Urban Growth and the Location of Economic Activity in Cities

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Abstract

We first document several empirical regularities regarding the evolution of urban structure in the largest US metropolitan areas over the period 1980-1990. These regularities relate to changes in resident and employment population, occupations, as well as the number and size of establishments in different sections of the city. We then propose a theory of urban structure that emphasizes the location and integration decisions of firms. In particular, firms can decide to locate their headquarters and operation plants in different regions of the city. Given that cities experienced positive population growth throughout the 1980s, we show that our theory can account for the various facts documented in the first part of the paper.

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1. INTRODUCTION

The internal structure of US metropolitan areas has evolved dramatically over the last three decades. This evolution exhibits striking patterns that hold for a wide range of cities. If one divides cities into a center county and edge counties, employment and resident levels have increased both at the center and at the edge. However, over this period, we also observe an important increase in the share of city residents and employment, as well as the number of establishments, at the edge. This shift in economic activity to the edge of the city is more pronounced for non-management occupations than for managers. In addition, the size of establishments decreased in both areas throughout this period. The first part of this paper is devoted to documenting these changes in US urban structure.

What accounts for the evolution of US urban structure over this period? The urban literature has argued that the migration of economic activity to the edge is the result of decreases in transport costs.[1] Explanations of this type, however, are not consistent with the migration of workers and firms to the edge. Furthermore, there exists a more fundamental problem with all available explanations for subsets of these phenomena in that they rely on mechanisms that decrease agglomeration forces, thereby explaining the migration of economic activity to the edge, but not the observed increase in the level of economic activity at the center. These theories are also silent regarding both functional (management versus non-management) and establishment shifts.[2]

[1] See Anas, Arnott and Small (1998), and Glaeser and Kahn (2003), for a general review of this literature, and Burchfield et. al. 2004 for a recent empirical study of urban sprawl in the US.

[2] In theories like Fujita and Ogawa (1982), or Lucas and Rossi-Hansberg (2002), a decline in transport costs tends to disentangle the location of business and residential areas thereby leading to employment concentration at the center. Of course, these theories incorporate a richer spatial dimension so that the implications of commuting costs or changes in externality parameters depend on the exact definition of the center. These theories do not incorporate occupational choices or firms' integration decisions.

This paper proposes a theory aimed at addressing the full set of facts described above. This theory relies on the ability of firms to break down their production process into headquarters and production plants, where either can locate in different parts of the city. As city population changes, some firms modify their organizational structure. Standard agglomeration forces motivate firms to keep at the center only the workers that can benefit from interactions in downtown locations (knowledge spillovers or, in general, production externalities). As a larger proportion of firms decides not to integrate their operations, employment at the center grows, but this growth is driven by increases in the number of headquarters, and therefore establishments and managers. Since each manager in a headquarter supervises several workers, and the production plants of these new firms are located at the edge, employment at the edge grows even more, thereby leading to a decline in the share of employment at the center. Moreover, because land rents are lower at the edge, many firms decide to integrate their operations at the edge as population grows. Hence, the number of managers at the edge also increases thereby reenforcing the fall in the share of center employment. The combined set of changes leads to a concentration of managers versus non-managers, as well as a decrease in the share of establishments, at the center. City centers are becoming manager hubs. These changes also lead to a decline in establishment sizes as more establishments become non-integrated firms.

Chatterjee and Carlino (2001) seek to explain systems of cities and argue that the deconcentration of U.S. metropolitan employment is the result of an increase in aggregate employment. The theory they present is one where an aggregate increase in employment raises densities in small metropolitan areas more than in large ones, since large metropolitan areas have larger employment densities and are not able to accommodate the new workers cheaply enough. Our paper shares with Chatterjee and Carlino (2001) the focus on population growth as the main engine driving the structural change of US cities. In contrast to their work, however, we model just one city, and not the interaction between cities, and focus on its internal structure. That is, we study the spatial allocation of employment within a city. We also analyze the location of establishments and agents in different occupations. The new set of facts we uncover leads us to emphasize firms' integration decision, which we argue rationalize observed changes in city structure as a result of population growth.

Duranton and Puga (2004) argue that cities have moved from being sectorally specialized to becoming functionally specialized. They contend that decreases in the cost of communication between headquarters and plants have lead to the location of headquarters in cities and the location of production plants in smaller towns. Our view of the changes in city structure shares many elements with Duranton and Puga (2004). In particular, they also model explicitly the decision to integrate the firm's headquarters and production plants. However, in their view, this integration decision has implications across metropolitan areas. We argue that firms' ability to separate headquarters and plants is also key in explaining the changes in the internal structure of cities. Our paper differs from Duranton and Puga (2004) in that we do not view changes in communication technology as the force underlying changes in urban structure, but instead rationalize the latter changes as the result of observed urban population growth. Our framework further has implications for the share of establishments located in different parts of the city that are consistent with the data. Davis and Henderson (2004) provide evidence that complements our findings. They find that firms take advantage of services and production externalities (which decline with distance) at the city's business sectors by locating their headquarters at the center and their operation plants in other parts of the city.

The rest of the paper is organized as follows. Section 2 presents our data organized in nine different stylized facts. Section 3 presents a simple urban framework that incorporates the firm's integration decision. Section 4 shows that population growth leads to changes in city structure consistent with the stylized facts that we document, and Section 5 concludes. The proofs of all propositions are included in the Appendix.

2. SOME FACTS ON THE EVOLUTION OF CITY STRUCTURE

This section documents a set of regularities in the evolution of city structure that we care to rationalize with the simple theory proposed in this paper. We document these facts for the decade spanning the 80's, although most of the empirical regularities hold from 1970 to 2000. The reason is that for some of these regularities, in particular the ones that involve the location of agents with different occupations, we do not have data covering a longer period. Thus, we chose to homogenize the time period and document our stylized facts over the same decade.

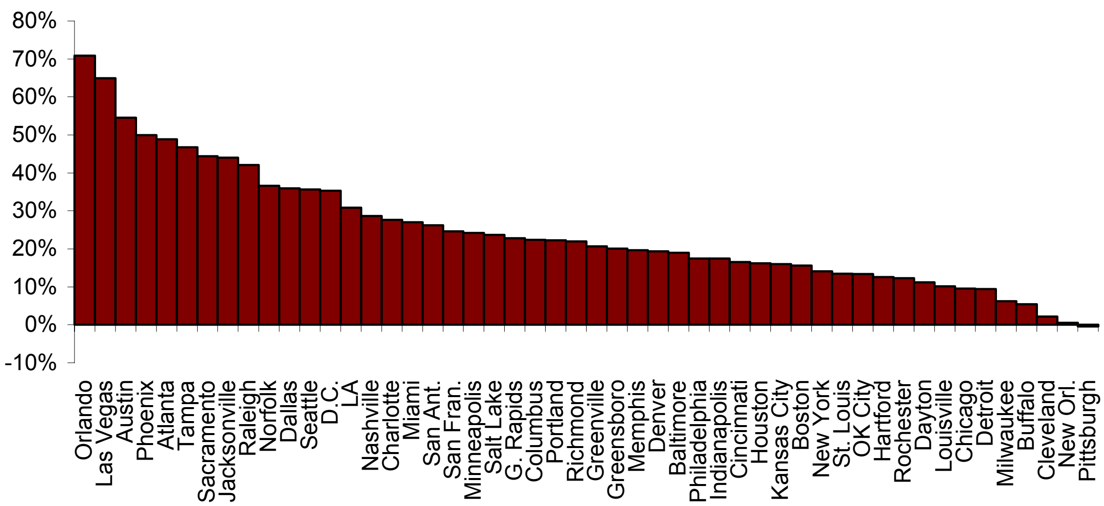
Given our focus on the structure of cities, we separate the city into two locations: center and edge. The center is the area encompassed by the central county of the city in question. The edge is the set of counties that surround the central county. The central county always includes the central business district of the city, or the downtown area, and in general is much larger than just the downtown area. We use the 50 largest Metropolitan Statistical Areas (MSAs) in the US according to their 1999 population.

Our data originates from four sources: the Census Bureau "Commute to Work" data, the Census County Business Patterns, the Housing and Urban Development State of the Cities database, and the BEA Regional Economic Information System. Since the theory we propose is that of a city rather than a system of cities, and since we wish to abstract from idiosyncratic city characteristics, our analysis focuses only on time changes in city structure during the 80's, and not on the state of city structure across US cities at a given point in time.

1.1 Changes in absolute population levels

The first set of facts we mention are well known. As shown in Figure 1, overall population increased throughout the 80's in all but one city in our sample. City population growth averaged 21.3% over that period, while Pittsburgh grew at only -0.38%. All averages presented in this section are weighted averages using population shares.

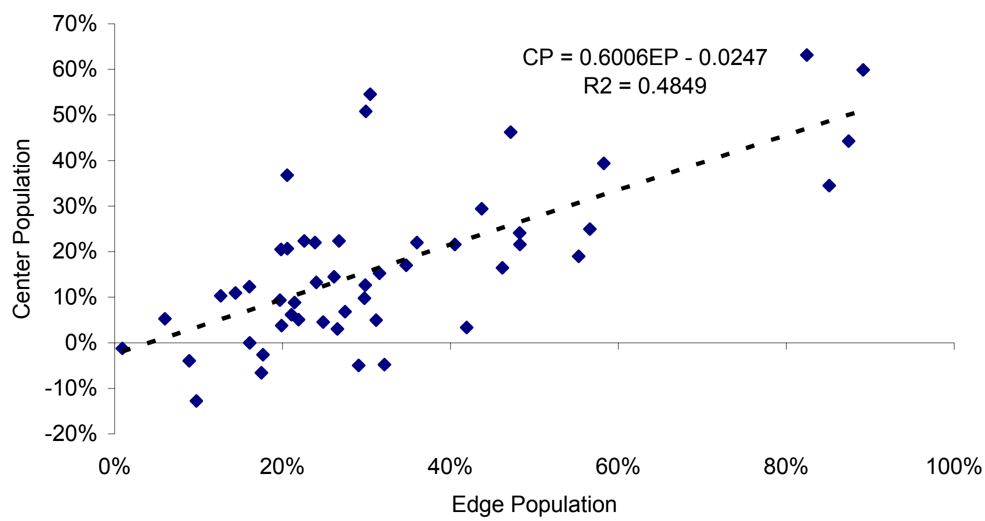
Figure 1: MSA Population Growth, 1980-1990



Furthermore, population in most cities increased both at the center and at the edge. To illustrate this point, Figure 2 plots population changes in these areas from 1980 to 1990. Observe that in almost all cities changes are in fact positive both at the center and at the edge and, for some cities, very large. The population of Las Vegas, for example, grew by more than 80% in the edge counties and more than 50% at the center. A 1% increase in population at the edge is associated with a 0.6% increase in population at the center, as the trend-line in the graph suggests. The correlation between changes in population at the center and at the edge is 0.69. All 50 cities in our sample grew in terms of population at the edge, and only 7 declined in terms of population at the center. In the latter cases, this decline is always small except for New Orleans whose population fell by 12.8 % at the center, but increased by 9.7% at the edge.

