Rare Events and the Equity Premium*

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Abstract

The allowance for low-probability disasters, suggested by Rietz (1988), explains a lot of asset-pricing puzzles, including the high equity premium, low risk-free rate, and the volatility of stock returns. Another mystery that may be resolved is why expected real interest rates were low in the United States during major wars, such as World War II. The rare-disasters framework achieves these explanations while maintaining the tractable framework of a representative agent, time-additive and iso-elastic preferences, and complete markets. The results hold with i.i.d. shocks to productivity growth in a Lucas-tree type economy and also when capital formation is considered.

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The Mehra-Prescott (1985) article on the equity risk-premium puzzle has received a great deal of attention. An article published three years later by Rietz (1988) purported to solve the puzzle by bringing in the potential for low-probability disasters. I think that Rietz's basic reasoning is correct, but the profession seems to think differently, as gauged by the continued attempts to find more and more complicated ways to resolve the equity-premium puzzle.

In this study, I extend Rietz's analysis and argue that it provides a plausible resolution of the equity-premium and related puzzles. Included in these other puzzles are the low risk-free rate and the volatility of stock returns. Another mystery that may be resolved is why expected real interest rates were low in the United States during major wars, such as World War II.

I. Representative-Agent Model of Asset Pricing

A. Setup of the model

Following Mehra and Prescott (1985), I use a version of Lucas's (1978) representative-agent, fruit-tree model of asset pricing with exogenous, stochastic production. Output of fruit in each period is A_t. In the initial version of the model, the number of trees is fixed, that is, there is neither investment nor depreciation. Since the economy is closed and all output is consumed, consumption, C_t, equals A_t.

One form of asset in period *t* is an equity claim on period *t+1*'s output, A_{t+1}. (This asset is a claim on one dividend, not the tree itself.) If the period *t* price of this asset in units of period *t*'s fruit is denoted by P_{t1}, the one-period gross return on equity is

(1) $$R^e_{t1} = A_{t+1}/P_{t1}.$$

I consider later claims in period *t* on output in periods *t+2, t+3*, and so on. I assume that property rights are secure, so that the claim ensures ownership over next period's fruit, A_{t+1}, with probability one.

There is also an asset, which I view as a short-term government bill, that guarantees a risk-free return in normal times but on which partial default occurs on rare occasions. I detail later the assumptions about default probability.

The representative consumer maximizes a time-additive utility function with iso-elastic utility:

(2) $$U_t = E_t \sum_{i=0}^{\infty} [e^{-\rho i} \cdot u(C_{t+i})],$$

where

(3) $$u(C) = (C^{1-\theta} - 1)/(1 - \theta).$$

In these expressions, $\rho \geq 0$ is the rate of time preference and $\theta > 0$ is the magnitude of the elasticity of marginal utility (and the coefficient of relative risk aversion). The intertemporal elasticity of substitution for consumption is $1/\theta$.

The usual first-order optimization condition implies

(4) $$u'(C_t) = e^{-\rho} \cdot E_t[u'(C_{t+1}) \cdot R_{t1}],$$

where R_{t1} is the one-period gross return on any asset traded at date *t*. Using Eq. (4), substituting C = A for periods *t* and *t+1*, and replacing R_{t1} by the formula for R_{t1}^e in Eq. (1) yields

(5) $$(A_t)^{-\theta} = e^{-\rho} \cdot (1/P_{t1}) \cdot E_t[(A_{t+1})^{1-\theta}].$$

Therefore, the price of the one-period equity claim is given by

(6) $$P_{t1} = e^{-\rho} \cdot (A_t)^{\theta} \cdot E_t[(A_{t+1})^{1-\theta}].$$

I assume that the log of output (productivity) evolves as a random walk with drift,

(7) $$\log(A_{t+1}) = \log(A_t) + \gamma + u_{t+1} + v_{t+1},$$

where $\gamma \geq 0$. The random term u_{t+1} is assumed to be i.i.d. normal with mean 0 and variance σ^2. This term will give results similar to those of Mehra and Prescott. I assume that γ and σ are known. Weitzman (2005) argues that learning about σ is important for asset pricing—this idea is not pursued here. However, Weitzman's learning model generates "fat tails" that have effects analogous to the low-probability disasters considered in my model.

The random term v_{t+1} picks up low-probability disasters, as in Rietz (1988). In these events, output contracts sharply, such as in the Great Depression I assume that the probability of a disaster is the known amount $p \geq 0$ per unit of time, where p is a constant and is therefore independent of u_t and u_{t+1}. The probability of more than one disaster in a period is assumed to be small enough to neglect—later, I allow the arbitrary period length to shrink to zero. If a disaster occurs, output contracts proportionately by the amount b, where $0 \leq b \leq 1$. The idea is that the probability of disaster in a period is small but b is large. The distribution of v_{t+1} is therefore

probability e^{-p}: $v_{t+1} = 0$,

probability $1- e^{-p}$: $v_{t+1} = \log(1-b)$.

This specification creates negative skewness in the distribution of A_{t+1}, because disasters are not offset in a probabilistic sense by bonanzas. However, the asset-pricing results are similar for a symmetric specification in which favorable events of size b also occur with probability p. With diminishing marginal utility of consumption, bonanzas do not count nearly as much as disasters for the pricing of assets.

B. Economic disasters in the United States and other countries

Actual and potential economic disasters could reflect economic events (the Great Depression, financial crises), wartime destruction (the world wars, a nuclear conflict), natural disasters (tsunamis, hurricanes, earthquakes, asteroid collisions), and epidemics of disease (SARS, avian flu, Black Death). I examine the 20th century history of actual economic disasters to determine reasonable settings for disaster probabilities, sizes of contractions, and default probabilities.

A prototype economic disaster is the Great Depression, which featured a large, global economic decline that did not trigger default on assets such as government bills.[1] However, from the standpoint of sizes of world economic disasters, war has been more important than purely economic contractions in the 20th century. For the United States, at least since 1815 and aside from the Confederacy during the Civil War, wars did not involve massive destruction of domestic production capacity. In fact, some of the wars, especially World War II, were times of robust economic activity. However, the history for many other OECD countries is very different, notably for World Wars I and II.

Part A of Table 1 shows all episodes of 15% or greater decline in real per capita GDP in the 20th century for 20 advanced countries covered over a long period by Maddison (2003).[2] This group comprises the major economies of Western Europe plus

[1] The rise in the gold price and abrogation of gold clauses in bond contracts may be viewed as forms of partial default—see McCulloch (1980).

[2] In the present model, which lacks investment, government purchases, and net exports, GDP and consumption coincide. More generally, it would be preferable to measure consumption rather than GDP in order to relate the data to the model. Unfortunately, for long-term analyses, data on GDP are much more plentiful than data on consumer expenditure.

Australia, Japan, New Zealand, and the United States—all members of the OECD since the 1960s.[3]

In the model, the occurrence of an economic disaster shows up as a downward jump in per capita GDP in an instant of time. In the data, the declines are stretched out over time, for example, over 1939-45 for some countries during World War II. In using the data to calibrate the model in a rough sense, there is no reason to focus on falls in per capita GDP that applied over a fixed interval of time, such as a calendar year (the period over which the long-term data happen to be computed). My approach in Table 1 is to measure declines in real per capita GDP that applied to consecutive years, such as 1939-45. My reasoning is that the start of a major war, such as World War II for Western European countries in 1939, puts a country into a regime where, with much higher probability than usual, output falls sharply over the next several years. The exact outcome depends on whether the country wins or loses, the extent of destruction of property and life, and so on. These features and the length of the war are unknown at the outset.

A reasonable way to model this kind of disaster is that, with probability p per unit of time, a country enters into a war that leads eventually to a contraction in per capita GDP by the proportionate amount b. The length of time that it takes to resolve the uncertainty about the extent of contraction—for example, one year or five—is, I think, secondary. That is why the cumulative decline in real per capita GDP during each war

[3] Kehoe and Prescott (2002) extend the concept of a depression to cases where the growth rate of real per capita GDP falls well below the historical average for an extended period. Thus, they classify as depressions the periods of slow economic growth in New Zealand and Switzerland from the 1970s to the 1990s. Hayashi and Prescott (2002) take a similar approach to Japan in the 1990s. These experiences can be brought into the present framework by allowing for a small probability of a substantial cutback in the productivity growth parameter, γ. However, the potential for this kind of change turns out not to "work," because—with the parameter θ in the reasonable range where $\theta > 1$—a decline in γ (applying to the whole world) turns out to *raise* the price-earnings ratio for stocks. Also, the equity premium is independent of γ.

should provide a reasonable estimate of b. The associated disaster probability corresponds to the number of wars (say, per century) that featured these sharp cumulative contractions, rather than the fraction of years in which a country was involved in this kind of war.

I take a similar approach to purely economic depressions, such as the Great Depression. These events often involve financial crises, which are similar in some respects to wars. However, I think that an appropriate procedure is less clear in these cases than for wars.

Nine of the contractions in part A of Table 1 were associated with World War II, eight with World War I, eight with the Great Depression, and one or two with the Spanish Civil War.[4] There are also four aftermaths of major wars—three following World War I and one after World War II. However, these experiences involved demobilizations with substantial declines in government purchases, work effort, and capital utilization and—with the possible exception of Canada after World War I—did not feature substantial decreases in consumer expenditure.[5] Therefore, except possibly for Canada in 1917-21, these cases are not applicable to my analysis. I exclude the four war aftermaths from my subsequent analysis.

Although 15% or greater declines in real per capita GDP are rare events, only 2 of the 20 OECD countries (Norway and Switzerland) lack any such events in the 20th century, and these came close (see the notes to Table 1). The striking observation from

[4] I am unsure whether the fall in per capita GDP in Portugal by 9% in 1935-36 reflected spillovers from the Spanish Civil War. Per capita GDP happened also to decline by 6% in Portugal in 1934-35.

[5] For the United States, data from Bureau of Economic Analysis show that real consumer expenditure did not decline from 1944 to 1947. The same holds for real consumer expenditure from 1918 to 1921 in the United Kingdom (see Feinstein [1972]) and Italy (see Rossi, Sorgato, and Toniolo [1993]). Long-term national-accounts data for Canada from Urquhart (1993) do not break down GDP into expenditure components. However, my estimate from Urquhart's data is that real consumer expenditure per person fell by about 18% from 1917 to 1921, compared to the decline by 30% in real GDP per person in Table 1.

part A of Table 1 is the dramatic decreases in real per capita GDP during the major wars and the Great Depression. The falls during World War II ranged between 45% and 64% for Italy, France, Japan, the Netherlands, Austria, Greece, and Germany. Moreover, the deviations from trend real per capita GDP (which would have risen over the several years of war) were even greater. In addition, the sharp expansions of government purchases during the wars suggest that consumption fell proportionately by even more than GDP (although investment likely declined sharply and net imports may have increased in some cases).

