls the allocation of labor. Employers choose total labor input to set the marginal product of labor to the sticky wage. In that case, the sticky wage is the marginal product,  $w$ , as well. As far as I know, the literature lacks a detailed, rigorous account of the resulting equilibrium in the labor market comparable to the MP model. One simple view is that employed workers work  $h(\lambda, w)$  hours and that the number employed,  $n$ , is the total number of hours demanded divided by  $h(\lambda, w)$ . Unemployment of the rent-seeking type in Harris and Todaro (1970) results whenever  $n$  falls short of the labor force. In that case, the unemployed

are those queued up for scarce jobs. The arguments of the employment function  $n(\cdot)$  include  $\lambda$ ,  $w$ , and the other determinants of labor demand. But  $n$  depends *negatively* on  $\lambda$  because a higher value results in more hours of work by the employed and thus fewer jobs. And  $n$  depends negatively on  $w$  for a similar reason and because labor demand falls with  $w$ . Finally,  $n$  depends on the other determinants of labor demand, such as the capital stock. Thus, because they drop the key assumption of perfectly elastic labor demand, allocational sticky-wage models have rather different implications for the employment function. In particular, labor-market outcomes depend on more than the two variables  $\lambda$  and  $w$ .

In the class of models where employment depends just on  $\lambda$  and  $w$ , a value of  $w$  that is high in relation to  $\lambda$  tightens the labor market and results in high employment. An important implication of this property is that the response of unemployment to changes in  $w$  is stronger when  $\lambda$  remains constant—a transitory change in  $w$ —than when the change is permanent and  $\lambda$  changes as well. Pissarides (1987) made this point early in the development of the MP literature, though without a full development of the underlying preferences. Blanchard and Gali (2007) make the same point for the special case of separability between hours and consumption, and with consumption entering as the log.

The equilibrium model plainly belongs to this class. In that model, labor supply is perfectly elastic at a value of  $w$  dictated by  $\lambda$ . The employment function  $n(\lambda, w)$  is a correspondence mapping the two variables into 1.0 if  $w$  is above the critical value, into the unit interval at that value, and into zero below the value. On the other hand, allocational sticky-wage models are not in the class because they require that employment shifts along with the non-wage determinants of labor demand.

A quick summary of this discussion is that sticky-compensation models in the extended MP class are consistent with the model in this paper, while sticky-wage models are not.

I will proceed on the assumption that a function  $n(\lambda, w)$  that gives the employment rate  $n$  in an environment where marginal utility is  $\lambda$  and the marginal product is  $w$  is a reasonable way to think about the employment rate. The next step is to measure the response of the rate to the two determinants.

## 5 Research on Preferences

The empirical approach in this paper rests on using prior information about preferences from research on individual behavior. This section relates the three functions  $h(\lambda, w)$ ,  $c_e(\lambda, w)$ ,

and  $c_u(\lambda)$  to that research.

Consider the standard intertemporal consumption-hours problem without unemployment,

$$\max \mathbb{E}_t \sum_{\tau=0}^{\infty} \delta^\tau U(c_{t+\tau}, h_{t+\tau}) \quad (17)$$

subject to the budget constraint,

$$\sum_{\tau=0}^{\infty} (w_{t+\tau} h_{t+\tau} - p_{t+\tau} c_{t+\tau}) = 0. \quad (18)$$

Here  $p_\tau$  is the price of the consumption good. Both the wage  $w_\tau$  and the price  $p_\tau$  are quoted in units of abstract purchasing power, as of time  $t$ —they are Arrow-Debreu prices.

I let  $C(\lambda p, \lambda w)$  be the Frisch consumption demand and  $H(\lambda p, \lambda w)$  be the Frisch supply of hours per worker. See Browning, Deaton and Irish (1985) for a complete discussion of Frisch systems in general. They satisfy, for consumption and hours at time zero,

$$U_c(C(\lambda p_0, \lambda w_0), H(\lambda p_0, \lambda w_0)) = \lambda p_0 \quad (19)$$

and

$$U_h(C(\lambda p_0, \lambda w_0), H(\lambda p_0, \lambda w_0)) = -\lambda w_0 \quad (20)$$

Here  $\lambda$  is the Lagrange multiplier for the budget constraint. Consumption in period  $t$  is  $C(\lambda_t p_t, \lambda_t w_t)$  and similarly for hours. I will focus on time  $t$  and drop the time subscript in what follows.

The Frisch functions have symmetric cross-price responses:  $C_2 = -H_1$ . They have three basic first-order or slope properties:

- *Intertemporal substitution in consumption*,  $C_1(\lambda p, \lambda w)$ , the response of consumption to changes in its price
- *Frisch labor-supply response*,  $H_2(\lambda p, \lambda w)$ , the response of hours to changes in the wage
- *Consumption-hours cross effect*,  $C_2(\lambda p, \lambda w)$ , the response of consumption to changes in the wage (and the negative of the response of hours to the consumption price). The expected property is that the cross effect is positive, implying substitutability between consumption and hours of non-work or complementarity between consumption and hours of work.

Each of these responses has generated a body of literature, which I will draw upon. In addition, in the presence of uncertainty, the curvature of  $U$  controls risk aversion, the subject of another literature.

Consumption and hours are Frisch complements if consumption rises when the wage rises (work rises and non-work falls)—see Browning et al. (1985) for a discussion of the relation between Frisch substitution and Slutsky-Hicks substitution. People consume more when wages are high because they work more and consume less leisure. Browning et al. (1985) show that the Hessian matrix of the Frisch demand functions is negative semi-definite. Consequently, the derivatives satisfy the following constraint on the cross effect controlling the strength of the complementarity:

$$C_2^2 \leq -C_1 H_2. \tag{21}$$

To understand the three basic properties of consumer-worker behavior listed earlier, I draw primarily upon research at the household rather than the aggregate level. The first property is risk aversion and intertemporal substitution in consumption. With additively separable preferences across states and time periods, the coefficient of relative risk aversion and the intertemporal elasticity of substitution are reciprocals of one another. But there is no widely accepted definition of measure of substitution between pairs of commodities when there are more than two of them. Chetty (2006) discusses two natural measures of risk aversion when hours of work are also included in preferences. In one, hours are held constant, while in the other, hours adjust when the random state becomes known. He notes that risk aversion is always greater by the first measure than the second. The measures are the same when consumption and hours are neither complements nor substitutes.

The Appendix summarizes the findings of recent research on the three key properties of the Frisch consumption demand and labor supply system. The own-elasticities have been studied extensively. The literature on measurement of the cross-elasticity is sparse, but a substantial amount of research has been done on an equivalent issue, the decline in consumption that occurs when a person moves from normal hours of work to zero because of unemployment or retirement. I believe that a fair conclusion from the research is that a person in the middle of the joint distribution of the three properties has a Frisch elasticity of consumption demand of  $-0.5$ , a Frisch elasticity of hours supply of  $0.9$ , and a Frisch cross-elasticity of  $0.3$ . I use informative priors for these parameters. I use much less informative priors for parameters that have received less attention in past research—the elasticities of



the employment function with respect to  $\lambda$  and  $w$ , the variances of the stochastic elements, and the correlation of  $\lambda$  and  $w$ .