Figures 1 and 2 are indicative of overall city population growth, and the fact that this population locates both at the center and at the edge. A question which then arises is: has city population growth lead to a change in the link between employment location and residential location? As cities become larger, one might expect residential sprawl (i.e. residents locating at the boundary of the city) and employment concentration at the center (see Fujita and Ogawa [1982] and Lucas and Rossi-Hansberg [2002]). In general, our view of the data is that this phenomenon did not dominate in the US: changes in employment are in general paralleled by changes in the number of residents in both areas of the city.

Figure 2: Population Growth, 1980-1990



A fact consistent with this view is that net commuting, as a percentage of total population, between the center and the edge hardly changed at all throughout the

80's. Net commuting represented 8.98% of total MSA population in 1980 and 8.38% in 1990. Average residential growth in the 80's amounted to 15.4% at the center and 25.7% at the edge. Similarly, average employment growth in the 80's was 14.8% at the center and 28.3% at the edge. This similarity in the size of changes in employment and residents across city areas is surprising, and suggests that the link between employment and residential location is a key component of urban structure. Moreover, this evidence suggests that commuting costs did not decline in any significant way during the 80's.

Figure 3: Employment Growth, 1980 - 1990

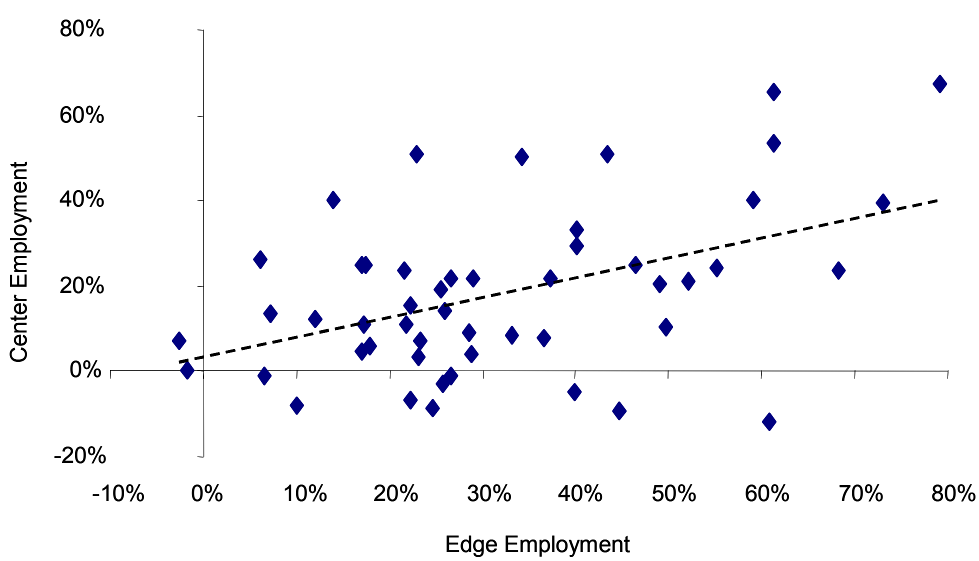
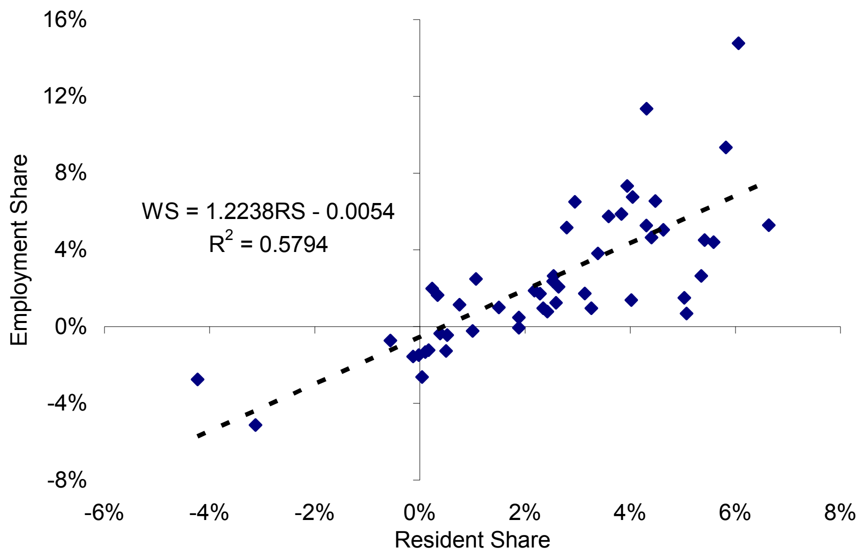


Figure 3 depicts employment growth across all cities in our sample (resident growth is identical to population growth, which is presented in Figure 2). The figures indicate that residential population and employment grew in the large majority of cities during this period. More specifically, resident and employment growth is positive at the edge in all cities. At the center, employment growth is negative in nine cities while resident growth is negative in only seven cities.

1.2 Changes in population shares across city areas

The facts we have just presented relate to absolute quantities of employment and resident growth. We now address changes in the shares of residents and employment at the edge during the 80's. Our data shows that the share of both residents and employment has generally increased at the edge. That is, while levels grew everywhere, population shares have shifted from the center to the edge of US cities. The average increase in the share of employment at the edge during the 80's was 3.11% while the average increase in the share of edge residents was 2.64%. Put simply, economic activity in the US is moving to the periphery. Figure 4 depicts changes in resident and employment shares at the edge during this period. It is clear from the graph that in most US cities, the share of individuals who both reside and work at the edge increased.

Figure 4: Change in Resident and Employment Share at the Edge, 1980-1990



Given the shift in employment shares towards the periphery, one might wonder whether the increase in edge employment was driven by particular industries. In other words, the facts above could have resulted from specific industries moving away from city centers while other industries, perhaps less labor and land intensive, moved to the city center. This does not seem to be the case. The average employment share at the center declined from 0.42 to 0.38 in manufacturing and from 0.47 to 0.43 in services. That is, average employment shares decline by about the same percentage in both sectors.

To gain further insight into the change in employment shares at the center, we examine the change in employment across occupations. In particular, we divide employment into two classes: management and non-management occupations. The first class includes managers and professional workers. The second includes what the Housing and Urban Development State of the Cities database classifies as non-management workers. The latter category includes individuals working as technicians, in sales, as administrative support, as precision workers, as laborers and machine operators, and in services.

The share of managers at the center declined in all but 3 cities in our sample. The average change in the manager share during the 80's was -4.71%. The share of non-managers at the center also declined in all but a handful of cities during this period. The average decline was -5.63%. Even though these facts are informative about changes in city structure, we are more interested in facts that compare the *relative* location of managers and non-managers. Figure 5a presents the difference between the fall in manager and non-manager shares. The average difference is 0.92% indicating that, on average, manager shares at the center fell less that non-manager shares during the 80's.[3] There are only 14 cities where the reverse is true. Some of these cities are, however, among the largest US cities including New York, Los Angeles, and Chicago. Figure 5b presents the change in the ratio of managers to

[3]The fact comes out even more clearly if we focus on a narrow definition of non-management workers, like laborers and machine operators or technicians.

non-managers at the center less the change in the same ratio at the edge: another way of looking at the same phenomenon.

Consistent with the previous observation, the change in the ratio of managers to non-managers at the center was larger than at the edge in all but 8 cities. The average change in the difference of these ratios was 2.82%. In general, these data suggest that city centers are becoming management hubs, with managers heading operation plants at the boundary of the city where land is cheap. We shall use this interpretation of Figures 5a and 5b extensively in the model we present below. Even though managers tended to be more concentrated in city centers throughout the 80's, the total number of managers increased relative to total employment in all cities over this period. Thus, cities as a whole are also becoming manager hubs. The change in total management over population is presented in Figure 5c, with an average of 2.8%. The figure shows that the proportion of agents that work as managers increased in all cities in our sample. The evidence in Figure 5c is consistent with the evidence in Duranton and Puga (2004) that argue that cities are becoming functionally specialized.

Figure 5a: Change in Management Share minus Non-Management Share at the Center, 1980-1990

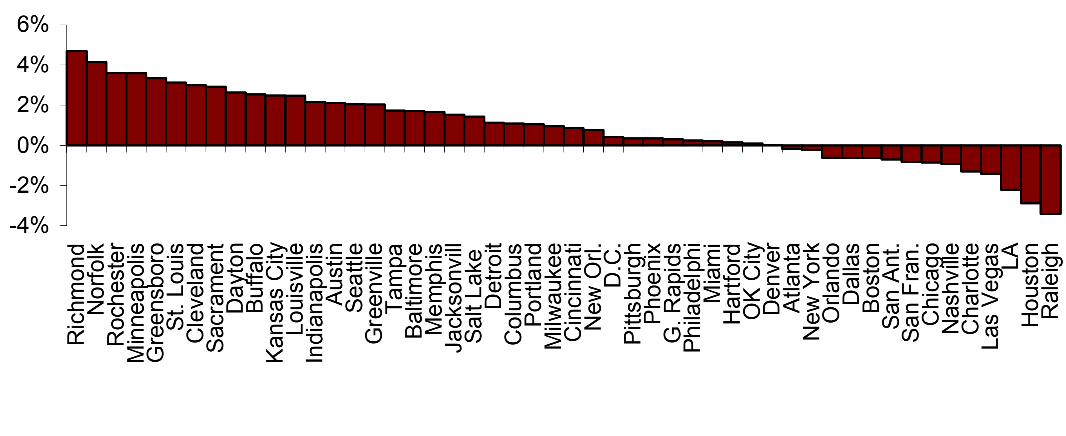


Figure 5b: Change in Management / Non-Management Employment Ratio at the Center minus the Edge, 1980-1990

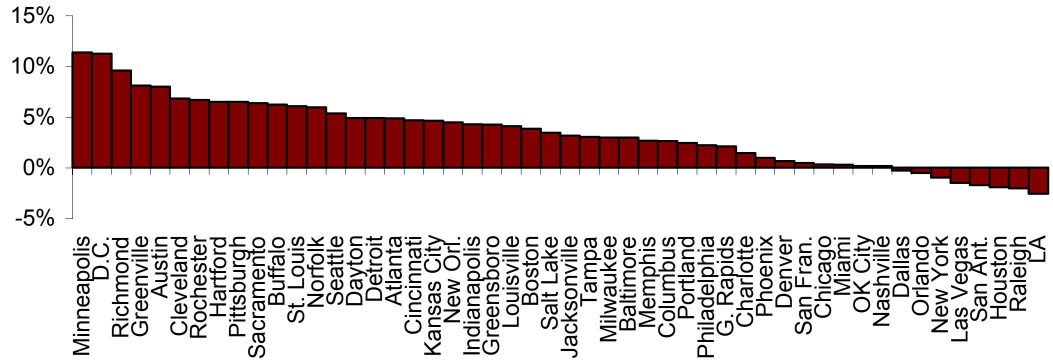
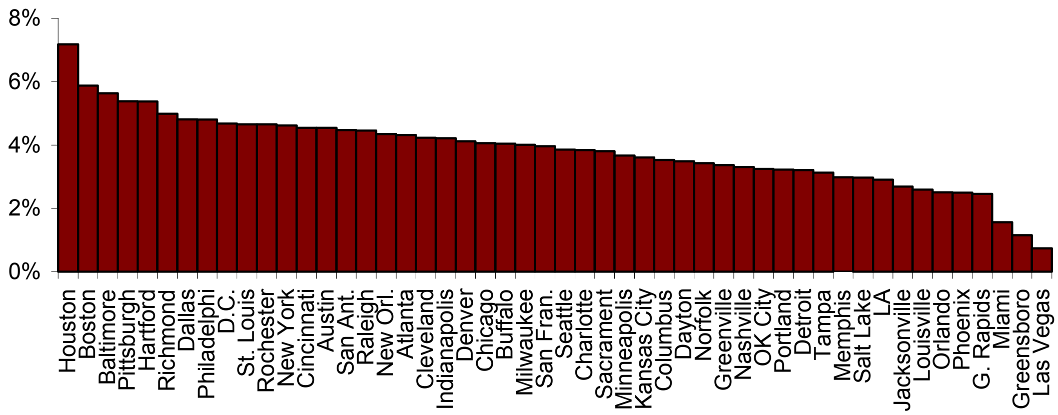


