Dual inflation and the real exchange rate in new open economy macroeconomics

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Abstract

This paper studies how the models of the new open economy macroeconomics, which usually focuses on the relationship between the nominal exchange rate and the external real exchange rate, can explain the coexistence of permanent dual inflation, namely diverging inflation rates for tradable and non-tradable goods, and real appreciation in emerging market economies.

It is shown that the impact of asymmetric sectoral productivity growth on the real exchange rate heavily depends on the market structure, and that the models of new open economy macroeconomics can be reconciled with the Balassa - Samuelson effect only if pricing to market is added to models.

It is demonstrated that the presence of nominal rigidities and frictions in capital accumulation helps to explain the appreciation of the external real exchange rate, and the slow adjustment of the relative price of non-tradabables to tradables.

Keywords: dual inflation, real exchange rate, new open economy macroeconomics, Balassa - Samuelson effect.
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1 Introduction

The traditional approach in international macroeconomics has attempted to explain real exchange rate behavior by the movements of domestic relative prices, that is, by the *internal real exchange rate*. This was a consequence of the assumptions they employed: strong homogeneity in international goods markets, where *purchasing power parity* (PPP) is dominant and the only source of heterogeneity is the distinction between *tradables* and *non-tradables*. In recent years, however, the literature has switched sides. According to the recent approach consumer markets are segmented, PPP has little explanatory power, and the main determinant of real exchange rate movements is the *external real exchange rate*, which is the relative price of domestic and foreign tradables. This new focus of research was initiated on the basis of empirical findings, see, e.g., the papers of Engel (1999) and Rogoff (1996). It appeared that, as Obstfeld (2001) put it "apparently, consumer markets for tradables are just about as segmented internationally as consumer markets for non-tradables."

After the collapse of the Bretton Woods system, floating exchange rate regimes became widespread. This enabled scrutiny of the relationship between nominal and real exchange rate behavior: It turned out, as first forcefully documented by Mussa, that nominal and real exchange rates were strongly correlated, and moving from fixed to floating exchange rate regimes resulted in a dramatic rise in the variability of the real exchange rate. The need for a comprehensive explanation for the aforementioned empirical findings stimulated the birth of *new open economy macroeconomics* (NOEM), initiated by the seminal paper of Obstfeld and Rogoff (1995), which combines the heterogeneity of goods with *nominal rigidities* in models with microfoundations.

Although the empirical literature related to NOEM revealed the importance of the external real exchange rate, in fast-growing and emerging market countries there are considerable movements of the internal real exchange rate. Permanent *dual inflation*, namely a significant divergence of inflation rates for tradable and non-tradable goods, is a frequent phenomenon of such markets: the inflation rate of non-tradables is permanently higher than that of tradables, which results in long-run real appreciation. This phenomenon was documented by Ito et al. (1997) for the case of Japan and some Southeast Asian countries, as well as by Coricelli and Jazbec (2001), Halpern and Wyplosz (2001), Égert (2002), Égert et al. (2002) and Kovács (2002) for European post-communist countries. Of course, this does not mean that in these countries the empirical phenomena emphasized in the NOEM literature are not present. For example, the required disinflation efforts, related to future EMU accession, have revealed that the connection between the consumer price index and the nominal exchange rate is weak, which, of course, violates the PPP and implies a strong co-movement of nominal and real exchange rates.

The objective of this paper is to build a NOEM model which is able to replicate both sets of empirical facts observable in emerging markets: the strong correlation of the nominal and real exchange rate, and dual inflation accompanied by real appreciation.

The problem is the following: The majority of empirical studies explain emerging markets' dual inflation by the *Balassa - Samuelson* (BS) effect, i.e. the relatively rapid productivity growth in the tradable sector. However, dual inflation accompanies real appreciation only if growth in tradable productivity does not result in a significant depreciation of the external real exchange rate. The external real exchange rate does not depreciate considerably if the common currency prices of domestically produced and foreign tradables cannot strongly deviate from each other, i.e. if domestically produced and foreign tradables are close substitutes. On the other hand, the strong co-movement of the nominal and real exchange rates stressed by the NOEM literature requires considerable deviations in the short run between domestic and foreign tradable prices (denominated in the same currency). Yet this requirement can be fulfilled only if the products of the aforementioned sectors are distant substitutes and/or *pricing to market* (PTM) is possible.

The paper demonstrates that no intermediate degree of international substitution exists that simultaneously guarantees the operation of the BS effect and strong co-movement of the nominal and real exchange rate. One possible remedy is an assumption of PTM. In this case it is possible that domestically produced export goods are close substitutes of foreign tradables, which ensures the existence of the BS effect. On the other hand, with PTM the common currency price of the exported and locally sold domestically produced goods can be substantially different over the short run. Hence, nominal-exchange-rate movements can influence the behavior of the real exchange rate.

The paper also shows that a certain combination of real and nominal rigidities has significant impact on the magnitude of the difference between sectoral inflation rates. As a consequence, the size of the effect of asymmetric sectoral productivity growth, in line with empirical observations, becomes smaller than predicted by the models of the traditional approach.

The paper is structured as follows. *Section 2* surveys the empirical literature which initiated the research of this study. *Section 3* presents the model and the solution technique employed. In *section 4* the Balassa – Samuelson hypothesis is examined; under study is how the model can reproduce the coexistence of dual inflation and real appreciation, and the relationship between asymmetric productivity growth and the magnitude of sectoral inflation differentials is examined. *Section 5* presents the conclusions.

2 Previous empirical results

This section briefly reviews the empirical literature which initiated the research of this paper. First, findings related to the internal real exchange rate are surveyed. On this issue the evidence is ambiguous. In developed economies, internal-real-exchange-rate movements are negligible, while in several emerging economies dual inflation is an important phenomenon. Second, findings on the strong relationship between the nominal and real exchange rates are considered, which are relevant in both developed and emerging economies.

2.1 Dual inflation and real appreciation

As mentioned in the *Introduction*, NOEM literature focuses on the behavior of the external real exchange rate, instead of the internal one, which was mainly studied by the previous traditional literature. This switch of interest was partly initiated by the findings of Engel (1999), who, using US data, showed that the volatility of the real exchange rate can be explained nearly perfectly by the movements of the external real exchange rate.

However, the validity of this finding is not general. Even in developed countries one can observe significant movements of the internal real exchange rate, as De Gregorio and Wolf (1994), or more recently López-Salido et al. (2005) have documented, but the real importance of this phenomenon is manifested in high growth and emerging market countries. Several empirical studies demonstrate that the Balassa - Samuelson (BS) effect plays a significant role in these countries.

Balassa (1964) and Samuelson (1964) formulated the hypothesis that the difference in productivity growth rates in tradable and non-tradable sectors results in dual inflation, and as a consequence real appreciation.[1] Ito et al. (1997) showed that mainly in Japan, Korea, and Taiwan, but to some extent in other Southeast Asian countries as well, the BS effect was determinant at particular stages of their development process. It also plays an important role in the transition of European post-communist countries, as the empirical studies of Coricelli and Jazbec (2001), Halpern and Wyplosz (2001), Égert (2002) Égert et al. (2002), and Kovács (2002) have documented.

[1] On the Balassa - Samuelson effect see Obstfeld and Rogoff (1996, chapter 4).

Coricelli and Jazbec (2001) examined the determinants of the real exchange rate in nineteen transition economies between 1991 and 1998.[2] Halpern and Wyplosz (2001) studied the relevance of the BS effect in nine European post-communist countries by estimating a panel regression for the period 1991-98.[3] Égert (2002) used time series and panel cointegration techniques to study the BS effect in five east European accession countries between 1991 and 2001.[4] Égert et al. (2002) examined the BS effect in nine European accession countries by panel cointegration techniques on a data set covering the period from 1995 to 2000.[5] The paper edited by Kovács (2002) summarizes the results of research on the BS effect conducted by the central banks of central European accession countries.[6]

The above studies demonstrate that in most European post-communist countries the coexistence of dual inflation and real appreciation can be observed in their transition period. In addition, dual inflation is related to sectoral productivity growth differentials, and real appreciation is due to the appreciation of both the external and internal real exchange rates.

Coricelli and Jazbec (2001), Halpern and Wyplosz (2001), Égert (2002), and Égert et al. (2002) estimated the relationship between the relative price of non-traded to traded goods and the sectoral productivity differential.[7] Their findings are summarized in *Table 1*.

[2] The examined countries were Armenia, Azerbaijan, Belarus, Bulgaria, Croatia, Czech republic, Estonia, Hungary, Kazakhstan, Kyrgyzstan, Latvia, Lithuania, Poland, Romania, Russia, Slovakia, Slovenia, Ukraine and Uzbekistan.

[3] The countries in the sample were the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Russia, Romania and Slovenia.

[4] The examined countries are the Czech Republic, Hungary, Poland, Slovakia and Slovenia.

[5] The studied countries are Croatia, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia and Slovenia.

[6] The examined countries and the length of the data set: the Czech Republic (1994-2001), Hungary (1992-2001), Poland, (1990-2001), Slovakia (1995-2000) and Slovenia (1992-2001).

[7] Since reliable estimates of total factor productivity were not available, due to the lack of capital stock data, they used labor productivity measures.

Table 1
Empirical long-run relationship between sectoral prices and productivity measures

	Type of regression	Estimated coefficient
Coricelli – Jazbec (2001)	price differential on productivity differential	0.87
Égert (2002)	panel, price differential on productivity differential	0.72
Égert (2002)	individual, price differential on productivity differential	0.49-0.95
Égert et al. (2002)	price differential on productivity differential	0.73-1
Halpern – Wyplosz (2001)	tradable price on tradable productivity	0.43
Halpern – Wyplosz (2001)	non-tradable price on non-tradable productivity	0.32

According to Coricelli and Jazbec (2001, equation 19), if the productivity differential rises by 1 per cent, the relative price rises by 0.87 per cent. Égert (2002, Table 1-7) found significant cointegration relationship between the relative price and productivity differential. The cointegration coefficient measuring the long-run relationship between the relative prices and productivity factors varies from 0.49 to 0.95 in individual country estimates, and 0.72 is the common estimate for the coefficient provided by the panel cointegration analysis. In Égert et al. (2002, Table 5) the same cointegration coefficient ranges from 0.73 to 1, depending on the applied definition of tradable and non-tradable sectors. Unlike the previous studies, Halpern and Wyplosz (2001, Table 7) estimated the effects of tradable and non-tradable productivity developments separately. They found significant coefficients with correct signs, although the estimated coefficients are quite small. If tradable productivity rises by 1 per cent, the sectoral relative price rises by 0.24 per cent in the short run and by 0.43 per cent in the long run. A 1 per cent rise of non-tradable productivity results in a 0.18 per cent decrease of the relative price in the short run and a 0.32 per cent decrease in the long run.

In summary: All papers found a significant relationship between sectoral prices and productivity measures. Magnitudes of estimated coefficients locate in quite a wide range. However, according to all but one estimates, productivity differentials are greater than the accompanying price differentials.

According to the original BS hypothesis, productivity induced real appreciation of the internal real exchange rate results in CPI-based real appreciation, since the external real exchange rate is fixed due to the assumed validity of PPP.

Kovács (2002, Table 1-1) documented that between 1993 and 2002 the annual average real appreciation of the examined countries varied from 2.2 to 5.8 per cent. However, the BS effect does not fully explain the observed CPI-based appreciations. Only 33-72 per cent of it can be attributed to productivity growth induced internal real exchange rate movements, the rest can be assigned to the external real exchange rate. Égert (2002, Table 9) also reveals that productivity induced appreciation of the internal real exchange rate cannot completely explain real appreciation. According to his panel analysis it is responsible for 38-60 per cent of CPI-based appreciation. He also stresses the importance of a trend appreciation of the external real exchange rate to explain the observed phenomena. Égert et. al (2002) presented similar findings and reinforced the conclusions of the above papers.

Although in this paper I studies only productivity induced dual inflation, I should mention that studies analyzing the BS effect have often detected other non-productivity factors in the determination of the sectoral relative price. Moreover, Arratibel et al. (2002) do not simply provide alternative explanations for dual inflation, they deny the role of productivity factors in the determination of the examined countries. However, the authors admit that one should interpret this result with caution because of the poor quality of productivity data.[8]

2.2 The co-movement of the nominal and real exchange rates

As mentioned in the *Introduction*, the NOEM literature was partly initiated by the empirical findings of Mussa (1986), who first documented the strong connection between the nominal and real exchange rates. Using Monacelli (2004), I summarize some important findings. The post-1971 data from 12 developed countries reveal that the unconditional correlation of real and nominal depreciation rates is 0.98. In flexible exchange rate regimes the

[8] In their paper they studied the inflation processes in 10 European post-communist countries. Their results support the existence of dual inflation in these countries. However, according to their estimations a positive productivity shock negatively influences the inflation rate in the non-tradable sector.

unconditional variance of the real depreciation rate is nearly equal to the unconditional variance of the nominal depreciation rate.

Violation of purchasing power parity (PPP) is a necessary condition for the above findings. Moreover, the violation of PPP is not a transitory phenomenon, as several empirical studies have shown. Chari et al. (2002) studied the persistency of the real-exchange-rate shocks using HP-filtered quarterly data for the USA and 11 developed European countries for the period 1973:1-2000:1. Their estimated quarterly autocorrelation is 0.84.[9] Though the above empirical results are all related to developed countries, the violation of PPP can also be detected in European post-communist countries, which are the primary focus of this study,[10] although the supporting evidence is mainly only stylized facts.

