Preliminary and Incomplete

Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles

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Abstract

In this paper we model dividend and consumption growth rates as having a small long-run predictable component. Such a component, for which we provide empirical support, plays a very significant role in determining asset prices. We show that, in equilibrium, these growth rate dynamics in conjunction with 'plausible' parameter configurations of the Epstein and Zin (1989) preferences can explain key observed asset markets phenomena. In particular, the model is capable of justifying the observed equity premium, the low risk free rate, and the volatilities of the market return and the real risk free rate.

1 Introduction

An enduring theme in economics is that asset prices are determined as an appropriately discounted value of the cash-flows. Further, in equilibrium the ex-ante rates of return are determined by the preferences of the agents and the time-series properties of the cash-flows. It is well recognized by now that a wide range of general equilibrium models find it difficult to simultaneously justify observed key features of asset markets data. Shiller (1981) and LeRoy and Porter (1981) argue that the observed dividend series is too smooth to justify the observed volatility of the market return (approximately 16% standard deviation per-annum). Mehra and Prescott (1985), Weil (1989), and Hansen and Jagannathan (1991), document the serious difficulties that standard economic models have in explaining the relatively large equity premium, and the low real risk free rate (approximately 7% and 1% respectively).

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These results suggest that from the perspective of a representative investor, with reasonably calibrated CRRA preferences, the systematic risks embodied in aggregate dividends time series are too small to justify the large equity premium. In addition, it is equally hard to justify that the volatility of the riskfree rate is less than 2% that of the market return and that risk premia seem to vary across the business cycle. In this paper we present a general equilibrium model where the interaction between dividend growth rates, which we show contains a small long-run predictable component, and non-expected utility preferences (as in Epstein and Zin (1989) and Weil (1989)) reproduces these asset markets phenomena.

It is quite standard in the asset pricing literature to model dividends (the fundamental cash flows in most asset pricing models) as a unit root process or some stationary stochastic process around a deterministic trend. The resulting model for growth rates, typically, has the feature that news regarding cash-flows alter investors perceptions regarding expected growth rates only for very short horizon, if any at all. With standard preferences (as in Mehra and Prescott (1985)), it is not too surprising that investors view such cash flows as relatively riskless and, therefore, demand small risk-premiums for holding it. In this paper we propose to model uncertainty in cash flow dynamics by decomposing cash-flows into cyclical and stochastic trend components. This characterization of the data, which we provide empirical support for, implies that cash-flow news should significantly alter the perceptions of agents regarding expected growth rates for the long run. The typical size of cash-flow news (i.e, its volatility) is quite small, and its impact on expected growth rates even smaller. However, the fact that such news has a long lasting impact on expected growth rates makes the capitalized value of the cash-flow quite risky, and one that warrants a large risk premium.

Along with our specification for the dividend (consumption) dynamics we also need the Epstein and Zin (1989) preferences to accommodate separation between risk-aversion and the elasticity of substitution parameters. In the absence of separation between risk aversion and the elasticity of intertemporal substitution, as is the case in time separable expected utility, there is an important tension between matching the equity premium, the risk-free, and the of volatility of stock returns. In particular, with risk aversion greater than one the standard model implies the counter-intuitive feature that a positive innovation to expected growth rate in cash flows implies a reduction in the price of the stock relative to current dividend. If risk-aversion is smaller than one then the equity market volatility and the risk premium may potentially be too low. Hence, the Epstein and Zin (1989) specification is important to our model just as our departure point for the cash-flow process.

In the simplest version of our model, the growth rate process for dividends is modeled and estimated in the time-series as an ARMA(1,1) with homoskedastic Gaussian innovations. We show that this specification for growth rates can be motivated as an outcome of a simple stochastic trend and cyclical variation model for the level of the dividends. Given this growth rate process, the Epstein and Zin (1989) preferences, and the Campbell and Shiller (1988) log-linearization for continuous ex-post returns, we analytically solve for equilibrium asset prices. For 'plausible' parameter values for preferences and the growth rate process, we find that the model can produce the observed level of the risk-free rate, the equity premium, the volatilities of stock returns and the risk free rate, and the observed auto-correlation of the price-dividend ratio.

To allow for time-varying risk premium, we augment the above model by incorporating conditional volatility in the dividend-consumption growth rate process. The conditional volatility of the growth rate process is assumed to follow a GARCH(1,1) as in Bollerslev (1986). Empirically, we find considerable support for a GARCH(1,1) volatility process for dividend growth rates (for related evidence see also Bollerslev and Hodrick (1995)). Using this modified specification we analytically solve for asset prices. Now in equilibrium, in addition to the growth rate risk, volatility risk is also priced, and risk-premiums are time-varying. Further, the ex-post market return in our model also inherits the GARCH(1,1) stochastic volatility structure. This is consistent with a large body of work which documents that the market return volatility can be well characterized as such a process (see Bollerslev, Engle, and Wooldridge (1988). This version of the model is fully capable of justifying many of the key level and volatility anomalies discussed above. In addition, it also justifies the common empirical finding that a rise in the price-dividend ratio predicts a fall in the market risk-premium.

There is voluminous literature which addresses the aforementioned asset market anomalies. Notable example, Abel (1990), Abel (1999), Bansal and Coleman (1997), Campbell (1996), Campbell and Cochrane (1999), Cechetti, Lam, and Mark (1990), Constantinides (1990), Constantinides and Duffie (1996), Hansen, Sargent, and Tallarini (1999), Heaton (1995), Heaton and Lucas (1996), Kandel and Stambaugh (1990), address various aspects of the asset market anomalies discussed above. The approaches taken to address these asset market phenomena include transaction costs, incomplete markets, and time-non-separable preferences. Note that in the context of frictionless markets, variation in price-dividend ratios can come about either due to variation in expected growth rates of dividends or variations in ex-ante rates of return (discount rates). Indeed a recurring theme in this literature is to ascribe much of the variation in price-dividend ratios to variation in discount rates, where it is commonly assumed that dividend growth rates are *i.i.d.* However, in our model the main source of the variability in the price-dividend ratios is variation in expected growth rates. Indeed, in the simpler version of our model we explain many of the aforementioned asset market puzzles with very little variation in discount rates.

It is natural and perhaps important to ask, what additional evidence, particularly at the micro level, supports our contention that growth rates of dividends have component with significant long run implications. We feel there is considerable micro-level evidence to support this view, for example, Easton and Zmijewski (1989) and Kormendi and Lipe (1987) show that news about earnings have significant impact on returns and valuation ratios. This impact of fundamental news on valuation ratios and returns is consistent with our model, where such news significantly alter perceptions about long-term growth rates. An alternative view to justify this price reaction would be a drop in the cost of capital to positive news about earnings. It is not evident, however, that such news can lower the cost of capital significantly enough to justify the large changes in price dividend ratios. In addition, it seems to us that the current debate (since the mid 90's) about the price variability of many high-tech stocks is intimately tied to the changing perceptions regarding their expected long-term growth rates. While this may be more evident in the context of these stocks, we argue that at a broader level, these risks also determine the key magnitudes of the asset market returns at large.

The paper is organized as follows. Section 2 presents our model. In section 3 we present a closed form solution for asset prices. Section 4 presents our estimation results and discusses the empirical evidence. regarding the model. Section 5 contains some concluding remarks.

2 An Economic Model for Asset Markets

Consider a representative agent with the following Epstein and Zin (1989) - Weil (1989) recursive preferences:

$$U_t = \{(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{1/\theta}\}^{\frac{\theta}{1-\gamma}}$$

where $0 < \delta < 1$ is the rate of time preference. Let $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$, where $\gamma \geq 0$ is the risk-aversion (sensitivity) parameter, and $\psi \geq 0$ is the intertemporal elasticity of substitution. The sign of θ is determined by the magnitudes of the risk-aversion and the elasticity of substitution. In particular if $\psi > 1$ and $\gamma > 1$ then θ will be negative. Note that when $\theta = 1$, that is $\gamma = (1/\psi)$, the above recursive preferences collapse to the standard case of expected utility, with $U_t^{1-\gamma} = (1-\delta)\sum_{j=0}^{\infty}\delta^j C_{t+j}^{1-\gamma}$. Further, when $\theta = 1$ and in addition $\gamma = 1$, we get the standard case of log-utility.

The above representative agent maximizes life-time utility subject to the period budget constraint

$$C_t + P_t' h_{t+1} = D_t' h_t + P_t' h_t \equiv W_t$$

P_t' refers to the vector of asset price per share at date t that offers a real dividend stream of

$D'_{t+j}, j = 1, \cdots \infty$. h_t is vector of asset holdings at the end of time-period $t-1$ (note that this vector also includes the payoff 1 from the risk-free asset). Given the above information note that at date t the wealth of the agent is W_t. The above budget constraint can also be written as

$$(W_t - C_t) * R_{a,t+1} = W_{t+1}$$

where $W_t - C_t = P'_t h_{t+1}$, equals the amount of capital invested in the asset markets, and $R_{a,t+1} = \frac{P'_{t+1}h_{t+1}+D'_{t+1}h_{t+1}}{P'_t h_{t+1}} = \frac{W_{t+1}}{(W_t - C_t)}$ is the return on portfolio held by the agent. As in Lucas (1978), we normalize the supply of all equity claims to be one and the risk-free asset to be in zero net supply. In equilibrium, aggregate dividends in the economy equal aggregate consumption of the representative agent, that is $D'_t \iota = C_t$.

For this economy, Epstein and Zin (1989) show, that the asset pricing restrictions for asset return $R_{i,t+1}$ satisfy,

$$E_t[\delta^\theta G_{t+1}^{-\frac{\theta}{\psi}}(1 + R_{a,t+1})^{-(1-\theta)}(1 + R_{i,t+1})] = 1 \tag{1}$$

where G_{t+1} is the aggregate gross growth rate of consumption. Assuming the return on the aggregate portfolio of the representative agent coincides with the return on the market portfolio, it follows that,

$$E_t[\delta^\theta G_{t+1}^{-\frac{\theta}{\psi}}(1 + R_{m,t+1})^{\theta}] = 1 \tag{2}$$

From the definition of a return, it follows that

$$1 + R_{m,t+1} = \frac{(1 + P_{m,t+1}/D_{t+1})}{P_{m,t}/D_t}(D_{t+1}/D_t) \tag{3}$$

where we refer to $(P_{m,t}/D_t)$ as Z_t. An equilibrium for this economy is a solution for Z_t that solves the functional equation (2).