To derive the relation between the Frisch functions and the corresponding functions used in the extended MP model, I normalize the price as  $p_t = 1$ . Thus in period  $t$ , values are stated in terms of units of period- $t$  output. Further,  $\lambda_t$  becomes marginal utility in period  $t$  under this normalization. Then

$$c(\lambda, w) = C(\lambda, \lambda w) \quad (22)$$

and

$$h(\lambda, w) = H(\lambda, \lambda w). \quad (23)$$

Notice that the response of consumption to a change in marginal utility  $\lambda$  is:

$$c_1 = C_1 + wC_2 \quad (24)$$

and for hours:

$$h_1 = -C_2 + wH_2. \quad (25)$$

## 6 Latent Factor Model

Because the disturbances in the model stated in levels are nonstationary, I work in first differences of logs, that is, rates of growth. I approximate the consumption demands, hours supply, and employment functions as log-linear, with  $\beta_{c,c}$  denoting the elasticity of consumption with respect to its own price (the elasticity corresponding to the partial derivative  $c_1$  in the earlier discussion),  $\beta_{c,h}$  the cross-elasticity of consumption demand and hours supply, and  $\beta_{h,h}$  the own-elasticity of hours supply. I further let  $\beta_{n,\lambda}$  denote the elasticity of employment with respect to marginal utility  $\lambda$  and  $\beta_{n,w}$  the elasticity with respect to the marginal product  $w$ .

### 6.1 Hours and employment

The factor equation for hours is:

$$\Delta \log h = (-\beta_{c,h} + \beta_{h,h})\Delta \log \lambda + \beta_{h,h}\Delta \log w + \epsilon_h \quad (26)$$

and for employment is:

$$\Delta \log n = \beta_{n,\lambda}\Delta \log \lambda + \beta_{n,w}\Delta \log w + \epsilon_n. \quad (27)$$

Here  $\beta_{h,h}$  and  $-\beta_{c,h}$  are the Frisch own- and cross-elasticities of hours supply for employed workers,  $\beta_{n,\lambda}$  and  $\beta_{n,w}$  are the elasticities of the employment function, and the  $\epsilon$ s are idiosyncratic random components.

## 6.2 Consumption

The model disaggregates the population by the employed and unemployed, who consume  $c_e$  and  $c_u$  respectively. Only average consumption  $c$  is observed. Observed consumption is the average of the two levels, weighted by the employment and unemployment fractions:

$$c = nc_e + (1 - n)c_u. \quad (28)$$

Taking first differences of the log-linearization in the variables, around the point  $\bar{n}$ ,  $\bar{c}_e$ ,  $\bar{c}_u$ , and  $\bar{c}$ , I find

$$\Delta \log c = \frac{\bar{c}_e - \bar{c}_u}{\bar{c}} \bar{n} \Delta \log n + \bar{n} \frac{\bar{c}_e}{\bar{c}} \Delta \log c_e + (1 - \bar{n}) \frac{\bar{c}_u}{\bar{c}} \Delta \log c_u. \quad (29)$$

The consumption changes relate to latent factors as

$$\Delta \log c_e = (\beta_{c,c} + \beta_{c,h}) \Delta \log \lambda + \beta_{c,h} \Delta \log w \quad (30)$$

and

$$\Delta \log c_u = \beta_{c,c} \Delta \log \lambda. \quad (31)$$

Substituting equations (30) and (31) into equation (29), I find, now including an idiosyncratic disturbance  $\epsilon_c$ ,

$$\Delta \log c = \frac{\bar{c}_e - \bar{c}_u}{\bar{c}} \bar{n} \Delta \log n + \beta_{c,c} \Delta \log \lambda + \beta_{c,h} \bar{n} \frac{\bar{c}_e}{\bar{c}} (\Delta \log w + \Delta \log \lambda) + \epsilon_c. \quad (32)$$

Finally, I substitute equation (27) for  $\Delta \log n$  to get

$$\begin{aligned} \Delta \log c &= \left( \beta_{c,c} + \beta_{c,h} \frac{\bar{c}_e}{\bar{c}} \bar{n} + \beta_{n,\lambda} \frac{\bar{c}_e - \bar{c}_u}{\bar{c}} \bar{n} \right) \Delta \log \lambda \\ &\quad + \left( \beta_{c,h} \frac{\bar{c}_e}{\bar{c}} \bar{n} + \beta_{n,w} \frac{\bar{c}_e - \bar{c}_u}{\bar{c}} \bar{n} \right) \Delta \log w + \epsilon_c + \frac{\bar{c}_e - \bar{c}_u}{\bar{c}} \bar{n} \epsilon_n. \end{aligned} \quad (33)$$

I use relatively recent data for the ratios  $\frac{\bar{c}_e}{\bar{c}}$  and  $\frac{\bar{c}_u}{\bar{c}}$ . The accuracy of the log-linear approximation rests on the constancy of these ratios over the sample period. The model implies that an increase in  $w$  lowers  $\lambda$  more than in proportion—see Figure 2. Thus equations (30) and (31) imply that the gap between  $c_e$  and  $c_u$  should close over time as  $w$  trends

upward. I do not incorporate this trend in the calculation for the following reason: The model embodies a backward-bending uncompensated labor-supply function. In fact, hours were fairly constant over the sample period. From the condition

$$\frac{U_c(c_u, 0)}{U_c(c_e, h)} = 1, \quad (34)$$

it follows that with preferences where the marginal utility of consumption is homogeneous of any degree, such as the non-separable preferences in Hall and Milgrom (forthcoming), the ratio  $c_u/c_e$  remains constant if  $h$  remains constant. Thus the addition of any trend in preferences, presumably reflecting a trend in home technology for which the preferences are a reduced form, will deliver a constant  $c_u/c_e$  ratio at the same time that it delivers the right trend in hours. The model as estimated ignores trends because it is based on the covariances of the log first differences and does not consider the means.

### 6.3 Productivity

I measure productivity as the average product of labor,  $m = \frac{q}{h}$ , where  $q$  is output per worker. I let  $\alpha$  be the elasticity of the production function with respect to labor input. From

$$w = \frac{\partial F}{\partial H} = \alpha \frac{q}{h}, \quad (35)$$

I get the equation for the log-change in  $m$ :

$$\Delta \log m = \Delta \log w - \Delta \log \alpha. \quad (36)$$

Notice that  $\Delta \log \alpha = 0$  for a Cobb-Douglas technology. Finally, I define  $\epsilon_m$  to include  $-\Delta \log \alpha$  and any other disturbances, such as measurement error, so the equation for  $m$  in the model is

$$\Delta \log m = \Delta \log w + \epsilon_m. \quad (37)$$

### 6.4 Intuition about estimation

Equation (37) suggests the use of  $\Delta \log \hat{w} = \Delta \log m$  as the observed counterpart of the latent factor  $\Delta \log w$ . Given knowledge of the Frisch elasticity of hours supply,  $\beta_{h,h}$ , and of the cross-elasticity  $\beta_{c,h}$ , data on hours provide an observable counterpart for the latent factor  $\Delta \log \lambda$ :

$$\Delta \log \hat{\lambda} = \frac{1}{\beta_{h,h} - \beta_{c,h}} (\Delta \log h - \beta_{h,h} \Delta \log w) \quad (38)$$

Then, one could consider the coefficients from the regression of  $\Delta \log n$  on  $\Delta \log \hat{w}$  and  $\Delta \log \hat{\lambda}$  as estimates of the parameters  $\beta_{n,\lambda}$  and  $\beta_{n,w}$ . The procedure described in the next section is a close cousin of this approach. It uses prior distributions on  $\beta_{h,h}$  and  $\beta_{c,h}$ , rather than taking them as known, and it attends to the econometric issue of the correlation of the disturbance in the estimating equation with the right-hand variables. But the basic approach is to infer well-being, as measured by  $\lambda$ , from the observed choice of hours given the wage  $w$ , and then to examine the response of the employment rate  $n$  to the two determinants,  $\lambda$  and  $w$ .