3 The model

One of the main focuses of this paper concerns how to construct a model which can simultaneously guarantee the empirical regularities characterized in *section 2*, i.e. the co-movement of the nominal and real exchange rates and generate the Balassa - Samuelson (BS) effect, i.e. the coexistence of productivity based dual inflation and real appreciation

To guarantee the empirically observable correlation between the nominal and real exchange rates the model needs sticky prices and internationally segmented tradable markets. Obviously, to consider the BS effect it is necessary to have at least two sectors with different total factor productivities (TFP).

International market segmentation can be captured in different ways. I therefore compare whether model versions with different descriptions of market segmentation can generate the BS effect. I consider a version (version A) without pricing to market (PTM) and with the assumption that domestic and foreign tradables are imperfect substitutes. In version B PTM combined

[9]Diebold et al. (1991) and Lothian and Taylor (1996) using long annual time series of different currencies found much more persistent real-exchange-rate shocks than Chari et al. (2002). It is difficult to explain their findings purely by nominal rigidities. Rogoff (1996) refers to this phenomenon as the '*PPP puzzle*'. Engel and Morley (2001) built an empirical model, which may help to resolve this puzzle.

[10]Hornok et al. (2002) tried to perform econometric estimations on very short time series and the half-time they found is approximately 2.8 years. On the other hand, Darvas (2001) using the data of the Czech Republic, Hungary, Poland, and Slovenia found very short, less than one year, half-lives. But in the studied time periods narrow-band crawling peg regimes were typical in these countries, which may explain his results.

with local currency pricing (LCP) is added to the model.[11]

The other main topic of the paper is the relationship between the magnitude of sectoral relative price and productivity differentials. In frictionless, sectorally symmetric models the two quantities are equal. Yet this is not in line with empirical results, which reveal that the relative price of non-tradables to tradables is smaller than the sectoral productivity differential. Nominal rigidities help to explain this phenomenon: if prices are sticky the adjustment of the sectoral relative price is not immediate. In addition, as Woodford (2003, chapter 3) demonstrates, *decreasing returns* amplify the impact of sticky prices, making the adjustment process even slower, which provides a better fit in terms of empirical results.

One way of applying decreasing returns in the model is the assumption of fixed capital stock with a constant-returns-to-scale technology. However, one may criticize this approach in that in the relevant time horizon of the Balassa – Samuelson effect, which is longer than a usual business cycle phenomenon, it can be misleading to neglect capital accumulation.

Hence, I choose another way of generating decreasing returns. As Woodford (2005) shows, even if the technology exhibits constant returns to scale, the *lack of an economywide rental market* for physical capital and *frictions in investments formation* combined with sticky asynchronized price setting result in suboptimal input allocation, and as a consequence, scarcity and decreasing returns to scale in the short run.[12]

3.1 Households

The domestic economy is populated by a continuum of infinitely-lived identical households. To simplify the notation household indices are dropped, since this does not cause confusion. The utility accrued to a given household at date t is

$$\mathcal{U}\left(c_t, l_t\right) = u(c_t) - v(l_t),$$

where c_t is the consumption and l_t is the labor supply of the representative household at date t. Furthermore, $u(c) = c^{1-\sigma}/(1-\sigma)$ and $v(l) = l^{1+\varphi}/(1+\varphi)$, σ, $\varphi > 0$. Households discount the future at the rate $0 < \beta < 1$.

[11] Although it is rarely studied in the literature, there is a third logical possibility, namely PTM with producer currency pricing. For the sake of clear presentation I omit discussion of this case.

[12] There can be different explanations for the lack of a rental market for physical capital. One is based on the existence of *firm-specific* investments and capital goods. The literature of the theory of firms considers this factor very important: one can explain with this phenomenon the size and integration of firms, as Hart (1995) discusses.

The consumption good c_t is composed of *tradable* and *non-tradable* consumption goods:

$$c_t = \left[a_T^{\frac{1}{\eta}} \left(c_t^T \right)^{\frac{\eta-1}{\eta}} + a_N^{\frac{1}{\eta}} \left(c_t^N \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

where c_t^T is the tradable, c_t^N is the non-tradable consumption good, η and $a_T = 1 - a_N$ are non-negative parameters.

The intertemporal budget constraint of a given household is the following:

$$P_t^T c_t^T + P_t^N c_t^N + P_t^B B_t = \zeta_t B_{t-1} + W_t l_t + T_t,$$

where P_t^T and P_t^N are the price indices of tradables and non-tradables, B_t is the household's nominal portfolio at the beginning of date t, P_t^B is its price, and ζ_t is its stochastic payoff. W_t is the nominal wage, while T_t is a lump-sum tax/transfer variable.

It is well known that the linear homogeneity of function (1) implies that the households' problem can be solved in two steps. First they maximize the objective function

$$\sum_{t=1}^{\infty} \beta^{t-1} \mathrm{E}_0 \left[\mathcal{U} \left(c_t, l_t \right) \right],$$

with respect to c_t subject to the following modified budget constraint:

$$P_t c_t + P_t^B B_t = \zeta_t B_{t-1} + W_t l_t + T_t, \tag{2}$$

non-negativity constraints on consumption, and no-Ponzi schemes. In the budget constraint (2) the consumer price index P_t is defined by the following expression:

$$P_t = \left[a_T \left(P_t^T \right)^{1-\eta} + a_N \left(P_t^N \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{3}$$

Second, knowing c_t it is possible to determine c_t^T and c_t^N by the demand functions

$$c_t^T = a_T \left(\frac{P_t}{P_t^T} \right)^{\eta} c_t, \quad c_t^N = a_N \left(\frac{P_t}{P_t^N} \right)^{\eta} c_t. \tag{4}$$

The assumption of complete asset markets implies that the optimal intertemporal allocation of consumption is determined by the following condition in all states of the world:

$$\beta \frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}} = D_{t,t+1}, \tag{5}$$

where Λ_t is the marginal utility of consumption,

$$\Lambda_t = c_t^{-\sigma},$$

and $D_{t,t+1}$ is the stochastic discount factor, which satisfies the condition

$$P_t^B = \mathrm{E}_t \left[D_{t,t+1} \zeta_{t+1} \right].$$

Since in this economy the asset markets are also complete internationally, the foreign equivalent of equation (5) is also held:

$$\beta \frac{\Lambda_{t+1}^* e_t P_t^{F*}}{\Lambda_t^* e_{t+1} P_{t+1}^{F*}} = D_{t,t+1}, \tag{6}$$

where Λ_t^* is the marginal utility of foreign households, P_t^{F*} is the foreign consumer price index in foreign currency terms, and e_t is the nominal exchange rate. For simplicity P_t^{F*} is assumed to be constant. Combining equations (5) and (6) and applying recursive substitutions yields formula

$$\frac{\Lambda_t e_t P_t^{F*}}{\Lambda_t^* P_t} = \iota, \tag{7}$$

where ι is a constant, which depends on initial conditions.

The solution of the households' problem implies that the real wage is equal to the marginal rate of substitution between consumption and labor, i.e.

$$w_t = c_t^\sigma l_t^\varphi, \tag{8}$$

which determines labor supply decision.

3.2 Production

Final and intermediate goods production

There are two stages of production in the model: in the first step import goods and labor are transformed into differentiated intermediate goods in each sector,[13] while in the second step a homogenous final good is produced in each sector by intermediate products.

As mentioned above, one objective of this paper is to study how the different descriptions of international goods markets segmentation influence the operation of the BS effect. Therefore, two different model versions are considered and compared. In version A it is assumed that there is no PTM. That is, the domestically produced export goods and the domestically consumed tradable goods have the same prices, if they are measured in the same currency. In version B there is pricing to the market, i.e. the price of the

[13] Thus, I apply the approach of McCallum and Nelson (2001), Smets and Wouters (2002) and Laxton and Pesenti (2003), who consider imports as a production input.

domestically produced export goods and the domestically consumed tradable goods can be different, even if they are measured in the same currency.

To capture these characteristics in version A the assumption is made that the domestically produced export goods and the locally traded tradable goods are the same and produced by the same sector. Hence, two sectors are distinguished in version A: a tradable and a non-tradable one.

In version B there are two types of tradable goods: goods which are traditionally classified as tradable, but in practice they are *local goods*, and another type of tradables that are produced for export. As a consequence, prices of local tradables and export goods denominated in the same currency can be different. Local tradables and the export goods are produced by different sectors.[14]

Let us denote by y_t^s the production of a given sector, where $s = T, x, N$, with T referring to the tradable sector in version A and to the sector of local tradables in version B, x to the exports sector in version B, and N to non-tradables. The final goods are produced in competitive markets by constant-returns-to-scale technologies from a continuum of differentiated inputs, $y_t^s(i)$, $i \in [0, 1]$. The technology is represented by the following CES production function:

$$y_t^s = \left(\int_0^1 y_t^s(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$. As a consequence, the output price P_t^s is given by

$$P_t^s = \left(\int_0^1 P_t^s(i)^{1-\theta} \, di \right)^{\frac{1}{1-\theta}}, \tag{9}$$

where $P_t^s(i)$ denotes the prices of differentiated goods. The demand for good $y_t^s(i)$ is determined by

$$y_t^s(i) = \left(\frac{P_t^s}{P_t^s(i)} \right)^{\theta} y_t^s. \tag{10}$$

In each sector the continuum of good $y_t^s(i)$ is produced in a monopolistically competitive market. Each $y_t^s(i)$ is made by an individual firm using the following uniform technology:

$$y_t^s(i) = A_t^s k_t^s(i)^{\alpha} z_t^s(i)^{1-\alpha}, \tag{11}$$

[14]To guarantee PTM, of course, the distinction of local tradables and export goods is not necessary. I applied this assumption due to technical reasons. Otherwise in the presence of heterogeneous capital the price setting problem of firms would be intractable. On the other hand, this approach is not unique in the literature. For example, Burnstein et al. (2002) also assumed the existence of local and real tradables. But unlike me, they assumed a quality difference between the two groups: local goods are inferior.

where $0 < \alpha < 1$, A_t^s is total factor productivity of sector s, $k_t^s(i)$ is the stock of physical capital available for firm i at date t (it was produced in the previous period), and $z_t^s(i)$ denotes an individual firm's utilization of the composite input z_t^s defined in the following way:

$$z_t^s(i) = N_s l_t^s(i)^{n_s} m_t^s(i)^{1-n_s}, \tag{12}$$

where $l_t^s(i)$ is an individual firms' utilization of labor l_t, and $m_t^s(i)$ is the utilization of imported good m_t, n_s is a given non-negative parameters, and $N_s = n_s^{-n_s}(1-n_s)^{n_s-1}$. The price of z_t^s is given by

$$W_t^{z,s} = W_t^{n_s}\left(e_t P_t^{m*}\right)^{1-n_s}, \tag{13}$$

where P_t^{m*} is the foreign currency price of the imported good.

Cost minimization and input demand

It is assumed that there is no rental market for physical capital. The necessary capital goods are produced by the firms themselves. As a consequence, firms' optimal input allocation problem cannot be separated from the problem of capital accumulation and cannot be derived from a sequence of static cost minimization problems.

Instead they solve the following dynamic cost minimization problem. Suppose the trajectories of $y_t(i)$, P_t, $W_t^{z,s}$ and $D_{T,t}$ are given. Then a firm should minimize the objective function

$$\sum_{t=T}^{\infty} \mathrm{E}_T\left[D_{T,t}\left(W_t^{z,s} z_t^s(i) + P_t I_t^s(i)\right)\right],$$

with respect to $z_t^s(i)$, $I_t^s(i)$, $k_{t+1}^s(i)$, subject to the technological constraint (11) and the investment constraint

$$k_{t+1}^s(i) = (1-\delta)k_t^s(i) + \Phi_s\left(\frac{I_t^s(i)}{k_t^s(i)}\right)k_t^s(i), \tag{14}$$

where $I_t^s(i)$ is the investment of firm i at date t. Function Φ_s represents the adjustment costs for investments, and δ is the depreciation rate. As is common in the literature, it is assumed that $\Phi_s' > 0$, $\Phi_s'' < 0$, and that in the steady-state adjustment costs do not exist, i.e. $\Phi_s(I^s/k^s) = I^s/k^s$ and $\Phi_s'(I^s/k^s) = 1$, where variables without time indices refer to the steady-state values.

The first-order conditions of the cost minimization problem are

$$\frac{D_{T,t}P_t}{\nu_t^s(i)} = \Phi_s'\left(\frac{I_t^s(i)}{k_t^s(i)}\right), \tag{15}$$

where $\nu_t^s(i)$ is the Lagrange multiplier of the investment equation,[15] and

$$\nu_t^s(i) = \mathrm{E}_T\left[\nu_{t+1}^s(i)\left\{(1-\delta)+\phi_s\left(\frac{I_{t+1}^s(i)}{k_{t+1}^s(i)}\right)\right\}+D_{T,t+1}P_{t+1}r_{t+1}^s(i)\right], \quad (16)$$

where $\phi_s(y) = \Phi_s(y) - y\Phi_s'(y)$, and

$$r_{t+1}^s(i) = \frac{\alpha}{1-\alpha} w_{t+1}^{z,s} \frac{z_{t+1}^s(i)}{k_{t+1}^s(i)}. \quad (17)$$

In models with a rental market for physical capital $r_{t+1}^s(i)$ in equation (16) represents the rental rate of capital.[16]

The solution of the cost minimization problem provides equations (11), (14), (15) (16) and (17), which determine the paths of $z_t^s(i)$, $k_t^s(i)$, $I_t^s(i)$, $r_t^s(i)$, and $\nu_t^s(i)$ given the paths for $y_t^s(i)$, P_t, $w_t^{z,s}$ and $D_{T,t}$. Knowing $z_t^s(i)$ one can determine the labor and import demand of a particular firm by

$$l_t^s(i) = n_s \frac{W_t^{z,s}}{W_t} z_t^s(i), \quad (18)$$

$$m_t^s(i) = (1-n_s)\frac{W_t^{z,s}}{e_t P_t^{m*}} z_t^s(i). \quad (19)$$

Firms' investment good is a composition of (local) tradables and non-tradables. The investment good and aggregate consumption good c_t are defined by the same function:

$$I_t^s(i) = \left(a_T^{\frac{1}{\eta}} \mathcal{I}_t^{Ts}(i)^{\frac{\eta-1}{\eta}} + a_N^{\frac{1}{\eta}} \mathcal{I}_t^{Ns}(i)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \quad (20)$$

where $\mathcal{I}^{Ts}$ is the demand for (local) tradables of firm i in sector s, and $\mathcal{I}^{Ns}$ is the demand for non-tradables. The particular form of function (20) implies that

$$\mathcal{I}_t^{Ts}(i) = a_T\left(\frac{P_t}{P_t^T}\right)^{\eta} I_t^s(i), \quad \mathcal{I}_t^{Ns}(i) = a_N\left(\frac{P_t}{P_t^N}\right)^{\eta} I_t^s(i). \quad (21)$$

[15] That is, it is the shadow price of investment. $\nu_t^s(i)\,(D_{T,t}P_t)^{-1}$ is the equivalent of Tobin's q in this model.