Continuous versions of variables needed to characterize the solution for the model are written in lower case letters; hence, $ln(1+R_{m,t+1}) \equiv r_{m,t+1}$, $ln(G_{t+1}) \equiv g_{t+1}$, and $ln(Z_t) = z_t$. Note that (2) can be written in terms of the continuous variables as,

$$E_t[\exp^{\{\theta ln(\delta) - \frac{\theta}{\psi} g_{t+1} + \theta r_{m,t+1}\}}] = 1 \tag{4}$$

The continuous return can be written as $r_{m,t+1} = ln(1 + (P_{m,t+1}/D_{t+1})) - z_t + g_{t+1}$. To derive analytical solution to the model we use the standard approximation derived in Campbell and

Shiller (1988), Campbell (1993).

$$r_{m,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1} \tag{5}$$

where κ_0 and κ_1 are approximating constants and both depend only on the average level of z.[1]

Note that the inter-temporal marginal rate of substitution, or the "pricing kernel" in this model is $M_{t+1} = \delta^\theta G_{t+1}^{-\frac{\theta}{\psi}}(1 + R_{m,t+1})^{-(1-\theta)}$. The one step ahead innovation, in $log(M_{t+1})$, is

$$\eta_{M,t+1} = -\frac{\theta}{\psi}\eta_{t+1} - (1-\theta)\eta_{m,t+1}, \tag{6}$$

where η_{t+1} is the innovation in continuous growth rate of consumption and $\eta_{m,t+1}$ is the innovation in the continuous market return.

2.1 Growth Rate Process

To solve the model we first need to characterize the growth rate process. The growth rate process can potentially have large effects on the implied volatility of the equity return and the size of the equity premium. For convenience, it will be easier for us, at this stage, to capture the idea of stochastic trends and cyclical variation in dividends by characterizing the growth rate process as an ARMA(1,1) process. As shown later, this process for growth rates can be motivated by assuming that the stochastic trend for the level of dividends follows an exponential smoothing process and the cyclical component is a standard first order auto-regressive process. We subsequently show empirically that the ARMA(1,1) is a good description of the dividend growth rate. For simplicity, for now, we equate the dividends and consumption process and subsequently provide a model that treats them as separate processes.

$$g_t = \mu + \rho g_{t-1} + \eta_t - \omega\eta_{t-1} \tag{7}$$

Note that this process can be more conveniently written as

$$g_t = \frac{\mu}{(1-\omega)} + (\rho - \omega)\frac{g_{t-1}}{1-\omega L} + \eta_t \tag{8}$$

[1]Note that $\kappa_1 = exp(\bar{z})/(1+exp(\bar{z}))$. In our empirical work $\kappa_1 = 0.9969$, which is based on the magnitude of $\bar{z}$ in our sample. This is consistent with magnitudes used in Campbell and Shiller (1988). Further note that Campbell and Koo (1997) show that the solution to their model is not very sensitive to this approximation.

where L is the lag operator. It is assumed that g is stationary, and hence ρ and ω are less than one in absolute value. While this standard ARMA(1,1) process characterizes the dynamics for the growth rate,he relevant state variable that affects the present values of cash flows is the conditional mean of this process. The conditional mean of g_t at date $t-1$ is determined by the state variable x_{t-1}, where

$$x_{t-1} = \frac{\mu}{(1-\omega)} + (\rho - \omega)\frac{g_{t-1}}{1-\omega L} \tag{9}$$

Using (8) and (9), it follows that x_t, is an AR(1) process,

$$x_t = \mu + \rho x_{t-1} + (\rho - \omega)\eta_t \tag{10}$$

The parameter ρ determines the persistence of the process, and ω is the smoothing parameter that affects the construction of x_t. There are two cases of particular interest that the ARMA(1,1) representation accommodates. If $\rho = \omega$, then the conditional mean of g is a constant, and g is an $i.i.d$ process. Second, if $\omega = 0$, then g is a standard AR(1) process. Consider the revision in expected growth rates for horizon $n \geq 1$

$$E_t[g_{t+n}] - E_{t-1}[E_t(g_{t+n})] = \rho^{n-1}(\rho - \omega)\eta_t \tag{11}$$

Equation (11) shows that if $\rho - \omega \neq 0$, then rational agents will revise their long-run expected growth rates in the amount stated in (11). If the difference between ρ and ω is positive and small then the revision in long-run expected growth rates is quite small; in the extreme case when $\rho = \omega$ there is no revision in the expected growth rate at all. The "permanence" of the expectation revision is determined by ρ — if it happens to equal one, then the revision in expectation is identical across all horizons. When ρ is less than one, the revision is larger for shorter horizons and almost zero for very long horizons. An interesting case is where the difference between ρ and ω is small and positive, and ρ is large — in this case growth rate news leads to very small revisions to the long-run expected growth rate.

It is important to note that when ρ is even slightly bigger than ω the growth rate process will look very close to an $i.i.d$ process — the asset pricing implications, however, can dramatically differ from the case in which the growth rate is assumed to be $i.i.d$ (that is, ρ exactly equals ω). It is quite likely, that in finite samples, the data on dividend (or consumption) growth rate by itself may not sharply be able to distinguish across these different cases.[2] It would then seem that the different asset pricing implications of these

[2]Shephard and Harvey (1990) provide small sample evidence which shows that with population values of $\rho = 1$, and $1 - \omega$ small (ω around .9), standard estimation procedures, in finite samples, are biased

alternative growth rate specifications may prove to be valuable in sharper identification of the growth rate process itself. In a similar vein Cochrane and Hansen (1992) argue that asset markets data provide important information regarding preference parameters.

Barsky and De-Long (1993) use the classic Gordon Growth Formula and an expected growth rate process with a unit root (equation (10) with $\rho = 1$) to document that such a specification can explain fluctuations in the market index. Bansal and Lundblad (1999) consider the ARMA(1,1) specification for dividend growth rates (see equation (7)) in an international context and explore its implications for asset return cross-correlations across economies utilizing the market return based static-CAPM model.

To allow for time variation in risk premia, we further assume that there is stochastic volatility in the growth rate dynamics—where, $\sigma^2_{g,t}$ is the stochastic volatility of the growth rate. Following Bollerslev (1986) we model the stochastic volatility process as a GARCH(1,1). That is, the squared innovations in the growth rate, η^2_{t+1} follows an ARMA(1,1) process,

$$\begin{aligned}\eta^2_{t+1} &= \nu_0 + \nu_1 \eta^2_t + e_{t+1} - \omega_v e_t, \\ \sigma^2_{g,t+1} &= \nu_0 + \nu_1 \sigma^2_{g,t} + w_{t+1} \qquad (12)\end{aligned}$$

where $\sigma^2_{g,t} \equiv E_t[\eta^2_{t+1}]$, and $w_{t+1} \equiv (\nu_1 - \omega_v)(\eta^2_{t+1} - \sigma^2_{g,t}) = (\nu_1 - \omega_v)e_{t+1}$.[3] Further, we assume that w_t is normally distributed and is independent of the innovation in consumption growth rate η_t.[4]

3 Solving for Asset Prices

3.1 Solution

As stated earlier to solve the model we need to derive the process for $z_t \equiv log(P_{m,t}/D_t)$. The relevant state variables for the deriving the solution, in the absence of asset bubbles, are x_t and $\sigma^2_{g,t}$. To derive a solution for the endogenous variable z_t, we substitute (5) for $r_{m,t+1}$ in (4). To do so we conjecture that $z_t = A_0 + A_1 x_t + A_2 \sigma^2_{g,t}$. This conjecture, along with Euler

toward estimating values of ω equal to one. This suggest that it is difficult, in finite samples, to detect these permanent components. We suspect that the same problems are endemic to our case when ρ is moderately less than one, and $\rho - \omega$ is small.

[3]Equation (12) can be derived as follows, $\eta^2_{t+1} = \nu_0 + \nu_1 \eta^2_t + e_{t+1} - \omega_v e_t$ or $\eta^2_{t+1} = \frac{\nu_0}{1-\omega_v} + (\nu_1 - \omega_v)\frac{\eta^2_t}{1-\omega_v L} + e_{t+1}$. Note that $\sigma^2_{g,t} \equiv E_t[\eta^2_{t+1}] = \frac{\nu_0}{1-\omega_v} + (\nu_1 - \omega_v)\frac{\eta^2_t}{1-\omega_v L}$, which in turn implies $\sigma^2_{g,t} = \nu_0 + \nu_1 \sigma^2_{g,t-1} + w_t$

[4]For simplicity we will let w_t be a mean zero normal, however, one can assume some other distribution such a chi-square with one degree of freedom, this will not change the main results. The main assumption required is that the innovation in stochastic volatility process is homoskedastic, without this simplifying assumption, the solution to the model losses it simplicity, and may not be solvable analytically.

equation associated with the market return, (4), are used to solve for the unknown vector of coefficients $\mathbf{A} = [A_0, A_1, A_2]$. The details for the solution for these coefficients are provided in Appendix A.

The solution coefficient for A_1 is,

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho} \tag{13}$$

Note that an *a-priori* belief that higher expected growth in dividends raises the price-dividend ratio, would imply that A_1 is positive. With $|\rho| < 1$ one would require ψ to be bigger than one to ensure that A_1 is positive. The positivity of A_1 captures the usual intuition of the textbook Gordon Growth formula that higher expected growth, all else equal, should increase the valuation. Note that if $|\rho| < 1$ then then standard expected utility model, with risk aversion bigger than one (i.e., $\psi < 1$), would imply that A_1 is negative — a rise in expected growth rates leads to a fall in the price-dividend ratio.

