Modeling Sovereign Yield Spreads: A Case Study of Russian Debt

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Abstract

We construct a model for pricing sovereign debt, which accounts for the risks of both default and restructuring, and the compensation for illiquidity. Using a new and relatively efficient method, we estimate the model using Russian dollar-denominated bonds. We consider the determinants of the Russian yield spread, the yield differential across different Russian bonds, and the implications for market integration, relative liquidity, relative expected recovery rates, and implied expectations of different default scenarios.

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1 Introduction

In this paper we construct a model of the joint distribution of the term structures of risk-free and sovereign bond yields that incorporates the risks of default and restructuring, as well as for illiquidity. This is accomplished by extending the framework of Duffie and Singleton (1999) to allow for multiple types of credit events and for bond-specific credit factors that may reflect differential liquidities across bonds and the idiosyncratic treatment of bonds in the event of defaults. Although, in this paper, we apply this framework to price sovereign debt, it is also applicable to pricing problems, such as corporate debt, in which there are bond-specific risk or liquidity factors reflected in the relevant discount rates for pricing.

The determinants of default for a sovereign are quite different from those of a corporation. For example, a holder of sovereign debt may not have recourse to a bankruptcy code in the event of default, and a sovereign default is largely a political decision. A government trades off the cost of making debt payments against reputation costs (Eaton and Gersovitz 1981), the costs of having assets abroad seized, and the costs of having international trade impeded (Bulow and Rogoff (1989b) and Gibson and Sundaresan (1999)). A sovereign rarely makes an outright default. Rather, it forces a restructuring or renegotiation of the debt. Indeed, the same bond may be repeatedly renegotiated (Bulow and Rogoff 1989a). A sovereign also trades off the costs of default (or forced restructuring) of internal versus external debt. As we shall see, this trade-off has interesting implications for the pricing of different classes of Russian debt. We are not aware of any theoretical papers that address this issue — in fact almost all papers on sovereign debt are (implicitly) concerned with external debt only.

We consider a convenient "reduced-form model" of the *arrival intensities* of a wide range of *credit events*, including:

- Default/repudiation: The sovereign announces that it will stop making payments on its debt.
- Restructuring/renegotiation: The sovereign and the lenders "agree" to reduce (or postpone) the remaining payments.
- A "regime switch" such as a change of government or the default of another sovereign bond that changes the perceived risk of future defaults.

Building on the valuation framework in Duffie and Singleton (1999), we show that, under regularity, the promised cash flows of a sovereign bond can be discounted using a default-adjusted discount rate equal to the riskless rate plus a credit spread that reflects the arrival rate, and associated losses in market value upon arrival, of each of the preceding types of credit events. Additionally, we allow for differential yields across bonds due to *illiquidity*: sovereign bonds may have high transaction costs and be thinly traded, and traders may be asymmetrically informed. In particular, government insiders may have superior information.

When pricing sovereign bonds in the presence of these potential credit and liquidity events, it is important to accommodate the possibility that bonds issued by the same sovereign, of exactly the same type but possibly of different maturities, may be priced in the market using different discount factors. That is, there may not be a common discount-rate process that prices the entire term structure of sovereign bond yields. One reason why seemingly homogeneous bonds would not be priced with the same underlying discount rate is that the bond covenants do not include cross-default clauses that would force, upon the default of one bond, the simultaneous default of other bonds of the same type, but of a different maturity. For various strategic reasons related to internal or external political and/or economic considerations, sovereign issuers may choose to default on or renegotiate the terms of one bond (or set of bonds), but not on others. Further, as in the Russian case examined in this paper, portions of the outstanding debt may have been issued during different political regimes. In this situation, the current regime may feel, or be perceived to feel, a stronger *obligation* to make contractual payments on the debt issued during its own regime. Finally, for these reasons and because of possible clientele trading patterns, bonds of the same type may have different liquidity characteristics.

With these considerations in mind, we develop our pricing model allowing for bond-specific components in the discount factors used to present-value cash flows. Importantly, these factors are incorporated directly into the discount rates rather than being treated as additive pricing or measurement errors as has become common practice in the literature on dynamic term structure models.[1] In its most general form, our modeling framework is

[1]See, for example, Chen and Scott (1993), Duffie and Singleton (1997), Duffee (1999). Our modeling approach could also be used in fitting models of default-free term structures instead of having additive *measurement* errors, though it is most natural for studying time-series of yields on specific bonds rather than time-series on constant-maturity yields.

agnostic about the determinants of these bond-specific factors. In circumstances for which a researcher has data on issuer-specific or macroeconomic variables that are useful for distinguishing between various credit and liquidity components of sovereign spreads, such information could be incorporated into our econometric formulation.

Alternatively, one can proceed, as we do with our Russian example, to treat the composite credit/liquidity spread as an unobservable state variable and then, after estimation of the model, to study the time-series properties of the model-implied spreads. Specifically, we estimate the risk-free term structure using two- and ten-year swap rates, and estimate the Russian term structure using three Russian dollar-denominated coupon bonds. We allow the three Russian bonds to have differential expected recovery and liquidity. The model has a relatively good fit both for the securities used in the estimation and for the yields on several out-of-sample bonds. We examine a variety of implications of the estimated model including the determinants of the spread, the degree of integration of the markets for different Russian sovereign bonds, differences in both the liquidities and expected default recoveries of different bonds, and the implied expectations of investors about different default scenarios. We also study, using a standard vector autoregression, the correlation structure of these spreads with various macroeconomic time-series including data on the reserves of the Russian Central Bank and the price of oil.

In estimating our pricing models, we use a novel efficient estimation methodology, based on an approximation to the likelihood function,[2] that is applicable to a large class of affine diffusions.[3] The basic idea of our approach is to divide the state vector Y_t into the subvector Y_t^I that drives the stochastic volatility of Y_t and the vector of remaining state variables Y_t^D, as in the canonical representation of affine models proposed by Dai and Singleton (1999). For the special case in which the elements of Y_t^I are independent square-root processes,[4] the conditional density function of Y_t^I is known to

[2]Liu, Pan, and Pedersen (1997) propose a closed-from approximation to the likelihood function of a general affine jump-diffusion, Ait-Sahalia (1999) offers closed-from approximations to the likelihood function of a generic one-dimensional diffusion, and Pedersen (1995) derives a simulation-based approximation to the likelihood function of a generic diffusion (see also Brandt and Santa-Clara (1999)).

[3]See Duffie and Kan (1996) and Dai and Singleton (1999) for discussions of affine term-structure models.

[4]The assumption that the volatility of each $[Y_t^I]_i$ is determined by the square-root of

be the product of non-central chi-square distributions (Cox, Ingersoll, and Ross 1985). Furthermore, the distribution of Y_t^D conditional on Y_{t-1} and the sample path of Y_s^I, $t-1 < s \leq t$, is normal with conditional moments that are known in closed form. Using these properties (as well as Bayes Rule) allows us to derive an approximation to the density of Y_t conditional on Y_{t-1} that leads to computationally tractable estimators for the problems that we have examined.

Few other papers study credit-risky securities empirically. Duffee (1999) estimates a term structure for corporate bonds, Duffie and Singleton (1997) and Collin-Dufresne and Solnik (1997) estimate models of swap rates, and Merrick (1999) calibrates a discrete-time model to Russian and Argentinian bonds. Keswani (1999) estimates the models of Longstaff and Schwartz (1995) and Duffee (1999) using Brady bonds from three Latin-American countries.

The remainder of this paper is organized as follows. Section 2 reviews the market for Russian sovereign debt and the events of the last decade. Section 3 describes our model, Section 4 presents our estimation methodology, and Section 5 presents our empirical results for Russian bond data.

2 The Case of Russia

Our empirical analysis focuses on the properties of the prices of Russian bond leading up to the default in August, 1998. In order to both set up this empirical analysis and to motivate some of the modeling choices made in Section 3, we begin by reviewing the economic events leading up to the Russian default and by examining the behavior of the prices of Russian bonds over this time period.

The Russian government ran large budget deficits through the nineties and, with these deficits, the associated debt servicing costs became increasingly burdensome. The domestic debt was mostly short dated and, consequently, the Russian Government was frequently forced to raise new capital to make debt payments. On the revenue side, the Russian Government was having difficulty collecting taxes. More generally, the implementation of structural reforms to improve the economic strength of the country was

itself is essentially without loss of generality in affine diffusion models. As will be made precise subsequently, it is the independence assumption that leads us to lose generality by ruling out feedback among the Y_t^I through the drift.

proceeding poorly. The private sector was also having difficulties. Russian real GDP contracted by an estimated 33% from 1993 to the end of 1998 (IMF 1999b). Corruption was a problem throughout the economy. Facing weak earnings, Russian banks borrowed aggressively overseas and used the proceeds to buy high-yielding domestic bonds with a high (promised) yield. This strategy exposed the banking system to substantial domestic sovereign credit risk, and would later prove ruinous.

Making matters worse, the prices of Russian commodities — in particular, the price of Brent oil — started to decline in October, 1997. This decline continued into 1998. Partly as a consequence, in 1997, Russia's current account turned negative for the first time since the start of the reforms. The current account deteriorated further through the first half of 1998 (IMF 1999b).

These macroeconomic developments had significant impacts on structure of the outstanding debts of the Russian Government and the *market's* perceptions of the riskiness of these debts. Our overview of the structure of Russian sovereign debt begins in 1991, when the Soviet Union collapsed and defaulted on its debt. In the following decade Russia restructured this Soviet-era debt giving rise to what is commonly referred to as the "Paris Club" and "London Club" debts. The London Club represents more than 600 Western commercial lenders. In 1997, it agreed with Russia to restructure Soviet-era debt into two securities: $6 billion of principal notes (Prins) and $20 billion of interest-arrears notes (IANs). Similarly, the Paris Club consists of Western governments that have lent money to developing nations and have agreed not to accept restructuring terms less favorable than those offered to the London Club. In 1996, the Paris Club rescheduled around $40 billion of Soviet-era debt.

In addition to old Soviet-era debt, Russia has built up significant amounts of new debt to finance its budget deficits. This has been done partly by issuing Ruble-denominated Treasury bonds known as GKOs and OFZs. The GKOs are short-dated discount bonds. The OFZs are longer-dated coupon bonds. In mid 1998, Russia's Treasury debt had reached around $70 billion, of which about one third was held by nonresident investors (IMF 1998). In 1993, five dollar-denominated MinFin[5] bonds were issued as payment to Russian exporters for accounts in the Vnesheconombank that were frozen

[5]The name MinFin is derived from Ministry of Finance. The MinFins are also known as Taiga bonds.

in 1991. Many of these bonds were subsequently sold to foreigners. The MinFins are, technically, domestic debt under the jurisdiction of Russian law. In 1996, two additional MinFins were issued, and Russia issued its first Eurobond. This was followed by several additional Eurobond issues in 1997 and 1998. Eurobonds are denominated in various currencies, have cross-default triggers (that is, failure to make payments on one bond triggers default on all bonds), and in the case of default creditors may be able to seize Russian assets abroad. Renegotiation of Eurobonds is difficult since they are widely held around the globe. Finally, Russia was granted a number of loans by the International Monetary Fund (IMF) and other international financial institutions during the last decade.

In this paper, we focus on the MinFins and the (dollar-denominated) Eurobonds, because there are natural (essentially) default-free reference curves relative to which we can study Russian credit spreads. The spreads of these bonds over US Treasuries were high until the end of 1995, at which point they started to fall. (See Figure 3, Panel 7.) Spreads continued to fall until October 1997 due to some economic improvements and to reported[6] optimistic views of Russia's future. In October 1997, however, spreads started to increase with Russia's mounting economic problems. In the spring and summer of 1998, spreads were driven up further as Russia was downgraded by several credit agencies.

These developments did not prevent Russia from issuing new Eurobonds — in fact Russia issued five Eurobonds in 1998, one in March, one in April, two in June, and one in July. The first four of these were oversubscribed (IMF 1999a). Some market participants even talk about "feeding frenzies" in connection with new Russian issues. Yield-seeking investors evidently presumed that Russia was too important a country for major industrial nations to allow it to collapse. These Eurobonds were issued 1998 in anticipation of shortfalls in cash needed to make payments on short-term domestic debt and to service foreign debt. In addition, the IMF reluctantly agreed to give Russia additional loans. In August, 1998 it became clear that these measures were not enough to establish sufficient liquidity for Russia, and there was a substantial outflow of capital from Russia.