[16] If there is no adjustment costs for investments, then condition (16) becomes $P_t = \mathrm{E}_t\left[D_{t,t+1}P_{t+1}\left((1-\delta)+r_t^s(i)\right)\right]$. As a consequence, $r_t^s(i) = r_t^s = r_t$. In a deterministic setting the previous equation takes the form $1/\beta = r + 1 - \delta$, which is a simple arbitrage condition.

Price setting

So far, it has been shown how to find the optimal paths of $z_t^s(i)$, $k_t^s(i)$, $l_t^s(i)$, $m_t^s(i)$ conditional on the trajectories of $y_t^s(i)$ and $P_t^s(i)$. Now the optimal paths of the latter two variables will be determined.

Intermediate goods producers follow a sticky price setting practice. As in the model of Calvo (1983) each individual firm in a given time period changes its price in a rational, optimizing, forward looking manner with probability $1-\gamma_s$. Those firms which do not optimize at a given date follow a rule of thumb, as in Christiano et al. (2001) and Smets and Wouters (2003), and update their prices according to the past sectoral inflation rate.

All firms in sector $s = T$, N which follow the simple indexation rule at date T update their prices according to formula

$$P_t^s(i) = P_T^s(i)\left(\frac{P_{t-1}^s}{P_{T-1}^s}\right)^{\vartheta_s}.$$

Those which set their prices rationally take into account that $P_T^s(i)$ (the price they set at date T) will exist with probability γ_s^{t-T} at date t. Thus, they maximize the expected profit function

$$\sum_{t=T}^{\infty} \mathrm{E}_T\left[\gamma_s^{t-T} D_{T,t}\left\{(1-\tau_s)\, P_T^s(i)\left(\frac{P_{t-1}^s}{P_{T-1}^s}\right)^{\vartheta_s} - MC_t^s(i)\right\}\right]$$

with respect to $P_T^s(i)$ and $y_t^s(i)$ subject to constraint (10), where τ_s is a tax/transfer variable which modifies firms' markup, and $MC_t^s(i)$ is the marginal cost of firm i. In version B of the model the output price of the exports sector in foreign currency terms $P_T^{x*}(i)$ is sticky. Thus, the problem of the firms in the sector is

$$\max_{P_T^{x*}(i), y_t^x(i)} \sum_{t=T}^{\infty} \mathrm{E}_T\left[\gamma_x^{t-T} D_{T,t}\left\{(1-\tau_x)\, e_t P_T^{x*}(i)\left(\frac{P_{t-1}^{x*}}{P_{T-1}^{x*}}\right)^{\vartheta_x} - MC_t^x(i)\right\}\right],$$

subject to constraint (10), where τ_x is also a tax/transfer variable.[17] The log-linear approximations of the solutions of the above price setting problems can be found in *Appendix A.2*.

Since the capital stock available at a given date is predetermined, the variable cost of a firm is $W_t^{z,s} z_t^s(i) + P_t I_t^s(i)$. Thus, its marginal cost is

$$MC_t^s(i) = W_t^{z,s}\frac{\partial z_t^s(i)}{\partial y_t^s(i)}.$$

[17] Since the government's budget is balanced, the tax/transfer represented by τ_s ($s = T$, x, N) is compensated by T_t lump-sum tax/transfer variable in equation (2). In the present model the only role of τ_s is to simplify steady-state calculations, see *Appendix A.1*.

Expressing $z_t^s(i)$ by the technological constraint (11), and differentiating it with respect to $y_t^s(i)$ yields

$$MC_t^s(i) = W_t^{z,s} \left(\frac{y_t^s(i)}{k_t^s(i)}\right)^{\frac{\alpha}{1-\alpha}} (A_t^s)^{\frac{-1}{1-\alpha}}. \tag{22}$$

3.3 Exports demand

Foreign behavior is not modelled explicitly. It is assumed that the following *ad hoc* equation determines demand for exports:

$$x_t = \left(\frac{P_t^{FT*}}{P_t^{x*}}\right)^{\eta^*} x_t^*, \tag{23}$$

where x_t, P_t^{x*} is the foreign currency price of the export goods, P^{FT*} is the foreign currency price of the rival goods (which is constant by assumption), x_t^* is an exogenous parameter representing the volume of demand, and $\eta^* > 0$ is an exogenous parameter.

In version A of the model, exported goods are produced by the tradable sector, and $P_t^{x*} = P^T/e_t$. While in version B local tradables and export goods are different, hence their prices denominated in the same currency can be different, i.e. it is possible that $P_t^{x*} \neq P^T/e_t$.

3.4 Equilibrium conditions

In version A the equilibrium of the tradable sector is given by

$$y_t^T = c_t^T + \sum_{s=T,N} \mathcal{I}_t^{Ts} + x_t. \tag{24}$$

In version B the equilibrium conditions of the sector of local tradables and of the exports sector is given by

$$y_t^T = c_t^T + \sum_{s=T,x,N} \mathcal{I}_t^{Ts}, \quad y_t^x = x_t, \tag{25}$$

where $\mathcal{I}_t^{Ts} = \int_0^1 \mathcal{I}_t^{Ts}(i)\, di$. The equilibrium condition of the non-tradable sector is

$$y_t^N = c_t^N + \sum_s \mathcal{I}_t^{Ns}, \tag{26}$$

where $\mathcal{I}_t^{Ns} = \int_0^1 \mathcal{I}_t^{Ns}(i)\, di$. Finally, the labor market equilibrium condition is

$$l_t = \sum_s \int_0^1 l_t^s(i)\, di. \tag{27}$$

3.5 Real exchange rate indices

In this study the following real exchange indices will be considered:

$$q_t = \frac{e_t P_t^{F*}}{P_t}, \quad q_t^T = \frac{e_t P_t^{FT*}}{P_t^T}, \quad P_t^R = \frac{P_t^N}{P_t^T}, \tag{28}$$

where q_t is the CPI-based real exchange rate and q_t^T is the external real exchange rate. The movements of P_t^R, the domestic relative price of non-tradables to tradables, unambiguously determine the fluctuation of the internal real exchange rate, since it is assumed that P^{FT*} and P^{FN*} are constant.

3.6 The log-linearized model

To solve the model its log-linear approximation around the steady state is taken. The complete description of the log-linearized model and the derivation of its equations can be found in *Appendix A.3*. In this section, the most important equations of the system are reviewed. Variables without time indices refer to their steady-state values, and the tilde denotes the log-deviation of a variable from its steady-state value.

Aggregate demand

The path of the aggregate consumption is described by

$$\sigma \tilde{c}_t = \tilde{q}_t. \tag{29}$$

In version A exports demand is represented by

$$\tilde{x}_t = \eta^* \tilde{q}_t^T, \tag{30}$$

since in this version $\tilde{q}_t^T = \widetilde{P}_t^{x*}$. In version B the log-linearized exports demand becomes

$$\tilde{x}_t = -\eta^* \widetilde{P}_t^{x*}. \tag{31}$$

Demand for tradable goods depends on exports demand, aggregate consumption and investments, and the sectoral relative price. In version A it takes the form

$$\tilde{y}_t^T = \frac{x x_t + c\tilde{c}_t + I\tilde{I}_t + (c+I)\eta a_N \widetilde{P}_t^R}{c+x+I}, \tag{32}$$

where I_t denotes aggregate investments, and $\widetilde{\chi}_t^N$ is and exogenous shift of relative sectoral demand. In version B the demand for tradables is given by

$$\tilde{y}_t^T = \frac{c}{c+I}\tilde{c}_t + \frac{I}{c+I}\tilde{I}_t + \eta a_N \widetilde{P}_t^R. \tag{33}$$

Demand for non-tradables depends on the same factors:

$$\tilde{y}_t^N = \frac{c}{c+I}\tilde{c}_t + \frac{I}{c+I}\tilde{I}_t - \eta a_T \widetilde{P}_t^R. \tag{34}$$

Price setting

Following Woodford (2005), *Appendix A.2* presents the solution of the price setting problem of *section 3.2*. The path of the inflation rate in sector $s = T$, N is given by

$$\pi_t^d - \vartheta_d \pi_{t-1}^d = \beta \mathrm{E}_t \left[\pi_{t+1}^d - \vartheta_d \pi_t^d\right] + \xi_s \widetilde{mc}_t^s, \tag{35}$$

where $s = T, x, N$, and $d = x^*$ if $s = x$, otherwise $d = s$. Furthermore, $\pi_t^d = \widetilde{P}_t^d - \widetilde{P}_{t-1}^d$ is the sectoral inflation rate, and $\widetilde{mc}_t^s$ is the average real marginal cost of sector s and

$$\xi_s = \frac{(1-\gamma_s)(1-\beta\gamma_s)}{\gamma_s\left(1 + \frac{\alpha}{1-\alpha}\theta - \psi_s\right)}, \tag{36}$$

where parameter ψ_s is defined in *Appendix A.2*. It is assumed that the technology and the price setting parameters of the tradable and the exports sector are the same.

Marginal costs

The previous equations reveal that sectoral real marginal costs play a key role in the price setting process. I therefore summarize the determinants of such costs. The average real marginal cost in sector $s = T$, N is given by

$$\widetilde{mc}_t^s = \frac{\alpha}{1-\alpha}\left(\tilde{y}_t^s - \tilde{k}_t^s\right) - \frac{1}{1-\alpha}\widetilde{A}_t^s + n_s \widetilde{w}_t + (1-n_s)\tilde{q}_t + \chi_s \widetilde{P}_t^R, \tag{37}$$

where $\chi_T = a_N$ and $\chi_N = -a_T$. The real marginal cost in the exports sector is

$$\widetilde{mc}_t^x = \frac{\alpha}{1-\alpha}\left(x_t - \tilde{k}_t^x\right) - \frac{1}{1-\alpha}\widetilde{A}_t^T + n_T\left(\widetilde{w}_t - \tilde{q}_t\right) - \widetilde{P}_t^{x*}. \tag{38}$$

Policy rule

In this model monetary policy is represented by the following simple log-linear nominal exchange rate rule:

$$d\tilde{e}_t = -\omega\left(a_T \pi_{t-1}^T + a_N \pi_{t-1}^N\right) + \mathcal{S}_t^{de}, \tag{39}$$

where $d\tilde{e}_t = \tilde{e}_t - \tilde{e}_{t-1}$ is the nominal depreciation rate, and $\mathcal{S}_t^{de}$ is an exogenous nominal depreciation shock.

3.7 Model solution and parameterization

To solve the model Uhlig's (1999) implementation of the *undetermined coefficients* method is used, the numerical results being generated by the aforementioned author's MATLAB algorithm.

Benchmark values of the basic parameters are found in *Table 2*.

Table 2
Parameter values of
the benchmark economy

Parameter	
Name	Value
β	0.984
σ	1.000
φ	3.000
a_T	0.500
η	1.000
α	0.250
ρ_s	1.000
δ	0.025
ε_s	3.000
θ	10.80
γ_s	0.817
ϑ_s	0.365
ω	1.000

Note: $s = T$, x, N.

The value of β is taken from King and Rebello (1999). The value α is chosen in such a way that capital's share in GDP is 0.4.[18] The values of σ, φ, a_T, η, ρ_s, and δ, are widely accepted in the literature. The value of θ was chosen in such a way as to obtain the same degree of strategic complementarity of price setting as in Woodford (2003, 2005). Parameters ε_s measure the degree of investments adjustment costs in sector $s = T$, x, N, their values are taken from Woodford (2005). I take the values of γ_s and ϑ_s from the study of Galí et al. (2001), which also contains Euro area estimates.[19] The value of parameter η^* is not fixed: in the simulation

[18] In this model α is not equal to capital's share in GDP since one has to subtract the value of imports from the value of total output to obtain GDP.

[19] In that study they interpret inflation persistency differently from the approach I use. They use the model of Galí and Gertler (2000) and assume that each firm updates its price in a given period by probability $1-\gamma$. Hence, according to the law of large numbers in a given period $1-\gamma$ fraction of the firms change their prices. But only $1-\vartheta$ fraction of the price setters choose their prices in an optimal forward-looking manner, the rest update

exercises of *section 4* several different values are considered. Finally, ω was chosen in such a way that the model fits the empirical findings of *section 2*.

4 Examination of the Balassa – Samuelson effect

It was discussed in *section 2* that there is a strong relationship between the nominal and real exchange rates, and that asymmetric sectoral productivity growth results in dual inflation and real appreciation in developing countries. Under study in this section is how it is possible to reproduce both sets of evidence in a NOEM model.