The solution coefficient for for A_2 is

$$A_2 = \frac{0.5[\theta - \frac{\theta}{\psi} + \theta A_1 \kappa_1 (\rho - \omega)]^2}{\theta(1 - \kappa_1 \nu_1)} \tag{14}$$

Note that if θ is negative, then the effect of a rise in volatility is to lower the price to dividend ratio. Note that if $\psi > 1$ is bigger one, then $\theta < 0$ for risk aversions that are bigger than one. Hence, in the case when risk aversion is bigger than one, a rise in uncertainty would lead to a drop in the price-dividend ratios. An *a priori* belief that increased economic uncertainty should lead to a drop in asset prices suggest that if $\gamma > 1$, then $\psi > 1$ as well.

3.1.1 The Equity Premium

Given the solutions for $\mathbf{A}$, it is straightforward to derive (shown in the Appendix A) the expression for the market return from which it follows that the innovation in the market return is,

$$\eta_{m,t+1} = [1 + \kappa_1 A_1 (\rho - \omega)]\eta_{t+1} + \kappa_1 A_2 w_{t+1} \tag{15}$$

Further note that the conditional variance of the market return can be expressed as,

$$var_t[\eta_{m,t+1}] \equiv \sigma^2_{m,t} = [1 + \kappa_1 A_1 (\rho - \omega)]^2 \sigma^2_{g,t} + [\kappa_1 A_2]^2 \sigma^2_w \tag{16}$$

Also, the conditional covariance between the consumption innovation and the market return innovation is,

$$cov_t(\eta_{t+1}, \eta_{R,t+1}) = [1 + \kappa_1 A_1(\rho - \omega)]\sigma^2_{g,t}$$

If asset returns and the pricing kernel are conditionally log-normal, as is the case here, then the continuous risk premium is,

$$E_t[r_{i,t+1} - r_{f,t}] = -0.5\sigma^2_{r_i,t} - cov_t(ln(M_{t+1}), r_{i,t+1})$$

The arithmetic risk premium, $E[R_{i,t+1} - R_{f,t}]$ can be derived by adding the Jensen's effect piece, $0.5\sigma^2_{r_i,t}$, to both sides of the above expression. The risk premium on the market portfolio, the derivation of which is provided in appendix A, is,

$$E_t[R_{m,t+1} - R_{f,t}] = [\frac{\theta}{\psi}B + (1-\theta)B^2]\sigma^2_{g,t} + (1-\theta)[\kappa_1 A_2]^2\sigma^2_w \tag{17}$$

where $B = [1 + \kappa_1 A_1(\rho - \omega)]$. Note that B plays a critical role in the determination of the risk premium in equation (17). B, as shown later is bigger than one, captures the impact on long-term expected growth rates in response to innovations in dividend growth rates. The impact of this persistent component on the equity premium via B can be very large—implying a large equity premium. Further note that volatility risk is priced and effects the risk premium when $\theta \neq 1$.

To provide intuition regarding the various effects on the market risk premium, consider some special cases. First, consider the case of standard time-separable preferences where $\theta = 1$, or equivalently $\gamma = 1/\psi$. From (17) it is evident that the equity premium is

$$E_t[R_{m,t+1} - R_{f,t}] = \gamma B\sigma^2_{g,t} \tag{18}$$

Note that with expected utility volatility risk is not priced. With $\theta = 1$, the innovation in the market return does explicitly affect the innovation in the IMRS – see (6). This is an outcome of the fact that innovations to stochastic volatility do not affect the marginal utility of wealth with expected utility preferences. When $\rho = \omega$ and volatility is constant, that is, the dividend process is $i.i.d$ and consequently $B = 1$. In this case the risk premium of the market portfolio is product of the variance of growth rates and risk aversion. Further, $\gamma = (1/\psi) = 1$ (hence log utility) ensures that $B = 1$ even if $\rho \neq \omega$, and the resulting market return is exactly equal to the growth rate process.[5]

Next, consider the more general case where the risk aversion parameter need not equal

[5]Note that in the case of log-utility A_1 and A_2 are equal to zero and price dividend ratio is constant.

the reciprocal of the elasticity of substitution parameter (i.e., θ need not equal 1). In this case with $\rho = \omega$ (hence $B = 1$) the equity premium simplifies to $E_t[R_{m,t+1} - R_{f,t}] = \gamma\sigma^2_{g,t} + (1-\theta)[\kappa_1 A_2]^2\sigma^2_w$.[6] The effect of $\theta \neq 1$ is that volatility risk is priced. An interesting case is one where $\theta < 0$ and $B > 1$, in which case the magnification afforded by the term pre-multiplying $\sigma^2_{g,t}$ can be big enough to generate a large equity premium. This captures the intuition that small innovations in the growth rate lead to large changes in the market return, which in turn is positively correlated with the representative agent's consumption. To hold the market portfolio the agent needs to be compensated for bearing this risk by being offered a large equity premium.

Given the expression for the market volatility in (16), note that the geometric equity premium, the focus of our empirical analysis, is straightforward to derive,

$$E_t[r_{m,t+1} - r_{f,t}] = [\frac{\theta}{\psi}B + (1-\theta)B^2]\sigma^2_{g,t} + (1-\theta)[\kappa_1 A_2]^2\sigma^2_w - 0.5\sigma^2_{m,t} \tag{19}$$

3.1.2 The Risk Free Rate and Volatility

To derive the risk free rate we exploit the Euler condition in (4) and the fact that the pricing kernel is log-normally distributed. This allows us to derive the following expression for the risk free rate (details are given in Appendix A).

$$r_{f,t} = -\log(\delta) + \frac{1}{\psi}E_t[g_{t+1}] + \frac{(1-\theta)}{\theta}E_t[r_{m,t+1} - r_t] - \frac{1}{2\theta}Var_t[\frac{\theta}{\psi}g_{t+1} + (1-\theta)r_{m,t+1}] \tag{20}$$

As is standard in most models, a rise in expected growth rates increases the risk free rate here as well. The volatility of of the pricing kernel (the last term in the expression for $r_{f,t}$) can be fairly large if the return to the market volatility is large, which can significantly alter the implications for the level of the risk free rate. Further, if $\theta < 0$ a rise in the equity premium lowers the risk free rate.

The volatility of the risk free rate is the determined by the volatility of the expected growth rate process and the volatility of the conditional variance of dividend growth rate. In particular, we show that,

$$Var(r_{f,t}) = (\frac{1}{\psi})^2 Var(x_t) + \left\{\frac{1-\theta}{\theta}K_2 - K_1^2\frac{1}{2\theta}\right\}^2 Var(\sigma^2_{g,t}) \tag{21}$$

where the details of derivation and the constants K_1 and K_2 are given in Appendix A. Note that in the absence of stochastic volatility in the model, the volatility of the risk free rate is

[6]This follows from recognizing that the term pre-multiplying $\sigma^2_{g,t}$ in (17) collapses to γ. To see this recall that θ is equal to $\frac{1-\gamma}{1-\frac{1}{\psi}}$, and with $B = 1$ it follows that $[\frac{\theta}{\psi}B + (1-\theta)B^2]$ is equal to $(\frac{\theta}{\psi} + 1 - \theta) = \gamma$.

determined by the volatility of x and the elasticity of substitution, ψ — larger values of ψ lower the volatility of the risk free rate.

Finally, the volatility of the market return is,

$$Var(r_{m,t}) = B^2 Var(\eta_{t+1}) + (\frac{1}{\psi})^2 Var(x_t) + [A_2(\kappa_1\nu_1 - 1)]^2 Var(\sigma^2_{g,t}) + [A_2\kappa_1]^2\sigma^2_w \qquad (22)$$

where again, $B \equiv [1 + A_1\kappa_1(\rho - \omega)]$. The first order effect on the volatility of the market return is B^2. As discussed earlier, B captures the impact of dividend innovations on long-term expected growth rates. As long as $A_1 > 0$ an increase in ρ, with $\rho - \omega > 0$, increases B implying growth innovations have bigger impact on market volatility.

3.2 Separating Consumption and Dividends

In this section we model consumption and dividends as separate processes. We want to maintain the relatively parsimonious framework presented above, while capturing the idea that the consumption process is smoother than the dividends process. Consequently, we develop a one state variable framework in which consumption and dividends are not perfectly correlated

$$\begin{aligned} g_{c,t+1} &= \mu_c + x_t + \eta_{c,t+1} \\ g_{d,t+1} &= \mu_d + \lambda x_t + \eta_{d,t+1} \end{aligned} \qquad (23)$$

where x_t is already defined in (9). The parameter λ is equal to ratio of the unconditional standard deviation of the dividends and consumption growth rates; that is, $\lambda = (var(g_{d,t+1})/var(g_{c,t+1}))^{1/2}$. This implies that the R^2 for predicting g_c and g_d based on x is equal. Further, it follows that $var(\eta_d) = \lambda^2 var(\eta_c)$. Since, dividends are more volatile than consumption λ will be bigger than 1.

Let τ be the conditional correlation between dividend and consumption innovations. To allow for imperfect correlation between these innovations we further decompose the dividend innovation as follows,

$$\begin{aligned} \eta_{d,t+1} &= \tau(\frac{var(\eta_d)}{var(\eta_c)})^{1/2}\eta_{c,t+1} + \zeta_{t+1} \qquad (24) \\ var(\eta_d) &= \tau^2\lambda^2 var(\eta_c) + var(\zeta) \quad \text{which implies} \\ var(\zeta) &= (1 - \tau^2)\lambda^2 var(\eta_c) \end{aligned}$$

To solve the model, as in the previous discussion, we first need to compute the solution to the endogenous variable, $z_{c,t} = \log(P_{c,t}/C_t)$. This is the *consumption* counterpart to what we have referred to as $z_t = \log(P_{m,t}/D_t)$ for dividends. Consequently, the solution $z_{c,t}$ is identical to what is derived in the section 3.1. To solve for the market return, which now is the return on a claim on stream of *dividend*, we need to derive the solution for $z_{d,t} = \log(P_{m,t}/D_t)$. The solution for $z_{d,t}$ is,

$$\begin{aligned} z_{d,t} &= A_{0,d} + A_{1,d}x_t \quad \text{where} \\ A_{1,d} &= \frac{1-\frac{1}{\psi\lambda}}{1-\kappa_1\rho} = \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho} + \frac{\frac{\lambda-1}{\lambda\psi}}{1-\kappa_1\rho} = A_1 + \frac{\frac{\lambda-1}{\lambda\psi}}{1-\kappa_1\rho} \end{aligned} \tag{25}$$

The second term in $A_{1,d}$ captures the idea that dividends are levered relative to aggregate consumption. Note, that the market price to dividend ratio is likely to be more volatile than the market price for a claim on consumption, since $A_{1,d}$ is likely to be bigger than A_1. Following the same solution procedure as in the previous section the equity premium and the market return volatility respectively satisfy,

$$E_t[r_{m,t+1} - r_{f,t}] = B_d\left[\frac{\theta}{\psi} + (1-\theta)B\right]Var(\eta_c) - 0.5Var_t(r_{m,t+1}) \tag{26}$$

where $B_d \equiv [A_{1,d}\kappa_1(\rho-\omega) + \tau\lambda]$. The conditional innovation in the market return is $B_d\eta_{c,t+1} + \zeta_{t+1}$, and its conditional variance is $Var_t(r_{m,t+1}) = B_d^2 Var(\eta_c) + (1-\tau^2)\lambda^2 Var(\eta_c)$. The unconditional variance of the market return is

$$Var(r_{m,t}) = B_d^2 Var(\eta_c) + \left[\frac{1}{\lambda\psi} - 1 + \lambda\right]^2 Var(x_t) + (1-\tau^2)\lambda^2 Var(\eta_c), \tag{27}$$

where, again, all details are deferred to the appendix.