The regression of  $\Delta \log c$  on  $\Delta \log \hat{w}$  and  $\Delta \log \hat{\lambda}$  would similarly provide estimates of the compound coefficients of equation (33). The coefficient on  $\Delta \log \hat{\lambda}$  would estimate

$$\beta_{c,c} + \beta_{c,h} \frac{\bar{c}_e}{\bar{c}} \bar{n} + \beta_{n,\lambda} \frac{\bar{c}_e - \bar{c}_u}{\bar{c}} \bar{n}. \quad (39)$$

An estimate of  $\beta_{c,c}$  could be extracted from this coefficient, because all the other elements would be known at this stage. The coefficient on  $\Delta \log \hat{w}$  would estimate

$$\beta_{c,h} \frac{\bar{c}_e}{\bar{c}} \bar{n} + \beta_{n,w} \frac{\bar{c}_e - \bar{c}_u}{\bar{c}} \bar{n}. \quad (40)$$

This coefficient involves no further unknown parameter, so it appears that its value could help fix the values of the other parameters. But this appearance is false. The model is two parameters short of identification, so in a classical setting, one would need to assume known values for two of the parameters.

The estimation procedure I employ examines the response of consumption to the latent value of  $\Delta \log \lambda$  and interprets it as coming from three sources: (1) the direct substitution response controlled by  $\beta_{c,c}$  by the consumption of the employed and the unemployed, (2) the cross effect controlled by  $\beta_{c,h} \frac{\bar{c}_e}{\bar{c}} \bar{n}$  for the consumption of the employed only, and (3) the change in consumption associated with the change in employment and the higher consumption of the employed, controlled by  $\beta_{n,\lambda} \frac{\bar{c}_e - \bar{c}_u}{\bar{c}} \bar{n}$ . Similarly, the estimation procedure interprets the response of consumption to  $\Delta \log w$  as coming from two sources: (1) the cross effect from the wage on the consumption of the employed, controlled by  $\beta_{c,h} \frac{\bar{c}_e}{\bar{c}} \bar{n}$  and (2) the consumption change induced by employment change, controlled by  $\beta_{n,w} \frac{\bar{c}_e - \bar{c}_u}{\bar{c}} \bar{n}$ .

## 6.5 Statistical model

I assume that the idiosyncratic components,  $\epsilon$ , are uncorrelated with  $\lambda$  and  $w$ . This assumption is easiest to rationalize if the  $\epsilon$ s are measurement errors.

The model has 12 parameters: the 5  $\beta$  slope coefficients, the variances and correlation of the latent factors,  $\sigma_\lambda^2$ ,  $\sigma_w^2$ , and  $\sigma_{\lambda,w}$ , and the variances of the four idiosyncratic components,  $\sigma_{\epsilon,c}^2$ ,  $\sigma_{\epsilon,h}^2$ ,  $\sigma_{\epsilon,n}^2$ , and  $\sigma_{\epsilon,m}^2$ . The model implies 10 observed moments, the distinct elements of the covariance matrix of the observables, the employment-adjusted log-change in consumption and the log-changes of hours, employment, and productivity. It is further restricted by non-negativity of the 6 variances, by the Cauchy inequality for the covariance,

$$\sigma_{\lambda,w}^2 \leq \sigma_\lambda^2 \sigma_w^2, \quad (41)$$

and by the concavity condition, equation (21).

Under the assumption that the random variables  $\lambda$ ,  $w$ ,  $\epsilon_c$ ,  $\epsilon_h$ ,  $\epsilon_n$ , and  $\epsilon_m$  are multivariate normal, any parameter set that matches the sample moments achieves the maximum of the likelihood function. The likelihood has a plateau of equal height for any set of parameters with this property. The posterior distribution is governed by the prior everywhere on the plateau. Stripped of an inessential constant, the log-likelihood function is

$$-\frac{T}{2} \left[ \log \det \Omega + \text{tr} \left( \Omega^{-1} \hat{\Omega} \right) \right]. \quad (42)$$

$\Omega$  is the covariance matrix of the observables implied by the model and  $\hat{\Omega}$  is the sample covariance matrix. On the plateau,  $\Omega = \hat{\Omega}$  and the value of the log-likelihood is

$$-\frac{T}{2} \left( \log \det \hat{\Omega} + 4 \right). \quad (43)$$

The prior distribution is discrete. It takes the 12 parameters to be independent of one another. The marginal distribution of each parameter takes on equal values at four equally spaced points. Thus the posterior distribution is defined on a lattice of  $4^{12} = 16.8$  million points. I calculate the exact marginals of the posterior distribution by summation over these points.

## 6.6 Inferring the values of $\lambda$ and $w$

I write the model in matrix form as

$$\Delta x = \theta_\lambda \Delta \log \lambda + \theta_w \Delta \log w + \epsilon. \quad (44)$$

Here  $x$  is the vector of observed values of the logs of consumption, hours, employment, and productivity. I infer  $\lambda$  as a linear combination,  $\hat{\lambda} = a'x$ . I choose the weights  $a$  as the coefficients of the projection of  $\lambda$  on  $x$ , using the moments implied by the parameter values at the posterior mean. I calculate the inference of  $w$ ,  $\hat{w}$ , similarly.

<i>Parameter</i>	<i>Interpretation</i>	<i>Mean</i>	<i>Lowest value</i>	<i>Highest value</i>
$\beta_{c,c}$	Frisch own-price elasticity of consumption	-0.50	-0.6	-0.4
$\beta_{c,h}$	Frisch cross-price elasticity of consumption	0.30	0.0	0.6
$\beta_{h,h}$	Frisch wage elasticity of hours	0.90	0.8	1.0
$\beta_{n,\lambda}$	Elasticity of employment with respect to $\lambda$	0.50	0.0	1.0
$\beta_{n,w}$	Elasticity of employment with respect to $w$	1.00	0.0	2.0
$\sigma_{\lambda}^2$	Variance of latent $\lambda$	2.15	0.3	4.0
$\sigma_w^2$	Variance of latent $w$	2.15	0.3	4.0
$\rho$	Correlation of $\lambda$ and $w$	-0.70	-0.9	-0.5
$\sigma_c^2$	Variance of consumption noise	1.00	0.5	1.5
$\sigma_h^2$	Variance of hours noise	0.30	0.2	0.4
$\sigma_n^2$	Variance of employment noise	0.25	0.1	0.4
$\sigma_m^2$	Variance of productivity noise	0.75	0.3	1.2

Table 1: Priors

## 7 Prior Distributions

Table 1 shows the marginal prior distributions I use for the parameters. They are four-point distributions for all parameters. The priors are highly informative when drawn from the research summarized in Appendix A. They are less informative for parameters where earlier work is either sparse or nonexistent, for the variances of the random elements, and for the correlation of  $\Delta \log \lambda$  and  $\Delta \log w$ . I constrain the cross-elasticity  $\beta_{c,h}$  to satisfy concavity and the correlation of the latent factors to be greater than  $-1$ .