Part B of Table 1 shows declines of 15% or more in real per capita GDP for additional countries—eight in Latin America and seven in Asia—that have nearly continuous data from Maddison (2003) back at least before World War I. These data show ten sharp economic contractions in the post-World War II period (eight in Latin America), eight during the Great Depression, eight in World War II (six in Asia), and five around World War I.[6] Of the 15 countries considered, 3 lack 15% events (see the notes to the table).

Figure 1 summarizes the information from Table 1 in terms of the number of contractions of per capita GDP by at least 15% that occurred in the 35 countries during the 20th century. The histogram shows numbers of events for contraction intervals

[6] Data are available for a few additional countries starting in the 1920s and for many countries after World War II. In terms of 15% or greater events, this extension adds 6 cases associated with the Great Depression (Costa Rica, Cuba, El Salvador, Guatemala, Honduras, and Nicaragua), 4 during World War II (Costa Rica, Guatemala, Burma, and China), 1 aftermath of World War II (Paraguay), and 30 post-World War II depressions (about half war related) outside of sub-Saharan Africa. Among all of these additional cases, the largest contractions were 75% for Iraq (1987-91), 46% for Burma (1938-50), 45% for Iran (1976-81), and 44% for West Bank/Gaza (1999-2003). There were also 25 declines of 15% or more in real per capita GDP in the 1990s for transitions of former Communist countries. Stock-return data seem to be unavailable during any of the events mentioned in this footnote.

ranging from 15-19% to 60-64%.[7] There are 58 events for the 35 countries over 100 years (not counting the 4 war-aftermath experiences in Table 1a). Thus, overall, the probability of entering into a 15% or greater event was 1.7% per year. (Recall, however, that "events" applied to varying numbers of years.) For events of 30% or more contraction, the number was 23, whereas for 45% or more contraction, the number was 10.[8]

Table 2 reports realized real rates of return on stocks and government bills during the economic downturns enumerated in Table 1. Not many observations are available, partly because of the limited number of crises and partly because of missing financial data during the majority of these crises. Thus, out of the 58 cases in Table 1 (not counting the 4 war aftermaths), data are shown in Table 2 for only 13 cases. (One of these is for Thailand, not included in Table 1.)

For the Great Depression, I consider returns up to the full year before the rebound in the economy: Australia for 1929-30, France for 1929-31, Germany for 1929-31, and the United States for 1929-32. The averages of the arithmetic annual real rates of return for the four countries were -18.0% for stocks and 8.0% for bills. Thus, stocks did badly, and bills did well.

Consider now the post-World War II depressions shown in part B of Table 1 for countries with data on asset returns. For Argentina in 1998-2001, the average annual real

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[7] An allowance for trend growth would raise each of the proportionate contractions. For example, with a trend growth rate for per capita GDP of 2.5% per year, a 15% contraction in the level of per capita GDP over 5 years translates into a decrease relative to trend of 25%. A 45% contraction translates into a decline relative to trend of 51%.

[8] Since worldwide disturbances are the important force in the model, a different way to count is to consider only the three global shocks—World War I, the Great Depression, and World War II. The 20th century frequency of entering into this kind of global disturbance was 3% per year. The size of each global shock would be a weighted average of the declines in per capita GDP shown in Table 1 along with the smaller decreases for the roughly one-half of countries that did not experience 15% events at these times.

stock return was -3.6%, compared to 9.0% for bills.[9] For Indonesia in 1997-98 (during the Asian financial crisis), the respective returns were -44.5% and 9.6%. For the Philippines in 1982-84, the numbers were -24.3% and -5.0%. Given the scarcity of financial data during depressions, it seemed worthwhile to add the recent observation for Thailand (for which GDP data before 1950 are available only in scattered years). The contraction of real per capita GDP in 1996-98 was 14%, just short of the criterion used in Table 1. The average annual rates of return in 1996-97 were -48.9% for stocks and 6.0% for bills—similar to those for Indonesia in 1997-98. Thus, as in the Great Depression, stocks did badly, and bills did comparatively well.

For World War I, data on asset returns are available for only two of the countries with economic contractions in part A of Table 1. For 1914-18, the average annual real rate of return on stocks in France was -5.7%, while that on bills was -9.3%. For Germany, the values were -26.4% and -15.6%.[10] Thus, stocks and bills both performed badly in these countries that suffered economically from World War I. For bills, the reason was high inflation. There is no clear pattern of relative performance—stocks did better in France and worse in Germany.[11]

For World War II, data on asset returns are available for three countries with economic contractions: France, Italy, and Japan. The data are problematic for France, partly because the stock market was closed during parts of 1940 and 1941. The Italian

[9] Partial default on Argentine government bonds occurred later. For 1998-2002, the average real rates of return were -3.7% per year for stocks and -0.1% per year for bills.

[10] The impact of the German hyperinflation came later, 1920-23. For 1920-22, the average annual real rate of return on stocks was -50.7%, while that on bills was -56.2%. Thus, surprisingly, stocks did almost as badly in real terms as bills. The data for 1923, the peak year of the hyperinflation, are unreliable, though stocks clearly did far better in real terms than in 1922.

[11] This conclusion is the same for periods that correspond more closely to the years of economic downturn shown in Table 1, part A. For France from 1916 to 1918, the annual real rate of return on stocks was -0.3% while that on bills was -12.7%. For Germany from 1913 to 1915, the corresponding numbers were -16.6% and -3.5%.

data for the early part of the war also seem unreliable. I report information for 1943-45 in each case. All real rates of return were sharply negative—for bills, the reason again was high inflation. Stocks did worse than bills in France, better than bills in Italy, and about the same in Japan.

The overall conclusion is that government bills were clearly superior to stocks during purely economic crises, represented by the Great Depression and post-World War II depressions in Latin America and Asia. However, bills did not perform obviously better or worse than stocks during economic contractions related to major wars, notably World Wars I and II.[12]

C. Default probability

I argue that the disaster shock, v_t, can account for the behavior of stock returns during major economic contractions of the sort considered in Table 1. However, an explanation for the low returns on government bills during the wars requires another force, something akin to partial default or loss of property rights. Outright default on government paper does not typify the group of 20 advanced economies considered in part A of the table—which notably omits Czarist Russia and, from an earlier time, the American Confederacy. For example, France did not default after World War II on debts incurred by the Third Republic or the Vichy government. Similarly, Belgium and the Netherlands did not explicitly default after World War II on government bills and bonds but did have forced conversions into illiquid instruments. The most common mechanism for partial default was depreciation of the real value of nominal debt through

[12] Better performing assets in these circumstances would be precious commodities, such as gold and diamonds, and maybe Swiss bank accounts and human capital.

(unanticipated?) increases in price levels. These inflations occurred during and shortly after some of the wars.[13]

Default probability can be introduced into the model in a number of ways. I make the following assumptions. First, default never occurs in normal times but occurs with a constant probability $q \geq 0$ when a v-type disaster occurs. (Note that the probability q is a pure number, whereas the probability p is measured per unit of time.) Second, when a default occurs, the constant fraction d ($0 \leq d \leq 1$) of the gross return on bills is wiped out. Third, default never applies to equities.[14] Fourth, default does not affect productivity, A_{t+1}, and, hence, real GDP or the consumption of the representative agent. The proceeds from default on government bills are returned as lump-sum transfers to the representative consumer.

Given this specification for default probability, the gross return, R^b_{t1}, on government bills is as follows:

probability e^{-p}: R^b_{t1} = the specified face amount, R^f_{t1} ,

probability $1- e^{-p}$: probability 1-q: $R^b_{t1} = R^f_{t1}$,

probability q: $R^b_{t1} = (1\text{-}d)\cdot R^b_{t1}$.

[13]Notable are the hyperinflations in the early 1920s in Germany and Austria, likely due to Reparations payments imposed after World War I, rather than the war directly. High inflation also occurred during and after World War I in France and during or after World War II in Austria, Belgium, Finland, France, Greece, Italy, and Japan. In West Germany, suppressed inflation associated with World War II was effectively ratified by a 10:1 currency conversion and the lifting of price controls in 1948.

[14]In some circumstances, particularly involving war and changes of government, property rights on equity shares could be threatened. For example, during the German hyperinflation in 1922, the real value of stocks fell enormously. Bresciani-Turroni (1937, p. 265) observed that the capitalized value of Daimler Motors fell to the equivalent of 327 cars. Explanations that have been offered include potential future corporate liability for reparations payments and the possible loss of ownership rights due to the introduction of Communism.

The pricing formula from Eq. (4) yields an expression that involves the face return on bills, R^f_{t1}:

(8) $$(A_t)^{-\theta} = e^{-\rho}\cdot(R^f_{t1})\cdot E_t[(A_{t+1})^{-\theta}]\cdot[e^{-p} + (1 - e^{-p})\cdot(1 - qd)].$$

The default probability, q, and the size of default, d, enter only as the product qd because default does not affect the consumption of the representative agent.

D. Solution of the model

Given the probability distributions for u_{t+1} and v_{t+1}, Eqs. (6) and (8) determine the price of the (one-period) equity claim and the expected returns on equity and bills. The result for the equity claim follows from Eq. (6) as

(9) $$P_{t1} = A_t e^{-\rho-(\theta-1)\gamma+(1/2)(\theta-1)^2\sigma^2}\cdot[e^{-p} + (1 - e^{-p})\cdot(1-b)^{(1-\theta)}]\cdot$$

The expected gross return on equity is $E_t(R^e_{t1}) = E_t(A_{t+1})/ P_{t1}$. I focus on the expected rate of return on equity that applies asymptotically as the arbitrary period length approaches zero. The formula is

(10) $$\log[E_t(R^e_{t1})] = \rho+\theta\gamma-(1/2)\theta^2\sigma^2+\theta\sigma^2-p\cdot(1-b)\cdot[(1-b)^{-\theta}-1]\cdot$$

This expression is for a sample with the representative number of economic disasters. For a sample conditioned on no disasters, the expected rate of return on equity is

(11) $$\log[E_t(R^e_{t1})]_{\,|\,v_{t+1}=0} = \rho+\theta\gamma-(1/2)\theta^2\sigma^2+\theta\sigma^2-p\cdot[(1-b)^{(1-\theta)}-1]\cdot$$

The face rate of return on bills can be determined from Eq. (8). As the period length approaches zero, this rate of return approaches

(12) $$\log(R^f_{t1}) = \rho+\theta\gamma-(1/2)\theta^2\sigma^2-p\cdot[(1-qd)\cdot(1-b)^{-\theta}-1]\cdot$$

This expression gives the (risk-free) rate of return on bills conditioned on no default. The full expected rate of return, which includes the default possibility, is given by

(13) $$\log[E_t(R^b_{t1})] = \rho + \theta\gamma - (1/2)\theta^2\sigma^2 - p \cdot (1-qd) \cdot [(1-b)^{-\theta} - 1].$$

The $\rho + \theta\gamma$ part of the rates of return in Eqs. (10)-(13) corresponds to the usual formula for the steady-state real interest rate in the deterministic neoclassical growth model ($\sigma = p = 0$). The equity premium—the spread between the expected rates of return on equity and bills, $\log[E_t(R^e_{t1})] - \log[E_t(R^b_{t1})]$—is given from Eqs. (10) and (13) by

(14) $$\text{equity premium} = \theta\sigma^2 + p \cdot (b - qd) \cdot [(1-b)^{-\theta} - 1].$$

This formula applies to a sample that includes representative numbers of economic disasters and defaults on bills. Conditioned on no disasters (and, therefore, also no defaults), the premium can be calculated from Eqs. (11) and (12) as

(15) $$\text{equity premium}_{|\, v_{t+1}=0} = \theta\sigma^2 + p \cdot (b - qd) \cdot (1-b)^{-\theta}.$$

I assume that b > qd, which means that, on average, disasters are worse for equities than bills (as is true in Table 2). In this case, the equity premia in Eqs. (14) and (15) are increasing in p, b, and θ and decreasing in qd. These effects will be explored later.