First, it will be demonstrated that, unlike in the models of the traditional approach, in NOEM models productivity induced dual inflation is not necessarily accompanied by real appreciation, which contradicts the empirical findings discussed previously. It will be shown that the international substitution parameter η^* in equations (30) and (31) has a key role in generating real appreciation. On the other hand, η^* also influences the degree of co-movement of the nominal and real exchange rates. According to my numerical simulations, the assumption of pricing to market (PTM) is necessary to find such a value of η^* which ensures both the strong co-movement of the nominal and real exchange rates and the CPI-based real appreciation related to asymmetric productivity growth.

Second, it will be shown that it is difficult to reproduce the observable slow adjustment of the sectoral relative price to the sectoral productivity differential by frictionless models. However, the coexistence of heterogeneity in capital accumulation and sticky prices help to explain this phenomenon.

4.1 Productivity induced dual inflation and real appreciation

As discussed in *section 2.1*, in European post-communist countries in the 1990s the fast productivity growth of the tradable sector resulted in dual inflation, i.e. appreciation of the internal real exchange rate, which accompanied the appreciation of the external and the CPI-based exchange rate.

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their prices according to the past inflation rate. If $\beta = 1$, then the approach I use and the one used by Galí and Gertler coincides, if $\vartheta_s = \vartheta/\gamma$ and $(1-\gamma_s)^2\gamma_s^{-1} = (1-\vartheta)(1-\gamma)^2\gamma^{-1}$, $s = T, x, N$. Although in our case $\beta \neq 1$, as an approximation I used the above mentioned formula to determine the values of γ_s and ϑ_s.

Usually productivity induced coexistence of dual inflation and real appreciation, i.e. the BS effect is analyzed with models of the traditional approach. These models can successfully explain the coexistence of dual inflation and real appreciation, since in these models PPP is assumed, which prevents external real exchange rate movements. On the other hand, due to PPP they cannot reproduce the observable appreciation of the real exchange rate.

It seems that with NOEM models it is even more problematic to explain the discussed empirical phenomena. It is typical in NOEM models that although a positive productivity shock in the tradable sector results in real appreciation of the internal real exchange rate, at the same time, due to increasing productivity, domestic tradables become cheaper, i.e. the external real exchange rate also depreciates. As Beningno and Thoenissen (2002) demonstrated, the latter effect suppresses internal appreciation, hence the CPI-based real exchange rate also depreciates.

This possibility is especially important in version A. Consider the exports demand equation (30). If the international substitution parameter $\eta^* = +\infty$ then $\tilde{q}_t^T = 0$, i.e. the external real exchange rate becomes constant, and there will not be any relationship between the nominal and the real exchange rate, which contradicts empirical results. On the other hand, if η^* is low, and $\widetilde{P}_t^T$ is sticky, i.e. it responds to shocks slowly, then $\tilde{q}_t^T = \tilde{e}_t - \widetilde{P}_t^T$ will move together with the nominal exchange rate. However, in this case high tradable-productivity growth may cause strong external-real-exchange depreciation. The question is whether there is an intermediate value of η^* which can replicate both sets of empirical findings in version A of the model.

In version B even a high value of η^* can guarantee a strong co-movement of the nominal and real exchange rates. On the other hand, in this case the foreign currency price of domestically produced export goods $\widetilde{P}_t^{x*}$ does not deviate much from the prices of their foreign rivals. As a consequence, if other factors are kept fixed, the marginal costs of the domestic exports and the tradable sectors are similar, hence $\widetilde{P}_t^T - \tilde{e}_t$ remains relatively stable. Thus, the conjecture is that in version B it is possible to find appropriate values for the substitution parameter, which guarantee that asymmetric sectoral productivity growth results in real appreciation.

First, it is studied which value of the substitution parameter η^* is consistent with the strong co-movement of the nominal and real exchange rates discussed in *section 2*. In the simulation exercises the depreciation shock $\mathcal{S}_t^{de}$ is the only source of nominal-exchange-rate movements. This approach is supported by several empirical studies. In a closed economy context Smets and Wouters (2003) and Ireland (2004) demonstrated by their estimated models that nominal shocks have a primary role while technological shocks have only an auxiliary role in explaining business cycles. Clarida and Galí

(1994) showed that in open economies 35-41 per cent of real exchange rate movements can be attributed to nominal shocks. The prominent importance of the nominal-exchange-rate shocks in emerging markets is documented by Calvo and Reinhart (2002).

In the following simulations all parameters, except η^*, are set to their benchmark values (see *Table 2*). *Table 3* displays the results. Empirical values of the statistics in the table are taken from *section 2.2*.

Table 3
The relationship between nominal and
real exchange rates in the model economy

Version *A*

		Parameter values of η^*			
Statistics	Data	1	5	10	15
Autocorrelation of the real exchange rate					
1 quarter	0.84	0.84	0.76	0.71	0.67
1 year	0.50	0.55	0.41	0.32	0.27
2 years	0.25	0.41	0.28	0.22	0.19
The relative variance of the real and nominal depreciations	1	0.89	0.86	0.84	0.82

Version *B*

		Parameter values of η^*			
Statistics	Data	1	5	10	15
Autocorrelation of the real exchange rate					
1 quarter	0.84	0.83	0.81	0.80	0.80
1 year	0.50	0.53	0.49	0.47	0.46
2 years	0.25	0.40	0.34	0.32	0.30
The relative variance of the real and nominal depreciations	1	0.94	0.94	0.94	0.94

The time pattern of the reaction of the real exchange rate to the nominal-exchange-rate shock can be captured by the autocorrelation function of the real exchange rate. If $\eta^* = 1$ both versions of the model reproduce the 1-quarter and 1-year value of empirical autocorrelations quite well. However the simulated 2-year autocorrelation coefficients are higher than the observed one.[20]

In version A all the autocorrelation coefficients significantly diminish as η^* increases. In particular, the 1-year coefficient becomes very small compared to the data. On the other hand, in version B the autocorrelation coefficients are much less sensitive to the substitution parameter.

Another measure indicating the strength of the co-movement of nominal and real exchange rates is the relative variance of nominal and real depreciations. In version A this statistic decreases as η^* increases, and becomes significantly smaller than the empirical value. On the other hand, in version B the relative variance does not react to the change of the substitution parameter.

In summary: while model version B is quite insensitive to the change of η^*, version A is sensitive to the variation of the substitution parameter. It can reproduce the empirical results only if η^* has low values, i.e. domestically produced export goods and their foreign rivals are far substitutes.

The next issue is whether dual inflation induced by asymmetric sectoral productivity growth is accompanied by real appreciation. The role of the international substitution parameter η^* in equations (30) and (31) will be studied by numerical simulations.

In the simulation exercises I imitate some characteristics of productivity developments of transition countries. The model's steady state represents the state of the economy at the beginning of its transition process. Foreign productivity growth is normalized to zero, hence the productivity variables $\widetilde{A}_t^T$ and $\widetilde{A}_t^N$ represent relative productivity of the examined small open economy. In the model transition is driven by increasing productivity. The start of the process is captured by an unexpected productivity shock. It is assumed that during transition the growth rate of productivity is constant. After the transition process the growth rate of productivity in the small open economy will be equal to zero as well. The steady state belonging to the new level of productivity represents the after-transition state of the economy. However, this new state of the economy is beyond my focus. I assume that the transition process is mainly driven by tradable productivity, hence I assume that in the examined transition period the growth rate of non-tradable productivity is equal to zero. In the simulation exercises I set the growth rate of the tradable TFP $d\widetilde{A}_t^T = \widetilde{A}_t^T - \widetilde{A}_{t-1}^T = 1$.

Figure 1 and *2* display the simulation results for the benchmark economy

[20] This contradicts the simulation results of Chari et al. (2002), who found weaker simulated autocorrelations. However, Benigno (2004) demonstrated that if monetary policy is described by a rule with inertia, and the foreign and home country are asymmetric in such a way that monetary shocks result in terms of trade changes, then the required persistence can be attained by the model. These conditions are fulfilled in my model.

with $\eta^* = 1$ in version A and B. The first panels of the figures plot the difference between the growth rates of sectoral productivity factors $d\widetilde{A}_t^T - d\widetilde{A}_t^N$, and the inflation differential $\pi_t^R = \pi_t^N - \pi_t^T$. The latter determines the movements of the internal real exchange rate. If π_t^R is positive, then the internal real exchange rate appreciates. The second panels plot the depreciation of the real exchange rate $d\tilde{q}_t$, and the CPI-based external real exchange rate $d\tilde{q}_t^T$. Positive values of $d\tilde{q}_t$ and $d\tilde{q}_t^T$ mean deprecation. The third panels display $\tilde{y}_t^T - \tilde{k}_t^T$ and $\tilde{y}_t^N - \tilde{k}_t^N$. As equations (35) and (37) reveal, beyond productivity factors these quantities also influence sectoral inflation rates. Finally, the fourth panels plot the growth rates of the real wage and exports. All growth rates are expressed in annualized terms.

Simulation results reveal that although the internal real exchange rate appreciates, the real exchange rate depreciates since the effect of the depreciating external rate is stronger then that of the internal rate. The reason is that the productivity growth of the tradable sector is higher than those of the non-tradable sector and foreign tradable sectors. As a consequence, the relative price of domestically produced tradables to foreign tradables decreases. That is, the external real exchange rate depreciates. If domestically produced and foreign tradables were perfect substitutes, then the reduced relative price would induce a large instant increase of demand for domestic tradables. Hence, domestic real wages and tradable prices would increase and the prices of domestic and foreign tradables denominated in the same currency would equalize immediately. But in the studied case domestic and foreign tradables are far substitutes, hence increasing demand does not result in equalized prices.

Figures 3 and *4* plot simulation results belonging to an intermediate value of η^* in both versions. The figures reveal that if domestic and foreign tradables are closer substitutes than in the previous case, then the depreciation of the external real exchange rate becomes more moderate. Moreover, in the initial periods it appreciates. However, in the long run even these moderate levels of depreciation prevent appreciation of the CPI-based real exchange rate. As a consequence, even these values of the international substitution parameter η^* are insufficient to reproduce empirical findings.

Figures 5 and *6* display the results belonging to a relatively high value of η^*. Since in this case export goods are relatively close substitutes of their foreign rivals their prices cannot deviate much, hence the depreciation of the internal real exchange rate is moderate. As a consequence, the CPI-based real exchange rate appreciates in the long run.

Again, initial appreciation of the external exchange rate can be observed. One may ask whether this phenomenon is induced by movements of the nominal exchange rate. Hence I repeat the same exercises with fixed nominal exchange rates (parameter $\omega = 0$ in equation (39)). The results are displayed in *Figures 7* and *8*. In these cases the external real exchange rate still appreciates initially, although the appreciation is weaker.

Since the initial appreciation of the external real exchange rate cannot simply be explained by the policy rule, it is important to discuss what causes this phenomenon. Due to the lack of a rental market for physical capital, as equations (35) and (37) reveal, term $\tilde{y}_t^T - \tilde{k}_t^T$ influences price setting in the tradable sector. Relatively slow adjustment of capital and increasing relative demand for local tradables imply that the quantity $\tilde{y}_t^T - \tilde{k}_t^T$ strongly increases in initial periods, and this suppresses the price-reducing effect of tradable productivity growth. As a consequence, the external real exchange rate appreciates. However, in the long run capital accumulation is sufficient and the productivity effect becomes dominant.

In summary: It was demonstrated that the international substitution parameter η^* had a key role in reproducing empirical facts related to the BS effect. If η^* is low, i.e. domestic and foreign tradables are far substitutes, then the external real exchange rate depreciates too much, and prevents the appreciation of the CPI-based real exchange rate. Hence, relatively high values of parameter η^* are the only possible candidates to generate results consistent with empirical findings.[21] However, in version A, when PTM is not allowed, sufficiently high values of η^* result in insufficient and weak relationship between the nominal and real exchange rates. In version A to generate real appreciation at least $\eta^* = 15$ is necessary, but this parameter value induces small autocorrelation coefficients and relative variance of the real exchange rate (recall *Table 3*). Hence, PTM seems necessary to appropriately describe the BS effect in NOEM models.

As was discussed in *section 2.1*, in European post-communist countries the observed long-run appreciation of the real exchange rate is only partly caused by dual inflation, the long-run appreciation of the external real exchange rate also lies behind this phenomenon. The presented model is not able to reproduce the long-run appreciation of the external real exchange rate. However, due to the assumed frictions in capital formation and the related decreasing-returns-to-scale features of real marginal cost the model can explain initial appreciation of the external real exchange rate. To explain

[21] One may criticize the choice of the applied substitution parameters which are different from the ones used in other open economy models. For example, Backus et al. (1994) use much lower substitution parameter to replicate the empirically observable responses of the trade balance to productivity shocks. My conjecture is that if the inertia of the exports demand is increased, as in Laxton and Pesenti (2003), or the import requirement of exports production is increased, then my model would also be able to reproduce the short run behavior of the trade balance.

this phenomenon sufficiently it seems necessary to relax the assumption of fixed structure of goods in the model. As Ito et al. (1997) discussed, the export structure of fast developing countries changes, and higher value-added goods gain importance. If the process of improving quality and increasing variety is not properly captured by the statistical system tradable prices may dramatically rise, as Broda and Weinstein (2004) demonstrated.

One more remark. To simplify the exposition I did not discuss the possibility of PTM with producer currency pricing (PCP), but it is possible to show that in the present framework it provides practically the same results as version *B*. As a consequence, I would rather not take sides in the LCP vs. PCP debate since both approaches can be consistent with the BS effect.[22] PCP can be applied without the assumption of price discrimination. Moreover, in most cases PCP is applied without PTM, which is equivalent to applying version *A*. The reason for this is that the arguments of the supporters of PCP remain valid without PTM. However, my results point out that if one wants to capture the particularities of emerging markets, then the PCP approach cannot be applied without the assumption of international price discrimination.