The ratio of unemployment consumption  $\bar{c}_u$  to employment consumption  $\bar{c}_e$  reflects the same properties of preferences as does the Frisch cross-elasticity,  $\beta_{c,h}$ . Accordingly, I take the

joint prior for the two parameters to have perfect correlation, with  $\bar{c}_u/\bar{c}_e = 0.75\beta_{c,w}$ . The proportionality factor 0.75 is derived from a parametric utility function that matches the means of the priors of the Frisch elasticities—when the cross-elasticity is 0.20, the consumption ratio is 0.85.

## 8 Data

To avoid complexities from durables purchases and measurement error in the consumption of services, I use nondurables consumption as an indicator of consumption. I take the quantity index for nondurables consumption from Table 1.1.3 of the U.S. National Income and Product Accounts and population from Table 2.1. I take weekly hours per worker from series LNU02033120, Bureau of Labor Statistics, Current Population Survey, and the unemployment rate from series LNS14000000. I measure productivity as output per hour of all persons, private business, BLS series PRS84006093. For further discussion of the labor-market data, see Hall (forthcoming).

Table 2 shows the covariance and correlation matrixes of the log-differences of the four series. Consumption is correlated positively with both hours and employment—it is quite pro-cyclical. Consumption-hours complementarity can explain this fact. Not surprisingly, hours and employment are quite positively correlated. Consumption has surprisingly high volatility, a property not explained in this paper. Consumption also has by far the highest correlation with productivity.

The variance of the employment rate is about 70 percent higher than the variance of hours—the most important source for the added total hours of work in an expansion is the reduction in unemployment. Hours and the employment rate are not very correlated with productivity.

## 9 Results

Table 3 shows the means and standard deviations of the marginal prior and posterior distributions of the 12 parameters of the model. In general, the decline in the standard deviation from prior to posterior measures the information contributed by the sample evidence and the difference between the prior and posterior means indicates the direction of the influence of the evidence. For two key parameters, the Frisch own-elasticities of consumption and

	<i>Consumption</i>	<i>Hours</i>	<i>Employment</i>	<i>Productivity</i>
Covariances				
Consumption	2.08	0.54	1.03	0.81
Hours		0.76	0.63	0.10
Employment			1.26	0.27
Productivity				2.37
Correlations				
Consumption	1.000	0.511	0.702	0.363
Hours		1.000	0.645	0.075
Employment			1.000	0.159
Productivity				1.000

Table 2: Covariances and Correlations of Log-First Differences of Consumption, Hours, Employment, and Productivity

hours supply, the priors are highly informative, as they are based on a large body of existing research. For both of those parameters, the posterior mean is virtually the same as the prior mean and the posterior standard deviation is small, mainly because the prior standard deviation is small, but also because the sample evidence tends to confirm the prior.

For the Frisch cross-elasticity  $\beta_{c,h}$ , the prior is relatively uninformative and the sample evidence is influential, as indicated by the difference between the standard deviation of the prior, 0.36, and the standard deviation of the posterior, 0.09. The data suggest that this parameter is quite large—the posterior mean is 0.56, rather higher than the value suggested by research in household data, taken to be around 0.3. Hours-consumption complementarity is an important part of the story told by the results. The evidence against separability, with  $\beta_{c,h} = 0$ , is strong—the posterior distribution combines my summary of the evidence from earlier research with household data with the aggregate evidence used here to reach that conclusion.

No earlier research provides information about the two elasticities of the employment function,  $\beta_{n,\lambda}$  and  $\beta_{n,w}$ , so the priors have large standard deviations. The data are quite informative. The posterior mean of the elasticity of the employment rate,  $n$ , with respect to marginal utility,  $\lambda$ , is 0.73 with a standard deviation of 0.15, strong confirmation of the (non-obvious) proposition that fluctuations in long-term well-being have a separate influence on unemployment. In terms of the canonical MP model, this finding implies that the flow



<i>Parameter</i>	<i>Interpretation</i>	<i>Prior mean</i>	<i>Prior standard deviation</i>	<i>Posterior mean</i>	<i>Posterior standard deviation</i>
$\beta_{c,c}$	Frisch own-price elasticity of consumption	-0.50	0.12	-0.49	0.07
$\beta_{c,h}$	Frisch cross-price elasticity of consumption	0.30	0.36	0.53	0.09
$\beta_{h,h}$	Frisch wage elasticity of hours	0.90	0.12	0.95	0.06
$\beta_{n,\lambda}$	Elasticity of employment with respect to $\lambda$	0.50	0.61	0.73	0.15
$\beta_{n,w}$	Elasticity of employment with respect to $w$	1.00	1.21	1.60	0.33
$\sigma_{\lambda}^2$	Variance of latent $\lambda$	2.15	2.24	3.58	0.72
$\sigma_w^2$	Variance of latent $w$	2.15	2.24	1.14	0.57
$\rho$	Correlation of $\lambda$ and $w$	-0.70	0.24	-0.72	0.13
$\sigma_c^2$	Variance of consumption noise	1.00	0.61	1.18	0.23
$\sigma_h^2$	Variance of hours noise	0.30	0.12	0.35	0.05
$\sigma_n^2$	Variance of employment noise	0.25	0.18	0.25	0.11
$\sigma_m^2$	Variance of productivity noise	0.75	0.55	1.15	0.12

Table 3: Posterior Distribution

	$\lambda$	$w$
Consumption	0.31	1.13
Hours	0.42	0.95
Employment	0.73	1.60
Average product of labor	0.00	1.00

Table 4: Coefficients for Log-First Differences of Consumption, Hours, Employment, and Productivity on  $\lambda$  and  $w$

benefit of not working, usually called  $z$ , is not a fixed parameter but rather an endogenous variable. The posterior mean of the elasticity of  $n$  with respect to the marginal product of labor,  $w$ , is 1.60 with a standard deviation of 0.33. All models in the MP tradition agree that the employment rate responds positively to  $w$ , though they disagree on the magnitude. The data appear to compel the view that the response is quite strong.

The priors are uninformative about the six variance parameters. These are stated as variances of percentage changes (100 times log changes) of the variables. The data are moderately successful in pinning down the variances of the two latent factors,  $\lambda$  and  $w$ , and quite successful for the four variances of the noise components of the observed variables. The prior on the correlation of the two latent factors favors a strong negative correlation of  $-0.7$  and that data concur, so that the posterior mean is  $-0.72$  with a standard deviation of 0.13

Table 4 shows the coefficients relating the observed variables to the latent variables  $\lambda$  and  $w$  at the posterior means of the parameters. The coefficients for employment and for the response of hours to  $w$  are the elasticities reported in Table 3 and those for productivity are zero on  $\lambda$  and one on  $w$ . The more complicated relations are for consumption and for the response of hours to  $\lambda$ , from equations (26) and (29).