Because the shocks u_t and v_t are i.i.d., the results take the same form for all future periods. The price of an equity claim that pays A_{t+2} in period *t+2* looks like Eq. (9), except that the expression in the first brackets is multiplied by 2 and the expression in the second brackets enters as a square. Expected rates of return and equity premia are still given by Eqs. (10)-(15).

If an equity share is defined to pay the full stream of "dividends," A_{t+1}, A_{t+2}, …, the price of this asset equals the sum of the prices of the claims on each period's output. Defining

(16) $$\Phi \equiv e^{-\rho-(\theta-1)\gamma+(1/2)(\theta-1)^2\sigma^2} \cdot [e^{-p} + (1-e^{-p}) \cdot (1-b)^{(1-\theta)}],$$

which is the expression that appears in Eq. (9), the result for the share price is

$$P_t = A_t \cdot \Phi/(1-\Phi),$$

so that the "price-earnings ratio," P_t/A_t, is

(17) $$P_t/A_t = \Phi/(1-\Phi).$$

As the period length approaches zero, the P-E ratio approaches[15]

(18) $$P_t / A_t = \frac{1}{\rho + (\theta-1)\gamma - (1/2)(\theta-1)^2\sigma^2 - p \cdot [(1-b)^{(1-\theta)} - 1]}.$$

The model also determines the expected growth rate of the economy. Using Eq. (7) and allowing the period length to approach zero, the expected growth rate approaches

(19) $$\log[(E_t A_{t+1})/A_t] = \gamma + (1/2)\sigma^2 - pb.$$

Thus, this expected growth rate is decreasing in p and b. This result applies if the sample includes the representative number of economic disasters. For a sample conditioned on no disasters, the expected growth rate is

(20) $$\log[(E_t A_{t+1})/A_t]_{|\, v_{t+1}=0} = \gamma + (1/2)\sigma^2,$$

[15] Since $\Phi>0$, the formula in Eq. (17) is valid if $\Phi<1$. This condition guarantees that expected utility is finite, as in the model of Kocherlakota (1990). When $\sigma = p = 0$, the inequality $\Phi<1$ reduces to the usual transversality condition for the deterministic neoclassical growth model, which is $\rho+\theta\gamma > \gamma$—the real interest rate exceeds the growth rate. The condition $\Phi<1$ in the stochastic context is analogous. It is equivalent to the condition that $\log[E_t(R^e_{t1})]$ exceed $\log(E_t A_{t+1}/A_t)$. From Eq. (18), the condition is $\rho + (\theta-1)\gamma - (1/2)(\theta-1)^2\sigma^2 - p\cdot[(1-b)^{(1-\theta)}-1] > 0$.

which is independent of p and b.

E. Calibration of disaster parameters

Economic disasters are characterized by four parameters: the probability of disaster, p (per year), the size of a contraction during a disaster, b, the probability of default (contingent on the occurrence of disaster), q, and the extent of default, d. I use the historical patterns from Tables 1 and 2 and Figure 1 to generate reasonable values for these parameters.

The frequency of disasters (58 occurrences for 35 countries over roughly 100 years) suggests a baseline value for p of 1.7% per year. The sizes of proportionate contractions, b, in per capita GDP shown in Table 1 and Figure 1 indicate a range of 15% to 64%. The model simplifies by treating the parameter b as a single number. The empirical counter-part of this b parameter is not the mean of the proportionate contractions shown in Figure 1. Because of diminishing marginal utility of consumption, the larger contractions count a lot more than the smaller ones. For example, in the formula for the equity premium in Eq. (14), this effect is given by the term $[(1-b)^{-\theta} - 1]$. With a coefficient of relative risk aversion of $\theta = 3$, this term equals 7 when b = 0.5 but equals only 0.6 when b = 0.15. To go further, we need to make some assumption about the default term, qd, which also enters into Eq. (14).

The patterns in Table 2 suggest that some form of partial default on government bills is highly probable during major economic contractions associated with war, notably World Wars I and II. However, default would seem to be less probable for wartime contractions in which the country was not heavily involved in the conflict (and did not suffer wartime destruction). Experiences of this type in Table 1—for which I lack data on asset returns—are Portugal during the Spanish Civil War and Latin America during World Wars I and II. Based on this reasoning, 24 of the 58 events (or 41%) shown in Table 1 featured partial default on bills. Thus, a reasonable number for q is around 0.4.

Table 2 shows that, during the wartime contractions, average real returns on bills were similar to those on stocks. To get this pattern in the model, the size of partial default, d, has to be close to the size of economic contraction, b. Therefore, I assume d = b in the calibration exercises.

The specification q = 0.4 and d = b means that the term qd in Eqs. (12)-(15) equals 0.4·b and is therefore proportional to b. That is, a larger economic contraction, b, is associated one-to-one with a larger expected default loss on bills, given by qd. The results would be the same if qd varied with b because of differences in q, rather than d. However, the results would be different if, for example, qd varied more than in proportion with b.

If we assume qd = 0.4·b, the term multiplying p in the formula for the equity premium in Eq. (14) becomes $0.6 \cdot b \cdot [(1-b)^{-\theta} - 1]$. Thus, the way to use the empirical frequency distribution from Figure 1 to average the b's is to weight the individual values by the term $[(1-b)^{-\theta} - 1]$. With $\theta = 3$, the resulting weighted average for the observed b's is 0.45. The formulas for the expected rates of return on equity and bills (Eqs. [10] and [13]) involve the same term, $[(1-b)^{-\theta} - 1]$, and therefore also generate a weighted average for b of 0.45. Based on this calculation, I focus in the simulations on two alternative values of b: 0.5 and 0.35. The first value may be a little on the high side, and the second value is probably on the low side.

II. Replication of Mehra and Prescott

Results that ignore disasters, so that p = 0, accord with Mehra and Prescott (1985) in failing to fit observed equity premia. Table 3 has data for the G7 countries on average real rates of return on stocks and bills, based on arithmetic annual rates of return. Part 1 has long samples back as far as 1880 for six countries (excluding Germany, which has missing data[16]), and part 2 is for the seven countries from 1954 to 2004. For the long samples, the average real rate of return on stocks for the six countries is 0.074, whereas, from 1954 to 2004, the average for the seven countries is 0.087. Average real bill returns are 0.000 for the long samples and 0.017 for the post-1954 period. Thus, the average equity premium is around 0.07 for both samples.

In the model, the remaining parameters to specify are γ, σ, ρ, and θ. The values of γ and σ determine the mean and standard deviation of the growth rate of output and consumption in no-disaster periods. For annual U.S. data from 1890 to 2004,[17] the growth rate of real consumer expenditure per person has a mean of 0.020 and a standard deviation of 0.035.[18] For real per capita GDP, the values are 0.021 and 0.045. For a more recent period, the means are similar but the standard deviations are much smaller. For example, from 1954 to 2004, the growth rate of per capita consumer expenditure has a mean of 0.024 and a standard deviation of 0.017, whereas the values for real per capita

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[16] See the notes to Table 3 for a discussion of Germany. Clearly, the data problems that result in the exclusion of German data for the long samples are not exogenous with respect to events such as the German hyperinflation and World War II. This exclusion biases upward the average real rates of return on bills and stocks and biases downward the standard deviations of these returns. For a general discussion of this kind of sample selection problem in the context of stock returns, see Jorion and Goetzmann (1999).
[17] National-accounts data since 1929 are from Bureau of Economic Analysis. Earlier data are from Kendrick (1961) and Romer (1987, 1988).
[18] Since it makes little quantitative difference, I calculate average growth rates of consumption and real GDP in the usual geometric-average manner, rather than as arithmetic averages.

GDP are 0.021 and 0.022. One possible problem (observed by Romer [1987, 1988]) is that the higher volatility in the period before World War I may reflect poorer data.

Table 4 shows statistics for the growth rate of real per capita GDP for the G7 countries for 1890-2004 and 1954-2004. Standard deviations for the post-1954 period are similar to that for the United States, ranging between 0.02 and 0.03. Values for the longer samples are much higher, reflecting the events shown in Table 1 and probably to a minor extent the lower quality of the earlier data. Mean growth rates of per capita GDP in the post-1954 period are 0.02-0.03, except for Japan, which has 0.04. For the period since 1890, the mean growth rates range from 0.015 for the United Kingdom to 0.027 for Japan.

Table 4 also shows the sample kurtosis for growth rates in each country and period. For the 1954-2004 samples, the numbers are close to three, the value for a normal density. Standard tests, such as the Anderson-Darling test, accept the hypothesis of normality with p-values above 0.05. Hence, for these tranquil periods—in which crises of the sort shown in Table 1 did not occur—the growth-rate data seem reasonably described as normal. The situation is very different for the 1890-2004 samples, where the sample kurtosis always exceeds five and reaches astronomical levels for Germany and Japan.[19] These high values—indications of fat tails—reflect the kinds of disasters shown in Table 1, especially during World War II. Standard tests, including Anderson-Darling, reject normality at low p-values. That is, normality does not accord with samples such as

[19] In the model, the kurtosis can be expressed as a function of p, b, and σ. The kurtosis equals 3 when p=0 but is very sensitive to p. For example, with b = 0.5, the kurtosis peaks at 156 when p = 0.0016, then falls to 74 when p = 0.01 and 43 when p = 0.02.

1890-2004 that include occurrences of disasters.[20] The reasoning in this paper is that the potential for these disasters also affects asset pricing in tranquil periods, such as 1954-2004, where disasters happened not to materialize.

Based on the information in Table 4 for the post-1954 periods, I calibrate the baseline specification with $\gamma = 0.025$ and $\sigma = 0.02$. The equity premium in the model, given by Eq. (14), does not depend on γ.

The rate of time preference, ρ, does not affect the equity premium (Eq. [14]) but does affect the levels of the rates of returns on equity and bills and the price-earnings ratio. A typical assumed value for ρ is 0.02 per year. However, this value seems to derive more from observed real interest rates than from direct observations about time preference. Therefore, it seems reasonable to pick a value for ρ that allows the model to accord with observed levels of rates of return, particularly with average bill rates. I use $\rho = 0.03$ per year in the baseline specification.