4 Empirical evidence

4.1 Data

We construct a monthly series of real dividend growth rates using the CRSP data set. Specifically, dividends are imputed from the Value Weighted return on the NYSE including and excluding dividends. Using the market capitalization rate and the CPI index we construct a real valued dividend index for monthly observations from January 1935 to December 1998. The de-seasonalized level of dividend is a trailing 12 month average — this procedure is similar to that in Bollerslev and Hodrick (1995) and Heaton (1995), Hodrick (1992). For each date the monthly dividend growth rate is defined as the continuous growth rate of this de-seasonalized level of dividends.

The top panel in Table 1 provides summary statistics of the first 2 moments and first 2 autocorrelations of the data used in the paper. The table provides information on dividend growth rate, the Value return on NYSE, real return on the one-month Treasury Bill, and inflation. Note that the measured real risk free rate is constructed by subtracting a trailing 12-month moving average of inflation from the nominal one month T-bill rate. All the return series including inflation are taken from the CRSP data set. The stylized facts discussed in the introduction are evident in Table 1. That is the continuous mean equity premium and real risk free rate are 7.5% and .17% respectively, per-annum. In our sample the equity premium is particularly large while the real risk free rate is relatively small. The annualized standard deviation of the market return and the real risk free rate are about 16% and 1% respectively.

4.2 Cash Flow Dynamics

In Panel A of Table 2 we provide the ARMA(1,1) estimates for the dividend growth rate process. The AR(1) coefficient, ρ, is .965 and the MA(1) coefficient, ω, is .856. These correspond quite closely to the type of estimates discussed earlier in motivating the model. We also report the estimates for the ARMA(1,1) model with GARCH(1,1) stochastic volatility in the innovations of the dividend growth rate process. Note that the volatility process is quite persistent with $\nu_1 = 0.975$. The innovations in stochastic volatility, determined by the difference $\nu_1 - \omega_v = 0.032$, is quite small. The economic implications of these estimates for the conditional mean of the growth rate process are: first, that it is quite persistent, and second, that the optimal forecast of the conditional mean is revised in the amount of $0.965 - 0.865 = 0.1$ in the direction of the innovation in the growth rate. Analogously, the volatility forecasts are revised approximately in the amount of $\nu_1 - \omega_v$.

An often held view is that the log-levels of consumption and dividends are a random walk with a drift. Another commonly held view, seen extensively in the RBC literature, is that aggregate time-series can be meaningfully decomposed into a trend and business-cyclical components by a filter such as the HP-filter. Not surprisingly, the asset pricing implications will be significantly different across these two alternative characterization of the data. It is not clear, however, that the time series dynamics of consumption and dividends, in themselves, can speak to which of these two views is the *correct* one.

For example, in Panel C of Table 2 we decompose, using the HP-filter, the monthly dividend growth rate series into cyclical and stochastic trend components. In utilizing the HP filter we used the 'standard' RBC "smoothing" parameter of $\lambda = 14400$ (see Hodrick and Prescott (1997) for details). The first two columns in Panel C provide the first two autocorrelations of the growth rate of the trend and cyclical components. The results indicate that the trend growth rate component is very persistent while the growth in the cyclical component is not very persistent. The last column, denoted Var-Ratio reports the relative variance of each component to the overall growth rate variance. The results in that panel suggest that the growth rate in the "Trend" component is small in size but very persistent, where as the growth rate of the cyclical component is quite volatile but not persistent. Viewed from the perspective of the HP filter, it seems that shocks to the trend component will significantly alter the implications for long run expected growth rates, and consequently have serious implications for the equity premium and market volatility.

The ARMA(1,1) specification used in this paper can also be derived from a specific trend-cycle model for the level of dividends. For example, let $\log D_t \equiv Y_t = T_t + S_t$ where T_t is the trend component and S_t is the cyclical component. If one assumes that the trend for Y follows a exponential smoothing process, where $T_t = \omega T_{t-1} + (1-\omega)Y_t$, and the cyclical part follows an AR(1) process, with AR(1) parameter of ρ. Then the growth rate $g_{t+1} = Y_{t+1} - Y_t$ follows an ARMA(1,1) process as discussed in section 2.1. Note that this is one of the many ways to characterize the trend and cyclical components so that the implied growth rate process is an ARMA(1,1). This description of the trend and cycle is analogous to the HP filter which, as opposed to this, is a two-sided filter.

Finally, in figure 1 we plot the predicted trend growth rate component of the HP filtered series against the expected growth rate process implied by the estimated ARMA(1,1) process. The two track each other very well although the ARMA(1,1) is clearly not as smooth series the HP-filtered trend growth rate,which as stated earlier is a two-sided filter. Overall, the HP filter implications for the expected growth rate process and the ARMA(1,1) process, despite some differences, provides the same broad message—the expected growth rate process is very persistent.

4.3 Estimation and Asset Pricing Implications

We now turn to the empirical asset pricing implications of the model. We first provide estimation results for the one-state (i.e. constant volatility) and two-state variable models. We then proceed to a few calibration experiments designed to highlight the economics of the model.

4.3.1 One State Variable Model

In Table 3 we present results for the ARMA(1,1) constant volatility model. In Panel A of this table we estimate the preference parameters δ, γ, and ψ while holding fixed the estimated law of motion for dividend growth rate (given in the top row of Table 2). The results imply a risk aversion of 3 and an elasticity of intertemporal substitution of 3.95. In terms of the asset pricing implications, the resulting level of the equity premium and risk free rate are essentially identical to those observed in the data. The standard deviation of the market return is about 15.63 %, which is also comparable to that observed in the data. The standard deviation of the ex-ante risk-free rate is 0.5% in the model. The latter is some what lower than the volatility of the measured real risk-free rate.

In Panel B of Table 3 we jointly estimate the preference and the dividend growth rate parameters. The asset markets data, in addition to the observed dividend growth rate process, must contain valuable information regarding the dividend growth rate process. As reported in Table 3, the estimated ρ and ω, at 0.975 and 0.88 are marginally higher than their point estimates in the univariate estimation (where $\rho = 0.965$ and $\omega = 0.86$). However, this parameter configuration for of the dividend growth rate process, has an important by-product, that ψ and γ can now be lower—they are now estimated to be 2.94 and 2.89 respectively. Higher values of ρ imply that growth rate innovations have a larger long-run impact on expected growth rates, the same can be achieved by increasing ψ (see expression for A_1, that is equation (13), consequently, larger values of ρ permit smaller values for ψ and γ. Note that this parameter configuration, as with the last one one, fully justifies the size of the equity premium and the level of the risk-free rate observed in the data, and generates a market return standard deviation of 15.65% per annum, which is exactly the magnitude for the return volatility observed in the data. Finally, this specification also produces, a risk-free rate standard deviation of 0.7% p.a. which some what lower than the magnitude of 1% p.a. observed in the data.

In Panel C of Table 3 we report the implications of the consumption-dividend model described in section 3.2. As we do not observe the monthly consumption time-series for the period 1935.01-1998.12, we re-scale the observed dividend growth rate process to match the volatility of the consumption growth rate process for the period 1959.01-1998.12, for which we do observe both the time-series on a monthly frequency. In other words, we construct a pseudo consumption growth rate process $g_c = \bar{g} + (1/\lambda)(g_d - \bar{g}_d)$, where λ is equal to the standard deviation of dividend growth rate process divided by the standard deviation of the consumption growth rate process. The implied series has the same mean and standard deviation as the consumption growth rate process for the period 1959.01-1998.12. The ρ and

ω are unaltered by this transformation. By doing this simple transformation, we ensure that the volatility of the consumption growth rate process that is utilized to solve the model is similar to that observed in the data. Also we assume that the conditional correlation between the dividend growth rate and the consumption growth rate process is 0.44 percent per annum (which is equal to $1/\lambda$). Panel C of Table 3 shows that this model, in which consumption and dividends growth rates are imperfectly correlated and have volatilities that are comparable to that observed in the data, is also capable of justifying the means and volatilities of asset returns of interest. Note that the point estimate for ρ is marginally higher than the previous two panels. The magnitude of risk aversion is comparable to the previous cases while the ψ is in fact somewhat lower at 2.64. In summary, this relatively minor adjustment to our benchmark model illustrates that many of the positive results we obtained under the pure dividend model can be attained in a model with explicit separation between consumption and dividends. In other words, the ability of this framework to generate plausible asset pricing moments is not confined to the specification of exclusively using dividends.

4.4 Two State Variable model

In Table 4 we estimate the two-state variable model – that is the ARMA(1,1)-GARCH(1,1) model for dividend growth. In Panel A of this table we estimate only the preference parameters and use the growth rate process parameter estimates reported in Panel B of Table 2. This specification of the model allows us to address additional asset market issues such as time-varying risk premia and return predictability. The model in Panel A results in estimated risk aversion and elasticity of substitution parameters that are slightly lower than those in Table 3. This model can also generate many of the observed asset market phenomena of interest. In Panel B of Table 4 we jointly estimate the preference and ARMA-GARCH system. The magnitudes of ρ and γ estimated in this specification is somewhat lower than the comparable one-state variable specification where volatility is constant.