The biggest surprise in Table 4 is the positive response of consumption to marginal utility  $\lambda$ . Although one might think that marginal utility is a declining function of consumption, theory does not require that property in a Frisch demand system. Recall from equation (29) that the coefficient on  $\lambda$  in the consumption equation is

$$\beta_{c,c} + \beta_{c,h} \frac{\bar{c}_e}{\bar{c}} \bar{n} + \beta_{n,\lambda} \frac{\bar{c}_e - \bar{c}_u}{\bar{c}} \bar{n}. \quad (45)$$

The theoretical limit on the complementarity effect is, from equation (21),

$$\beta_{c,h} \leq \sqrt{-\beta_{c,c} \beta_{n,w}}. \quad (46)$$

From Table 3, the cross-elasticity is 0.53 while the square root is 0.68, comfortably larger. The key point is that the coefficient of consumption on  $\lambda$  is *not* the own-price effect, which is necessarily negative, but the own-price plus the cross-price effect, which can be positive if complementarity is strong enough. Because of the aggregation of consumption across workers and the unemployed, the complementarity effect has two components in equation (45). First, a higher  $\lambda$  (lower well-being) raises the consumption of workers through the direct effect of the complementarity, controlled by  $\beta_{c,h}$ . Second, a higher  $\lambda$  increases the employment rate. Because the employed consume more than the unemployed, average consumption rises on this account as well. The second effect is controlled by  $\beta_{n,\lambda}$ , whose posterior mean is 0.73.

Complementarity also explains the high response of consumption to the current marginal product of labor,  $w$ . Again from equation (29), this response is

$$\beta_{c,h} \frac{\bar{c}_e}{\bar{c}} \bar{n} + \beta_{n,w} \frac{\bar{c}_e - \bar{c}_u}{\bar{c}} \bar{n}. \quad (47)$$

The second term describes the stimulus to employment (decline in unemployment) that accompanies an increase in  $w$ . The direct effect through  $\beta_{c,h}$  is 0.53. The effect from employment change is  $\beta_{n,w} = 1.60$  multiplied by the consumption-difference effect, which is 0.32.

The effect of  $\lambda$  on hours, 0.42, is correspondingly weak. The coefficient is  $-\beta_{c,h} + \beta_{h,h}$ . Complementarity enters negatively, offsetting the relatively strong own-elasticity effect. An increase in  $\lambda$  raises the price of consumption as it raises the reward to work. Because non-work time is a substitute for consumption, people shift toward non-work when the price of consumption rises.

The cross-elasticity  $\beta_{c,h}$  plays an important role in explaining two features of the data shown in Table 3—the generally high correlation of consumption with other cyclical variables and the particularly high correlation, relative to the hours and employment, between consumption and productivity. Recall that productivity reveals the latent marginal product  $w$  except for its own noise.

The sample evidence is also influential about the elasticities of employment with respect to  $\lambda$  and  $w$ , a subject not previously investigated. The posterior reaches a sharp peak for the  $w$ -elasticity at 1.4. Despite the model’s lack of identification and the uninformative priors placed on these parameters (uniform from 0 to 1 for the first and from 0 to 2 for the second), the other priors combine with the sample evidence to provide useful information.

	<i>Inferred <math>\lambda</math></i>	<i>Inferred <math>w</math></i>
Consumption	-0.29	0.18
Hours	0.15	0.10
Employment	0.60	0.14
Average product of labor	-0.76	0.37

Table 5: Coefficients for Inference of  $\lambda$  and  $w$  from Log-First Differences of Consumption, Hours, Employment, and Productivity

## 9.1 Implied values of marginal utility and marginal product

Table 5 shows the coefficients of the projection of the latent factors on the observed variables, at the posterior means of the parameter values. As expected, the inference of marginal utility puts negative weights on consumption and productivity—increases in them signal improvements in well-being and thus lower values of marginal utility,  $\lambda$ . The inference puts a positive weight on employment. The reason is shown in Table 4. An increase in  $\lambda$  raises employment by more than it raises consumption and hours, relative to the coefficients for  $w$ . Therefore, on the average, an increase in employment signals that an increase in  $\lambda$  has occurred. The other feature of Table 5 worth noting is that the weight on productivity in the inference of  $w$  is 0.37, well below the loading of productivity on  $w$  of 1. This finding reflects the noise in productivity. The inference puts weight on all of the variables positively correlated with productivity to filter out as much noise as it can.

Figure 2 shows the estimates of the change in marginal utility,  $\Delta \log \lambda$ , and in the marginal product,  $\Delta \log w$ , resulting from the application of the coefficients in Table 5 to the data on the four observables. The figure shows a pronounced negative correlation between the changes in marginal utility and in the marginal product of labor. News that raises the current marginal product of labor tends to raise lifetime well-being and thus to lower  $\lambda$ . If the economy were perturbed by a single shock and households had no advance information about the shock, the correlation would be  $-1$ . With multiple shocks and advance information, the correlation would be less negative, in accord with the estimated correlation of  $-0.72$ .

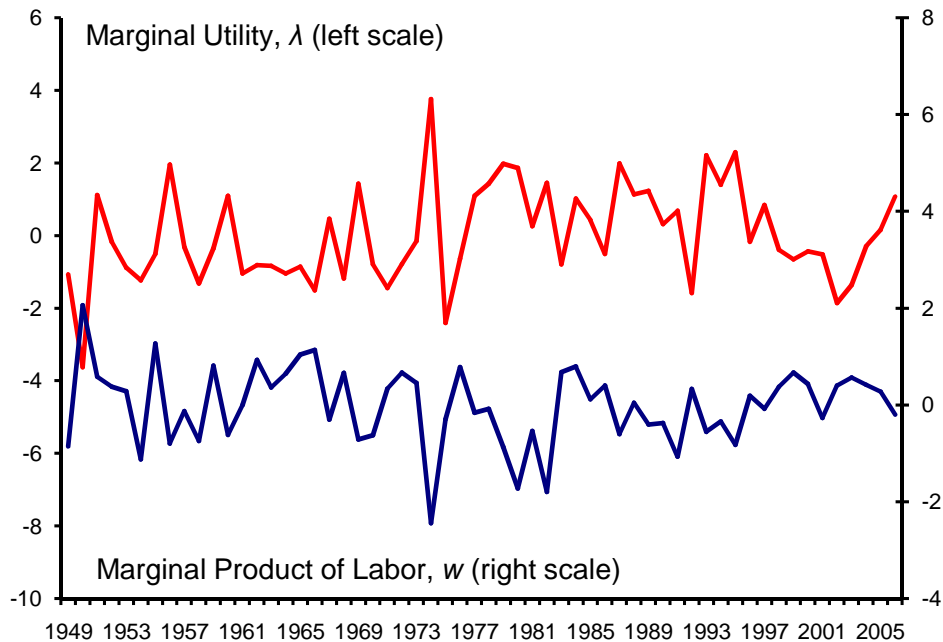


Figure 2: Inferred Growth of Marginal Utility,  $\lambda$ , and Marginal Product of Labor,  $w$ , Percentage Points

## 9.2 Fitted values for observables

Given the time series for  $\lambda$  and  $w$ , I can calculate the implied fitted values for the four observables. These are shown in Figure 3. The two-factor setup is highly successful in accounting for the observed movements of all four variables. Little is left to the idiosyncratic disturbances. Of course, two factors are likely to be able to account for most of the movement of four macro time series, especially when two of them, hours and employment, are fairly highly correlated. But the choices of the factors and the factor loadings are not made, as in principal components, to provide the best match. The loadings are influenced by the priors drawn from earlier research. The success of the model is not so much the good fit shown in Figure 4, but rather achieving the good fit with coefficients that satisfy economic reasonability.