The elasticity of marginal utility or coefficient of relative risk aversion, θ, matters more for the results. From the perspective of risk aversion, the usual view in the finance literature is that θ should lie in a range of something like 2 to 4. From the standpoint of intertemporal substitution of consumption, Barro and Sala-i-Martin (2004, Ch. 2) argue that a similar range for θ is needed to accord with country-level observations on levels and transitional behavior of saving rates.[21] If θ is much below 2, saving rates fall

[20] With respect to skewness, the data are consistent with the specification that emphasizes disasters over bonanzas. Negative skewness applies to 6 of the 7 countries for the 1890-2004 samples. The skewness is particularly large in magnitude for Germany and Japan (-5.2 and -5.6, respectively) and is positive only for France (0.5, because of a dramatic rise in per capita GDP in 1946). In contrast, for the 1954-2004 samples, skewness is negative for four countries and positive for three countries.

[21] In the present specification, the coefficient of relative risk aversion, θ, equals the reciprocal of the intertemporal elasticity of substitution for consumption. My view is that this restriction may, in fact, be satisfactory for investigating asset pricing and economic growth. Kocherlakota (1990) explores an asset-pricing model in which the two forces can be distinguished.

markedly as a country develops. If θ is much above 3, average levels of saving rates are counterfactually low. I use $\theta = 3$ in the initial specification.

Equation (14) shows that the equity premium in the model equals $\theta\sigma^2$ when $p = 0$. Therefore, when $\theta = 3$ and $\sigma = 0.02$, the premium is around 0.001, compared to the observed value of around 0.07. This spectacular failure, the central point of Mehra and Prescott (1985), has been often remarked on in the asset-pricing literature. For discussions, see Campbell (2000), Mehra and Prescott (2003), and Weitzman (2005). The main source of difficulty is that the real bill rate predicted by the model is way too high. Using the baseline parameters in Eq. (12) leads to a real bill rate of 0.093, much higher than the observed values of around 0.01 in Table 4.

The model cannot be fixed by reasonable modifications of the parameters θ and σ. To get an equity premium of 0.07 when $\theta = 3$, σ would have to be 0.15, way above observed standard deviations for annual growth rates of real per capita GDP and consumption. Alternatively, if $\sigma = 0.02$, θ would have to be 175 to get a spread of 0.07. However, a large θ does not accord with observed real rates of return on bills. If $\theta\sigma^2 = 0.07$, to get a real bill rate around 0.01 in Eq. (12) when $p = 0$, θ has to be close to 1. Then, to get the right equity premium, σ has to be around 0.26, even further above observed values.

III. Leverage

Mehra and Prescott (1985) considered how their results change when equity shares represent a claim on only a part of real GDP. Three ways to modify the model in this respect are to allow for labor income and taxes and to have a financial structure that includes fixed claims (bonds) as well as equities. Extensions to include labor income (or other forms of income not owned through corporate equity) and taxes tend not to affect the main results within the framework of a representative agent (see Barro [2005]). However, a modification to include leverage is more important.

Suppose that tree owners have a capital structure that involves partly equity and partly fixed claims in the form of one-period private bonds. I assume that these bonds are identical to government bills—they promise the gross face return R_{t1}^{f} but partial default in the proportion d occurs with probability q when a v-type crisis occurs. This specification seems reasonable if private bonds are nominally denominated, like most government bills, and if partial default occurs through unanticipated inflation. I assume that default on private bonds does not occur in other circumstances. In particular, I make assumptions that rule out bankruptcy.

I use the following setup to assess leverage. At the start of period t, the owner of a tree issues a one-period equity share that sells at the (equilibrium) price P_{t1}. Then the owner issues β units of one-period private bonds and gives the proceeds to the equity holder. Thus, the net price paid for levered equity is $P_{t1} - \beta$. In period t+1, the tree delivers output of A_{t+1}. The contractual payment to bondholders at t+1 is βR_{t1}^{f}. However, the proportion d of this gross payment is withheld with probability q if a v-type disaster occurs. I make an assumption so that, as the length of the period shrinks to zero, the output is sufficient to cover the debt payments with probability 1.

In accordance with Modigliani-Miller, leverage does not affect the overall market value of claims on the next period's output. This value is still given by the expression for P_{t1} in Eq. (9). However, the coefficient β affects the expected rate of return on levered equity, which is valued at P_{t1} - β. A formula for the expected rate of return can be obtained from an application of the pricing formula in Eq. (4).

The debt-equity ratio, λ, is given as the length of the period approaches zero by

(21) $$\lambda = \beta/(A_t - \beta).$$

Using this expression, the formula for the expected rate of return on levered equity turns out to be

(22) $$\log[E_t(R_{t1}^e)] = \rho + \theta\gamma - (1/2)\theta^2\sigma^2 + (1+\lambda)\cdot\left\{\theta\sigma^2 - p\cdot[1-b-(\frac{\lambda}{1+\lambda})\cdot(1-qd)]\cdot[(1-b)^{-\theta}-1]\right\}.$$

This result is a generalization of Eq. (10) to allow for leverage (λ > 0). I assume the condition 1 - b > λ/(1+λ), which ensures that output is sufficient to cover the contracted interest payments with probability one. This condition implies that the next-to-last bracketed term on the right-hand side of Eq. (22) is positive.

The expected rate of return on bills, log[$E_t(R_{t1}^b)$, is still given by Eq. (13). Therefore, the levered equity premium is given from the difference between Eqs. (22) and (13) as

(23) $$\textit{equity premium} = (1+\lambda)\cdot\{\theta\sigma^2 + p\cdot(b-qd)\cdot[(1-b)^{-\theta}-1]\}.$$

Thus, the levered equity premium is the multiple 1+λ of the unlevered premium given in Eq. (14).

According to the Federal Reserve's Flow-of-Funds Accounts, recent debt-equity ratios for the U.S. non-financial corporate sector are around 0.5. If λ is around 0.5, the presence of leverage does not affect the main conclusions in the model without rare disasters about the deviation between actual and predicted equity premia. The reason, from Eq. (23), is that the levered spread is the multiple 1+λ of the no-leverage spread. If the no-leverage spread is trivial—around 0.001—multiplying this number by 1.5 (corresponding to $\lambda = 0.5$) still generates a trivial number. However, in the model with rare disasters, λ does affect the quantitative results.

IV. Rare Disasters and Rates of Return

Inclusion of the Rietz (1988)-type disaster probability, p, gets the model into the right ballpark for explaining the high equity premium and low real rate of return on bills. Results from calibrations of the model are in Table 5. Based on the earlier discussion of disaster events in Table 1 and Figure 1, I start with a specification of $p = 0.017$ and $b = 0.5$. As noted before, I assume in the baseline specification that the default probability for bills, contingent on disaster, is $q = 0.4$. As also discussed before, I assume as a baseline that the proportion of default on bills is $d = b$, which equals 0.5 in the initial setting.

The potential for disasters affects the variance of the growth rate, A_{t+1}/A_t. As the length of period approaches zero, this variance can be determined from Eq. (7) as

(24) $$VAR(A_{t+1}/A_t) = \sigma^2 + pb^2.$$

Assuming again that $\sigma = 0.02$, $p = 0.017$, and $b = 0.5$, the standard deviation of the growth rate is 0.068. This value accords with the average of 0.061 for the standard deviation of the growth rates of real per capita GDP for the G7 countries from 1890 to 2004 (Table 4). These long samples can be viewed as containing the representative number of v-type disasters. In contrast, the tranquil period from 1954 to 2004, also shown in Table 4, has an average standard deviation for the growth rate of real per capita GDP in the G7 countries of only 0.023. This value can be thought of as the standard deviation when the samples are conditioned on observing no disasters. Hence, this standard deviation corresponds to σ, which is still set at 0.02.

The baseline specification in Table 5, column 1 shows that the allowance for rare disasters generates more reasonable equity premia and real bill rates. One consequence of raising p from 0 to 0.017 is that the expected real bill rate falls dramatically—from 0.103 (risk-free) to 0.008. The inverse relation between p and $\log[E_t(R^b_{t1})]$ applies generally in Eq. (13). With the baseline parameters for (θ, b, q, and d), the coefficient on p in Eq. (13) is -5.6.

Less intuitively, a rise in p also lowers the expected rate of return on equity, $\log[E_t(R^e_{t1})]$, given in Eq. (10) for a sample that includes the representative number of v-type disasters. If $\theta > 1$, this change reflects partly an *increase* in the price-earnings ratio in Eq. (18)—that is, the P-E ratio of 35.5 in Table 5, column 1 exceeds the value 12.6 that applies when p = 0. Intuitively, a rise in p motivates a shift toward the risk-free asset and away from the risky one—this force would lower the equity price. However, households are also motivated to hold more assets overall because of greater uncertainty about the future. If $\theta > 1$, this second force dominates, leading to a net increase in the equity price. Even if $\theta < 1$, the negative effect of p on $\log[E_t(R^e_{t1})]$ applies in Eq. (10). The reason is that a rise in p also lowers the expected growth rate of dividends, $E_t(A_{t+1}/A_t)$, and this force makes the overall effect negative as long as $\theta > 0$. In any event, the expected rate of return on equity falls by more than the expected bill rate, so that the spread increases. This property can be seen in Eq. (14).

With no leverage, the equity premium in the baseline specification in Table 5, column 1 is 0.037. With a debt-equity ratio of $\lambda = 0.5$, the premium becomes 0.055.

Thus, with the baseline parameters, the model's equity premium gets into the vicinity of the empirical observations in Table 3. In fact, if the coefficient of relative risk aversion were a little higher—$\theta = 3.3$—the model's equity premium would correspond to the empirical value of around 0.07.

The results are sensitive to the value of the disaster probability, p. In fact, since the term $\theta\sigma^2$ is small, the equity premium is nearly proportional to p in Eq. (14) and, for the levered case, in Eq. (23). With the baseline parameters for (θ, b, q, and d) and a debt-equity ratio of $\lambda = 0.5$, the coefficient on p in Eq. (23) is 3.2. Thus, Table 5, column 2 shows that if p falls by 0.005 to 0.012, the levered equity premium declines by 0.015 to 0.040.

The results depend a lot on how bad a disaster is, as gauged by the parameter b. This sensitivity can be seen in the formula for the levered equity premium in Eq. (23). For example, in Table 5, column 3, where b = 0.35, the expected bill rate of 0.065 is much higher than the baseline value of 0.008.[22] In this case, the levered equity premium is only 0.016. Another way to express this result is that, if the parameters for p and q are fixed at their baseline values, a value b = 0.35 requires a coefficient of relative risk aversion, θ, well above the baseline value of 3 to generate a levered equity premium in the neighborhood of the empirical number, 0.07. Given the specification for the other parameters in column 3, θ would have to equal 6.0 to generate a levered equity premium of 0.07.