Figure 5c: Change in Total Management / Population Ratio, 1980-1990

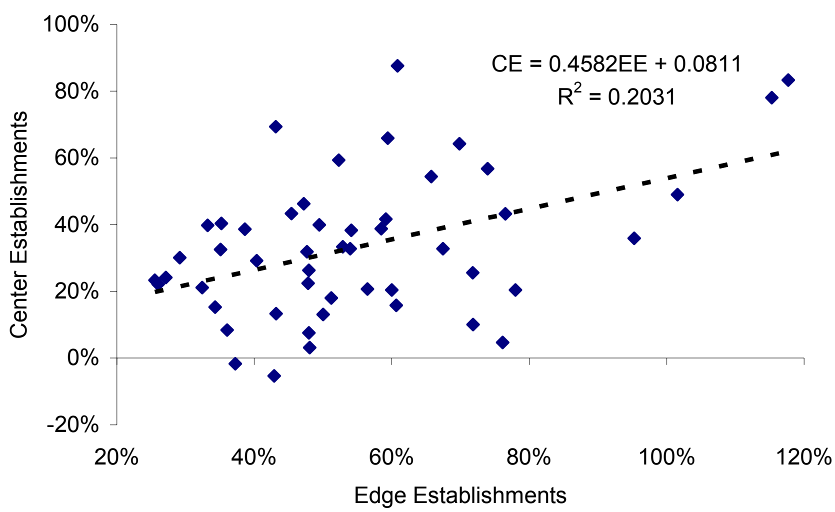


1.3 Changes in the location and characteristics of establishments

Having set out some facts regarding changes in population location within US cities, we now turn to the location of establishments in different parts of the city. Consistent with overall population and employment growth throughout the 80's, the number of establishments also increased in all but three US cities, both at the center and at the edge.

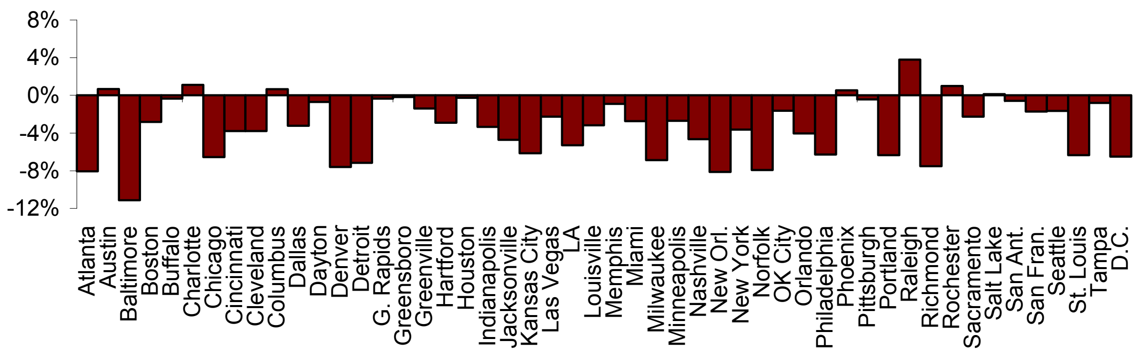
The number of establishments grew on average by 30.1% at the center and by 50.5% at the boundary. Hence, while firm or plant net entry is more pronounced at the city edge, firm entry is also substantial at the center. The correlation between establishment net entry at the center and at the edge is 0.45. Figure 6 illustrates these changes. Note that in some cities, net entry of establishments at the edge exceeds 80% over our sample period and, in some cases, reaches as high as 110%.

Figure 6: Establishment Growth at the Edge and the Center, 1980-1990



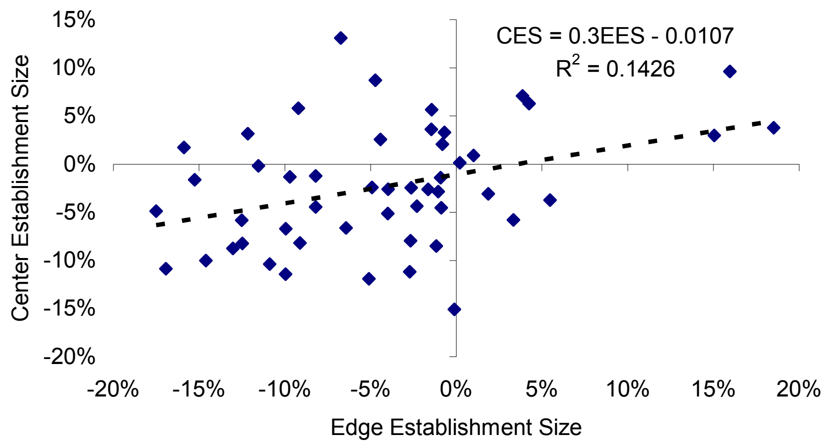
Although central city counties experienced significant net firm entry throughout the 80s, more establishments located in the periphery over that period. Indeed, the change in the share of establishments at the center is negative in all but a few cities, as Figure 7 illustrates. The average change in the share of establishments located at the center is -3.65%. In 64% of the cities, the share of establishments was larger at the center than at the edge in 1980, with an average establishment share of 54.9% at the center. Therefore, while more than one half of the establishments were located at the center in 1980, many new firms chose to locate at the city edge during the subsequent decade which lead to a significant decrease in the share of firms residing at the center.

Figure 7: Change in Establishment Share at the Center, 1980-1990



How were establishment sizes, measured in number of employees, affected during this period? In general, we find that establishment sizes declined over the 1980s. This finding is consistent with other evidence in the literature concerning the average size of firms in the US (see Garicano and Rossi-Hansberg [2003]). Establishment sizes declined on average by -4.08% at the center and -3.92% at the edge. Figure 8 shows changes in establishment sizes both at the center and at the edge. We can see that for most cities, establishment sizes fell in both regions. This finding, however, does not hold for all cities. About half of the cities in our sample experienced a decline in average establishment size in both regions simultaneously. Establishments tend to be larger at the center than at the boundary, with 21.4 employees per establishment at the center versus 17.1 employees per establishment at the edge in 1980.

Figure 8: Change in Establishment Size at the Edge and the Center, 1980-1990



The last characteristic of urban economic activity that we wish to establish in this section concerns the relationship between changes in the number of managers and the number of establishments across cities. Figures 9a and 9b illustrate this relationship.

Figure 9a: Change in Managers and Change in Establishments at the Center, 1980-1990

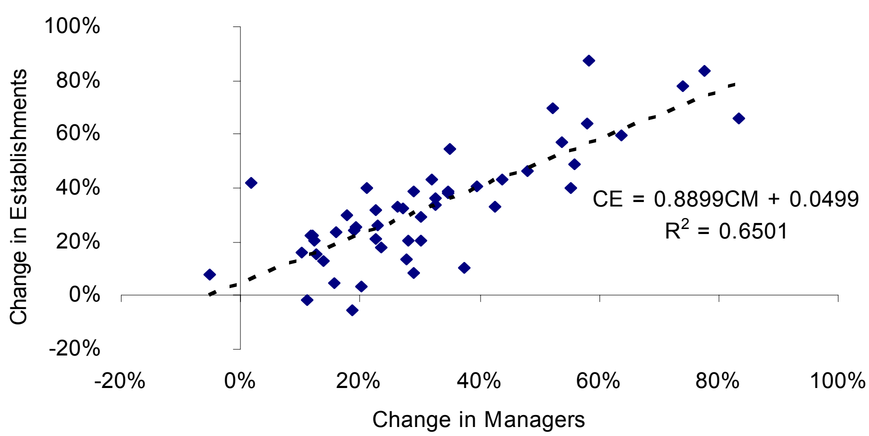
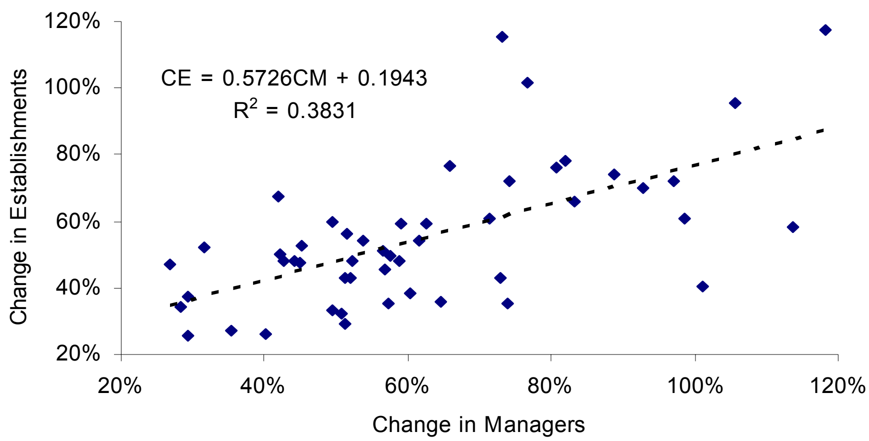


Figure 9b: Change in Managers and Change in Establishments at the Edge, 1980-1990



First, note that the number of establishments and managers are very highly correlated: the correlation is 0.81 at the center and 0.62 at the edge. Second, a 1% increase in the number of managers is associated with a 0.89% increase in the number of establishments at the center. At the edge, a 1% increase in the number of managers is associated with 0.57% more establishments. This evidence suggests that the number of establishment per manager at the center is larger than at the edge, and roughly constant across cities

Let us now summarize the set of stylized facts that we have presented for the fifty largest US cities over the 1980-1990 decade. Throughout the paper, we shall refer to these stylized facts by the number we assign to each below.

1. Overall population growth.

2. Population growth at the center and at the edge of cities.

3. Resident and employment growth at the center and at the edge.

4. A similar reduction in resident and employment shares at the center.

    (a) Present both in services and manufacturing.

5. City centers increasingly becoming management hubs

    (a) An increase in the share of managers relative to that of non-managers at the center.

    (b) An increase in the ratio of managers to non-managers at the center relative to the edge.

    (c) An increase in the total number of managers over total employment.