## 10 Interpretation

The results in the previous section achieve the main goal of the paper—to show that standard economic principles embodied in the Frisch consumption demand and hours supply, together with a model of unemployment in the extended Mortensen-Pissarides class, can account

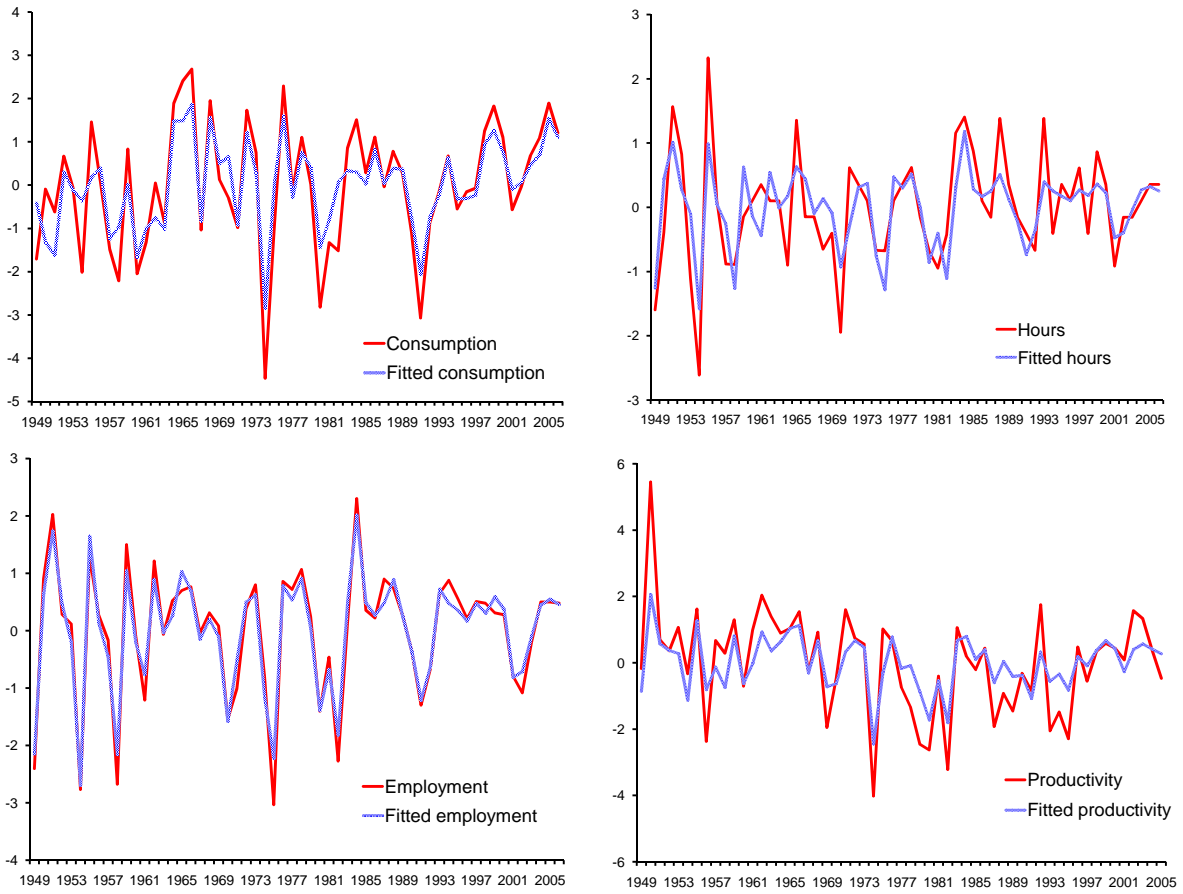


Figure 3: Actual and Fitted Values of the Four Observables

for the higher-frequency movements of those variables. The accounting does not rest on implausible values of any parameters. Most importantly, it does not rest on exaggerated ideas about the elasticity of hours supply. Because the business cycle dominates the higher-frequency movements of the variables, the results give a coherent account of the business cycle.

The main way that the model escapes reliance on unrealistic elasticity of hours supply is to recognize that the primary dimension of fluctuations in labor input at higher frequencies is in the employment rate. Research on labor supply in household data does not reveal the elasticities of employment, which are not features of household choice alone, but reflect an equilibrium involving employer actions as well. This paper is the first to provide estimates of an employment function of the type implied by the MP model, though one could interpret Hagedorn and Manovskii (forthcoming) in terms of its implications for the employment function. However, their approach rests on preferences that imply an extremely high wage elasticity of hours supply.

Thus the centerpiece of the account in this paper of movements in labor input over the cycle is the high elasticity of employment with respect to the marginal product of labor. Employment falls and unemployment rises in a contraction because  $w$  falls and the elasticity of employment with respect to  $w$  is something like 1.6. A rise in marginal utility offsets some of the decline in  $w$  in the typical recession, but its elasticity is only around 0.7.

Recent research in the MP tradition has focused on the finding in Shimer (2005) that an MP model with Nash bargaining and a reasonable set of parameter values cannot come close to generating realistic fluctuations in unemployment from the observed movements in productivity. In the vocabulary of this paper, the Nash bargain makes  $y$  virtually constant, so equation (5) implies that the employment rate  $n$  is also virtually constant. Many papers stimulated by Shimer's work make specific changes, such as adding on-the-job search, to raise the response of labor-market tightness to productivity. The approach here is different. I do not take a stand on the source of the high elasticity of the employment function with respect to  $w$ . In particular, I do not sponsor any particular bargaining principle in place of the Nash bargain. I take a purely empirical approach to the measurement of the elasticity. In a model that follows Mortensen and Pissarides in every respect except bargaining, my results imply that bargaining power shifts toward workers during recessions, or, to put it differently, that compensation is sticky. The upfront payment  $y$  falls when  $w$  falls, so compensation

is cushioned and does not fall as much. Because I take a purely empirical approach, there is nothing surprising or significant in itself in the model's ability to track variations in the employment rate or other measures of tightness.