The results are sensitive to the coefficient of relative risk aversion, θ. Again, the effects on the equity premium can be seen for the levered case in Eq. (23). If $\theta = 2$, as in column 4 of Table 5, the expected bill rate of 0.038 is well above the baseline value of

[22] I continue to assume d = b, so that d = 0.35 also applies in column 3.

0.008, and the levered equity premium is only 0.024. Another way to express this result is that, if the parameters for b and q are fixed at their baseline values, a value $\theta = 2$ requires a disaster probability, p, well above the baseline value of 0.017 to generate a levered equity premium of 0.07. The required value of p in this case is 0.051.

Figure 2 summarizes the results by using Eq. (23) to show the combinations of the three key parameters—p, b, and θ—needed to generate a levered equity premium of 0.07. (These calculations assume that d = b always holds.) The lowest graph applies when a disaster reduces output by b = 0.5, the middle graph when a disaster reduces output by b = 0.35, and the upper graph when a disaster reduces output by b = 0.25. Recall that the empirical frequency distribution in Figure 1 suggested a weighted-average value for b of around 0.45. Thus, b = 0.5 might be a little high, b = 0.35 is probably somewhat low, and b = 0.25 is very low relative to the disaster experience.

One way to use these results is to ask what value of θ is consistent with a levered equity premium of 0.07 when the disaster probability is p = 0.017. When the disaster event is 50%, the required value of θ is 3.3; when the disaster event is 35%, the required value is 6.0; and when the disaster event is 25%, the required value is 10.0. Alternatively, if θ is fixed at 3, we can ask what value of p is required to generate a levered equity premium of 0.07. These values go from 0.022 to 0.082 to 0.22 as the size of the disaster event goes from 50% to 35% to 25%. Thus, a value b = 0.25 would not work, but a value close to 0.5 delivers reasonable results.

Campbell (2000) and Weitzman (2005) observe that Rietz's low-probability disasters create a "peso problem" when disasters are not observed within sample. Indeed, data availability tends to select no-disaster samples, as observed by Jorion and

Goetzmann (1999). However, this consideration turns out not to be quantitatively so important in the model.

The results in Table 5 show the consequences of conditioning on samples that exclude disasters. In the baseline specification in column 1, the levered expected equity rate of 0.063 is less by 0.011 than the rate of 0.074 for a sample conditioned on no disasters. The expected bill rate of 0.008 is less by 0.003 than the face bill rate of 0.011. The equity premium of 0.055 is less by 0.008 than the value of 0.063 for a no-disaster sample. Finally, the expected economic growth rate of 0.017 (computed from Eq. [19]) is less by 0.008 than the rate of 0.025 for a no-disaster sample (computed from Eq. [20]). The point is that the selection of no-disaster samples—a selection that tends to be driven by data availability—has only moderate effects on long-run averages of returns on equity and bills, the equity premium, and economic growth rates. Nevertheless, the potential for these disasters has major effects on rates of return and the equity premium.

V. Disaster Probability and the Bill Rate

The results in Table 5 apply when p and q and the other model parameters are fixed permanently at designated values; for example, $p = 0.017$ per year and $q = 0.4$. However, the results also show the effects from permanent changes in any of the parameters, such as the probabilities p and q. I now use the model to assess the effects from changes in p and q. However, in a full analysis, stochastic variations in p and q—possibly persisting movements around stationary means—would be part of the model.

A fall in p raises the expected bill rate in Eq. (13) and the face bill rate in Eq. (12). The result in Table 5, column 2 shows considerable sensitivity: the expected bill rate rises from 0.008 to 0.036 when p falls (permanently) from 0.017 to 0.012. Mehra and Prescott (1988, p. 135) criticized the analogous prediction from Rietz's (1988) analysis:[23]

> "Perhaps the implication of the Rietz theory that the real interest rate and the probability of the extreme event move inversely would be useful in rationalizing movements in the real interest rate during the last 100 years. For example, the perceived probability of a recurrence of a depression was probably high just after World War II and then declined. If real interest rates rose significantly as the war years receded, that would support the Rietz hypothesis. But they did not. … Similarly, if the low-probability event precipitating the large decline in consumption were a nuclear war, the perceived probability of such an event surely has varied in the last 100 years. It must have been low before 1945, the first and only year the atom bomb was used. And it must have been higher before the Cuban Missile Crisis than after it. If real interest rates moved as predicted, that would support Rietz's disaster scenario. But again, they did not."

The general point about the probability of depression makes sense, but I am skeptical that this probability was high at the end of World War II. I consider later the likely movements in disaster probability, p, associated with the Great Depression itself. The observations about the probability of nuclear war confuse, using my terminology, the disaster probability, p, and the (conditional) expected default loss on bills, given by qd. A heightened chance of nuclear war likely raises p and qd—because defaults on bills would be highly probable and large in the wake of a nuclear conflict.[24] Although an increase in p lowers the rate of return on bills (Eqs. [12] and [13]), a rise in qd has the opposite effect. For the expected rate of return on bills in Eq. (13), the net effect depends on the term p·(1-qd). For a particular case—such as depression or war or an event such as the Cuban missile crisis—the key issue is whether the changes in p and qd lead to an increase or decrease in p·(1-qd).

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[23] Mehra and Prescott (2003, p. 920) essentially repeat this criticism.

[24] The probability of loss of property rights on equity claims—assumed to be zero in my analysis—would also become significant, especially if nuclear war led to the end of the world!

Changing probabilities of depression would likely isolate the effect of changing p, because defaults on bills are atypical in these situations. However, the analysis depends on identifying the variations in depression probability that occurred over time or across countries. From a U.S. perspective, the onset of the Great Depression in the early 1930s likely raised p (for the future). The recovery from 1934 to 1937 probably reduced p, but the recurrence of sharp economic contraction in 1937-38 likely increased p again. Less clear is whether the end of World War II had an effect on perceived future probability of depression.

Changing probabilities of nuclear war are unlikely to work—they would involve a mixture of increases in p with substantial increases in qd, and the net impact on bill rates is ambiguous.[25] The events shown in Table 1 suggest consideration of changing probability of the types of wars seen in history—notably World Wars I and II, which were massive but not the end of the world (for most people). My assumption is that the occurrence of this type of major war raised p and qd, that is, increased the perceived likelihood of future disasters and of future default losses on bills. However, starting from the baseline parameters ($p = 0.017$ per year, $q = 0.4$, $d = 0.5$), the effect from higher p leads to a net increase in $p\cdot(1-qd)$ if the inequality holds:

$$\Delta p > 0.021\cdot\Delta(qd). \tag{25}$$

For example, if a war leads to $\Delta p = 0.005$, this inequality would be violated only if qd more than doubled. I assume that the inequality in (25) holds during the U.S. wars that I

[25] I considered using the famous "doomsday clock," discussed by Slemrod (1986), to assess empirically the changing probability of nuclear war. The clock is available online from the *Bulletin of the Atomic Scientists*. I decided not to use these "data" because the settings are heavily influenced by an ideology that always identifies toughness with higher probability of nuclear war and disarmament with lower probability. For example, the clock was nearly at its worst point—three minutes to midnight—in 1984 shortly after President Reagan began his successful confrontation of the "evil empire" of the Soviet Union.

consider back to the Civil War. In that case, the model predicts that the expected bill rate in Eq. (13) and the face bill rate in Eq. (12) would each decline.

Figure 3 shows an estimated time series since 1859 of the expected real interest rate on U.S. Treasury Bills or analogous short-term paper.[26] The source of data on nominal returns is Global Financial Data, the same as in Table 3. Before the introduction of T-Bills in 1922, the data refer to high-grade commercial paper.

To compute the expected real interest rate, I subtracted an estimate of the expected inflation rate for the CPI. Since 1947, my measure of expected inflation is based on the Livingston Survey. From 1859 to 1946, I measured the "expected inflation rate" as the fitted value from an auto-regression of annual CPI inflation on a single lag.[27] Additional lags lack explanatory power, although there may be a long-run tendency over this period for the price level to adjust toward a stationary target.

One striking observation from Figure 3 is that the expected real interest rate tended to be low during wars—especially the Civil War, World War I, and World War II. The main exception is the Vietnam War.[28] Table 6 shows the nominal interest rate, expected inflation rate, and expected real interest rate during each war and the Great Depression. The typical wartime pattern—applicable to the Civil War, World Wars I and II, and the first part of the Korean War—is that the nominal interest rate changed little, while actual and expected inflation rates increased. Therefore, expected real interest rates declined, often becoming negative. Moreover, the price controls imposed

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[26] It would be preferable to look at returns on indexed bonds, but these instruments exist in the United States only since 1997.

[27] The inflation rate is the January-to-January value from 1913 to 1946. Before 1913, the CPI data are something like annual averages. The estimated lag coefficient is 0.62 (s.e. = 0.09). The R^2 for this regression is 0.35. In this context, I measured the inflation rate as the usual geometric value, $\log(P_{t+1}/P_t)$.

[28] Taxation of nominal interest along with an increase in the expected inflation rate may explain the Vietnam pattern. That is, expected after-tax real interest rates were low.

during World War II and the Korean War likely led to an understatement of inflation; therefore, the expected real interest rate probably declined even more than shown for these cases.

Figure 3 and Table 6 show that expected real interest rates fell in 2001-03 during the most recent war—a combination of the September 11th attacks and the conflicts in Afghanistan and Iraq. This period also has data on real yields on U.S. Treasury indexed bonds, first issued in 1997. The 10-year real rate fell from an average of 3.8% for 1/97-8/01 to 2.2% for 10/01-8/05.[29] Similarly, the 5-year real rate declined from an average of 3.2% for 12/00-8/01 to 1.6% for 10/01-8/05. These real rate reductions on indexed bonds accord with those shown for the short-term "expected real rate" in Table 6.[30]

The tendency for expected real interest rates to be low during U.S. wars has been a mystery, described in Barro (1997, Ch. 12).[31] Most macroeconomic models predict that a massive, temporary expansion of government purchases would raise expected real interest rates. In previous work, I conjectured that military conscription and mandated production might explain part of the puzzle for some of the wars. Mulligan (1997) attempted to explain the puzzle for World War II by invoking a large increase of labor supply due to patriotism. A complementary idea is that patriotism and rationing motivated declines in consumption and increases in saving, perhaps concentrated on war

[29] The indexed bonds data show that risk-free real interest rates are not close to constant. For 10-year U.S. indexed bonds, the mean for 1/97-8/05 was 3.0%, with a standard deviation of 0.9% and a range from 1.5% to 4.3%. For the United Kingdom from 2/83-8/05, the mean real rate on ten-year indexed bonds was 3.2%, with a standard deviation of 0.8% and a range from 1.5% to 4.6%.

[30] The real rate on 10-year indexed bonds peaked at 4.2% in May 2000 then fell to 3.3% in August 2001—perhaps because of the end of the Internet boom in the stock market but obviously not because of September 11 or the Afghanistan-Iraq wars. However, the rate then fell to 3.0% in October 2001 and, subsequently, to 1.8% in February 2003. The lowest level was 1.5% in March 2004.