6. An increase in the number of establishments at the center and at the edge.

7. A decline in establishment share at the center.

8. A decline in establishment size both at the center and at the edge

9. A greater number of establishments per manager at the center than at the edge.

    (a) The number of establishments per managers is roughly constant across cities, more so at the center than at the edge.

In the next section, we propose a simple model of city structure which argues that Fact 1 alone leads to stylized Facts 2 through 9. Put another way, it is possible to think of the changes in population, residents, employment, occupations, and establishments described above as the result of urban population growth. At the heart of the theory we present lies a decision on the part of firms to either integrate their operations in one location or separate them into headquarters and production plants. We believe that adding this additional margin to urban models is crucial in explaining the set of changes observed in the internal structure of US cities.

3. THE MODEL

This section presents a theory of the internal structure of cities simple enough to remain analytically tractable yet rich enough to address all the stylized facts outlined in the previous section. Since our goal is to illustrate the main forces that lead to these empirical regularities, we model cities as consisting of only two areas: the center of the city and what we call the edge. We think of these two areas as the model equivalent of the central and edge counties in the data. Given this parallel, we assume that the central area of the city is given exogenously by $L_c > 0$. The edge area is endogenous, and we denote it by $L_b > 0$, where b stands for the city boundary. We assume that residential land rents at the edge are given by an agricultural land rent, $\overline{R} \geq 0$, that represents its opportunity cost. Land rents at the center of the city are endogenous and determined in equilibrium. Total city population, P, is exogenously given. In fact, we shall argue below that the theory we develop can rationalize the set of stylized facts presented above simply as the result of population growth. Our theory, therefore, is a partial equilibrium theory that takes as given two key elements from national economic behavior, namely agricultural land rents and city population sizes. Any theory of city structure must take a stand on the variables to be determined at the aggregate level rather than the city level, and our choice is driven by the set of facts that we are aiming to explain.

The key insight of our model is that allowing firms to separate their location into headquarters and production plants implies that city growth leads to a set of empirical regularities regarding urban structure. Headquarters develop knowledge and, therefore, experience external effects from other headquarters. Knowledge transactions are carried out in headquarters which tend to agglomerate in high rent areas of the city. Production plants, in contrast, carry out routine tasks that do not lead to knowledge spillovers and, consequently, tend to locate in areas where land rents are low. Production plants in our framework can be interpreted as either manufacturing plants, retail stores, or other production facilities.

3.1 Firms

The city produces and consumes one good, the price of which we normalize to one. A firm is made up of a manager who hires workers to produce. The manager and her workers can locate at either the center or the edge of the city. We refer to the location of the manager as the firm's headquarters and the location of her workers as the firm's production plant. The number of workers a manager can hire is determined by whether the firm is located in only one location (an integrated firm), or whether the headquarters and production plants reside in different locations. In the former case, the manager finds it less costly to communicate and interact with workers that are located close by so that she can deal with a large set of workers, $n_{cc} = n_{bb} = \alpha\overline{\delta} > 1$, where n_{ij} denotes the number of employees of a firm with headquarters in area i and production plant in area j. In contrast, if the manager decides to set up a non-integrated firm, she needs to spend additional resources to monitor and interact with her workers, who are physically removed, and her span of control is given by $n_{cb} = n_{bc} = \alpha\underline{\delta} > 1$, where $\underline{\delta} < \overline{\delta}$. This assumption is motivated by Fact 9 above. In other words, in the data, changes in the number of managers are clearly positively correlated with changes in the number of establishments both at the center and at the edge. In fact, the correlation between changes in managers and changes in the number of establishments is significantly higher at the center than at the edge in the 80's, which is consistent with a constant number of managers per firm (abstracting from composition effects which are not present at the center).

In our model, the location of a firm's headquarters matters significantly in that it determines its productivity. In particular, firm productivity depends on the number of managers located in the area of the city where the firm's headquarters are also located: a production externality. Total output of a firm with headquarters in area i and production plant in area j is given by AE_in_{ij}, where E_i denotes the number of managers in i and A is a city-wide productivity parameter. A firm has to pay labor costs given by a city-wide wage w (since all agents in the city are assumed identical) times the number of workers it hires, n_{ij}, as well as land rents. We assume that the firm needs to hire one unit of land per worker in order to operate, so that total land rent paid by this firm is given by R_jn_{ij}. Firms are owned by managers whose earnings are, therefore, given by firm's profits: total revenue minus total costs. It follows that a manager who owns the firm we have just described earns

$$F_{ij} = (AE_i - w - R_j)n_{ij}. \tag{1}$$

The problem of a manager is then to choose the location of the firm's headquarter and production plant to solve

$$\begin{aligned} \overline{F} &= \max_{ij} \{F_{ij}\} \text{ for } i,j \in \{c,b\} \text{ and subject to} \\ n_{ij} &= \begin{cases} \alpha\overline{\delta} \text{ if } i = j \\ \alpha\underline{\delta} \text{ if } i \neq j \end{cases}. \end{aligned}$$

Put alternatively, managers decide whether to locate at the center or at the edge of the city and, from that location, whether to operate integrated or separate production facilities.

3.2 Individuals

As mentioned earlier, we consider a city occupied by a population P of identical agents. Agents consume the only good produced in the city and they live where they work. The latter assumption is justified by the fact that in the data, employment and residents in both areas of the city have moved to the edge in similar proportions, as summarized in Fact 4. Recall also that the share of net commuters between the center and edge stayed remarkably constant throughout our sample period. Consumers order consumption according to a linear utility function. Therefore, given that the price of consumption goods is normalized to one, they act to maximize income and solve

$$U = \max \left\{ \overline{F}, w \right\}. \tag{2}$$

Since all agents are identical and, in equilibrium, some agents become managers while others become workers, we know that $\overline{F} = w$. Furthermore, the fact that all agents have the option to set up integrated or non-integrated firms in any set of locations yields, in equilibrium, $\overline{F} = F_{ij}$ for all operating firms with headquarters in i and production plants in j.[4]

3.3 Equilibrium

We denote by E_{ij} the number of managers operating firms with headquarters in i and operation plants in j. Hence, the total number of managers at location i, E_i, is given by

$$E_i = E_{ii} + E_{ij}. \tag{3}$$

Since the number of units of land at the center is exogenously given by L_c, and firms rent one unit of land per worker, the number of workers at the center is given by

$$L_c = E_{cc} n_{cc} + E_{bc} n_{bc}. \tag{4}$$

Analogously, the number of workers at the edge is also given by the number of units of land used at the boundary,

$$L_b = E_{cb} n_{cb} + E_{bb} n_{bb}. \tag{5}$$

[4] We abstract from differences in agent's human capital or ability that may lead to differences in wages or managerial rents. We could introduce them and reproduce our findings in terms of efficiency units of labor.

Land use at the edge, however, is endogenous and the area occupied by the city expands or contracts as the economic environment changes. It follows that the total number of workers in the city is given by $W = L_c + L_b$. Total city population is given by these workers plus those individuals who become managers of firms with headquarters at the center and at the edge. Therefore, labor market equilibrium requires that

$$E_c + E_b + W = P. \tag{6}$$

We are now ready to define a competitive equilibrium for this city:

Given P, a competitive city equilibrium is a set of scalars $\{E_c,\ E_b,\ E_{cc,}\ E_{bb},\ E_{cb},\ E_{bc},\ L_b,\ R_c,\ \overline{F},\ F_{cc,}\ F_{bb},\ F_{cb},\ F_{bc}\}$ such that:

1. Agents solve (2), managers solve (1), and $w = \overline{F} = F_{ij}$ for all firms of type ij that operate in location, $i, j \in \{c, b\}$.

2. Equilibrium conditions (3), (4), (5), and (6) are satisfied.

3. Land available at the center is given by L_c, and land rates at the boundary are given by $R_b = \overline{R}$.

Observe that the number of establishments at the center is given by

$$S_c = E_{cc} + E_{cb} + E_{bc}, \tag{7}$$

where S_c counts integrated production units at the center, E_{cc}, headquarters at the center used by managers who operate plants in the periphery, E_{cb}, and production plants at the center run by managers residing at the boundary, E_{bc}. Similarly, the number of establishments at the edge is defined as

$$S_b = E_{bb} + E_{cb} + E_{bc}.$$

Within the framework of our model, average establishment sizes at the center and at the edge are given by $(L_c + E_c)/S_c$ and $(L_b + E_b)/S_b$ respectively. Note that establishments are of only three sizes: size one in the case of headquarters of non-integrated firms, size $\alpha\underline{\delta}$ in the case of the operation plant of a non-integrated firm, and size $1 + \alpha\overline{\delta}$ in the case of integrated firms.

The model we have just laid out potentially yields different types of equilibria. These types correspond to different sets of firms (i.e. integrated or not) operating in different areas of the city. As we now show, the fact that spans of control differ across integrated and non-integrated firms rules out equilibria where all types of firms coexist. In essence, if differences in spans of control are such that a firm finds worth it to locate its headquarters at the center and its production plant at the edge, then the reverse cannot be true. We formalize this result in the next proposition. All proofs are included in the Appendix.

Proposition 1 *There are no equilibria where both integrated and non-integrated firms operate at all locations.*

Of the remaining cases, the one corresponding to the type of city most often encountered in the data has most headquarters locating at the center (which in fact defines what we call the center and what is defined as a central county in the data). We now show that this case exists as an equilibrium of our model under a mild parameter restriction. In this equilibrium, there are no firms whose headquarters are at the edge but whose production plants reside at the center. Thus, the equilibrium we have just described is such that

$$\overline{F} = F_{cc} = F_{cb} = F_{bb} = w \text{ and } F_{bc} < \overline{F}, \tag{8}$$

so that $E_{cc}, E_{cb}, E_{bb} > 0$ and $E_{bc} = 0$. Since land rents are much lower at the edge than at the center in the data, firms that have operation plants at the center and headquarters at the edge are indeed very rare. Perhaps the most compelling reason to focus on this type of equilibrium is Fact 9. This fact shows that the number of managers is very tightly connected to the number of establishments, particularly at the center. In the equilibrium with $E_{bc} = 0$, the number of managers and establishments will move one for one at the center, as Figure 9a suggests. This will not be the case at the edge, however, where some firms will be operation plants. As in the model, Fact 9 shows that the changes in managers are related to smaller proportional changes in establishments at the edge. Hence, all results below are proven only for this case.