The primary focus of this paper is the demonstration of the consistency of a model grounded in the theory of household behavior and in the MP class of unemployment theory with the actual behavior of the key variables in the U.S. economy. The paper does not claim to reject other theories. What the model interprets as high complementarity of hours and consumption could arise from liquidity constraints that link current earnings to consumption more tightly than under the assumptions made here. Less-than-full insurance against the idiosyncratic risk of unemployment may contribute to the finding of high complementarity as well. With respect to unemployment, I noted earlier that the assumption that the determinants of the employment-payment bargain,  $y$ , are limited to those that are payoff-relevant, while often made in game-theoretic models, is not completely compelling. Until theory provides more guidance, it is hard to see how to characterize additional determinants of  $y$  and test for their exclusion. On the other hand, the evidence here of the positive effect of productivity on employment is inconsistent with the allocational sticky-wage model. This finding rests on my econometric identifying assumptions. Under an alternative identification strategy, as in Gali (1999), the effect of an innovation in productivity on employment is negative and therefore consistent with the allocational sticky-wage model. The debate on that topic remains unresolved.

## 11 Observable Variables Not Included in the Model

### 11.1 Compensation

The factor model does not consider the actual value of compensation paid to workers, despite the key role of compensation in the Mortensen-Pissarides class of employment models. In that class of models, compensation gains its influence over unemployment through the non-contractible, pre-match effort of employers in attracting workers. These efforts—which take the form of the creation of vacancies in the model—govern the tightness of the labor market and thus the unemployment rate. The difference between the marginal product and compensation, anticipated at the time of hiring, governs the employer's vacancy-creation efforts. The class of models has no further implications about the pattern of payment of compensation over the period of employment. The bargained level of compensation has no



allocational role once a job-seeker and an employer find each other—it only divides the surplus from the match. In particular, nothing rules out smoothing of compensation in relation to productivity. I am not aware of any way to introduce observed compensation, averaged over workers hired over the past 40 years, into the factor model without making special assumptions about the determination of compensation during the period of employment. Even if compensation is the result of period-by-period bargaining, one would have to take a stand on bargaining principles to pin down compensation.

## 11.2 Asset returns

Equation (11) implies:

$$\delta(1 + r_t) \mathbb{E} \frac{\lambda_{t+1}}{\lambda_t} = 1, \quad (48)$$

the asset-pricing condition of the consumption capital-asset pricing model. If the economy traded an asset with a stochastic return, its return ratio would be inside the expectation. In principle, this joint relation of asset returns would help pin down the latent variable,  $\Delta \log \lambda_t$ . I tested this idea in a standard way, with the equation,

$$\mathbb{E} \delta \frac{\lambda_{t+1}}{\lambda_t} r_{x,t} = 0, \quad (49)$$

where  $r_{x,t}$  is the excess return of the S&P 500 stock portfolio over one-year Treasury bills. The average value of  $r_{x,t}$  is the equity premium and is 6.6 percent per year over the period I used (1953 to 2003), with a standard error of 2.5 percent. The average value of the compound random variable  $\frac{\lambda_{t+1}}{\lambda_t} r_{x,t}$  over the same period is the same, 6.6 with a standard error of 2.5. The  $t$ -statistic for the hypothesis that the value is zero, as required by asset-pricing theory, is 2.6, indicating strong rejection. This finding replicates the famous conclusion of Mehra and Prescott (1985). Campbell and Cochrane (1999) discuss the challenge of constructing a successful asset-pricing variable out of aggregate consumption. The variable needs to have vastly higher volatility than would any marginal utility based on standard principles and reasonable risk aversion.

The alterations I have introduced in this paper to measure  $\lambda$  that make it differ from Mehra and Prescott’s simple calculation from standard preferences do not result in anything like the highly volatile variable needed to satisfy the asset-pricing condition for the equity premium or for other asset-pricing exercises. Until further progress is made in understanding the failure of the consumption capital-asset pricing model, I believe it would be a mistake to force the latent factor  $\lambda$  to satisfy any asset-pricing condition.

## 12 Concluding Remarks

Contrary to earlier impressions, one can make sense out of the fairly large cyclical fluctuations in hours of work per person without invoking either unreasonably high elasticity of labor supply—as in real business cycle models—or allocational sticky wages. A Frisch elasticity of labor supply of 0.95, at the upper end of the range found in recent research using household data, does the job.

About a third of the volatility of cyclical fluctuations in hours per person takes the form of volatility of hours of job-holders. I argue that movements in the marginal product of labor and in the marginal utility of consumption are plausible sources of the movements of hours. These are the arguments of the Frisch hours supply function.

The remaining larger part of cyclical fluctuations in labor input per person comes from unemployment. Labor input declines in recessions because fewer people work and more are looking for work. I show that the U.S. labor market appears to have a well-defined employment function with reasonable positive elasticities for both the marginal product of labor and the marginal utility of consumption. An extended version of the Mortensen-Pissarides model makes unemployment depend on just these two variables. Further work on the employment function, either in the framework of the extended MP model or outside that framework, is clearly in order.

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## A Appendix: Research on Properties of Preferences

### A.1 Approaches

Chetty (2006) considers the issues surrounding the calibration of household preferences. He shows that the value of the coefficient of relative risk aversion (or, though he does not pursue the point, the inverse of the intertemporal elasticity of substitution in consumption) is implied by a set of other measures. He solves for the consumption curvature parameter by drawing estimates of responses from the literature on labor supply. One is the third item on the list above, consumption-hours complementarity. The others are the compensated wage elasticity of static labor supply and the elasticity of static labor supply with respect to unearned income. These are functions of the derivatives listed above, so information about static labor supply does not add anything that those derivatives miss. In principle, as long as the mapping has adequate rank, one could take any set of measures of behavior and solve for the slopes of the Frisch functions or any other representation of preferences. My procedure links the empirical measures more directly to the underlying basic properties of preferences. I do, however, study the implications of my calibration for static labor supply. My calibration lies within the space of values that Chetty extracts from a wide variety of studies of static labor supply.

Basu and Kimball (2000) pursue an idea related to Chetty's. They calibrate preferences to an outside estimate of the intertemporal elasticity of substitution in consumption and to zero uncompensated elasticity of static labor supply with respect to the wage. They constrain the complementarity of consumption and hours to have the multiplicative form of King, Plosser and Rebelo (1988).

### A.2 Risk aversion

Research on the value of the coefficient of relative risk aversion (CRRA) falls into several broad categories. In finance, a consistent finding within the framework of the consumption capital-asset pricing model is that the CRRA has high values, in the range from 10 to 100 or more. Mehra and Prescott (1985) began this line of research. A key step in its development was Hansen and Jagannathan's 1991 demonstration that the marginal rate of substitution—the universal stochastic discounter in the consumption CAPM—must have extreme volatility to rationalize the equity premium. Models such as Campbell and Cochrane (1999) generate a

highly volatile marginal rate of substitution from the observed low volatility of consumption by subtracting an amount almost equal to consumption before measuring the MRS. I am skeptical about applying this approach in a model of household consumption.