[31] Barro (1987) finds that interest rates were high during U.K. wars from 1701 to 1918. However, this evidence pertains to nominal, long-term yields on consols. Short-term interest rates are unavailable for the United Kingdom over the long history. Realized short-term real interest rates in the United Kingdom were very low during World Wars I and II.

bonds. However, the low real interest rate in wartime seems to be too pervasive a phenomenon to be explained by these kinds of special factors. The rare-disasters framework offers a more promising explanation: expected real interest rates tend to fall in wartime because of increases in the perceived probability, p, of (future) economic disasters.

Table 6 also shows the behavior of the expected real interest rate in the United States during the Great Depression. According to the theory, the expected real rate should have declined if the probability, p, of disaster increased. Matching this prediction to the data is difficult because of uncertainty about how to gauge expected inflation during a time of substantial deflation.

The nominal return on Treasury Bills fell from over 4% in 1929 to 2% in 1930, 1% in 1931, and less than 1% from 1932 on. However, the inflation rate became substantially negative (-2% in 1930, -9% in 1931, -11% in 1932, -5% in 1933), and the constructed expected inflation rate also became negative: -4% in 1931 and -6% in 1932 and 1933. Therefore, the measured expected real interest rate was high for 1931-33. However, this construction is likely to be erroneous because the deflation in 1931-33 depended on a series of monetary/financial shocks, each of which was unpredictable from year to year. Hence, rational agents likely did not anticipate much of this deflation. This interpretation is supported by Hamilton's (1992) observation that futures prices on several commodities were typically well above spot prices during the early 1930s. Hence, I think that expected real interest rates were much lower in the early 1930s than the values reported in the table. From 1934 on, the inflation rate became positive. The combination of positive expected inflation with nominal interest rates close to zero generated low expected real interest rates for 1934-38. This period includes the sharp recession—and possible fears of a return to depression—in 1937-38.

VI. Local versus Global Shocks

Equation (18) shows that an increase in the disaster probability, p, raises the price-earnings ratio if $\theta > 1$. This relation applies when v_t is a global shock, for example, a world war or a global depression. It would also apply to a shock for a single country if the country were isolated from the rest of the world. However, the results are different for a local disaster when the locality is integrated into world asset markets. (Similarly, within a country, the probability of disaster for an individual company can be distinguished from an economy-wide shock.)

Suppose that, in addition to the world shock v_t, an idiosyncratic disaster shock v_{ti} applies for each economy i. With probability p_i per unit of time, output in country i declines by the proportion b_i. Assume, further, that each economy is too small to have a significant effect on the consumption, C_t, of the world's representative agent. In this case, the expected rate of return on equity claims for trees in country i is still given by Eqs. (10) and (11). That is, the individual country risk, represented by p_i, would be diversified away. Similarly, the (world) expected bill rate, still given by Eq. (13), does not depend on p_i.

The formula for the price-earnings ratio differs from Eq. (18) by the addition of a new term in the denominator, $p_i b_i$. Hence, an increase in p_i *lowers* the P-E ratio for equity claims in country i. That is, whereas a higher probability, p, of global disaster raises the P-E ratio (if $\theta > 1$), a higher probability, p_i, of local disaster lowers the ratio. At least the last result holds if the locality is fully integrated into global asset markets.

VII. Volatility of Stock Returns

The variance of the growth rate of A_t is given in Eq. (24). In the baseline model in Table 5, the price-earnings ratio is constant. Therefore, the standard deviation of stock returns equals the standard deviation of the growth rate of A_t, which equals 0.068 for the baseline parameters in column 1. This value applies to a sample with the representative number of disasters, such as the long samples displayed in the upper part of Table 4. However, the average standard deviation of stock returns over these periods was 0.23, way above the value predicted by the model. Similarly, the tranquil periods since 1954 displayed in the lower part of Table 4 should correspond to the model conditioned on the realization of no disasters. In this case, the model standard deviation of stock returns is 0.02 (the value for σ in the baseline specification), whereas the average standard deviation was again 0.23. These discrepancies correspond to the well-known excess-volatility puzzle for stock returns.

A natural way to resolve this puzzle is to allow for variation in underlying parameters of the model, notably the probability p of v-type disaster. Table 5 shows that the price-earnings ratio is highly sensitive to changes in p. For example, comparing columns 1 and 2, a decrease in p from 0.017 to 0.012 lowers the price-earnings ratio from 35.5 to 23.1. As already noted, this change in p refers to a once-and-for-all, permanent shift in the disaster probability. However, an extension of the model to allow for stochastic, persisting variations in p_t would likely account for the observed volatility of stock returns.

To get an idea of the magnitudes involved, the annual standard deviation of the residuals from an AR(1) process for the log of the U.S. P-E ratio is around 0.2 for December values from 1880 to 2004 or 1954 to 2004. Using the baseline parameters ($b = 0.5$ and $\theta = 3$) in the denominator of the formula for the P-E ratio in Eq. (18), the coefficient on p equals -3. This coefficient implies that the effect of a change in p on the proportionate change in the P-E ratio is 106 in the neighborhood of the baseline values, including $p = 0.017$. Therefore, to generate an annual standard deviation for the log of the P-E ratio of 0.2, the annual standard deviation of p has to be around 0.002.

Variations in p also induce changes in the expected rate of return on bills. In Eq. (13), the coefficient on p with the baseline parameters equals -5.6. Therefore, an annual standard deviation for p of 0.002 generates a standard deviation of the expected bill rate of 0.011. The annual standard deviation of the residuals from an AR(1) process for December values of realized real rates of return on U.S. Treasury Bills or short-term commercial paper is 0.018 from 1880 to 2004 and 0.016 from 1954 to 2004. These values are in the ballpark of the predicted standard deviation, 0.011. Thus, it seems likely that an extension of the model to allow for variations in p would accord with observed variations in P-E ratios and real interest rates.

Another way to look at the results is in terms of the Sharpe ratio for equity, discussed, for example, in Campbell (2000). The Sharpe ratio is the risk premium on equity divided by the standard deviation of the excess return on equity. In the data for the long samples shown in Table 3, the Sharpe ratio is around 0.3. In the model, using the baseline parameters (Table 5, column 1), the ratio is 0.055/XXX = XXX. Thus, the risk premium is "too high" relative to the volatility of returns. My conjecture is that the introduction of variations in p_t would generate a closer match between observed and theoretical Sharpe ratios. That is, the standard deviation of stock returns would increase, but the risk premium may not change greatly. However, variability in disaster probabilities might also affect the risk premium. A key issue is how the variations in probabilities co-vary with consumption: do increases in p_t tend to occur when the economy is doing badly or, instead, in a manner roughly orthogonal to current GDP and consumption?

VIII. Capital Formation

The model neglected investment, that is, changes in the quantity of capital in the form of trees. To put it another way, growth and fluctuations resulted from variations in the productivity of capital, A_t, with the quantity of capital, K, assumed fixed. To assess the implications of capital formation, it is convenient to consider the opposite setting, that is, a fixed productivity of capital, A, with the quantity of capital, K_t, allowed to vary.

The production function takes the "AK" form:

(26) $$Y_t = AK_t,$$

where Y_t is the output of fruit, K_t is the quantity of trees, and $A > 0$ is constant.[32] Output can be consumed (as fruit) or invested (as seed). The process of creating new trees through planting seeds is assumed, unrealistically, to be rapid enough so that, as in the conventional one-sector production framework, the fruit price of trees (capital) is pegged

[32] Temporary fluctuations in A can be added to the model without affecting the main results. However, a positive trend in A tends now to generate rising growth rates of output and capital stock.

at a price normalized to one. In other words, I ignore costs of adjustment for investment. This setting corresponds to having "Tobin's q" always equal to one—unlike in the previous model, where the market price of trees was variable.

Depreciation of trees occurs at the rate $\delta_t > 0$. This rate includes a normal depreciation rate, $\delta > 0$, plus a stochastic term, v_t, that reflects the types of disasters considered before. With probability $p > 0$ in each period, a disaster occurs that wipes out the fraction b ($0 < b < 1$) of the existing trees. As before, the idea is that p is small but b is large.

Since the price of trees is fixed at one, the one-period gross return on tree equity can be calculated immediately as

$$(27) \qquad R_{t1}^{e} = 1 + A - \delta - v_{t+1}.$$

Therefore, the assumed distribution for v_{t+1} implies that the expected gross return on equity is

$$(28) \qquad E_t(R_{t1}^{e}) = 1 + A - \delta - pb.$$

The usual asset-pricing formulas still apply. For equity—which has to be priced in equilibrium at one—the formula is

$$(29) \qquad (C_t)^{-\theta} = e^{-\rho}\cdot E_t[(C_{t+1})^{-\theta}\cdot(1+A-\delta-v_{t+1})],$$

where I used the expression for R_{t1}^{e} from Eq. (27). For the risk-free bill return, the result is

$$(30) \qquad (C_t)^{-\theta} = e^{-\rho}\cdot R_{t1}^{f}\cdot E_t[(C_{t+1})^{-\theta}],$$

where R_{t1}^{f} is the one-period gross risk-free return. (A probability of default on bills could readily be added to this model.)

To determine the risk-free return from Eq. (30), we have to know how output, Y_t, divides up each period between consumption, C_t, and gross investment, I_t. In the present model, a change in the single state variable, K_t, will generate equi-proportionate responses of the optimally chosen C_t and I_t. That is, I_t will be a constant proportion, ν, of K_t. Using this fact in the context of Eq. (29) allows for a determination of ν. The result, as the length of the period approaches zero, is

$$\nu = \delta + (1/\theta)\cdot[A - \rho - \delta + p\cdot(1-b)^{1-\theta} - p]. \quad (31)$$

Since $0 < b < 1$, an increase in p raises ν—the saving rate—if $\theta > 1$.

The result for ν allows for the determination of the risk-free bill return, R_{t1}^{f}, from Eq. (30). When the period length approaches zero, the gross risk-free return is given by

$$R_{t1}^{f} = 1 + A - \delta - pb\cdot(1-b)^{-\theta}. \quad (32)$$

Therefore, the spread between the expected return on equity, given in Eq. (28), and the risk-free bill return is

$$E_t(R_{t1}^{e}) - R_{t1}^{f} = pb\cdot[(1-b)^{-\theta} - 1]. \quad (33)$$

This formula is the same as the one in the original model (Eq. [14]), except for the omission of the terms involving background noise (σ^2) and default probability on bills (qd).

The model also determines the growth rate of the economy, that is, the growth rate of the number of trees, $K_{t+1}/K_t - 1$, which equals the growth rate of output, $Y_{t+1}/Y_t - 1$. As the period length approaches zero, the growth rate approaches

$$\begin{aligned} K_{t+1}/K_t - 1 &= \nu - \delta - v_{t+1} \\ &= (1/\theta)\cdot[A - \rho - \delta + p\cdot(1-b)^{1-\theta} - p] - v_{t+1}. \end{aligned} \quad (34)$$

Given the probability distribution of v_{t+1}, the expected growth rate can be determined as

(35) $E_t(K_{t+1}/K_t - 1) = (1/\theta)\cdot(A - \rho - \delta) + p\cdot\{(1/\theta)\cdot[(1-b)^{1-\theta} - 1] - b\}$.