3.4 Equilibrium allocation

Given the restriction $E_{bc} = 0$, we now construct an equilibrium allocation for our model. From (3), we know that $E_b = E_{bb}$ since $E_{bc} = 0$, and that $E_{cb} = E_c - E_{cc}$ where, by equation (4), $E_{cc} = L_c / \left(\alpha\overline{\delta}\right)$. Then, the number of workers in the city is given by

$$L_c + L_b = W$$

and so

$$L_c \left(1 - \frac{\underline{\delta}}{\overline{\delta}}\right) + E_c \alpha \underline{\delta} + E_b \alpha \overline{\delta} = W \tag{9}$$

Condition (8) implies that integrated and non-integrated firms at the center earn equal profits,

$$(AE_c - w - R_c)\alpha\overline{\delta} = (AE_c - w - \overline{R})\alpha\underline{\delta}, \tag{10}$$

as do integrated firms across locations,

$$(AE_c - w - R_c)\alpha\overline{\delta} = (AE_b - w - \overline{R})\alpha\overline{\delta}$$

which implies that

$$E_b = E_c + \frac{\overline{R} - R_c}{A}. \tag{11}$$

Equality between rents and wages then implies that

$$(AE_c - w - R_c)\alpha\overline{\delta} = w. \tag{12}$$

From equations (10) and (12), it follows that

$$(AE_c - w - R_c)\alpha\overline{\delta} = w = (AE_c - w - \overline{R})\alpha\underline{\delta}.$$

Consequently,

$$\overline{R} - R_c = \left(\frac{1}{\alpha\overline{\delta}} - \frac{1}{\alpha\underline{\delta}}\right) w = \underbrace{-\left(\frac{\overline{\delta} - \underline{\delta}}{\alpha\overline{\delta}\underline{\delta}}\right)}_{\Lambda<0} w, \tag{13}$$

so that $R_c > \overline{R}$ under our maintained assumptions regarding the span of control, $\overline{\delta} > \underline{\delta}$. That is, land rents are larger at the center than at the edge. This implication results from the assumption that $E_{bc} = 0$. Moreover, the fact that in most cities land rents decrease as we move away from the center is consistent with our focusing on an equilibrium with this feature.

From equation (12), and substituting for R_c using (13), we obtain, after some manipulations,

$$\begin{aligned} E_c &= \frac{1 + \frac{1}{\alpha\overline{\delta}}}{A} w + \frac{R_c}{A}, \\ &= \frac{1 + \frac{1}{\alpha\underline{\delta}}}{A} w + \frac{\overline{R}}{A} \end{aligned} \tag{14}$$

which implies that $E_c'(w) > 0$, which is to say that the number of managers at the center increases with city wages. Using (13) and equation (11), we can solve for the set of managers at the edge as a function of wages,

$$\begin{aligned} E_b &= E_c + \left(\frac{\Lambda}{A}\right) w \\ &= \frac{1 + \frac{1}{\alpha\overline{\delta}}}{A} w + \frac{\overline{R}}{A}. \end{aligned} \tag{15}$$

Therefore, the number of managers at the edge also increases with wages, but at a slower rate since the rent differential decreases with wages which reduces the incentives to locate at the boundary.

Now consider the market clearing equation (6) given by,

$$L_c\left(1-\frac{\delta}{\overline{\overline{\delta}}}\right)+E_c\alpha\underline{\delta}+E_b\alpha\overline{\delta}=W=P-E_c-E_b$$

Substituting for the number of managers in both regions, the demand for workers becomes

$$\begin{aligned} W &= L_c\left(1-\frac{\delta}{\overline{\overline{\delta}}}\right)+E_c\alpha\underline{\delta}+E_b\alpha\overline{\delta} \\ &= L_c\left(1-\frac{\delta}{\overline{\overline{\delta}}}\right)+\left(\frac{\alpha\underline{\delta}+\alpha\overline{\delta}}{A}\right)\overline{R}+\frac{2+\alpha\underline{\delta}+\alpha\overline{\delta}}{A}w \end{aligned} \tag{16}$$

which is linear and increasing in the wage. The supply of workers is given by

$$P-E_c-E_b=\left(P-\frac{2\overline{R}}{A}\right)-\frac{2+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}}{A}w, \tag{17}$$

which again is linear but decreasing in w. The equilibrium wage can then be found by equating (16) and (17). That is

$$AP-AL_c\left(1-\frac{\delta}{\overline{\overline{\delta}}}\right)-\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)\overline{R}=\left(4+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}\right)w$$

so that city wages are given by

$$w=\frac{AP-AL_c\left(1-\frac{\delta}{\overline{\overline{\delta}}}\right)-\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)\overline{R}}{4+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}}. \tag{18}$$

With the equilibrium wage in hand, we can then solve for all equilibrium variables of the model. For the comparative statics in the next section, it is helpful to denote the denominator of (18) as $D>0$. Moreover, an equilibrium of the type we are interested in exists only if

$$\frac{P}{L_c}>\left(1-\frac{\delta}{\overline{\overline{\delta}}}\right)+\frac{\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)\overline{R}}{AL_c} \tag{19}$$

This restriction essentially requires city population densities that are large enough to make the creation of a city profitable in this location given land rents at the edge. Population densities must also be large enough so that agglomeration effects guarantee that some non-integrated firm' headquarters choose to locate at the center. The following proposition is a direct result of the equilibrium wage computed above.

Proposition 2 *The equilibrium wage is an increasing function of population size, P, city-wide productivity, A, and the supply of land at the center, L_c. The equilibrium wage is a decreasing function of the span of control parameter, α, and edge land rents, $\overline{R}$.*

Note that the wage increases with population size. While this finding matches the fact that wages are generally higher in larger cities, it nevertheless seemingly conflicts with the standard intuition that wages fall as the supply of workers increases. There are three characteristics of our set up that contribute to overturning this intuition. First, the wage is the compensation of workers but, since all agents in the city are identical, it also represents profits of entrepreneurs or managers. Hence, since an increase in overall population creates new managers, the demand for workers also increases. Second, the production externality in our framework implies that the more managers operate in a given location, the higher the productivity (output per worker) of all firms in that location. As population grows, and more firms operate in the city, this effect contributes to raising manager rents and worker wages. As we have just mentioned, the prediction regarding wages and city size can be directly contrasted with data. In 2003, for instance, the average wage in the largest five cities in our sample was \$42,976, as compared to just \$34,340 for the five smallest cities.

The effect of the model's other parameters on wages are more standard. Wages increase with city-wide productivity, and also increase with the amount of land available at the center. The latter effect reflects the fact that as L_c increases, more integrated firms choose to locate at the center, which raises production externalities at the center and reduces the cost of locating in that area. As rents at the boundary increase, the advantage of setting up a non-integrated firm fall, which reduces externalities at the center and, therefore, wages. Finally, as firms' span of control, α, increases, firms become larger, less agents become managers, and externalities fall along with wages and managerial rents.

4. ADDRESSING THE STYLIZED FACTS

This section shows that the model we have developed naturally leads to the changes in city structure laid out in Section 2. From Fact 1, we know that population growth was positive in virtually all cities in our sample. Recall that the first set of facts referred to population size both at the center and at the edge. Since, in our model, agents live and work at the same location, the model's predictions concerning the growth in residents and employment are identical. Therefore, if we can show that as population grows, employment increases both at the center and at the edge (Fact 2), then the model will immediately satisfy Fact 3.

To address Fact 2, observe that two effects emerge as population grows. First, the number of managers at the center increases, as a result of the increase in agglomeration effects, which raises population at the center (Recall that the number of workers in the center is pinned down at level L_c by assumption). Second, since the number of managers at the boundary also increases as population grows, again as a result of larger externalities, so does the number of workers given the fixed span of control. Therefore, total employment must also increase at the edge. Note that a portion of the additional workers at the boundary will work for managers that head firms with headquarters at the center. Ultimately, this reasoning implies that the model is consistent with Fact 2.

Total employment at the center and at the edge is given by $E_c + L_c$ and $E_b + L_b$ respectively. Simple differentiation then leads to the following proposition, consistent with Facts 2 and 3. All the proofs in this section are relegated to the Appendix.

Proposition 3 *Total employment at the center and at the edge increases with population growth.*

The proposition above provides conclusions regarding the level of employment in both areas of the city. We are also interested in the share of employment in each area. As population grows, we know that the number of managers at the center increases. All the new managers, however, lead non-integrated firms. The reason is that the number of workers at the center cannot expand given the fixed amount of land and the technology that requires one unit of land per worker. Managers choose non-integrated rather than integrated firms because rents at the boundary do not grow with population, since they are pinned down by the opportunity cost of land, $\overline{R}$. In contrast, since the center section of the city remains constant, the price of land at the center increases which motivates some managers to send their operation plants to the edge. Thus, rents at the center rise until the number of center managers who choose to operate integrated firms reverts back to its initial equilibrium. In fact, as equation (13) makes clear, the increase in land rents at the center is proportional to the increase in wages.

If the share of managers at the center is larger than one half (which is indeed the case in the data on average), the increase in population at the center (caused solely by the increase in managers) is always smaller than the increase in the number of managers and workers at the edge. This reasoning implies that employment shares at the center fall with population growth. We formalize this result in the next proposition, consistent with Fact 4.

Proposition 4 *The employment share at the center decreases with population growth if the share of establishments at the center is larger than* $1/2$ *(that is* $\alpha\left(\overline{\delta}-\underline{\delta}\right)\overline{R} < AL_c\left(\alpha\underline{\delta}+1\right)$*).*

To summarize thus far, an increase in overall city population leads to findings for the levels and shares of employment in different areas of the city that are consistent with the data. In particular, population growth leads to an increase in employment levels everywhere, but to a shift in employment from the center to the edge in shares. These results follow directly from firms having the opportunity to locate headquarters at the center and operation plants at the edge.

It is difficult to understand how the observed changes in levels and shares could be the result of forces that are not related to an expansion of city size (i.e. population growth). If other forces where responsible for these changes, and since one needs to introduce scale effects in order to generate cities, reductions in the share of employment at the center will generally lead to reductions in the level of employment as well. Hence, it seems that two elements are needed to obtain models that can reproduce these dimensions of the data. First, we need models where these changes are the result of city growth. Second, we need models where employment densities are endogenous. If densities were not endogenous, given that the center county area has not changed in the data, these models would necessarily predict employment at the center to remain constant.

The advantage of writing down a model in which agents' occupations are explicitly considered is that it has predictions for the effect of changes in exogenous variables on the location of different occupations within cities. In Section 2, we presented a set of facts that are related to the locations of agents working in different occupations. Specifically, we showed that the changes in manager shares at the center were generally larger than those in non-manager shares (Fact 5a). We also showed that the change in the manager to non-manager ratio was larger at the center than at the edge (Fact 5b). Both these facts imply that managers are increasingly concentrated at the edge relative to the boundary. Note that all these facts are expressed in terms of differences in shares and ratios between the center and the edge, and not in term of levels, shares, or ratios directly. Evidently, spans of control will depend on the different sets of industries operating in a city. Therefore, a model with only one industry is not suited to explain how the levels of managers and workers in different areas of the city are determined. Our model is also a model of only one city, not a system of cities. Hence, it is silent about cross-sectional differences between cities. This cross-sectional differences would in principle be the result of different values of L_c, $\overline{R}$, α, or A across cities. Other forces – such as transport and communication costs across cities or between cities and rural areas – may also increase, say, the total numbers of managers in the city beyond the increase in population growth (as in Duranton and Puga [2004]). Fact 5c shows that the fraction of urban population in management positions increased during this period. Our model is consistent with this fact, but these other dimensions working at a more aggregated level may reinforce it as well. Hence, we focus primarily on the predictions of our model for changes in the difference between the center and edge management shares as well as manager to non-manager ratios driven by city population growth.