On August 17, 1998, Russia announced a compulsory restructuring of the domestic government debt (GKO and OFZ). A 90-day moratorium was placed on foreign commercial debt, and the exchange-rate band was devalued.

[6]See Section 5.3.

At this time, it was unclear whether Russia would also default on external sovereign debt. Subsequently, however, Russia defaulted on the London Club Debt, the Paris Club Debt, and the principal of the MinFin 3, which matured in 1999. Russia remains current on interest payments on the remainder of the MinFins and Eurobonds as of the date of this paper.

Figure 1, Panel 1 displays the prices of five MinFin bonds from 7/31/1998 (just prior to default) to 11/12/1999. Most of these bonds recovered approximately 10 cents for each dollar of face value. The MinFin 3, which matured in 1999, recovered more than 20 cents to the dollar. Panel 3 shows the prices for the same period of five Russian dollar-denominated Eurobonds. Panels 2 and 4 show the prices in Panels 1 and 3, respectively, normalized so that they all have a market value of 100 at 7/31/1998. For both types of bonds, we see that normalized prices follow a more homogeneous pattern than do the unnormalized. Looking ahead to our model formulation, we will assume that the market priced these bonds as though, in the event of a default, bondholders would recover an exogenous fraction of the market value (RMV) just prior to default. The patterns in Figure 1 suggest that this is not an unreasonable assumption relative, say, to the alternative that bondholders would recover an exogenous fraction of the bond's face value.[7] We note that the MinFins recovered about 20-30% of their previous market value after default, whereas the Eurobonds recovered around 30-40% of their value. Hence, since the August, 1998 default, investors may have been viewing Eurobonds as being of higher quality than MinFins. We shall examine below whether this was reflected in pre-default prices.

3 Pricing Sovereign Debt

Models for pricing default risk can be divided into two groups: *intensity* and *first-passage* models. In first-passage models, default occurs when incentives suggest that it is optimal for the issuer to default, or when payment on debt is impossible. This is typically modeled as the event that the debtor's cash-flows or asset-liability ratio falls below some cut-off level for the first time. In intensity models, the actual time of default is a surprise event not directly linked to (observed) decision variables of the debtor. What is modeled is the

[7] When modeling the prices of credit risky securities, it is usually assumed that after a credit event, recovery is at an exogenous fraction of pre-default market value (Duffie and Singleton 1999), or at an exogenous fraction of face value (Lando 1998).

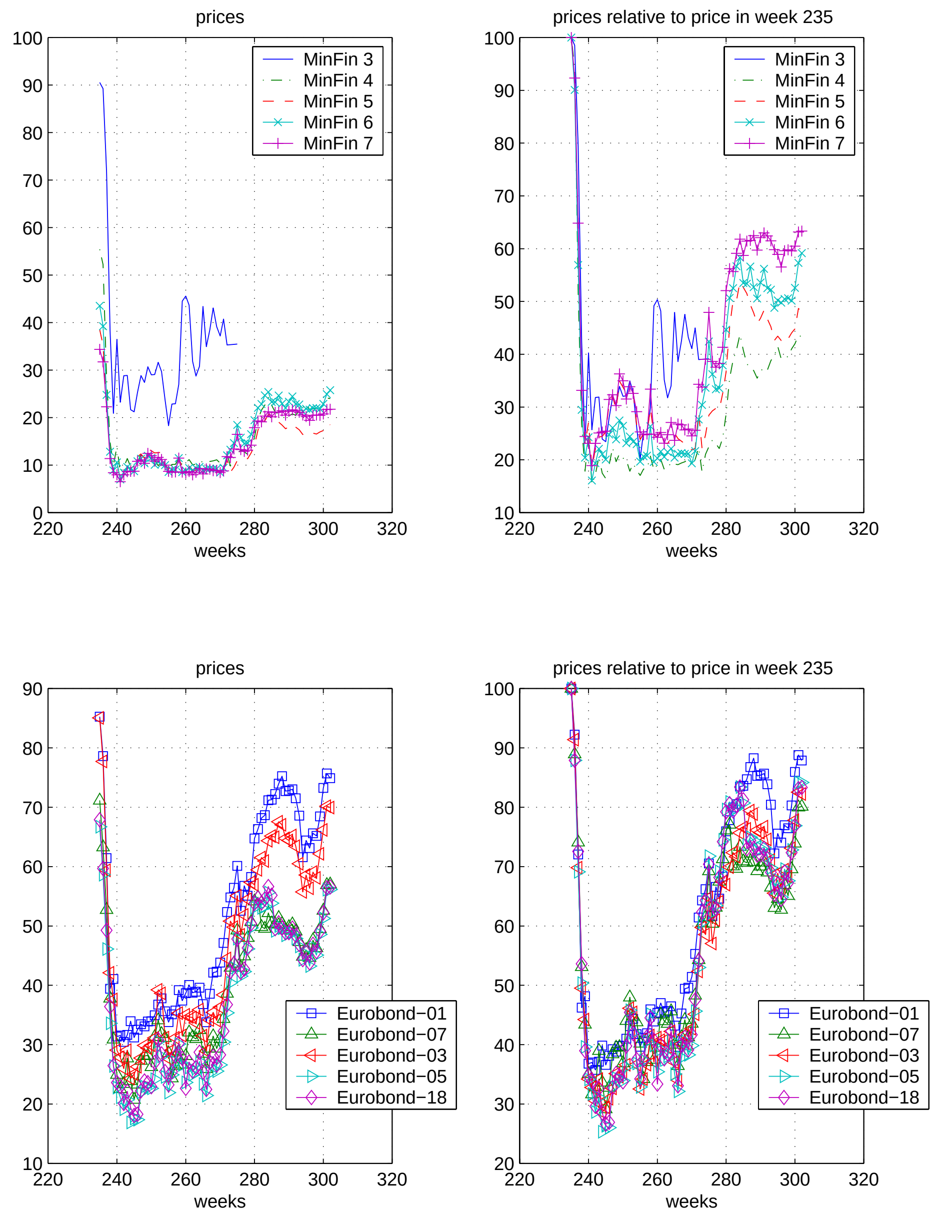

Figure 1: Panel 1 (top left) shows the prices of the Russian MinFin 3–7 bonds in the time period from 7/31/1998 (just prior to default) to 11/12/1999. Panel 2 (top right) shows the prices of the same bonds normalized so that they all have a market value of 100 at 7/31/1998. Panels 3 (bottom left) and 4 (bottom right) show the prices and normalized prices, respectively, of five Russian Eurobonds.

risk, that is the intensity, of this event. The intensity can be modeled as an exogenous process or can be a function of economic variables. Duffie and Lando (2000) show that can even arise through a first-passage-type model of the issuer's incentives, when some decision variables are unobserved.

While first-passage models have intuitive appeal when applied to corporate bonds, they are sometimes difficult to estimate for sovereign debt. This is due to the fact that measuring a sovereign's assets and cash flows is difficult, and more importantly that the incentives of a sovereign to default are relatively complex. As an example of the difficulties in measuring a developing country's macro-economic variables, Deutsche Bank Research (1998b) states: "the Russian budget itself is of course largely fictitious in the sense that it can make the deficit pretty much what you want..."

When a sovereign defaults it may lose assets held abroad, but assets held within the country need not be seized as collateral. Rather, the country loses reputation for failure to make payments, which worsens its access to international capital markets, and may impede international trade. Politicians trade these costs against the cost of making payments on the debt, along with other political considerations. For instance, Deutsche Bank Research (1998a) writes: "We continue to maintain that a default depends far more on Russia's willingness to pay versus its ability to pay its debt." The incentives of a corporation are usually simpler: It default (in theory) when it cannot make its payments, or when equity holders find that assets have become worth less than liabilities. In the light of these considerations, we pursue here an intensity model for studying sovereign securities.

We are interested in the price, P_t, of a security that promises to pay a (possibly random)[8] amount X at time T. We take as given a short-rate[9] process r and an equivalent martingale measure Q. In modeling sovereign spreads, we extend the fractional recovery of market value model in Duffie and Singleton (1999), developed for a single credit event (default), to a model that accommodates many types of credit and liquidity events, in particular those enumerated in the introduction. This yields a pricing formula similar to one derived (in a less general setting) by Schönbucher (1998), who considers

[8] All random variables are defined on a probability space $(\Omega, \mathcal{F}, Pr)$. Also, we fix a family $\{\mathcal{F}_t : t \geq 0\}$ of increasing σ-algebras satisfying the usual conditions (see Protter (1990)).

[9] The short rate process, r, is a predictable process with $\int_0^t |r_s|\, ds < \infty$ a.s., and $E^Q \left[\exp(-\int_0^t r_s\, ds)\right] < \infty$ for all t. See Protter (1990) for definitions not provided here.

multiple defaults.

We let N_t be the number of credit events that has happened by time t, where N is a counting process with risk-neutral intensity h. We assume that at the n'th credit event, the promised payment of the security is lowered to a fraction, Y^n, of its pre-credit-event value. Also, at a credit event, the sovereign pays a cash amount, which is a fraction, Z^n, of the pre-credit-event market value.[10] This way of modeling credit events captures several typical events. First, the event that $Y = 0$ can describe a situation where the sovereign makes an outright repudiation, that is it liquidates its debt completely. If the sovereign makes a cash payment, or if there is a collateral to collect, then $Z > 0$. This is the kind of event that the literature on default risk usually considers. Another way of interpreting an event with $Y = 0$ and $Z > 0$ is as a regime switch. If the economic environment changes completely, one can think of the security as liquidated, and that the holders are paid in terms of securities in the new regime whose market value is Z times the pre-regime-switch value. Second, the event that $Z = 0$ and $Y \in (0, 1)$ describes a restructuring or renegotiation, where the promised payment is lowered, and no cash is paid. Finally, there are all the cases in between, where both Y and Z are different from zero. For instance, a sovereign could write down the principal significantly, but pay a cash amount up front.

The total fractional loss at the n-th credit event is $1 - Y^n - Z^n$. At time t, the risk-neutral expected loss rate of the next credit event is denoted by L_t.[11] Hence, the mean loss rate (due to credit events) is hL. We also allow liquidity to affect pricing. We make the simplistic assumption that illiquidity of the security translates into a fractional cost rate of l, where l is a predictable process. Hence, the total mean loss rate of the security is $hL + l$. Furthermore, after N credit events the security is worth the fraction $\prod_{n \leq N} Y^n$ of an otherwise identical security that has not experienced any credit events. This gives the intuition for the pricing formula in the following theorem.

[10] Here, Y^n and Z^n are random variables measurable with respect to $\mathcal{F}_\tau$, where $\tau = \inf\{t : N_t \geq n\}$. We assume that h, Y, and Z are exogenous in the sense that they do not depend on the price, P, of the security.

[11] To be rigorous, L is a predictable process such that for each $n \in \mathbb{N}$, $L_\tau = E^Q(1 - Y^n - Z^n \mid \mathcal{F}_{\tau-})$ where τ is the stopping time $\tau = \inf\{t : N_t \geq n\}$. Dellacherie and Meyer (1978) show that such a process exists. (See Duffie (1998) for further details on this construction.)

Theorem 1 *Suppose that* $V_t = E_t^Q\left(e^{-\int_t^T R_u\,du} X\right)$ *is well defined, and that (a.s.)* $\Delta V_t \Delta N_t = 0$ *for all* t*. Then,*

$$P_t = E_t^Q\left(e^{-\int_t^T R_u\,du} X\right) \prod_{n \le N_t} Y^n, \tag{1}$$

where $R = r + s$ *and*

$$s = hL + l. \tag{2}$$

We refer to the process s in (2) as the short (or instantaneous) spread. The (technical) condition $\Delta V \Delta N = 0$ means that V does not jump at any event time. This condition is automatically satisfied in our model of Russian bond prices, where V is a diffusion process (and hence continuous). In Theorem 1 we have lumped all credit events into a single counting process N, but we can also think of different credit events separately. Suppose, for instance, that a liquidation event has intensity h^1 and expected loss rate L^1, that a restructuring has intensity h^2 and expected loss rate L^2, and that a regime switch happens with intensity h^3 and expected loss rate L^3. Supposing that these events cannot happen at the same time, the total intensity of a credit event is $h = h^1 + h^2 + h^3$, and the expected loss at such an event is the intensity weighted average, $L = \frac{h^1}{h}L^1 + \frac{h^2}{h}L^2 + \frac{h^3}{h}L^3$. Hence, by Theorem 1, the spread is,

$$s = h^1L^1 + h^2L^2 + h^3L^3 + l. \tag{3}$$

It would be interesting to able to identify each term of (3) separately, but this is a difficult task.