4.2 The adjustment of the relative price of non-tradables to tradables

As discussed in *section 2.1* and displayed in *Table 1*, according to most of the estimations of Coricelli and Jazbec (2001), Halpern and Wyplosz (2001), Égert (2002) and Égert et al. (2002) in the long-run the magnitude of the relative price of non-tradables to tradables P_t^R is significantly smaller than that of the sectoral productivity differential $\widetilde{A}_t^T - \widetilde{A}_t^N$. In addition, Halpern and Wyplosz found that the short-run adjustment of the relative price was very slow.

It is difficult to explain these fact by models of the traditional approach. Applying classical assumptions to the present model,[23] it is easy to show that the relative price is determined by

$$\widetilde{P}_t^R = \frac{n_N}{n_T}\widetilde{A}_t^T - \widetilde{A}_t^N, \tag{40}$$

[22] LCP vs. PCP is one of the most important undecided debates in the NOEM literature, since the choice of the optimal exchange rate is not independent of this problem. One can read pro LCP arguments in Engel (2002a, 2002b). Obstfeld (2001, 2002) and Obstfeld and Rogoff (2000) presents arguments supporting the PCP approach. Two recent studies on this topic are Bergin (2004), which provides evidence supporting LCP, and Koren et al. (2004) with findings reinforcing PCP.

[23] Flexible price setting, internationally homogeneous goods and capital markets.

where n_T and n_N are the labor utilization parameters in the technological equation (12). If the tradable productivity process $\widetilde{A}_t^T$ is dominant, then the only way to reproduce the aforementioned empirical long-run relationship is to assume that the tradable sector is more labor intensive than the non-tradable one. But this is counterfactual. Beyond this, the above formula implies instant adjustment of the relative price to the productivity differential.

In this section I show that the presence of nominal and real rigidities helps to explain the above empirical findings, even if $n_N \geq n_T$. For expositional simplicity, I assume that $n_N = n_T$. Combine the sticky price equations (35) and real marginal cost formulas (37), and for expositional simplicity assume that $\xi_T = \xi_N = \xi$ and $\vartheta_T = \vartheta_N = \vartheta$. Then the inflation differential $\pi_t^R = \pi_t^T - \pi_T^N$ is determined by

$$\begin{aligned} \pi_t^R - \vartheta\pi_{t-1}^R &= \beta \mathrm{E}_t\left[\pi_{t+1}^R - \vartheta\pi_t^R\right] + \frac{\xi}{1-\alpha}\left(\widetilde{A}_t^T - \widetilde{A}_t^N\right) \\ &+ \frac{\xi\alpha}{1-\alpha}\left(\tilde{y}_t^N - \tilde{k}_t^N - \tilde{y}_t^T + \tilde{k}_t^T\right) - \xi\widetilde{P}_t^R. \end{aligned} \tag{41}$$

Terms $\tilde{y}_t^N - \tilde{k}_t^N$ and $\tilde{y}_t^T - \tilde{k}_t^T$ appear in the above equation, since due to imperfections in capital accumulation real marginal cost functions have decreasing-returns-to-scale features. In the constant-returns-to-scale version of the present model only the productivity factors $\widetilde{A}_t^T$, $\widetilde{A}_t^N$ and the relative price $\widetilde{P}_t^R$ would influence the evolution of the inflation differential.

Obviously, speed of the adjustment of $\widetilde{P}_t^R$ depends on the magnitude of parameter ξ. The smaller ξ is, the slower the adjustment process. The presence of terms $\tilde{y}_t^N - \tilde{k}_t^N$ and $\tilde{y}_t^T - \tilde{k}_t^T$ also influences the adjustment process. Suppose $\widetilde{A}_t^T$ increases, then formula (41) implies that π_t^R and $\widetilde{P}_t^R$ increase as well. As a consequence, the demand for $\tilde{y}_t^T$ will rise and for $\tilde{y}_t^N$ will decrease. But according to the above formula this change of demand will diminish the rise of π_t^R and $\widetilde{P}_t^R$, hence the adjustment process will be slower.

Relative price adjustment in the presence of sticky prices is definitely slower than in the flexible price models of the traditional approach represented by formula (40). However, nominal rigidities without frictions in capital accumulation are not sufficient to reproduce the empirical estimates, as the simulation exercise belonging the upper panel of *Figure 9* demonstrates. The figure plots the adjustment process of the relative price to the sectoral productivity differential: it displays the fraction of the relative price to the productivity differential, i.e. $\widetilde{P}_t^R/(\widetilde{A}_t^T - \widetilde{A}_t^N)$. In the simulation exercise I apply the same productivity process as previously, and use version B with $\eta^* = 15$, but I assume that capital accumulation is frictionless, i.e. real marginal cost functions exhibit constant-returns-to-scale features, hence terms

$\tilde{y}_t^T - \tilde{k}_t^T$ and $\tilde{y}_t^N - \tilde{k}_t^N$ are missing from formula (41). To compare simulation results with empirical estimates I calculated the OLS regression

$$\widetilde{P}_t^R = \rho\left(\widetilde{A}_t^T - \widetilde{A}_t^N\right) + u_t$$

using the simulated ten-year-long time series. The obtained OLS coefficient ρ represents the empirical 'long-run' estimates of the studied relationships. The magnitude of the OLS coefficient ρ is also displayed on the figure. *Figure 9* reveals that the adjustment of $\widetilde{P}_t^R$ is not instant, however ρ is nearly equal to 1. However, with one exception the empirical estimates are significantly smaller than this.

If there are frictions in capital accumulation, and real marginal cost functions have decreasing-returns-to-scale features, the adjustment process becomes slower, since in the constant-returns-to-scale case the adjustment parameter ξ is greater, and terms $\tilde{y}_t^T - \tilde{k}_t^T$ and $\tilde{y}_t^N - \tilde{k}_t^N$ are not present.[24] The lower panel of *Figure 9* illustrates this. In this simulation exercise I used the original form of version B with heterogeneous capital ($\eta^* = 15$). The figure reveals that now the adjustment is slower and ρ becomes smaller. However, it is 0.967, which is still quite far from the majority of the empirical estimates.

One possible way of reducing the speed of the adjustment process is assuming that price setting is more rigid in the non-tradable sector. However, according my numerical simulations one would have to assume an unrealistically high price setting parameter γ_N to reproduce empirical results. That is why I choose another possibility. Using equations (56) and (57) one can express term $\tilde{y}_t^N - \tilde{y}_t^T$ in formula (41) as

$$\tilde{y}_t^N - \tilde{y}_t^T = -\eta \widetilde{P}_t^R.$$

Parameter η measures the elasticity of substitution between local tradables and non-tradables. The above expression reveals its importance in the adjustment process of the sectoral relative price. If η is high, i.e. tradables and non-tradables are close substitutes, the adjustment becomes slow, since it is more difficult to deviate the prices of close substitutes. It is important to note that this mechanism does not work if terms $\tilde{y}_t^T$ and $\tilde{y}_t^N$ are not present in real marginal cost functions, i.e. in the constant-returns-to-scale case. The following simulation exercise demonstrates the importance of this mechanism. I use version B with $\eta^* = 15$, but instead of the benchmark value of $\eta = 1$, I use $\eta = 15$. The upper panel of *Figure 10* displays the results. Now $\rho = 0.865$, which approximates the empirical estimates quite

[24]In the constant-returns-to-scale case term $1 + \theta\alpha/(1-\alpha) - \psi_s$ is missing from the denominator of formula (36).

well.[25] Exceptions are the findings of Halpern and Wyplosz (2001), but their results are rather different from the others. It is important to note that the increase of the value of η does not alter the results of the previous section, both the relationship between the nominal and real exchange rates and the behavior of the external and CPI-based real exchange rates remain the same.

One can further reduce the speed of adjustment if asymmetry of sectoral investments adjustment costs is introduced in the model economy of the previous simulation exercise. Assume that $\varepsilon_N = 3$ as in the benchmark economy, but $\varepsilon_T = \varepsilon_x = 10$. The lower panel of *Figure 10* plots the results: ρ becomes 0.782.

In summary: Although both flexible price models and sticky price models with flexible capital accumulation can roughly capture the relationship between sectoral price and productivity differentials, they fail to reproduce the exact empirical magnitudes. Frictions in capital accumulation and the accompanying decreasing-returns-to-scale features of real marginal cost help to explain the observed phenomena. However, to reproduce the estimated regularities one has to assume that tradables and non-tradables are not far substitutes. Asymmetry of sectoral investments adjustment costs can also improve the ability of the model to replicate empirical findings.

5 Conclusions

This paper has reviewed how the models of the new open economy macroeconomics (NOEM) can explain the permanent dual inflation and the accompanying real appreciation often observed in emerging markets.

The coexistence of dual inflation and real appreciation is usually explained by the Balassa - Samuelson (BS) effect, i.e. by the faster productivity growth in the tradable sector. Traditionally, the BS effect is derived from models with flexible prices and internationally homogenous tradable goods markets. On the other hand, NOEM models assume sticky prices and/or wages and heterogeneous goods markets. The traditional approach focuses on the determinants of the internal real exchange rate, while NOEM emphasize the importance of the external real exchange rate.

It was shown that a NOEM model can simultaneously guarantee the strong correlation of nominal and real exchange rates and generate the BS effect only if there is pricing to market in the model.

[25] Altig et al. (2005) emphasize the magnitude of the adjustment parameter ξ in firm-specific-capital models to reconcile micro and macro evidence. However, in the present model instead of the magnitude of ξ, the presence of output terms in real marginal cost functions is the key factor in slackening the adjustment process.

The study also looks at how the presence of nominal rigidities and frictions in capital accumulation modify the effects of asymmetric productivity growth on dual inflation and the external real exchange rate. The paper demonstrated that in the presence of the aforementioned nominal and real rigidities sectoral real marginal cost functions have decreasing-returns-to-scale features, which help to explain the appreciation of the external real exchange rate, and the slow adjustment of the relative price of non-tradables to tradables observable in post-communist European countries.

Although it was not studied in this paper, it is worth mentioning here that decreasing-returns-to-scale features can also explain the role of demand factors in generating dual inflation documented in Arratibel et al. (2002) and López-Salido et al. (2005).

A Appendix

A.1 The steady state

In this section the non-stochastic steady state of the benchmark model is described. Variables without time indices refer to their steady-state values.

In the steady state there is no difference between the two model versions since $eP^x = P^{x*}$, the technologies of the tradable and the exports sector are the same, and in the steady state nominal rigidities do not exist. Hence, in this section it is sufficient to discuss version A: thus index T will refer both to local and exported tradables. In the steady state there is no intra-household and intra-sector heterogeneity. Therefore the index i of firms are omitted to simplify the notations.

It is assumed that $P = P^T = P^N = 1$. Then equations (4) and (21) imply that

$$c^T = a_T c, \quad c^N = a_N c, \quad \mathcal{I}^{TT} + \mathcal{I}^{TN} = a_T I^T, \quad \mathcal{I}^{NT} + \mathcal{I}^{NN} = a_T I^N. \tag{42}$$

Furthermore, it is assumed that $Px = eP^{m*}m$. Hence,

$$GDP = a^T \left(c^T + \mathcal{I}^{TT} + \mathcal{I}^{TN}\right) + a^N \left(c^N + \mathcal{I}^{NT} + \mathcal{I}^{NN}\right) = c + I,$$

where $I = I^T + I^N$.

Since $\Phi_s(I^s/k^s) = I^s/k^s$ and $\Phi'_s(I^s/k^s) = 1$, in the steady state investments do not have adjustment costs and, as was mentioned, in the steady state nominal rigidities do not exist. Hence, firms' optimization problem will be the same as in the case when there is a rental market for physical capital and the real rental rate of capital is determined by the real interest rate and the depreciation rate. Equation (5) implies that the real interest rate is equal to $1/\beta - 1$. If the real rental rate of physical capital, which is uniform in all sectors, is denoted by r, then

$$r = \frac{1}{\beta} - 1 + \delta.$$

This formula represents a special case of equation (16). I set the values of τ_T and τ_N in such a way that the markups

$$1 = \tau_s \frac{\theta}{\theta - 1}, \quad s = T,\, N.$$

Then it is true for all sectors that the marginal product of capital is equal to r. Thus, equation (11) implies that

$$\varkappa = \left(\frac{r}{\alpha}\right)^{\frac{1}{1-\alpha}},$$

where $\varkappa = z^T/k^T = z^N/k^N$. Furthermore, equations (11), (24), and (26) imply that

$$c^T + I^T + x = k^T \varkappa^{1-\alpha}, \quad c^N + I^N = k^N \varkappa^{1-\alpha}. \tag{43}$$

Beyond this, in the steady-state equation (14) takes the form $I^s = \delta k^s$. Thus, if one defines the $k = k^T + k^N$ aggregate capital stock, then $I = \delta k$.