The net effect of p on the expected growth rate is ambiguous—the positive effect of p on ν (if $\theta > 1$) is offset by the direct negative impact of p on the expected growth rate. For the baseline parameters used before—$\theta = 3$ and $b = 0.5$—the net effect is positive. More generally, if $\theta = 3$, the effect is positive if $b > 0.23$. If $b = 0.5$, the effect is positive if $\theta > 2$.

IX. Concluding Observations

The allowance for low-probability disasters, suggested by Rietz (1988), explains a lot of puzzles related to asset returns and consumption. Moreover, this approach achieves these explanations while maintaining the tractable framework of a representative agent, time-additive and iso-elastic preferences, complete markets, and i.i.d. shocks to productivity growth. The framework can also be extended from Lucas's fixed-number-of-trees model to a setting with capital formation.

A natural next step is to extend the model to incorporate stochastic, persisting variations in the disaster probability, p_t, and the default probability, q_t. Then the empirical analysis could be extended to measure p_t and q_t and to relate these time-varying probabilities to asset returns and consumption. Options prices on the overall stock market might help in the measurement of the disaster probability, p_t.[33] Other possibilities include insurance premia and contract prices in betting markets.

Other extensions include the following. The asset menu could be expanded to include precious commodities, such as gold and diamonds, which are likely to be

[33] Xavier Gabaix made this suggestion. Santa-Clara and Yan (2005) use S&P options prices to gauge the probability of jumps, which relate to the rare events that I consider.

important as hedges against disasters. The trees in the Lucas model can also be readily identified with real estate, so that housing prices could be related to disaster probabilities. The model's structure could be generalized to allow for variations in the growth-rate parameter, γ. Some of this variation could involve business-cycle movements—then the model might have implications for cyclical variations in rates of return and the equity premium. In an international context, the distinction between local and global disasters could be applied to events such as regional financial crises.

In an international setting, the model also has implications for failures of interest-rate parity conditions. For example, Kugler and Weder (2005) observe that interest rates on Swiss franc denominated assets have been lower in the long run than those on deposits denominated in other major currencies after taking account of observed variations in exchange rates. Their favored explanation, consistent with the rare-disasters framework, is a "reverse-peso" problem. That is, investors anticipated that the Swiss franc would appreciate relative to other currencies in response to a disaster event—such as a major war—which happened not to materialize within the sample. Possibly the application of the rare-disasters framework can also explain short-run deviations from interest-rate parity conditions among the other OECD countries.

Table 1 Declines of 15% or More in Real Per Capita GDP in the 20th Century			
Part A: 20 OECD Countries in Maddison (2003)			
Event	**Country**	**Years**	**% fall in real per capita GDP**
World War I	Austria	1913-15	23
	Belgium	1916-18	30
	Denmark	1914-18	16
	Finland	1914-18	32
	France	1916-18	31
	Germany	1913-15	21
	Netherlands	1916-18	15
	Sweden	1914-18	17
Great Depression	Australia	1929-31	17
	Austria	1929-33	23
	Canada	1929-33	33
	France	1929-32	16
	Germany	1929-32	17
	Netherlands	1929-34	16
	New Zealand	1929-32	18
	United States	1929-33	31
Spanish Civil War	Portugal?	1934-36	15
	Spain	1935-38	31
World War II	Austria	1944-45	58
	Belgium	1939-43	24
	Denmark	1939-41	24
	France	1939-44	49
	Germany	1944-46	64
	Greece	1939-45	64
	Italy	1940-45	45
	Japan	1943-45	52
	Netherlands	1939-45	52
Aftermaths of wars	Canada	1917-21	30
	Italy	1918-21	25
	United Kingdom	1918-21	19
	United States	1944-47	28

Part B: Eight Latin American & Seven Asian Countries in Maddison (2003)			
Event	**Country**	**Years**	**% fall in real per capita GDP**
World War I	Argentina	1912-17	29
	Chile	1912-15	16
	Chile	1917-19	23
	Uruguay	1912-15	30
	Venezuela	1913-16	17
Great Depression	Argentina	1929-32	19
	Chile	1929-32	33
	Mexico	1926-32	31
	Peru	1929-32	29
	Uruguay	1930-33	36
	Venezuela	1929-32	24
	Malaysia	1929-32	17
	Sri Lanka	1929-32	15
World War II	Peru	1941-43	18
	Venezuela	1939-42	22
	Indonesia*	1941-49	36
	Malaysia**	1942-47	36
	Philippines***	1940-46	59
	South Korea	1938-45	59
	Sri Lanka	1943-46	21
	Taiwan	1942-45	51
Post-WWII Depressions	Argentina	1979-85	17
	Argentina	1998-02	21
	Chile	1971-75	24
	Chile	1981-83	18
	Peru	1981-83	17
	Peru	1987-92	30
	Uruguay	1998-02	20
	Venezuela	1977-85	24
	Indonesia	1997-99	15
	Philippines	1982-85	18

Notes to Table 1

Part A covers 20 OECD countries for 1901-2000: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and United States. Those with no 15% events are Norway (-14% in 1916-18, -13% in 1941-44) and Switzerland (-11% in 1915-18). Satisfactory data for Ireland are unavailable until after World War II. Data for Greece are missing around World War I, 1914-20.

Part B covers eight Latin American and seven Asian countries that have nearly continuous data from Maddison (2003) at least from before World War I. The sample is Argentina, Brazil, Chile, Colombia, Mexico, Peru, Uruguay, Venezuela, India, Indonesia, Malaysia, Philippines, South Korea, Sri Lanka, and Taiwan. Data for Argentina and Uruguay after 2001 are from Economist Intelligence Unit, *Country Data*. Countries with no 15% events are Brazil (-13% in 1928-31, -13% in 1980-83), Colombia (-9% in 1913-15), and India (-11% in 1916-20, -12% in 1943-48). Data for Peru appear to be unreliable before the mid 1920s.

Adjustments were made by Maddison to account for changes in country borders.

*No data available for 1942-48.
**No data available for 1941-45.
***No data available for 1943-46.

Table 2 Stock and Bill Returns during Economic Crises		
Event	**average real rate of return on stocks (% per year)**	**average real rate of return on bills (% per year)**
World War I		
France, 1914-18	-5.7	-9.3
Germany, 1914-18	-26.4	-15.6
Great Depression		
Australia, 1929-30	-13.1	9.7
France, 1929-31	-20.5	1.4
Germany, 1929-31	-22.2	11.2
United States, 1929-32	-16.3	9.7
World War II		
France, 1943-45	-29.3	-22.1
Italy, 1943-45	-33.9	-52.6
Japan, 1943-45	-12.9	-13.7
Post-WWII Depressions		
Argentina, 1998-01	-3.6	9.0
Indonesia, 1997-98	-44.5	9.6
Philippines, 1982-84	-24.3	-5.0
Thailand, 1996-97*	-48.9	6.0

Note: The table shows real rates of return on stocks and government bills over periods with available financial data that correspond to the economic downturns shown in Table 1. Rates of return are computed as averages of arithmetic annual real rates of return. Data are from Global Financial Data, except for Indonesia, where the real rate of return on bills comes from data on money-market interest rates from EIU *Country Data*. Stock-return data for France and Italy prior to 1943 during World War II appear to be problematic. Therefore, I used the periods 1943-45 for these cases, although the economic downturns began earlier.

*Thailand's contraction of real per capita GDP by 14% for 1996-98 falls just short of the 15% criterion used in Table 1.

Table 3

Stock and Bill Returns for G7 Countries

(averages of arithmetic annual returns, standard deviations in parentheses)

Country & time period	real stock return	real bill return	spread
1. Long samples			
Canada, 1934-2004	0.074 (0.160)	0.010 (0.036)	0.063
France, 1896-2004	0.070 (0.277)	-0.018 (0.095)	0.088
Italy, 1925-2004	0.063 (0.296)	-0.009 (0.128)	0.072
Japan, 1923-2004	0.092 (0.296)	-0.012 (0.138)	0.104
U.K., 1880-2004	0.063 (0.183)	0.016 (0.055)	0.047
U.S., 1880-2004	0.081 (0.189)	0.015 (0.048)	0.066
Means for 6 countries	0.074 (0.234)	0.000 (0.083)	0.073
2. 1954-2004			
Canada	0.074 (0.165)	0.024 (0.024)	0.050
France	0.091 (0.254)	0.019 (0.029)	0.072
Germany	0.098 (0.261)	0.018 (0.015)	0.080
Italy	0.067 (0.283)	0.016 (0.034)	0.051
Japan	0.095 (0.262)	0.012 (0.037)	0.083
U.K.	0.097 (0.242)	0.018 (0.033)	0.079
U.S.	0.089 (0.180)	0.014 (0.021)	0.076
Means for 7 countries	0.087 (0.235)	0.017 (0.028)	0.070

Note: Indexes of cumulated total nominal returns on stocks and government bills or analogous paper are from Global Financial Data. See Taylor (2005) for a discussion. The nominal values for December of each year are converted to real values by dividing by consumer price indexes. Annual real returns are computed arithmetically based on December-to-December real values. CPI data since 1970 are available online from Bureau of Labor Statistics and OECD. Earlier data are from Bureau of Labor Statistics, U.S. Department of Commerce (1975), Mitchell (1980, 1982, 1983), and Mitchell and Deane (1962). German data for a long sample were omitted because the German CPI has breaks corresponding to the hyperinflation in 1923-24 and the separation into East and West in 1945. German data on dividend yields are also unavailable for 1942-52.

Table 4 Growth Rates of Real Per Capita GDP in G7 Countries							
	Canada	**France**	**Germany**	**Italy**	**Japan**	**U.K.**	**U.S.**
	growth rate of real per capita GDP, 1890-2004						
mean	0.021	0.020	0.019	0.022	0.027	0.015	0.021
standard deviation	0.051	0.069	0.090	0.059	0.082	0.030	0.045
kurtosis	5.4	5.4	40.6	10.4	49.0	5.8	5.8
	growth rate of real per capita GDP, 1954-2004						
mean	0.022	0.026	0.027	0.030	0.043	0.021	0.021
standard deviation	0.023	0.017	0.024	0.022	0.034	0.018	0.022
kurtosis	3.4	2.5	3.9	2.8	2.4	3.1	2.6

Note: Except for the U.S., data are from Maddison (2003), updated through 2004 using information from Economist Intelligence Unit, *Country Data*. For the U.S., the sources are noted in the text. The GDP series for Germany has a break in 1918; hence, the growth-rate observation for 1918-19 is missing.