Theorem 1 gives the price of a security with a single cash flow. The price of a security with multiple cash flows, however, is just the sum of the prices of the individual cash flows.[12] As an example, we consider the price of a sovereign coupon bond that promises to pay a coupon, c, at times $t_1, t_2, \ldots, t_m$, and a face value of 1 at maturity, t_m. For some sovereign bonds, the market may believe that coupons and principal have different default characteristics. In particular, the sovereign may be more likely to be able to make the coupon payments than to pay the entire principal. For

[12] This additivity follows from our assumptions of exogenous intensity, loss rate, and liquidity.

instance, Russia defaulted on the principal of MinFin 3, when it matured in 1999, but paid the coupons on the other MinFins and Eurobonds. Hence, we let Y^c and Y^f denote the write-down factors for coupons and face value, respectively, and s^c and s^f the corresponding short spreads. The price of the bond at time t is,

$$P_t = \sum_{\{i:t_i>t\}} E_t^Q \left(e^{-\int_t^{t_i} (r_u + s_u^c)\, du} \right) c \prod_{n \le N_t} Y_n^c + E_t^Q \left(e^{-\int_t^{tm} (r_u + s_u^f)\, du} \right) \prod_{n \le N_t} Y_n^f.$$

The two spreads, s^c and s^f, can be identified (under certain non-degeneracy conditions) if one has data on the prices of two bonds of different cash-flow structure with identical default characteristics.

The case of $Y^c = Y^f$ is when restructuring results in a reduction of the number of bonds outstanding, with the original terms remaining unchanged. If in addition the fractional cash payments are identical, $Z^c = Z^f$, then there is a single short-spread process, $s = s^c = s^f$, which can be used to price all of the bond's cash flows. We make this assumption in the empirical analysis of Russia. This is done for tractability and because we believe there may be differences between the bonds that imply that they have different short spreads. Reasons for this are outlined in the introduction, and will be discussed further when we get to the empirical analysis. We accommodate such bond-specific discounting in our econometric analysis by treating the spread for each bond as the sum of the short spread of a "benchmark bond" and an idiosyncratic short-spread component. Although this modeling approach does not allow us to identify all of the different components in (3) separately, this decomposition may be useful in interpreting the model-implied spread differentials. Clearly, in other settings, if one had data on variables that allowed for separate identification of the components $h^1 L^1$, $h^2 L^2$, and so on, then these components could be parameterized directly in terms of the state vector.

An attractive feature of the pricing relation (1) is that valuation can proceed using familiar term structure frameworks once the functional relation between the spread s in (2) and the state vector is chosen. In particular, all of the tractability of affine term structure models (Duffie and Kan (1996), Dai and Singleton (1999)) can be exploited for pricing sovereign debt. The next section presents one possible formulation of an affine model of sovereign bond prices, the one we use for studying Russian bond data.

3.1 The Parametric Model

Within the affine family of term structure models we can, in principle, allow for general affine dependence among the state variables driving the riskless term structure and the sovereign spreads s (possibly indexed by bond), subject to the requirement that the model give well-defined prices. However, practical considerations often lead one to work with "recursive" parameterizations and to trade off estimator efficiency against computational tractability. In the case of defaultable securities, one is often faced with the problem of pricing a wide variety of securities all of which are priced relative to a common default-free reference curve. Estimation of a model using all of the defaultable bond yields and a collection of default-free yields simultaneously would be computationally challenging. Instead, it is convenient to estimate the default-free term structure in a first stage, and then estimate the parameters of the spread factors taking as given those of the process r. In this manner, the parameters of the default-free curve are held fixed across the analyses of various defaultable securities. This section presents the recursive model we use in our empirical analysis.[13]

We assume a two-factor affine model for the riskless term structure with the dynamics of the short rate r given by:

$$d \begin{bmatrix} v_t \\ r_t \end{bmatrix} = \begin{bmatrix} K^{vv} & 0 \\ K^{rv} & K^{rr} \end{bmatrix} \left(\begin{bmatrix} \theta^v \\ \theta^r \end{bmatrix} - \begin{bmatrix} v_t \\ r_t \end{bmatrix} \right) dt + \sqrt{v_t} \begin{bmatrix} 1 & 0 \\ \Sigma^{rv} & \Sigma^{rr} \end{bmatrix} dW_t^{v,r}, \tag{4}$$

where $W^{v,r} = (W^v, W^r)^\top$ is a two-dimensional standard Brownian motion. The two factors are correlated through both the drift and diffusion terms. The first factor, v, serves as a source of stochastic volatility for itself as well as for r. As we shall see below, this model fits the dynamics of the risk-free term structure of dollar yields rather well.

To accommodate idiosyncratic components in the credit risk-adjusted discount rate R in (1), we select one sovereign as the "benchmark" bond and model the joint behavior of the riskless term structure and the benchmark

[13] Duffee (1999) faced these same considerations in his study of yields on individual corporate bonds. He used a somewhat different recursive model.

short spread as:

$$d\begin{bmatrix} v_t \\ r_t \\ s_t \end{bmatrix} = \begin{bmatrix} K^{vv} & 0 & 0 \\ K^{rv} & K^{rr} & 0 \\ K^{sv} & K^{sr} & K^{ss} \end{bmatrix} \left(\begin{bmatrix} \theta^v \\ \theta^r \\ \theta^s \end{bmatrix} - \begin{bmatrix} v_t \\ r_t \\ s_t \end{bmatrix} \right) dt + \begin{bmatrix} 1 & 0 & 0 \\ \Sigma^{rv} & \Sigma^{rr} & 0 \\ \Sigma^{sv} & \Sigma^{sr} & \Sigma^{ss} \end{bmatrix} \begin{bmatrix} \sqrt{v_t} & & \\ & \sqrt{v_t} & \\ & & 1 \end{bmatrix} dW_t, \tag{5}$$

where $W = (W^v, W^r, W^s)$ is a standard Brownian motion in $\mathbb{R}^3$. The fact that s can take on negative values is not logically inconsistent with our theoretical model because of the possibility of a negative liquidity factor, l, in (2). We also allow this possibility of a negative spread in order to identify possible "mis-pricings" by the market (see Section 5.4), although that is outside the scope of this model. Representation (5) is equivalent to assuming that s is an affine function of a trivariate state vector (r, v, η) and this vector follows an affine diffusion with (possibly) correlated components. Additional state variables that drive s, but not r directly, can of course be accommodated. However, given our limited sample size and the fact that we are allowing for each bond spread to have an idiosyncratic component, we proceed with this parsimonious representation.

For any non-benchmark bond, numbered $i \in \{1, \ldots, I\}$, issued by the same sovereign, we assume its short spread process is given by $s^i = s + \gamma^i$, where

$$d\gamma_t^i = \kappa^i \left(\theta^i - \gamma_t^i\right) dt + \sigma^i d\xi_t^i, \tag{6}$$

where ξ^i is a standard Brownian motion independent of $\{W, \xi^j, j \neq i\}$. The state variable γ^i captures differences in liquidity, hazard rates, and recovery rates, relative to the benchmark bond. We experimented with alternative parameterizations of γ^i, including a square-root process, but this Gaussian formulation seemed to fit the data best.

To complete the model, we specify the market prices of risk, or equivalently, the risk-neutral behavior of the state variables. For our analysis we specify the risk-neutral distribution of the Brownian motions as:

$$\begin{aligned} dW_t &= -\begin{bmatrix} \sqrt{v_t} & & \\ & \sqrt{v_t} & \\ & & 1 \end{bmatrix} \begin{bmatrix} \lambda_t^v \\ \lambda_t^r \\ \lambda_t^s \end{bmatrix} dt + d\tilde{W}_t \\ d\xi_t^i &= \left(\kappa^i - \tilde{\kappa}^i\right) \gamma_t^i dt + d\tilde{\xi}_t^i, \end{aligned} \tag{7}$$

where $\tilde{W}, \tilde{\xi}^1, \ldots, \tilde{\xi}^I$ are independent standard Brownian motions under the given "risk-neutral" measure Q. This specification of the market prices of risk $\lambda = (\lambda_v, \lambda_r, \lambda_s)$ is standard for affine models (see, for example, Dai and Singleton (1999)). However, the market prices of risk for the Gaussian idiosyncratic factors, $\gamma^1, \ldots, \gamma^I$, are non-standard in that adjusting for the risk premium induces different rates of mean-reversion under the objective and risk-neutral measures for these factors, a feature that we found to be important in the empirical analysis.[14]

4 Estimation Method

We estimate our model in two steps. First, we estimate the risk-free model using an approximate-maximum-likelihood method. Then, taking the parameters of the riskfree process as given, we estimate the parameters of the spread process again by the method of maximum likelihood. The parameters of the riskfree term structure and spread processes could also be estimated simultaneously, assuming that this is computationally feasible. Readers that are interested mainly in the empirical results may skip directly to Section 5.

We denote the parameter vector of the risk-free model by

$$\psi = (\theta^v, \theta^r, K^{vv}, K^{rv}, K^{rr}, \Sigma^{rr}, \Sigma^{rv}, \lambda^v, \lambda^r),$$

and the parameter vector for the spread relations by

$$\phi = \left(\theta^s, K^{sv}, K^{sr}, K^{ss}, \Sigma^{sv}, \Sigma^{sr}, \Sigma^{ss}, \lambda^s, (\kappa^i, \theta^i, \sigma^i, \tilde{\kappa}^i)_{i=1,\ldots,I}\right).$$

4.1 An Approximate, Maximum Likelihood Estimator

From the literature on the estimation of default-free, affine term structure models, we know the functional relation between the vector of default-free yields used in estimation, c_t, and the state vector Y determined by $c_t = g(Y_t; \psi)$, where g known in closed-form.[15] Thus, if the functional form of the

[14] In different settings, Duffee (2000) and Dai and Singleton (2000) found that risk premiums that affect both the long-run mean and rate of mean reversion of Gaussian factors improves the forecasting ability of dynamic term structure models.

[15] See, for example, Chen and Scott (1993), Pearson and Sun (1994), and Duffie and Singleton (1997). In affine models, the prices of zero-coupon bonds are known to be exponential-affine functions of the state variable (Duffie and Kan 1996). Further, the

conditional transition density of the state vector, $p(Y_t^{\psi} \mid Y_{t-1}^{\psi}; \psi)$, is known then standard change-of-variable arguments lead us to the likelihood function

$$p(c;\psi) = \prod_{t=1}^{T} p(Y_t^{\psi} \mid Y_{t-1}^{\psi};\psi) \frac{1}{|\det Dg(Y_t^{\psi};\psi)|}, \tag{8}$$

where the model-implied state variables are given by

$$Y_t^{\psi} := g(\,\cdot\,;\psi)^{-1}(c_t), \tag{9}$$

and, for affine models, the Jacobian Dg can easily be computed in closed form.

Unfortunately, the transition density of an affine diffusion is not generally known in closed form, outside of the special cases of Gaussian and independent square-root diffusions.[16] However, we have found that a simple approximation to the likelihood function leads to reliable estimates in a large sub-class of affine models, including the models examined in this paper. We proceed to outline this estimation strategy, leaving formal details to Appendix B.