In Panel C, we report the estimates from the two-state variable pseudo-consumption based model. Specifically, we generate a consumption series using a scaled version of the dividend series such that the resulting consumption series will have the same volatility at that observed for consumption growth in the data. To retain simplicity, at least for this version of the paper, we treat this pseudo consumption series as the the dividend series also. Note this simplification relative to our consumption-dividend model of section 3.2 makes it *a-priori* more difficult for this specification to match asset market data. The estimated ρ at 0.984 is somewhat higher than in the dividend based model. Consequently, this model generates an equity premium that is somewhat higher that he dividend model and the volatility of the market return at 13.2% is marginally lower than what is observed in the data. The level of the risk free rate and its volatility are comparable to magnitudes observed in data.

An additional important dimension in which the model performs quite well is justifying the predictability of future returns on current price-dividend ratios. In Panel A of Table 5 we report these predictability regressions for horizons 1,12, and 24 in the data and the model. Note that the model captures the negative relation between expected returns and price-dividend ratios. It is also evident from Panel B in Table 5 that the model's price-dividend ratio displays the same persistence as observed in the data.

There is a large literature which documents that market return volatility displays a GARCH(1,1) pattern with fairly persistent volatility shocks (see Bollerslev (1986)). Note that this feature of the data is easily be reproduced in our model. Equation (16) implies that $\sigma_{m,t}^2$, the market volatility process is

$$\sigma_{m,t}^2 = B^2 \frac{w_t}{1-\nu_1 L} + \kappa_1 A_2^2 \sigma_w^2,$$

that is the market volatility is also a GARCH(1,1) process the persistence of which is ν_1. Further note that innovations to market volatility relative to dividend growth rate volatility are magnified by the amount B^2, which is bigger than 1. Finally, the implications of our model for the time-series of the conditional equity premium are shown Figure 2. As growth rate uncertainty is high during recessions it follows, from the perspective of our model, that the equity premium also rises during these time periods.

4.4.1 Calibration

To generate intuition for the way various parameters effect the asset prices in our model we provide some additional sensitivity analysis which are reported in Tables 6-9.

Table 6 provide calibration results for the case in which preferences exhibit non-expected utility and cash flow dynamics follow the ARMA(1,1) specification with no stochastic volatility. All parameters are set to the estimated parameters of Table 3 except γ and ψ which take various values. We let γ range between 1 and 4. It is clear that for a given risk aversion, increasing the elasticity of substitution results in a rise in the equity premium, a rise in the market volatility, and lowers the risk-free rate and its volatility. In other words, given a level of risk aversion, higher levels of ψ help the model in justifying the observed magnitudes of interest. Further note that in the absence of stochastic volatility, for a given ψ, the volatility of asset returns is unaffected as one changes the level of risk aversion. Table 7 replicates the analysis of Table 6 except that $\rho = .985$ rather than then .965. It can be easily seen that a higher ρ for a given γ and ψ increases the equity premium, lowers the risk free rate and increases the market return volatility. This observation in conjunction the first observation suggests that a larger ρ will obviate the need to choose a larger ψ to match the asset returns moments of interest.

Table 8 provides results for the case in which cash-flow dynamics are i.i.d (that is $\rho = \omega$) and preferences still exhibit the non-expected utility. In this case the price-dividend ratio is constant. Consequently, the market return volatility mirrors the volatility of dividend growth rates. Further, the equity premium is small and unaltered by changing ψ, and the level of the risk free rate is too high.

Table 9 provides results for the standard time-non-separable preferences. Panel A which assumes cash-flow dynamics are $i.i.d$ reproduces the well documented inability of this model to explain asset market data. In Panel B dividend growth rates follow our constant volatility ARMA(1,1) specification. In interpreting these results it is useful to recall the expression for A_1 in (13). For values of γ (which in this case equals the reciprocal of ψ) bigger than 1, A_1 is negative — a rise in expected growth rates lowers the price-dividend ratio. A result of this feature is that when there is a positive shock to consumption (dividends) there is a negative shock to market returns. Hence, the covariance between consumption and the market return is negative. In this case the market insures the agent's consumption and therefore the required equity premium is negative. This feature underscores the importance of separation between risk aversion and elasticity of substitution in the context of our model. Overall, the key message of these tables is that neither non-expected utility nor the AMRA(1,1) specification for growth rates are sufficient by themselves to produce results that accord well with the stylized facts of the data. In our model the two channels must co-exist if the model is to match the equity premium, the level of the risk free rate, and the volatilities of the risk free rate and the market return.

5 Conclusions

In this paper we explore the idea that news about consumption and dividend growth rates continuously alters perceptions regarding long-term expected growth rates, and that in equilibrium this feature is important for explaining various asset market anomalies. If news about dividends has non trivial impact on long-term expected growth rates, then the capitalized value of this cash-flow would be fairly sensitive to small news. Further, if dividends are positively correlated with consumption than the dividend cash-flow may warrant a large equity premium.

We document that the interaction between dividend growth rate dynamics, which incorporate this idea of long term risks, in conjunction with the preferences as developed in Epstein and Zin (1989)-Weil (1989) can indeed explain many outstanding asset market puzzles. In particular, we show that such a model is capable of justifying the observed magnitudes of the equity premium, the low risk free rate and the volatility of market return and risk free rate. Further, we provide empirical support for the view that the observed aggregate dividend growth process contains a persistent component that may impose such long term risks.

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Table 1 : Data Description

	π	r_f	r_m	Div. Growth - g
		Panel A: Monthly Frequency		
μ	3.896	0.166	7.649	3.826
	(0.210)	(0.132)	(1.968)	(0.668)
σ	1.688	1.056	15.761	5.351
	(0.144)	(0.049)	(0.730)	(0.483)
ρ_1	0.415	0.980	0.038	0.151
	(0.072)	(0.012)	(0.048)	(0.077)
ρ_2	0.343	0.952	0.044	0.133
	(0.063)	(0.019)	(0.042)	(0.029)

r_f is the real risk free rate derived by subtracting a trailing 12-month average of inflation (π) from the one month T-Bill. r_m is the continuous real return on the Value Weighted NYSE taken from CRSP. We construct a monthly dividend series using the Value Weighted return on NYSE. The de-seasonalized level of dividends are constructed using a trailing 12 month moving average of real dividends. The statistics reported above are for continuous (i.e. log) growth rates constructed from this time series for the level of dividends. The sample period is 1935:01-1998:12. All return data is from CRSP and inflation is CPI based on BLS. Standard errors are Newey and West (1987) corrected using 12 lags.

Table 2 : Estimating Dividend Growth Rate Dynamics

	μ	ρ	ω	σ_η	ν_0	ν_1	ω_v
Panel A: Monthly Frequency							
Estimates	.000089	.965	.860	.0148			
S.E.	(.0001)	(.0068)	(.0375)				
Estimates	.000092	.965	.856	.0148	.00000485	.978	.946
S.E.	(.00011)	(.007)	(.038)		(.000008)	(.038)	(.072)
Panel B: HP-filtered dynamics- monthly data							
	ρ_1	ρ_2	Var-Ratio				
Growth in HP-trend	.998	.993	7.3%				
Growth in HP-cyclical	.024	.004	87%				

The model for dividend growth rate is:

$$g_{t+1} = \mu + \rho g_t + \eta_{t+1} - \omega \eta_t$$

The GARCH (1,1) model for η implies the stochastic volatility model (12) :

$$\sigma^2_{g,t+1} = \nu_0 + \nu_1 \sigma^2_{g,t} + (\nu_1 - \omega_v)(\eta^2_{t+1} - \sigma^2_{g,t}).$$

The moments used in estimation of the ARMA(1,1) model are: $E[\eta_t g_{t-1}, \eta_t \eta_{t-1}, \eta_t] = \mathbf{0}$. The ARMA(1,1) parameters underlying the GARCH(1,1) specification are estimated in analogous manner. The parameters ρ_1, ρ_2 are the first and second autocorrelations and Var-Ratio is the variance of the HP-filtered component growth rate relative to the observed growth rate. Standard Error are Newey and West (1987) corrected using 12 lags.

Table 3 : Estimation Results: One State Variable Model

ρ	ω	δ	γ	ψ	$E[r_m - r_f]$	$E[r_f]$	$\sigma(r_m)$	$\sigma(r_f)$
Panel A: One State Variable-only preference parameters								
.965	.86	.997	3.08	3.95	7.48	.17	15.63	0.52
		(.001)	(.74)	(1.05)				
Panel B: One State Variable-dividend growth rate and preference parameters								
.975	.88	.997	2.89	2.94	7.48	.16	15.65	.70
(.017)	(.055)	(.001)	(.929)	(1.95)				
Panel C: One State Variable-consumption growth rate and preference parameters								
.987	.86	.998	3.12	2.64	7.48	.16	15.76	0.60
(.0068)	(.036)	(.003)	(.84)	(3.20)				

The *one* state variable dynamics for dividend growth are given by the ARMA(1,1) equation (7). In Panel A we estimate the preference parameters while the parameters governing dividend growth, $\rho, \omega, \sigma_\eta^2$ are given by the ARMA(1,1) estimates in the first row in Table 2. The moment conditions used in the estimation of Panel A are based on minimizing the difference between those observed in the data and the corresponding population moments implied by the model for $[E[r_m - r_f]$, $E[r_f]$, $\sigma^2(r_m)$, $\sigma^2(r_f)]$. In Panel B, the dividend growth process is estimated jointly with the preference parameters, using the additional moments for estimating the dividend growth rate parameters as discussed in Table 2. All estimations use a two-step GMM procedure with a Newey and West (1987) covariance matrix with 12 lags.