Consider first the difference in the share of managers and non-managers at the center. We know from the analysis we just carried out that population growth leads to an increase in the number of managers at the center. In fact, it also leads to an increase in the number of managers at the boundary. Some of the new managers establish themselves at the center because of the production externalities caused by all the managers that locate there. Others take advantage of the larger spans of control, as well as lower rents, at the boundary. However, the share of managers at the center increases unambiguously because managers do not use land and so do not drive land rents up. As we argued earlier, all employees under the supervision of new entrepreneurs at the center work in operation plants located at the edge. In addition, all new managers at the boundary head integrated firms. Hence the share of workers at the center must decrease, therefore leading to an increase in the difference of manager and non-manager shares as the result of population growth.

It should be remarked that the increase in the share of managers at the center is actually a counter-factual implication of our model. Specifically, the theory over-emphasizes the concentration of managers at the center. This implication could be corrected if managers also had to rent land at the headquarter's location. This extension, however, would come at the cost of a much more complicated setup. In any case, our take is that the decrease in manager share at the center is the result of the overall increase in the number of agents in management occupations, as in Fact 5c.

Our model also predicts that the difference in the ratio of managers to non-managers at the center relative to the edge increases with population growth. Since the number of managers at the center increases and the number of workers is fixed, it is clear that this ratio increases at the center. At the edge, since the number of workers increases in part because of the increase in the managers of non-integrated firms at the center, the ratio always decreases. Together these results directly lead to Fact 5b. As population increases, all of the new managers at the center run non-integrated firms. Given that non-integrated firms have fewer workers per manager (smaller spans of control), this leads to an increase in the share of city residents that become managers, as in Fact 5c. These results are stated formally in the next proposition, consistent with Fact 5.

Proposition 5 *The number of managers per resident increases with population growth. Furthermore, the difference between the manager and non-manager share at the center increases with population growth; and the difference between the ratio of managers to non-managers at the center relative to the edge increases with population growth.*

Our model also has predictions for the number of establishments at the center and at edge of the city. First, observe that the number of establishments at the center is equal to the number of managers at the center, since establishments at the center are either headquarters or non-integrated firms, but under our assumptions never just operation plants. It follows immediately that the number of establishments at the center increases with population growth. Furthermore, since new managers at the center operate only non-integrated firms, the boundary also sees an increase in operation plants. These last two findings are consistent with Fact 6. Because every new manager at the center is associated with an additional operation plant at the edge, and the edge also experiences entry of new integrated firms following population growth, the number of establishments at the edge must increase and, in fact, must increase by more than the increase in center establishments. This implies that the share of establishment at the center must fall, as in Fact 7. As in Proposition 4, this reasoning assumes that the share of establishments at the center is larger than one half, as is the case on average in the data. We impose this condition to formalize the results given in the next proposition, consistent with Facts 6 and 7.

Proposition 6 *The number of establishments at the center and at the edge increase with population growth, and the number of establishments increases more at the edge than at the center. Furthermore, if the share of establishments at the center is larger than* $1/2$ *(that is* $\alpha\left(\overline{\delta}-\underline{\delta}\right)\overline{R} < AL_c\left(\alpha\underline{\delta}+1\right)$*) the share of establishments at the center decreases with population growth.*

As discussed earlier, establishments are of three different sizes in our set up: Headquarters of size one, operation plants of size $\alpha\underline{\delta}$, and integrated firms of size $1+\alpha\overline{\delta}$. Hence, the specific combination of firms of each type in a given region of the city will determine average establishment size in that region. Since there are no operation plants in the center, firms can only be of size one or $1+\alpha\overline{\delta}$ in that section of the city. As population increases, the number of managers at the center increases and so does the share of establishments of size one. Therefore, population growth contributes to a decrease in average firm size in the center region, as in Fact 8. In contrast, the boundary comprises only establishments of size $\alpha\underline{\delta}$ and $1+\alpha\overline{\delta}$. First, this implies that the average size of establishments is larger at the edge than at the center, unless most firms are production plants and many integrated firms reside at the center. The latter case would make our model consistent with the larger establishment sizes at the center that we observe in 1980. As population grows, the set of establishments that are production plants increases at the edge , and so does the set of integrated firms. In the next proposition, we show that the increase in the number of production plants dominates, thereby leading to a decrease in firm size at the edge, consistent with Fact 8.

Proposition 7 *Population growth leads to a decrease in the average size of establishments both at the center and at the edge.*

Thus, our model predicts that population growth reduces establishment sizes in both regions, due to composition effects. The data, however, shows that the changes in both regions are negative for about half of the cities in our sample. What happened to firms in the other cities? A possible answer given the theory we have laid out is that decreases in communication costs have led to larger spans of control (an increase in α) and, therefore, larger firms throughout the city. The evidence suggests that this phenomenon did not dominate changes in average firm size in most cities in the 80's, but may nevertheless be important in more recent time periods for some cities (as argued by Garicano and Rossi-Hansberg [2003] for the late 1990s).

5. CHANGES IN CITY STRUCTURE AND POPULATION GROWTH

In Section 2, we presented a set of stylized facts on the evolution of city structure. We then argued in the two subsequent sections that observed increases in population alone could help rationalize those facts. At this stage, therefore, it is natural to ask whether one could establish the implications of our model more directly in the data? As a first pass, we can use the data to assess whether the changes in city structure presented in Facts 2 through 9 are in fact correlated with population growth. However, note that one should be cautious in the interpretation of such an empirical exercise. First, the theory predicts that the effect of changes in population size should lead to Facts 2 through 9 only if all cross-sectional characteristics of cities are properly controlled for. In particular, the theory has predictions for the sign of these correlations after controlling for center county land sizes (L_c), land rents at the boundary ($\overline{R}$), spans of control (α, $\overline{\delta}$ and $\underline{\delta}$), and productivity (A), as well as any changes in these variables during the 80's. At this point, we do not have residential land rents at the boundary for 1980, or a suitable proxy. However, we can control for L_c and, to some degree, for productivity as well as spans of control using the 1980 level of per capita income and ratio of managers to population respectively. Since our theory also abstracts from available city infrastructure and other idiosyncratic city characteristics, we take city age into consideration by using the decade in which the city became one of the largest 50 cities in the US. This last variable helps but cannot obviously capture all the cross-sectional characteristics omitted from the model. Because of the size of our sample, we do not control for changes in any of these variables. Finally, a key problem with calculating simple correlations is that our theory does not predict a linear response of city structure to population changes. Despite these many caveats regarding the mapping between these correlations and our theoretical results, Table 1 presents encouraging results that our theory is broadly consistent with the framework introduced in this paper.

Table 1 presents correlations between population growth and the residuals obtained from running an OLS regression of the various changes in city structure laid out in Section 2 against the controls discussed above. Observe that all but the last two correlations, the ones related to establishment size, have the sign predicted by our theory. Some of these correlations are admittedly low. Nevertheless, our framework does surprisingly well given that increases in only one independent variable, namely population growth, were shown to be consistent with eleven changes in the internal structure of cities. The wrong sign on the correlation of center and edge establishment sizes with population growth is somewhat disappointing but understandable. The model does not contain a force that would lead to larger firms in larger cities. Here, firms are larger only if they decide to integrate but there are no differences across integrated firms. In practice, larger cities have larger firms partly because demand for the firm's varieties is larger, a dimension from which we have abstracted.

Table 1

Correlations with Population Growth

	Population Growth
Center Population Growth	0.511
Edge Population Growth	0.558
Center Employment Growth	0.496
Edge Employment Growth	0.538
Change in Edge Population Share	0.127
Change in Edge Employment Share	0.155
Change in Management - Non-Management Shares	0.027
Change in Management over Non-Management Ratio	0.053
Change in Center Establishments	0.381
Change in Edge Establishments	0.572
Change in Center Establishment Share	-0.188
Change in Center Establishment Size	0.467
Change in Edge Establishment Size	0.129

6. CONCLUSIONS

This paper makes three distinct contributions. First we document a set of facts regarding changes in urban structure experienced by US cities in the 80's. These facts include overall population growth; an increase in residents and employment at the center and city boundaries; a reduction in the share of employment and residents in center region of cities; a concentration of managers relative to non-managers at the center; an increase in establishments in both areas of the city but a decrease in establishment shares at the center; and a decline in establishment size both at the center and at the edge of cities. Second, we propose a theory that incorporates firms' location and integration decisions and characterize the implications of such a theory for urban structure. Third, we show that population growth alone is consistent with the set of changes observed in the 80's, thereby highlighting the effects of population growth on urban structure.

The theory we present has urban policy implications that differ from more standard theories of urban structure. In particular, we provide a framework that could potentially be used to analyze the kinds of policies aimed at "reviving city centers" that have been put in practice in many cities across the US. Given that our framework includes agglomeration forces in the form of externalities, we know that some urban policies can improve equilibrium allocations in our setup. However, the specific type of policy in place is critical. For example, Au and Henderson (2004) show that restrictions on urban migration have had important efficiency costs in China. For now, the question of whether zoning restrictions or location subsidies, as in Rossi-Hansberg (2004), are optimal in our setup, and in general the design of these policies, is left for future research.

In order to underscore the importance of firms' location decisions, as well as their decision regarding whether or not to integrate their operations, our model abstracts from important elements of cities typically emphasized in the urban literature. One such element is a spatial setup in which multiple sub-centers may arise (see Fujita and Ogawa [1982], and Lucas and Rossi-Hansberg [2002]). Other dimensions, such as the effect of durable housing structures, as in Glaeser and Gyourko (2004), and urbanization patterns in a system of cities, as in Henderson (2003), and Henderson and Wang (2004), are undoubtedly important as well. In addition, our theory focuses on one particular type of agglomeration force. However, as argued by Rosenthal and Strange (2003), different agglomeration forces interact in metropolitan areas.

One could, in principle, study any of these forces along with the firm's location and integration decisions we emphasize. The explanatory power gained by incorporating these firm decisions with respect to the facts we document in this paper will, hopefully, push the urban literature to add these margins to the rich set of frameworks available.