With little loss of generality we can assume[17] that $Y = (Y^I, Y^D) \in \mathbb{R}^n \times \mathbb{R}^m$, where Y^I is a positive vector process driving the volatility, Y^D is an unbounded process, and Y solves the stochastic differential equation:

$$\begin{aligned} dY_t^I &= \left(k^I - K^{II} Y_t^I\right) dt + \sqrt{\Delta(Y_t^I)}\, dW_t^I, \\ dY_t^D &= \left(k^D - K^{DI} Y_t^I - K^{DD} Y_t^D\right) dt + \sqrt{\Delta(\alpha^D + B^{DI} Y_t^I)}\, dW_t^D, \end{aligned}$$

where $\Delta(x)$ denotes a diagonal matrix whose diagonal elements are the respective elements of the vector x, and where (W^I, W^D) is a standard Brownian motion in $\mathbb{R}^n \times \mathbb{R}^m$. We wish to evaluate the transition density $p(Y_\delta \mid Y_0)$, where δ is the time between consecutive observations. We take advantage of Bayes' Rule, and the fact that Y^I is itself a Markov process, to obtain

$$\begin{aligned} p(Y_\delta \mid Y_0) &= p(Y_\delta^I \mid Y_0)\, p(Y_\delta^D \mid Y_0, Y_\delta^I) \\ &= p(Y_\delta^I \mid Y_0^I)\, p(Y_\delta^D \mid Y_0, Y_\delta^I). \end{aligned} \tag{10}$$

yields on coupon bonds are simple functions of the zero prices, so g is known in closed-form up to the solution of certain ordinary differential equations underlying the zero-coupon pricing.

[16] For this reason, researchers have often relied on method-of-moments estimators outside of these special cases. See, for example, Dai and Singleton (1999).

[17] Under regularity, most of the affine diffusions studied in the literature on dynamic term structure models can be transformed to this form. See Dai and Singleton (1999).

Our method is based on the assumption that the matrix K^{II} is diagonal, or equivalently that the the elements of Y^I are independent. In our two-factor model of the riskless term structure, this assumption is trivially satisfied, since the Y^I is the scalar v. However, when $n > 1$, this independence assumption represents a restriction on the state process that, if violated, rules out the application of our approach. Under this independence assumption, the conditional distribution of Y^I_δ given Y^I_0 is a product of non-central chi-squares (Cox, Ingersoll, and Ross 1985), with a density given explicitly in Appendix B.

There is no known explicit expression for $p(Y^D_\delta \mid Y_0, Y^I_\delta)$. We base an approximation on the following observation: The distribution of Y^D_δ conditional on Y^D_0 *and the entire path of* Y^I *from time* 0 *to time* δ is a known normal distribution:

$$p(Y^D_\delta \mid Y_0, Y^I_s, s \in [0,\delta]) = \varphi(Y^D_\delta, m_\delta, V_\delta), \tag{11}$$

where $\varphi(\,\cdot\,, m, V)$ is the density of a normal with mean m and variance V given in Appendix B. By the law of iterated expectations,

$$\begin{aligned} p(Y^D_\delta \mid Y_0, Y^I_\delta) &= E\left(p(Y^D_\delta \mid Y_0, Y^I_s, s \in [0,\delta]) \mid Y_0, Y^I_\delta\right) & (12)\\ &= E\left(\varphi(Y^D_\delta, m_\delta, V_\delta) \mid Y_0, Y^I_\delta\right). & (13) \end{aligned}$$

Thus, completing the specification of the conditional density function of the state amounts to approximating the expectation in (13). The simplest approximation that we consider is to approximate $p(\,\cdot \mid Y_0, Y^I_\delta)$ as the conditional density of Y^D_δ given Y^I, evaluated at an outcome of the path of Y^I that is linear between Y^I_0 and Y^I_δ. We have found this approximation to be tractable and accurate for our application. In Appendix B we show how to make this approximation more precise, at the expense of some of the tractability.

4.2 Estimation of the Risk-free Term Structure Model

Using this approximation, and returning to our specific application, the log-likelihood of the yield vector c is thus

$$L(c;\psi) = \frac{1}{T}\sum_{t=1}^{T}\left[\log p(v^\psi_t, r^\psi_t \mid v^\psi_{t-1}, r^\psi_{t-1}) - \log|\det Dg(v^\psi_t, r^\psi_t;\psi)|\right].$$

The maximum likelihood estimator, $\hat{\psi}$, for ψ is then defined by

$$\hat{\psi} = \arg\max_{\psi} L(c; \psi).$$

Under standard technical conditions (see, for instance, Davidson and MacKinnon (1993)), $\hat{\psi}$ is consistent and efficient with respect to the data $(c_1, \ldots, c_T)$. That is, as $T \to \infty$,

$$\begin{aligned} \hat{\psi} &\overset{a.s.}{\to} \psi, \\ T^{1/2}(\hat{\psi} - \psi) &\overset{\mathcal{L}}{\to} N(0, \Omega), \end{aligned} \tag{14}$$

where $\overset{\mathcal{L}}{\to}$ denotes convergence in law, and

$$\Omega^{-1} = E\left(\frac{\partial L}{\partial \psi}\frac{\partial L}{\partial \psi}^{\top}\right) = -E\left(\frac{\partial^2 L}{\partial \psi \partial \psi^{\top}}\right). \tag{15}$$

Moreover, Ω can be estimated consistently, under these conditions, by the sample counterpart of (15).

4.3 Estimation of the Spread Parameters

We now turn to the estimation of the full set, (ψ, ϕ), of parameters. For this, we use data on $I + 1$ (we have $I = 2$ in our application) defaultable bond prices, denoted $b = (b_t)_{t=T_1,\ldots,T_2}$, where $1 \le T_1 < T_2 \le T$. The prices of the bonds implied by the model are of the form $H(v_t, r_t, s_t, \gamma_t; \psi, \phi)$, where H is a sum of known exponential-affine functions of the state vector $(v_t, r_t, s_t, \gamma_t)$, and $\gamma_t = (\gamma_t^1, \ldots, \gamma_t^I)$. In estimating ϕ, we fix the parameters of the risk-free term structure at the maximum likelihood estimates based on c alone, and set (r_t, v_t) to the values implied by the model evaluated at the maximum likelihood estimates $\hat{\psi}$, $(\hat{v}_t, \hat{r}_t) = (v_t^{\hat{\psi}}, r_t^{\hat{\psi}})$. We then estimate ϕ as

$$\hat{\phi} = \arg\max_{\phi} L(c, b; \hat{\psi}, \phi). \tag{16}$$

Under technical conditions, $\hat{\phi}$ is consistent and asymptotically normally distributed. That is, as $T \to \infty$,

$$\hat{\phi} \overset{a.s.}{\to} \phi, \tag{17}$$

$$T^{1/2}(\hat{\phi} - \phi) \overset{\mathcal{L}}{\to} N(0, \Gamma), \tag{18}$$

where Γ will be given explicitly in a subsequent revision of this paper.

5 Empirical Results

This section presents our empirical results.

5.1 Data

We estimate the default-free model using two- and ten-year swap rates for the time period 1/1/1988 to 7/23/1999. There are several advantages of using swap rates over using treasury yields. Swap rate data are commonly available at constant times to maturity. The maturity of a given treasury, on the other hand, changes during the sample.

A potential problem with using swaps is the fact a counterparty may default on a swap contract. Swap rates are nevertheless little affected by default risk because of their contractual netting provisions (Duffie and Huang 1996). In fact, the effect of default risk on swap rates is orders of magnitude smaller than the effect on rates for comparable-quality bonds, and the fundamental default risk in the swap market is in turn orders of magnitude smaller than that of the sovereign-debt markets that we are considering in this paper. Hence, the default risk associated with swaps is negligible for our purposes. The fact that floating-rate swap coupons are contractually set at market short-dated LIBOR rates, which are slightly above risk-free rates, may, however, cause swap rates to be higher than the risk-free rate.

The spread between the treasury yield curve and the swap yield curve is generated in large part by such factors as the convenience yield associated with holding treasuries, the specialness of treasuries, liquidity differences, and because of tax issues (see Collin-Dufresne and Solnik (1997) and Duffie and Singleton (1997)). The spread between treasuries and swaps is sometimes not insignificant (especially recently), and our could in principle be contaminated to some extend by institutional and market factors specific to swaps. In any case, we judge it to be appropriate to compare the yield curve of defaultable bonds with the swap yield curve.

In order to estimate the Russian term structure we use data on the MinFin 3, 4, and 5 bonds for the time period 2/4/1994 to 7/31/1998. Russia defaulted on August 17th, and we are using only pre-default data. We choose these bonds because they are the dollar-denominated Russian bonds with the longest time series. We use the MinFin 4 as our benchmark bond because it has the largest amount outstanding. Descriptive information on these bonds is supplied in Table 1. This table also contains information about four bonds

Issue	Issue Date	Maturity	Coupon	Amount Issued (Billions of US$)
MinFin 3	5/14/1993	5/14/1999	3.00	1.32
MinFin 4	5/14/1993	5/14/2003	3.00	3.38
MinFin 5	5/14/1993	5/14/2008	3.00	2.84
MinFin 6	5/14/1996	5/14/2006	3.00	1.75
MinFin 7	5/14/1996	5/14/2011	3.00	1.75
Eurobond-01	11/27/1996	11/27/2001	9.25	2.40
Eurobond-07	6/26/1997	6/26/2007	10.00	1.00

Table 1: Contractual Characteristics of Russian Dollar-denominated MinFins

that we do not use in estimation, namely two other MinFin bonds and two Eurobonds. Later, we compare the observed prices of these bonds with those predicted by the model. The source of all data is Datastream.

5.2 Risk Free Term Structure

We estimate the default-free term structure using maximum likelihood. We use the first-step approximation to the likelihood function described in Section 4. We impose three over-identifying restrictions on the parameters to reduce the dimension of the parameter space and increase efficiency for the remaining parameters. (This approach is subject to the obvious danger of misspecification.) We set the market price of volatility risk, λ^v, to zero. (We have a non-zero market price, λ_r, of changes in the level of the short rate.) We set $\theta^r = \frac{1+\varepsilon}{2K^{rr}}$. That is, we set the long-run mean of the volatility so that the Feller condition[18] is satisfied with a margin of ε. This condition is imposed because an unrestricted estimation leads to a binding Feller condition. The choice[19] of ε does not greatly affect the results since the likelihood function is relatively flat in this dimension. Lastly, we set the long-run mean, θ^r, of the short rate, r, equal to its implied sample mean. This is done in two steps. First, we fix θ^r at the historical mean of the one-year treasury yield between 2/1/1962 and 7/9/1999, which is 6.82%. Then, we estimate the model and compute the sample mean of the implied short rate to be 5.68%, fix θ^r at this

[18]The Feller condition, that $K^{vv}\theta^v > \frac{1}{2}$, ensures that v_t is strictly positive for all t with probability one.

[19]We use $\varepsilon = 0.001$.

	est.	SD1	SD2
K^{vv}	0.0047	(0.0030)	(0.0021)
K^{rv}	−0.0268	(0.0078)	(0.0109)
K^{rr}	0.3384	(0.1378)	(0.1548)
θ^{v}	121.8293	(—)	(—)
θ^{r}	5.6800	(—)	(—)
Σ^{rv}	0.0436	(0.0126)	(0.0145)
Σ^{rr}	0.1145	(0.0353)	(0.0249)
λ^{v}	0.0000	(—)	(—)
λ^{r}	−0.0764	(0.0372)	(0.0412)

Table 2: The first column contains the estimates. The second and third columns contains standard deviations estimated using first (SD1) and second (SD2) derivatives, respectively.

level, and reestimate. Due to the fact that the likelihood function is rather insensitive to this parameter, the second step parameters are similar to those of the first step. Indeed, the sample mean of the short rate remains, at the second step, at 5.68%. The parameter estimates are reported in Table 2. We note that all[20] parameters are significant at the 5% level. We also note that the standard deviations computed using the first and second derivatives are similar, indicating that the asymptotic analysis may be appropriate.

Panel 1 of Figure 2 shows the average term structure of swap yields, up to 10 years, that is implied by the model along with those observed in the market, using data on 6-month LIBOR, as well as 2, 3, 4, 5, 7, and 10 year swap rates.[21] The model implies a term structure that has essentially the same average shape as that observed in the market. Panel 2 shows the empirical and model-implied term structures of volatility. We see that the term structure of volatility is matched well except at the short-maturity end. Panel 3 shows the average difference between model yields and observed swap yields for maturities of up to 15 years. It also shows the one-standard-deviation bands. Most of the model's pricing errors are well within 20 basis point for swaps of maturities shorter than 15 years.