Table 4 : Estimation Results: Two state Variable

ρ	ω	ν_0	ν_1	ω_v	δ	γ	ψ	$E[r_m - r_f]$	$E[r_f]$	$\sigma(r_m)$	$\sigma(r_f)$
Panel A: Two State Variable-only preference parameters											
.965	.86	4.85e-6	.978	.946	.996	2.88	3.31	7.83	.58	15.57	1.36
					(.001)	(.603)	(.871)				
Panel B: Two State Variable-dividend growth rate dynamics and preference parameters											
.974	.88	6.32e-6	.973	.926	.997	2.53	3.44	7.42	.28	15.91	1.63
(.019)	(.056)	(1.1e-6)	(.055)	(.092)	(.001)	(1.21)	(2.91)				
Panel C: Two State Variable-preference parameters with consumption as state variable											
.984	.90	9.84e-7	.980	.939	.996	3.81	4.79	8.77	.25	13.19	1.31
.(.009)	(.049)	(9.1e-6)	(.224)	(.287)	(.001)	(1.11)	(3.11)				

The *two* state variable dynamics for dividend growth include the GARCH(1,1) specification given in (7). In panel A, only the preference parameters are estimated while the GARCH(1,1) parameters are given by the estimates in Panel B in (2). The moment conditions used in the estimation of Panel A are based on minimizing the difference between those observed in the data and the corresponding population moments implied by the model for $[E[r_m - r_f],\ E[r_f],\ \sigma^2(r_m),\ \sigma^2(r_f)]$. In Panel B, the ARMA(1,1)-GARCH(1,1) dividend growth process is estimated jointly with the preference parameters, using the additional moments for estimating the dividend growth rate parameters as discussed in Table 2. In Panel C we re-estimate the model re-interpreting consumption as following the estimated ARMA(1,1)-GARCH(1,1) process for dividend *except* that we adjust the unconditional variance of the process to match that of observed aggregate consumption (non-durables and services) data. All estimations use a two-step GMM procedure with a Newey and West (1987) covariance matrix with 12 lags.

Table 5 : Return Predictability (Price-Dividend) Regressions

Panel A: Predictability Regressions

Horizon (Months)	β_{model}	β_{data}
1	-0.014	- 0.006
12	-0.079	-0.014
24	-0.027	-0.005

Panel B: Autocorrelation of log Price-Dividend Ratio

Lag	Model	Data
1	.962	.979
2	.924	.956

The entries in this table are based on the following regression:

$$r_{m,t} = \alpha + \beta \log(P_{t-horizon}/D_{t-horizon}) + \epsilon_t$$

applied to data generated by the GARCH(1,1) model estimated in Panel B in Table 4.

Table 6 : Calibration - Non Expected Utility, Dividend Growth $\rho \neq \omega$ and $\rho = .965$

γ	ψ	$E[r_m - r_f]$	$E[r_f]$	$\sigma(r_m)$	$\sigma(r_f)$
1.50	0.50	-0.46	11.40	10.25	4.14
1.50	1.00	0.70	7.20	5.95	1.97
1.50	1.50	1.41	5.66	9.80	1.38
1.50	2.00	1.89	4.69	12.14	1.04
1.50	2.50	2.22	4.08	13.55	0.83
1.50	3.00	2.44	3.66	14.49	0.69
1.50	5.00	2.92	2.79	16.39	0.41
1.50	10.00	3.30	2.12	17.82	0.21
2.00	0.50	-1.37	11.37	10.25	4.14
2.00	1.00	1.24	6.70	5.95	1.97
2.00	1.50	2.34	5.04	9.80	1.38
2.00	2.00	3.06	3.99	12.14	1.04
2.00	2.50	3.52	3.33	13.55	0.83
2.00	3.00	3.83	2.88	14.49	0.69
2.00	5.00	4.50	1.96	16.39	0.41
2.00	10.00	5.02	1.24	17.82	0.21
3.00	0.50	-3.17	11.33	10.25	4.14
3.00	1.00	2.32	5.71	5.95	1.97
3.00	1.50	4.21	3.79	9.80	1.38
3.00	2.00	5.39	2.59	12.14	1.04
3.00	2.50	6.12	1.84	13.55	0.83
3.00	3.00	6.62	1.33	14.49	0.69
3.00	5.00	7.65	0.29	16.39	0.41
3.00	10.00	8.45	-0.52	17.82	0.21
4.00	0.50	-4.98	11.28	10.25	4.14
4.00	1.00	3.40	4.72	5.95	1.97
4.00	1.50	6.08	2.54	9.80	1.38
4.00	2.00	7.71	1.19	12.14	1.04
4.00	2.50	8.72	0.35	13.55	0.83
4.00	3.00	9.41	-0.22	14.49	0.69
4.00	5.00	10.80	-1.39	16.39	0.41
4.00	10.00	11.88	-2.28	17.82	0.21

This table provides sensitivity analysis for the various effects γ and ψ have on asset prices. The calibration results are based on the one-state variable model where all parameters other than γ and ψ are set at the point estimates of row 1 in Table 2.

Table 7 : Calibration - Non Expected Utility, Dividend Growth $\rho \neq \omega$ and $\rho = .985$

γ	ψ	$E[r_m - r_f]$	$E[r_f]$	$\sigma(r_m)$	$\sigma(r_f)$
1.50	0.50	-1.12	9.95	27.01	6.41
1.50	1.00	1.16	6.83	6.50	3.05
1.50	1.50	3.56	4.21	14.62	2.14
1.50	2.00	5.31	2.32	19.61	1.60
1.50	2.50	6.49	1.07	22.64	1.28
1.50	3.00	7.32	0.18	24.66	1.07
1.50	5.00	9.12	-1.72	28.71	0.64
1.50	10.00	10.57	-3.26	31.76	0.32
2.00	0.50	-5.69	11.49	27.01	6.41
2.00	1.00	2.16	5.98	6.50	3.05
2.00	1.50	6.08	2.70	14.62	2.14
2.00	2.00	8.72	0.43	19.61	1.60
2.00	2.50	10.42	-1.05	22.64	1.28
2.00	3.00	11.61	-2.09	24.66	1.07
2.00	5.00	14.11	-4.29	28.71	0.64
2.00	10.00	16.10	-6.06	31.76	0.32
3.00	0.50	-14.83	14.57	27.01	6.41
3.00	1.00	4.16	4.27	6.50	3.05
3.00	1.50	11.11	-0.32	14.62	2.14
3.00	2.00	15.52	-3.34	19.61	1.60
3.00	2.50	18.29	-5.28	22.64	1.28
3.00	3.00	20.19	-6.63	24.66	1.07
3.00	5.00	24.11	-9.44	28.71	0.64
3.00	10.00	27.16	-11.66	31.76	0.32
4.00	0.50	-23.96	17.64	27.01	6.41
4.00	1.00	6.16	2.56	6.50	3.05
4.00	1.50	16.15	-3.33	14.62	2.14
4.00	2.00	22.33	-7.12	19.61	1.60
4.00	2.50	26.16	-9.51	22.64	1.28
4.00	3.00	28.77	-11.16	24.66	1.07
4.00	5.00	34.10	-14.59	28.71	0.64
4.00	10.00	38.21	-17.26	31.76	0.32

This table provides sensitivity analysis for the effects ρ has on asset prices. This table is a replica Table 8 with $\rho = .985$ rather than the estimated parameter of .965. The calibration results are based on the one-state variable model where all the rest of the parameters are set at the point estimates of row 1 in Table (2).

Table 8 : Calibration-Non Expected Utility, Dividend Growth is i.i.d, $\rho = \omega$

γ	ψ	$E[r_m - r_f]$	$E[r_f]$	$\sigma(r_m)$	$\sigma(r_f)$
1.50	0.50	0.29	11.50	5.35	0.00
1.50	1.05	0.29	7.57	5.35	0.00
1.50	1.50	0.29	6.50	5.35	0.00
1.50	2.00	0.29	5.87	5.35	0.00
1.50	2.50	0.29	5.49	5.35	0.00
1.50	3.00	0.29	5.24	5.35	0.00
1.50	5.00	0.29	4.74	5.35	0.00
1.50	10.00	0.29	4.37	5.35	0.00
2.00	0.50	0.43	11.29	5.35	0.00
2.00	1.05	0.43	7.43	5.35	0.00
2.00	1.50	0.43	6.38	5.35	0.00
2.00	2.00	0.43	5.76	5.35	0.00
2.00	2.50	0.43	5.39	5.35	0.00
2.00	3.00	0.43	5.15	5.35	0.00
2.00	5.00	0.43	4.66	5.35	0.00
2.00	10.00	0.43	4.29	5.35	0.00
3.00	0.50	0.72	10.86	5.35	0.00
3.00	1.05	0.72	7.15	5.35	0.00
3.00	1.50	0.72	6.14	5.35	0.00
3.00	2.00	0.72	5.55	5.35	0.00
3.00	2.50	0.72	5.19	5.35	0.00
3.00	3.00	0.72	4.96	5.35	0.00
3.00	5.00	0.72	4.49	5.35	0.00
3.00	10.00	0.72	4.13	5.35	0.00
4.00	0.50	1.00	10.43	5.35	0.00
4.00	1.05	1.00	6.87	5.35	0.00
4.00	1.50	1.00	5.90	5.35	0.00
4.00	2.00	1.00	5.33	5.35	0.00
4.00	2.50	1.00	4.99	5.35	0.00
4.00	3.00	1.00	4.77	5.35	0.00
4.00	5.00	1.00	4.31	5.35	0.00
4.00	10.00	1.00	3.97	5.35	0.00