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APPENDIX

Proof of Proposition 1: The proof proceeds by contradiction. Suppose that both integrated and non-integrated firms exist in both areas of the city. Then by (2), it must be the case that

$$F_{bb} = F_{cc} = F_{cb} = F_{bc}. \tag{20}$$

The fact that $F_{cc} = F_{bb}$ implies that

$$\begin{aligned} (AE_c - w - R_c)\alpha\overline{\delta} &= (AE_b - w - \overline{R})\alpha\overline{\delta} \text{ or} \\ AE_c - AE_b &= R_c - \overline{R}. \end{aligned} \tag{21}$$

Similarly, the fact that $F_{cb} = F_{bc}$ implies that

$$\begin{aligned} (AE_c - w - \overline{R})\alpha\underline{\delta} &= (AE_b - w - R_c)\alpha\underline{\delta} \text{ or} \\ AE_c - AE_b &= \overline{R} - R_c. \end{aligned} \tag{22}$$

Equations (21) and (22) can only hold if $R_c = \overline{R} = R$, in which case $E_c = E_b = E$. It follows that profits for an integrated firm are $(AE - w - R)\alpha\overline{\delta}$ while those of a non-integrated firm are $(AE - w - R)\alpha\underline{\delta}$. That is, $F_{cc} = F_{bb} > F_{cb} = F_{bc}$ which contradicts (20).■

Proof of Proposition 2: Simple partial derivatives imply that

$$\begin{aligned} \frac{\partial w}{\partial P} &= \frac{A}{D} > 0, & \frac{\partial w}{\partial A} &= \frac{P - L_c\left(1 - \frac{\underline{\delta}}{\overline{\delta}}\right)}{D} > 0, \\ \frac{\partial w}{\partial L_c} &= \frac{A}{D}\left(1 - \frac{\underline{\delta}}{\overline{\delta}}\right) > 0, & \frac{\partial w}{\partial \overline{R}} &= \frac{-\left(2 + \alpha\underline{\delta} + \alpha\overline{\delta}\right)}{D} < 0, \end{aligned}$$

and

$$\frac{\partial w}{\partial \alpha} = -\frac{\left(\underline{\delta} + \overline{\delta}\right)\overline{R}}{D} - w\frac{\underline{\delta} + \overline{\delta} - \frac{1}{\alpha^2\underline{\delta}} - \frac{1}{\alpha^2\overline{\delta}}}{D} < 0$$

The sign of the last term is guaranteed since the span of control of integrated and non-integrated firms is greater than $\alpha\overline{\delta} > \alpha\underline{\delta} > 1$.■

Proof of Proposition 3: Employment at the city center is given by

$$E_c + L_c = \frac{1 + \frac{1}{\alpha\underline{\delta}}}{A} w + \frac{\overline{R}}{A} + L_c.$$

Differentiating with respect to P, we obtain that

$$\frac{\partial \left(E_c + L_c\right)}{\partial P} = \frac{1 + \frac{1}{\alpha\underline{\delta}}}{D} > 0,$$

which captures the increase in population at the center. Total employment at the edge is given by

$$\begin{aligned} E_b + L_b &= \left(E_c - \frac{L_c}{\alpha\overline{\delta}}\right) \alpha\underline{\delta} + E_b \left(1 + \alpha\overline{\delta}\right) \\ &= \left(E_c - \frac{L_c}{\alpha\overline{\delta}}\right) \alpha\underline{\delta} + \left(E_c + \left(\frac{\Lambda}{A}\right) w\right) \left(1 + \alpha\overline{\delta}\right) \\ &= \frac{\overline{R}}{A} \left(1 + \alpha\underline{\delta} + \alpha\overline{\delta}\right) - L_c \frac{\underline{\delta}}{\overline{\delta}} + \frac{3 + \alpha\underline{\delta} + \alpha\overline{\delta} + \frac{1}{\alpha\overline{\delta}}}{A} w \end{aligned}$$

so that

$$\frac{\partial \left(E_b + L_b\right)}{\partial P} = \frac{3 + \alpha\underline{\delta} + \alpha\overline{\delta} + \frac{1}{\alpha\overline{\delta}}}{D} > 0$$

which yields the increase in population at the boundary.■

Proof of Proposition 4: Employment share at the center is given by

$$\frac{E_c + L_c}{P} = \left(\frac{1 + \frac{1}{\alpha\underline{\delta}}}{A} w + \frac{\overline{R}}{A} + L_c\right) \frac{1}{P}$$

Hence, the derivative with respect to population is

$$\begin{aligned} \frac{\partial \frac{E_c + L_c}{P}}{\partial P} &= -\frac{\overline{R}}{P^2 A} \left(1 - \left(2 + \alpha\underline{\delta} + \alpha\overline{\delta}\right) \frac{1 + \frac{1}{\alpha\underline{\delta}}}{D}\right) - \frac{L_c}{P^2} \left(1 - \frac{1 + \frac{1}{\alpha\underline{\delta}}}{D} \left(1 - \frac{\underline{\delta}}{\overline{\delta}}\right)\right) \\ &< -\frac{\overline{R}}{A} \frac{1}{P^2} \left(\frac{\begin{array}{c} \left(\alpha^2 \underline{\delta}\overline{\delta} - 1\right) \left(\overline{\delta} - \underline{\delta}\right) + \alpha^3 \underline{\delta}\overline{\delta} \left(\overline{\delta}^2 - \underline{\delta}^2\right) \\ + \alpha\overline{\delta}^2 \left(\alpha\underline{\delta} - 1\right) - \alpha\underline{\delta}^2 \left(\alpha\underline{\delta} - \frac{\overline{\delta}}{\underline{\delta}}\right) + \alpha\underline{\delta} \left(\overline{\delta} - \underline{\delta}\right) \end{array}}{\left(4\alpha\underline{\delta}\overline{\delta} + \alpha^2 \underline{\delta}^2 \overline{\delta} + \alpha^{2\overline{\delta}2} \underline{\delta} + \overline{\delta} + \underline{\delta}\right) \left(\alpha\underline{\delta} + 1\right)} \right) < 0 \end{aligned}$$

where the first inequality comes from imposing that the share of establishments at the center is larger than 1/2 or

$$\frac{\left(\frac{\overline{\delta}}{\underline{\delta}}-1\right)}{\left(1+\frac{1}{\alpha\underline{\delta}}\right)}\frac{\overline{R}}{A}<L_c,$$

and the second from $\overline{\delta}-\underline{\delta}>0$, which yields the result.■

Proof of Proposition 5: The share of managers at the center is given by

$$\begin{aligned}\frac{E_c}{E_c+E_b} &= \frac{\frac{1+\frac{1}{\alpha\underline{\delta}}}{A}w+\frac{\overline{R}}{A}}{2E_c+\left(\frac{\Lambda}{A}\right)w} \\ &= \frac{\left(1+\frac{1}{\alpha\underline{\delta}}\right)w+\overline{R}}{\left(2+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}\right)w+2\overline{R}}\end{aligned}$$

Hence

$$\begin{aligned}\frac{\partial\frac{E_c}{E_c+E_b}}{\partial P} &= \frac{\left(\left(1+\frac{1}{\alpha\underline{\delta}}\right)-\left(\frac{E_c}{E_c+E_b}\right)2\left(1+\frac{1}{2\alpha\underline{\delta}}+\frac{1}{2\alpha\overline{\delta}}\right)\right)\frac{\partial w}{\partial P}}{2\left(\left(1+\frac{1}{2\alpha\underline{\delta}}+\frac{1}{2\alpha\overline{\delta}}\right)w+\overline{R}\right)} \\ &> \frac{\left(\left(1+\frac{1}{\alpha\underline{\delta}}\right)-\frac{\left(1+\frac{1}{\alpha\underline{\delta}}\right)w}{\left(1+\frac{1}{2\alpha\underline{\delta}}+\frac{1}{2\alpha\overline{\delta}}\right)w}\left(1+\frac{1}{2\alpha\underline{\delta}}+\frac{1}{2\alpha\overline{\delta}}\right)\right)\frac{\partial w}{\partial P}}{2\left(\left(1+\frac{1}{2\alpha\underline{\delta}}+\frac{1}{2\alpha\overline{\delta}}\right)w+\overline{R}\right)}=0\end{aligned}$$

which captures the increase in manager share at the center. The share of workers at the center is given by

$$\frac{L_c}{L_c+L_b}=\frac{L_c}{W}=\frac{L_c}{L_c\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)+\left(\frac{\alpha\underline{\delta}+\alpha\overline{\delta}}{A}\right)\overline{R}+\frac{2+\alpha\underline{\delta}+\alpha\overline{\delta}}{A}w}.$$

Therefore,

$$\frac{\partial\frac{L_c}{L_c+L_b}}{\partial P}=-\left(\frac{L_c}{W^2}\frac{2+\alpha\underline{\delta}+\alpha\overline{\delta}}{A}\right)\frac{\partial w}{\partial P}<0$$

which captures the fall in worker share at the center. In order for the decrease in worker share to be larger than the decrease in manager share as population grows, it must be that

$$\left(\frac{\frac{L_c}{L_c\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)+\left(\frac{\alpha\underline{\delta}+\alpha\overline{\delta}}{A}\right)\overline{R}+\frac{2+\alpha\underline{\delta}+\alpha\overline{\delta}}{A}w}\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)}{\left(AL_c\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)+\left(\alpha\underline{\delta}+\alpha\overline{\delta}\right)\overline{R}+\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)w\right)}\right)$$

$$> \quad \frac{\left(\frac{\left(1+\frac{1}{\alpha\underline{\delta}}\right)w+\overline{R}}{\left(\left(1+\frac{1}{2\alpha\underline{\delta}}+\frac{1}{2\alpha\overline{\delta}}\right)w+\overline{R}\right)}\left(1+\frac{1}{2\alpha\underline{\delta}}+\frac{1}{2\alpha\overline{\delta}}\right)-\left(1+\frac{1}{\alpha\underline{\delta}}\right)\right)}{2\left(\left(1+\frac{1}{2\alpha\underline{\delta}}+\frac{1}{2\alpha\overline{\delta}}\right)w+\overline{R}\right)},$$

or, after some manipulation,

$$\frac{-\alpha\underline{\delta}\overline{\delta}\overline{R}\left(\overline{\delta}-\underline{\delta}\right)\left(L_cA\left(\overline{\delta}-\underline{\delta}\right)+\left(\underline{\delta}+\overline{\delta}\right)\overline{R}\alpha\overline{\delta}+\left(2+\alpha\underline{\delta}+\overline{\delta}\alpha\right)w\overline{\delta}\right)}{\left(\left(2\alpha\underline{\delta}\overline{\delta}+\overline{\delta}+\underline{\delta}\right)w+2\overline{R}\alpha\underline{\delta}\overline{\delta}\right)^2}$$

$$< \quad \frac{L_c\overline{\delta}^2A\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)}{\left(L_cA\left(\overline{\delta}-\underline{\delta}\right)+\left(\underline{\delta}+\overline{\delta}\right)\alpha\overline{\delta}+\left(2+\alpha\underline{\delta}+\overline{\delta}\alpha\right)\overline{\delta}w\right)}.$$

This inequality is trivially satisfied since $\overline{\delta}-\underline{\delta}>0$. Hence,

$$\frac{\partial\frac{E_c}{E_c+E_b}}{\partial P}>\frac{\partial\frac{L_c}{L_c+L_b}}{\partial P}$$

which leads to the first result.