The Russian bonds that we consider have maturities of less than 15 years.

[20]The parameter K^{vv} is significant when we use the standard error computed using the second derivative, but not with that computed using the first derivative.

[21]These are the only swap rates that we have data on for the entire sample period. Hence, we cannot make this graph for swap yields with longer maturities.

Our model's pricing errors corresponding to the risk-free part of these securities are small, allowing us to study the term structure of spreads without obvious distortions due to misspecification of the underlying risk-free rates.

	est.	SD1	SD2
θ^s	0.0200	(18.0348)	(20.2950)
θ^1	0.0000	(—)	(—)
θ^2	0.0000	(—)	(—)
K^{sv}	0.0230	(0.0165)	(0.0159)
K^{sr}	0.0898	(0.1575)	(0.1484)
K^{ss}	0.2112	(0.0193)	(0.0107)
κ^1	1.3433	(0.8149)	(0.9836)
κ^2	1.8690	(0.9614)	(1.0859)
Σ^{sv}	0.0000	(—)	(—)
Σ^{sr}	0.0000	(—)	(—)
Σ^{ss}	7.8895	(0.5213)	(0.3713)
σ^1	2.4126	(0.2610)	(0.1915)
σ^2	0.3518	(0.1887)	(0.0892)
λ^s	−0.2547	(0.4827)	(0.5320)
$\tilde{\kappa}^1$	−0.1989	(0.0572)	(0.0394)
$\tilde{\kappa}^2$	−0.2301	(0.0642)	(0.0305)

Table 3: The first column contains the estimates. The second and third columns contain standard deviations estimated using first (SD1) and second (SD2) derivatives, respectively. These standard errors are not corrected for the facts that the swap parameters have been estimated, and that the Russian bond data are non-stationary because of changing maturities.

5.3 The Russian Term Structure

In this section, we lay out the estimated term-structure model for Russian sovereign bonds and discuss various implications for the model. The model is estimated using maximum likelihood, as discussed in Section 4. The estimated parameters are reported in Table 3. We have imposed four over-identifying restrictions because of the high dimension of the parameter space. We set the long-run means, θ^1 and θ^2, of the idiosyncratic factors, γ^1 and γ^2, to zero. Likewise, we let $\Sigma^{sv} = \Sigma^{sr} = 0$. The model allows the bench-

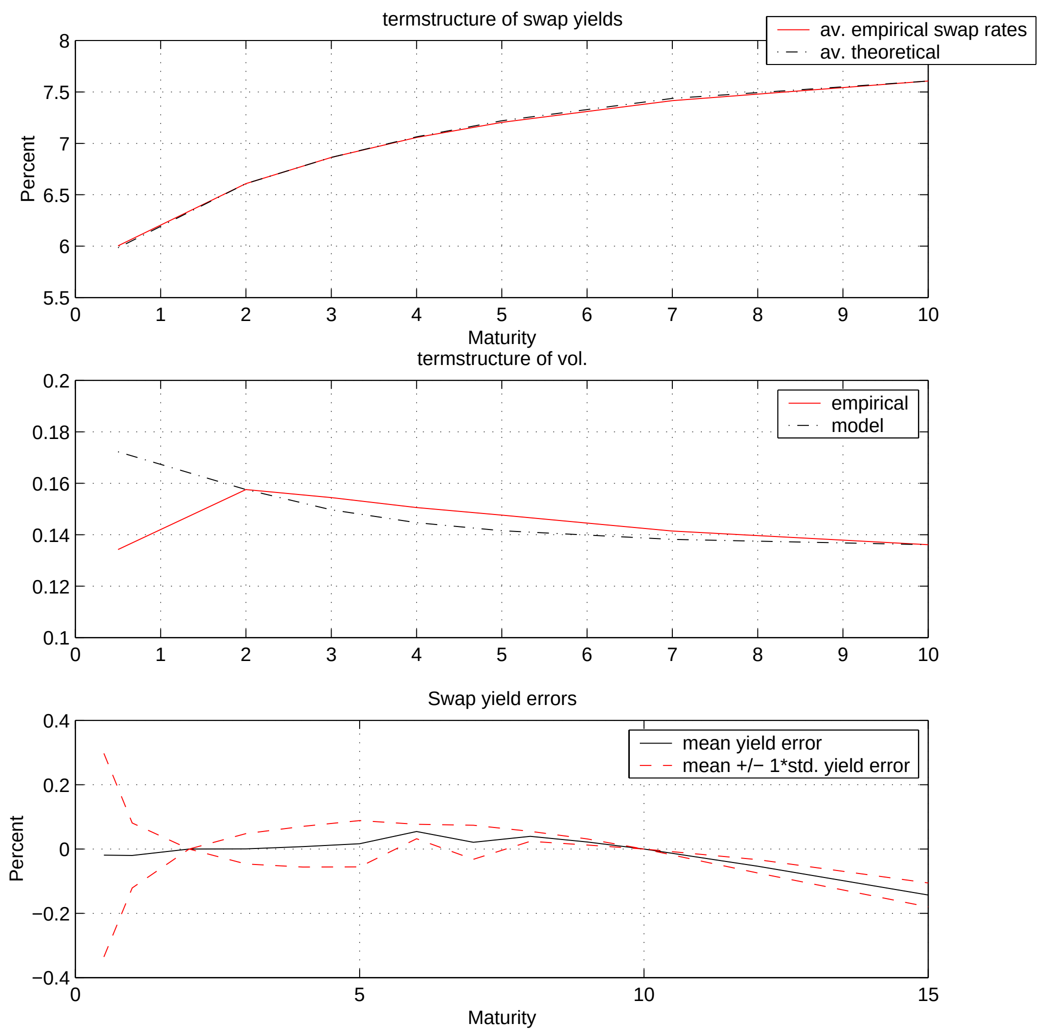

Figure 2: Panel 1 shows the average term structure of swap rates observed in the market and implied by our model. Panel 2 shows the average term structure of volatility of swap rates, also both observed in the market and implied by the model. Panel 3 shows the average difference between model rates and observed rates as well as one-standard-deviation bands.

mark spread factor, s, to be correlated with both r and v, since K^{sv} and K^{sr} may be non-zero. We re-estimated the model after freeing θ^1 and θ^2. These parameters were estimated to be insignificantly different from zero at conventional levels of significance, the model had essentially the same qualitative results as the restricted one. Moreover, freeing up θ^1 and θ^2 in this way gave other parameters higher standard errors.

Panel 1 of Figure 3 shows the mean term structure of Russian MinFin 4 zero-coupon yield spreads implied by the model. This term structure is computed as follows. The model implies that the yield of a riskless zero-coupon bond with maturity τ is of the the form $A(\tau)+B^v(\tau)v_t+B^r(\tau)r_t$, and that the yield of a zero-coupon bond with the same credit risk as the MinFin 4 is of the form $\bar{A}(\tau)+\bar{B}^v(\tau)v_t+\bar{B}^r(\tau)r_t+\bar{B}^s(\tau)s_t$, where A, B^v, B^r, $\bar{A}$, $\bar{B}^v$, $\bar{B}^r$, $\bar{B}^s$ can be found as solutions to ordinary differential equations (Duffie and Kan 1996). Hence, the term structure of zero-coupon yield spreads at time t is given, at maturity τ, by,

$$\bar{A}(\tau)-A(\tau)+\left(\bar{B}^v(\tau)-B^v(\tau)\right)v_t+\left(\bar{B}^r(\tau)-B^r(\tau)\right)r_t+\bar{B}^s(\tau)s_t. \tag{19}$$

The mean term structure is found by replacing the state variables by their implied sample means. The graph of Panel 1 shows that the estimated mean term structure is downward sloping as is common for low-quality bonds. It is not, however, downward sloping during the entire sample period. In 1997, the spreads were low. During this period the term structure was upward sloping. This behavior is consistent with a view by market participants that spreads are mean reverting (at least risk neutrally).

Panel 2 decomposes the spread into its components, in that it shows each term of (19) with the coefficients replaced by their estimates and state variables replaced by their implied sample means. Panel 3 decomposes the variation of the term structure. This is done as follows. Each component of the term structure of spreads is evaluated at the mean of the corresponding state variable, plus or minus twice its sample standard deviation. We see that changes in the level of the instantaneous risk-free rate have almost no effect on the term structure of yield spreads, for given values of v and s. When the volatility of the short rate increases, however, the spread on long-maturity bonds narrows. Variation in the volatility of the risk-free rate contributes only marginally to the overall variation in the spread. Most of the variation of the spread is due to changes in the spread factor itself. The short rate and its volatility may have an indirect effect on the spread through their

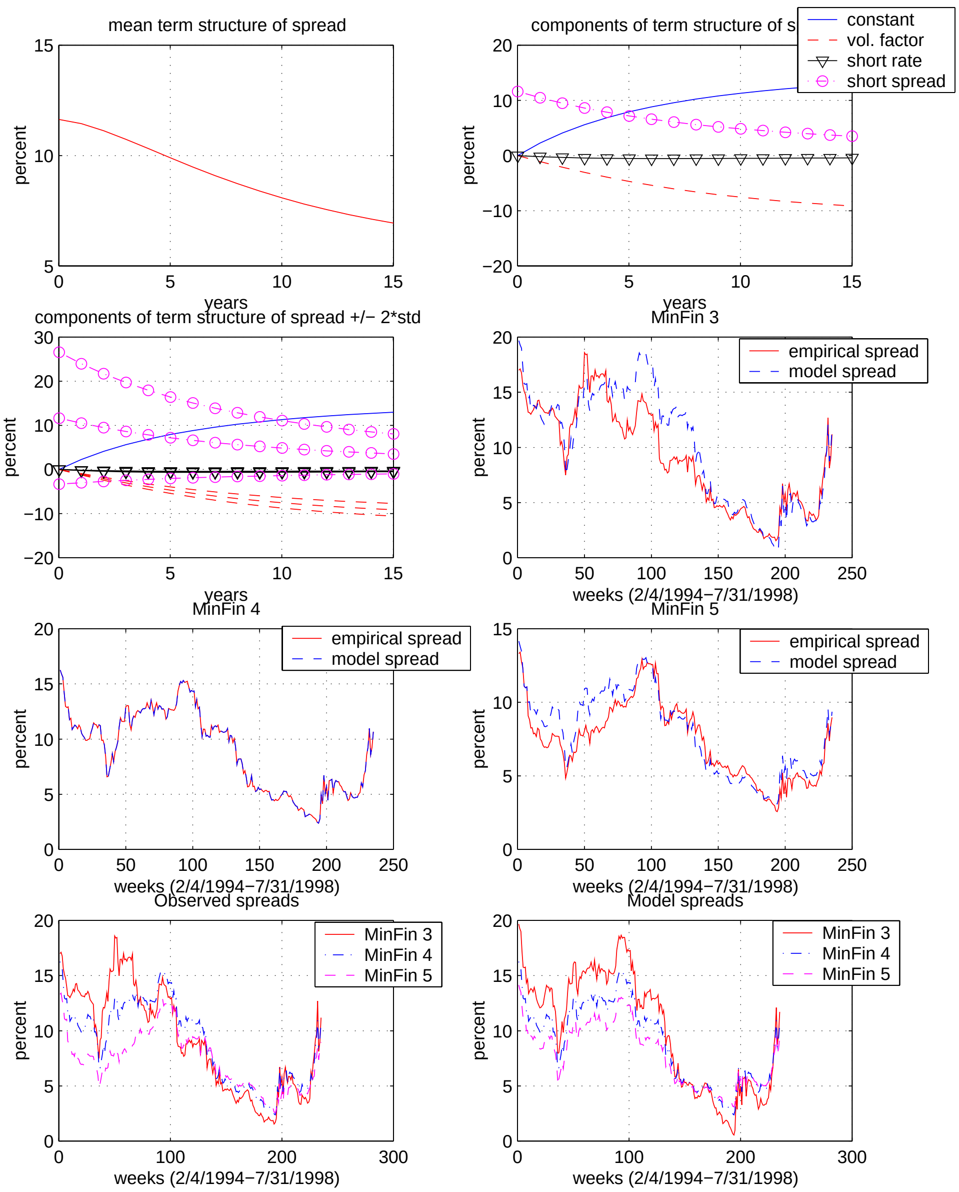

Figure 3: Panel 1 (top left) shows the sample mean of the zero-coupon term structure of yield spreads of bonds with the same credit risk as MinFin 4. Panel 2 (top right) shows the components of this term structure of spreads, and Panel 3 shows the components of the spread variation. Panels 4–6 contain yield spreads observed empirically, and implied by the model without idiosyncratic factors, for MinFin 3, 4 and 5, respectively. Panel 7 has the observed yield spreads of MinFins 3–5, and Panel 8 have those implied by the model without idiosyncratic factors.

correlations with the spread. The sample correlation between r and s is -0.18, the sample correlation of v and s is 0.46. This indicates that higher US short rates, and lower US short-rate volatilities, are associated with lower spreads on Russian bonds.