It is assumed that $w = W = eP^{m*}$, then equation (13) implies that $w^z = w$. Since in each sector w^z is equal to the marginal product of z^s

$$w = (1-\alpha)\varkappa^{-\alpha}.$$

In the benchmark economy $w = 1.212$. Let us denote the exogenous exports/GDP ratio by s_x, and I set $s_x = 0.6$. Since $x = eP^{m*}m$,

$$s_x = \frac{x}{c+I} = \frac{eP^{m*}m}{c+I}. \tag{44}$$

It is assumed that in the benchmark economy $n_N = n_T = n$. Then the imports demand equation (19) implies that

$$m = (1-n)\left(z^T + z^N\right).$$

Then one can show that

$$m = (1-n)\varkappa\left(k^T + k^N\right) = (1-n)\varkappa k. \tag{45}$$

Using the formula $I = \delta k$, the previous expression for m, and equation (44) yields

$$c = \mathsf{K}k, \tag{46}$$

where

$$\mathsf{K} = eP^{m*}(1-n)\varkappa s_x^{-1} - \delta.$$

By equation (43) one can similarly show that

$$k\varkappa^{1-\alpha} = c + \delta k + x = \left(eP^{m*}(1-n)\varkappa s_x^{-1} - \delta\right)k + \delta k + eP^{m*}(1-n)\varkappa k.$$

This implies that

$$n = 1 - \frac{\varkappa^{1-\alpha}}{eP^{m*}\varkappa\left(1 + s_x^{-1}\right)}.$$

In the benchmark economy $n = 0.526$.

In the steady state the labor supply function of households (8) takes the form

$$w = c^\sigma l^\varphi. \tag{47}$$

As for imports, one can derive a similar expression for labor:

$$l = n\varkappa k. \tag{48}$$

Substituting equations (46) and (48) into equation (47) yields an expression for the capital stock:

$$k = \left[w\mathsf{K}^{-\sigma} (n\varkappa)^{-\varphi} \right]^{\frac{1}{\sigma+\varphi}}.$$

Using this expression one can calculate the steady-state value of the capital stock and investments. In the benchmark economy $k = 21.008$, and $I = \delta k = 0.525$. Then using formula (46) yields the value of consumption, $c = 2.076$, and equation (48) provides the value of labor, $l = 1.43$.

A.2 Price setting

Following Woodford (2005), one can show that the log-linearized solution of the price setting problem of *section 3.2* takes the form

$$\pi_t^d - \vartheta_s \pi_{t-1}^d = \beta \mathrm{E}_t \left[\pi_{t+1}^d - \vartheta_s \pi_t^d \right] + \xi_s \widetilde{mc}_t^s,$$

where $s = T$, x, N, and $d = x*$, if $s = x$, otherwise $d = s$.

Furthermore, $\widetilde{mc}_t^s$ is the average real marginal cost of sector s, and

$$\xi_s = \frac{(1-\gamma_s)(1-\beta\gamma_s)}{\gamma_s (1 + \hat{\alpha}\theta - \psi_s)},$$

where $\hat{\alpha} = \alpha(1-\alpha)^{-1}$, which is the elasticity of capital in equations (66) and (68).

Parameter ψ_s can be obtained in the following way. First define λ_s which is the solution of the quartic equation

$$\begin{aligned} 0 &= \left[(1+\hat{\alpha}\theta)(1-\beta\gamma_s\lambda_s)^2 - \gamma_s^2 \beta \hat{\alpha} \Xi_s \lambda_s \right] \\ &\times \left\{ \beta^2 \lambda_s^2 - \left[1 + \beta + (1-\beta(1-\delta))\bar{\alpha}\varepsilon_s^{-1} \right] \beta\lambda_s + \beta \right\} \\ &+ \beta(1-\gamma_s)(1-\beta\gamma_s)\hat{\alpha}\Xi_s\lambda_s, \end{aligned}$$

where $\bar{\alpha} = (1-\alpha)^{-1}$, which is the coefficient of capital in equations (63) and (64), and

$$\Xi_s = \frac{(1-\beta(1-\delta))\,\bar{\alpha}\theta}{\varepsilon_s}.$$

In addition λ_s satisfies a set of three inequalities,

$$\begin{aligned} \lambda_s &< \gamma_s^{-1}, \\ \lambda_s &> \frac{\gamma_s}{\beta(1+\gamma_s)} \left\{ \beta^2 \lambda_s^2 - \left[1 + \beta + (1-\beta(1-\delta))\bar{\alpha}\varepsilon_s^{-1} \right] \beta\lambda_s + \beta \right\} - 1, \\ \lambda_s &< \frac{\gamma_s}{\beta(1+\gamma_s)} \left\{ \beta^2 \lambda_s^2 - \left[1 + \beta + (1-\beta(1-\delta))\bar{\alpha}\varepsilon_s^{-1} \right] \beta\lambda_s + \beta \right\} + 1. \end{aligned}$$

Then

$$\psi_s = \hat{\alpha}\frac{\beta\gamma_s^2\Xi_s\lambda_s}{(1-\beta\gamma_s\lambda_s)^2}.$$

A.3 The complete log-linearized model

To solve the model described in *section 3* its log-linear approximation around the steady state is taken. In this section the log-linearized version is described. Variables without time indices refer to their steady-state values, and the tilde denotes the log-deviation of a variable from its steady-state value.

The log-linearization of the price index formula (3) yields

$$\widetilde{P}_t = a_T\widetilde{P}_t^T + a_N\widetilde{P}_t^N, \tag{49}$$

where I used the assumption that $P = P^T = P^N$.

The log-linearized versions of the real exchange rate indices in equation (28), and the assumption that P_t^{F*}, P_t^{FT*} and P_t^{FR} are constant are used for the derivation of the following formulas:

$$\pi_t^T = d\tilde{e}_t - \left(\tilde{q}_t^T - \tilde{q}_{t-1}^T\right), \tag{50}$$

$$\pi_t^N = \pi_t^T + \widetilde{P}_t^R - \widetilde{P}_{t-1}^R, \tag{51}$$

$$\pi_t^{x*} = \widetilde{P}_t^{x*} - \widetilde{P}_{t-1}^{x*}, \tag{52}$$

$$\tilde{q}_t = \tilde{q}_t^T - a_N\widetilde{P}_t^R, \tag{53}$$

where $d\tilde{e}_t = \tilde{e}_t - \tilde{e}_{t-1}$ is the depreciation rate of the nominal exchange rate.

Log-linearizing equations (8), (13), and using the assumption that $W = eP^{m*}$ yields

$$\widetilde{w}_t^{z,s} = n_s\left(\sigma\tilde{c}_t + \varphi\tilde{l}_t\right) + (1-n_s)\tilde{q}_t, \tag{54}$$

for $s = T,\, N$. It is assumed that $n_x = n_T$, hence it is not necessary to have a separate equation for the exports sector.

Using the log-linearized version of equations (4), (21), (28), and using equation (49) one can obtain the following expressions:

$$\tilde{c}_t^T = \eta a_N\widetilde{P}_t^R + \tilde{c}_t,$$

$$\widetilde{\mathcal{I}}_t^{Ts}(i) = \eta a_N\widetilde{P}_t^R + \tilde{I}_t^s(i).$$

Let us define $I_t^s = \int_0^1 I_t^s(i)\,di$. Then one can show[26] that

$$\widetilde{\mathcal{I}}_t^{Ts} = \eta a_N\widetilde{P}_t^R + \tilde{I}_t^s.$$

[26]If a variable is defined in the following manner: $\mathfrak{z} = \int_0^1 \mathfrak{z}(i)\,di$ then its log-linear approximation yields $\tilde{\mathfrak{z}} = \int_0^1 \tilde{\mathfrak{z}}(i)\,di + o^2$, where o^2 denotes those second and higher order errors, which were neglected in the approximation process.

The above formulas imply that the log-linearized version of the equilibrium condition (24) takes the form

$$\tilde{y}_t^T = \frac{x\tilde{x}_t + \mathsf{c}\tilde{c}_t + I\tilde{I}_t + (c+I)\,\eta a_N \widetilde{P}_t^R}{c+I+x}, \tag{55}$$

where $I_t = \sum_s I_t^s$. Similarly, the log-linearized equilibrium condition (25) takes the form

$$\tilde{y}_t^T = \frac{c}{c+I}\tilde{c}_t + \frac{I}{c+I}\tilde{I}_t + \eta a_N \widetilde{P}_t^R. \tag{56}$$

Finally, the log-linear approximation of the equilibrium condition (26) is

$$\tilde{y}_t^N = \frac{c}{c+I}\tilde{c}_t + \frac{I}{c+I}\tilde{I}_t - \eta a_T \widetilde{P}_t^R. \tag{57}$$

The log-linearization of equations (7) and (28) yields the expression which determines the trajectory of aggregate consumption:

$$\sigma\tilde{c}_t = \tilde{q}_t. \tag{58}$$

In version A of the model $\widetilde{P}_t^T - \tilde{e}_t = \widetilde{P}_t^{x*}$, hence the log-linearized version of the exports demand equation (23) is

$$\tilde{x}_t = \eta^* \tilde{q}_t^T, \tag{59}$$

where equation (28) was used. In version B the log-linearized exports demand becomes

$$\tilde{x}_t = -\eta^* \widetilde{P}_t^{x*}. \tag{60}$$

Define the aggregate stock of physical capital in sector s as $k_t^s = \int_0^1 k_t^s(i)\,di$. Log-linearizing the investment equation (14) yields

$$\tilde{k}_{t+1}^s = (1-\delta)\tilde{k}_t^s + \delta\tilde{I}_t^s,$$

where the steady-state properties of Φ_s are used. As a consequence, the log-linearized equation for the aggregate investment is

$$\delta\tilde{I}_t = \sum_s \frac{I^s}{I}\left[\tilde{k}_{t+1}^s - (1-\delta)\tilde{k}_t^s\right], \tag{61}$$

where in version A $s = T, N$, in version B $s = T, x, N$.

Let us combine the log-linearized versions of equations (8), (11), (18), (27), (28), and equation (54). Then aggregating the result yields an expression for aggregate labor demand:

$$\tilde{l}_t = \sum_{s=H,x,N} \frac{l^s}{l}\left[(1-n_s)\left(\tilde{q}_t - \sigma\tilde{c}_t - \varphi\tilde{l}_t\right) + \bar{\alpha}\left(\tilde{y}_t^s - \widetilde{A}_t^s\right) - \hat{\alpha}\tilde{k}_t^s\right], \tag{62}$$

where, again, in version A $s = T, N$, and in version B $s = T, x, N$, furthermore, $\bar{\alpha} = (1-\alpha)^{-1}$ and $\hat{\alpha} = \alpha\bar{\alpha}$.

Log-linearizing and combining equations (5), (15) and (16) results in

$$\begin{aligned}
&\widetilde{\Lambda}_t - \mathrm{E}_t\left[\widetilde{\Lambda}_{t+1}\right] + \varepsilon_s\left(\tilde{k}^s_{t+1}(i) - \tilde{k}^s_t(i)\right) = \\
&\mathrm{E}_t\left[[1-\beta(1-\delta)]\tilde{r}^s_{t+1} + \beta\varepsilon_s\left(\tilde{k}^s_{t+2}(i) - \tilde{k}^s_{t+1}(i)\right)\right],
\end{aligned}$$

where $\varepsilon_s = -\Phi''_s(\delta)\delta$. Log-linearizing and combining equations (11) and (17) yields

$$\tilde{r}^s_t(i) = \widetilde{w}^s_t + \bar{\alpha}\left(\tilde{y}^s_t(i) - \widetilde{A}^s_t - \tilde{k}^s_t(i)\right).$$

Combining the above two equations, aggregating the result, and using the definition of Λ_t results in the equation which determines the evolution of physical capital in sector $s = T,\ N$:

$$\begin{aligned}
&-\sigma\tilde{c}_t + \sigma\mathrm{E}_t\left[\tilde{c}_{t+1}\right] + \varepsilon_s\left(\tilde{k}^s_{t+1} - \tilde{k}^s_t\right) \\
&= \Delta\mathrm{E}_t\left[\widetilde{w}^{z,s}_{t+1} + \bar{\alpha}\left(\tilde{y}^s_{t+1} - \widetilde{A}^T_{t+1} - \tilde{k}^s_{t+1}\right)\right] + \beta\varepsilon_s\mathrm{E}_t\left[\tilde{k}^s_{t+2} - \tilde{k}^s_{t+1}\right],
\end{aligned} \tag{63}$$

where $\Delta = [1-\beta(1-\delta)]$. For the exports sector it is

$$\begin{aligned}
&-\sigma\tilde{c}_t + \sigma\mathrm{E}_t\left[\tilde{c}_{t+1}\right] + \varepsilon_T\left(\tilde{k}^x_{t+1} - \tilde{k}^x_t\right) \\
&= \Delta\mathrm{E}_t\left[\widetilde{w}^{z,T}_{t+1} + \bar{\alpha}\left(\tilde{x}_{t+1} - \widetilde{A}^T_{t+1} - \tilde{k}^x_{t+1}\right)\right] + \beta\varepsilon_T\mathrm{E}_t\left[\tilde{k}^x_{t+2} - \tilde{k}^x_{t+1}\right],
\end{aligned} \tag{64}$$

where the second equilibrium condition of equations (25) is used.

In *Appendix A.2* it was shown that the inflation rate in sector $s = T,\ N$ is determined by

$$\pi^s_t - \vartheta_s\pi^s_{t-1} = \beta\mathrm{E}_t\left[\pi^s_{t+1} - \vartheta_s\pi^s_t\right] + \xi_s\widetilde{mc}^s_t. \tag{65}$$

The average real marginal cost is defined by

$$mc^s_t = \frac{MC^s_t}{P^s_t},$$

and by using equation (22) it can be expressed as

$$\widetilde{mc}^s_t = \hat{\alpha}\left(\tilde{y}^s_t - \tilde{k}^s_t\right) - \bar{\alpha}\widetilde{A}^s_t + \widetilde{w}^{z,s}_t + \chi_s\widetilde{P}^R_t, \tag{66}$$

where $\chi_T = a_N$ and $\chi_N = -a_T$.