The next claim in the proposition states that $E_c/L_c - E_b/L_b$ increases with P. We analyze each term in turn. Since L_c is fixed, the statement implies that E_c increases with P, which is clearly true given that

$$\frac{\partial\left(\frac{E_c}{L_c}\right)}{\partial P}=\frac{1+\frac{1}{\alpha\underline{\delta}}}{L_cD}>0.$$

To analyze the second term, notice that E_b/L_b is given by

$$\frac{E_b}{L_b} = \frac{E_b}{\left(E_c-\frac{L_c}{\alpha\overline{\delta}}\right)\alpha\underline{\delta}+E_b\alpha\overline{\delta}}$$

$$> \frac{\left(1+\frac{1}{\alpha\overline{\delta}}\right)w+\overline{R}}{\left(2+\alpha\overline{\delta}+\alpha\underline{\delta}\right)w+\alpha\left(\underline{\delta}+\overline{\delta}\right)\overline{R}}$$

Hence, $\frac{A}{D}$

$$
\begin{aligned}
\frac{\partial \frac{E_b}{L_b}}{\partial P} &= \frac{1}{DL_b}\left(1+\frac{1}{\alpha\overline{\delta}}-\frac{E_b}{L_b}\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)\right) \\
&< \frac{1}{DL_b\alpha\overline{\delta}}\left(1-\frac{\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)w+2\alpha\overline{\delta}\overline{R}}{\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)w+\alpha\left(\underline{\delta}+\overline{\delta}\right)\overline{R}}\right)<0.
\end{aligned}
$$

The last result we need to prove is that $\left(E_c+E_b\right)/P$ increases with P. To show this first note that

$$
\frac{E_c}{P}=\left(\frac{1+\frac{1}{\alpha\underline{\delta}}}{A}w+\frac{\overline{R}}{A}\right)\frac{1}{P}.
$$

Hence, the derivative with respect to population is

$$
\begin{aligned}
\frac{E_c}{P} &= \frac{1+\frac{1}{\alpha\underline{\delta}}}{D}\left(1-\frac{L_c}{P}\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)\right)+\left(1-\frac{1+\frac{1}{\alpha\underline{\delta}}}{D}\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)\right)\frac{\overline{R}}{AP} \\
\frac{\partial \frac{E_c}{P}}{\partial P} &= \frac{1}{P^2}\left(\frac{1+\frac{1}{\alpha\underline{\delta}}}{D}L_c\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)-\left(1-\frac{1+\frac{1}{\alpha\underline{\delta}}}{D}\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)\right)\frac{\overline{R}}{A}\right) \\
&= \frac{1}{P^2}\left(\frac{1+\frac{1}{\alpha\underline{\delta}}}{D}L_c\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)+\left(\frac{3+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{2}{\alpha\underline{\delta}}+\frac{\overline{\delta}}{\underline{\delta}}}{4+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}}-1\right)\frac{\overline{R}}{A}\right)>0
\end{aligned}
$$

where the inequality follows from $\overline{\delta}>\underline{\delta}$. Also note that

$$
\begin{aligned}
\frac{E_b}{P} &= \frac{1+\frac{1}{\alpha\overline{\delta}}}{PA}w+\frac{\overline{R}}{PA} \\
&= \frac{1+\frac{1}{\alpha\overline{\delta}}}{D}\left(1-\frac{L_c}{P}\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)-\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)\frac{\overline{R}}{PA}\right)+\frac{\overline{R}}{PA}
\end{aligned}
$$

Hence,

$$
\frac{\partial \frac{E_b}{p}}{\partial P}=\frac{1}{P^2}\left(\frac{\left(1+\frac{1}{\alpha\overline{\delta}}\right)\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)}{D}L_c+\left(\frac{3+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{2}{\alpha\overline{\delta}}+\frac{\underline{\delta}}{\overline{\delta}}}{4+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}}-1\right)\frac{\overline{R}}{P^2A}\right),
$$

and so

$$
\begin{aligned}
\frac{\partial \frac{E_c-E_b}{P}}{\partial P} &= \frac{\left(2+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}\right)\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)}{P^2D}L_c \\
&\quad+\left(\frac{6+2\alpha\underline{\delta}+2\alpha\overline{\delta}+\frac{2}{\alpha\underline{\delta}}+\frac{2}{\alpha\overline{\delta}}+\left(\frac{\overline{\delta}}{\underline{\delta}}+\frac{\underline{\delta}}{\overline{\delta}}\right)}{4+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}}-2\right)\frac{\overline{R}}{P^2A}
\end{aligned}
$$

which is positive if

$$\frac{\overline{\delta}}{\underline{\delta}} + \frac{\underline{\delta}}{\overline{\delta}} > 2$$

or if

$$\overline{\delta}^2 - 2\overline{\delta}\underline{\delta} + \underline{\delta}^2 = \left(\overline{\delta} - \underline{\delta}\right)^2 > 0$$

which is trivially true. ■

Proof of Proposition 6: The total number of establishments at the center is given by

$$S_c = E_c + E_{bc} = E_c = \frac{1 + \frac{1}{\alpha\underline{\delta}}}{A} w + \frac{\overline{R}}{A}.$$

It follows that

$$\frac{\partial S_c}{\partial P} = \frac{1 + \frac{1}{\alpha\underline{\delta}}}{D} > 0,$$

which depicts the rise in establishments at the center. The number of establishments at the edge is given by

$$\begin{aligned} S_b &= E_b + E_{cb} = E_b + E_c - E_{cc} \\ &= \left(\frac{\frac{1}{\alpha\overline{\delta}} - \frac{1}{\alpha\underline{\delta}}}{A}\right) w + 2E_c - \frac{L_c}{\alpha\overline{\delta}} \\ &= \frac{2 + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\delta}}}{A} w + 2\frac{\overline{R}}{A} - \frac{L_c}{\alpha\overline{\delta}}. \end{aligned}$$

Therefore

$$\begin{aligned} \frac{\partial S_b}{\partial P} &= -\left(\frac{\frac{1}{\alpha\underline{\delta}} - \frac{1}{\alpha\overline{\delta}}}{A}\right) w + 2\left(\frac{1 + \frac{1}{\alpha\underline{\delta}}}{A} w + \frac{\overline{R}}{A}\right) - \frac{L_c}{\alpha\overline{\delta}} \\ &= \frac{2 + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\delta}}}{D} > 0 \end{aligned}$$

which establishes the increase in establishments at the edge. It is also the case that

$$\frac{\partial S_c}{\partial P} = \frac{1 + \frac{1}{\alpha\underline{\delta}}}{D} < \frac{2 + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\delta}}}{D} = \frac{\partial S_b}{\partial P},$$

so that the number of establishments increases more at the edge. Finally, the share of establishments at the center is given by

$$
\begin{aligned}
\frac{S_c}{S_c+S_b} &= \frac{\frac{1+\frac{1}{\alpha\underline{\delta}}}{A}w+\frac{\overline{R}}{A}}{\frac{1+\frac{1}{\alpha\underline{\delta}}}{A}w+\frac{\overline{R}}{A}-\left(\frac{\frac{1}{\alpha\underline{\delta}}-\frac{1}{\alpha\overline{\delta}}}{A}\right)w+2\left(\frac{1+\frac{1}{\alpha\underline{\delta}}}{A}w+\frac{\overline{R}}{A}\right)-\frac{L_c}{\alpha\overline{\delta}}} \\
&= \frac{\frac{1+\frac{1}{\alpha\underline{\delta}}}{A}w+\frac{\overline{R}}{A}}{\frac{3+\frac{1}{\alpha\overline{\delta}}+2\frac{1}{\alpha\underline{\delta}}}{A}w+3\frac{\overline{R}}{A}-\frac{L_c}{\alpha\overline{\delta}}}.
\end{aligned}
$$

It follows that

$$
\frac{\partial\frac{S_c}{S_c+S_b}}{\partial P}=\frac{1}{S_c+S_b}\left(\left(1-\frac{S_c}{S_c+S_b}\right)\frac{\partial S_c}{\partial P}-\left(\frac{S_c}{S_c+S_b}\right)\left(\frac{\partial S_b}{\partial P}\right)\right)
$$

which, given what we have just shown, is negative if

$$
\frac{S_c}{S_c+S_b}>1/2,
$$

arguably the empirically relevant case. In terms of the parameters of the model, the sufficient condition for the share of establishments at the center to decrease is such that the term

$$
\frac{\partial\frac{S_c}{S_c+S_b}}{\partial P}=-\frac{1}{\frac{3+\frac{1}{\alpha\overline{\delta}}+2\frac{1}{\alpha\underline{\delta}}}{A}w+3\frac{\overline{R}}{A}-\frac{L_c}{\alpha\overline{\delta}}}\left(\frac{S_c}{S_c+S_b}\frac{3+\frac{1}{\alpha\overline{\delta}}+2\frac{1}{\alpha\underline{\delta}}}{D}-\frac{1+\frac{1}{\alpha\underline{\delta}}}{D}\right)
$$

is negative. This is the case if

$$
\frac{S_c}{S_b}>\frac{1+\frac{1}{\alpha\underline{\delta}}}{2+\frac{1}{\alpha\overline{\delta}}+\frac{1}{\alpha\underline{\delta}}},
$$

or alternatively if

$$
\overline{R}<\frac{AL_c\left(\alpha\underline{\delta}+1\right)}{\alpha\left(\overline{\delta}-\underline{\delta}\right)}.
$$

Hence if the opportunity cost of land at the boundary is small enough, so that $\overline{R}$ is small (or zero), then the share of establishments at the center decreases with MSA population growth.■

Proof of Proposition 7: Average establishment size at the center is given by

$$\frac{L_c + E_c}{S_c} = \frac{L_c + E_c}{E_c} = \frac{L_c + \frac{1+\frac{1}{\alpha\underline{\delta}}}{A} w + \frac{\overline{R}}{A}}{\frac{1+\frac{1}{\alpha\underline{\delta}}}{A} w + \frac{\overline{R}}{A}}$$

so that

$$\frac{\partial \frac{L_c+E_c}{E_c}}{\partial P} = -\frac{\partial E_c}{\partial P}\frac{L_c}{(E_c)^2} = -\frac{1+\frac{1}{\alpha\underline{\delta}}}{D}\frac{L_c}{(E_c)^2} < 0.$$

Hence, average firm size at the center decreases with population growth. Average establishment size at the edge is analogously given by

$$\frac{E_b + L_b}{S_b} = \frac{\overline{R}\left(1 + \alpha\underline{\delta} + \alpha\overline{\delta}\right) - AL_c\frac{\underline{\delta}}{\overline{\delta}} + \left(3 + \alpha\underline{\delta} + \alpha\overline{\delta} + \frac{1}{\alpha\overline{\delta}}\right) w}{\left(2 + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\delta}}\right) w + 2\overline{R} - A\frac{L_c}{\alpha\overline{\delta}}}.$$

It follows that

$$\frac{\partial \frac{E_b+L_b}{S_b}}{\partial P} = \frac{1}{S_b}\frac{A}{D}\left(\left(3 + \alpha\underline{\delta} + \alpha\overline{\delta} + \frac{1}{\alpha\overline{\delta}}\right) - \frac{E_b + L_b}{S_b}\left(2 + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\delta}}\right)\right)$$

which is negative as long as

$$\frac{3 + \alpha\underline{\delta} + \alpha\overline{\delta} + \frac{1}{\alpha\overline{\delta}}}{2 + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\delta}}} < \frac{E_b + L_b}{S_b}$$

or

$$\frac{AL_c}{\overline{\delta}\alpha} > -\left(\frac{\overline{\delta} - \underline{\delta} + \alpha\left(\overline{\delta} - \underline{\delta}\right)^2}{1 + \overline{\delta}^2\alpha^2 - \underline{\delta}\overline{\delta}\alpha^2 + 2\overline{\delta}\alpha - \underline{\delta}\alpha}\right)\frac{\overline{R}}{\underline{\delta}}$$

which holds since $1 + \overline{\delta}^2\alpha^2 - \underline{\delta}\overline{\delta}\alpha^2 + 2\overline{\delta}\alpha - \underline{\delta}\alpha > 0$. Consequently, average firm size at the edge also falls.■