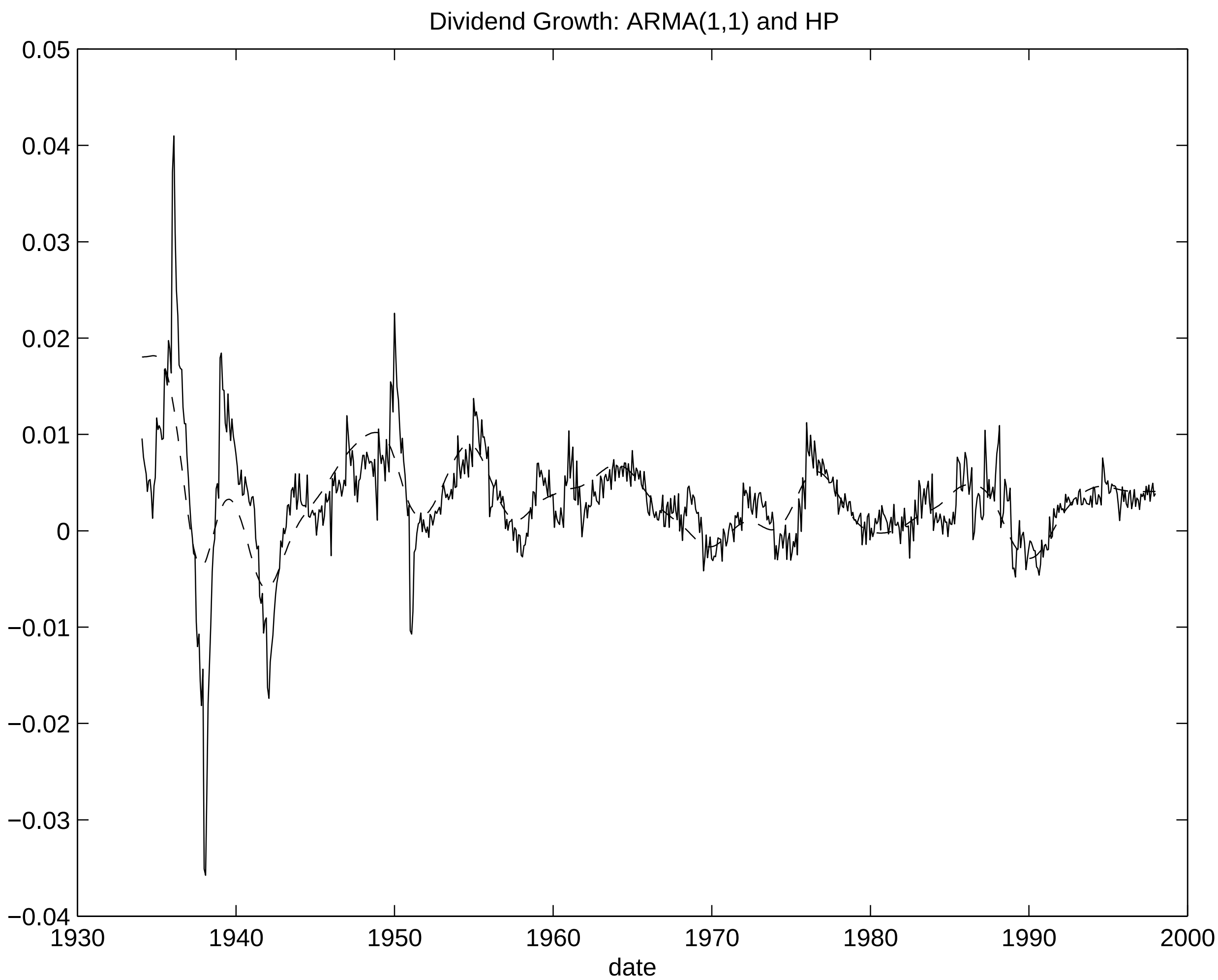
This tables provide sensitivity analysis for the effect *i.i.d* dividend growth rate has on asset prices when preferences are Epstein and Zin (1989). The calibration is based on the one state model. All parameters are from row 1 in Table 2 except ω that set equal to ρ.

Table 9 : Calibration - Expected Utility

γ	$\psi = \frac{1}{\gamma}$	$E[r_m - r_f]$	$E[r_f]$	$\sigma(r_m)$	$\sigma(r_f)$
Panel A: Dividend Growth is i.i.d, $\rho = \omega$					
0.50	$\frac{1}{0.50}$	0.00	6.08	5.35	0.00
2.00	$\frac{1}{2.00}$	0.43	11.29	5.35	0.00
5.00	$\frac{1}{5}$	1.29	19.76	5.35	0.00
20.00	$\frac{1}{20}$	5.58	23.47	5.35	0.00
40.00	$\frac{1}{40}$	11.31	-71.79	5.35	0.00
Panel B: Dividend Growth, $\rho \neq \omega$					
0.50	$\frac{1}{0.50}$	-0.42	6.09	12.05	1.03
2.00	$\frac{1}{2.00}$	-1.33	11.37	10.07	4.11
5.00	$\frac{1}{5}$	-26.05	20.29	52.60	10.27
20.00	$\frac{1}{20}$	-607.81	31.90	266.75	41.07
40.00	$\frac{1}{40}$	-2571.29	-38.05	552.36	82.22

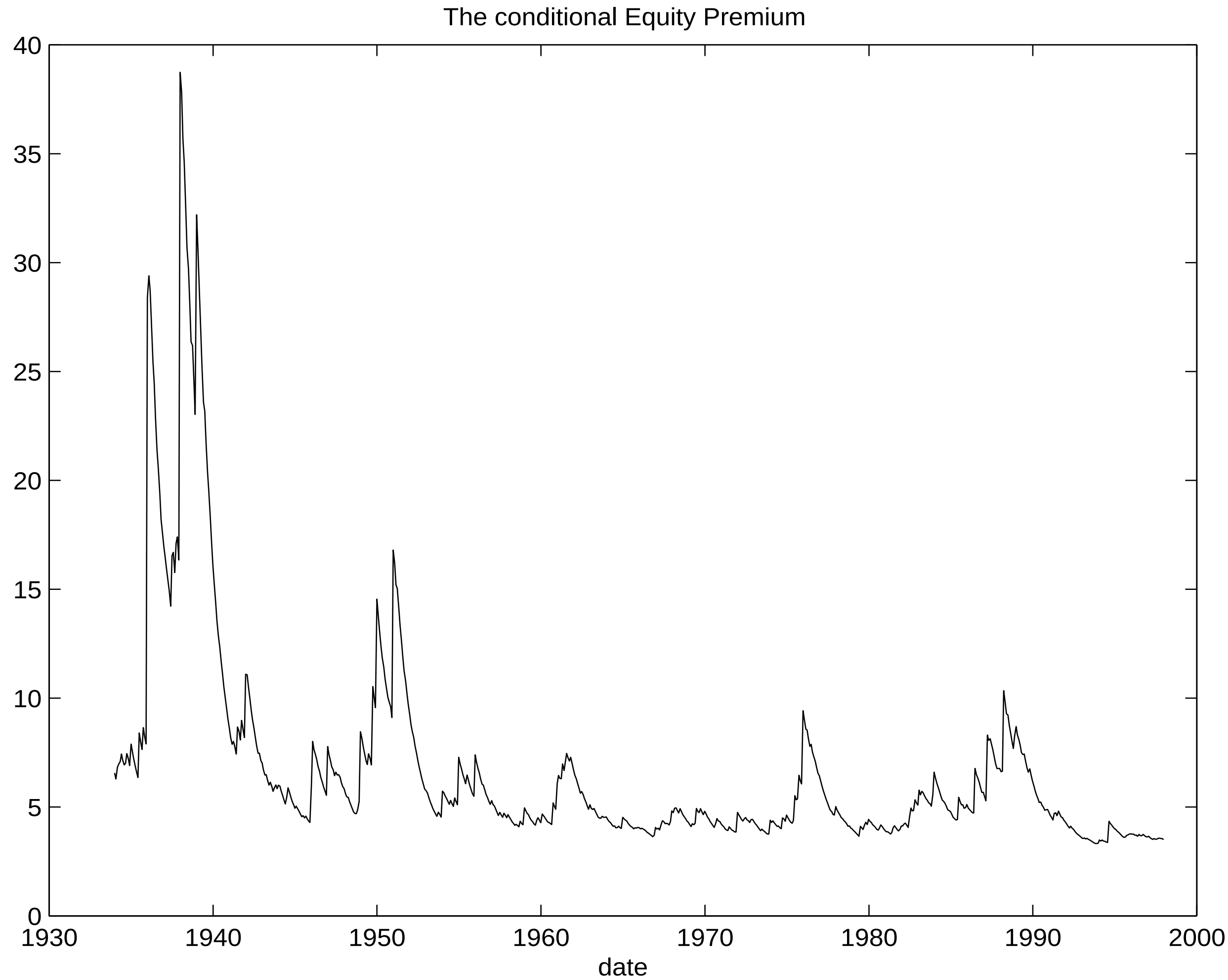
This table provides sensitivity analysis of the effect of risk aversion on asset prices under expected utility preferences. In Panel A dividend growth rate is i.i.d so $\omega = \rho$, and the rest of the parameters are from row 1 in Table 2. In Panel B dividend growth rate follows the estimated process in row 1 of Table 2.

Figure 1
HP filtered trend and fitted ARMA(1,1)



The figure depicts the fitted process for dividend growth rates from the trend component of the HP filter and the ARMA(1,1) estimation (Table 2).

Figure 2
The Conditional Equity Premium



The figure depicts the conditional equity premium based on the estimated GARCH(1,1) model in Panel B of Table 4. The conditional equity premium is in terms of percent per-annum.

6 Appendix-A

6.1 Solving for vector of coefficients A

To derive the solution for the endogenous variable z_t we substitute the approximation $r_{m,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}$ into (4), and conjecture that $z_t = A_0 + A_1 x_t + A_2 \sigma^2_{g,t}$.

Given the conditional normality of g, x and $\sigma^2_{g,t}$, the conditional mean in(4) must equal $\exp(c_{y,t} + var_{y,t}/2) = 1$, where $c_{y,t}$ is the conditional mean of $y_{t+1} \equiv \theta ln(\delta) - \frac{\theta}{\psi} g_{t+1} + \theta r_{m,t+1}$, and $var_{y,t}$ is its conditional variance. Using the conjectured solution, the approximation for the return, the process for the state variables, and (4), it follows that the conditional mean $c_{y,t}$ is,

$$\theta ln\delta - [\frac{\theta}{\psi}(\mu + x_t)] + \theta\{[\kappa_0 + \kappa_1(A_0 + A_1((\rho - \omega)\mu + \rho x_t) + A_2(\nu_0 + \nu_1 \sigma^2_{g,t}))] - A_0 - A_1 x_t - A_2 \sigma^2_{g,t} + x_t + \mu\}$$

and the conditional variance $var_{y,t}$

$$var_t[-\frac{\theta}{\psi}\eta_{t+1} + \theta(\eta_{z,t+1} + \eta_{t+1})] = var_t[-\frac{\theta}{\psi}\eta_{t+1} + \theta A_1 \kappa_1 (\rho - \omega)\eta_{t+1} + \theta A_2 w_{t+1} + \theta \eta_{t+1}]$$

hence,

$$var_{y,t} = [\theta - \frac{\theta}{\psi} + \theta A_1 \kappa_1 (\rho - \omega)]^2 \sigma^2_{g,t} + \theta^2 A_2^2 \sigma^2_w$$

Since $c_{y,t} + var_{y,t}/2$ must equal zero for all realizations of the state variables, the coefficients A can be solved for by matching the coefficients on the state variables.