The equation for the inflation rate in the exports sector in version B can be derived as

$$\pi_t^{x*} - \vartheta_T \pi_{t-1}^{x*} = \beta \mathrm{E}_t \left[\pi_{t+1}^{x*} - \vartheta_T \pi_t^{x*} \right] + \xi_T \widetilde{mc}_t^x. \tag{67}$$

To derive the above equation it is assumed that the technology and the price setting parameters of the tradable and the exports sector are the same. Hence, the coefficients of these two equations are the same as in equation (65). The average real marginal costs are defined as

$$mc_t^x = \frac{MC_t^x}{e_t P_t^{x*}}.$$

Hence, using equation (22) provides the log-linearized real marginal cost formula of the exports sector:

$$\widetilde{mc}_t^x = \hat{\alpha} \left(x_t - \tilde{k}_t^x \right) - \bar{\alpha} \widetilde{A}_t^T + n_T \left(\sigma \tilde{c}_t + \varphi \tilde{l}_t - \tilde{q}_t \right) - \widetilde{P}_t^{x*}. \tag{68}$$

As mentioned in *section 3.6*, exchange rate policy is represented by the following simple rule:

$$d\tilde{e}_t = -\omega \left(a_T \pi_{t-1}^T + a_N \pi_{t-1}^N \right) + \mathcal{S}_t^{de}, \tag{69}$$

where $d\tilde{e}_t = \tilde{e}_t - \tilde{e}_{t-1}$ is the nominal depreciation rate, and $\mathcal{S}_t^{de}$ is the shock of an exogenous nominal depreciation.

Version A of the model (no PTM) contains 18 equations: (50), (51), (53), (55), (57)–(59), (61), (62), (69), and the two-equation systems of formulas (54), (63), (65), and (66). This system determines the trajectories of the following 18 endogenous variables: $\tilde{c}_t$, $\tilde{x}_t$, $\tilde{I}_t$, $\tilde{y}_t^T$, $\tilde{y}_t^N$, $\tilde{l}_t$, $\tilde{k}_t^T$, $\tilde{k}^N$, $\widetilde{mc}_t^T$, $\widetilde{mc}_t^N$, $\widetilde{w}_t^{z,T}$, $\widetilde{w}_t^{z,N}$, $\tilde{q}_t$, $\tilde{q}_t^T$, $\widetilde{P}_t^R$, $d\tilde{e}_t$, π_t^T, π_t^N.

To obtain version B (PTM, LCP) replace equations (55) and (59) by equations (56) and (60). Furthermore, add equations (52), (64), (67), and (68) to the system. This is a system of 22 equations. It determines the paths of the variables belonging to version A, plus the trajectories of $\tilde{k}_t^x$, $\widetilde{mc}_t^x$, $\widetilde{P}_t^{x*}$, π_t^{x*}.

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Figure 1
Balassa-Samuelson effect
No PTM – version *A*
η*=1

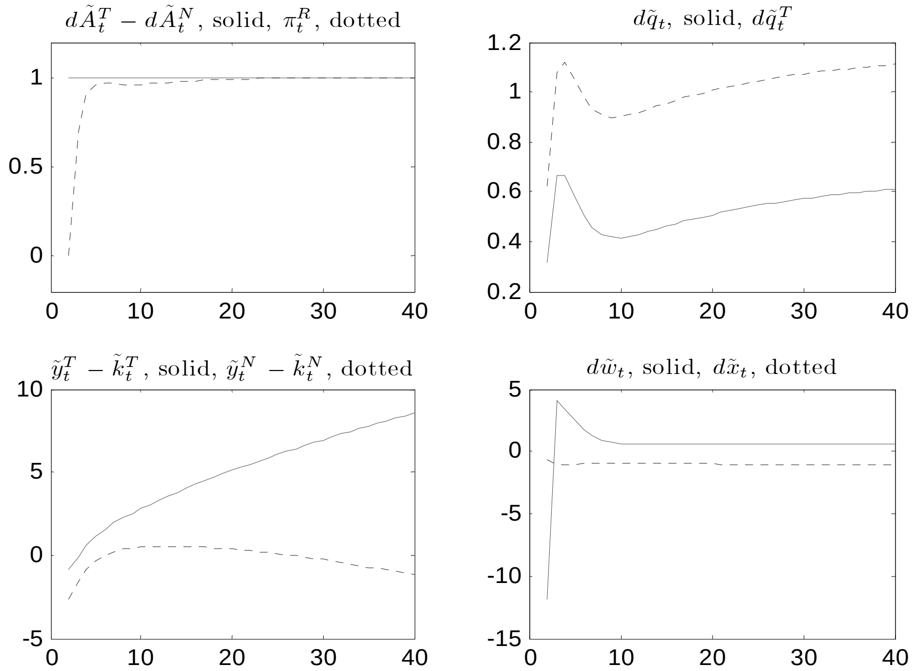
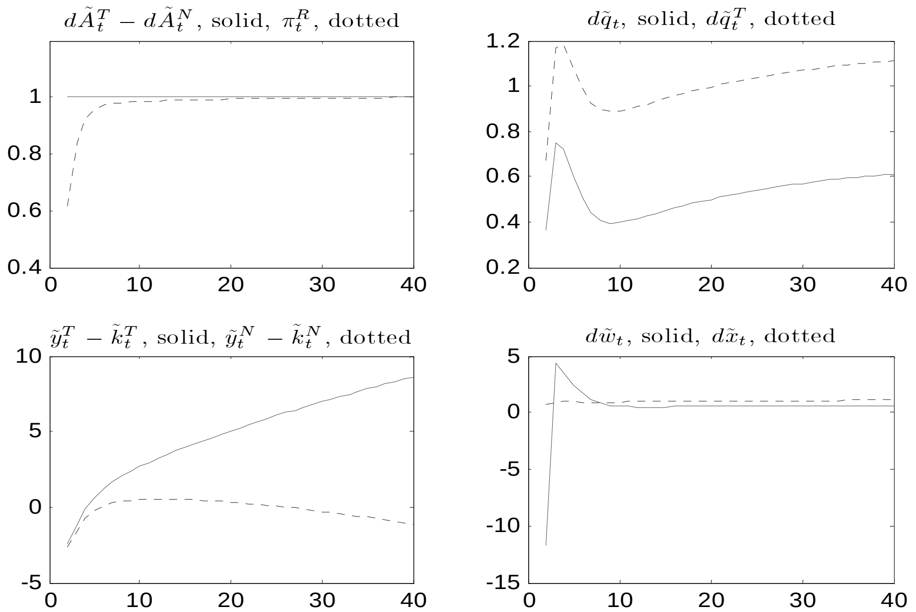


Figure 2
Balassa-Samuelson effect
PTM with LCP – version *B*
η*=1



Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Growth rates are displayed in annualized terms.

Figure 3

Balassa-Samuelson effect

No PTM – version *A*

η*=5

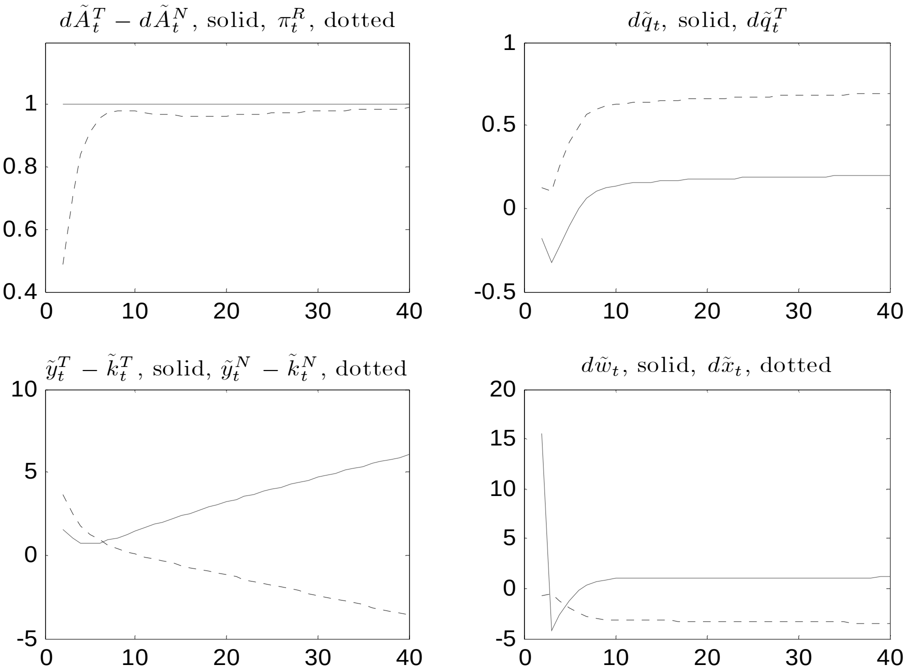
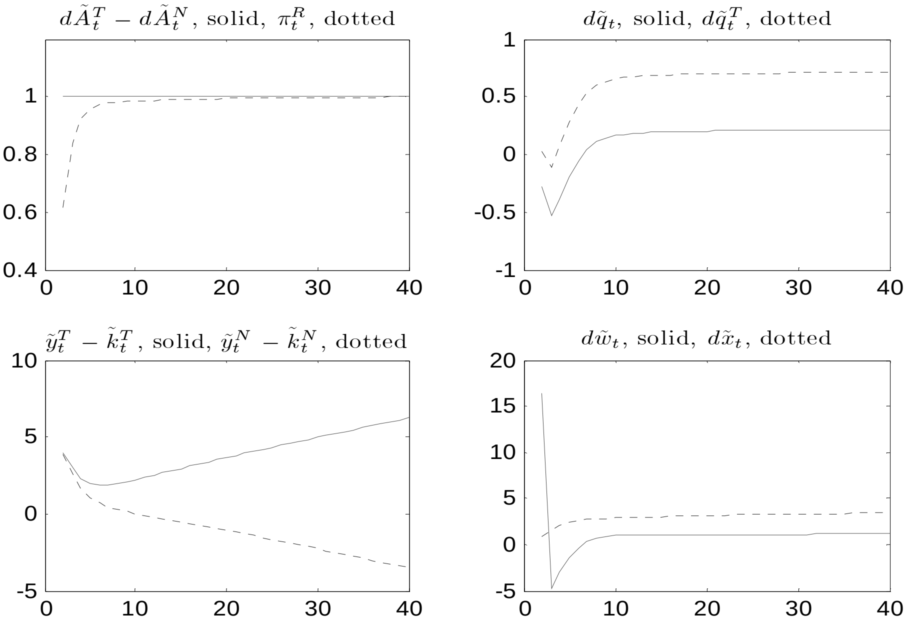


Figure 4

Balassa-Samuelson effect

PTM with LCP – version *B*

η*=5



Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Growth rates are displayed in annualized terms.

Figure 5
Balassa-Samuelson effect
No PTM – version *A*
η*=15

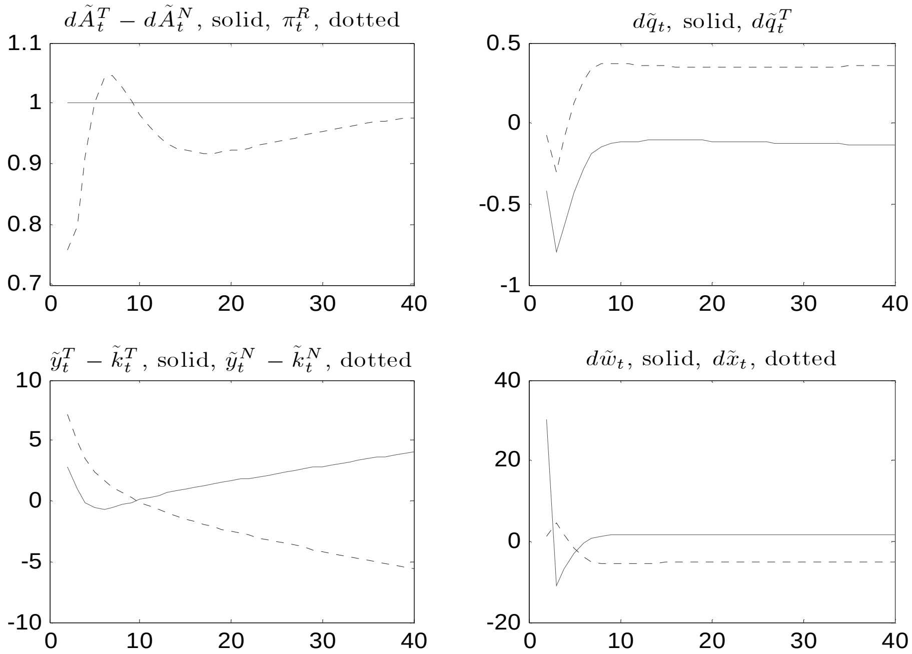
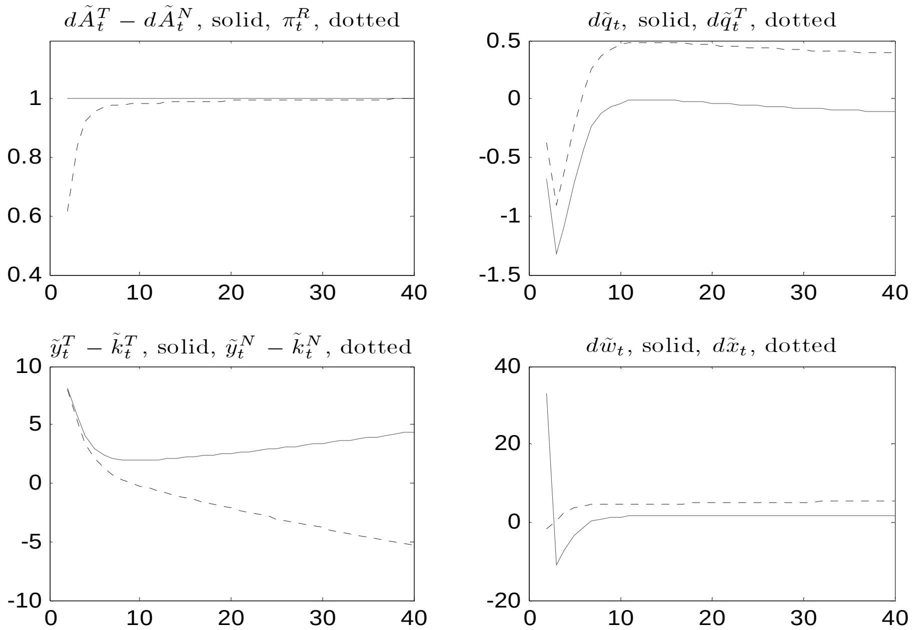


Figure 6
Balassa-Samuelson effect
PTM with LCP – version *B*
η*=15



Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Growth rates are displayed in annualized terms.