Table 5 Calibrated Model for Rates of Return							
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**
	base-line	**low p**	**low b and d**	**low θ**	**high γ**	**high q**	**high d**
	parameters (same as baseline if not shown)						
θ (coefficient of relative risk aversion)	**3**			**2**			
σ (s.d. of growth rate, no disasters)	**0.02**						
ρ (rate of time preference)	**0.03**						
γ (growth rate, deterministic part)	**0.025**				**0.030**		
p (disaster probability)	**0.017**	**0.012**					
b (disaster size)	**0.5**		**0.35**				
q (bill default probability in disaster)	**0.4**					**0.5**	
d (bill default proportion in disaster)	**0.5**		**0.35**				**0.6**
	variables						
expected equity rate	0.045	0.062	0.075	0.054	0.060	0.045	0.045
expected bill rate	0.008	0.036	0.065	0.038	0.023	0.014	0.013
equity premium	0.037	0.026	0.011	0.016	0.037	0.031	0.032
expected equity rate, conditional	0.053	0.068	0.081	0.063	0.068	0.053	0.053
face bill rate	0.011	0.038	0.067	0.042	0.026	0.018	0.017
equity premium, conditional	0.042	0.030	0.014	0.021	0.042	0.035	0.037
price-earnings ratio	35.5	23.1	17.9	26.5	26.2	35.5	35.5
expected growth rate	0.017	0.019	0.019	0.017	0.022	0.017	0.017
expected growth rate, conditional	0.025	0.025	0.025	0.025	0.030	0.025	0.025
	levered results (debt-equity ratio = 0.5)						
expected equity rate	0.063	0.076	0.081	0.063	0.078	0.060	0.061
equity premium	0.055	0.040	0.016	0.024	0.055	0.046	0.048
expected equity rate, conditional	0.074	0.083	0.088	0.074	0.089	0.071	0.072
equity premium, conditional	0.063	0.045	0.021	0.032	0.063	0.053	0.055

Note: The expected rate of return on equity is from Eq. (10). The expected rate of return on bills is from Eq. (13). The equity premium is the difference between these two rates. The expected rate of return on equity conditioned on no disasters is from Eq. (11). The face bill rate is from Eq. (12). The equity premium conditioned on no disasters is the difference between these two rates. The price-earnings ratio comes from Eq (18). The expected growth rate is from Eq. (19). The expected growth rate conditioned on no disasters is from Eq. (20). The expected rate of return on levered equity is from Eq. (23). The levered equity premium is the difference between this rate and the expected rate of return on bills.

Table 6 Interest and Inflation Rates during Wars and the Great Depression in the U.S.			
Year	**nominal return**	**expected inflation rate**	**expected real return**
Civil War			
1860	0.070	0.006	0.063
1861 (start of war)	0.066	0.026	0.039
1862	0.058	0.063	-0.005
1863	0.051	0.082	-0.031
1864	0.062	0.128	-0.066
1865	0.079	0.050	0.029
Spanish-American War			
1897	0.018	0.015	0.004
1898 (year of war)	0.021	0.006	0.015
World War I			
1914	0.047	0.021	0.026
1915	0.033	0.011	0.022
1916	0.033	0.026	0.007
1917 (U.S. entrance)	0.048	0.075	-0.028
1918	0.059	0.116	-0.057
Great Depression			
1929	0.045	0.000	0.044
1930 (start of depression)	0.023	0.006	0.016
1931	0.012	-0.038	0.050
1932	0.009	-0.059	0.068
1933 (worst of depression)	0.005	-0.057	0.062
1934	0.003	0.022	-0.020
1935	0.002	0.025	-0.023
1936	0.002	0.015	-0.014
1937 (onset of sharp recession)	0.003	0.018	-0.016
1938	0.001	0.012	-0.012
World War II			
1939	0.000	-0.005	0.006
1940	0.000	0.005	-0.005
1941 (U.S. entrance)	0.001	0.014	-0.012
1942	0.003	0.072	-0.068
1943	0.004	0.053	-0.049
1944	0.004	0.024	-0.021
1945	0.004	0.021	-0.017

Table 6, continued			
Year	**nominal return**	**expected inflation rate**	**expected real return**
Korean War			
1950	0.012	0.014	-0.002
1951	0.016	0.026	-0.010
1952	0.017	0.005	0.012
1953	0.019	-0.009	0.028
Vietnam War			
1964	0.036	0.011	0.025
1965	0.041	0.012	0.029
1966	0.049	0.018	0.031
1967	0.044	0.022	0.022
1968	0.055	0.029	0.026
1969	0.069	0.032	0.037
1970	0.065	0.036	0.029
1971	0.044	0.035	0.008
1972	0.042	0.033	0.009
Gulf War			
1990	0.077	0.039	0.038
1991 (year of war)	0.054	0.035	0.020
1992	0.035	0.034	0.001
Afghanistan-Iraq War			
2000	0.058	0.025	0.033
2001 (September 11)	0.033	0.025	0.008
2002 (start of Afghanistan war)	0.016	0.022	-0.006
2003 (start of Iraq war)	0.010	0.017	-0.006
2004	0.014	0.018	-0.004

Note: Nominal returns on U.S. Treasury Bills or commercial paper (before 1922) are calculated as in Table 3. The expected inflation rate for the CPI is constructed as described in the notes to Figure 3. The expected real return is the difference between the nominal return and the expected inflation rate.

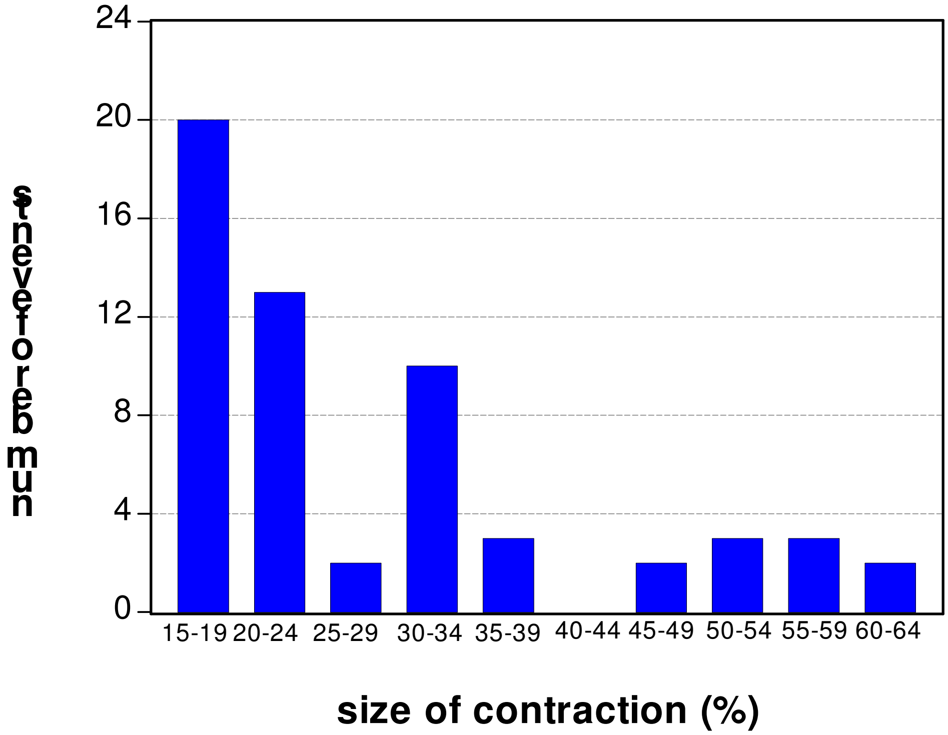

Figure 1

Note: The histogram applies to the 35 countries covered over the 20th century in Table 1. The horizontal axis has intervals for percentage declines in real per capita GDP. The vertical axis shows the number of economic contractions in each interval.

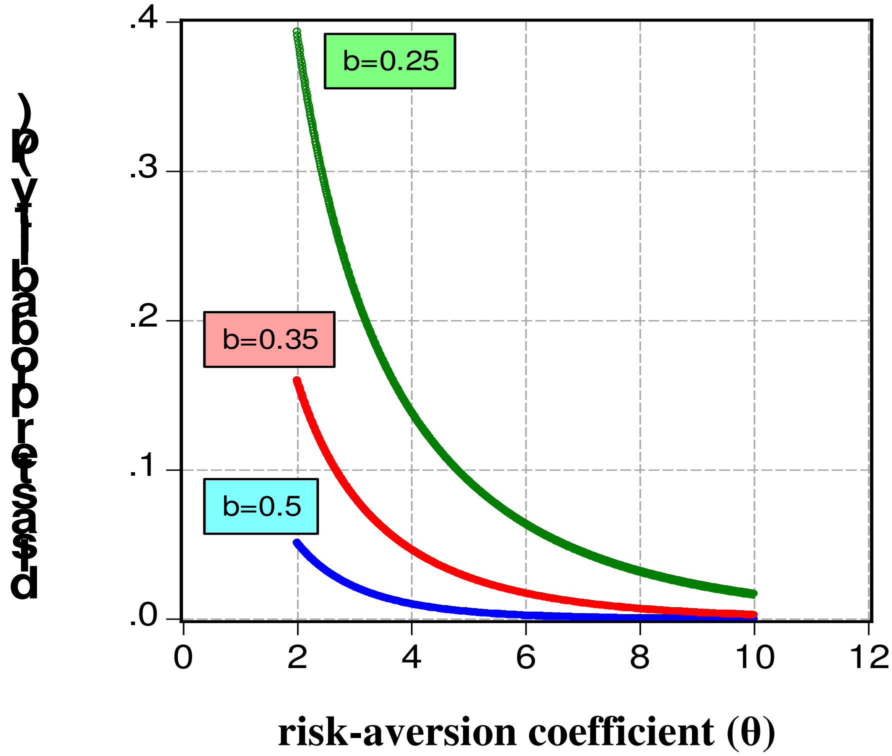

Figure 2

Iso-Premium Curves

The graphs show the combinations of the disaster probability (p) and the coefficient of relative risk aversion (θ) needed to generate a levered equity premium of 0.07 in Eq. (23) with a leverage coefficient, λ, of 0.5. In the lowest graph, a disaster reduces output by 50%, in the middle graph, a disaster reduces output by 35%, and in the upper graph, a disaster reduces output by 25%.

Figure 3

Expected Real Interest Rate on
U.S. T-Bills/Commercial Paper, 1859-2004

Note: Data on nominal returns on U.S. Treasury Bills (1922-2004) and Commercial Paper (1859-1921) are from Global Financial Data. See the notes to Table 4. From 1947-2004, expected real returns are nominal returns less the Livingston expected inflation rate for the CPI (using six-month-ahead forecasts from June and December). For 1859-1946, the expected real return is the nominal return less a constructed estimate of expected inflation derived from a first-order auto-regression of CPI inflation rates for 1859-1946. The CPI data are from Bureau of Labor Statistics (January values since 1913, annual averages before 1913) and U.S. Department of Commerce (1975).

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