The model is designed to match exactly the yield spreads of the three MinFin bonds. If we price these bonds without including the effects of the idiosyncratic factors, γ^1 and γ^2, however, then the yield spreads of the MinFin 3 and 5 are not matched exactly. The spreads computed in this way can be interpreted as the spreads of the MinFin 3 and 5 *if they had the same liquidity and default recovery rate as does the MinFin 4*. If these MinFin 4-implied spreads were close to those observed in the market, it would mean that the markets for the different MinFins are well integrated, and that these bonds have similar recovery and liquidity. (Of course, this rests on an assumption that the model is well specified and the estimates are relatively accurate.) Panel 4 of Figure 3 shows the yield spread of the MinFin 3 observed in the data and that implied by the model when γ^1 is not used. Similarly, Panels 5 and 6 show the yield spreads of the MinFin 4 and MinFin 5. We see that the markets are well integrated during some time periods. During other time periods, however, there are several hundred basis points between the market yield and the non-adjusted modeled yield. Panel 7 shows the yield spreads of all three MinFins observed in the data. Panel 8 shows the yields implied by the MinFin 4 according to the model with no idiosyncratic factors. We note that the yield curve implied by the model flips from inverted to normal at approximately the same time as that observed in the data, a positive sign of the fit of the model, even without idiosyncratic factors.

5.4 Discussion and Attribution of Spreads

Panel 1 of Figure 4 shows the time series of the implied benchmark short spread. This MinFin 4 short spread showed large variation over the time period that we consider. This spread was over 25% at the beginning of 1994, after which it generally decreased until October 1997. Then, the short spread tended to increase until the default of August 1998. The short spread was negative between 9/19/1997 and 10/24/1997, a potentially troubling result. One interpretation of this is that Russian MinFins were "too expensive" in this period. This possible sign of "over pricing" in the market is consistent with observations made by some market participants. For instance, Malleret, Orlova, and Romanov (1999) write:

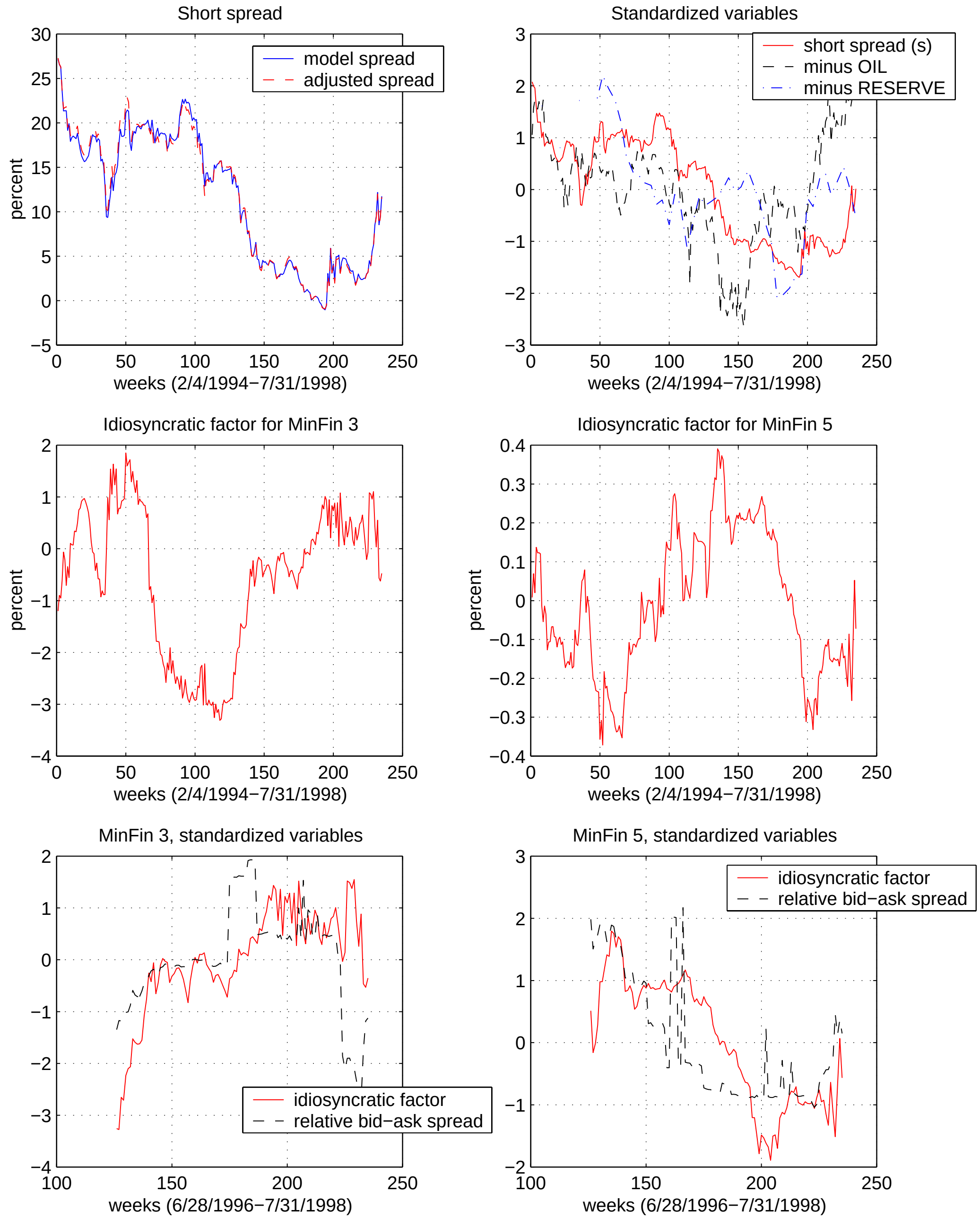

Figure 4: Panel 1 (top left) shows the time series of the implied MinFin 4 short spread. Panel 2 (top right) show the short spread, minus the Brent oil price, and minus the Russian reserves. These variables have been standardized to have mean zero and unit standard deviation. Panels 3 and 4 show the idiosyncratic factors corresponding to MinFin 3 and 5, respectively. Panels 5 and 6 contain the both idiosyncratic factors and empirical relative bid-ask spreads again for MinFin 3 and 5, respectively.

> "In 1996 and 1997, Russia was the darling of international investors. (...) the investors had been buying not the unappealing reality, but Russia's expected bright future."

The IMF (1999a) writes:

> "there is clear evidence that Russia represents a case where many investors bought securities that they did not fully understand (...) Some investors in MinFins appear to have lacked a full understanding of their legal status, including the jurisdiction in which they where issued."

We note that the yield spreads of all bonds are positive — it is the instantaneous spread that becomes negative. Only the dynamic nature of the model allows us to pick up this possibility of inconsistent pricing during this time period. Of course, we must also entertain the explanation of model misspecification or inaccurate parameter estimates. One potential problem could be that our model does not match the risk-free short rate perfectly. To determine the extent of this problem, we compute an adjusted short spread as $s_t + r_t - r_t^{obs}$, where r_t^{obs} is the 1-month LIBOR rate. Panel 1 shows that the difference between the short spread and its adjusted version is small. Indeed, the adjusted short spread is also negative when the short spread is.

The spread of Russian sovereign debt is mostly determined by the risk of a Russian default or restructuring. What determines this risk? The most important factors are the Russian budget deficit, current account, reserves of hard currency, required future debt service, and expectations of future levels of these variables. The IMF's International Financial Statistic contains monthly data on Russia's budget deficit (for part of our sample period), quarterly data of the current account, monthly data of the total reserves minus gold, among other economic variables. The (monthly) data on the budget deficit have extreme fluctuations, and may not be reliable. The data on the current account is of frequency (quarterly) too low for our purposes. Instead of using these variables, we use data on Brent oil prices. This variable is both reliable and available at high frequency, and it proxies for some of the macro-economic variables in which we are interested. Fuel products constituted between 40% and 50% of Russia's exports in the period that we consider, and oil products contributed to more than half of this (IMF 1999b). Hence, the oil price is important in determining both present and future current accounts. Also, the oil price is related to Russia's budget deficit. In addition, we consider the effect of the reserves on the Russian spread. Panel 2 of Figure 4 shows the short spread together with (minus) the Brent oil price index and (minus) the reserves. All of these time series are standardized so that they have zero means and unit standard deviations. We see that there is a somewhat close connection between these variables. A simple way of considering the effect of reserves and oil price on spreads is to do the following regression:

$$\underset{}{s_t =} \underset{(8.9)}{26.7} - \underset{(0.52)}{0.277} * OIL_t - \underset{(0.0002)}{0.0010} * RESERVE_t + \varepsilon_t,$$

with an R^2 of 81.4%. Here, OIL is the Brent oil price index, $RESERVE$ is the Russian total reserves (minus gold), and the numbers in brackets are standard errors, which are corrected for heteroscedasticity using the Newey-West method. The regression is done for monthly data. The regression coefficients are both negative, although only the $RESERVE$-coefficient is statistically significant on conventional levels. These signs have the natural implications that the spread is low in periods when the oil price is high and the reserves are high. The R^2 is high considering how few variables that we have included, but this could be related to the fact that the time series are rather persistent. Regressing in differences, we get

$$\Delta s_t = \underset{(.33)}{-0.30} + \underset{(0.13)}{0.0013} * \Delta OIL_t - \underset{(0.0002)}{0.00022} * \Delta RESERVE_t + \varepsilon_t,$$

with an R^2 of 4.4%. In this regression the sign of the OIL-coefficent has changed, but it is highly insignificant. The $RESERVE$-coefficient has the expected sign, but is not significant. This can be due to the relatively small sample size (53 months). The R^2 is rather modest which says that, on a month to month basis, there are many other factors that determine spreads than those considered here. To determine the longer-run effects, we estimate a VAR for the three variables. Figures 5 and 6 contain orthogonalized impulse-response functions. We see that shocks to the spread itself die out during a few months. Shocks to the oil price and reserves, on the other hand, build up over almost a year. As we would expect, positive shocks to oil price and reserves implies that the spread decreases in the following period.

Panels 3 and 4 of Figure 4 shows the idiosyncratic factors corresponding to the MinFin 3 and MinFin 5, respectively. These factors can be attributed to differences in either liquidity or expected recovery. It is interesting to consider which of these causes is the more important. This question is hard to answer, though, since we have no data on investor expectations of recovery, and only poor data on the liquidity of the MinFins. To determine the liquidity of these bonds, their repo rates, volumes of trade, and bid-ask spreads are relevant variables. We have not yet been able to get data on the former two, and have only obtained data for the last half of the sample period for the bid-ask spread. We call the bid-ask spread divided by the mid price the "relative bid-ask spread." Panel 5 of Figure 4 shows the relative bid-ask spread of the MinFin 3 minus that of the MinFin 4. Panel 5 also shows the implied idiosyncratic short-spread factor, γ^1, corresponding to the MinFin 3. Both of these time series are standardized. Panel 6 shows the same for the MinFin 5. Both graphs show some connection between the two measures of relative liquidity. We note, however, that this is mostly driven by the fact that the relative bid-ask spread has the price in the denominator — the bid-ask spread itself hardly contributes. In fact, constructing these graphs as if the bid-ask spreads are constant gives a more striking similarity. (This is not shown in the figure.) If idiosyncratic short-spread factors are not related to differences in liquidity, then they should be driven by differences is the risk of and expected recovery at restructurings (or other credit events). To get a idea of whether this is the case, we try to explain the pattern of the idiocyncratic factors in light of the macro economic events that happened. We focus on the idiosyncratic factor, γ^1, of MinFin 3 since it is bigger in magnitude, and it moves according to a somewhat clear pattern: First, it varies around zero, then from May 1995 to August/October 1996 it is negative, and after October 1996 it is close to zero. The macro-economic developments in this period are described by IMF (1999b) in the following way:

> "...the authorities resolved in 1995 to achieve a decisive reduction in inflation via a tight monetary policy supported by a halving of the federal budget deficit to under 6 percent of GDP. ... Finally, to foster stability of the ruble, they adopted an exchange rate band system from July 1995. Despite political pressures, the authorities stuck to their program, and their main goals were achieved."