A second body of research considers experimental and actual behavior in the face of small risks and generally finds high values of risk aversion. For example, Cohen and Einav (2007) find that the majority of car insurance purchasers behave as if they were essentially risk-neutral in choosing the size of their deductible, but a minority are highly risk-averse, so the average coefficient of relative risk aversion is about 80. But any research that examines small risks, such as having to pay the amount of the deductible or choosing among the gambles that an experimenter can offer in the laboratory, faces a basic obstacle: Because the stakes are small, almost any departure from risk-neutrality, when inflated to its implication for the CRRA, implies a gigantic CRRA. The CRRA is the ratio of the percentage price discount off the actuarial value of a lottery to the percentage effect of the lottery on consumption. For example, consider a lottery with a \$20 effect on wealth. At a marginal propensity to consume out of wealth of 0.05 per year and a consumption level of \$20,000 per year, winning the lottery results in consumption that is 0.005 percent higher than losing. So if an experimental subject reports that the value of the lottery is one percent—say 10 cents—lower than its actuarial value, the experiment concludes that the subject’s CRRA is 200!

Remarkably little research has investigated the CRRA implied by choices over large risky outcomes. One important contribution is Barsky, Juster, Kimball and Shapiro (1997). This paper finds that almost two-thirds of respondents would reject a new job with a 50 percent chance of doubling income and a 50 percent chance of cutting income by 20 percent. The cutoff level of the CRRA corresponding to rejecting the hypothetical new job is 3.8. Only a quarter of respondents would accept other jobs corresponding to CRRA’s of 2 or less. The authors conclude that most people are highly risk-averse. The reliability of this kind of survey research based on hypothetical choices is an open question, though hypothetical choices have been shown to give reliable results when tied to more specific and less global choices, say, among different new products.

### **A.3 Intertemporal substitution**

Attanasio, Banks, Meghir and Weber (1999), Attanasio and Weber (1993), and Attanasio



and Weber (1995) are leading contributions to the literature on intertemporal substitution in consumption at the household level. These papers examine data on total consumption (not food consumption, as in some other work). They all estimate the relation between consumption growth and expected real returns from saving, using measures of returns available to ordinary households. All of these studies find that the elasticity of intertemporal substitution is around 0.7.

Barsky et al. (1997) asked a subset of their respondents about choices of the slope of consumption under different interest rates. They found evidence of quite low elasticities, around 0.2.

Guvenen (2006) tackles the conflict between the behavior of securities markets and evidence from households on intertemporal substitution. With low substitution, interest rates would be much higher than are observed. The interest rate is bounded from below by the rate of consumption growth divided by the intertemporal elasticity of substitution. Guvenen's resolution is in heterogeneity of the elasticity and highly unequal distribution of wealth. Most wealth is in the hands of those with elasticity around one, whereas most consumption occurs among those with lower elasticity.

Finally, Carroll (2001) and Attanasio and Low (2004) have examined estimation issues in Euler equations using similar approaches. Both create data from the exact solution to the consumer's problem and then calculate the estimated intertemporal elasticity from the standard procedure, instrumental-variables estimation of the slope of the consumption growth-interest rate relation. Carroll's consumers face permanent differences in interest rates. When the interest rate is high relative to the rate of impatience, households accumulate more savings and are relieved of the tendency that occurs when the interest rate is lower to defer consumption for precautionary reasons. Permanent differences in interest rates result in small differences in permanent consumption growth and thus estimation of the intertemporal elasticity in Carroll's setup has a downward bias. Attanasio and Low solve a different problem, where the interest rate is a mean-reverting stochastic time series. The standard approach works reasonably well in that setting. They conclude that studies based on fairly long time-series data for the interest rate are not seriously biased. My conclusion favors studies with that character, accordingly.

I take the mean of the prior distribution of the Frisch own-elasticity of consumption demand to be  $-0.5$ . Again, I associate the evidence described here about the intertemporal

elasticity of substitution as revealing the Frisch elasticity, even though many of the studies do not consider complementarity of consumption and hours explicitly.

#### **A.4 Frisch elasticity of labor supply**

The second property is the Frisch elasticity of labor supply. Pistaferri (2003) is a leading recent contribution to estimation of this parameter. This paper makes use of data on workers' personal expectations of wage change, rather than relying on econometric inferences, as has been standard in other research on intertemporal substitution. Pistaferri finds the elasticity to be 0.70 with a standard error of 0.09. This figure is somewhat higher than most earlier work in the Frisch framework or other approaches to measuring the intertemporal elasticity of substitution from the ratio of future to present wages. Here, too, I proceed on the assumption that these approaches measure the same property of preferences as a practical matter. Kimball and Shapiro (2003) survey the earlier work.

Mulligan (1998) challenges the general consensus among labor economists about the Frisch elasticity of labor supply with results showing elasticities well above one. My discussion of the paper, published in the same volume, gives reasons to be skeptical of the finding, as it appears to flow from an implausible identifying assumption.

Kimball and Shapiro (2003) estimate the Frisch elasticity from the decline in hours of work among lottery winners, based on the assumption that the uncompensated elasticity of labor supply is zero. They find the elasticity to be about one. But this finding is only as strong as the identifying condition.

Domeij and Floden (2006) present simulation results for standard labor supply estimation specifications suggesting that the true value of the elasticity may be double the estimated value as a result of omitting consideration of borrowing constraints.

Pistaferri studies only men and most of the rest of the literature in the Frisch framework focuses on men. Studies of labor supply generally find higher wage elasticities for women. Consequently, I take the prior mean of the Frisch own-elasticity of labor supply to be 0.9.

#### **A.5 Consumption-hours complementarity**

The third property is the relation between hours of work and consumption. A substantial body of work has examined what happens to consumption when a person stops working, either because of unemployment following job loss or because of retirement, which may be

the result of job loss.

Browning and Crossley (2001) appears to be the most useful study of consumption declines during periods of unemployment. Unlike most earlier research in this area, it measures total consumption, not just food consumption. They find a 14 percent decline on the average from levels just before unemployment began.

A larger body of research deals with the “retirement consumption puzzle”—the decline in consumption thought to occur upon retirement. Most of this research considers food consumption. Aguiar and Hurst (2005) show that, upon retirement, people spend more time preparing food at home. The change in food consumption is thus not a reasonable guide to the change in total consumption. Hurst (2008) surveys this research.

Banks, Blundell and Tanner (1998) use a large British survey of annual cross sections to study the relation between retirement and nondurables consumption. They compare annual consumption changes in 4-year wide cohorts, finding a coefficient of  $-0.26$  on a dummy for households where the head left the labor market between the two surveys. They use earlier data as instruments, so they interpret the finding as measuring the planned reduction in consumption upon retirement.

Miniaci, Monfardini and Weber (2003) fit a detailed model to Italian cohort data on non-durable consumption, in a specification of the level of consumption that distinguishes age effects from retirement effects. The latter are broken down by age of the household head. The pure retirement reductions range from 4 to 20 percent. This study also finds pure unemployment reductions in the range discussed above.

Fisher, Johnson, Marchand, Smeeding and Terrey (2005) study total consumption changes in the Consumer Expenditure Survey, using cohort analysis. They find small declines in total consumption associated with rising retirement among the members of a cohort. Because retirement in a cohort is a gradual process and because retirement effects are combined with time effects on a cohort analysis, it is difficult to pin down the effect.

I take the prior mean of the Frisch cross-price elasticity of demand to be 0.3, which corresponds to a difference in consumption between workers and non-workers of 15 percent when the two Frisch own-price elasticities are at their prior means.