The solution coefficient for the expected growth rate x_t, that is A_1, involves the solution to the following

$$-\frac{\theta}{\psi} x_t + \theta[\kappa_1 A_1 \rho x_t - A_1 x_t + x_t] = 0.$$

It immediately follows that,

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho}$$

which is (13) in the main text.

The solution coefficient for for $\sigma^2_{g,t}$, that is A_2, satisfies

$$\theta[\kappa_1 \nu_1 A_2 \sigma^2_{g,t} - A_2 \sigma^2_{g,t}] + \frac{1}{2}[\theta - \frac{\theta}{\psi} + \theta A_1 \kappa_1 (\rho - \omega)]^2 \sigma^2_{g,t} = 0$$

which implies that

$$A_2 = \frac{0.5[\theta - \frac{\theta}{\psi} + \theta A_1 \kappa_1 (\rho - \omega)]^2}{\theta(1 - \kappa_1 \nu_1)}$$

the solution given in (14).

6.2 Market Return

Given the solutions for **A**, the expression for the market return is given by

$$r_{m,t+1} = \kappa_0 + \kappa_1 (A_0 + A_1 x_{t+1} + A_2 \sigma^2_{g,t+1}) - (A_0 + A_1 x_t + A_2 \sigma^2_{g,t}) + g_{t+1}$$

and the innovation in the market return is

$$\begin{aligned}\eta_{m,t+1} &= \kappa_1 A_1(\rho-\omega)\eta_{t+1} + \kappa_1 A_2 w_{t+1} + \eta_{t+1} \\ &= [1+\kappa_1 A_1(\rho-\omega)]\eta_{t+1} + \kappa_1 A_2 w_{t+1}\end{aligned}$$

First, note that

$$var_t[\eta_{m,t+1}] \equiv \sigma^2_{m,t} = [1+\kappa_1 A_1(\rho-\omega)]^2\sigma^2_{g,t} + [\kappa_1 A_2]^2\sigma^2_w \tag{28}$$

Further, the conditional covariance between the consumption innovation and the market return innovation is,

$$cov(\eta_{t+1}, \eta_{R,t+1}) = [1+\kappa_1 A_1(\rho-\omega)]\sigma^2_{g,t}$$

Since the pricing kernel and asset returns are conditionally log-normal, it follows that

$$E_t[r_{i,t+1} - r_{f,t}] = -0.5\sigma^2_{r_i,t} - cov_t(ln(M_{t+1}), r_{i,t+1})$$

After shifting to arithmetic returns as discussed in the main text, it follows that the equity premium is equal to

$$E_t[R_{m,t+1} - R_{f,t}] = -cov_t(-\frac{\theta}{\psi}\eta_{t+1} - (1-\theta)\eta_{m,t+1}, \eta_{m,t+1})$$

where $\eta_{m,t+1}$ is the innovation in the market return. Using (28), and the covariance between the consumption and market return innovation, shown above, the risk-premium on the market must equal

$$E_t[R_{m,t+1} - R_{f,t}] = \frac{\theta}{\psi}[1+\kappa_1 A_1(\rho-\omega)]\sigma^2_{g,t} + (1-\theta)\sigma^2_{m,t} \tag{29}$$

Finally, after substituting (28) into (29), it follows that the equity premium is,

$$E_t[R_{m,t+1} - R_{f,t}] = [\frac{\theta}{\psi}B + (1-\theta)B^2]\sigma^2_{g,t} + (1-\theta)[\kappa_1 A_2]^2\sigma^2_w \tag{30}$$

6.3 The Risk Free Rate and Volatility

To derive the risk free rate start with (4) and plug the risk-free rate for r_i.

$$r_{f,t} = -\theta\log(\delta) + \frac{\theta}{\psi}E_t[g_{t+1}] + (1-\theta)E_t r_{m,t+1} - \frac{1}{2}Var_t[\frac{\theta}{\psi}g_{t+1} + (1-\theta)r_{m,t+1}] \tag{31}$$

subtract $(1-\theta)r_{f,t}$ from both sides and divide by θ, where it is assumed that $\theta \neq 0$. It then follows that,

$$r_{f,t} = -\log(\delta) + \frac{1}{\psi}E_t[g_{t+1}] + \frac{(1-\theta)}{\theta}E_t[r_{m,t+1} - r_t] - \frac{1}{2\theta}Var_t[\frac{\theta}{\psi}g_{t+1} + (1-\theta)r_{m,t+1}]$$

To solve the above expression we need an expression for $Var_t[\frac{\theta}{\psi}g_{t+1} + (1-\theta)r_{m,t+1}]$. Recall that $\eta_{t+1} = g_{t+1} - E_t[g_{t+1}]$, is the innovation in the growth rate process with conditional variance of $\sigma^2_{g,t}$. The solution for $Var_t[\frac{\theta}{\psi}g_{t+1} + (1-\theta)r_{m,t+1}]$ is,

$$\begin{aligned} &= Var_t[\frac{\theta}{\psi}\eta_{t+1} + (1-\theta)(\rho-\omega)\kappa_1 A_1\eta_{t+1} + (1-\theta)\eta_{t+1} + (1-\theta)\kappa_2 A_2 w_{t+1}] \\ &= Var_t[\frac{\theta}{\psi}\eta_{t+1} + (1-\theta)\left\{(\rho-\omega)\kappa_1 A_1\eta_{t+1} + \eta_{t+1} + \kappa_2 A_2 w_{t+1}\right\}] \\ &= (\frac{\theta}{\psi} + (1-\theta)[1+(\rho-\omega)\kappa_1 A_1])^2\sigma^2_{g,t} + (1-\theta)^2(\kappa_1 A_2)^2\sigma^2_w \end{aligned} \tag{32}$$

Further, if the innovation in growth rate process is homoskedastic, the above expression simplifies as $\sigma_w^2 = 0$. The unconditional mean of $r_{f,t}$ is derived by substituting the expression for the market risk-premium, and (32) into (31). This substitution yields,

$$E(r_t) = -\log(\delta) + \frac{1}{\psi}E(g) + \frac{(1-\theta)}{\theta}E[r_{m,t+1} - r_t] - \frac{1}{2\theta}[K_1^2 E(\sigma_{g,t}^2) + (1-\theta)^2(\kappa_1 A_2)^2\sigma_w^2]$$

where $K_1 \equiv \left(\frac{\theta}{\psi} + (1-\theta)[1 + (\rho - \omega)\kappa_1 A_1]\right)$.

The unconditional variance of $r_{f,t}$ is,

$$Var(r_t) = (\frac{1}{\psi})^2 Var(x_t) + \left\{\frac{1-\theta}{\theta}K_2 - K_1^2\frac{1}{2\theta}\right\}^2 Var(\sigma_{g,t}^2)$$

where $K_2 \equiv [\frac{\theta}{\psi}B + (1-\theta)B^2]$.

To find the unconditional variance of r_m substitute the solution for z and g into the definition of r_m to derive,

$$\begin{aligned} r_{m,t+1} &= const + A_1(\kappa_1 x_{t+1} - x_t) + A_2(\kappa_1 \sigma_{g,t+1}^2 - \sigma_{g,t}^2) + x_t + \eta_{t+1} \\ &= [1 + A_1\kappa_1(\rho - \omega)]\eta_{t+1} + \frac{1}{\psi}x_t + A_2[\kappa_1\nu_1 - 1]\sigma_{g,t}^2 + A_2\kappa_1 w_{t+1} \end{aligned}$$

The unconditional variance of the market return then follows,

$$Var(r_{m,t}) = B^2 Var(\eta_{t+1}) + (\frac{1}{\psi})^2 Var(x_t) + [A_2(\kappa_1\nu_1 - 1)]^2 Var(\sigma_{g,t}^2) + [A_2\kappa_1]^2\sigma_w^2 \tag{33}$$

6.4 Consumption-Dividend Model

The market return process can be derived in a manner analogous to our discussion in the previous section. Specifically,

$$\begin{aligned} r_{m,t+1} &= \kappa_0 + \kappa_1(A_{0,d} + A_{1,d}x_{t+1}) - (A_{0,d} + A_{1,d}x_t) + g_{d,t+1} \\ & \quad \text{substituting the } g \text{ and } x \text{ process it follows} \\ r_{m,t+1} - Er_{m,t+1} &= \kappa_1\rho A_{1,d}x_t - A_{1,d}x_t + (\rho - \omega)\kappa A_{1,d}\eta_{c,t+1} + \lambda x_t + \tau\lambda\eta_{c,t+1} + \zeta_{t+1} \\ &= \left[\frac{1}{\lambda\psi} - 1 + \lambda\right] x_t + B_d\eta_{c,t+1} + \zeta_{t+1} \end{aligned} \tag{34}$$

where $B_d \equiv [A_{1,d}\kappa_1(\rho - \omega) + \tau\lambda]$. Therefore the unconditional variance of the market return is

$$Var(r_{m,t}) = B_d^2 Var(\eta_c) + \left[\frac{1}{\lambda\psi} - 1 + \lambda\right]^2 Var(x_t) + (1 - \tau^2)\lambda^2 var(\eta_c) \tag{35}$$

as the conditional innovation in the market return is $B_d\eta_{c,t+1} + \zeta_{t+1}$, the conditional variance of $r_{m,t+1}$, $var_t(r_{m,t+1}) = B_d^2 Var(\eta_c) + (1 - \tau^2)\lambda^2 var(\eta_c)$. It follow that the equity premium on the market return is

$$E_t[r_{m,t+1} - r_{f,t}] = B_d\left[\frac{\theta}{\psi} + (1-\theta)B\right] var(\eta_c) - 0.5 var_t(r_{m,t+1}) \tag{36}$$