Furthermore, Yeltsin's re-election in July 1996 increased trust in stability, and helped to strenghten the market-oriented reformers. In this period the optimism was growing, and January-September 1997 is called "The Zenith of

Expectations" (IMF 1999b). The following story would reconcile the macroeconomic events with the idiosyncratic factor implied by our model: In 1995, Russia implements reforms, which are viewed positively by the market. These are expected to have mostly shorter-term effects, though. Therefore, the shortest bond, MinFin 3, is expensive (compared to MinFin 4) in this period. This corresponds to the period where the γ^1 is negative. In late 1996, after Yeltsin's election, the market accepts these improvements as more permanent, and the MinFin 3 aligns with the MinFin 4. Although this story may partly explain the implied path of γ^1, it is hard to tell whether the idiosyncratic factors really describe differences is credit quality. A final possibility is that the idosyncratic factors are driven by model misspecification. The fact that γ^1 and γ^2 are somewhat negatively correlated may suggest that it might be desirable to have an additional factor for the (benchmark) spread model to describe the slope of the term structure of spreads.

In Figure 7, we apply the model to price bonds that were not used in estimation, namely the MinFin 6, MinFin 7, Eurobond-01, and Eurobond-07. Descriptive information about these bonds is found in Table 1. For each of these out-of-sample bonds, we compute two model prices: one using no idiosyncratic factor, and one using the idiosyncratic factor, γ^i, corresponding to that bond whose maturity is nearest. It is seen that both of these models match the two out-of-sample MinFin spreads reasonably well during most of the sample period. The model-implied spread of the Eurobonds is too high prior to September 1997 (week 190), after which time the model-implied spreads match well the observed Eurobond spreads. This indicates that the market considered Eurobonds to be of a higher quality than MinFins before September 1997, and of the same quality after that time. Our interpretation of these spread differences is as follows. In case of a Russian crisis, three stylized scenarios can be imagined: *(i)* Russia defaults on all sovereign debt, *(ii)* Russia defaults on global debt only, and *(iii)* Russia defaults on internal debt only. MinFin bonds are considered domestic bonds and are partly held by residents of Russia, whereas Eurobonds are global bonds held abroad. Hence, spread differentials are likely to be caused by differences in the relative likelihoods of cases *(ii)* and *(iii)*. Russia is more likely to default on internal debt when the country as a whole is productive and needs a good relationship with other countries in order to attract capital and encourage trade. Russia is more likely to default on external debt when the country is doing so poorly that it is unlikely to attract much capital anyway, and the government may decide that making payments to residents is more important than to foreign

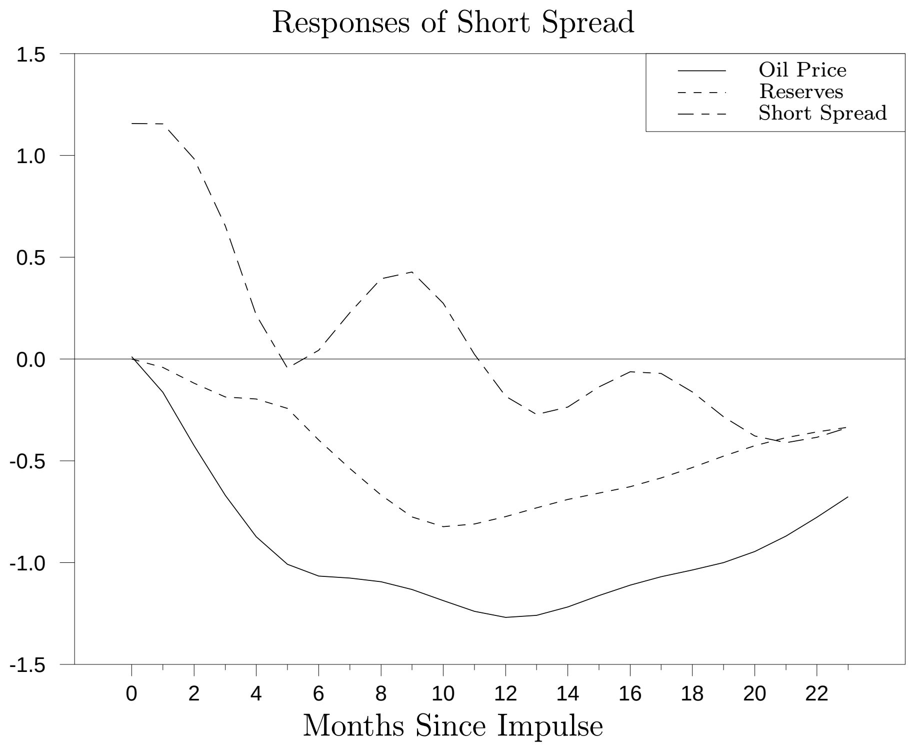

Figure 5: This graph shows the orthogonalized impulse-response function of the short rate based on a VAR of the short rate, the oil price, and the Russian reserves.

institutions. Also, political favors can be particularly important in such poor times. After Yeltsin's victory in 1996 and until September 1997, there was a high degree of confidence in Russia's political stability and economy. This is why investors expected case *(iii)* to be the more likely scenario and therefore Eurobonds were trading at lower spreads than MinFins. When the Russian economy's problems mounted in late 1997 and 1998, investors re-evaluated the Russian risk — they became unsure about whether case *(ii)* or case *(iii)* was more likely. This story is consistent with the fact that MinFins and Eurobonds were trading at similar spreads during this period. In August 1998, Russia defaulted on its Ruble-denominated Treasuries. This, according to the story, convinced investors that case *(iii)* was the likelier one, and MinFin prices dropped more severely than did Eurobond prices.

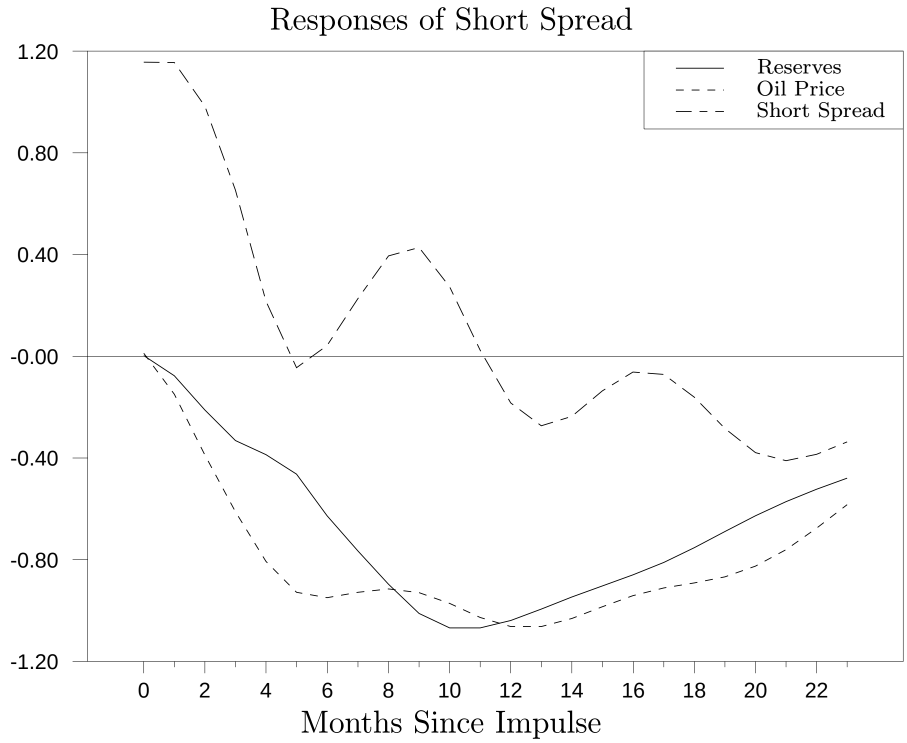

Figure 6: This graph shows the orthogonalized impulse-response function of the short rate based on a VAR of the short rate, the oil price, and the Russian reserves. Here, the variables have been orthogonalized in a different order than in Figure 5.

Figure 8, Panel 1 shows the implied risk-neutral default hazard rates for the MinFin 4 bond under the assumption that the recovery of market value (RMV) is constant and equal to 0.2, and that there is no compensation for illiquidity. When Russia defaulted in August 1998 the MinFin 4 fell to a value which was around 15–25% of its pre-default value. Although the implied hazard rate is increasing in the weeks prior to default, it is not exploding. This shows that the default may indeed be viewed as a "surprise event," consistent in spirit with our intensity-based modeling approach, according to which default times are inaccessible stopping times, meaning roughly that they cannot be foretold immediately beforehand. Panels 2 shows the implied expected rates of recovery of market value for the MinFin 3, 4, and 5 assuming that they have the same hazard rates. Similarly, Panel 3 shows the implied

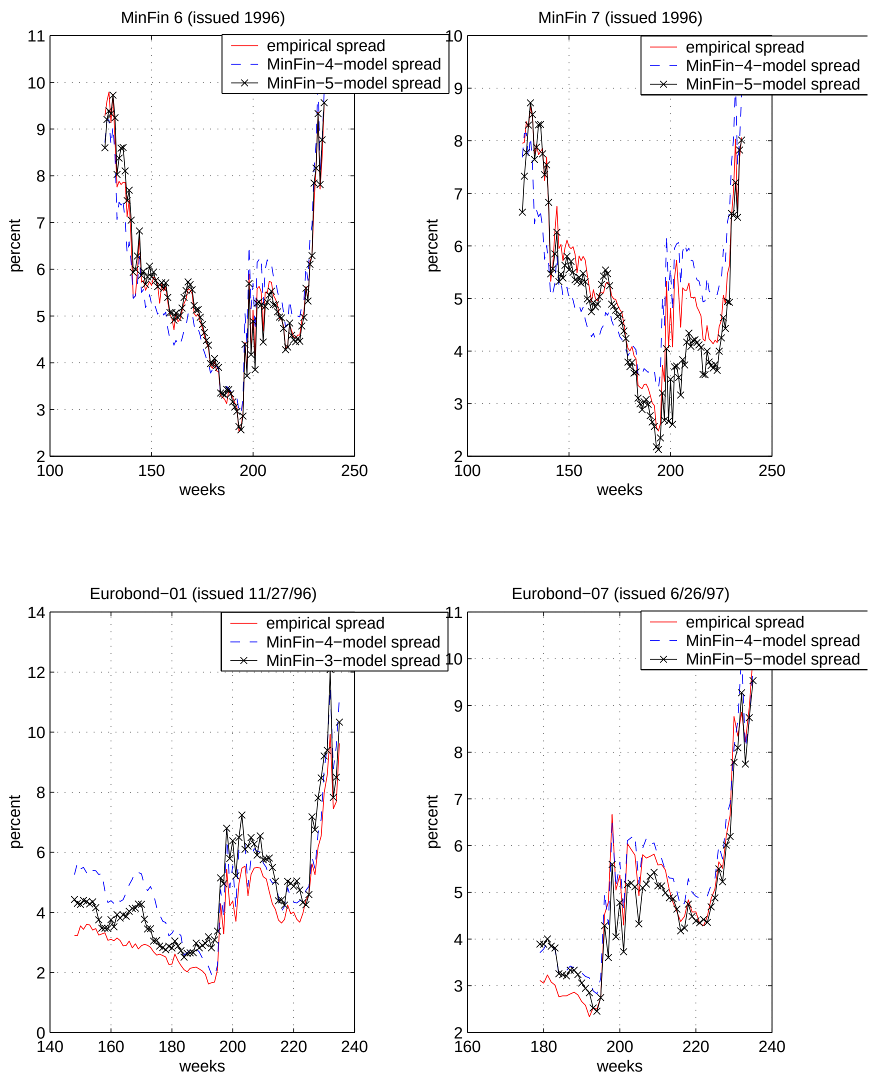

Figure 7: Panels 1 (top left) and 2 (top right) contain the empirical and model-implied yield spreads of MinFin 6 and 7, respectively. The model-implied spreads are computed both using the MinFin 4 model, and the model corresponding to MinFin 3 or 5 depending on which one has the nearer maturity to the bond showed in the graph. Panels 3 and 4 show these spreads for Eurobond-01 and Eurobond-07, respectively.

expected rates of recovery of face value (RFV).