Figure 7
Balassa-Samuelson effect
No PTM – version *A*
η*=15, fixed nominal exchange rate

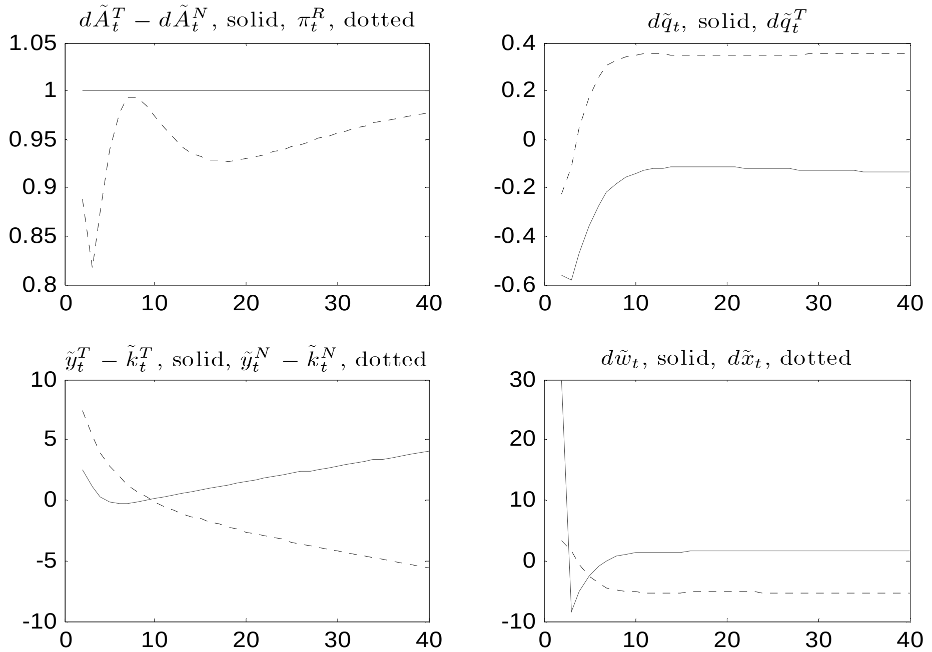
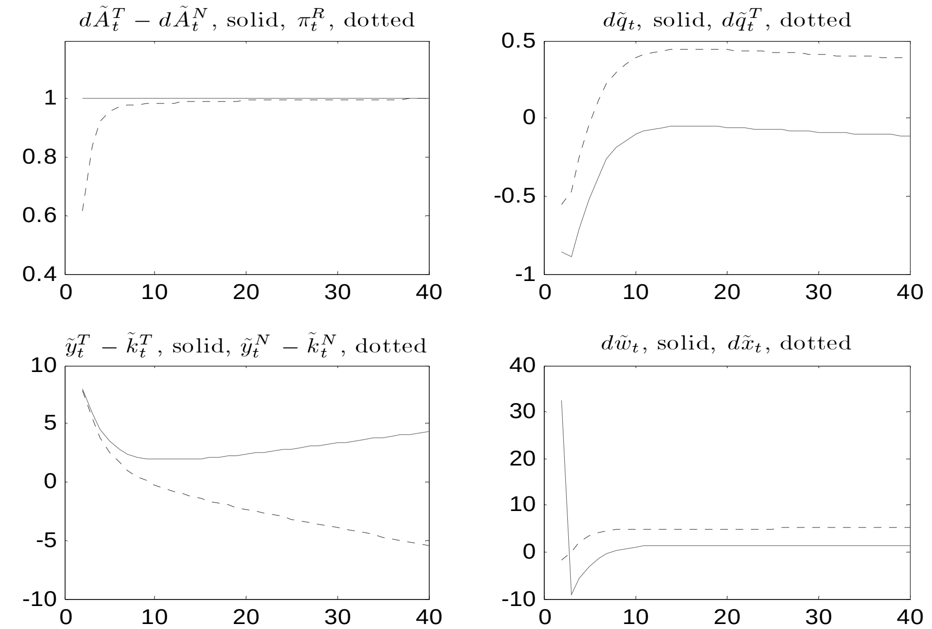
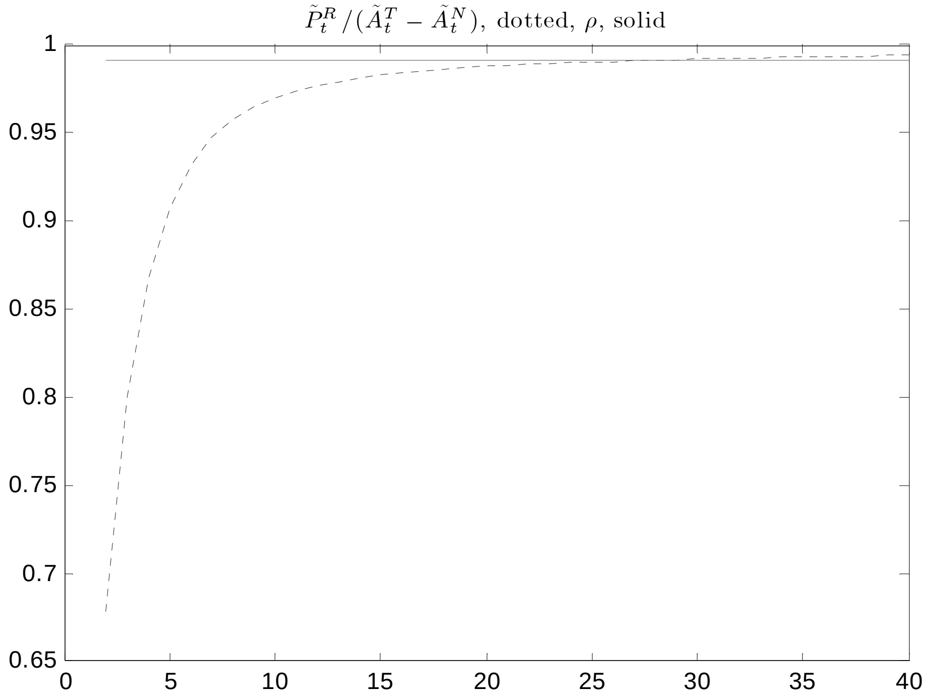


Figure 8
Balassa-Samuelson effect
PTM with LCP – version *B*
η*=15, fixed nominal exchange rate

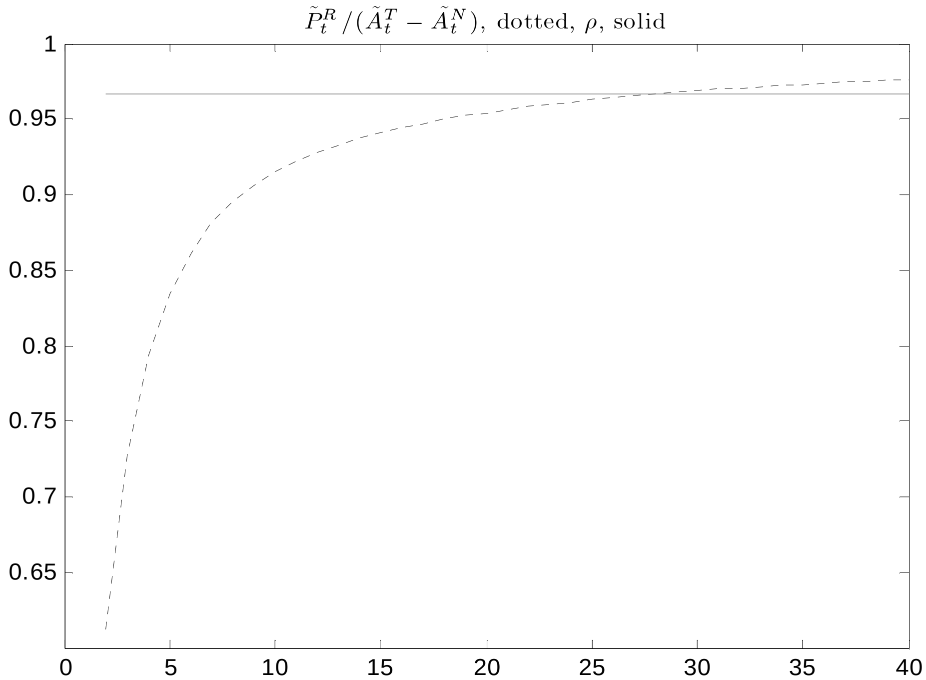


Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Growth rates are displayed in annualized terms.

Figure 9
Adjustment of the relative price of non-tradables to tradables P_t^R
Frictionless capital accumulation

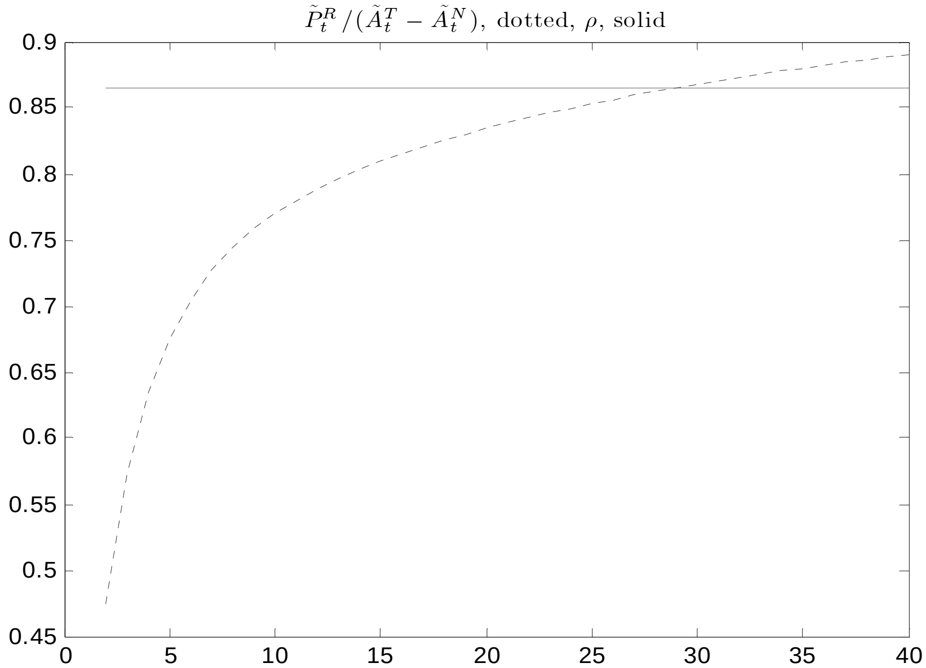


version B, benchmark economy, η*=15

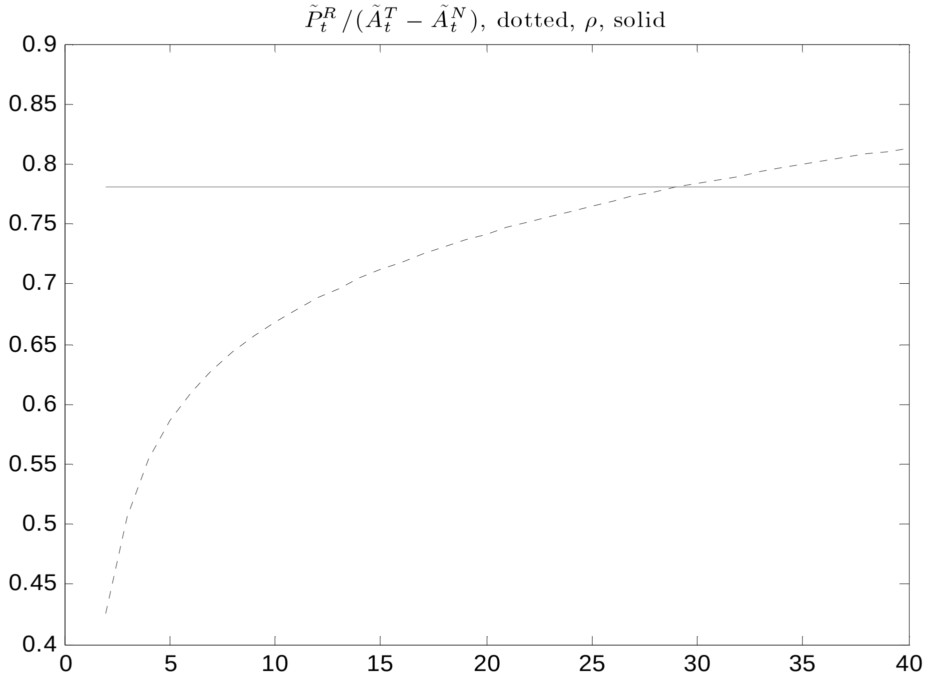


Units on a horizontal axis represent quarters, on a vertical axis percentage points.

Figure 10
Adjustment of the relative price of non-tradables to tradables P_t^R
version B, η*=15, η=15



version B, η*=15, η=15, high investments adustment costs in sector *T* and *x*



Units on a horizontal axis represent quarters, on a vertical axis percentage points.