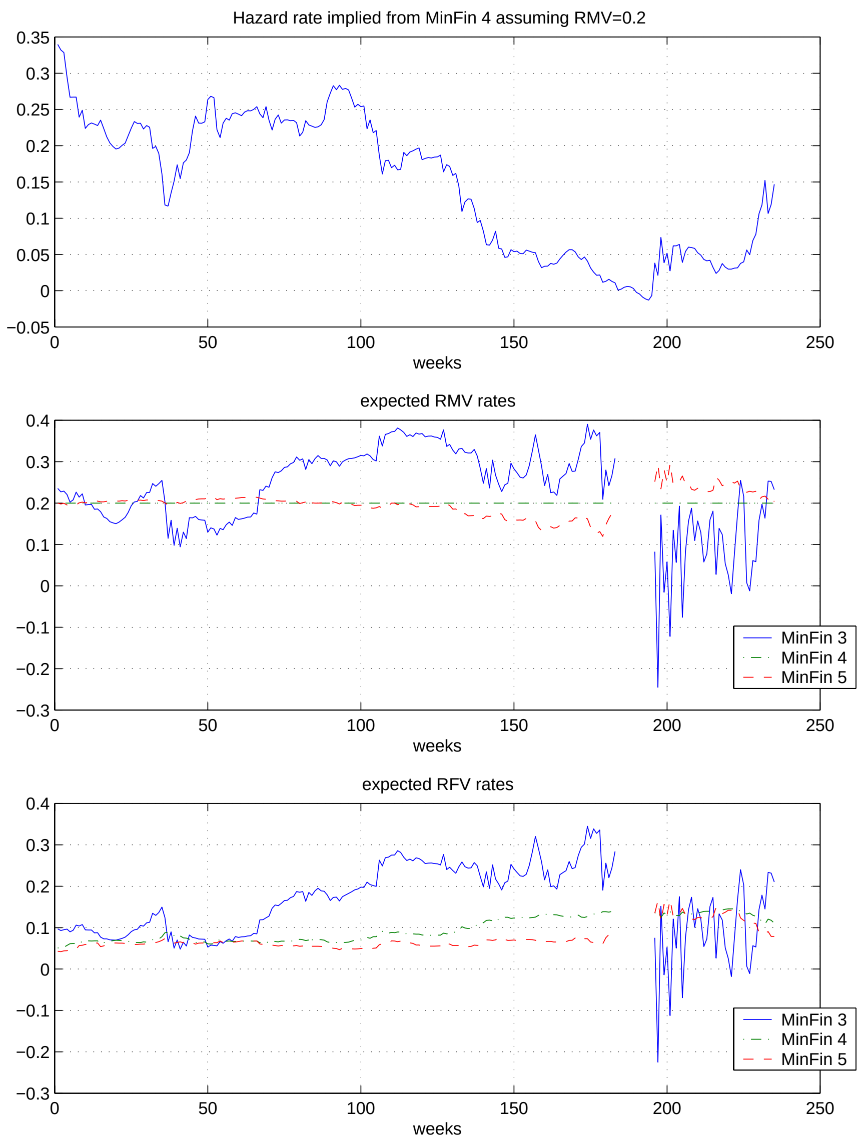

Figure 8: Panel 1 shows the model-implied (MinFin 4) default hazard rate assuming that the expected recovery at market value (RMV) is constant and equal to 0.2. Panel 2 shows the model-implied expected rates of recovery of market value of MinFins 3, 4, and 5 assuming that all three bonds have the same default hazard rate and that the MinFin 4 has a RMV of 0.2. Panel 3 shows the implied expected rates of recovery of face value (RFV) under the same assumptions.

Appendix A: Proof of Theorem 1

The proof of this theorem uses standard techniques and is only sketched. The discounted gain process is

$$G_t = -\int_0^t e^{-\int_0^v R_u\,du} l_v P_v\,dv + \int_0^t e^{-\int_0^v R_u\,du} Z_{N_v} P_{v-}\,dN_v + e^{-\int_0^t R_u\,du} P_t.$$

Let M be the Q-martingale defined by $M_t = E_t^Q\left(e^{-\int_0^T R_u\,du} X\right)$.[22] Using Itô's Formula, we get

$$\begin{aligned} dG_t &= e^{-\int_0^t R_u\,du} P_t\left[(s_t - l_t)\,dt - (1 - Y_{N_t} - Z_{N_t})\,dN_t\right] \\ &\quad + e^{-\int_0^t s_u\,du}\left(\prod_{n<N_t} Y^n\right) dM_t. \end{aligned}$$

Under the conditions assumed, the gain process is seen to be a Q-martingale using that $(1 - Y_{N_t} - Z_{N_t})\,dN_t = hL\,dt + dM_t^N$, where M^N and is a Q-martingale.

Appendix B: Transition Density

In this section, we derive an arbitrarily good approximation to the transition density

$$p(Y_\delta \mid Y_0) = p(Y_\delta^I \mid Y_0^I)\, p(Y_\delta^D \mid Y_0, Y_\delta^I). \tag{20}$$

The first factor of the transition density is known to be of the form

$$p(Y_\delta^I \mid Y_0^I) = \prod_{i\in I} p(Y_\delta^i \mid Y_0^i), \tag{21}$$

where

$$p(Y_\delta^i \mid Y_0^i) = c_i e^{c_i(Y_\delta^i + e^{-K_{ii}\delta} Y_0^i)} \left(\frac{Y_\delta^i}{e^{-K_{ii}\delta} Y_0^i}\right)^{q_i/2} \tag{22}$$

$$\cdot I_{q_i}\left(2c_i\left(Y_\delta^i Y_0^i e^{-K_{ii}\delta}\right)^{1/2}\right), \tag{23}$$

[22]The proces M is well defined because V is assumed to be well defined.

with $c_i = 2K_{ii}\left(1 - e^{-K_{ii}\delta}\right)^{-1}$, $q_i = 2k_i - 1$, and I_q denotes the modified Bessel function of the first kind of order q. We focus on computation of the second part of the transition density.

The distribution of Y_δ^D conditional on Y_0^D *and the entire path of* Y^I *from time* 0 *to time* δ is known to be normal. Specifically,

$$p(Y_\delta^D \mid Y_0, Y_s^I, s \in [0,\delta]) = \varphi(Y_\delta^D, m_\delta, V_\delta), \tag{24}$$

where $\varphi(\,\cdot\,, m, V)$ is the density of a normal with mean m and variance V,

$$\varphi(x, m, V) = \left((2\pi)^N |\det(V)|\right)^{-\frac{1}{2}} e^{-\frac{1}{2}(x-m)^\top V^{-1}(x-m)}, \tag{25}$$

and

$$m_\delta = e^{-K^{DD}\delta}\left[\int_0^\delta e^{K^{DD}s}(k^D - K^{DI}Y_s^I)\,ds + Y_0^D\right] \tag{26}$$

$$V_\delta = e^{-K^{DD}\delta}\left[\int_0^\delta e^{K^{DD}s}\Delta(\alpha^D + B^{DI}Y_s^I)e^{K^{DD\top}s}\,ds\right]e^{-K^{DD\top}\delta}. \tag{27}$$

Hence, by the law of iterated expectations,

$$p(y_\delta^D \mid Y_0, Y_\delta^I) = E\left(p(y_\delta^D \mid Y_0, Y_s^I, s \in [0,\delta]) \mid Y_0, Y_\delta^I\right) \tag{28}$$

$$= E\left(\varphi(y_\delta^D, m_\delta, V_\delta) \mid Y_0, Y_\delta^I\right). \tag{29}$$

We may approximate (29) by simulating the path of Y^I conditional on its starting point, Y_0^I, and its endpoint, Y_δ^I. This is done as follows.

We divide the interval $[0,\delta]$ into J sub-intervals, and let Y^J be the piecewise-linear approximation to Y^I corresponding to this division of the interval. (We could have used other interpolation schemes.) We define m^J and V^J similarly to m and V, respectively, but with Y^I replaced with Y^J. This is like approximating the integrals in (26) and (27) with Riemann sums. Hence, we have (a.s.) that $(m_\delta^J, V_\delta^J) \to (m_\delta, V_\delta)$, as $J \to \infty$. Using this result as well as dominated convergence,[23]

$$p(y_\delta^D \mid Y_0, Y_\delta^I) = E\left(\varphi(y_\delta^D, m_\delta, V_\delta) \mid Y_0, Y_\delta^I\right) \tag{30}$$

$$= E\left(\lim_{J\to\infty}\varphi(y_\delta^D, m_\delta^J, V_\delta^J) \mid Y_0, Y_\delta^I\right) \tag{31}$$

$$= \lim_{J\to\infty} E\left(\varphi(y_\delta^D, m_\delta^J, V_\delta^J) \mid Y_0, Y_\delta^I\right) \tag{32}$$

$$= \lim_{J\to\infty} p^J(y_\delta^D \mid Y_0, Y_\delta^I), \tag{33}$$

[23]When $\alpha^D > 0$, V_δ^J is bounded away from zero and dominated convergence is immediate. Otherwise, it may be shown under regularity.

where p^J is defined by the last equation. This gives us a sequence, $\{p^J\}$, of approximations to the density, that converges to the true density. The J'th approximation, p^J, can be computed by Monte Carlo integration:

$$p^J(y_\delta^D \mid Y_0, Y_\delta^I) = \lim_{Z\to\infty} \frac{1}{Z} \sum_{z=1}^{Z} \varphi(y_\delta^D, m_\delta^{Y^{J,z}}, V_\delta^{Y^{J,z}}). \tag{34}$$

Here, $(Y^{J,z})_{z=1,2,\ldots}$ are independent simulations of $(Y_0^I, Y_{\delta/J}^I, \ldots, Y_\delta^I)$, drawn from their joint conditional distribution given Y_0^I and Y_δ^I. We take $m^{Y^{J,z}}$ and $V^{Y^{J,z}}$ to be the corresponding mean and variance, based on linear interpolations. We an compute $m^{Y^{J,z}}$ and $V^{Y^{J,z}}$ explicitly since we need only compute integrals of exponentials. We know the transition densities of Y^I in closed form, but simulating from this density is not trivial. Instead, we can simulate from another (possibly incorrect) density for $Y^{J,z}$, say $\hat{f}(\,\cdot\,)$, and use "importance sampling" to compute p^J as

$$p^J(y_\delta^D \mid Y_0, Y_\delta^I) = \lim_{Z\to\infty} \frac{1}{Z} \sum_{z=1}^{Z} \varphi(y_\delta^D, m_\delta^{Y^{J,z}}, V_\delta^{Y^{J,z}}) \frac{f(Y^{J,z})}{\hat{f}(Y^{J,z})}, \tag{35}$$

where $(Y^{J,z})_{z=1,2,\ldots}$ are independent draws from $\hat{f}(\,\cdot\,)$, and $f(\,\cdot\,)$ denotes the true density of $(Y_0^I, Y_{\delta/J}^I, \ldots, Y_\delta^I)$ conditional on Y_0^I and Y_δ^I.

The first-order ($J = 1$) density approximation p^1 *does not require simulation*, since it is based on the (linear) interpolation between Y_0^I and Y_δ^I. We have found this approximation to be tractable and accurate for our application.

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